CHAPTER 5
CONCRETE DESIGN THEORY

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CHAPTER 5
CONCRETE DESIGN THEORY

5.1 INTRODUCTION

Concrete is the most commonly used material in California highway structures, especially after the wide acceptance of prestressing technology in the 1950s. Nowadays, concrete bridges, prestressed or non-prestressed, account for about 90% of all bridges in the California highway system. Such dominancy is attributable to the many advantages that concrete offers:

- Ability to be cast in almost any shape
- Low cost
- Durability
- Fire resistance
- Energy efficiency
- On-site fabrication
- Aesthetic properties

Concrete design has evolved from Allowable Stress Design (ASD), also Working Stress Design (WSD), to Ultimate Strength Design (USD) or Load Factor Design (LFD), to today's Limit State Design (LSD) or Load and Resistance Factor Design (LRFD). Concrete design takes on a whole new look and feel in the *AASHTO LRFD Bridge Design Specifications* (AASHTO 2007). New concepts that had been ruminating amongst concrete experts for decades reached a level of maturity appropriate for implementation. While not perfect, the new methods are more rational than those in the *AASHTO Standard Specifications*, (AASHTO 2002) and entail an amount of effort appropriate given today's technology compared to that available when the LFD was developed. Changes include:

- Unified design provisions for reinforced and prestressed concrete
- Modified compression field theory for shear and torsion
- Alternative Strut and Tie modeling techniques for shear and flexure
- End zone analysis for tendon anchorages
- New provisions for segmental construction
- Revised techniques for estimating prestress losses

Chapter 5 will summarize the general aspects of concrete component design using the AASHTO LRFD Specifications with California Amendments, while Chapter 7 will give a detailed description of the design procedure for post-tensioned box girder bridges, and Chapter 8 will cover the design of precast prestressed girder bridges. Concrete decks are covered in Chapter 10.

5.2 STRUCTURAL MATERIALS

5.2.1 Concrete

The most important property of concrete is the compressive strength. Concrete with 28-day compressive strength $f_c' = 3.6$ ksi is commonly used in conventionally reinforced concrete structures while concrete with higher strength is used in prestressed concrete structures. The California Amendments (Caltrans 2008) specify minimum design strength of 3.6 ksi for prestressed concrete, although AASHTO-LRFD (AASHTO 2007) Article 5.4.2.1 requires minimum design strength of 4.0 ksi. When a higher strength is specified for a project, designers should consider various factors including cost and local availability.

5.2.2 Reinforcing Steel

Steel reinforcing bars are manufactured as plain or deformed bars. In California, the main reinforcing bars are always deformed. Plain bars are usually used for spirals and ties.

Reinforcing bars must be low-alloy steel deformed bars conforming to requirements in ASTM A 706/A 706M with a 60 ksi yield strength, except that deformed or plain billet-steel bars conforming to the requirements on ASTM A 615/A 615M, Grade 40 or 60, may be used as reinforcement in some minor structures as specified in Caltrans Standard Specifications (Caltrans 2006a).

5.2.3 Prestressing Steel

Two types of high-tensile strength steel used for prestressing steel are:

1. Strands: ASTM A 416 Grades 250 and 270, low relaxation.
2. Bars: ASTM A 722 Type II

All Caltrans designs are based on low relaxation strands using either 0.5 in. or 0.6 in. diameter strands.

5.3 DESIGN LIMIT STATES

Concrete bridge components are designed to satisfy the requirements of service, strength, and extreme-event limit states for load combinations specified in AASHTO LRFD Table 3.4.1-1 with Caltrans revisions. The following are the four limit states into which the load combinations are grouped:

I. *Service Limit States*
Concrete stresses, deformations, and cracking, distribution of reinforcement, deflection and camber are investigated at service limit states.

Service I: Crack control and limiting compression in prestressed concrete
Service III: Crack control/tension in prestressed concrete
Service IV: Post-tensioned precast column sections

II. *Strength Limit States*
Axial, flexural, shear strength and stability of concrete components are investigated at strength limit states. Resistance factors are based on AASHTO LRFD 5.5.4.2 (AASHTO 2007).

Strength I: Basic load (HL-93)
Strength II: Owner specified load (Permit)
Strength III: Wind on structure
Strength IV: Structure with high DL/LL (>7)
Strength V: Wind on structure and live load

III. *Extreme Event Limit States*
Concrete bridge components and connections must resist extreme event loads due to earthquake and appropriate collision forces, but not simultaneously.

IV. *Fatigue Limit States*
Fatigue of the reinforcement need not be checked for fully prestressed concrete members satisfying requirements of service limit state. Fatigue need not be investigated for concrete deck slab on multi-girder bridges. For fatigue requirements, refer to AASHTO LRFD 5.5.3 (AASHTO 2007).

5.4 FLEXURE DESIGN

5.4.1 Strength Limit States

5.4.1.1 Design Requirement

In flexure design, the basic strength design requirement can be expressed as follows:

$$M_u \leq \phi M_n = M_r$$

where M_u is the factored moment at the section (kip-in.); M_n is the nominal flexural resistance (kip-in.); and M_r is the factored flexural resistance of a section in bending (kip-in.).

