Petroleum Economics and Engineering

Third Edition

Edited by Hussein K. Abdel-Aal Mohammed A. Alsahlawi



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Preface

Technology and economics are two areas that have the capacity to transform our world. Petroleum technology from the engineering point of view is subject of this book, along with economic analysis. Technology is the great enabler that has made exploration for oil more effective, drilling more efficient, and production more prolific, safer, and less intrusive to the environment than ever.

The application of engineering principles to practical ends such as the design, manufacture, and operation of efficient and economical plants, machines, and processes exemplifies the leading role of technology in the petroleum sector. The largest volume products of the industry are fuel oil and gasoline. Petroleum is also the raw material for many chemical products, including pharmaceuticals, solvents, fertilizers, pesticides, and plastics.

In oil production, virtually every oil field decision is founded on profitability. With no control of oil and gas prices and facing steadily rising costs and declining reserves, companies' basic decisions are based on constantly moving targets. Drilling, completing, and producing oil and gas is an extremely complex business. One might think that the world's unshakable thirst for cheap, abundant energy resources makes profitability a sure thing, but this might not be so.

Petroleum economics and engineering is the application of economic techniques and analysis to the evaluation of design and engineering alternatives encountered in the petroleum industry. It includes the systematic evaluation of the economic merits of proposed solutions to engineering problems. Part of the role of petroleum economics and engineering is to assess the appropriateness of a given project, estimate its value, and justify it from an engineering standpoint.

The philosophy in this book is the same as in previous editions in that the fundamentals of economics as applied to engineering problems in the petroleum industry are emphasized. The text focuses on the fact that engineers seek solutions to problems; the economic viability of each potential solution is considered along with its technical merits. This is typically true for the petroleum sector, which includes the global processes of exploration, production, refining, and transportation (often by oil tankers and pipelines).

Fully revised and updated to reflect major changes over the past two decades, this third edition offers thorough coverage of every sector in oil operations, focusing on engineering problems encountered in the oil industry. Sound economic decision making to solve these problems is the main target of the book. Section 1 consists of introductory materials. All principles, methods, and techniques of engineering economics (as applied to the petroleum industry) are presented in Section 2.

An eminent group of contributors, in addition to the editors of the book, wrote chapters for which they are specially qualified and possess valuable experience. The book's twenty chapters are arranged in three parts.

Section 1, Introduction to the Oil and Gas Industry, consists of three chapters. World petroleum and gas supply and demand patterns are examined in Chapter 1. The activities and structure of the oil and gas industry are examined in Chapter 2. Chapter 3 provides information about the different types of crude oil, the composition of natural gas, and the properties of petroleum products.

Section 2, Principles, Methods, and Techniques of Engineering Economics Analysis, is the backbone of the book. Economic principles are revised and presented in this new edition. This part consists of seven chapters. Chapter 4 is concerned with interest and time relationships; Chapters 5 through 7 are devoted to the calculation of depreciation and depletion costs, profitability analysis, and comparison of alternatives and replacements, respectively. Risk evaluation and decision analysis, breakeven and sensitivity analysis, and optimization techniques are covered in Chapters 8 through 10. Many examples are worked out in this part using Excel.

Section 3, Applications and Case Studies, covers the hydrocarbon industry. It treats in chronological order the three major components that characterize the oil industry:

- The upstream component including all subsurface operations
- The midstream component, known as surface petroleum operations
- The downstream component, known as refining/processing operations

Section 3 consists of ten chapters. Technology aspects and engineering background are described first in each chapter, followed by selected case studies with applications to demonstrate how to apply economic analysis for many engineering problems encountered in various sectors of petroleum operations. Chapters 11 through 13 handle upstream (subsurface) operations, covering exploration and drilling, reserves and reserve estimates, and production, respectively. Chapters 14 through 16 deal with midstream (surface) operations, covering gas/oil separation, crude oil treatment, and gas treatment and conditioning, respectively. Chapters 17 through 20 are concerned with downstream operations (refining/processing), covering crude oil refining by physical separation and chemical conversion, gas processing, and transportation of oil and gas, respectively.

The techniques of economic analysis employed throughout the text are used to the fullest extent, and details are carefully presented, covering each sector of the oil industry. Many application examples are included to illustrate various theoretical solutions.

The purpose of the book goes beyond description and systematization of economic problems in oil engineering. Engineers and managers may combine principles drawn from the chapters to solve problems and evaluate oil economic projects of which they are in charge.

The economic principles and techniques covered in Section 2, in combination with the technological descriptions of different phases encountered in the oil industry and the illustrative examples and case studies in Section 3, impart the required skills for effective economic evaluation of most practical oil engineering problems. In addition, concepts and techniques of analysis useful in evaluating the worth of petroleum systems are considered. The answers to frequently asked questions such as "which petroleum projects are worthwhile?" and "which project should have a higher priority?" are presented.

This book is invaluable to senior and graduate students majoring in petroleum engineering, chemical engineering, and economics. It is a helpful resource for practicing engineers and production people working in the petroleum industry who have the responsibility of planning and decision making in oil or gas field development. It also may be used as a reference volume for managers, executives, and other personnel engaged in this field. Although the book is focused on petroleum engineering economics, most of its contents should be equally applicable to other engineering disciplines. The text can be adopted, accordingly, as a principal or supplemental resource book in allied courses such as engineering economics, petroleum economics and policy, project evaluation, and plant design.

Since all aspects of the field of engineering economy in the petroleum industry cannot be covered in detail in a single book, every effort has been made by the editors and the authors to expose the readers to the nature of the problems that are typical of the oil industry. This book by no means presents a complete description of the design of any part of these processes. Many details have been omitted in order to summarize a vast subject. Errors of exposition and inelegances of expression undoubtedly remain. These are our responsibility.

References are grouped together at the end of the book to serve as a subject bibliography. They do not represent complete citation of the authorities for all the statements given in the text. Conversion factors are included in Appendix A, and Appendix B lists the economic factors as a function of the interest rate and the number of years that are used extensively in Section 2.

We are pleased to acknowledge the help we have received over the years from colleagues and students, and in particular from established sources and texts on the same topic. We are greatly indebted to the many firms and publications that have allowed us to use their materials as references.

The editors are grateful to Taylor & Francis Group for their enthusiasm in the publication of this new edition of our book. It is our pleasure to acknowledge the help provided by Allison Shatkin, Jill Jurgensen, and Amy Rodriguez throughout this task.

> Hussein Abdel-Aal Mohammed Alshlawi

The Editors

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Abdel-Aal worked in the oil industry (1956–1960) as a process engineer in Suez oil refineries before doing his graduate studies in the United States. On returning to Cairo, he joined NRC for the period 1965 to 1970, followed by one year at University of Manchester Institute of Science and Technology (UMIST), United Kingdom, as a postdoctoral scholar.

He then joined the Department of Chemical Engineering at KFUPM, Dhahran, Saudi Arabia (1971–1985). He was the head of the department from 1972 to 1974 and was a visiting professor with the Chemical Engineering Department at Texas A&M from 1980 to 1981. From 1985 to 1988, Abdel-Aal assumed the responsibilities of the head of the Solar Energy Department, NRC, Cairo, before rejoining KFUPM again from 1988 to 1998.

Abdel-Aal has contributed to over 90 technical papers and is the editor of *Petroleum Economics and Engineering* and the main author of *Petroleum and Gas Field Processing* (Marcel Dekker, New York, 1992 and 2003, respectively).

Abdel-Aal is listed in *Who's Who in the World*, 1982, is a member of the American Institute of Chemical Engineers (AIChE), Sigma Si, and Phi Lambda Upsilon. He is Fellow and founding member of the board of directors of the International Association of Hydrogen Energy, Miami, Florida.

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Section 1

Introduction to the Oil and Gas Industry

1

World Oil and Gas Supply and Demand

M.A. Al-Sahlawi

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The oil and gas industry has invested billions of dollars in finding, discovering, developing, producing, transporting, and refining hydrocarbons for more than a century and has long been an enormous source of wealth creation.

In this chapter, world petroleum and gas supply-and-demand patterns are examined. Current statistics on reserves are also reviewed. It is noted that estimates of proven reserves change from year to year, and high oil prices stimulate searching for oil and gas, which tends to increase the amount of proven reserves around the world. Regarding oil supply and demand, the United States is still the main oil producer and consumer in the world. It also leads the world in refining capacity. Oil demand is predicted to continue to increase despite the high price of oil. There are many sources of demand for oil. As countries develop and industrialize, their oil consumption grows with their economy.

Today China and India are the big players when it comes to growing economies. The world has never seen economic growth like it has with these two countries, and the impact on oil demand has already begun. Developed countries are also yet to seriously change their oil habits but will likely adapt at a faster pace if oil prices continue to rise.

1.1 Introduction

This chapter provides a general review of oil and gas reserves and the patterns of production and consumption. Recent statistics on world distribution of oil proven reserves indicate that oil is found in many regions, but the Middle East accounts for the highest share of the world total. Proven gas reserves are distributed around the world with more concentrating in the Middle East and Eastern Europe including Russia. High oil prices induce more discoveries which increase the amount of proven reserves.

In 2011, 242 oil and gas discoveries were made globally. This was 45 percent less than global oil and gas discoveries made in 2009. The factors that led to this decrease include an increase in exploration activities in technically challenging areas, such as deep offshore and ultra-deep offshore areas and the Arctic, as well as a lack of required technical equipment, environmental protests, and government restrictions.

As far as oil supply and demand are concerned, the United States has been the principal oil producer. Over the years different producers have emerged in Latin America, the Middle East, and North Africa. The Middle East alone produced more than one-fourth of world oil production in 1960. The trends continue in recent years with more than 10 mbd from Saudi Arabia alone in 2010. The production of refined oil products, however, is concentrated near the consuming areas. The United States has led the world in refining capacity. Together with Western Europe they produce almost half the world refined oil products.

On the other hand, world oil demand has increased substantially over the last three decades. The growth in demand has been noticed in the industrialized countries. The demand for oil has increased recently in the developing countries of the Middle East, South Asia, and China as a result of rapid economic growth and high population. The gas supply picture shows an increasing share for Organization of the Petroleum Exporting Countries (OPEC), mainly Iran and Qatar, while maintaining the position of Russia and the United States as leading suppliers, with the demand for gas concentrated in the United States, Russia, and Europe.

1.2 Oil Reserves

Oil reserves can generally be classified into cumulative production to date, proven reserves, and probable reserves. Proven reserves, however, are defined as the part of oil in place which can be produced under current economic and technical conditions without reasonable doubt. This includes all successfully tested areas as well as reserves that have been developed for production. At the end of 2010, world proven oil reserves were estimated to be 1.47 trillion barrels (bbl).



FIGURE 1.1



Figure 1.1 shows world distribution of oil proven reserves at the end of the year over the period 1960 to 2010. Until 1950 North America constituted the largest share of world oil proven reserves, but after 1960 other areas have emerged, such as the Middle East, which has the highest share. Its share rose to 65 percent of the world total by 1990 and continues to rise, while Latin America and Russia have followed, with 13 percent and 9 percent, respectively. The distribution of world oil proven reserves by OPEC nations compared to the rest of the world is presented in Figure 1.2. It shows the proven



FIGURE 1.2 (See Color Insert)

OPEC share of world oil reserves end 2010. (From *OPEC Annual Statistical Bulletin*, Vienna, 2011. With permission.)

oil reserves for countries at the end of 2010. Venezuela is at the top of the list, followed by Saudi Arabia, Iran, Iraq, and Kuwait. However, oil proven reserves for these countries have been revised upward lately. Changes in these estimates from year to year are due to changes in production levels, new discoveries, and extensions of the existing fields.

To measure the expected life of oil reserves, a ratio of proven reserves to annual production is calculated given certain assumptions. These assumptions include constant rate of production, stagnant oil demand, and no additional discoveries. For example, in the Gulf countries such as Saudi Arabia, the expected life of an oil reserve is around 90 years, where the average for the world is about 40 years. However, this ratio is changing over time as a result of changing oil prices and the state of technology. Rising oil prices in the 1970s and subsequent periods have stimulated more investment in exploration, even in relatively high-cost areas, which in turn has raised the proven reserves. This indicates that there is a positive correlation between oil prices and oil reserves.

1.3 World Oil Supply

1.3.1 Crude Oil Production

Since the 1850s, oil has been produced in different parts of the world. The United States was the major producer; it produced over 90 percent of world production until 1875. Over the years and with the increasing importance of oil, new regions have emerged as key oil producers. The Middle East share of world's oil production has increased from 4.8 percent in 1940 to more than 25 percent in 2000, while the United States share reduced to around 10 percent in 2000 from 62 percent in 1940. Table 1.1 shows the share of crude

			Ye	ar			2010 Share
Region	1960	1970	1980	1990	2000	2010	of Total
North America	9.20	13.26	14.10	13.85	13.90	13.88	16.6%
Latin America	2.90	4.83	3.75	4.51	6.81	6.91	8.9%
Western Europe	0.30	0.46	2.6	3.70	4.10	4.2	21.8%
Eastern Europe	3.20	7.60	12.31	12.4	10.5	13.81	21.8%
Middle East	5.30	13.90	22.02	17.54	23.55	25.18	30.2%
Africa	0.28	6.11	6.79	6.72	7.80	10.10	12.2%
Asia and Pacific	0.60	1.99	5.11	6.73	7.87	8.35	10.2%
Total	21.78	48.09	66.05	65.46	74.89	82.10	

TABLE 1.1

Share of World Crude Oil Production by Region from (mbd) 1960 to 2010

Source: BP Statistical Review of World Energy, London, 2011. With permission.

oil production by regions for the period 1960 to 2010. Oil production by Eastern Europe including Russia exceeded America's in 1990, but the latter spurted ahead after that, where in 2010 Eastern European production reached 12.6 mbd compared to 6.7 mbd for North America. Latin American countries, specifically Mexico, started production in 1920, followed by Venezuela in 1930, with a production share equal to 16.2 percent of world production. After World War II, the Middle East emerged as an important producing region. Middle Eastern oil producers produced more than 25 percent of world output by 1960. Indonesia was the largest oil producer in the Asia-Pacific area; its production was mainly for export and constituted around 4.5 percent of world production in 1960 and increased around 10 percent by 2010. African output, starting with very small quantities in 1920, became significant after the expansion of Algerian production. With the output of Libya and Nigeria, African production totaled more than 13 percent of world production in 1970 and maintained almost the same percent in 2010.

As far as oil production compared with production of other forms of energy is concerned, Table 1.2 shows that between 1960 and 2010 the pattern of primary energy production changed between different forms of energy.

The share of oil in world energy production reached its maximum in 1970 with more than 60 percent. This was caused by the decrease in coal production in major parts of the world. In the 1990s, however, the share of oil production declined to less than 40 percent as a result of its replacement by other forms of energy such as coal.

TABLE 1.2

World Primary Energy Production in Percent Share (Energy Mix in Production), 1960–2010 Year

			Y	ear		
Energy Source	1960	1970	1980	1990	2000	2010
Oil	54.53	60.19	46.45	39.40	39.00	38.50
Natural gas	22.28	25.62	18.41	20.51	21.50	21.70
Coal ^a	20.36	11.56	26.18	28.07	28.1	28.20
Hydroelectric power	02.82	02.46	06.35	06.58	06.00	06.50
Nuclear power	0.01	00.17	02.60	05.43	05.40	05.10
Total	100	100	100	100	100	100

^a Commercial solid fuels only (i.e., bituminous coal and hard coal, lignite and brown coal). Sources: Annual Energy Review 1988, U.S. Energy Information Administration, Washington, DC, with permission; Jenkins, Gilbert, Oil Economist's Handbook, 4th ed., Elsevier Applied Science, New York, 1986, with permission; Energy Statistics Yearbook, 1982, UN Statistics Division, New York, with permission; Basic Petroleum Data Book, Vol. VIII, No. 3, American Petroleum Institute, Washington, DC, September 1988, with permission; data for 2000 and 2010 are based on author's estimation.



FIGURE 1.3 (See Color Insert)

Production of refined products (mb/d), 1960–2010. (From OPEC Annual Statistical Bulletin, Vienna, 2010. With permission.)

1.3.2 Production of Refined Oil Products

Production of refined oil products is determined by several factors, mainly the supply of crude oil, refining capacity, oil prices, environmental regulations, and world economic growth. However, adequate supplies of oil products depend on the optimal allocation between types of crude oils and an increasing supply of natural gas, which affect the sources of refinery feedstock. The type of crude oil with respect to its density and sulfur contents determines refining yields and refining processes. For example, light crude with lesser density will yield a higher proportion of more valuable final oil products such as gasoline and will require a less complex refining process.

In 2010 world production of refined oil products was estimated to be around 82.3 mbd with an average annual increase of 6 percent from 1960 to 2010. Figure 1.3 presents world production of refined products by regions over the period 1960 to 2010 compared to OPEC's share. The United States and Western Europe produce almost half of the world total. On the other hand, Latin America, Eastern Europe, and mainly Russia produced around 10 percent each of world production in 2010 over the same period. The Middle East, which is the largest producer of crude oil, however, produces almost 8 percent of world production of refined products. This indicates that refineries were located near the consuming areas rather than producing areas, except in the case of the United States and Europe which are both major producers and consumers. Refineries located near the markets are known as market refineries, in contrast to resource refineries which are located near producing oil fields. Refinery locations can be determined by certain factors including product types and transport costs as well as political considerations.

Table 1.3 gives the distribution of world refining capacity by regions for the period 1965 to 2010. Before 1965 the United States led the world in refining capacity with a share of 67 percent of total world refining capacity.

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World Oil Refinery Capacity by Region (1000 b/d)^a, 1965-2010

			Yea	ar			Change 2010	2010 Share
Regions	1965	1970	1980	1990	2000	2010	over 2009	of Total
North America	11,896	14,818	21,982	19,195	19,937	20,971	-0.7%	22.8%
South and Central America	3562	4808	7251	6009	6271	6707	0.3%	7.3%
Europe and Eurasia	13,194	21,968	32,136	27,909	25,399	24,516	-1.0%	26.7%
Middle East	1702	2466	3528	5260	6491	7911	1.2%	8.6%
Africa	560	697	2102	2804	2897	3292	8.9%	3.6%
Asia Pacific	3600	6588	12,364	13,470	21,478	28,394	2.7%	30.9%
World	34,514	51,344	79,363	74,647	82,473	91,791	0.8%	100.0%
Of which:								
OECD	22,852	34,591	49,833	40,542	44,761	45,124	-1.3%	49.2%
Non-OECD	11,662	16,754	29,530	34,105	37,712	46,667	3.0%	50.8%
European Union ^b	8413	15,119	20,669	15,239	15,456	15,240	-2.0%	16.6%
Former Soviet Union	4518	6105	10,190	11,217	8574	8033	0.9%	8.8%
^a Atmospheric distillation	capacity on	a calendar-da	ay basis.					

Note: Annual changes and shares of total are calculated using thousand barrels daily figures. ^b Excludes Lithuania prior to 1985 and Slovenia prior to 1991.

Source: BP Statistical Review of World Energy, London, 2011. With permission.



FIGURE 1.4

World oil refining capacity (mbd), 2000–2020. From Alsahlawi M., Global Refining Industry Outlook, 2nd Annual Global Refining Technology Forum, 19 March 2012, Doha, Qatar.

This trend has continued with a decreasing rate as the refining industry has been directed toward markets of refined oil products. Supporting this argument, refining capacity in Western Europe and Asia has increased substantially, and their shares in world refining capacity have increased to 27 percent and 30 percent in 2010, respectively. Over the last four decades the world refining capacity rose to reach more than 91 mbd in 2010 from 51 mb/d in 1970. The major contributors to this rise were Europe and the Far East. The Middle East as a major crude oil producer has increased its refining capacity from 1.7 mbd in 1965 to 7.9 mbd in 2010. However, against expectations, its share in world refining capacity has not increased substantially. As a matter of fact, it did not exceed 8.6 percent in 2010.

In forecasting refined oil products supply, it is assumed that world economic growth rates would be 2 percent per year from 2010 to 2015 and 3 percent from 2015 to 2020. Oil prices, however, would be around \$180 over the period 2010 to 2015 and would be in the range of \$100 to \$110 during the years 2015 to 2020. Figure 1.4 shows the future projections of world oil refining capacity for the years 2000 through 2020.

1.4 World Oil Demand

1.4.1 Crude Oil Consumption

Table 1.4 presents the percentage of world energy consumption by energy sources for the years 1960, 1970, 1980, 1990, 2000, and 2010. Coal was the dominant source until 1970 when it was replaced by oil. The displacement

)	lear		
Energy Source	1960	1970	1980	1990	2000	2010
Oil	34.21	46.06	43.55	38.70	39.10	38.00
Natural gas	14.00	20.01	18.95	20.20	21.00	21.90
Coal ^a	49.84	33.79	29.11	29.5	28.60	28.30
Hydroelectric power	01.93	02.08	06.00	06.50	06.20	06.04
Nuclear power	0.006	00.13	02.39	05.10	5.10	5.4
Total	100	100	100	100	100	100

TABLE 1.4

World Primary Energy Consumption in Percent Share (Energy Mix in Consumption), 1960–2010

^a Commercial solid fuels only (i.e., bituminous coal and hard coal, lignite and brown coal).

Sources: Annual Energy Review 1988, U.S. Energy Information Administration, Washington, DC, with permission; Jenkins, Gilbert, Oil Economist's Handbook, 4th ed., Elsevier Applied Science, New York, 1986, with permission; BP Statistical Review of World Energy, London, June 1988, with permission; author estimations for 2000 and 2010.

of coal by oil continued, but because of high oil prices and the implementation of energy conservation and environmental policies in oil-consuming countries, the share of oil has reduced to 38.7 percent in 1990. Consumption of other forms of energy has also increased, which enhanced the lower consumption of oil. Yet compared to other energy sources, oil is still the most important source for energy consumption, with the highest share.

As shown in Table 1.5, total world oil consumption has increased from 22.9 mb/d in 1960 to 87.4 mb/d in 2010. Percentage-wise, this can be translated to an average 6 percent increase per year.

TABLE 1.5

			Y	ear			2010 Share
Region	1960 ^a	1970	1980	1990	2000	2010	of Total
North America	11.70	16.59	20.00	20.32	23.57	23.45	25.8%
Latin America	1.20	20.87	33.22	36.23	48.55	6.10	7%
Western Europe	4.10	13.20	16.28	16.20	15.50	14.12	18%
Eastern Europe and Russia ^b	3.33	5.02	8.62	8.20	4.30	5.40	5%
Middle East	0.70	1.16	2.04	3.60	5.12	7.82	8.9%
Africa	0.30	0.72	1.37	1.94	2.44	3.29	3.9%
Asia and Pacific	2.00	6.65	10.48	13.82	21.13	27.24	31.5%
Total	22.93	45.41	61.12	66.50	76.60	87.38	100%

Share of World Crude Oil Consumption by Region from 1960 to 2010

^a Author's estimation.

^b It is calculated by subtracting the consumption of Western European countries from the available total consumption of Europe.

Source: BP Statistical Review of World Energy, London, 2011. With permission.

The United States alone had more than 55 percent of world oil consumption in 1960, which means that it was the largest oil consumer in the world. However, the U.S. share of world oil consumption has been declining over the years to around 26 percent in 2010 in the face of increasing consumption from other regions such as Europe and the Far East. The consumption share of the first group has increased from less than 10 percent to 18 percent, while the consumption share of the second group, including Japan, has increased from 7.4 percent to 31.5 percent over the same period. Western Europe's share of world oil consumption reached its maximum in the mid-1970s up to 27 percent and started to decrease afterward as a result of substituting oil by other types of energy and applying oil conservation measures.

1.4.2 Consumption of Refined Oil Products

Free world consumption of refined oil products is found to be equal to 2373.6 million tons in 1988. As indicated in Table 1.6, it has increased by 81.5 percent from 1965 to 1988. Gasoline, middle distillates, and fuel oil are the major products, which represent around 83 percent of total refined oil products consumed.

The United States is the largest consumer of oil products. It alone consumes more than a third of total free world consumption. Its consumption share, however, was higher before 1965. This increase in consumption with a declining rate is due to an increase in consumption by other regions, such as Western Europe and Japan. Western Europe has raised its consumption of refined oil products by 54 percent from 1965 to 1988. Japan's consumption has also increased by almost threefold for the same period.

Table 1.5 indicates that oil product consumption by industrialized countries is very high compared with developing countries or crude oil producers. The consumption share of the industrial world constitutes about 84 percent of total world consumption in 1965. Their consumption, however, has risen in absolute terms, but the share relative to world consumption declined to 72 percent in 1988.

The demand outlook for refined oil products for the period 2000 to 2020 is presented in Figure 1.5. This outlook is based on the same assumptions of economic growth and oil prices that led to the projections of world oil products capacity. The forecast of oil product consumption shows different growth rates, ranging from 0.5 percent for the United States to 1.5 percent for Asia Pacific and the Middle East.

1.4.3 Natural Gas Reserves, Production and Consumption

As indicated in Figure 1.6, natural gas proven reserves increased from 19 trillion cubic meters in 1960 to about 192 trillion cubic meters in 2010. OPEC's share of natural gas world reserves increased from 38 percent in 1960 to 49 percent in 2010. Although natural gas has been known for many centuries, its commercial use is quite recent. Even today, natural gas is an important and relatively clean fossil fuel, but its use is constrained by the capital costs

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			Yea	rr			Change 2010	2010 Share
Region	1965	1970	1980	1990	2000	2010	over 2009	of Total
North America								
Light distillates	5584	6975	8148	8782	10090	10,949	1.0%	46.8%
Middle distillates	3166	4099	5116	5496	6809	6548	4.2%	28.0%
Fuel oil	1773	2464	2908	1796	1491	826	3.1%	3.5%
Others	2404	3055	3835	4242	5184	5095	1.5%	21.8%
Total North America	12,927	16,593	20,008	20,316	23,574	23,418	2.1%	100.0%
Of which: United States								
Light distillates	5108	6336	7077	7651	8813	9305	0.5%	48.6%
Middle distillates	2704	3484	4270	4757	5852	5449	4.0%	28.5%
Fuel oil	1508	2087	2416	1224	893	547	7.7%	2.9%
Others	2202	2802	3299	3357	4143	3848	2.2%	20.1%
Total United States	11,522	14,710	17,062	16,988	19,701	19,148	2.0%	100.0%
South and Central America								
Light distillates	400	566	886	1135	1443	1835	4.9%	30.1%
Middle distillates	418	560	973	1109	1628	2203	7.7%	36.1%
Fuel oil	554	640	875	672	728	757	-1.7%	12.4%
Others	234	316	587	708	1056	1309	3.5%	21.4%
Total South and Central America	1606	2082	3322	3623	4855	6104	4.7%	100.0%
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Oil Products Consumption by	Region (100	0 bd), 1965–	2010					
			Yea	L			Change 2010	2010 Share
Region	1965	1970	1980	1990	2000	2010	over 2009	of Total
Europe								
Light distillates	1636	2678	3682	4338	4309	3401	-1.3%	22.4%
Middle distillates	2510	4473	5470	5626	6760	7663	1.6%	50.5%
Fuel oil	2951	4458	4223	2456	1843	1230	~0.0~	8.1%
Others	1134	1766	2238	2452	2927	2867	-2.9%	18.9%
Total Europe	8231	13,375	15,612	14,871	15,838	15,161	-0.9%	100.0%
Former Soviet Union								
Light distillates	600	872	1634	1639	942	1287	4.3%	29.6%
Middle distillates	1247	1812	3214	2490	1003	1293	6.8%	29.7%
Fuel oil	1026	1491	2511	2662	726	390	-2.3%	9.0%
Others	441	652	979	1584	1072	1379	5.3%	31.7%
Total Former Soviet Union	3314	4826	8338	8376	3743	4349	4.7%	100.0%
Middle East								
Light distillates	162	193	354	552	949	1783	5.8%	22.8%
Middle distillates	316	402	786	1235	1640	2387	3.2%	30.5%
Fuel oil	304	369	706	1090	1397	2035	5.3%	26.0%
Others	172	194	199	683	1036	1616	7.7%	20.7%
Total Middle East	954	1158	2044	3559	5021	7821	5.2%	100.0%

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Africa								
Light distillates	118	174	330	478	576	802	3.8%	24.4%
Middle distillates	211	299	576	763	1030	1486	3.8%	45.2%
Fuel oil	144	159	290	376	403	447	4.1%	13.6%
Others	55	87	176	327	430	555	-0.7%	16.9%
Total Africa	527	720	1371	1943	2439	3291	3.0%	100.0%
Asia Pacific								
Light distillates	632	1232	1997	3130	5842	8326	5.7%	30.6%
Middle distillates	778	1601	3151	4990	7795	9836	6.2%	36.1%
Fuel oil	1377	2925	3830	3385	3571	3163	2.0%	11.6%
Others	437	894	1504	2310	3927	5912	5.1%	21.7%
Total Asia Pacific	3224	6652	10,482	13,814	21,135	27,237	5.3%	100.0%
World								
Light distillates	8532	11,818	15,397	18,414	24,150	28,383	2.8%	32.5%
Middle distillates	7398	11,434	16,072	19,218	26,665	31,417	4.4%	36.0%
Fuel oil	7103	11,015	12,831	9775	10,159	8849	0.7%	10.1%
Others	4436	6312	8539	10,721	15,631	18,734	2.7%	21.4%
Total World	27,469	40,580	52,839	58,127	76,605	87,382	3.1%	100.0%
<i>Notes</i> : Annual changes and sl gasolines and light dis marine bunkers). <i>Fuel</i>	hares of total are ca tillate feedstock (L) <i>oil</i> includes marine	lculated usin DF). <i>Middle di</i> to bunkers and	ig thousand istillates cor l crude oil	l barrels da nsist of jet a used direct	uly figures. Ind heating Iy as fuel. (Light distillat kerosenes, ar <i>Others</i> consist	es consist of av nd gas and dies of refinery gas	iation and motor sel oils (including s, liquefied petro-
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World Oil and Gas Supply and Demand

leum gas (LPG), solvents, petroleum coke, lubricants, bitumen, wax, other refined products, and refinery fuel and loss.

Source: BP Statistical Review of World Energy, London, 2011. With permission.



FIGURE 1.5

World demand for oil products (mbd), 2000–2020. From Alsahlawi M., Global Refining Industry Outlook, 2nd Annual Global Refining Technology Forum, 19 March 2012, Doha, Qatar.

required in production, transportation, and regasification. In addition to the financial costs, the difficulty of moving gas to a fragmented market induced flaring natural gas where it is produced. However, liquefied natural gas (LNG) is starting to play a major role in the natural gas industry and facilitates the supply of gas to different markets. The basic structure of the natural gas industry consists of exploration, production, transportation, processing, and distribution. The production levels reflect the supply, and consumption represents the demand. Generally the balance between supply and demand indicates market price for natural gas with some differences due to different regional markets. Table 1.7 presents natural gas production for the period from 1960 to 2010. In the 1970s, North America was the major producer of natural gas, but Russia and Eastern Europe surpassed North



FIGURE 1.6 (See Color Insert)

Proven gas reserves (trillion cm), 1960–2010. (From OPEC Annual Statistical Bulletin, Vienna, 2011. With permission.)

			Year			Change 2010	2010 Share
Region	1970	1980	1990	2000	2010	over 2009	of Total
North America	663.0	649.9	640.0	763.7	826.1	3.0%	26.0%
South and Central America	18.1	34.0	58.1	100.2	161.2	6.2%	5.0%
Europe and Eurasia	281.9	618.5	961.2	938.9	1043.1	7.6%	32.6%
Middle East	19.9	37.5	101.3	208.1	460.7	13.2%	14.4%
Africa	2.8	24.0	68.8	130.3	209.0	4.9%	6.5%
Asia Pacific	15.7	70.2	150.9	272.1	493.2	10.5%	15.4%
World	1001.5	1434.3	1980.4	2413.4	3193.3	7.3%	100.0%
Of which:							
OECD	746.0	852.3	851.7	1073.9	1159.8	2.9%	36.5%
Non-OECD	255.5	582.0	1128.7	1339.5	2033.5	9.9%	63.5%
European Union ^b	101.7	197.2	185.1	231.9	174.9	2.0%	5.5%
Former Soviet Union	179.1	393.8	747.7	654.2	757.9	9.7%	23.7%
^a Excluding gas flared or recy	rcled.						

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^b Excludes Estonia, Latvia, and Lithuania prior to 1985 and Slovenia prior to 1991.

derived directly from tonnes of oil equivalent using an average conversion factor, they do not necessarily equate with gas volumes expressed in specific national terms. Annual changes and shares of total are calculated in million tonnes oil Notes: As far as possible, the data above represent standard cubic meters measured at 15°C and 1013 millibar (mbar). As they are equivalent figures.

Source: BP Statistical Review of World Energy, London, June 2011. With permission.

World Oil and Gas Supply and Demand

Natural Gas: Production^a (billion cubic meters), 1970–2010

TABLE 1.7

			Yea	r			Change 2010	2010 Share of
Region	1965	1970	1980	1990	2000	2010	over 2009	Total
North America	44.9	62.4	61.6	61.7	76.7	81.9	4.7%	26.9%
South and Central America	1.4	1.8	3.4	5.6	9.3	14.3	9.3%	4.7%
Europe and Eurasia	15.1	28.1	61.5	94.3	95.1	110.0	7.2%	35.8%
Africa	0.1	0.2	1.9	3.8	5.6	10.2	6.1%	3.3%
Asia Pacific	0.6	1.4	6.9	15.0	28.1	54.9	12.6%	17.9%
World	63.0	95.5	138.7	189.7	232.7	306.6	7.4%	100.0%
Of which:								
OECD	47.4	71.3	87.7	96.8	130.8	149.6	6.4%	48.9%
Non-OECD	15.5	24.2	51.0	92.8	101.9	157.0	8.4%	51.1%
European Union ^a	3.8	10.5	26.2	31.6	42.5	47.7	7.4%	15.5%
Former Soviet Union	11.2	17.5	34.8	62.2	50.5	57.7	6.8%	18.8%
^a Excludes Estonia, Latvia, and Notes: Annual changes and sha figures and the world pr	d Lithuania pi ares of total ar roduction stat	rior to 1985 and e calculated in istics is due to	Slovenia prior million tonnes variations in st	r to 1991. of oil equival ocks at storae	ent figures. Tl e facilities and	ne difference b	etween these worl	d consumption th unavoidable
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TABLE 1.8Natural Gas: Consumption (billion cm/d)

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Source: BP Statistical Review of World Energy, London, June 2011. With permission.

American production levels by the 1990s. The Middle Eastern and Asia Pacific countries emerged as important suppliers by the end of the 20th century with more LNG production capacities from Iran, Qatar, and Indonesia. Recently, Australia was expected to emerge as the global leader in LNG production by holding abundant reserves and a significant majority of upcoming projects.

In an overview of gas consumption patterns, Table 1.8 shows daily consumption by region. North America and Europe including Russia are the main consumers of natural gas worldwide. In 2010, their shares of total world consumption were 26.9 percent and 35.8 percent, respectively. However, natural gas consumption in non-OECD countries increased by 8.4 percent in 2010 over 2009 and accounts for 51.1 percent of total world consumption in 2010.

1.5 Summary

The production and consumption patterns of oil and natural gas over the past 50 years have been reviewed. In the energy mix, oil and gas will remain the main forms of energy in the future, despite the economic and technological factors. Traditionally, the United States has been the major oil and gas producer and consumer. Oil and gas from the Middle East and the Arabian Gulf in particular have noticeably increased over the years. From the consumption side, new emerging economies such as China and India show high oil and gas consumption rates in recent years.

2

Structure of the Oil and Gas Industry

M.A. Al-Sahlawi

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In this chapter, the structure of the oil and gas industry is analyzed, showing the oil and gas industry moving from exploration and development through production, transportation to crude oil refining and processing, then marketing. Historical reviews of the involved market structures and pricing mechanisms are provided to show how prices are arrived at in this complex industry.

2.1 Petroleum Industry Stages

The oil industry, like any industry, develops its products through different stages, as shown in Figure 2.1, but with more complexity than most industries. The main sectors of the oil industry are reviewed here to provide an overview of the operating elements and cost structure of each stage and to lay the groundwork for market structure analysis. The main stages in oil are exploration and development, production, refining, transportation, and marketing. Transportation is discussed in Chapter 20.


FIGURE 2.1

Petroleum industry stages from exploration to marketing.

2.1.1 Exploration and Development

Exploration for oil and gas begins with several kinds of geological and geophysical surveys. Seismic surveys have turned out to be the most useful. However, exploration and reservoir development remain a challenging stage in the petroleum industry in terms of economics and technology. This stage requires more integrated seismic programs, advanced data analysis systems, and sophisticated operational techniques. Examples of new technologies in

Company	1980	1985	1990	1995	2000	2005	2010
BP	7,409	9,617	9,844	8,380	11,171	14,149	23,016
Upstream	5,018	6,656	5,592	5,261	6,853	10,398	17,753
Downstream	1,964	2,079	3,271	2,989	3,959	2,859	4,029
ExxonMobil	11,565	13,525	11,988	12,862	11,168	17,699	32,226
Upstream	6,974	9,167	6,273	6,986	6,973	14,470	27,319
Downstream	2,830	2,924	4,504	4,724	4,086	3,149	4,720
Total	n.a.	1,679	3,933	2,544	7,677	13,928	21,573
Upstream	n.a.	1,206	1,172	1,294	5,191	10,091	17,510
Downstream	n.a.	305	2,470	1,196	2,217	3,600	3,956
Royal Dutch/ Shell	7,959	7,334	9,360	10,965	6,209	15,916	26,940
Upstream	4,974	5,021	3,736	4,477	2,292	4,770	4,523
Downstream	2,498	2,042	4,875	6,163	2,292	4,770	4,523
Chevron	6,674	6,859	7,679	7,928	9,520	11,063	21,755
Upstream	4,273	4,902	4,243	4,651	6,251	8,301	18,904
Downstream	1,302	1,201	3,097	3,075	2,226	2,301	2,552
Total Majors	33,603	39,014	42,804	42,709	45,745	72,755	125,510
Upstream	21,244	26,952	21,016	22,669	28,559	54,206	103,809
Downstream	8,594	8,551	18,226	18,142	14,855	16,666	19,780

TABLE 2.1

Capital and Exploration Expenditures of the Major Oil Companies (Million U.S. Dollars), 1980–2010^a

^a Capital and exploration expenditures include upstream, downstream, and other business corporate.

Note: BP and Amoco merged to create BP Amoco in December 1995 (name changed to BP in 2002). Exxon and Mobil merged to create ExxonMobil in November 1999. Total/Fina and Elf Agvitane merged to TotalFina Elf in February 2000 (name changed to Total in May 2003). Chevron and Texaco merged to Chevron Texaco in October 2001 (name changed to Chevron in May 2005). Upstream: exploration/production; downstream: refining, marketing, transportation, chemicals, and other downstreams.

Source: OPEC Annual Statistical Bulletin, Vienna, 2012 (based on oil companies' annual reports). With permission.

exploration and production (E&P) are 3-D and 4-D seismic imaging, basin modeling, remote sensing integration, and slim-hole drilling. These technical improvements are aimed at reducing the costs of E&P and increasing efficiency with less environmental impact.

Drilling a test well is the necessary next step, to ensure the presence of oil. Drilling methods vary from one area to another. Rotary drilling is more popular in the West; triple drilling is generally used in the former Soviet Union. Drilling is a very expensive operation. Table 2.1 gives the capital and exploration expenditures by major oil companies in 1980, 1985, 1990, 1995, 2000, 2005, and 2010.

The cost of exploration and production by major oil companies has increased over the last three decades by more than 400 percent. This is due mainly to expansion of the oil exploration and production activities beyond the traditional areas to new regions such as Africa and Asia Pacific. In addition to the monopolistic nature of the oil industry, capital and exploration expenditure has increased as a result of the high price of new technologies and the shortage of skilled human resources. Given its high production level and number of wells drilled, the United States accounts for more than 30 percent of world capital and exploratory expenditure. Its cost per well drilled was estimated to be \$2 million in 2006, while the cost in Western Europe is almost 10 times higher because of offshore drilling.

One of the reasons drilling is expensive is that in addition to drilling a test well, more confirmation wells have to be drilled near the discovery well to confirm the amount of oil present. Development comes next, when commercial discovery is demonstrated. The process of development consists first in identifying the field based on its geological structure, then drilling development wells, and then establishing gathering systems and other necessary facilities.

From a market structure point of view, oil prices are directly related to the cost of exploration and development. However, rising oil prices since the 1970s stimulated more investment in exploration, even in relatively high-cost areas such as the North Sea and Alaska. This can be seen in Table 2.2, which shows total world exploratory well completions compared to Organization of the Petroleum Exporting Countries (OPEC). The number of wells has

Country	1980	1985	1990	1995	2000	2005	2010
Algeria	249	40	80	95	137	198	258
Angola	24	59	60	60	40	45	118
Ecuador	29	22	38	72	48	131	176
Iran	25	50	24	67	150	183	186
Iraq	67	60	113	10	14	15	71
Kuwait	36	12	7	45	138	67	185
Libya	192	65	98	88	109	115	200
Nigeria	114	64	80	119	85	95	94
Qatar	57	13	23	30	66	62	35
Saudi Arabia	223	96	98	187	257	373	386
United Arab Emirates	109	208	75	112	87	109	146
Venezuela	819	373	236	550	691	1,281	890
Total OPEC	1,998	1,063	932	1,455	1,862	2,690	2,820
World ^b	84,192	91,654	50,880	52,242	60,095	97,430	97,140

Wells Completed in OPEC Member Countries and in the World, 1980–2010^a

^a Includes development and exploration wells.

^b Excluding Eastern Europe.

Source: OPEC Annual Statistical Bulletin, Vienna, 2012. With permission.

TABLE 2.2

increased by 30 percent over the period 1980 to 2010. However, as oil prices decline, the total number of exploratory well completions begins to fall.

2.1.2 Production

It is hard to separate production from exploration and development from the operating point of view as well as from the point of view of cost structure. After a field has been tested commercially, oil production begins. Normally for new fields, oil comes to the surface by natural drilling force as long as the well's surface pressure is less than the pressure in the reservoir. The source of this self-driving force is water or gas that is contained in the reservoir, or both. However, this natural flow will decline as the well gets older and cumulative production increases. Thus, secondary recovery methods such as water and gas injections and late tertiary recovery are applied.

The main objective is to maximize utilization of the oil reservoir. More advanced techniques have been applied in planning oil extraction, such as 3-D visualization modeling. Enhanced oil recovery (EOR) has become a challenging task in order to increase oil recovery rate and reduce the trapped hydrocarbons in the reservoir. Figure 2.2 illustrates the overall oil and gas production process.

The production process starts from the well head to metering, storage, and export through gathering, separation, and gas compression, including several facilities in addition to the utility systems of providing water, air,



FIGURE 2.2 Typical oil and gas production process.

and energy. For onshore wells, there is a gathering network connected to a gas oil separation plant (GOSP). In the case of offshore wells, the facilities depend on field size and water depth. Different types of platforms are used which range from fixed self-contained platforms to gravity platforms and floating, tension leg platforms where oil and gas can be extracted at the seabed.

Oil pricing should, in principle, be determined by the relationship of oil supply and demand. Given the curve of demand, the supply curve will be drawn based on production cost. The exploration and development stage is part of the overall production operation in the oil industry. Thus, production has large fixed costs (FCs), which are mainly the costs of exploration and development, and variable costs (VCs), which are mainly operating costs. In the oil industry, variable costs tend to be much lower than fixed costs. This would imply that long-run average total cost (LATC) declines with increasing production. This characterizes "natural monopoly" industries, and is true for the giant oil fields such as those of the Middle East.

In the oil industry, cost structure alone does not determine market structure. Market size and government policies are important. There are also a number of small fields that tend to have higher operating costs and cause LATC to rise. Given the demand, and assuming perfect competition, a simple model of the world oil market in the short run can be presented when the world oil supply is drawn as the upper part of a marginal cost curve above the Average Variable Cost (AVC), as shown in Figure 2.3. The intersection of this supply curve with the demand curve will give the equilibrium market oil price (P) and quantity (Q).



FIGURE 2.3 A static model of the world oil market.

The supply of crude oil is generally regarded as inelastic with respect to price, which means that oil production responds slowly to price changes. Price elasticity of supply is defined as the percentage change in quantity supplied as a result of one percentage change in price. Accordingly, the supply is considered elastic when price elasticity of supply is greater than one and inelastic when the elasticity is less than one. The inelasticity of oil supply is caused mainly by the high fixed costs involved in the production stage. The short-run supply elasticity with respect to oil price is estimated to be 0.02. However, high oil price will encourage development of high-cost regions.

On the other hand, the demand for crude oil is a derived demand, which depends on refined oil products demand. In general, demand for refined oil products is inelastic with respect to price; therefore, demand for crude becomes inelastic especially in the short run, as shown in Figure 2.3. Price elasticity of demand is defined as the percentage change in quantity demanded resulting from one percentage change in price.

2.1.3 Refining

Refining is a series of physical and chemical processes that convert crude oil into many finished oil products. Physical processes are those that depend on atmospheric and vacuum distillations. For the chemical processes, many different methods have been used, such as thermal and catalytic cracking, hydrogen catalytic process, polymerization, alkylation, and isomerization. After that, blending and treatment processes make oil products ready for use. Table 2.3 shows historical development of refining processes with its purposes.

The number of operating refineries in different parts of the world has increased in total from 646 in 1989 to 700 in 2008 refineries located in 120 countries over the last decade. Table 2.4 presents the number of world oil refineries by region for the years 1984, 1989, 1996, 2003, 2008. Most refineries are located near oil product markets.

The oil industry, including refining, used to be controlled by the major oil companies. This structure, however, has changed since the 1970s when oil-producing countries took over most oil operations except refining, which is still generally under the oil companies' control or as joint ventures with national oil companies. Figure 2.4 shows the distribution capacity by regions at the end of 2011.

The refining industry is located mostly where oil is consumed. For example, the Western Hemisphere and Western Europe have 21.4 and 24.6 percent of world refining capacity, respectively. The share of Asia and Pacific world refining capacity is growing, and reached 29.1 percent in 2011.

Most of the world refineries operate on average at about 85 percent of refined capacity. This may sound high, but in fact indicates a problem of excess capacity, which has tended to prevent oil producers from increasing their refining capacities or building new refineries. During the 1980s and 1990s, the excess capacity was clearly high; a result of the drop in world oil

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Type of Petroleum Refining Processes

Year	Process Name	Purpose	By-Products, etc.
1862	Atmospheric distillation	Produce kerosene	Naphtha, tar, etc.
1870	Vacuum distillation	Lubricants (original), cracking feedstocks (1930s)	Asphalt, residual-coker feedstocks
1913	Thermal cracking	Increase gasoline	Residual bunker fuel
1916	Sweetening	Reduces sulfur and odor	Sulfur
1930	Thermal refining	Improve octane number	Residual
1932	Hydrogenations	Remove sulfur	Sulfur
1932	Coking	Produce gasoline base stocks	Coke
1933	Solvent extraction	Improve lubricant viscosity index	Aromatics
1935	Solvent dewaxing	Improve pour point	Waxes
1935	Catalytic polymerization	Improve gasoline yield and octane number	Petrochemical feedstock
1937	Catalytic cracking	Higher octane gasoline	Petrochemical feedstock
1939	Visbreaking	Reduce viscosity	Increased distillate, tar
1940	Alkylation	Increase gasoline octane and yield	High-octane aviation gasoline
1940	Isomerization	Produce alkylation feedstock	Naphtha
1942	Fluid catalytic cracking	Increase gasoline yield and octane	Petrochemical feedstocks
1950	Deasphalting	Increase cracking feedstock	Asphalt
1952	Catalytic reforming	Convert low-quality naphtha	Aromatics
1954	Hydrodesulfurization	Remove sulfur	Sulfur
1956	Inhibitor sweetening	Remove mercaptan	Disulfides
1957	Catalytic isomerization	Convert to molecules with high octane number	Alkylation feedstocks
1960	Hydro cracking	Improve quality and reduce sulfur	Alkylation feedstocks
1974	Catalytic dewaxing	Improve pour point	Wax
1975	Residual hydro cracking	Increase gasoline yield from residual	Heavy residuals

Source: U.S. Department of Labor, Occupational Safety and Health Administration, Chapter 2, Petroleum Refining Processes. With permission.

demand to less than 55 million barrels per day in the mid-1980s. This caused some refineries to close, but with recent growth in oil demand, capacity utilization increased and ultimately the refining margin improved.

There are plans to expand refining capacities and build new refineries, especially in emerging markets. However, recent upgrading activities will be

TABLE 2.4

Number of World Operating Refineries

Country	1984 ^a	1989 ^a	1996 ^b	2003ь	2008ь
North America	259	241	184	168	164
Western and Central Europe	124	107	147	134	125
Asia and Pacific	109	104	170	188	189
Latin America	79	78	78	75	75
Middle East	46	51	41	44	47
Africa	48	65	45	44	43
Eastern Europe	n.a.	n.a.	42	46	45
Central Asia	n.a.	n.a.	11	12	12
World	663	646	718	711	700

^a From Shell Briefing Service, N6, 1989. With permission.

^b From Ivica Billege, NAFTA 60 (97-8) 401-403, MSc, 2009. With permission.

reinforced by a growing shortage of basic refining capacity in major consuming areas. This shortage is likely to improve the profitability of source-based refineries in producing countries.