In assessing the nominal resistance for flexure, the AASHTO LRFD provisions (AASHTO 2007) unify the strength design of conventionally reinforced and prestressed concrete sections based on their behavior at ultimate limit state. In the old LFD Specifications, a flexure member was designed so that the section would fail in a tension-controlled mode. Thus, there was a maximum reinforcement ratio. Whereas, in the new LRFD specifications, there is no explicit upper bound for reinforcement. There is a distinction of compression and tension-controlled section based on the strain in the extreme tension steel. To penalize for the undesirable behavior of compression-controlled sections, a lower value of resistance reduction factor ϕ is assigned to "compression-controlled" sections compared to "tension-controlled" sections. The new procedure defines a transition behavior region in which the resistance factor ϕ, to be used for strength computation, varies linearly with the strain in the extreme steel fibers. The design of sections falling in this behavior region may involve an iterative procedure.

Here are a few terms used to describe the flexural behavior of the reinforced section:

Balanced strain condition: Strain in extreme tension steel reaches its yielding strain as the concrete in compression reaches its assumed ultimate strain of 0.003.

Compression-controlled strain limit: Net Tensile Strain (NTS) (excluding effect of prestressing, creep, etc.) in the extreme tension steel at the balanced condition. It may be assumed equal to 0.002 for Grade 60 reinforcement and all prestressed reinforcement.

Compression-controlled section: NTS $\leq$ compression-controlled strain limit just as the concrete in compression reaches its assumed strain limit of 0.003. When a section falls into this situation, it behaves more like a column than a beam. Thus, the component shall be properly reinforced with ties and spirals as required by AASHTO LRFD 5.7.2.1 with appropriate resistance factor (Caltrans 2008).

$\phi = 0.75$

Tension-controlled section: NTS $\geq$ 0.005 just as the concrete in compression reaches its assumed strain limit of 0.003.

Resistance factors are as follow:
$\phi = 1.0$ for precast prestressed members
$\phi = 0.95$ for cast-in-place prestressed members
$\phi = 0.90$ for non-prestressed members

Transition region: Compression controlled strain limit < NTS < 0.005. For the transition region, the resistance factor is calculated by using linear interpolation. Caltrans Amendments require that reinforced concrete sections in flexure be designed so that NTS ≥ 0.004. This requirement is to ensure that the section will not fail in compression-controlled modes.

Figure 5.4-1 Illustrates those three regions and equations for resistance factors of flexural resistance (Caltrans 2008).

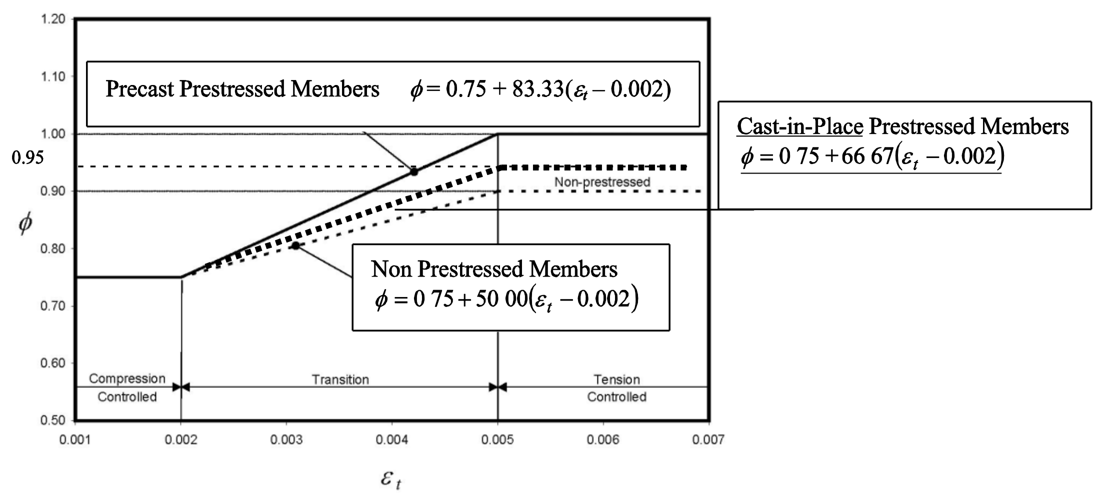

Figure 5.4-1 Resistance Factor Variation for Grade 60 Reinforcement and Prestressing Steel.

5.4.1.2 Nominal Flexural Resistance

The provisions for conventionally reinforced and prestressed concrete are now one-and-the-same. The basic assumptions used for flexural resistance (AASHTO LRFD 5.7.2.2) are as follows:

- Plane section remains plane after bending, i.e., strain is linearly proportional to the distance from the neutral axis, except the deep members.
- For unconfined concrete, maximum usable strain at the extreme concrete compression fiber is not greater than 0.003. For confined concrete, the maximum usable strain exceeding 0.003 may be used if verified.
- Stress in the reinforcement is based on its stress-strain curve.
- Tensile strength of concrete is neglected.
- Concrete compressive stress-strain distribution is assumed to be rectangular, parabolic, or any shape that results in predicted strength in substantial agreement with the test results. An equivalent rectangular compression stress

block of $0.85 f_c^{'}$ over a zone bounded by the edges of the cross-section and a straight line located parallel to the neutral axis at the distance $a = \beta_1 c$ from

the extreme compression fiber may be used in lieu of a more exact concrete stress distribution, where c is the distance measured perpendicular to the neutral axis and

$$0.65 \leq \beta_1 = 1.05 - 0.05 f_c^{'} \leq 0.85$$

where $f_c^{'}$ is in ksi

For a T-beam section, there are two cases (Figure 5.4-2) depending on where the neutral axis falls into:

- Case 1: flanged section when the neutral axis falls into the web
- Case 2: rectangular section when the neutral axis falls into the flange

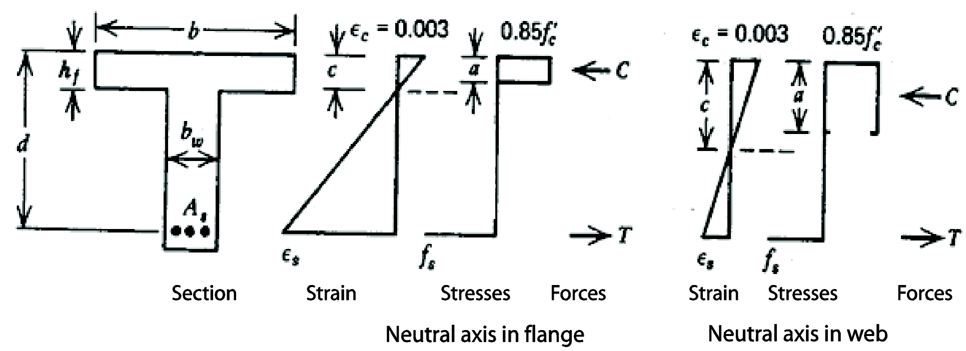

Figure 5.4-2 Stress and Strain Distribution of T-Beam Section in Flexure (shown with mild reinforcement only).

For flanged sections, the M_n can be calculated by the following equation assuming the compression flange depth is less than $a = \beta_1 c$:

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_s\left(d_s - \frac{a}{2}\right) - A_s^{'} f_s^{'}\left(d_s^{'} - \frac{a}{2}\right) + 0.85 f_c^{'}(b - b_w)h_f\left(\frac{a}{2} - \frac{h_f}{2}\right)$$

(AASHTO 5.7.3.2.2-1)

where a is the depth of equivalent rectangular stress block (in.); c is the distance from the extreme compression fiber to the neutral axis (in.); b is the width of the compression face of the member (in.); b_w is the web width (in.); h_f is the thickness of flange (in.); d is the distance from compression face to centroid of tension reinforcement (in.); d_s is the distance from compression face to centroid of mild tensile reinforcement (in.) and d_p is the distance to the centroid of prestressing steel

(in.); A_s is the area of mild tensile reinforcement (in.^{2}) and A_{ps} is the area of prestressing steel (in.^{2}); A'_s is the area of mild compressive reinforcement (in.^{2}); f_s is the stress in mild tensile steel (ksi); f'_s is the stress in the mild steel compression reinforcement (ksi) and f_{ps} is the stress in prestressing steel (ksi).

For rectangular sections, let $b_w = b$. The last term of the above equation will be dropped.

For circular and other nonstandard cross-sections, strain-compatibility must be used. WinConc (Caltrans 2005) is a suitable tool, and has been modified for LRFD.

To evaluate the prestressing stresses, the following equations can be used:

For bonded reinforcement and tendons:

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right) \qquad \text{(AASHTO 5.7.3.1.1-1)}$$

In which:

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right) \qquad \text{(AASHTO 5.7.3.1.1-2)}$$

For flanged sections:

$$c = \frac{A_{ps} f_{pu} + A_s f_s - A'_s f'_s - 0.85 f'_c (b - b_w) h_f}{0.85 f'_c \beta_1 b_w + k A_{ps} \dfrac{f_{pu}}{d_p}} \qquad \text{(AASHTO 5.7.3.1.1-3)}$$

For rectangular sections:

$$c = \frac{A_{ps} f_{pu} + A_s f_s - A'_s f'_s}{0.85 f'_c \beta_1 b + k A_{ps} \dfrac{f_{pu}}{d_p}} \qquad \text{(AASHTO 5.7.3.1.1-4)}$$

For unbonded tendons:

$$f_{ps} = f_{pe} + 900\left(\frac{d_p - c}{l_e}\right) \le f_{py} \qquad \text{(AASHTO 5.7.3.1.2-1)}$$

In which:

$$l_e = \frac{2l_i}{2 + N_s} \qquad \text{(AASHTO 5.7.3.1.2-2)}$$

For flanged sections:

$$c = \frac{A_{ps} f_{ps} + A_s f_s - A_s' f_s' - 0.85 f_c'(b - b_w)h_f}{0.85 f_c' \beta_1 b_w} \qquad \text{(AASHTO 5.7.3.1.2-3)}$$

For rectangular sections:

$$c = \frac{A_{ps} f_{ps} + A_s f_s - A_s' f_s'}{0.85 f_c' \beta_1 b} \qquad \text{(AASHTO 5.7.3.1.2-4)}$$

where f_{py} and f_{pu} are the yield and ultimate tensile strength of prestressing steel respectively; f_{pe} is the effective stress in prestressing steel after loss (ksi); l_e is the effective tendon length (in.); l_i is the length between anchorage (in.); and N_s is the number of support hinges crossed by the tendon between anchorages.

5.4.1.3 Reinforcement Limits

As mentioned before, there is no explicit limit on maximum reinforcement. Sections are allowed to be over reinforced but shall be compensated for reduced ductility in the form of a reduced resistance reduction factor.

The minimum reinforcement shall be provided so that, M_r, at least equal to lesser of M_{cr} and 1.3 M_u .

5.4.2 Service Limit States

Service limit states are used to satisfy stress limits, deflection, and cracking requirements. To calculate the stress and deflection, the designer can assume concrete behaves elastically. The modulus of elasticity can be evaluated according to the code specified formula such as AASHTO LRFD 5.4.2.4. The reinforcement and prestressing steel are usually transformed into concrete. For normal weight concrete with w_c = 0.145 kcf, the modulus of elasticity, may be taken as:

$$E_c = 1{,}820\sqrt{f_c'} \qquad \text{(AASHTO C5.4.2.4-1)}$$

For prestressed concrete members, prestressing force and concrete strength are determined by meeting stress limits in the service limit states, and then checked in the strength limit states for ultimate capacity. All other members are designed in accordance with the requirements of strength limit states first, the cracking requirement is satisfied by proper reinforcement distribution.