Catalytic cracking and catalytic reforming have been the oil industry's basic upgrading processes ever since World War II. In general, they have been adequate to meet moderate levels of unleaded gasoline octane ratings. The United States leads the world in basic upgrading capacity and has the



FIGURE 2.4

Distribution of refining capacity by region (million barrels/day) as of the end of 2011. (From *BP Statistical Review of World Energy*, London, 2012. With permission.)

largest recent increases in the most sophisticated refining capability, such as alkylation and aromatic isomerization. U.S. refining technology has transferred to major oil producers in the form of joint ventures to build new refineries near either the sources or the markets.

2.1.4 Oil Marketing

Marketing is the most complex sector of the world oil industry. Oil marketing may be viewed in many ways, including wholesale markets, in which large sales are made to sellers of small volumes, and versus retail markets, which sell to final consumers. Sometimes sales are on a spot or single-sale basis and sometimes on short- or long-term contracts. There are also differences between crude oil and oil product markets.

Historically, until the early 1970s, crude oil was marketed through integrated company systems. Sometimes producing/refining companies would exchange oil, usually on a barrel-for-barrel basis. Some crude oil, around 5 percent, was sold by producers through spot markets to refiners. This situation is now changed. Most of the world's equity crude has disappeared from the market, largely as a result of nationalization of the assets of most major oil producers. Although the traditional concessionary companies have retained preferred access to crude oil through service contracts, the amount of oil traded on a spot basis has increased to above 50 percent. This trend has been accentuated by the development of formal oil exchange markets such as New York, London, Hong Kong, and Dubai. In recent years oil exchange markets allow for movement away from physical crude oil markets to paper markets which consist of futures options and forwards. Such movement has increased market speculation and price volatility rather than the fundamentals of the supply and demand forces.

Marketing was relatively simple for oil products in the past. There were essentially three main products: motor gasoline, heating oil, and heavy oil. Motor gasoline markets were, and remain, the most fragmented among the world's oil products. In the United States, which consumes about half of the world's gasoline supply, private service stations tend to be the main marketing distributors. In the rest of the world, major private or government companies own the outlets. However, company- or government-owned service stations tend not to compete on a price basis, but on advertising and locational advantages.

For the middle distillates, mainly heating oil, diesel fuel, and aviation jet fuel, the situation is more complex. For heating oil, competition is less among suppliers, which implies less emphasis on advertising and brand identification. Diesel fuel sale, however, is mostly for trucks and other heavy equipment such as railroad engines, construction equipment, and marine diesel engines. Because sales tend to be in larger volume than for motor gasoline, marketing relies on price differentials. Aviation fuel tends to be an especially profitable marketing area. This is due to the large volume involved and requirements for high-quality product. Heavy fuel oil is mainly used for electric power generation. It is always sold on a wholesale basis, often under long-term contracts, with prices related to the prices of coal and natural gas.

Oil product pricing generally depends on crude oil price and the quality of crude in terms of sulfur content and density. The high quality of crude yields higher-value products which increases the refinery margins given the refinery process and configuration. However, beyond supply and demand, product pricing is affected by the degree of market competition, the way oil products are traded in the financial markets, and the governments' regulations.

2.2 Oil and Gas Market Structures

Here we provide a general review of the industrial structure of world oil and gas markets to explain the forces that shape the oil and gas industry and influence pricing.

2.2.1 Structure of Oil Industry

Before World War I, the world oil market was dominated by four major international oil companies: Shell, Standard Oil, Nobel, and Rothschild. The latter two companies were in Russia and were liquidated as private companies by the 1917 Russian Revolution. Another major company, founded by the British government, was the Anglo-Persian Company (now British Petroleum). In the 1920s, the oil market was essentially controlled by these three companies. In the 1930s, new major oil companies developed as offshoots of the old Standard Oil Company. They were Gulf, Texaco, Standard of California, Sohio, and Mobil. With these new entrants, the degree of competition in the world oil market increased, but only to a certain extent. In the 1940s and 1950s, the seven sisters (Gulf, Texaco, Standard of California, Sohio, Mobil, British Petroleum, Shell) had balanced the supply and demand mainly by market-sharing and joint producing agreements. To some extent these agreements distorted world market competition, resulting in an oligopoly market structure characterized by substantial differences between production cost and market price.

The deviation of oil prices from production costs allowed for vertical integration and controlling the market all the way from exploration to marketing. The share of the major oil companies in world oil production refining and marketing was about 60 percent. This concentration ratio, which indicates the degree of competition in the world oil market, has declined dramatically, especially in the production sector. This is due to the increased participation of oil-producing countries in production and to the evolution of the national oil companies. Table 2.5 shows the shares of the largest seven international oil companies in different sectors of the oil industry over the last three decades. It is clear that the market power of the majors has reduced,

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Shares of the Largest International Oil Companies in Oil Industry Activities (Thousand Barrels per Day), 1990–2010

		% of		% of		% of
Company Activity	1990	World	2000	World	2010	World
BP						
Crude oil reserves ^a	7,313	0.26	6,508	0.20	5,559	0.14
Crude oil product	2,104	3.56	1,928	2.93	2,374	3.40
Crude oil processed	2,783	4.39	2,928	4.04	2,426	2.95
Refined products sold	3,837	5.77	5,859	7.65	5,927	6.82
ExxonMobil						
Crude oil reserves ^a	10,181	0.37	12,171	0.36	11,673	0.29
Crude oil product	2,491	4.22	2,553	3.88	2,422	3.47
Crude oil processed	4,952	7.80	5,692	7.79	5,253	6.38
Refined products sold	7,283	10.94	7,993	10.44	6,414	7.40
Total						
Crude oil reserves ^a	2,731	0.10	6,960	0.21	5,987	0.15
Crude oil product	411	0.69	1,433	2.17	1,340	1.92
Crude oil processed	832	1.31	2,411	3.33	2,009	2.44
Refined products sold	1,487	2.23	3,695	4.83	3,776	4.34
Royal Dutch/Shell						
Crude oil reserves ^a	10,107	0.37	6,907	0.21	5,179	0.13
Crude oil product	1,820	3.10	2,274	3.45	1,619	2.32
Crude oil processed	3,218	5.17	2,923	4.03	3,197	3.88
Refined products sold	4,962	7.46	5,574	7.28	6,460	7.43
Chevron						
Crude oil reserves ^a	5,909	0.21	8,519	0.25	4,270	0.10
Crude oil product	1,745	2.95	1,997	3.03	1,923	2.75
Crude oil processed	3,285	5.19	2,540	3.51	1,894	2.30
Refined products sold	4,680	7.03	5,188	6.78	3,113	3.58
Total Majors						
Crude oil reserves ^a	36,241	1.31	41,065	1.23	32,668	0.80
Crude oil product	8,571	14.51	10,185	15.46	9,678	13.85
Crude oil processed	15,070	23.79	16,494	22.70	14,779	17.96
Refined products sold	21,961	33.00	28,309	40.00	25,690	29.56
Total World						
Crude oil reserves ^a	2,759,106	100.0	3,330,425	100.0	4,076,000	100.0
Crude oil product	59,077	100.0	65,863	100.0	69,840	100.0
Crude oil processed	63,336	100.0	72,439	100.0	82,305	100.0
Refined products sold	66,539	100.0	76,537	100.0	86,900	100.0

^a Reserves are one in million barrels as of year-end.

Notes: BP and Amoco merged to BPAmoco in December 1998 (names changed to BP in 2002). Exxon and Mobil merged to ExxonMobil in November 1999. Total Fina and Elf Aquitaine merged to TotalFina Elf in February 2000 (name changed to Total in May 2003). Chevron and Texaco merged to ChevronTexaco in October 2001 (name changed to Chevron in May 2005).

Source: Compiled from OPEC Annual Statistical Bulletin, Vienna, 2012. With permission.

yet they still control around 25 percent of world oil refining and about 35 percent of marketing activity.

Oil producers' participation in the oil industry began in 1960 when OPEC was established. OPEC was formed by five major oil-exporting countries: Iran, Iraq, Kuwait, Saudi Arabia, and Venezuela. Qatar joined in 1961 and was followed by Indonesia and Libya in 1962. By 1979, the number of OPEC members totaled 13, including the United Arab Emirates which joined in 1967, Algeria in 1969, Nigeria in 1971, Ecuador in 1973, and Gabon in 1975. From December 1992 to October 2007, Ecuador suspended its memberships, while Gabon terminated its membership in 1995. By January 2009, Indonesia suspended its membership, and Angola joined in the same year. Currently, OPEC has a total of 12 member countries.

In the 1960s, several national oil companies of the producing nations were established, although in most cases without significant market power. However, in the 1970s to 1990s, national oil companies gained more power over the oil industry and extended even more to refining and marketing.

2.2.2 Crude Oil Pricing

Before World War II, the world oil market (mainly the United States, the world's largest producer, consumer, and a net exporter) was controlled by the major oil companies. Thus, the single basing-point price system was applied. Under this system the price is quoted only for the point of delivery. It equaled the f.o.b. price at the base, which was the U.S. coast of the Gulf of Mexico, plus transport and insurance costs to its destination. This system tended to prevent competition and lower prices. After the war and the emergence of new suppliers from the Middle East, the price structure changed to a dual basing point system. The second basing point was the Arabian Gulf. By this system Middle Eastern oil was priced based on f.o.b. prices from the Arabian Gulf, which were agreed upon by the company and producing governments as equal to f.o.b. U.S. Gulf parity prices plus the transport cost from the Arabian Gulf to destination. This was about equivalent to the U.S. Gulf price plus the transport cost from some point near Malta in the Mediterranean. With the increase in demand for Middle Eastern crude oil, especially in Western Europe, oil companies moved the "parity point" westward to London, then to New York, in order to maintain low competitive prices among the various producer countries exporting to Europe.

In the 1950s, real oil prices tended to decline, except for the years 1956 to 1957 when the Suez Canal was closed. In this atmosphere of price volatility, OPEC was formed in 1960. The two-basing-point system was abandoned, at least for crude oil. Yet OPEC did not succeed in stabilizing oil prices and preventing them from falling. OPEC's first effective attempt to raise prices in line with demand growth and inflation took place in February 1971, when the Tehran agreement was signed. As a result of this agreement, the price of 40° API Arabian Gulf crude increased by 33 ç/bbl plus 2 q/bbl in settlement of freight disparities.

Until that time, oil prices were posted by the major integrated oil companies. However, these were realized or market selling prices, which were determined by giving discounts of posted prices. The posted prices, however, served as a basis for oil-producing governments to calculate their royalty interests and income taxes from the oil companies operating in their countries. OPEC was able to seize the initiative, and official OPEC prices emerged.

After October 1973 (34° API)—as a marker crude—Saudi Arabia light became OPEC's official reference crude oil. OPEC set a price for Saudi Arabia light and let member governments set their own prices for the different crudes reflecting the different locational, physical, and chemical characteristics of each crude.

Supply disruption from the Arabian Gulf because of the Iran Revolution in 1979–1980 caused spot oil prices to jump to over \$40/bbl and official prices of OPEC's crudes to rise accordingly. In the early 1980s, spot and future markets were widely used at the same time. In those conditions spot and official prices declined (Table 2.6). This led OPEC members to follow market-based pricing systems. In February 1987, OPEC effectively terminated market-priced sales, and oil prices tended to stabilize around a target price of \$18/bbl as OPEC's reference basket price or oil-pricing benchmark.

The current basket is composed of 12 crudes: Algerian Sahara blend, Angola's Girassol, Ecuador's Oriente, Iran's heavy, Iraq's Basra light, Kuwait's export, Libya's Essider, Nigeria's Bonny light, Qatar's Marine, Saudi Arabia's Arab light, United Arab Emirates' Murban, and Venezuela's Merey. Theoretically, this is a return to fixed price system. However, in March 2000, the reference basket price was set at a range of \$22 to \$28/bbl to reflect market forces. The market-based pricing system was enhanced by the development of derivative instruments such as forwards, futures options, and swaps. Trading oil became either through paper markets, where deals are futures and swaps ,or physical oil trading through spot market and long-term contracts, where the price of a cargo in long-term contracts is linked to spot price. Such financial and electronic revolutions caused massive market speculation and more fluctuation in oil prices. The period from 1990 to 2010 witnessed a wide variation in the exchange value of the U.S. dollar, which increased the volatility of oil prices. Beyond oil supply and demand, the effect of the U.S. dollar as the oil pricing currency and the increased role of paper trading of oil have substantially changed the structure of the oil market.

2.2.3 Oil Products Pricing

In principle and to a large extent, prices for oil products can be regarded as reflecting the economic value added in the chain from production to marketing. Product prices are linked to crude prices through the full-barrel refiner's margin, which can be considered as value added in the processing of crude oil.

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	OPEC								
	Reference		UK	Norway	Mexico	Colombia C.			
Year	Basket	Oman	Brent	Ekotisk	Isthmos	Limon	USA WTI	Russia Vrals	China Daging
1990	22.26	20.89	23.61	25.78	22.05	22.46	24.46	22.54	23.03
1991	18.62	17.14	20.06	20.34	18.48	19.18	21.55	19.03	19.03
1992	18.44	17.72	19.33	19.55	18.05	18.56	20.58	18.10	19.01
1993	16.33	15.77	17.00	17.18	15.93	16.56	18.45	15.39	17.54
1994	15.53	15.12	15.80	15.89	15.36	16.05	17.19	15.19	15.59
1995	16.86	16.34	17.01	17.19	16.73	17.58	18.42	16.62	17.59
1996	20.29	19.12	20.70	21.06	20.65	21.62	22.20	20.11	20.81
1997	18.68	18.52	19.06	19.22	18.26	19.28	20.56	18.28	19.33
1998	12.28	12.06	12.71	12.71	12.08	12.69	14.36	11.79	12.34
1999	17.48	17.27	17.91	17.93	17.29	17.95	19.30	17.27	17.76
2000	27.60	26.50	28.44	28.42	27.80	28.57	30.37	26.58	28.74
2001	23.12	22.75	24.46	24.39	22.22	22.82	26.00	22.97	24.78
2002	24.36	23.94	25.03	24.88	24.12	23.55	26.13	23.80	25.45
2003	28.10	27.14	28.81	28.89	28.25	27.78	31.09	27.02	29.52
2004	36.05	34.35	38.23	38.18	37.01	35.53	91.44	34.47	36.72
2005	50.64	50.48	54.44	54.48	50.35	51.31	56.51	50.79	52.65
2006	61.08	62.59	65.16	65.30	59.87	62.72	66.04	61.37	63.33
2007	69.08	68.86	72.55	73.20	67.55	69.67	72.29	69.55	71.30
2008	94.45	94.51	97.37	99.40	95.22	98.94	100.00	94.87	96.72
2009	66.06	62.06	61.68	62.67	60.85	64.38	61.88	61.22	59.97
2010	77.45	78.34	77.60	80.52	77.86	79.75	79.42	78.39	78.45
2011	105.5	106.63	14.36	112.74	105.64	108.30	41.99	109.19	110.46
Note: Spot cru Source: OPE	ude oil prices a C Annual Static	tre nominal] stical Bulletin	prices. t (oil prices), ⁷	Vienna, 2012. (Bi	ased on Platt's,	Direct communicat	tion, Reuters, a	nd Secretariat's a	ssessments). With
pern	nission.		•						

For perhaps two decades after World War II, the major refining companies "posted" prices for the major fuel products at which they were willing to sell to any wholesaler or distributor. With stable crude prices, the major product prices remained stable for long periods of time except for the summer/ winter fluctuations in heating oil and motor gasoline prices.

There have been at least three markets for oil products: spot sales, term contracts, and wholesale transactions. In oil surplus situations, which characterize the world oil market except for supply crises of 1972–1974 and 1978–1981, spot sales tend to command the lowest markup over crude oil costs and wholesale transactions the highest. Term contract sales, however, justify some discounting for outlet security, and therefore fall between wholesale and spot sales. Nonetheless, the existence of a spot market generated the need for some kind of reporting service. Platt's price assessment service developed to fill this need. Table 2.7 lists spot prices of oil products in major markets over the period from 1980 to 2010.

Individual product value-added in refining varies among different products. It also varies among market areas and over time. These variations require refiners to be competitive even during periods of supply surplus. More recently, competitive pressures on product prices generated different kinds of discounts from official crude selling prices.

Government regulations and different oil product pricing schemes in different countries are affecting the oil products market. As far as the market structure is concerned, spot and futures markets have been widely developed for oil product trading and transactions.

2.2.4 Structure of the Gas Industry

Natural gas is a mixture of hydrocarbon gases with almost 85 percent methane. It comes from oil wells as associated gas and non-associated when it is produced from gas wells. Before transporting natural gas to consumers, it has to be processed to separate all included hydrocarbons and obtain pure methane or dry gas.

For its economic and environmental advantages, natural gas has gained preference, and its share in the energy mix has been increasing since the 1980s. Consumption of natural gas has increased by more than 2 percent a year over the last 30 years. Since 2009, world demand for natural gas has declined as a result of the global economic crisis, but the supply of natural gas has increased due to capacity growth of liquefied natural gas (LNG) and new development of unconventional gases such as shale gas, especially in the United States.

As the global LNG demand dropped substantially, the supply of LNG improved as new LNG liquefaction plants opened in Qatar, Yemen, Indonesia, and Russia. During 2009 to 2016, approximately 233.5 million tonne per annum (MMTPA) of new LNG liquefaction capacity are expected to come on-stream. The major contributors to this increase are Australia,

,									
		U.S. Gulf			Singapore			Rotterdam	
Year	Gasoline Reg. UNL. 87	Gasoil 0.2% sulfur	Fuel oil 30% sulfur	Gasoline Prem. 15 G/L	Gasoil ^a 0.5% sulfur	Fuel oil 380 CST	Gasoline ^b Prem. 15 G/L	Gasoil ^a 0.2% sulfur	Fuel oil 3.5% sulfur
1980	41.43	38.94	24.88	34.53	42.37	26.59	42.91	40.41	25.06
1983	32.12	30.98	22.52	26.99	30.33	23.19	31.66	31.65	22.93
1990	29.65	27.63	14.64	32.58	29.32	15.56	31.79	28.08	14.94
1995	21.31	19.95	13.77	22.10	21.64	13.99	20.60	20.17	14.14
2000	35.10	34.04	20.77	32.55	32.46	23.04	35.16	33.76	20.47
2005	67.25	70.71	36.42	62.10	68.66	38.38	62.58	70.91	34.59
2010	89.54	89.55	70.45	90.05	90.35	72.28	92.35	90.85	70.55
From From	2005, gas oil with (2005, unleaded 95	0.05% sulfur. RON 0.05% sul	fur.						

Spot Prices of Oil Products in Major Markets (U.S. Dollar/Barrel), 1980-2010

TABLE 2.7

Note: Prices in case of U.S. Gulf and Singapore apply to cargo lots and in case of Rotterdam to barges. Gasoline specification in Singapore is 97 Research Octane number with 0.15 gram lead per liter. Fuel oil viscosity is 380 centistokes with 3 to 5 percent sulfur. Gasoline specification for Rotterdam is 97-98 RON with 0.15 gram per liter lead.

Source: OPEC Annual Statistical Builtetin, Vienna, 2012. (Information gathered from Platts Oilgram, and Reuters.) With permission.

Country	Natural Gas Exports	Natural Gas Imports	LNG Exports	LNG Imports
Natural Gas Prod	ucers			
Russia	195	_		_
Canada	103	_		_
Norway	96	_	_	_
Netherlands	62	_		_
Qatar	58	_	39	_
Algeria	57	_	20	_
Turkmenistan	54	_	_	_
Indonesia	37	_	28	_
Malaysia	28	_	31	_
United States	28	—	—	—
Natural Gas Cons	umers			
United States	_	113		10
Japan	_	95		95
Germany	_	92		_
Italy	_	77	_	_
Ukraine	_	53		_
France	_	48	_	10
Spain	_	39		28
United Kingdom	_	37	_	_
Turkey	_	37	_	5
Korea	_	37	_	37

TABLE 2.8

Natural Gas and LNG Exports and Imports (Billion cm), 2008

Source: Natural Gas Information, International Energy Agency (IEA), Paris, 2009. With permission.

Iran, Nigeria, and Qatar. The excess of natural gas supplies in the world has led LNG spot prices to hit new lows. The drop in spot LNG prices has made buyers rethink long-term LNG contracts. Importers can now easily tap the global market for spot cargoes at lower prices than the long-term supply agreements.

Table 2.8 shows natural gas and LNG exports and imports by leading natural gas producers and consumers in 2008. The United States and Russia are the leading countries in natural gas production and consumption, while Japan and Korea the major importers of LNG.

In the United States, the gas industry has been regulated since the beginning of gas discovery. From time to time, such regulation created a supply shortage. However, in competitive markets, the price of natural gas reflects the interaction between the demand and supply, which are inelastic with respect to price in the short run. This market structure was enhanced by



*Prices are based on delivery at the Henry Hub in Louisiana, from the trading floor of NYMEX, the delivery month is the calendar month following the trade date.

FIGURE 2.5

Henry Hub Gulf Coast natural gas spot prices (dollar/MMBTV), mid-year 1998–2012. (Prices are based on delivery at the Henry Hub in Louisiana, from the trading floor of NYMEX. The delivery month is the calendar month following the trade date.) (From *Annual Energy Review* (several issues), Energy Information Administration, US Department of Energy, Washington DC. With permission.)

the drop in natural gas prices because of the decline in demand for natural gas as a result of the 1970s energy crisis and energy conservation policies. This allowed for direct deals between suppliers and buyers, which opened the door for natural gas spot markets. With more fluctuations in natural gas prices, the futures market for natural gas has developed. The New York Mercantile Exchange (NYMEX) became the trading floor for short- and long-term futures contracts. Spot prices reflect market conditions where prices for the contracts are based on delivery at the Henry Hub in Louisiana. Figure 2.5 presents natural gas spot prices and short-term futures contracts.

Natural gas prices outside the United States are basically linked to oil prices through long-term contracts. In the United Kingdom, the market is liberalized and subject to arbitrage between spot gas traded on the national balancing point (NBP) and continental European long-term contracts. In the continental European market, gas contracts are based on oil products prices. For Asia, natural gas prices are based on government-regulated levels with spot pricing for LNG. There are price differentials between these natural gas markets attributed to different market conditions and price formation whether spot prices or long-term gas contracts are related to oil prices. The financial crisis of 2008 caused a fall in spot gas prices as a result of a drop in gas consumption. The two major spot markets, Henry Hub and NBP, recorded lows at ranges of \$3.5/MBTU and \$4/MBTU, respectively, between 2009 and 2012.

2.3 Summary and Conclusions

The main sectors of the oil industry have been reviewed and it has been shown that high oil prices stimulate more investment in exploration. The exploration and development stage have been shown to be part of the overall production operation in the oil industry. Because of the high fixed cost of exploration and development, the oil industry tends to be a decreasing cost industry. Crude oil has to go through refining processes to convert it into the useful finished product. Refining facilities are located mainly near the consuming areas.

Crude oil and oil products have in the past been marketed quite differently than they are today. With the increased fragmentation of the oil industry, crude oil marketing is becoming more like product marketing of the past. This has been encouraged by the emergence of official exchanges in major oil trading centers.

Until the early 1970s, the world oil market was controlled by the major international oil companies. Oil producers' participation in the world oil industry started with the formation of OPEC in 1960, and oil pricing mechanisms have changed accordingly. Oil prices were previously posted by the majors, but after 1973 official OPEC prices emerged. In the early 1980s, spot and future markets were widely used in the face of price volatility.

Natural gas consumption has increased over the past 30 years as a result of its economic and environmental advantages. The natural gas industry has been regulated in the United States except for periods when the demand is low, which allows for spot and futures market deals. Outside the United States, natural gas prices are linked to oil prices through long-term contracts.

3

Characteristics of Crude Oils and Properties of Petroleum Products

Saad Al-Omani

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The petroleum industry generally classifies crude oil by three criteria: the geographic location where it is produced (e.g., West Texas Intermediate, Brent, or Oman), its API gravity (an oil industry measure of density), and its sulfur content. This classification is important because it affects both the transportation costs to the refinery and the refining costs to meet sulfur standards imposed on fuels in the consuming countries. The largest volume products of the industry, on the other hand, are fuel oil and gasoline.

In this chapter, the composition of crude oils, their qualities, and the major factors included in determining their values are highlighted. Crude oil classification systems are covered as well. The major types of refined petroleum products produced and utilized and their economic importance are described.

3.1 Introduction

Petroleum or crude oil is a viscous brown-to-black liquid mixture. Historically, the word *petroleum* comes from two Latin words: *petra*, meaning "stone/ rock," and *oleum*, meaning "oil." In Arabic countries *crude oil* is called *alnaft*.

Generally we classify the crudes into three types, or families, based on their density: light, medium, and heavy. Crude oils with gravity >33 API are considered light crudes. Heavy crudes, those with gravity <28 API, tend to have more asphaltenes and are usually rich in aromatics (American Petroleum Institute, Case No. 800205-9, 2011). Examples of crude classifications with some crude names in the industry, along with their main characteristics, are presented in Table 3.1.

The quality of the oil is normally presented in a structured format called *crude assay*, which gives the various components of the crude along with the expected percentage of recovered products for each specific type of crude, which will determine the value of the crude, especially for traders and refiners. It is based on laboratory testing and experimental plant methods. It demonstrates the true boiling point (TBP) curve, specific density, API, sulfur content, sediment water, and other important properties. The price will be determined according to the crude oil quality, even if it is from the same crude family, as each formation will have its specific quality (e.g., light crudes will have different price structure based on the given crude assay).

Classification	API Range	Examples Crude Name, API
Light	>33	 Saudi super light, 39.5 Nigerian light, 36 North Sea Brent, 37
Medium	28–33	• Kuwait, 31 • Venezuela light, 30
Heavy	<28	• Saudi heavy, 28 • Venezuela heavy, 24

TABLE 3.1

Classification of Petroleum Crude

Typically, leading producers such as Saudi Arabia, Kuwait, and Iran will issue one comprehensive crude assay for all the light fields, as it would be logistically difficult to segregate the crudes from the same family, considering the required assets and capital investment needs.

The main constituent of crude oils is a hydrocarbon mixture with varying amounts of non-hydrocarbon compounds. Table 3.2 presents a summary of all hydrocarbon constituents found in crude oil as well as its associated gas. All hydrocarbon classes except alkenes are present in crude oils. Alkanes, cycloalkanes, and mono- and polynuclear aromatics have been identified in crude oils. The ratio of these classes, however, differs appreciably from one type of crude to another. Light hydrocarbon gases such as methane and ethane may be present in small amounts dissolved in the crude or in large amounts as in associated gas. Associated gas is mainly constituted of methane, ethane, and propane. This gas is a valuable raw material for many petrochemicals.

In addition to the hydrocarbon mixture, crude oils contain variable amounts of non-hydrocarbon compounds such as sulfur, nitrogen, vanadium, and oxygen compounds, and sometimes mercury is present. These compounds are sometimes referred to as impurities; they affect the handling, shipping, and processing of the crude as well as its market value.

Additional processing schemes may be required to reduce sulfur and nitrogen compounds in the intermediates and the products. A refiner would prefer to process a low-sulfur crude since this will reduce the cost of hydrogen required for hydrotreatment of the products. Crude oils also contain trace amounts of heavy metals in the form of organometallic compounds and some inorganic salts, mainly sodium chloride. Some of these heavy metals, such as vanadium and nickel, are poisonous to some processing catalysts and should be reduced to low levels. Crude oils having a high salt content should also be desalted before refining to reduce corrosion problems, as discussed in Chapter 18. The composition of crude oils, by major elements, is shown in Table 3.3.

At ambient temperatures, crude oil products may be light saturated hydrocarbon gases, such as methane, or ethane, unsaturated gases, such as

Characteristics of Crude	: Oils					
Tests	Test Methods	Arab Extra Light (BERRI)	Arab Light	Arab Medium	Arab Heavy	
Gravity, OAPI	ASTM D-287	38	33.8	31.2	28.1	
Ash, ppm	ASTM D-482	21	43	56	108	
Pour Point, OF	ASTM D-97	0	IJ	10	-10	
Hydrogen Sulfide, ppm	IP 103T	70	36	56	N11	
Sediment & Water, Vol %	ASTM D-96	Trace	0.1	Trace	Trace	
Sulfur, wt. %	ASTM D-129	1.17	1.75	2.48	2.83	
Viscosity, SUS @1000F	ASTM D-445	38	50	61.8	108.8	
Con. Carbon, Wt%	ASTM D-189	3.65	3.83	5.71	7.86	
Distillation Yield	ASTM D-285					
Vol. % @350OF			27.7	23.3	21.7	18
375OF			30.3	26.3	23.7	20
4000F			33.3	29.2	26	22
425OF			36.3	32	28.7	25
450OF			39.3	35	31	26
475OF			42.3	36.7	33.3	28
5000F			45.7	40	36	31
525OF			48.7	42	39	33
Source: Farahat, Mohamm	ad Ali, Ul Hasan, Mi	sbah, and Saleem, Mohamad, Di	istribution of Sulfu	r Compounds in Ara	b Crudes, SPE 99	583, 1981.

TABLE 3.2 Characteristics

44

Element	Percent by Weight (%)
Carbon	83–87
Hydrogen	11–14
Sulfur	$0.05-2.5^{a}$
Nitrogen	0.1–2ª
Oxygen	0–2ª
Minerals and salts	0–0.1
Courses Equalsot Mahammaa	Ali III Hasan Mishah and Salaam

TABLE 3.3

Composition of Petroleum Crude

Source: Farahat, Mohammad Ali, Ul Hasan, Misbah, and Saleem, Mohamad, Distribution of Sulfur Compounds in Arab Crudes, SPE 9583, 1981. With permission.

^a Regarded as impurities.

ethylene, propylene, or liquid products, such as naphtha or jet fuel, a semisolid, such as asphalt, or a solid, such as petroleum coke.

The major utilization of petroleum products globally is for fuel use and energy generation. However, a small fraction (about 7 percent) of a barrel of crude is used as a raw material for the production of chemicals. The current and future refining trend is toward chemical and petrochemical along with refining integration for various reasons to become a more profitable investment where the main products are olefins and aromatics. It has been estimated that the rate of return from a chemical refinery would be higher than that from a conventional refinery producing only fuels, even if the fuel products' prices are increased by 15 percent and the chemical prices decreased by 15 percent (John, 1977).

3.2 Crude Oils and Product Composition

The composite mixture forming crude oils and its products is a complex one. However, the compounds found in crude oils and its products generally belong to three broad classes: hydrocarbon compounds, non-hydrocarbon compounds, and metallic compounds, discussed below.

3.3 Hydrocarbons

The major constituents of most crude oils and their products are hydrocarbon compounds made of hydrogen and carbon only. These compounds belong to one of the following subclasses: alkanes, cycloalkanes, alkenes, and aromatics.



3.3.1 Alkanes (Paraffins)

Alkanes are saturated compounds having the general formula (C_nH_{2n+2}). The simplest hydrocarbon compound, methane, may be present in small amounts dissolved in the crude or may be produced during the refining process. Alkanes are relatively unreactive compounds in comparison to alkenes and aromatics. However, they are the main components of feeds used in olefin production. Alkanes may either be straight-chain or branched compounds. Branched hydrocarbons in the naphtha range are more valuable than the nonbranched (straight) isomers since they are useful for gasoline production because of their higher octane rating. Isomerization of the C₅/C₆ fraction is customarily applied to be enriched with branched isomers.

Straight-chain alkanes (*n*-alkanes) in the kerosene range are valuable compounds. They may be isolated, and those in the C_{12} - C_{14} range are used for the production of biodegradable detergents.

3.3.2 Cycloalkanes (Cycloparaffins, Naphthenes)

Cyclo- and bicycloalkanes are normally present in crude oils and its fractions in variable proportions. Cyclohexane, substituted cyclohexanes, and substituted cyclopentanes found in the naphtha range are important precursors for aromatic production via isomerization and dehydrogenation reactions. The presence of large amounts of these cyclic compounds in the naphtha range has its economic merits since the rate of aromatization of these compounds is much faster than from alkanes and branched alkanes. Catalytic reforming is a process for enriching the naphtha with aromatics and isoparaffins. Naphtha having a high percentage of naphthenes would be aromatized at a relatively lower temperature than low-naphthene naphtha. This will constitute an energy saving for the refiner (Matar, 1986).

3.3.3 Alkenes (Olefins)

Alkenes are unsaturated hydrocarbon compounds with the general formula (C_nH_{2n}) . These compounds are quite active, and react by addition to many simple reagents such as chlorine, hydrochloric acid, and water. The simplest alkene, ethylene, is an important monomer in petrochemical production. Light olefinic hydrocarbons are generally used for the production of many chemicals and polymers.

Alkenes are typically not present in crude oils, but they are produced during the processing of crude oils at high temperatures. Catalytic cracking is the main refining process that produces alkenes. However, due to the need for large amounts of light olefins for petrochemical use, they are produced by a noncatalytic steam cracking of ethane, propane, naphtha, gas oil, or residues. Light olefins are then separated and purified for chemical use. Butadiene is a by-product of hydrocarbon steam cracking. Butadiene is an active conjugated diolefin and is considered the most important monomer for synthetic rubber production. Olefins and diolefins generally react by addition.

Light olefins and diolefins may react and produce high-molecular-weight commercial polymers. For example, polyethylene is the most important thermoplastic, and polybutadiene is the most widely used synthetic rubber. High-molecular-weight olefins may be present in heavy petroleum fractions from cracking processes. The presence of large amounts of olefins in these fractions may be unfavorable because of their instability and tendency to polymerize and to get oxidized.

3.3.4 Aromatic Compounds

Aromatic compounds are normally present in crude oils and their products. However, only mononuclear aromatics in the range of C_6 – C_8 have gained commercial importance. The simplest aromatic compound, benzene (C_6H_6), is very reactive and is one of the basic raw materials for petrochemical production.

Aromatics in this range are not only important petrochemical feedstocks but are also valuable motor fuels. The main process for producing aromatics is catalytic reforming of naphtha. The product reformate is highly rich in C_6 - C_8 aromatics which increase the octane rating of the reformate.

Di- and polynuclear aromatic compounds are present in heavier petroleum fractions and residues. Asphaltenes, which are concentrated in heavy fuel oils and asphalt, are polynuclear aromatics of a complex structure. The molecular nature of asphaltenes has been extensively discussed. It has been confirmed by mass spectroscopic techniques that condensed-ring aromatic hydrocarbons and heterocyclic compounds are the major compounds of asphaltenes.

3.4 Non-Hydrocarbon Compounds

Many types of non-hydrocarbon compounds occur in crude oil and refinery streams. The most important are sulfur, nitrogen, and oxygen compounds. Metals are also present in trace amounts, mainly in the form of organometallics.

Sulfur present in crude oils is mainly in the form of organosulfur. These compounds are generally not acidic. Sour crudes are those containing a high percentage of hydrogen sulfide. However, many of the organic sulfur compounds are not thermally stable, and hydrogen sulfide is produced during crude processing. The content of sulfur in crude oil can vary from 0.05 to 10 percent. Sulfur compounds present in crude oil include thiols (RSH), sulfides (RSR), polysulfides (RSSR), thiophenic, and others (Hua, 2004). High-sulfur crudes are less in demand by refineries since an extra cost would be incurred for treating refinery stream from acidic hydrogen sulfide and mercaptans, which are corrosive. Experiments have proven that there can be over 10,000 species of sulfur compounds present in a middle distillation faction of crude oil (ranging from 150 to 450 C) (Hua, 2004).

High-sulfur heavy-petroleum feedstocks to catalytic cracking units should be hydrodesulfurized before being cracked to avoid poisoning of the catalyst. Naphtha feed to catalytic reformers is hydrotreated to reduce sulfur compounds to very low levels (1 ppm) to ensure a long life-cycle for the expensive platinum catalyst.

Nitrogen compounds in crude oils are usually low and are thermally more stable than sulfur compounds. Only trace amounts of nitrogen compounds are found in light streams. Nitrogen compounds in crudes are normally in the form of heterocyclic compounds such as pyridine and pyrrole. They may have a complex structure, as in porphyrins, which are usually found in heavy fuel oils and residues. Nitrogen compounds in petroleum have not yet proved to have any special commercial value. During hydrotreatment (hydrodesulfurization) of petroleum streams, hydrodenitrogenation takes place and the nitrogen content is reduced to acceptable levels in the feeds to catalytic processes.

Oxygen compounds in crude oils are more complex than sulfur compounds. However, oxygen compounds are not poisonous to processing catalysts. Most oxygen compounds are weakly acidic, such as phenol, cresylic acid, and naphthenic acids. The total acid content of many crudes is very low; however, it can reach a high value (3 percent), as in some California crudes (Tiratsoo, 1973; COQG, 2004). Nonacidic compounds such as esters, ketones, and amides are less abundant than acidic compounds. Many of these oxygen compounds, however, are concentrated in the heavier portion of the crude. Some of the oxygen compounds in the naphtha and kerosene fractions are of commercial value, such as naphthenic acids and cresylic acid. They are extracted by the use of an aqueous sodium hydroxide solution. However, hydrotreatment of these fractions reduces these weak acids to low levels to the extent that their extraction would not be feasible.

3.5 Metallic Compounds

Many metals are found in crude oils. Some of the more abundant are sodium, calcium, magnesium, aluminum, iron, vanadium, and nickel. These normally occur in the form of inorganic salts soluble in water, as in the case of sodium chloride, or in the form of organometallic compounds, as in the case of iron, vanadium, and nickel compounds, or in the form of salts of carboxylic acids (soaps), as in the case of calcium and magnesium. The organometallic compounds are usually concentrated in the heavier fractions and in crude oil residues. However, during processing crude oils, some of the volatile organometallic compounds are found in the lighter fractions. The presence of high concentrations of vanadium compounds in naphtha used in catalytic-reforming feeds produces permanent poisons. These feeds should be hydrotreated not only to reduce the metallic poisons but also to desulfurize and denitrogenate the sulfur and nitrogen compounds.

Hydrotreatment may also be used to reduce the metal content in heavy feeds to catalytic cracking. A lot of research is currently being invested to reduce the metal content in heavy products and resids.

3.6 Crude Oil Properties

Properties of crude oils vary appreciably and depend mainly on the origin of the crude. No two crudes would have exactly the same characteristics even if they are from the same family or field. However, many physical and chemical tests have been developed to help in establishing some general criteria to relate the crudes to one another and to help the refiner to select the best sequence of refining for maximum profitability as well as the energy users in the boilers. Other tests have also been developed to test for the quality of the products in relation to their utilization. Some of these tests are also important in controlling some of the harmful compounds that pollute the environment. Table 3.4 summarizes the most important factors or properties that affect the quality of crude oils.

The following are some of the most important tests used for the purpose of classifying the crude quality, along with a brief explanation about each specification.

3.6.1 Specific Gravity and API Gravity

The specific gravity of crude oils is sometimes used as a rough indication of the quality of the crude. A high specific gravity of a crude oil

Property	Comment	Range or Limitation
API gravity	°API = 141.5/sp.gravity 131.5	From 20 to 45; for Middle East oils it is around 35
Sulfur content	High-sulfur oils require extensive processing	Maximum of 0.5% by weight
Carbon residue	Related to the asphalt content in oil	A lower carbon residue means a higher quality oil
Salt content	Severe corrosion takes place if the salt content is high	Up to 15 lb/1000 bbl of oil
Nitrogen content	Not desirable if high; it causes poisoning of catalysts	Up to 0.25% by weight
Pour point	The temperature in °F at which an oil will no longer flow from a standard test tube	The lower the pour point, the lower the paraffin content of the oil

TABLE 3.4

Factors Affecting Quality of Crudes

would normally mean a lower percentage of the valuable light and middle fractions.

Specific gravity is also used to calculate the mass or weight of crude oils and its products. Usually crudes and products are first measured on a volume basis and then changed to the corresponding masses using the specific gravity.

Another useful measure to determine the relative weights of crude oils is the API gravity. The API gravity in degrees can be calculated mathematically using the following equation:

Higher API gravity indicates a lighter crude or product, while a low API gravity would mean heavy crude or product.

The following diagram illustrates the relation among the crude quality, specific gravity, and the API as well as the price fundamentals for light/ heavy crudes.



3.6.2 Ash Content

Ash content of a crude oil or a fuel oil is an indication of metals and salts present in the test sample. The ash is usually in the form of metal oxides, stable salts, and silicon oxides. The crude sample is usually burned in an atmosphere of air, and the ash is the material left unburned.

3.6.3 Salt Content

The salt content, expressed as sodium chloride, indicates the amount of salt dissolved in water. The water in crudes is found in variable amounts (normally small) in emulsion form. Salt in crudes and in heavier products may create serious corrosion problems, especially in the top-tower zone and the overhead condenser (due to hydrolysis of Cl⁻ to HCl).

3.6.4 Carbon Residue

Carbon residue is a rough indication of the asphaltic compounds and the materials that do not evaporate under conditions of the test, such as metals and silicon oxides. Carbon residue is more important for diesel fuels, lubricating oils, and heavy fuel oils since it may affect engine performance if high carbon deposition takes place.

3.6.5 Sulfur Content

Total sulfur content in crudes and heavy products is determined by burning a sample in a stream of air. The produced sulfur dioxide is further oxidized to sulfuric acid which is titrated with a standard alkali. Identification of individual sulfur compounds in crude oils or its products is not necessary.

The sulfur content of crude oils may be taken in consideration in addition to the specific gravity in determining their commercial value. It has been observed that denser crudes also have higher sulfur content (Hatch and Matar, 1981).

3.7 Crude Oil Classification

Although there is no specific method for classifying crude oils, it is useful for a refiner to establish some simple criteria by which the crude can be classified. A broad classification of crudes has been developed based on some simple physical and chemical properties. Crude oils are generally classified into three types depending on the relative amount of the hydrocarbon class that predominates in the mixture:

1. *Paraffinic* constituents are predominantly paraffinic hydrocarbons with a relatively lower percentage of aromatics and naphthenes.

- 2. *Naphthenics* contain a relatively higher ratio of cycloparaffins and a higher amount of asphalt than in paraffinic crudes.
- 3. *Asphaltics* contain a relatively large amount of fused aromatic rings and a high percentage of asphalt.

The price of petroleum crude oil is basically determined by four factors: American Petroleum Institute (API) gravity, sulfur content, viscosity, and capillarity. API gravity is the density, or specific gravity, of crude oil as measured against a common denominator, in this case an equal amount of water at 60°F. Sulfur content is described as the percentage of sulfur impurity in a sample of petroleum crude oil. Viscosity is a measure of the fluidity or resistance-to-flow characteristics of crude oil. Viscosity, like API gravity, is measured against water as a standard reference. Finally, capillarity is a measure of the adherence properties of crude oil.

Of these four factors that affect crude oil prices, only API gravity and sulfur content are of concern to the refiner. API gravity essentially tells the refiner how much crude oil he is getting for his money. The higher the API gravity, the greater the potential value of the crude oil. Therefore, the price of crude oil is adjusted through a differential to equalize the quality in density to the price.

Sulfur content tells the refiner the amount of basic impurity that is in the crude oil. Even though the API gravity figure might be attractively high, this might reflect a high sulfur content. The sulfur content for most crude oils falls between 1 and 2.5 percent, where 1 percent sulfur content is considered "sweet" crude and 2.5 percent sulfur content is considered "sour" crude.

At present there is no universally accepted differential similar to the quality differential system used for API gravity adjustments for adjusting the price of crude oil to the amount of sulfur content. Some refineries simply do not purchase "sour" crude (high sulfur content) at any price because they are not equipped to process them. Recently, however, the ecology push has forced refiners to reduce the amount of sulfur in their finished products, and the energy crisis has forced them to refine whatever they can get to go through their fractionating tower. As a result of this dilemma and the uncertainty of the future, the refineries have not been able to establish a price differential system for sulfur impurities.

While API gravity and sulfur content are major factors in determining the value of petroleum crude oil at the refinery, viscosity and capillarity are not of particular concern to the refiner. These latter properties describe the resistance properties of crude oil which affect the rate at which it will flow through pipelines, and they are therefore of primary importance to the producer and the transporter of petroleum crude oil in determining its value. All four factors, however, are used in establishing the value of oil at the wellhead. Wax is made of high-molecular-weight paraffins. A high wax content indicates the predominance of paraffinic compounds in the crude, and it would be classified as paraffinic.

3.7.1 Classification Systems

There are a number of classifications based on physical and chemical properties of crude oil.

3.7.1.1 Classification by Chemical Composition

Petroleum contains a large number of chemicals with different compositions depending on the location and natural processes involved. Petroleum composition (molecular type and weight) varies from one oil field to another, from one well to another in the same field, and even from one level to another in the same well.

A correlation index has been introduced to indicate the crude type or class. The following relationship between the mid-boiling point of the fraction and its specific gravity gives the correlation index, known as the *Bureau of Mines Correlation Index*:

$$BMCI = \frac{48640}{K} + (473.7d - 456.8)$$

where *K* is the mid-boiling point of a fraction in Kelvin degrees, and *d* is the specific gravity of the fraction at $60/60^{\circ}$ F.

Crudes can be classified as paraffinic, naphthenic (mixed), or asphaltic according to the calculated values using the above relationship. A zero value has been assumed for paraffins and 100 for aromatics.

3.7.1.2 Classification by Density

Density gravity (specific gravity) has been extensively applied to specify crude oils. It is a rough estimation of the quality of a crude oil. Density of petroleum fractions is expressed in terms of API gravity by the following relationship:

Another index used to indicate the crude type is the Watson characterization (UOP) factor. This also relates the mid-boiling point of the fraction in Kelvin degrees to the density.

Watson correlation factor =
$$\frac{(K^{1/3})}{d}$$

Property	Paraffin Base	Asphalt Base
API gravity	High	Low
Naphtha content	High	Low
Naphtha octane number	Low	High
Naphtha odor	Sweet	Sour
Kerosene smoking tendency	Low	High
Diesel-fuel knocking tendency	Low	High
Lube-oil pour point	High	Low
Lube-oil content	High	Low
Lube-oil viscosity index	High	Low

TABLE 3.5

General Properties of Crude Oils

A value higher than 10 indicates the predominance of paraffins, while a value lower than 10 indicates a predominance of aromatics. Properties of crude oils will thus vary according to their base type, as shown in Table 3.5.

The typical analysis of some crude is given in Table 3.6. The qualities are monitored as indicated earlier by the crude assay that is provided by the producers or seller to the consumer to ensure quality and sustainability.

This change in crude oil properties causes a corresponding change in the chemical composition of the products or the main fractions. Accordingly, the receivers or refiners modify operating parameters to handle and process the crude in order to get the desired products, as shown in Table 3.7 and as discussed in the following section.

Typical Analysis of Some Crude Oi

	Arab Heavy (Saudi Arabia)	Alamein (Egypt)
Specific gravity at 60/60°F API	28.0	33.4
Carbon residue (wt%)	6.8	4.9
Sulfur content (wt%)	2.8	0.86
Nitrogen content (wt%)	0.15	0.12
Pour point (°F)	-11.0	35.0
Ash content (ppm)	120.0	40.0
Iron (ppm)	1.0	0.0
Nickel (ppm)	9.0	0.0
Vanadium (ppm)	30.0	15.0
Paraffin wax content (wt%)	—	3.3

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Chemical Composition of Petroleum Fractions

	Boiling Range 50%	Parafi	in-Base Crude	(wt%)		Asphaltic-Ba	se Crude (wt%	•
Fraction	ASTM Distillation °F	Paraffin	Naphtha	Aromatic	Paraffin	Naphtha	Aromatic	Unsaturates
Gasoline	280	65	30	ъ	635	55	10	
Kerosene	450	60	30	10	25	50	25	
Gas oil	600	35	55	15		65	33	7
Heavy distillate	750	20	65	15		55	43	2

3.8 Crude Oil Products

Products from crude oils are diversified and may be produced either directly by distilling the oil in an atmospheric distillation unit where physical separation of different fractions takes place or by further processing one or more of these fractions or the residue from the atmospheric distillation in a differing and more complex processing unit. All time economics will play the major determining element in running the refinery and will determine where a specific stream will be utilized, processed, or blended. For example, the residual fuel oil from the atmospheric distillation may be utilized directly as a burner fuel or may be used as feed to a vacuum distillation unit, which is an optimum option, for producing more gas oil and lubricating oil base stocks, or alternatively may be introduced to a catalytic cracking unit for maximizing gasoline and middle distillates production, which will substantially improve the refinery economics and place it as a complex refinery configuration.

Fractions obtained from an atmospheric distillation unit are gases, naphtha, kerosene, or jet fuel and gas oil. The last three products are known as mid-distillates. The bottom product is residual fuel oil. For a refinery, maximizing fuel production is the main objective to provide a positive margin. The trend has changed toward integrating a fuel refinery with a chemical plant as one complex for maximum profit. An integrated fuel-chemical



FIGURE 3.1 (See Color Insert)

Semi-integrated chemical-fuel refinery.

refinery, as illustrated in Figure 3.1 may include, in addition to atmospheric and vacuum distillation units, a catalytic reforming unit with an aromatic extraction process, a catalytic cracking or hydrocracking unit to upgrade heavy fractions and residues, and possibly a steam cracking unit for olefin production. Many refineries may also include a visbreaking unit (a mild noncatalytic cracking unit) for improving the viscosity of heavy fuels, and one or more hydrotreating units for hydrodesulfurization of feeds to catalytic processes. The refining process is described in detail in Chapter 18. Table 3.8 provides a summary for the major refinery process units along with their feed and product streams. Some properties of petroleum products in relation to their end uses are described below.