To design the prestressed members, the following stress limits listed in Table 5.4-1 should be satisfied.

Table 5.4-1 Stress Limits for Concrete.

Condition	Stress	Location	Allowable Stress
Temporary Stress before loss	Tensile	In area other than Precompressed Tensile Zone and without bonded tendons or reinforcement	$0.0948\sqrt{f_c'} \leq 0.2$ (ksi)
		In area with bonded tendons or reinforcement sufficient to resist the tensile force in the concrete computed assuming an uncracked section, where reinforcement is proportioned using a stress of $0.5f_y$, not to exceed 30 ksi	$0.24\sqrt{f_c'}$ (ksi)
	Compression	All locations	$0.6\,f_c'$
Final Stress after loss at service load	Tensile	In the Precompressed Tensile Zone, assuming uncracked section:	
		• Components with bonded tendons or reinforcement, and are located in Caltrans' Environment Areas I and II	$0.19\sqrt{f_c'}$ (ksi)
		• Components with bonded tendons or reinforcement, and are located in Caltrans' Environment Area III	$0.0948\sqrt{f_c'}$ (ksi)
		• Components with unbonded tendons	0
	Compression	All locations due to:	
		• Permanent loads and effective prestress loads	$0.45\,f_c'$
		• Live load plus one-half permanent loads and effective prestress load	$0.4\,f_c'$
		• All load combinations	$0.6\,f_c'$
Permanent loads only	Tensile	Precompressed Tensile Zone with bonded prestressing tendons or reinforcement	0

5.4.3 Fatigue Limit States

As per AASHTO 5.5.3.1 (AASHTO 2007), the stress range in reinforcing bars due to the fatigue load combination should be checked and should satisfy:

$$f_f \leq 24 - 0.33 f_{\min} \qquad \text{(AASHTO 5.5.3.2-1)}$$

where:

f_f = Stress range (ksi)

$f_{\min}$ = Minimum live load stress (ksi) resulting from the fatigue load combined with the more severe stress from either the permanent loads or the permanent loads, shrinkage, and creep-induced external loads; positive if tension, negative if compression

For the fatigue check:

- The fatigue load combination is given in CA Amendment Table 3.4.1-1. A load factor of 0.875 is specified on the live load (Fatigue truck) for finite fatigue life and a load factor of 1.75 for the infinite fatigue life.
- A fatigue load is one design truck with a constant 30-ft. spacing between the 32.0-kip axles as specified in AASHTO 3.6.1.4 (AASHTO 2007).
- Apply the IM factor to the fatigue load.
- There is no permanent load considered in this check.
- Check both top and bottom reinforcements to ensure that the stress range in the reinforcement under the fatigue load stays within the range specified in the above equation.

5.5 SHEAR DESIGN

5.5.1 Basic Concept of Modified Compression Field Theory

Perhaps the most significant change for concrete design in AASHTO LRFD Specifications is the shear design methodology. It provides two methods: Sectional Method, and Strut and Tie Method. Both methods are acceptable to Caltrans. The Sectional Method, which is based on the Modified Compression Field Theory (MCFT), provides a unified approach for shear design for both prestressed and reinforced concrete components. For a detailed derivation of this method, please refer to the book by Collins and Mitchell (1991).

The two approaches are summarized as follows:

- Sectional Method
  - Plane section remains plane – Basic Beam Theory
  - Based on Modified Compression Field Theory (MCFT)

- Used for most girder design, except disturbed-end regions
- Used for any undisturbed regions

- Strut and Tie Method
  - Plane section does not remain plane
  - Used in "disturbed regions" and deep beams
  - Examples of usage: Design of Bent Caps (clear span to depth ratio less than 4); pile caps; anchorage zones (general or local); area around openings

In this chapter, only the Sectional Method will be outlined. The Strut and Tie Method will be discussed in other chapters.

Compression Field Theory (CFT) is highlighted as follows:

- Angle for compressive strut (or crack angle) is variable
- Plane section remains plane (for strain compatibility)
- Strength of concrete in tension is ignored
- Element level strains incorporate the effects of axial forces, shear and flexure
- Equations are based on element level stresses and strains
- The shear capacity is related to the compression in diagonally cracked concrete through equilibrium

This theory is further modified by including the strength of concrete in tension, and it is referred to as the Modified Compression Field Theory (MCFT).

5.5.2 Shear Strength

According to AASHTO LRFD 5.8.3.3, the nominal shear resistance, V_n, shall be determined as:

$$V_n = V_c + V_s + V_p \qquad \text{(AASHTO 5.8.3.3-1)}$$

But total resistance by concrete and steel: $V_c + V_s$ should be no greater than 0.25 $f_c' b_v d_v$. In the end region of the beam-type element when it is not built integrally with the support, $V_c + V_s$ should not exceed 0.18 $f_c' b_v d_v$. If it exceeds this value, this region should be designed using the Strut and Tie Method and special consideration should be given to detailing.

$$V_c = 0.0316\beta\sqrt{f_c'}b_v d_v \qquad \text{(AASHTO 5.8.3.3-3)}$$

$$V_s = \frac{A_v f_y d_v(\cot\theta + \cot\alpha)\sin\alpha}{s} \qquad \text{(AASHTO 5.8.3.3-4)}$$

where V_p is the component in the direction of applied shear of the effective prestressing force (kip); b_v is the effective web width (in.); and d_v is the effective shear depth (in.).