3.8.1 Refinery Gases

The lightest materials from the crude distillation tower are light hydrocarbon gas mixtures from methane to butanes and some pentanes. These are further processed for separation of the propane-butane mixture for use as LPG. Propane LPG is used as a prime portable fuel in homes, transportation, and agriculture. It may be used or marketed alternatively as a feed to cracking units for olefin production. Recently, a new catalytic process based on LPG to aromatics in high selectivity has been proposed. The Cyclar process (BP-UOP) converts propane-butane or a blend to a high aromatic-enriched hydrocarbon

TABLE 3.8

Process	Feed	Major Products	Catalyst
Crude distillation	Raw crude Naphtha Kerosene Diesel Fuel oil	LPG	None
Vacuum	Fuel oil, distillation bottoms	VGO Fuels	None
Platformer/rheniformer	Naphtha	Platformate 93–95 octane	Yes
Continuous catalytic reforming (CCR)	Naphtha	Platformate 100+ octane	Yes
Fluidized catalytic cracking (FCC)	Vacuum gas oil (VGO)	Platformate	Yes
Hydrocracker (HCK)	Atmospheric residue	Naphtha Diesel	Yes
Hydro-de-sulfurization (HDS)	High sulfur diesel	Low sulfur diesel	Yes
Isomerization	C4/C5 paraffins	Gasoline blending component	Yes
Vis-breaker	Vacuum bottoms	Fuel oil blends	None
Asphalt oxidizer	Vacuum bottoms	Asphalt	None

The Major Refinery Process Units
mixture (Doolan and Pujado, 1989). However, the economic ramifications of this process versus catalytic reforming have yet to be evaluated.

Butanes, in addition to their use as fuels, are also utilized to adjust the vapor pressure of reformate gasoline. This adds to their economic value, since gasoline generally holds high sale prices. A chemical butane mixture may also be used as an alternative route to acetic acid production or may be dehydrogenated to butenes and to butadiene.

Refinery gases produced in large modern refineries include, in addition to the saturated light hydrocarbons mentioned above, unsaturated compounds ranging from ethylene to butylenes, which are valuable petrochemical feedstocks. These are mainly produced from different cracking processes, such as catalytic cracking, delayed and fluid coking, and viscosity breaking processes. Hydrogen is mainly produced from catalytic reforming processes. Hydrogen sulfide, which is produced from all refinery processes, and specifically from hydrotreating processes, is used for the production of sulfur. The following is a brief description of some refinery gases.

3.8.2 Hydrogen

Hydrogen is a valuable coproduct; with reformer gasoline, the main catalytic reforming reactions are hydrogen producing (dehydrogenation). Product hydrogen from catalytic reformers is not pure and is always mixed with methane and ethane. Another source of refinery hydrogen (small amount with other gases) is the catalytic cracking process. Refinery hydrogen is mainly utilized in hydrodesulfurization, hydrocracking, hydrodealkylation, and isomerization processes.

Excess hydrogen after refinery needs may be marketed to fertilizer firms for the production of ammonia. Chemical hydrogen, however, is produced from the steam reforming or partial oxidation of any carbonaceous material, including crude oils and their products. Methane is the preferred feedstock for the production of synthesis gas (a mixture of H_2 and CO). Hydrogen is a valuable raw material for ammonia and urea synthesis and with carbon monoxide for the production of methanol.

3.8.3 Hydrogen Sulfide

Although hydrogen sulfide is a by-product from refinery operations and its presence in petroleum products is harmful, a substantial amount of elemental sulfur is produced from it via the Claus reaction:

$$H_2S + 3/2 O_2 \rightarrow SO_2 + H_2O$$
$$SO_2 + 2H_2S \rightarrow 2S + 2H_2O$$

The main source of hydrogen sulfide is refinery gases, which should be treated to remove hydrogen sulfide. This may be done through either a physical adsorption or a chemical absorption process. The stripped and purified fuel gas is then burned as a fuel for the refinery or may be used to make additional hydrogen for process needs.

The major use of sulfur is the production of sulfuric acid. However, sulfur has many other diversified uses, such as vulcanization of rubber, as an additive for paving asphalt, as a dusting powder for roses, and in pharmaceuticals. Sulfur–asphalt blends produce excellent road surfaces which have double the strength of conventional pavement.

3.8.4 Liquid Products

Liquid products from conventional refining processes include light fractions such as naphtha and jet fuel, and middle distillates such as gas oil and lube oil base stocks. Heavier liquid products are residual fuel oils. Asphalt is considered a semisolid petroleum product.

3.8.4.1 Naphtha

Naphtha is the lightest side stream from an atmospheric distillation unit. Light naphtha (boiling range approximately 36 to 80°C) is added to the reformate to adjust its volatility. Light naphtha is not used as a feed to catalytic reformers since it has a high percentage of low-molecular-weight hydrocarbons that are not suitable for aromatization.

Heavy naphtha contains a heavier mixture of hydrocarbons (boiling range approximately 70 to 160°C) than light naphtha. Reforming heavy naphtha in a catalytic reformer is a process to change the molecular structure of C_6 - C_8 to aromatic hydrocarbons and to isomerize straight-chain molecules to the branched isomers. Aromatic hydrocarbons and branched straight-chain alkanes have high octane ratings. Depending on the crude oil base, naphtha rich in cyclohexanes and substituted cyclopentanes and cyclohexanes would be a more suitable feed to catalytic reformers than naphtha rich in paraffinic compounds. This is attributed to the ease of dehydrogenation of naphthenes rather than the dehydrocyclization of paraffins to naphthenes followed by dehydrogenation.

Heavy naphtha is also the preferred feedstock for olefin production in Europe. The price of naphtha will always depend on the demand for ethylene production and the availability of ethane as an alternative feed to cracker units, as well as the gasoline demand and market supply.

3.8.4.2 Gasoline

Straight-run naphtha is not suitable for direct use in motor gasoline engines because of its unfavorable knocking characteristics. The addition of antiknock additives increases the octane rating of the naphtha. A maximum limit has been designated for the lead additive due to its toxicity. In many parts of the world, the use of lead alkyls has been limited to very low levels or banned for use in new automobiles. Unleaded or no-lead gasolines are the major grades now used in the United States and Canada.

Marketable gasoline is usually a blend of light naphtha and a high-octane hydrocarbon mixture from any of the processes that produce high-octane hydrocarbons called reformate. The main processes used for this purpose are platforming, catalytic cracking, hydrocracking, alkylation of olefins, isomerization of C_5/C_6 naphtha cut, and polymerization of C_3-C_4 olefins. A mixture of one or more of the products in the gasoline range from these processes plus light naphtha makes the gasoline pool. The vapor pressure of the mixture is adjusted according to the specifications (summer grade, intermediate, or winter) by adding natural gasoline or pentanes. The two most important characteristics for marketable gasoline are the octane rating and the volatility.

The octane rating of a hydrocarbon is a measure of a property of the hydrocarbon to produce a knock when used in a gasoline internal combustion engine. This property depends on the molecular structure of the hydrocarbon. Straight-chain hydrocarbons in the gasoline range have a much lower knock characteristic in comparison to aromatic hydrocarbons. Branched alkanes have higher octane ratings compared to their straight-chain isomers. Since gasolines are a mixture of compounds from different hydrocarbon classes, the octane rating of the mixture is approximately the sum of octane ratings of the individual components according to their ratios.

Table 3.9 shows the octane ratings of some hydrocarbons and some oxygenated compounds. A substantial amount of research has been invested to use alcohols either as additives to the gasoline pool or as alternate motor fuels (Matar, 1982). Methyl and ethyl alcohols have high octane rating. However, some problems are still associated with the widespread use of alcohols or their blends as motor fuels. This subject has been reviewed thoroughly by Keller (1979).

Volatility of gasolines is a property related to cold startability of the engine in winter and to vapor lock problems generally encountered in hot driving conditions. Vapor lock is a set of engine operating difficulties attributable to an excess of extremely volatile constituents in the motor fuel or to high temperatures in the gasoline engine system. Fuel supply is interrupted by formation of bubbles of vapor in supply lines, pumps, or carburetor passages.

Two tests are generally used to indicate the volatility of gasolines: Reid vapor pressure and distillation. The Reid vapor pressure test (ASTM D-323) is the vapor pressure at 100°F of a gasoline sample placed in a bomb in which the liquid volume is one-fifth of the total volume. Although Reid vapor pressure is not the same as the true vapor pressure, it is a simple test for indicating vapor lock tendencies and the explosion and evaporation hazards. Normally Reid vapor pressure values are specified for gasolines used during winter and summer.

TABLE 3.9

Octane Ratings of Hydrocarbons and Some Oxygenated Compounds in the Gasoline Range

Compound	Boiling Point (°C)	Research Octane Number	Motor Octane Number
<i>n</i> -Pentane	36.1	61.7	61.9
2-Methylbutane	27.8	92.3	90.3
2,2-Dimethylbutane	50.0	91.8	93.4
2,3-Dimethylbutane	58.3	103.5	94.3
<i>n</i> -Hexane	68.9	24.8	26.0
2-Methylpentane	63.3	73.4	73.5
3-Methylpentane	60.0	74.5	74.3
<i>n</i> -Heptane	97.8	00.0	00.0
<i>n</i> -Octane	125.6	-19.0	-15.0
2,2,4-Trimethylpentane (isooctane)	99.4	100.0	100.0
Benzene	80.0		114.8
Toluene	110.6	120.1	103.5
Ethylbenzene	136.7	107.4	97.9
<i>O</i> -Oxylene	144.4	120.0	103.0
<i>m</i> -Xylene	139.4	145.0	124.0
<i>p</i> -Xylene	138.3	146.0	127.0
Reformate gasoline (100 RON)	0.0	100.0	88.
Methyl alcohol	0.0	112.0	92.0
Ethyl alcohol	0.0	110.0	90.0
Methyl-tertiary butyl-ether (MTBE)	0.0	118.0	0.0
Tertiary-amyl-methyl-ether (TAME)	0.0	112.0	0.0

The ASTM distillation is also used to indicate the volatility of hydrocarbons as well as a smooth supply of the fuel to the carburetor. The front end mixture is indicated by the initial boiling point (IBP) and the temperature at which 10 percent has been distilled. A few other tests are also required to qualify the gasoline for marketing, such as oxidation stability, sulfur content, and gum formation tendency.

3.8.4.3 Benzene, Toluenes, and Xylenes (BTX)

Another important objective for catalytically reforming naphtha is the production of aromatics. Benzene, toluene, ethylbenzene, and xylene mixtures are the important aromatics in the gasoline range. Benzene is the most important aromatic hydrocarbon for petrochemical production. Although benzene does not polymerize, many other monomers are derived from it, such as styrene for polystyrene production, caprolactam for nylon 6, and adipic acid for nylon 6/6. Benzene is also a precursor for many valuable chemicals and intermediates.

Toluene is also an important petrochemical feedstock. However, fewer chemicals are produced from toluene than from benzene. For this reason, a substantial amount of toluene is hydrodealkylated to benzene or disproportionated to benzene and xylenes.

Xylenes are produced as a mixture and their separation is not simple, due to the closeness of the boiling points. The most valuable xylene is the paraisomer, which is used for the production of terephthalic acid for polyester manufacture. Polyesters are the most widely used synthetic fibers.

3.8.4.4 Kerosene

Kerosene is the fraction heavier than naphtha (BP range approximately 150 to 290°C). It may be produced from atmospheric distillation or from hydrocracking units. Kerosene produced from catalytic cracking and delayed coking units contains unsaturated hydrocarbons which affect its stability. Hydrotreatment is used to saturate olefinic compounds and to hydrogenate sulfur and nitrogen compounds in kerosenes from cracking units.

Kerosenes have been extensively used in heating purposes. However, most of the kerosene produced in the United States and Europe is used for the production of jet fuels. Since all the important uses of kerosene involve burning under specified conditions, its physical and chemical properties are important. The types of hydrocarbons present, the sulfur content, and the corrosive sulfur compounds and amount of residue left after burning can affect the burning quality of kerosene. For example, aromatic hydrocarbons are known to be more smoke-forming than paraffinic hydrocarbons. The smoke point test and the flush point and thermal stability are normally done for kerosenes and jet fuels.

Kerosene is a clear, almost colorless liquid that does not stop flowing in cold weather except below -30° C. Its specific gravity ranges between 0.79 and 0.81. It has been used in the past as a fuel for lamps and is still used for that purpose in remote areas where electricity is expensive. It is also used as a burning fuel and for heating purposes. *n*-Paraffins in the range of C₁₂–C₁₄ may be extracted from kerosenes, especially those from paraffinic base crudes. These paraffins are used in the production of biodegradable detergents.

Jet fuels or aircraft turbine fuels are hydrocarbon mixtures in the kerosene range. Due to their use in engines at much lower temperatures, the freezing point of these fuels is important. The aromatic content of jet fuels is also important since it affects the burning quality of the fuel. This is indicated by the smoke point, which is defined as the height of the flame in millimeters beyond which smoking takes places (ASTM D-1322). Military-type jet fuel (JP-4) is principally naphtha containing some kerosene. Accordingly, it has a wider boiling range than commercial jet fuels such as JP-A-1.

3.8.4.5 Gas Oils (Diesel)

Gas oil or diesel is a heavier petroleum product than kerosene. It may be produced from atmospheric distillation units, atmospheric gas oil (AGO), with a boiling range of approximately 190 to 325°C or from vacuum distillation (VGO) having a boiling range of approximately 299 to 500°C. Atmospheric gas oil has a relatively lower density and lower sulfur content than vacuum gas oil produced from the same origin. The aromatic content of gas oil ranges from about 10 percent for light gas oil to about 48 percent for vacuum gas oil. The sulfur content of gas oils differs appreciably, depending on the type of the crude used. However, as a general rule vacuum gas oil contains about three times as much sulfur as atmospheric gas oil. There can be over 10,000 species of sulfur compounds present in a middle distillation faction of crude oil (ranging from 150 to 450°C) (Hua, 2004). Nevertheless, stringent specifications on sulfur level have been placed in the market to preserve the environment, where some countries have mandate as low as 5 PPM. Accordingly, technology providers have worked diligently with refiners to explore all possible alternatives and innovative solutions to treat and remove the sulfur in diesel.

Gas oils are used as heating oils and fuels for metallurgical and other manufacturing purposes. Gas oils containing cracked products are unstable and produce more smoke and more deposits than atmospheric gas oil.

The specific gravity of gas oils usually falls within a wide range of approximately 0.81 to 0.86. This usually depends on the economics of the process, which is governed by the market demand for a lighter or a heavier product according to the end use.

Substantial amounts of gas oils are used as feeds to catalytic cracking units and to steam cracking for olefin production. Gas oil used for ethylene production normally requires a pretreatment step to remove the sulfur compounds.

3.8.4.6 Fuel Oil

Fuel oil is the term generally used for the residue produced from atmospheric distillation. However, this term is sometimes used for other fuels such as kerosene and gas oil (types No. 1 and No. 2 fuel oils, respectively). Fuel oil may also be produced from catalytic, hydrocracking, and steam cracking units. Fuel oils produced from cracking units are not very stable and produce more smoke and more deposits that may clog the burner orifices.

Pour point is a test that indicates the flow properties under specified conditions and is normally performed for fuel oils. Viscosity and sulfur content are important as well, since a very viscous fuel oil may require preheating before use, and a high sulfur content fuel oil will require additional cutter blending.

Due to the low sale value of fuel oil and need to refine the barrel for money making, it is normally upgraded by being either thermally cracked in a fluid or delayed coking unit or catalytically cracked or hydrocracked for producing more valuable light and middle distillates.

3.8.4.7 Lube Oil Base Stocks

Lube oil base stocks are special petroleum products used in lubrication. The base stocks produced from vacuum distillation units are further treated to remove asphaltenes (propane deasphalting), waxes (solvent extraction), and finally are mildly hydrotreated. The base stocks, which are of different grades depending basically on their viscosities, are mixed with special additives according to their final use. Lube oils, however, are beyond the scope of this chapter.

3.8.5 Solid Products

3.8.5.1 Asphalt

Asphalt is a dark brown solid or semisolid material. It is usually produced from vacuum distillation of topped crudes and from propane deasphalting of heavy lube oil base stocks. Petroleum asphalt is classified into several commercial grades of widely varying consistency. Consistency of asphalt is normally indicated by such tests as penetration, softening point, and viscosity. These tests determine the temperature required for the application and final hardness of the finished product. An important test for asphalt is the ductility, which measures the elongation or stretch suitable for paving purposes and ability to resist vibration.

The major use of asphalt is for paving. Asphalt is also used for roofing, as in paint asphalt.

Cutbacks are mixtures of hard asphalt that has been diluted with a light hydrocarbon liquid to permit its use as a liquid without excessive heating.

Air-blown asphalts are more resilient, have lower specific gravity, have lower ductility, and have lower tensile strength. They are used for roofing and insulation purposes.

3.8.5.2 Petroleum Coke

Petroleum coke is practically the bottom of a barrel of oil. The primary purpose of coking was to upgrade heavy residuals that cannot be used as feeds for catalytic cracking processes. Coke used to be considered a "no-value" product. In recent years, however, the emphasis is to produce a quality product for anodes used by the aluminum industry.

The quality of petroleum coke differs appreciably from one oil base to another. In the coking process, essentially all the nonvolatiles, metals, and sulfur in the feed remain in the coke. The quality of coke is essentially related to the quality of the feedstock.

Raw petroleum coke contains 5 to 30 percent volatile matter. For use as an industrial carbon, raw coke has to be calcined in rotary kilns at about 1200 to 1400°C. Specifications of petroleum coke for aluminum electrodes should be low in sulfur and metal content.

Class	Number of Products
Fuel gas	1
Liquefied gases	13
Gasolines	40
Motor	19
Aviation	9
Other (tractor, marine, etc.)	12
Gas turbine (jet) fuels	5
Kerosenes	10
Distillates (diesel fuels and light fuel oils)	27
Residual fuel oils	16
Lubricating oils	1156
White oils	100
Rust preventatives	65
Transformer and cable oils	12
Greases	271
Waxes	113
Asphalts	209
Cokes	4
Carbon blacks	5
Chemicals, solvents, miscellaneous	300
Total number of products	2347

TABLE 3.10

Products Made by the U.S. Petroleum Industry

3.8.5.3 Carbon Black

Carbon black is considered the most important industrial carbon. It is produced under conditions permitting control of its particle size and its surface area. Carbon black is constituted mainly of elemental carbon with varying amounts of volatile matter and ash.

Important properties of carbon black are its surface area, particle size, and pH. These properties depend mainly on the type of feed used and on the production process.

Feeds suitable for carbon black production include natural gas, decant oil, fractionator bottoms, etc. Heavy hydrocarbon compounds rich in aromatics are more suitable feeds. The furnace process utilizes either natural gas or liquid feeds (gas furnace versus oil furnace).

Carbon black is a valuable petroleum product. It is mainly used for tire production. Other important uses are in ink, enamels and paints, gravitized electrodes, and plastic building materials.

A survey of the petroleum industry in the United States, made by the API, revealed over 2000 different refined oil products made from crude oils, as shown in Table 3.10.

Section 2

Principles, Methods, and Techniques of Engineering Economics Analysis

Basic knowledge and techniques for performing investment analysis are presented in this section. The tools for tackling economic and investment problems in the petroleum sector are provided, keeping in mind that the economic viability of each potential solution is considered along with the technical aspects. Some fundamental principles and concepts are described as follows.

Principle 1: One dollar now is worth more than a dollar at a later time. This explained by the Time Value of Money (TVM).

Principle 2: Three parameters influence the TVM:

Inflation

Risk

Cost of money (interest)

- Of these, the cost of money is the most predictable, and hence it is the essential component in our economic analysis.
- **Principle 3:** Additional risk is not taken without the expected additional return.

Fundamental engineering economic concepts are discussed in Chapters 4 through 7, along with example problems to illustrate the use of various theoretical solutions. This covers the following:

Basic Tools: The mathematical and practical "tools" used in investment analysis for evaluating profitability, known as economic decision criteria, are:

- Annual Rate of Return (ARR)
- Payout or Payback Period (PP)
- Discounted Cash Flow Rate of Return (DCFR)
- Net Present Value (NPV)

Basic Concepts

- Cash flow
- Interest rate and time value of money
- Equivalence technique
- **Cash-Flow Concepts:** Cash flow is the stream of monetary (dollar) values, costs (inputs), and benefits (outputs) resulting from a project investment.
 - Cost of money is represented by (1) money paid for the use of a borrowed capital, or (2) a return on investment.
 - Considering the time value of money is central to most engineering economic analyses. Cash flows are discounted using an interest rate, *i*, except in the most basic economic studies.

Cost of money is determined by an interest rate.

- Time value of money is defined as the time-dependent value of money stemming both from changes in the purchasing power of money (inflation or deflation) and from the real earning potential of alternative investments over time.
- **Cash-Flow Diagrams:**^{*} The easiest way to approach a problem in economic analysis is to draw a picture that shows three items:
- A time interval divided into an appropriate number of equal periods
- All cash outflows (deposits, expenditures, etc.) in each period
- All cash inflows (withdrawals, income, etc.) for each period

[•] Unless otherwise indicated, all such cash flows are considered to occur at the end of their respective periods.

4

Time Value of Money (TVM) in Capital Expenditures

M. Bassyouni

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This chapter is devoted to an examination of some basic relationships that relate interest to a given capital of money (either expenditure or income) over a given period of time. It is essential to consider the effect time has on capital, as capital must always produce some yield. Time value of money (TVM) is the value of money figuring at a given amount of interest earned over a given amount of time. TVM is the central concept in finance theory.

Most economic problems in the petroleum industry involve determining what is economical in the long run; that is, over a period of time. A dollar now is worth more than the prospect of a dollar next year or at some later date.

All of the standard calculations for TVM are based on the most basic algebraic expression for the present value *P*, of a future sum *F*, discounted at an interest rate *i*. This is formulated mathematically by the expression:

$$P = F/(1+i)$$

4.1 Basic Definitions

4.1.1 Capital Investment

Money invested in oil projects is used for the following purposes:

- 1. To purchase and install the necessary machinery, equipment, and other facilities. This is called *fixed capital investment*. This investment is *depreciable*.
- 2. To provide the capital needed to operate an oil field or a refinery as well as the facilities associated with them. This is what is called *working capital*. Principally it is capital tied up in raw material inventories in storage, process inventories, finished product inventories, cash for wages, utilities, etc. This is a non-depreciable investment.

Working capital must not be ignored in a preliminary estimate of needs for capital investment, because it is usually a sizable amount of any total investment, and no economic picture of oil processing is complete without inclusion of working capital. Investment in working capital is no different than investment in fixed capital except that the former does not depreciate.

Working capital can theoretically be recovered in full when any refinery or oil field shuts down. Capital is "tied up" when the refinery is operating and capital must be considered part of the total investment. Working capital, however, is normally replaced as it is used up by sales dollars the oil company receives for crudes or refined oil products. Therefore we can safely say that this capital is always available for return to owners. This is not the immediate case with depreciable capital.

4.1.2 Interest

Interest may be defined as the compensation paid for the use of borrowed capital. The recognized standard is the *prime interest rate*, which is charged by banks to their customers. This definition is the one adopted by engineers; the classical definition describes interest as the money returned to the investors

for the use of their capital. This would mean that any profit obtained by using this capital is considered interest, which is not true. Instead a distinction is to be made between interest and the rate of return on capital.

4.2 Types of Interest

As shown in Figure 4.1, interest may be described as *simple* or *compound*. Simple interest, as the name implies, is not compounded; it requires compensation payment at a constant interest rate based only on the original principal. In compound interest, the interest on the capital due at the end of each period is added to the principal; interest is charged on this converted principal for the next time period. Most oil economics are based on compound interest.

4.3 Interest Calculation

If *P* represents the principal (in dollars), *n* the number of time units (in years), and *i* the interest rate based on the length of one interest period, then:





Mathematical definition and classification of interest.

Using simple interest: the amount of money to be paid on the borrowed capital *P*, is given by: (*P*) (*i*) (*n*).

Hence the sum of capital plus the interest due after n interest periods will be denoted by:

$$F = P + Pin = P(1 + in) \tag{4.1}$$

where *F* is the future value of the capital *P*.

Using compound interest: the amount due after any *discrete* number of interest periods can be calculated as follows:

	Principal Capital Available	Interest Earned on P	Principal Plus Interest
For the first period	Р	Pi	P(1 + i)
For the second period	P(1 + i)	P(1+i)i	$P(1 + i)^2$
	:	:	:
For the <i>n</i> th period	$P(1+i)^{n-1}$	$P(1+i)^{n-1}i$	$P(1 + i)^{n}$

Thus the general equation is given by:

$$F = P(1+i)^n \tag{4.2}$$

A simple illustration of interest calculations is shown in Example 4.1.

Example 4.1

A sum of \$1,000 is deposited into an account where the interest rate is 10% compounded annually; compare the future values of the deposit for the two cases of simple and compound interest after 4 years.

	Simple Interest ($P_o = P_1 = P_2 = \dots P_n$)	Compound Interest (<i>P</i> is changing from year to year)
For first year	$F_1 = P_o + \text{interest}$ = 1,000 + (1,000)(0.1) = 1,000(1 + 0.1)	$F_1 = P_o + \text{interest}$ = 1,000 + (1,000)(0.1) = 1,000(1 + 0.1)
For second year	$\begin{split} F_2 &= F_1 + P_o(0.1) \\ &= P_o(1+0.1) + P_o(0.1) \\ &= 1,000[1+(2)(0.1)] \\ \vdots \end{split}$	$F_2 = F_1 + (F_1)(0.1)$ = $F_1(1 + 0.1)$ = $1,000(1 + 0.1)^2$:
For fourth year	$F_4 = 1,000[1 + 4(0.1)]$ = \$1,400	$F_4 = 1,000(1+0.1)^4$ = \$1,464.1

From this example, it is concluded that compound interest increases future value faster than simple interest as shown in Figure 4.2.



FIGURE 4.2 (See Color Insert)

Values of the deposit for simple and compound interest after 4 years.

4.4 Effective Interest

In Figure 4.1, it is seen that the discrete compounding interest can be further classified as *effective* or *nominal* depending on the time period at which money is compounded. In other words, if the length of the discrete interest period is *1 year*, the interest rate is known as the *effective* one, while if other time units *less than 1 year* are used, the interest rate is described as *nominal*.

In common engineering practice, 1 year is assumed as the discrete interest period; however, there are many cases where other time units are employed. Thus the way interest rates are quoted affects the return on investment. For instance, the future value after 1 year of \$1,000 compounded annually at 6% is \$1,060, while if compounding is done quarterly (every 3 months), the return will be \$1,061 (i.e., 1.5% four times a year). A rate of this type would be referred to as "6 percent compounded quarterly." This is known as the *nominal interest rate*. The effective interest rate in this case is definitely greater than 6%, since we are making more money (compare \$61 to \$60).

The effective interest rate, " i_e " is related to the nominal interest rate " \overline{i} " as follows:

If " \overline{i} " is the nominal interest rate stated under the conditions for "*m*" compounding time periods per year, then the interest rate for one period is given by \overline{i}/m . Hence the future value after 1 year is

$$F_1 = P(1 + \bar{i}/m)^m$$
(4.3)

The future value F_1 can be expressed at the same time in an alternate form as

$$F_1 = P(1 + i_e) \tag{4.4}$$

Equating Equations (4.3) and (4.4), the effective interest rate " i_e " is related to \overline{i} and *m* as given by Equation (4.5):

$$i_e = \left(1 + \frac{\overline{i}}{m}\right)^m - 1 \tag{4.5}$$

To find the future worth after n years using the nominal interest rate, Equation (4.3) takes the following form:

$$F = P(1 + \overline{i} / m)^{mn} \tag{4.6}$$

Example 4.2

To illustrate the value of knowledge of the effective interest rate to oil management, assume that a short-term loan for 1 year only could be arranged for an oil company in temporary distress. The company needs \$100,000 for immediate working capital at either a nominal rate of 12% compounded monthly or a nominal rate of 15% compounded semiannually. The oil company wants to know which arrangement would provide the oil company with the lower debt at the end of the short-term loan period. The use of the effective interest rate formula gives the answer.

SOLUTION

On a nominal 12% rate compounded monthly, and using Equation (4.5):

Effective interest rate =
$$\left(1 + \frac{0.12}{12}\right)^{12} - 1$$

= $(1.01)^{12} - 1$
= $1.127 - 1 = 0.127$, or 12.7%



FIGURE 4.3 (See Color Insert)

Interest at the end of one year for compounded monthly and semiannual interest.

On a nominal 15% rate, compounded semiannually:

Effective interest rate =
$$\left(1 + \frac{0.15}{2}\right)^2 - 1$$

= $(1.075)^2 - 1$
= $1.156 - 1 = 0.156$, or 15.6%

The loan at 12% compounded monthly has the lower effective interest rate, or 12.7% and 15.6% for the loan arrangement using a nominal rate of 15% compounded semiannually. Thus the oil company will borrow \$100,000 for 1 year at 12% interest compounded monthly, paying back the loan at the end of 1 year with \$112,700, which includes \$12,700 in interest, instead of borrowing at 15% compounded semiannually, which would cost \$15,600 in interest as illustrated in Figure 4.3, and a total of \$115,600. Thus the oil company saves \$2,900 by borrowing at 12% compounded monthly.

CONTINUOUS INTEREST

The final type of interest to be discussed here is what is known as *continuous compounded interest*. So far, we have considered payments to be charged at periodic and discrete intervals—a year, a month, a week. As the time interval for this discrete compounding interest is allowed to become infinitesimally small (i.e., approaches zero), the interest is said to be compounded continuously.

Equations are derived as follows:

If *m* approaches infinity, Equation (4.6) is rewritten in the following form:

$$F = P \lim_{m \to \infty} (1 + \overline{i} / m)^{mn}$$

= $P \lim_{m \to \infty} [(1 + \overline{i} / m)^{m/\overline{i}}]^{(\overline{i}n)}$, hence (4.7)
$$F = P e^{\overline{i}n}$$

where $\lim_{m \to \infty} (1 + \overline{i}/m)^{m/\overline{i}} = e$, the base of the natural system of logarithms.

The effective interest rate "ie," could be determined as a function of \overline{i} for this case as follows:

For one year,
$$F_1 = Pe^i = P(1 + i_e)$$
.
Therefore, $i_e = (e^{\overline{i}} - 1)$.
$$(4.8)$$

Example 4.3

- 1. What is the accumulated sum after 1 year for a \$1,000 principal compounded *daily* at a nominal interest rate of 20%?
- 2. Repeat if compounding is done *continuously*.
- 3. Calculate " i_e " for both cases.

SOLUTION

1. Using Equation (4.3), where P = \$1,000, i = 0.20, m = 365:

$$F_1 = 1,000 \left(1 + \frac{0.20}{365}\right)^{365} = \$1,221.3$$

2. Using Equation (4.7), where n = 1:

$$F_1 = 1,000 \ e^{(0.20)(1)} = \$1,221.4$$

3.

"*i_e*," compounding interest =
$$\left(1 + \frac{\overline{i}}{m}\right)^m - 1$$

= $\left(1 + \frac{0.2}{365}\right)^{365} - 1$
= 0.2213, or 22.13%
"*i_e*," continuous interest = $e^{\overline{i}} - 1 = 1.2214 - 1$
= 0.2214, or 22.14%

For a comparison between these different types of interests, the future value of \$100 is calculated at a nominal interest rate of 5% using these methods. The change in values with time (in years) is shown in Figure 4.4.

4.5 Annuities and Periodic Payments

Compound interest and discount factors are defined as:

$$C = (1+i)^n \tag{4.9}$$

$$D = (1+i)^{-n} \tag{4.10}$$



FIGURE 4.4 (See Color Insert) Change in future values of \$100 principal with time, using different types of interest.

Other conversion factors covering both single payments and uniform payments are needed to solve many of our engineering economy problems. Most important are the sinking fund factor and capital recovery factor, to be derived in this section.

An annuity is a series of equal payments occurring at equal time intervals, normally at the end of the period. Payments of this type are used to accumulate a desired amount of capital as in depreciation calculations, where engineers face the problem of an unavoidable decrease in value of equipment. The amount of an annuity is the sum of all payments plus interest if allowed to accumulate at a definite rate of interest during the annuity term.

4.5.1 Derivation of the Basic Equation (Sinking Fund Factor)

Assume that the amount of annuity at the end of n years is F, while A is the uniform yearly periodic payment to be invested at i interest yearly rate.

By the end of the annuity term:

The 1st payment *A* should have a value $F_1 = A(1 + i)^{n-1}$

The 2nd payment *A* should have a value $F_2 = A(1 + i)^{n-2}$

÷

The last payment *A*, should have a value $F_n = A$

Finally the sum of all payments will be *F*, where

$$F = A(1+i)^{n-1} + A(1+i)^{n-2} + \dots + A(1+i) + A$$

Hence it can be shown that

$$F = \frac{A[(1+i)^n - 1]}{1} \tag{4.11}$$

The above factor $[(1 + i)^n - 1]/i$ is known as the compound amount factor or sinking fund factor.

4.5.2 Applications of the Annuity Technique

4.5.2.1 Determining the Annual Depreciation Costs

This is a typical application of an ordinary annuity. The value of the depreciable equipment (heat exchanger, separating vessel, pump, etc.) is set equal to the amount of annuity F in Equation (4.11); call it F_o . Then A, to be denoted by A_d , is directly calculated as follows:

$$A_{d} = F_{o} \left[\frac{i}{(1+i)^{n} - 1} \right]$$
(4.12)

4.5.2.2 Determining the Annual Capital Recovery Costs

The annual capital recovery cost (A_r) is defined as the annual amount of money that, if put aside in an annuity, would generate the sum of the original principal (capital investment) plus the interest on it. This sum is called *F*. In other words, A_r is related to this value of *F* by:

$$A_r = F\left[\frac{i}{\left(1+i\right)^n - 1}\right] \tag{4.13}$$

The present worth of this annuity, on the other hand, is defined as the original principal (P) which would have to be invested at the present time at i to yield (F) after n years. In other words:

$$F = P(1+i)^n$$
(4.14)

Substituting the value of *F* as given by Equation (4.14) in Equation (4.13) and solving for A_r :

$$A_r = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$
(4.15)

where $[i(1 + i)^n/(1 + i)^n - 1]$ is known as the capital recovery factor.

Mathematically speaking, A_r is related to A_d by going through the following solved example.

Example 4.4

The capital cost of a small portable pump is \$7,000, with a lifetime of 20 years. If money can be invested at 6% (annual interest rate), calculate the annual depreciation costs and the annual capital recovery and compare between the two values.

SOLUTION

In order to protect the original capital (principal), the annual depreciation costs are calculated using Equation (4.12).

$$A_d = 7000 \left[\frac{0.06}{\left(1.06 \right)^{20} - 1} \right] = \$190.29$$

In other words, the sum of these A_d 's plus the interest accumulating in a sinking-fund annuity will generate exactly the \$7,000.

What about the "cost" of using the capital? Provision must be made in order to create an incentive in using this investment. If this \$7,000 had been deposited in a bank, it would have generated a return or interest as follows:

$$F = 7,000(1 + 0.06)^{20} = $22,450$$

Since this *F* is the same future worth of the annuity we are looking for, and developed above by Equation (4.13), the value of A_r is calculated directly as given by Equation (4.15):

$$A_r = P\left[\frac{i(1+i)^n}{(1+i)^n}\right]$$
$$= 7000\left[\frac{0.06(1.06)^{20}}{(1.06)^{20}-1}\right] = $610.3$$

It is seen now that the sum of these A_r 's plus the interest accumulating in a sinking-fund annuity will generate \$22,450. Specifically, the annual capital recovery costs will include the annual depreciation costs plus the annual interest cost on the principal.

To relate A_d to A_r :

$$A_r/A_d = \frac{\text{Equation (4.15)}}{\text{Equation (4.12)}}$$

and since F_o in Equation (4.12) is by definition the original principal or capital investment, *P*, this gives:

$$A_r / A_d = (1 + i)^m$$

Therefore,
$$A_r = A_d (1+i)^n$$
 (4.16)

For our example:

$$A_r$$
(to be calculated) = 190.29(1.06)²⁰ = 610.3

This is a double check on the value of A_r calculated previously.

This example has been solved in block-diagram form for further illustration of some of these concepts, as shown in Figure 4.5.

Equation (4.15) should prove to be a very powerful and useful tool for decision making in many oil operations involving capital investments. Capital recovery, or recovery of capital investment *with interest* (the profit to the investor on the investment), is a matter of vital concern to the oil investor. He usually looks for assurance that any risk he takes with his investment is proportional to the interest earned.

Capital recovery is thus important in any study of oil economics, since it is repayment to the oil investor of his investment plus interest. Capital recovery is the reward to the oil investor for the reward of his money, and for the risk he was willing to take.



FIGURE 4.5

Solution of Example 4.4 to illustrate the concept of capital recovery, where n = 10 years and i = 0.06.

4.6 Capitalized Costs

4.6.1 Calculation of Capitalized Costs of an Asset to Be Replaced Perpetually

Here, we have to establish what is known as "perpetuity." In an annuity, periodic payments were made for a definite number, *n* years. However, in perpetuity, the periodic payments continue indefinitely:

Annuity $\rightarrow n$ years Perpetuity $\rightarrow \infty$ years

To establish a perpetuity based on capitalized costs for equipment, we should have an accumulated amount of money, *K*, in order to provide funds for:

- 1. The capital cost of the new equipment, C_v
- 2. The capital investment *P*, the present worth of the same asset, such that at the end of *n* years, this *P* should have generated enough money for replacing the equipment, perpetually, i.e., to provide C_R .

Hence,
$$K = C_V + P$$
 (4.17)

In addition,
$$F = P(1+i)^n = P + C_R$$
 (4.18)

Solving Equation (4.18) for *P*, we get:

$$P = [C_R / (1+i)^n - 1]$$
(4.19)

Substituting the value of *P* as given by Equation (4.19) into Equation (4.17), the capitalized cost *K* is given by Equation (4.20).

$$K = V_s + \left[\frac{C_R (1+i)^n}{(1+i)^n - 1}\right]$$
(4.20)

where $C_v = C_R + V_s$; that is, the cost of new equipment equals the replacement cost plus the salvage value (V_s).

4.6.2 Calculation of the Capitalized Costs of a Perpetual Annual Expense

In determining what the value is at the present time for a perpetual series of annual payments in the future, the equipment "capitalized cost" of

annual operating costs, such as repairs and maintenance, that must be paid in an indefinite number of periods in the future in order to continue the given services is considered in this section. Thus if repairs and maintenance in an oil field cost \$300,000 yearly on average, the capitalized costs of such continuous expenses at an interest rate of, say, 8% will be: 300,000/0.08 = \$3,750,000. This is the equivalent cost of a series of annual operating costs.

To generalize this approach, the capitalized cost in this case is defined as follows:

Capitalized cost =
$$\frac{\text{total annual operating expenses}}{\text{average interest rate}}$$
 (4.21)

If the capitalized cost of an asset involves annual operating expenses, then Equation (4.20) should be rewritten in its general form as follows:

$$K = V_s + \left[\frac{C_R (1+i)^n}{(1+i)^n - 1}\right] + \frac{\text{annual operating expenses}}{i}$$
(4.22)

Example 4.5

Management of an oil company is considering purchase of a benchscale reverse-osmosis desalination unit. The installed cost of the unit is \$12,000, its lifetime is 10 years, and the salvage value is \$2,000. You have been asked to calculate the capitalized cost of the perpetual service of this unit, assuming that interest is compounded at 6% annually.

SOLUTION

Using Equation (4.20) (since no annual operating expense is involved in this case), the capitalized cost is calculated directly:

$$K = 2000 + \left[\frac{10,000(1.06)^{10}}{(1.06)^{10} - 1}\right]$$
$$= $24,650$$

A detailed illustration of how the perpetual replacement takes place is given in Figure 4.6.

Steps (1) and (2) are carried out only once at the initiation of the project (for the first period only, i.e., for the first 10 years). Steps (3) through (7) are what we call a *perpetual loop* that goes on and on. One period is made up of 10 years.



FIGURE 4.6

Solution of Example 4.5 illustrating the concept of perpetuity and capitalized costs. Steps (1) and (2) are carried out only once at the initiation of the project (for the first period only, i.e., for the first 10 years). Steps (3) through (7) are what we call a *perpetual loop* that goes on and on. One period is made up of 10 years.

4.7 Equivalence

The knowledge of equivalent values can be of importance to oil companies. The concept of equivalence is the cornerstone for comparisons of time values of money comparisons. Incomes and expenditures are identified with time as well as with amounts. Alternatives with receipts and disbursements can be compared by use of equivalent results at a given date, thus aiding in decision making.

The concept that payments that differ in total magnitude but that are made at different dates may be equivalent to one another is important in engineering economy. Specifically, three factors are involved in the equivalence of sums of money:

- 1. Capital investment involved
- 2. Time
- 3. Interest rate

Examples 4.6 and 4.7 illustrate the concept of equivalence.

Example 4.6

A sum of \$10,000 is borrowed by a refining oil company. Propose four different equivalent plans of money payments for this capital over a period of 10 years assuming the interest rate is 6%.

SOLUTION

As shown in Table 4.1, Plan 1 involves the annual payment of interest only (\$600) until the end. Plans 2 and 3 involve systematic reduction of the principal of the debt (\$10,000). For Plan 2 this is done by uniform repayment of principal (\$1,000/yr) along with diminishing interest, while for Plan 3 a scheme is devised to allow for uniform annual payment for both capital and interest all the way through until the end (\$1,359). For Plan 4, on the other hand, payment is done only once at the end of the 10th year. The equivalence of the four payments is further illustrated in Figure 4.7.

Example 4.7

Show how \$100,000 received by an oil company today can be translated into equivalent alternatives. Assume money is worth 8%.

SOLUTION

Cash flow is translated to a given point in time by determining the present value or the future value of the cash flow. Accordingly, \$100,000 today is equivalent to \$215,900 10 years from now (using the formula Find *F*/Given *P* in the next section or the tables in Appendix A). Also, \$100,000 today is equivalent to \$25,046 received at the end of each year for the next 5 years (using the formula Find A_r /Given *P*). Many other options can be selected for different periods of time. Figure 4.8 illustrates this concept.

4.8 Formulas and Applications: Summary

4.8.1 Formulas

The fundamental formulas dealing with interest can be summarized as follows:

1. Find *F*/Given *P*
$$F = P(1+i)^n$$
 (4.23)

Year	Investment	Plan 1 (\$)	Plan 2 (\$)	Plan 3 (\$)	Plan 4 (\$)
0	\$10,000				
1		600	1,600	1,359	
2		600	1,540	1,359	
3		600	1,480	1,359	
4		600	1,420	1,359	
5		600	1,360	1,359	
6		600	1,300	1,359	
7		600	1,240	1,369	
8		600	1,180	1,359	
9		600	1,120	1,359	
10		10,600	1,060	1,359	17,910

TABLE 4.1

Summary of the Four Plans for Solving Example 4.6

2. Find P/Given F $P = F(1+i)$	(4.24)
--------------------------------	--------

3. Find A/Given F
$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$
(4.25)

4. Find
$$A_r / \text{Given } P \quad A_r = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$
 (4.26)

5. Find *P*/Given
$$A_r$$
 $P = A_r \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$ (4.27)

6. Find
$$A_r$$
 / Given A_d $A_r = A_d (1+i)^n$ (4.28)

where

i represents interest rate per interest period

n represents number of periods of interest payments (year, month, etc.)

P represents value of principal, \$ (present),

A represents annual payments or receipts, \$/yr

F represents future value, \$

In Equation (4.25), A can stand for the annual depreciation costs and is designated as A_d (as given by Equation 4.12) or it can represent the annual capital recovery costs and is referred to as A_r (as given by Equation 4.13).







FIGURE 4.8 Solution of Example 4.7.

4.8.2 Practical Applications and Case Studies

Additional examples illustrating the practical applications of each of these interest formulas are presented next.

Example 4.8

In 10 years, it is estimated that \$144,860 (future value) will be required to purchase several cooling towers. Interest available at the bank is 8% compounded annually. Calculate the annual annuity payment that will amount to the given fund after 10 years of deposit.

SOLUTION

Using the compound interest tables in Appendix A, and the formula Find A/Given F (Equation 4.25) for 8% and 10 years, we get:

A = (144, 860)(0.06903)

= \$10,000 yearly

Year	Payment into Fund (\$)	Compound Interest Factor	Compound Interest	Payment with Interest into Fund (col.2* 4) (\$)	Amount in Sinking Fund (\$)
1	10,000	$(1+i)^{n-1}$	$(1 + 0.08)^9$	19,990	19,990
2	10,000	$(1+i)^{n-2}$	$(1 + 0.08)^8$	18,510	38,500
3	10,000	$(1+i)^{n-3}$	$(1 + 0.08)^7$	17,140	55,640
4	10,000	$(1+i)^{n-4}$	$(1 + 0.08)^6$	15,870	71,510
5	10,000	$(1+i)^{n-5}$	$(1 + 0.08)^5$	14,690	86,200
6	10,000	$(1+i)^{n-6}$	$(1 + 0.08)^4$	13,600	99,800
7	10,000	$(1+i)^{n-7}$	$(1 + 0.08)^3$	12,600	112,400
8	10,000	$(1+i)^{n-8}$	$(1 + 0.08)^2$	11,660	124,060
9	10,000	$(1+i)^{n-9}$	$(1 + 0.08)^1$	10,800	134,860
10	10,000	$(1+i)^{n-10}$	$(1 + 0.08)^0$	10,000	144,860
Totals	\$100,000		\$144,860		

TABLE 4.2

Tabulation	h of	Results	for	Exami	ole	48
rabulation	1 01	Results	101	LAUIU	JIC	1. 0

Thus each year a payment or deposit of \$10,000 should be made into the sinking fund at 8% compounded annually. After 10 years, the fund will contain \$144,860 with which the oil company can purchase cooling towers as provided for by the fund. Table 4.2 tabulates the future value at the end of 10 years of \$144,860, with total deposits of \$100,000. At the end of the second year the fund shows a total of \$38,500, and at the end of the fifth year a total of \$86,200. Amounts into the fund, including interest, decrease as each year progresses, with no interest being included in the 10th payment.

Example 4.9

A sinking fund is to be established to cover the capitalized cost of temperature recorders. The recorders cost \$2,000 and must be replaced every 5 years. Maintenance and repairs come to \$200 a year. At the end of 5 years the accumulated sinking fund deposits are expected to cover the capitalized cost of continuous expense for these recorders. How much money must be deposited each year, at an interest rate of, say, 5%, to cover the capitalized costs at the end of 5 years?

SOLUTION

Two methods are proposed to solve this problem:

1. Using Equation (4.22), where $V_s = 0$ and $C_R =$ \$2,000:

K, total capitalized cost = $2000 \frac{(1.05)^5}{(1.05)^5 - 1} + \frac{200}{0.05}$ = 9,238.5 + 4,000 = \$13,238.5 The capitalized cost due to the replacement of the equipment only is 9,238.5 - 2,000 = \$7,238.5.

The annual expenditures corresponding to this sum = (7,238.5)(0.05) = \$362.

In other words, the *total annual* expenditures to be deposited will be 362 + 200 = \$562.

2. Using the formula Find A/Given F (Equation 4.25), we get:

A = (2,000)(0.18097) = \$362 per year

Then adding \$362 to \$200, we get \$562.

If these annual deposits of \$362 are invested in a sinking fund deposit at 5%, they will be worth exactly \$2,000 at the end of 5 years.

Again, the addition of \$200 to this \$362 will give the required annual deposit of \$562 obtained by the first method.

A further check is done on the total capitalized cost as follows:

562/0.5 = \$11,240

Hence 11,240 + 2,000 = \$13,240.

Example 4.10

In Example 4.9, the annual sum of money of \$362 was calculated, which recovers the principal value of the temperature recorders if deposited in a sinking fund (\$2,000). What about the cost of the capital (interest on capital)?

Calculate the annual capital recovery costs (A_r) and compare with the annual depreciation cost (A_d).

SOLUTION

In solving Example 4.9, the annual depreciation cost was calculated by:

$$A_d = 2,000 \left[\frac{0.05}{(1.05)^n - 1} \right]$$
$$= \$362/\text{year}$$

In order to calculate the annual capital recovery costs, use is made of Equation (4.26) as follows:

A_r = 2,000
$$\left[\frac{0.05(1.05)^5}{(1.05)^5 - 1} \right]$$
 = 2,000(0.23097)
= \$462/yr

Now, the difference between A_r and A_d equals 462 - 362 = 100/year.

This \$100 accounts for the annual cost (interest) on the capital (\$2,000), which makes:

$$(100/2000)(100) = 5\%$$

Also, using Equation (4.28), we can check the value of A_{rr} given A_{d} :

$$A_r = (362)(1.05)^5$$

= \$462

Example 4.11

An oil production company wishes to repay in 10 installments a sum of \$100,000 borrowed at 8% annual interest rate. Determine the amount of each future annuity payment A_r required to accumulate the given present value (debt) of \$100,000 for a number of payments of 10 years.

SOLUTION

Find A_r /Given *P*:

$$A_r = (100, 000)(0.14903)$$

= \$14, 903/year

Thus for 10 years, \$149,030 would have been paid: \$100,000 as principal and \$49,030 as interest.

The \$100,000 is the present value of the 10-year annuity and the \$14,903 is the annual payment, or the annual capital recovery by the creditor.