In these equations, θ is the angle of crack and β is a factor. Unlike in the old LFD code where the angle of crack was assumed as a constant 45º, the MCFT method assumes it is a variable, which is a more accurate depiction of actual behavior.

For members with minimum transverse reinforcement, β and θ values calculated from the MCFT are given as functions of ε_x, shear stress v_u, and f'_c in AASHTO LRFD Table 5.8.3.4.2-1. ε_x is taken as the calculated longitudinal strain at mid-depth of the member when the section is subjected to M_u, N_u, and V_u.

$$\varepsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps} f_{po}\right)}{2(E_s A_s + E_p A_{ps})} \qquad \text{(AASHTO 5.8.3.4.2-1)}$$

For members without transverse reinforcement, β and θ values calculated from the MCFT are given as functions of ε_x, and the crack spacing s_{xe} in AASHTO LRFD Table 5.8.3.4.2-2. ε_x is taken as the largest calculated longitudinal strain which occurs within the web of the member when the section is subjected to M_u, N_u, and V_u.

$$\varepsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps} f_{po}\right)}{(E_s A_s + E_p A_{ps})} \qquad \text{(AASHTO 5.8.3.4.2-2)}$$

If the value of ε_x from either equation above is negative, the strain is taken as:

$$\varepsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps} f_{po}\right)}{2(E_c A_c + E_s A_s + E_p A_{ps})} \qquad \text{(AASHTO 5.8.3.4.2-3)}$$

The crack spacing parameter s_{xe}, is determined as:

$$s_{xe} = s_x \frac{1.38}{a_g + 0.63} \leq 80 \text{ in.} \qquad \text{(AASHTO 5.8.3.4.2-4)}$$

where a_g is maximum aggregate size (in.); and s_x is the lesser of either d_v, or the maximum distance between layers of longitudinal crack control reinforcement (in.).

As one can see, ε_x, β and θ are all inter-dependent. So, design is an iterative process:

1. Calculate shear stress demand v_u at a section and determine the shear ratio (v_u/f'_c)
2. Calculate ε_x at the section based on normal force (including p/s), shear and bending and an assumed value of θ
3. Longitudinal strain ε_x is the average strain at mid-depth of the cross section
4. Knowing v_u/f'_c & ε_x, obtain the values of β and θ from the table
5. Recalculate ε_x based on revised value of θ; repeat iteration until convergence in θ is achieved.

where v_u, the shear stress on the concrete, should be determined as:

$$v_u = \frac{\left|V_u - \varphi V_p\right|}{\varphi b_v d_v} \qquad \text{(AASHTO 5.8.2.9-1)}$$

To simplify this iterative approach, Professors Bentz, Vecchio, and Collins have proposed a simplified method (Bentz, E. C. et al, 2006).

5.5.3 Flexure - Shear Interaction

In the MCFT model, the concrete is essentially modeled as a series of compression struts in resisting shear forces. The horizontal components of these diagonal forces have to be resisted by horizontal ties - longitudinal reinforcement. Therefore, after the design of flexure and shear is completed, the longitudinal reinforcement is checked for such interaction. Provide additional reinforcement if required.

The following equations should be used for checking the adequacy of longitudinal reinforcement:

$$A_{ps} f_{ps} + A_s f_y \geq \frac{\left|M_u\right|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right) cot\theta \qquad \text{(AASHTO 5.8.3.5-1)}$$

$$V_s \leq \frac{V_u}{\phi}$$

$$A_{ps} f_{ps} + A_s f_y \geq \left(\frac{V_u}{\phi_v} - 0.5V_s - V_p\right) cot\theta \qquad \text{(AASHTO 5.8.3.5-2)}$$

- Requirement for the interaction check depends on the support / load transfer mechanism (direct supports or indirect supports)

- Maximum flexural steel based on moment demand need not be exceeded in / near direct supports
- Interaction check is required for simple spans made continuous for live load or where longitudinal steel is not continuous
- Equation (5.8.3.5-2) is required to be satisfied at the inside edge of the bearing area of simple support

Direct Support / Direct Loading

Figure 5.5-1 shows some of the examples of direct support and direct loading:

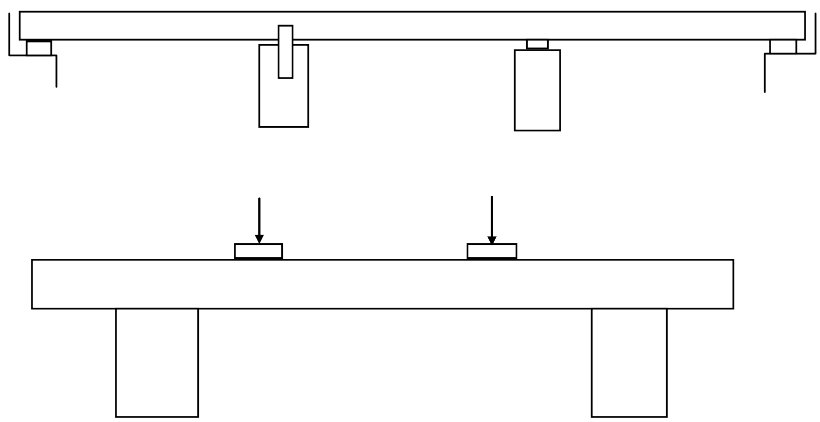

Figure 5.5-1 Examples of Direct Support and Direct Loading.