Example 4.12

An oil-exploration company plans to take over offshore operations 7 years from now. It is desired to have \$250,000 by that time. If \$100,000 is available for investment at the present time, what is the annual interest rate the company should require to have that sum of money?

SOLUTION

Using Equation (4.23), where

$$P = $100,000$$

 $F = 25,000$
 $n = 7$ years
 $i =$ to be found
 $250,000 = 100,000(1 + i)^7$

Solving for *i*: The interest rate = 14%, which is rather high to realize.

Example 4.13

During the treatment of associated natural gas it was decided to install a knockout drum in the feedline of the plant. This vessel can be purchased and installed for \$40,000 and will last for 10 years. An old vessel is available and can be used but needs to be repaired. However, the repairing has to be done every 3 years. If it is assumed that the two vessels (the new and the old ones) have equal capitalized costs, how much does the maintenance department have to spend repairing the old knockout drum? Assume interest is 10%.

SOLUTION

Assuming the salvage value, $V_s = 0$. Equation (4.20) gives: $K = C_R$. Comparing the *new vessel* with the *old vessel*:

C_{R} (\$)	10,000	Unknown
n (years)	10	3
i	0.1	0.1

Now, on the basis of equal capitalized costs:

10,000	$\left(\frac{(1.1)^{10}}{(1.1)^{10}-1} \right)$	$= C_R$	$\left(\frac{(1.1)^3}{(1.1)^3 - 1}\right)$	$\left(\right)$
--------	--	---------	--	------------------

Solving for C_R , it is found that the maximum amount the maintenance department can spend on repairing the old vessel (perpetual service) is \$4,047.

In concluding this chapter, steps in the use of compound interest factors or formulas involving F, P, and A for measurement and determination of time values of money for expansion or replacement of older assets are given as follows:

- 1. Determine what is wanted—*F*, *P*, or *A*.
- 2. Determine what is given—*F*, *P*, or *A*.
- 3. Then apply the formula as to what is given and what is desired, or use the appropriate compound factor for the formula (found in Appendix A) with the desired rate of interest (*i*).

Notation

- A, Annual payment (\$/yr)
- A_d , Annual payment, sinking fund depreciation (\$/yr)
- A_{rr} Annual capital recovery (\$/yr)
- *C*, Compound interest factor $(1 + i)^n$
- $C_{v'}$ Original value of equipment (\$), also denoted as (V_o)

 $C_{\scriptscriptstyle R\prime}$ Cost of replacement of equipment after n years of operation (\$)

D, Discount factor $(1 + i)^{-n}$

F, Future value of capital (\$)

i, Interest rate (%) per time period

 i_{e} , Effective interest rate

 i_{ν} Nominal interest rate for *m* periods

K, Total capitalized cost (\$)

n, Number of years

m, Compounding time periods per year

P, Present value of capital (\$)

 $V_{s'}$ Salvage value of equipment (\$)

5

Depreciation and Depletion in Oil Projects

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Economic analysis of the expenditures and revenues for oil operations requires recognition of two important facts: (1) physical assets decrease in value with time, i.e., they *depreciate*, and (2) oil resources, like other natural resources, cannot be renewed over the years, and they are continuously *depleted*. Depreciation, or amortization, is described as the systematic allocation of the cost of an asset from the balance sheet to a depreciation expense on the income statement over the useful life of an asset.

In this chapter the role played by depreciation/depletion in the oil industry is introduced. Then methods of determining depreciation costs are examined, including straight line, declining balance, sum-of-the-digits, and the sinking fund. Comparison between these methods and evaluation of each are presented as well. Depletion allowances are then computed using either the fixed percentage basis or the cost-per-unit basis.
5.1 Introduction and Basic Definitions

- *Depreciation* (from the accounting point of view): A system that aims to distribute the cost or other basic value of tangible capital assets (less salvage if any), over the estimated useful life of the unit. It is considered a process of *allocation*, not of valuation. Depreciation itself as a process is simply defined as the unavoidable loss in value of a plant, equipment, and materials.
- *Depletion* (from the accounting point of view): Depletion costs are made to account for or compensate for the loss in value of the mineral or oil property because of the exhaustion of the natural resources. Depletion is defined as the capacity loss due to materials consumed or produced.
- *Service life of an asset (equipment)*: The useful period during which an asset or property is economically feasible to use. The U.S. Bureau of Internal Revenue recognizes the importance of depreciation as a legitimate expense for industrial organizations. It is for this reason that the Bureau publishes an official listing of the estimated service lives of many assets. Table 5.1 includes the service lives of equipment and assets used in different sectors, both manufacturing and nonmanufacturing.
- Salvage value/junk (scrap) value: The value of the asset by the end of its useful life service. The term *salvage* would imply that the asset can be of use and is worth more than merely its scrap or junk value. The latter definition is applicable to cases where assets are dismantled and have to be sold as junk. The estimation of these values, including the lifetime, is generally based on the conditions of the asset when installed. In many cases, zero values are designated to the salvage and junk values.
- *Book value, present asset value, or unamortized cost*: The value of an asset or equipment as it appears in the official accounting record (book) of an oil organization. It is equal to the original cost minus all depreciation costs made to date.
- *Market value*: The value obtained by selling an asset in the market. In some conditions, if equipment is properly maintained, its market value could be higher than the book value.
- *Replacement value*: As the name implies, it is the cost required to replace an existing asset, when needed, with one that will function in a satisfactory manner.

TABLE 5.1

Estimated Service Life of Assets

	Life (Years)
Group I: General Business Assets	
1. Office furniture, fixtures, machines, equipment	10
2. Transportation	
a. Aircraft	6
b. Automobile	3
c. Buses	9
d. General-purpose trucks	4-6
e. Railroad cars (except for railroad companies)	15
f. Tractor units	4
g. Trailers	6
h. Water transportation equipment	18
3. Land and site improvements (not otherwise covered)	20
4. Buildings (apartments, banks, factories, hotels, stores, warehouses)	40-60
Group II: Nonmanufacturing Activities (Excluding Transportation, Communications, and	
Public Utilities)	
1. Agriculture	
a. Machinery and equipment	10
b. Animals	3–10
c. Trees and vines	Variable
d. Farm buildings	25
2. Contract construction	
a. General	5
b. Marine	12
3. Fishing	Variable
4. Logging and sawmilling	6–10
5. Mining (excluding petroleum refining and smelting and refining of minerals)	
6. Recreation and amusement	10
7. Services to general public	10
8. Wholesale and retail trade	10
Group III: Manufacturing	
1. Aerospace industry	8
2. Apparel and textile products	9
3. Cement (excluding concrete products)	20
4. Chemicals and allied products	11
5. Electrical equipment	
a. Electrical equipment in general	12
b. Electronic equipment	8
6. Fabricated metal products	12
7. Food products, except grains, sugar, and vegetable oil products	12
8. Glass products	14

(Continued)

TABLE 5.1 (Continued)

Estimated Service Life of Assets

	Life (Years)
9. Grain and grain-mill products	17
10. Knitwear and knit products	9
11. Leather products	11
12. Lumber, wood products, and furniture	10
13. Machinery not otherwise listed	12
14. Metalworking machinery	12
15. Motor vehicles and parts	12
16. Paper and allied products	
a. Pulp and paper	16
b. Paper conversion	12
17. Petroleum and natural gas	
a. Contract drilling and field service	6
b. Company exploration, drilling, and production	14
c. Petroleum refining	16
d. Marketing	
18. Plastic products	11
19. Primary metals	
a. Ferrous metals	18
b. Nonferrous metals	14
20. Printing and publishing	11
21. Scientific instruments, optical, and clock manufacturing	12
22. Railroad transportation equipment	12
23. Rubber products	14
24. Ship and boat building	12
25. Stone and clay products	15
26. Sugar products	18
27. Textile mill products	12–14
28. Tobacco products	15
29. Vegetable oil products	18
30. Other manufacturing in general	12
Group IV: Transportation, Communication, and Public Utilities	
1. Air transport	6
2. Central steam production and distribution	28
3. Electric utilities	
a. Hydraulic	50
b. Nuclear	20
c. Steam	28
d. Transmission and distribution	30

TABLE 5.1 (Continued)

Estimated Service Life of Assets

	Life (Years)
4. Gas utilities	
a. Distribution	35
b. Manufacture	30
c. Natural-gas production	14
d. Trunk pipelines and storage	22
5. Motor transport (freight)	8
6. Motor transport (passengers)	8
7. Pipeline transportation	22
8. Radio and television broadcasting	6
9. Railroads	
a. Machinery and equipment	14
b. Structures and similar improvements	30
c. Grading and other right-of-way improvements	Variable
d. Wharves and docks	20
10. Telephone and telegraph communications	Variable
11. Water transportation	20
12. Water utilities	50

Source: Peters, Max, Timmerhaus, Klaus, and West, Ronald, *Plant Design and Economics for Chemical Engineers*, 5th Edition, McGraw-Hill, New York, 2003. With permission.

5.2 Valuation of Assets Using Depreciation and Depletion: General Outlook

Petroleum company management frequently must determine the value of oil engineering properties. An adequate discussion of the methods used to arrive at the correct value of any property would require at least good-sized volume, so only a few of the principles involved will be considered here—those intimately connected with the subjects of depreciation and depletion.

There are many reasons for determining the value of oil field and refinery assets after some usage. For instance, these values may be needed to serve as a tax base or to establish current value for company statement purposes. Taking depreciation first, the primary purpose of depreciation is to provide for recovery of capital that has been invested in the "physical" oil property. Depreciation is a cost of production; therefore, whenever this production causes the property to decline in value, depreciation must be calculated. Indirectly, depreciation gives a method of providing capital for replacement of depreciated oil equipment. In short, depreciation can be considered as a cost for the protection of the depreciating capital, without interest, over the given period (minimum set by government) during which the capital is used. Finally, the process of valuation is usually an attempt either to make an estimate of present value of future oil profits which will be obtained through ownership of a property, or to determine what would have to be spent to obtain oil property capable of rendering the same service in the future at least as efficiently as the property being valued.

Investment of depreciable capital is used for one of two purposes in the oil fields:

- 1. As working capital for everyday operating expenses such as wages, materials, and supplies
- 2. To buy oil drilling machinery, rigs, etc., used in development and production of oil wells

Normally, working capital is replaced by sales revenue as it is used up. Thus, this part of investment capital is always available for return to investors.

Investment used for oil drilling machinery, well casings, etc.—that is, fixed capital—cannot be converted directly to original capital invested in oil equipment and machinery, because these physical properties decrease in value as time progresses. They decrease in value because they depreciate, wear out, or become obsolete. Recovery of this investment of fixed capital, with interest for the risks involved in making the investment, must be assured to the investor. The concept of capital recovery thus becomes very important.

The valuation of oil resources in the ground is something else. Oil resources cannot be renewed over a period of years like some other natural resources, such as timber or fish. Also, oil resources cannot be replaced by repurchase as such depreciable physical properties as machinery and equipment can be. Some provision is thus needed to recover the initial investment, or value, of oil reserves and reservoirs, sometimes referred to as an oil lease if purchased by others who are not owners of the land.

One way for investors to recover capital investment in an oil lease—known as depletion capital—is to provide a depletion allowance with annual payments made to the owners of the oil lease. Payments are based on the estimated life of the resource where such an estimate can be made with some degree of accuracy.

Another way to recover capital investment in an oil lease or other depletable capital is to set up a sinking fund with annual deposits based on one interest rate for the depletable capital plus another interest rate or profit on the investment.

In the case of exploration costs and development costs, or money spent for exploration and operations preliminary to actual recovery (production) of oil, such costs are usually recovered by "writeoffs" (an accounting term) against other revenues in the year they occur or through a depletion allowance. In the case of foreign oil companies—that is, foreign investors with other outside revenues—these costs can be subtracted from their other revenues along with other expenses in arriving at net income for tax purposes in their own countries. For example, in exploring and developing new leases in the Arabian Gulf area, which could involve millions of dollars before production or perhaps even with little chance of production success, oil companies could write off these costs against their overall revenues. This would reduce their taxable income and thereby reduce income taxes they would be liable to pay in their home countries.

To illustrate how both depreciation and depletion costs are calculated, several methods of determining depreciation and depletion are given, with examples of each.

5.3 Methods for Determining Depreciation

There are several ways of determining depreciation for a given period. The following are some of the more popular methods used in most industries. Some are more applicable to the oil industry than others. In general, these methods can be classified into two groups, as shown in Figure 5.1. This classification is based on either neglecting the interest earned on the annual depreciation costs, such that the sum accumulated at the end of the lifetime will equal the depreciable capital, or to take into consideration this interest.

5.3.1 Straight-Line Depreciation (S.L.D.)

Mathematically speaking, it is assumed that the value of the asset decreases linearly with time. Now, if the following variables are defined—d = annual



FIGURE 5.1

Methods used to calculate depreciation cost.

depreciation rate, \$/year; V_o , V_s = original value and salvage values of asset, \$; and n = service life, years—

then the annual depreciation $\cot = \frac{\text{depreciable capital}}{n}$, or

$$d = \frac{V_o - V_s}{n} \tag{5.1}$$

The asset value V_a — at the year a — is given by Equation (5.2):

$$V_a = V_o - (a)(d) \tag{5.2}$$

The straight-line method is widely used by engineers and economists working in the oil industry because of its simplicity. The need may arise to use what is called *multiple straight-line depreciation* (M.S.L.D.). This is true if reestimation of n and V_s are justifiable during the life period of the asset. Then, for each new estimated period, straight-line-depreciation calculation is carried out, to have M.S.L.D. as shown in Figure 5.2.



FIGURE 5.2 Comparison of different depreciation methods.

For oil operations, straight-line depreciation may be applied differently. Instead of using the lifetime of the asset, depreciation could be based on the *units of production* or *capacity output* of an oil well, a gas-oil separator, a stabilization unit, or a refining plant. Using this method, depreciation is computed by dividing the depreciable capital cost by the number of barrels to determine the "unit cost of depreciation." Then the total amount of depreciation in any given time period during the lifetime of this equipment is found by multiplying the unit cost by the number of units produced in that time period.

Example 5.1

An example of where this method might be used in the oil industry is a heat exchanger. Suppose that the heat exchanger as shown in Figure 5.3 has a depreciable cost of \$60,000 and will last for, say, 20 million bbl. Calculate the annual depreciation cost of the heat exchanger if it is processing 600,000 bbl yearly.

SOLUTION

The depreciation factor = 60,000/20,000,000 = \$0.003 per bbl. The annual depreciation, d = (600,000)(0.003) = \$1,800.

Obviously, the amount of depreciation per time period can vary greatly, depending on the activity level achieved by the oil company in that period. As demand for oil increases, there is an increase in depreciation expense resulting from the increased use of the equipment.

5.3.2 Declining Balance Depreciation (D.B.D.)

The declining balance method assumes that the equipment in question will contribute more to the earning of revenues in the early stage of useful life than it will as the equipment gets older.



FIGURE 5.3 Shell and tube heat exchanger.

A valid use of a declining pattern of depreciation occurs when it is felt that obsolescence will exert a strong influence on the life of the equipment, but there is no way of predicting when it will occur. In a simpler way this method is used where utility is higher in the earlier years of life. For example, a computer becomes obsolete within a certain period of time due to advancements in technology. In this method, a fixed percentage factor f is applied to the new asset value to calculate the annual depreciation costs, which will differ from year to year.

The formula relating "f" to V_a is derived as follows:

By the end of the first year: $V_1 = V_o(1 - f)$ By the end of the second year: $V_2 = V_o(1 - f)^2$ By the end of *a* year: $V_a = V_o(1 - f)^a$ By the end of the *n* year: $V_n = V_o(1 - f)^n$ or $V_s = V_o(1 - f)^n$,

since V_n represents value at the end of service life.

Finally solving for the value of *f*:

$$f = 1 - (V_s / V_o)^{1/n}$$
(5.3)

Examining Equation (5.3), one concludes the following:

- 1. The declining balance method permits the asset investment to be paid off more rapidly during the early years of life. This persuades oil companies starting new ventures to use the D.B.D., because it allows a reduction in income taxes at the early years of their operations.
- 2. The equation as such is seldom used practically for two reasons:
 - a. The equation is strongly dependent on the value of V_s .
 - b. The equation is not applicable if V_s is zero.

Because of these drawbacks, use is made of what is called *double declining balance depreciation* (D.D.B.D.). In this method, a fixed percentage factor giving a depreciation rate equivalent to twice the minimum rate with the straight-line method is to be selected. For example, any equipment lasting 5 years would have a 20% straight-line percentage and thus an allowable 40% for purposes of making the double declining balance calculation.

The declining balance depreciation method is more attractive because of its flexibility, ease of application to partial periods, and some common characteristics with depreciation laws (Berg et al., 2001).

Example 5.2

An example of how the double declining balance method is calculated is given here. If we assume that an acid injection unit had an original

Year	Depreciation Expense	Book Value	Remaining Depreciable Cost
Start	\$25,000	\$22,000	(with \$3,000 salvage value off)
After first year	\$10,000 (40% of \$25,000)	\$15,000	\$12,000 (\$22,000-\$10,000)
After second year	\$6,000 (40% of \$15,000)	\$9,000	\$6000 (\$12,000-\$6,000)
After third year	\$3,600 (40% of \$9,000)	\$5,400	\$2,400 (\$6,000-\$3,600)
After fourth year	\$2,160 (40% of \$5,400)	\$3,240	\$240 (\$2,400-\$2,160)
After fifth year	\$240 (depreciation before salvage value)	\$3,000	0

TABLE 5.2

Depreciation Schedule for Example 5.2

cost of \$25,000 and its lifetime is 5 years, it is necessary to calculate the annual depreciation costs and the book value for this unit. The salvage value, V_{sr} is taken to be \$3,000.

SOLUTION

Since n = 5 years, the annual depreciation using S.L.D. will be 20%, and the allowable fixed percentage to be applied using D.D.B.D. will be (2) (20%) = 40%. The depreciation schedule would then be as shown in Table 5.2.

5.3.3 Sum-of-the-Digits Depreciation (S.D.D.)

The S.D.D. method could be classified as a declining pattern depreciation. It is similar to the double declining balance depreciation, since larger costs are charged for the depreciation of an asset during the early years. It permits the asset to depreciate to a zero value or a given V_s by the end of its lifetime. The annual depreciation cost d_a for a given year a is calculated as follows:

- 1. For a given year a, calculate the number of years remaining in service, which equals (n a + 1).
- 2. Calculate the arithmetic series of the numbers from 1 to *n*, that is, $\sum_{n=1}^{n} y$.
- 3. Calculate the factor

$$"\bar{f}" = \frac{n-a+1}{\sum_{y=1}^{n} y}.$$
(5.4)

4. d_a would be = $(\overline{f})(V_o - V_s)$.

Finally,

$$d_a = \left(\frac{n-a+1}{\sum_{y=1}^n y}\right) (V_o - V_s) \tag{5.5}$$

$$=\frac{2(n-a+1)}{n(n+1)}(V_o - V_s)$$
(5.6)

This method is as aggressive as double declining balance and does not pose the problems. However there are several disadvantages of this method. It is not commonly used so it lacks comparability with competitors and familiarity with financial statement users. If the depreciation period does not align to the fiscal year it looks awkward. Some researchers believe that this method is predominantly applicable in the financial and regulated industries (Noland, 2011).

Example 5.3

A flow or recording control valve installed on the feed line of a causticsoda treating unit costs \$4,000, with a service life of 5 years and scrap value of \$400. Calculate the annual depreciation cost using the S.D.D.

SOLUTION

The sum of arithmetic series of numbers from 1 to 5 = 1 + 2 + 3 + 4 + 5 = 15. Using Equation (5.5) or (5.6), we get:

$d_1 = \left(\frac{5}{15}\right)(4000 - 400)$	= \$1,200
$d_2 = \left(\frac{4}{15}\right)(3600)$	= \$960
d_3	= \$720
d_4	= \$480
d_5	= \$240
Sum	= \$3,600

The bar chart clearly depicts that annual depreciation is constantly decreasing as the years pass as shown in Figure 5.4.



FIGURE 5.4 (See Color Insert)

Annual depreciation per year.

Example 5.4

An automobile part had an original cost of \$17,000 and its lifetime is 5 years. Calculate the annual depreciation cost and book value using the S.D.D. The salvage value is taken to be \$2,000.

SOLUTION

Calculations are shown in Table 5.3.

5.3.4 Sinking Fund Depreciation (S.F.D.)

This is the only method in which interest is considered on the accumulated annual depreciation costs. In other words, the use of compound interest is involved by establishing an annuity plan as discussed in Chapter 4. Equation (4.12) can be rewritten as:

$$A_d = (V_o - V_s) \frac{i}{(1+i)^n - 1}$$
(5.7)

TABLE 5.3

Depreciation Schedule for Example 5.4

Year	Depreciation Expense + \$2,000 for Salvage	Book Value	Remaining Depreciable
Start	0	\$17,000	\$15,000
After first year	\$5,000 (5/15 of \$15,000)	\$12,000	\$10,000
After second year	\$4,000 (4/15 of \$15,000)	\$8,000	\$6,000
		0	
After third year	\$3,000 (3/15 of \$15,000)	\$5,000	\$3,000
After fourth year	\$2,000 (2/15 of \$15,000)	\$3,000	\$1,000
After fifth year	\$1,000 (1/15 of \$15,000)	\$2,000	0

where $(V_o - V_s)$ is the sum of the annuity accumulated in *n* years, which represents the amount of depreciable investment of an asset.

After *a* years, the total amount of depreciation can be calculated using the following equation:

$$V_o - V_a = A_d = A_d \frac{(1+i)^n - 1}{i}$$
(5.8)

Substituting for A_d in Equation (5.8) by its corresponding value given by Equation (5.7) and solving for V_a :

$$V_a = V_o - (V_o - V_s) \frac{(1+i)^a - 1}{(1+i)^n - 1}$$
(5.9)

It is to be noted that the book values obtained by S.F.D. are always higher than the ones calculated using the straight-line method.

As far as the application of this method, the S.F.D. has limited utilization; however, it is useful for decision making on alternative investments and replacements.

Example 5.5

Assume a petroleum company investment of \$10 million for an expansion to a current refinery, allocated \$1,000,000 for land and \$7,000,000 for fixed and other physical properties subject to depreciation. Additional capital of \$2,000,000 is available for operation purposes, but this sum is not subject to depreciation. Investors want a 15% interest rate (or earning rate to investors) on their money for a 10-year period. The sinkingfund method will be used, with depreciation figured at 15% per year. No income taxes are involved in order to simplify the example.

SOLUTION

First-year profit before deducting the sinking-fund depreciation charge made at the earning rate of 15% interest, and assuming no salvage value for the physical properties, is $0.15 \times \$1,000,000$, or \$150,000 per year.

But the oil company must earn enough additional money annually to pay for the depreciation occurring on the depreciable capital of \$700,000.

Using sinking-fund depreciation and a 15% interest rate for the sinking fund, the annual deposit in the fund is given by:

$$A_d = \frac{\$700,000 \times 0.15}{(1.15)^{10} - 1} = \$34,440$$

Thus, company profits before depreciation must total \$184,440 (\$150,000 + \$34,440) and not merely \$150,000 in the first year. Actually, the \$184,440 in the first year represents:

\$34,440 = the sum of annual depreciation charge

\$105,000 = the 15% interest on the un-depreciated part of the depreciable capital which is, in the first year or before any deductions, 0.15 × \$700,000 \$45,000 = the 15% interest on the non-depreciable capital, or $0.15 \times $300,000$ \$184,440 = the total for the first year

Thus \$139,440 (\$105,000 + \$34,440) is needed to cover (1) the depreciation deposit in the sinking fund and (2) the interest on the depreciable capital for that year. This is also calculated by using:

$$A_r = \$700,000 \frac{0.15(1.15)^{10}}{(1.15)^{10} - 1} = \$139,440$$

In each succeeding year, the book value of the depreciable capital decreases, but the depreciation reserve increases in such a manner that the sum of the two always equals \$700,000, and the total annual interest remains constant at \$105,000 even though the interest charges on each component vary.

The biggest drawback to the actual use of the sinking-fund method in business is the fact that businesses rarely maintain an actual depreciation sinking fund. The interest rate that could be obtained on such deposits would be small, probably not over 6% in the petroleum business, according to financial experts in the oil industry. An active business, such as an oil company operation, is constantly in need of working capital. This capital will usually earn much more than 6%.

A reasonable rule is that all values should be kept invested in the oil business and not remain idle. As a result, a fictitious depreciation fund is often used. The amounts charged to depreciation are actually left in the business in the form of assets, and a "reserve for depreciation" account is used to record these funds.