Special Notes:

- If flexural reinforcement is not curtailed (eg: in bent cap), then there is no need to check for interaction.
- If flexural reinforcement is curtailed (eg: in a superstructure), check for interaction:
  - Check at 1/10 points and/or at curtailment locations.
  - If reinforcement per equation is inadequate, extend primary flexural reinforcement.
  - Area of tensile reinforcement need not exceed that required for maximum moment demand acting alone.

Indirect Support / Integral Girders

Figure 5.5-2 shows some of the examples of indirect support and integral girders. The girders framing into the bent cap are indirectly supported while the bent cap itself is directly supported by the columns.

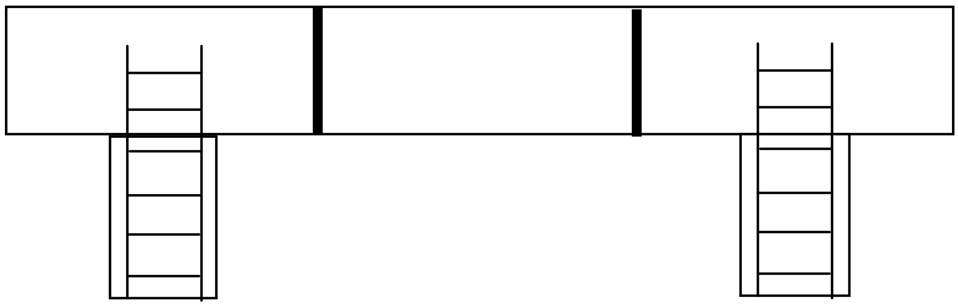

Figure 5.5-2 Example of Indirect Support and Integral Girder.

Special Notes:

- In bent caps, check interaction at 10 points, and at the girder locations / locations of major concentrated loads
- Check for interaction at face of integral supports
- If interaction is not satisfied, then adopt one of the following:
  - Increase flexural reinforcement
  - Increase shear reinforcement
  - Combination of the above

5.5.4 Transverse Reinforcement Limits

- Minimum Transverse Reinforcement

Except for segmental post-tensioned concrete box girder bridges, the area of steel should satisfy:

$$A_v \geq 0.0316\sqrt{f_c'}\frac{b_v s}{f_y} \qquad \text{(AASHTO 5.8.2.5-1)}$$

- Maximum Spacing of Transverse Reinforcement

The spacing of the transverse reinforcement should not exceed the maximum spacing, s_{max}, determined as:

- If $v_u < 0.125\, f'_c$ then:
  $s_{max} = 0.8d_v \leq 24.0$ in. (AASHTO 5.8.2.7-1)

- If $v_u \geq 0.125\, f'_c$ then:
  $s_{max} = 0.4d_v \leq 12.0$ in. (AASHTO 5.8.2.7-2)

5.6 COMPRESSION DESIGN

As stated previously, when a member is subjected to a combined moment and compression force its resulting strain can be in a compression-controlled state. Compression design procedure applies. The following effects are considered in addition to bending: degree of end fixity; member length; variable moment of inertia; deflections; and duration of loads. This chapter will only cover the two basic cases: pure compression, and combined flexure and compression ignoring slenderness. AASHTO LRFD 5.7.4.3 provides an approximate method for evaluating slenderness effect.

5.6.1 Factored Axial Compression Resistance – Pure Compression

The factored axial resistance of concrete compressive members, symmetrical about both principal axes, is taken as:

$$P_r = \phi P_n \qquad \text{(AASHTO 5.7.4.4.-1)}$$

In which: P_n , the nominal compression resistance, can be evaluated for the following two cases:

For members with spiral reinforcement:

$$P_n = 0.85[0.85\, f'_c\,(A_g - A_{st} - A_{ps}) + f_y A_{st} - A_{ps}\,(f_{pe} - E_p\,\varepsilon_{cu})] \qquad \text{(AASHTO 5.7.4.4-2)}$$

For members with tie reinforcement:

$$P_n = 0.80[0.85\, f'_c\,(A_g - A_{st} - A_{ps}) + f_y A_{st} - A_{ps}\,(f_{pe} - E_p\,\varepsilon_{cu})] \qquad \text{(AASHTO 5.7.4.4-3)}$$

where A_g is the gross area of the section (in.^{2}); A_{st} is the total area of longitudinal mild reinforcement (in.^{2}); A_{ps} is the area of prestressing steel (in.^{2}); E_p is the modulus of elasticity of prestressing steel (ksi); and ε_{cu} is the failure strain of concrete in compression.

In order to achieve the above resistance, the following minimum spiral shall be supplied:

$$\rho_s \geq 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_{yh}} \qquad \text{(AASHTO 5.7.4.6-1)}$$

where f_{yh} is the specified yield strength of transverse reinforcement (ksi).

To achieve more ductility for seismic resistance, Caltrans has its own set of requirements for spirals and ties. For further information, please refer to the current version of the Seismic Design Criteria (Caltrans 2010).

5.6.2 Combined Flexure and Compression

When a member is subjected to a compression force, end moments are often induced by eccentric loads. The end moments rarely act solely along the principal axis. So at any given section for analyzing or design, the member is normally subjected to biaxial bending as well as compression. Furthermore, to analyze or design a compression member in a bridge substructure, many load cases need to be considered.

Under special circumstances, the Specifications allow designers to use an approximate method to evaluate biaxial bending combined with axial load (AASHTO LRFD 5.7.4.5). Generally, designers rely on computer programs based on equilibrium and strain compatibility, such as WinYield (Caltrans 2006b), to generate a moment-axial interaction diagram. For cases like noncircular members with biaxial flexure, an interaction surface is required to describe the behavior. Figure 5.6-1 shows a typical moment-axial load interaction surface for a concrete section (Park and Pauley 1975).

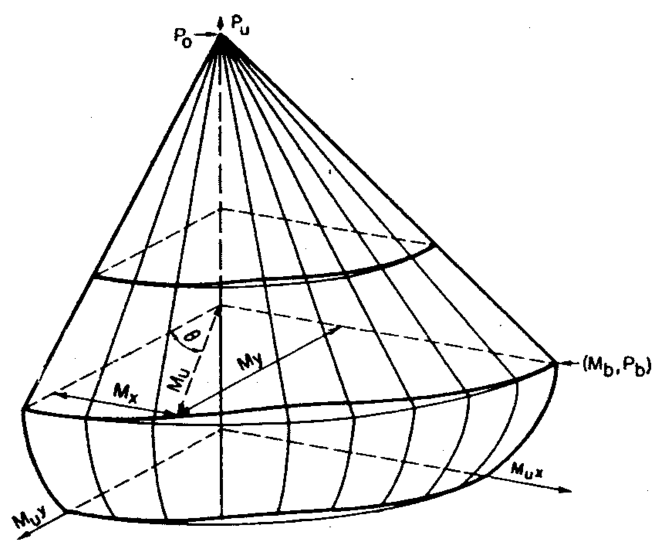

Figure 5.6-1 Moment-Axial Interaction Surface of a Noncircular Section.

In day-to-day practice, such a surface has little value to designers. Rather, the design program normally gives out a series of lines, basically slices of the surface, at fixed intervals, such as 15º. Figure 5.6-2 is an example plot from WinYield (Caltrans 2006b).

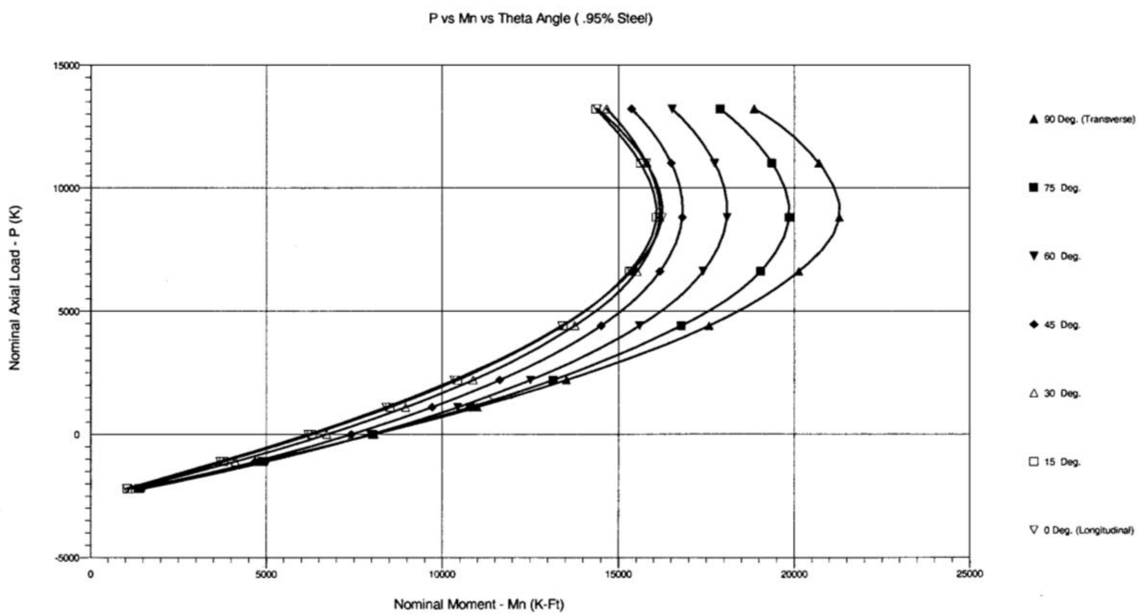

Figure 5.6-2 Interaction Diagrams Generated by WinYield.

From these lines, it can be seen that below the balanced condition the moment capacity increases with the increase of axial load. So, when designing a column, it is not enough to simply take a set of maximum axial load with maximum bending moments. The following combination needs to be evaluated:

1. $M_{ux\,max}$, corresponding M_{uy} and P_u
2. $M_{uy\,max}$, corresponding M_{ux} and P_u
3. A set of M_{ux} and M_{uy} that gives largest M_u combined, and corresponding P_u
4. $P_{u\,max}$ and corresponding M_{ux} and M_{uy}

Special Notes:

- Columns will be more thoroughly covered in Chapter 13.
- For load cases 1 through 3, the load factor γ_p corresponding to the minimum shall be used.
- P_n and M_n shall be multiplied by a single ϕ factor depending on whether it is compression controlled or tension controlled, as illustrated previously and as shown in AASHTO LRFD Figure C5.5.4.2.1-1.
- Slenderness effect shall be evaluated with appropriate nonlinear analysis program or the use of approximate methods such as AASHTO LRFD 5.7.4.3.
- In California, the column design is normally controlled by seismic requirements. That topic is not covered in this chapter.

5.6.3 Reinforcement Limits

The maximum area of prestressed and non-prestressed longitudinal reinforcement for non-composite compression members is as follows:

$$\frac{A_s}{A_g} + \frac{A_{ps} f_{pu}}{A_g f_y} \leq 0.08 \qquad \text{(AASHTO 5.7.4.2-1)}$$

And

$$\frac{A_{ps} f_{pe}}{A_g f_c'} \leq 0.30 \qquad \text{(AASHTO 5.7.4.2-2)}$$

The minimum area of prestressed and non-prestressed longitudinal reinforcement for non-composite compression members is as follows:

$$\frac{A_s f_y}{A_g f_c'} + \frac{A_{ps} f_{pu}}{A_g f_c'} \geq 0.135 \qquad \text{(AASHTO 5.7.4.2-3)}$$

Due to seismic concerns, Caltrans put further limits on longitudinal steel in columns. For such limits, please refer to the latest version of the Caltrans Seismic Design Criteria (Caltrans 2010).

NOTATION

A_c	=	area of core of spirally reinforced compression member measured to the outside diameter of the spiral (in.^{2})
A_g	=	gross area of section (in.^{2})
A_{ps}	=	area of prestressing steel (in.^{2})
A_s	=	area of non-prestressed tension steel (in.^{2})
A'_s	=	area of compression reinforcement (in.^{2})
A_{sh}	=	cross-sectional area of column tie reinforcement (in.^{2})
A_{st}	=	total area of longitudinal mild steel reinforcement (in.^{2})
A_v	=	area of transverse reinforcement within distances (in.^{2})
a	=	depth of equivalent rectangular stress block (in.)
b	=	width of compression face of the member (in.)
b_v	=	effective web width taken as the minimum web width (in.)
b_w	=	web width (in.)
c	=	distance from the extreme compression fiber to the neutral axis (in.)
D	=	external diameter of the circular members (in.)
d	=	distance from compression face to centroid of tension reinforcement (in.)
d_b	=	nominal diameter of a reinforcing bar (in.)
d_e	=	effective depth from extreme compression fiber to the centroid of tensile force in the tensile reinforcement (in.)
d_p	=	distance from extreme compression fiber to centroid of prestressing strand (in.)
d_s	=	distance from extreme compression fiber to centroid of non-prestressed tensile reinforcement (in.)
d_v	=	effective shear depth (in.)
E_c	=	modulus of elasticity of concrete (ksi)
E_p	=	modulus of elasticity of prestressing tendons (ksi)
E_s	=	modulus of elasticity of reinforcing bars (ksi)
f'_c	=	specified compressive strength of concrete (ksi)
f_{cpe}	=	compressive stress in concrete due to effective prestress force only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)
f_{pe}	=	effective stress in prestressing steel after losses (ksi)

f_{ps}	=	average stress in prestressing steel at the time for which the nominal resistance of members is required (ksi)
f_{pu}	=	specified tensile strength of prestressing steel (ksi)
f_{py}	=	yield strength of prestressing steel (ksi)
f_r	=	modulus of rupture of concrete (ksi)
f_s	=	stress in mild tensile reinforcement at nominal flexural resistance (ksi)
f_s'	=	stress in mild compression reinforcement at nominal flexural resistance (ksi)
f_y	=	specified minimum yield strength of reinforcing bars (ksi)
f_{yh}	=	specified yield strength of transverse reinforcement (ksi)
h_f	=	thickness of flange (in.)
l_e	=	effective tendon length (in.)
l_i	=	length of tendon between anchorages (in.)
M_b	=	nominal flexural resistance at balanced condition (kip-in.)
M_{cr}	=	cracking moment (kip-in.)
M_n	=	nominal flexural resistance (kip-in.)
M_{dnc}	=	total unfactored dead load moment acting on the monolithic or non-composite section (kip-ft.)
M_r	=	factored flexural resistance of a section in bending (kip-in.)
M_u	=	factored moment at the section (kip-in.)
M_{ux}	=	factored moment at the section in respect to principal x axis (kip-in.)
M_{uy}	=	factored moment at the section in respect to principal y axis (kip-in.)
N_u	=	factored axial force (kip)
N_s	=	number of support hinges crossed by the tendon between anchorages or discretely bonded points
P_n	=	nominal axial resistance of a section (kip)
P_o	=	nominal axial resistance of a section at 0 eccentricity (kip)
P_r	=	factored axial resistance of a section (kip)
P_u	=	factored axial load of a section (kip)
s	=	spacing of reinforcing bars (in.)
V_c	=	nominal shear resistance provided by tensile stresses in the concrete (kip)
V_n	=	nominal shear resistance of the section considered (kip)
V_p	=	component in the direction of the applied shear of the effective prestressing forces; positive if resisting the applied shear (kip)

V_r = factored shear resistance (kip)

V_s = shear resistance provided by the shear reinforcement (kip)

V_u = factored shear force (kip)

v_u = average factored shear stress on the concrete (ksi)

S_c = section modulus for the extreme fiber of the composite section where tensile stress is caused by externally applied loads (in.^{3})

S_{nc} = section modulus for the extreme fiber of the monolithic or non-composite section where tensile stress is caused by externally applied loads (in.^{3})

α = angle of inclination of transverse reinforcement to longitudinal axis (º)

β = factor relating effect of longitudinal strain on the shear capacity of concrete, as indicated by the ability of diagonally cracked concrete to transmit tension

β_1 = ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength limit state to the depth of the actual compression zone

γ = load factor

ε_{cu} = failure strain of concrete in compression (in./in.)

ε_x = Longitudinal strain in the web reinforcement on the flexural tension side of the member (in./in.)

θ = angle of inclination of diagonal compressive stress (º)

ϕ = resistance factor

ϕ_c = resistance factor for compression

ϕ_f = resistance factor for moment

ϕ_v = resistance factor for shear

ρ_s = ratio of spiral reinforcement to total volume of column core

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