Where such a "depreciation reserve" is used, the company is actually borrowing its own depreciation funds. Therefore, there is no place from which interest on these values could be obtained except from the business itself. This would create a situation in which a business pays itself interest for the use of its own money. To accomplish this, the cost of depreciation equal to the sinking-fund deposit has to be charged as an operating expense, and then interest on the accumulated sinking fund has to be charged as a financial expense. Such a procedure accurately accounts for all expenses but might require considerable explanation to government income tax authorities. Hence interest is not used when sinking fund deposits are not made to an outside source.

Example 5.6

Rework Example 5.5 to compare S.L.D. and S.F.D.

SOLUTION

Table 5.4 illustrates depreciation over 10 years for the investment in Example 5.5 as calculated by both the sinking-fund and straight-line methods. Figure 5.5 compares the book values obtained by the two

1 Start 2	c	15% of Column 2	Deposit	Charge	End of Year	Annual Charge	book Value
Start 1 2	1	3	4	IJ	9	7	80
7 1	0	0	0	0	\$700,000	0	\$700,000
7	\$34,440	0	\$34,440	\$34,440	665,560	\$70,000	630,000
	74,040	\$51,600 (15% of	34,440	39,600	625,960	70,000	560,000
		74,040)					
ю	119,580	11,100	34,440	45,540	580,420	70,000	490,000
4	171,960	17,940	34,440	52,380	528,040	70,000	420,000
сл	232,200	25,800	34,440	60,240	467,800	70,000	350,000
9	301,540	34,900	34,440	69,340	398,460	70,000	280,000
7	381,280	45,300	34,440	79,740	318,720	70,000	210,000
œ	472,920	57,200	34,440	94,640	227,080	70,000	140,000
6	577,960	70,600	34,440	105,040	122,040	70,000	70,000
10	700,000	86,600	34,440	22,040	0	70,000	0
		\$355,600	\$344,400	\$700,000		\$700,000 is	
						a constant	
						deduction	

Solution of Example 5.4 Using Straight-Line and Sinking-Fund Depreciation Methods

TABLE 5.4



FIGURE 5.5 Comparison of straight-line and sinking-fund methods of calculating depreciation.

methods as a line graph. As Table 5.4 and Figure 5.5 show, at the end of the second year the depreciation deposit into the sinking fund is \$34,440, but interest on the previous deposit is $0.15 \times $34,440$ (deposit for the first year), or \$5,160. This is repeated for the third year with 15% interest on \$39,600 (\$34,440 + \$5,160), and so on for each year. Before the petroleum company can earn interest of \$150,000 for the second year it must deposit \$34,440 in the sinking fund and pay \$5,160 interest on a total of \$39,600 to the sinking-fund depreciation reserve.

Figure 5.5 shows how the straight-line and sinking-fund methods differ. The curve of the sinking fund bulges from the straight-line method curve, yet both eventually meet at the end of the 10th year. Before turning to the subject of depletion, let us compare the different depreciation methods described so far, trying to evaluate each of them.

COMPARISON BETWEEN THE DEPRECIATION METHODS

The choice of the best depreciation is not a straightforward task. It is not our purpose to explore here the details of depreciation accounting methods. Suffice it to say that the following factors are important in choosing one method of depreciation and not the other:

- 1. Type and function of property: lifetime, salvage value
- 2. Time value of money (interest)
- 3. Simplicity

Choose the one for which the present worth of all depreciation charges is a maximum.

In the absence of guidelines and for quick results, the following rules are recommended:

- 1. Use straight-line depreciation (simple).
- 2. Take the useful lifetime of the asset = 10 years.
- 3. Assume salvage value = zero.

Now, we can make the following specific comparison:

Straight-Line and	Sinking Fund	versus 1	Declining	Balance	and
Sum-of-Digits	-		_		

- Annual depreciation costs are *constant*.
- The asset value is higher for S.F.D. because of the effect of *i*, as compared to S.L.D.
- S.L.D. is simple and widely used.
- S.F.D. is seldom used. It is applicable for assets that are sound in performance and stand little chance of becoming obsolete.

- Annual depreciation costs are *changing*, greater in early life than in later years.
- Used for equipment where the greater proportion of production occurs in the early part of life, or when operating costs increase with age.
- Both methods are classified as "accelerated depreciation" type. They provide higher financial protection.
- For D.D.B.D., the annual fixed percentage factor is constant, while for S.D.D. it is changing.

Berg et al. (2001) worked on a model in selecting the best method for calculation between the straight-line depreciation method and an accelerated depreciation method like sum-of-the-digits and double declining methods. They found that straight-line depreciation can be better than other depreciation methods, as the other methods are usually considered in empirical literature on accounting method choice. They also concluded that while making a selection between straight-line and accelerated method, it is necessary to consider the uncertainty in future cash flows and the structure of the tax system.

Noland (1997) states that the declining balance method is the most prominent type of accelerated depreciation used in financial reporting. However, he adds a drawback that at the end of the asset's useful life this method depreciates the asset to its salvage value. Different companies use various ways to adjust this problem.

5.4 Methods for Determining Depletion

5.4.1 Background

When limited natural resources such as crude oil and natural gas are consumed, the term *depletion* is used to indicate the decrease in value which has occurred. As some of the oil is pumped up and sold, the reserve of oil shrinks and the value of the oil property normally diminishes. Unless some provision such as depletion charges is made to recover the invested capital as the crude oil is pumped and sold, the net result will be loss of capital. This is prevented by charging each barrel, or ton, of crude with the depletion it has caused.

As shown in Figure 5.6, for oil production operations we have two phases where capital investment has to be spent. The first phase, called the





pre–oil-production phase, involves preliminary preparation, exploration, drywell drilling, and development. The property is now ready for the second phase, where money is spent in providing necessary assets and equipment for the production stage and the post oil production stage. The question is: How can we recover the capital spent in the pre-oil-production phase and the production and postproduction phases as well?

In the second phase, physical assets can be tangibly verified in a property; hence *depreciation accounting* can be applied to recover this capital investment. The first phase, on the other hand, exhibits the contrary: Intangible costs were invested, because no physical assets can count for them. In this case *depletion accounting* is introduced in order to recover the development costs that were spent for exploration and other operations preliminary to the actual production and the recovery of oil and gas. In other words, depletion allowance is a depreciation-like charge applied to account for the exhaustion of natural resources.

5.4.2 Methods

If a depletion allowance is to be used, there are two possible methods of calculating its value:

- 1. Fixed percentage method
- 2. Cost-per-unit basis

For the fixed percentage method, the percentage depletion is usually set by government ruling (in the United States it has been 22% of net sales), but in no case can the fixed percentage exceed 50% of net income before deduction of depletion.

In the cost-per-unit method, the amount of depletion charged to each barrel, or ton, of crude produced is determined by the ratio of intangible development cost plus the depletable costs divided by the estimated total units potentially recoverable. This then gives a cost per unit, which is in either barrels or tons depending on how the estimated total units potentially recoverable are given.

The total units recoverable may be estimated if the number of years of production and the production rates can be estimated. For oil and gas wells the calculations vary with the nature of the production curve and the allowable flow permitted by conservation authorities of the government of the oil-producing country. A mathematical analysis is used for estimating the total barrels of oil potentially recoverable under certain assumed conditions.

Example 5.7

Given the following:

The intangible development costs, excluding a \$1,000,000 bonus to land owner, all occur in the first year = \$8,000,000.

Depreciable capital such as casing, machinery, derricks, rigs, etc. = \$45,000,000.

Estimated life of equipment = 9 years.

Assume that 1,500,000 bbl of crude oil are produced and sold the first year at \$100/bbl.

Assume the annual operating expenses (and others) = \$2,500,000.

Estimate the depletion charge using a fixed percentage rate of 27.5% of net sales.

SOLUTION

The depletion charge is based on a 3-year period. Cost items for the first year (\$):

Net sales for 1,500,000 bbl at \$100/bbl = 15,000,000 Annual depreciation (\$45,000,000/g) = 5,000,000 First-year expenses = 2,500,000 The depletion allowance = (0.275)(15,000,000) = 412,250,000

In order to check on the criterion that the depletion allowance (\$412,250,000) does not exceed 50% of the net income (before allowing for depletion), the following calculations are carried out (in \$):

Net sales (revenue)		150,000,000
First-year expenses	2,500,000	
Development expenses	8,000,000	
Annual depreciation charges	<u>5,000,000</u>	
Total expenses		15,500,000
Total net income (profit)		134,500,000
50% of net income		67,250,000

Thus the maximum allowable depletion will be \$67.25 million and not \$41.25 million. The \$1,000,000 bonus in this problem is recovered as part of the depletion charge.

Example 5.8

Solve Example 5.7 using the cost-per-unit method, and then compare the two methods used in calculating the depletion allowance.

SOLUTION

The depletion charge using cost-per-unit method:

$$= \frac{\text{Sum of development and bonus costs}}{\text{recoverable oil reserves}}$$
$$= \frac{8000000 + 100000}{720000}$$
(5.10)
$$= \$12.5 \text{ /bbl}$$

The allowable depletion based on the cost-per-unit method for the first year:

= (12.5) (1,500,000) = \$18,750,000

This amount of \$18,750,000 would be allowed even if it exceeded the value permitted by the fixed-percentage method. However, the cost-perunit method must be used *each year* once it has been adapted.

5.4.3 Summary and Comparison

	0	1	5	
Basis	Percentage Depletion (\$)		Cost per Unit Depletion (\$)	
Annual depreciation	5,000,000		5,000,000	
Development expenses	8,000,000		Included by using Equation (5.10)	
First-year depletion	67,250,000		18,750,000	
Total first-year charges for capital recovery	80,250,000		23,750,000	
Second: Net Income fo	or First-Year (N	et Revenue – To	otal Costs)	
Net revenue (sales)	150,000,000		150,000,000	
Operating expenses	2,500,000		2,500,000	
Development expenses (100% incurred in first year)	8,000,000		(included)	
Depreciation expenses	5,000,000		5,000,000	
Depletion expenses	67,250,000		18,750,000	
Total costs		82,750,000		26,250,000
Net Income		67,250,000		123,750,000
Third: Net Income for	the Second Ye	ar ^a		
Net revenue (sales)		150,000,000		150,000,000
Operating expenses	2,000,000 ^b		2,000,000	
Depreciation expenses	5,000,000		5,000,000	
Depletion	41,250,000°		18,750,000	
Total costs		48,250,000		25,750,000
Net Income		101,750,000		124,250,000
- 4 1	· · · 1 · C· · ·			

First: Allowable First-Year Charges for Capital Recovery

^a Assume the same sales as in the first year.

^b Operating expenses for the second year are assumed to be less than for the first year.

^c For this case \$41,250,000 calculated by the fixed percentage method (27.5%) is less than the 50% criterion: 150,000,000 - 7,000,000(0.5) = \$146,500,000.

One can conclude from the above calculations that the percentage depletion method promotes the recovery of a greater amount of oil-reserve depleted value, or \$67,250,000 to \$18,750,000 for the cost-per-unit method. But there are no more development costs incurred after the first year's \$8,000,000. The \$800,000 was a large factor in determining the amount of depletion. Of course, new development charges could be incurred in other years and would then be included in determining the amount of depletion. Also, any additional development costs or any changes in estimated recoverable oil will require a recalculation of the cost-per-unit depletion rate, which is then used to determine depletion in subsequent years.

Although the net income by the cost-per-unit depletion method is greater in both the first and second years for the example given, the total net income plus capital recovery for 2 years added together by the percentage depletion is equal to the cost-per-unit depletion method total.

However, the percentage method has an advantage in a lower profits tax over the cost-per-unit method with reported lower net incomes for each year (in the first year, \$67,250,000 to \$123,750,000, and in the second year, \$101,750,000 to \$124,250,000). But the cost-per-unit method does have an economic advantage where rights to oil resources are purchased outright or leased at a relatively higher price to the seller because the net income figures are greater with this method. A comparison of both methods is shown in Figures 5.7 and 5.8. In these figures, bar charts for the years using both methods are shown.

Accounting for depletion can be complicated because of the uncertainties of future development costs, uncertainties about actual recoveries of oil from proven reserves, uncertainties about future value of oil reserves as selling prices go down or up, and uncertainties about the scale of operations, that is, the magnitude of production of oil. Variations in all or any of these factors may result in changes in the depletable value, necessitating separate calculations each year for the depletion charges.



FIGURE 5.7 (See Color Insert) Comparison for the first year.



FIGURE 5.8 (See Color Insert) Comparison for the second year.

When there is an increase in value of oil reserves as opposed to an increase in amount of proven oil reserves, or a big increase in selling prices, accretion rather than depletion is practiced to show the increase or "growth" in the oil reserve. When such an increase in value results, an allowance for it must be made in the accounts of the oil company.

Notation

- A_d , A_r , Annual depreciation and annual capital recovery defined by Equations (4.12) and (4.15), respectively
- *a*, A specific year in the useful lifetime (*n*)
- d, Annual depreciation rate (\$/yr)
- d_{a} , Annual depreciation rate for the year (a)
- *f*, Fixed percentage factor defined by Equation (5.3)
- \overline{f} , Accelerated depreciation factor defined by Equation (5.4)
- *n*, Number of useful (service) years of life
- $V_{a'}$ Value of an asset at year a
- $V_{o'}$ Original value of an asset (\$)
- V_{s} , Salvage value of an asset (\$), also referred to as V_{n}

6

Financial Measures and Profitability Analysis

Maha Abd El-Kreem

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The basic aim of financial measures and profitability analysis is to provide some yardsticks for the attractiveness of a venture or a project, where the expected benefits (revenues) must exceed the total production costs. There are many different ways to measure financial performance, but all measures should be taken in aggregation.

Profitability measures the extent to which a business generates a profit from the use of resources, land, labor, or capital. Behind the need for profitability is the fact that any business enterprise makes use of invested money to earn profits. Simply stated, profitability is measured by dividing the profits earned by the company by the investment (or money) used by the company

In this chapter, the mathematical methods for evaluating profitability—or the economic indicators—are presented. Their applications in solving problems encountered in the oil industry are illustrated and amplified with the help of many examples that are solved using Excel spreadsheets.

6.1 Introduction

Capital expenditure proposals must be sufficiently specific to permit their justification for exploration and production operations, surface petroleum operations, petroleum refining, and expansion purposes or for cost reduction improvements and necessary replacements. In reality, an evaluation of capital expenditure proposals is both technical and economic in nature. First, there are the technical feasibilities and validities associated with a project, and next come economic evaluation and viability.

In the economic phase of evaluation, oil management may find that it has more investment opportunities than capital to invest, or more capital to invest than investment opportunities. Whichever situation exists, oil management needs to resort to some economic criteria for selecting or rejecting investment proposals. Management's decision in either case is likely to be based largely on the measures of financial return on the investment.

The most common measures, methods, and economic indicators of economically evaluating the return on capital investment discussed in this chapter are:

- 1. Rate of return, or return on investment (R.O.I.)
- 2. Payment period (P.P.)
- 3. Discounted cash-flow rate of return (D.C.F.R.) and present value index (P.V.I.)
- 4. Net present value (N.P.V.)

No one method is by itself a sufficient basis for judgment. A combination of more than one profitability standard is needed to approve or recommend a venture. In addition, it must be recognized that such a quantified profitability measure would serve as a guide. Many unpredictable factors and uncertainties cannot be accounted for, specifically those in exploration and production operations.

6.2 Mathematical Methods for Evaluating Profitability

Classification of these methods into two groups is considered, where the time value of the cash flow received from a project is the criterion used in this classification:

1. Time value of money is neglected. Two methods fall in this group. They are known as the annual rate of return (R.O.I.) and the payment period (P.P.).

2. Time value of money is considered. Two methods represent this group. They are known as the discounted cash flow of return (D.C.F.R.) and the net present value (N.P.V.).

Based on this classification, the R.O.I. and P.P. are described as "rough" or "crude" quick methods, while the D.C.F.R. and N.P.V. are known to be accurate, realistic, and time-demanding indicators.

6.2.1 Annual Rate of Return (Return on Investment, R.O.I.)

The annual rate of return is defined by the equation:

Consideration of income taxes is provided in calculating the R.O.I. by using either "net" profit or "gross" profit.

For oil ventures, where the cash flow extends over a number of years, the average rate of return is calculated using an average value for the profit, by dividing the sum of the annual profits by the useful lifetime:

R.O.I. =
$$\left[\frac{\sum_{1=y}^{n} \text{ annual profits}}{n}\right] / (\text{capital investment})(100) \quad (6.1b)$$

The main drawback of this method is the fact that money received in the future (cash flow) is treated as money of present value (which is less, of course).

Example 6.1

It is necessary to calculate the R.O.I. for two projects involving the desalting of crude oil; each has an initial investment of \$1 million. The useful life of project 1 is 4 years and of project 2 is 5 years. The earnings pattern is given in Table 6.1.

SOLUTION

The average rate of return is calculated for both projects as shown in Table 6.1. The final answers are:

R.O.I. for project 1 = 16.25% R.O.I. for project 2 = 22.2%

		Income before	Depreciation	Net Earnings after	Income before	Depreciation	Net Earnings after
		Depreciation	Allowance	Depreciation	Depreciation	Allowance	Depreciation
	1	\$400,000	\$250,000	\$150,000	\$75,000	\$200,000	-\$125,000
	2	\$350,000	\$250,000	\$100,000	\$180,000	\$200,000	-\$20,000
Year	ю	\$300,000	\$250,000	\$50,000	\$300,000	\$200,000	\$100,000
	4	\$275,000	\$250,000	\$25,000	\$400,000	\$200,000	\$200,000
	5				\$600,000	\$200,000	\$400,000
Sum				\$325,000			\$555,000
Average Investment				\$500,000			\$500,000
Average Earning				\$81,250			\$111,000
Average Rate of Return			16.25%			22.20%	

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TABLE 6.1



A comparison is made between the two projects, as illustrated next.

Project 1

6.2.2 Payout Period (P.P.), Payback Time, or Cash Recovery Period

Payout period is defined as the time required for the recovery of the depreciable capital investment in the form of cash flow to the project. Cash flow would imply the total income minus all costs except depreciation.

Mathematically, this is given by Equation (6.2), where the interest charge on capital investment is neglected:

Payout period (years) (P.P.) =
$$\frac{\text{depreciable capital investment}}{\text{average annual cash flow}}$$
 (6.2)



FIGURE 6.1 Illustration of payout period (P.P.).

A hypothetical cumulative cash flow diagram, shown in Figure 6.1, illustrates some of the basic concepts, including the payout period. It is briefly described as follows:

- Investment for land (if needed) comes first, followed by investment for the depreciable asset throughout the construction period (points 1 and 2).
- The need for the working capital comes next for startup and actual production (points 2 and 3).
- Production starts now at point 3 (zero time) and goes all the way profitably to cross the zero cash line at point 4. This point corresponds to the time spent to recover the cumulative expenditure, which consists of capital of land + capital cost of depreciable assets + working capital. The payout period will accordingly be defined by point 4 that is, the time required to recover the depreciable capital only.
- Point 4 could be considered an alternative way (but different in value) to define payout period as the time needed for the cumulative expenditure to balance the cumulative cash flow exactly.

Usually oil companies seek to recover most of their capital investments in a short payback period, mostly because of uncertainty about the future and the need to have funds available for later investments. This becomes especially important when the company is short of cash—emphasis on rapid recovery of cash invested in capital projects may be a necessity.

The payback period is used by oil companies in ascertaining the desirability of capital expenditures, because it is a means of rating capital proposals. It is particularly good as a "screening" means relative to various capital proposals. For example, expenditures for units may not be made by an oil refinery unless the payback period is no longer than 3 years. On the other hand, the proposed purchase of a subsidiary may not be considered further unless the payback period is 5 years or less.

But payback has its drawbacks. For example, payback ignores the actual useful length of life of a project. Also, no calculation of income beyond the payback period is made. Payback is not a direct measure of earning power, so the payback method can lead to decisions that are really not in the best interests of an oil company.

Example 6.2

Calculate the payout period for the two alternatives of capital expenditures involving an investment of \$2 million each for a sulfur removal plant, as given in Table 6.2. The life of project 1 and project 2 is 6 and 10 years, respectively.

		Cash	Flow (S)
Year		Project 1	Project 2
0		2,000,000	2,000,000
1		1,500,000	200,000
2		500,000	300,000
3		400,000	400,000
4		350,000	400,000
5		250,000	400,000
6		200,000	400,000
7		100,000	400,000
8		—	400,000
9		_	400,000
10		_	400,000
	Cash flow	\$3,300,000	\$3,700,000
	Annual cash flow (\$/yr)	471,429	370,000
	P.P (yr)	4.24	5.41

TABLE 6.2

Cash Flow for the Sulfur Removal Plant (Example 6.2)

SOLUTION

From the cash flow given in payout Table 6.2, the payout period (P.P.) is calculated as follows:

 $(P.P)1 = 2 \times 10^{6}/471,429 = 4.24$ years

where \$471,429 is the average annual cash flow.

 $(P.P.)2 = 2 \times 106/370,000 = 5.41$ years

where \$370,000 is the average annual cash flow.



The pay period index would thus recommend project 1 in favor of project 2 (fewer years are required to recover the same initial capital increment). However, project 1, as shown in Table 6.2, ceases to generate any cash flow after the sixth year, while project 2 continues, through the added cash flow, to generate \$400,000 each year after the investment has been paid back in full at the end of the sixth year (P.P. is 7 years). It is pointless to select project 1 on the ground that over the period from year 7 to year 10, \$1.2 million would be generated by project 2, which makes a total of \$0.8 million more by project 2 over project 1 for the 10-year period.

Example 6.3

With reference to the investment made to procure boilers for surface facilities in an oil field, as shown in Table 6.3, calculate the payback period for each alternative and give reasons for selecting one and not the other.

SOLUTION

P.P. is readily calculated using Equation 6.2 as follows:

P.P. = 50,000(\$)
$$\left/ \frac{50,000 ($/yr)}{4} \right.$$

= 4 years, for both cases

	Cash	Flow
Year	Boiler 1	Boiler 2
0	50,000	50,000
1	20,000	5,000
2	15,000	10,000
3	10,000	15,000
4	5,000	20,000
Total cash flow	50,000	50,000
Payback period	P.P. 1 =	4 Years
	P.P. 2 =	4 Years

IABLE	0.3		

Comparison of Two Boiler Investment (Solution of Example 6.3)

As far as the P.P. as a criterion for choice, the number of years to recover the depreciable capital is the same for both types of boilers. However, the recovery of investment for boiler 1 is faster than for boiler 2 (for example, compare \$20,000 to \$5,000 for the first year). Therefore, from the standpoint of cost of money (time value of money), investment in boiler 1 is preferable to investment in boiler 2.



This example points out that when using the payout period method, oil management should also observe the rapidity of cash flows between alternatives. The alternatives may have the same number of years-topay-back as they do here, but one may be more favorable than the other because the largest amount of cash flow comes in the first few years. This could be an excellent point in favor of investment in one alternative over another when both have approximately the same payout periods. It could be a strong factor in selection of one especially if a greater amount of cash "back" is needed early in the investment.

6.2.3 Discounted Cash-Flow Rate of Return (D.C.F.R.) and Present Value Index (P.V.I.)

If we have an oil asset (oil well, surface treatment facilities, a refining unit, etc.) with an initial capital investment *P*, generating annual cash flow over a

lifetime *n*, then the D.C.F.R. is defined as the rate of return, or interest rate that can be applied to yearly cash flow, so that the sum of their present value equals *P*.

From the computational point of view, D.C.F.R. cannot be expressed by an equation or formula, similar to the previous methods. A three-step procedure involving trial and error is required to solve such problems. Example 6.4 illustrates the basic concepts.

Solved Example 6.4

Assume an oil company is offered a lease of oil wells which would require a total capital investment of \$110,000 for equipment used for production. This capital includes \$10,000 working money, \$90,000 depreciable investment, and \$10,000 salvage value for a lifetime of 5 years.

Cash flow to project (after taxes) gained by selling the oil is as given in Figure 6.2. Based on calculating the D.C.F.R., a decision has to be made: should this project be accepted?

Two approaches are presented to handle the D.C.F.R.

Year	Cash Flow (\$10 ³)
0	-110
1	30
2	31
3	36
4	40
5	43



FIGURE 6.2 Cash flow pattern.



FIGURE 6.3

D.C.F.R. for investment in a lease of oil wells.

First Approach: Using the Future Worth

Our target is to set the following equity: By the end of 5 years, the future worth of the cash flow recovered from oil sales (as shown in Figure 6.3) should break even with the future worth of the capital investment, had it been deposited for compound interest in a bank at an interest rate *i*.

This amounts to:

$$F_o = F_B \tag{6.3}$$

where $F_B = 110,000(1 + i)5$, for banking, and $F_o = \sum_{i=1}^5 F_i$ for oil investment which represents the cash flow to the project, compounded on the basis of end-of-year income.

Hence,

$$F_o = 30,000(1+i)^4 + 31,000(1+i)^3 + 36,000(1+i)^2 + 40,000(1+i) + 43,000 + 20,000$$

Notice that the \$20,000 represents the sum of working capital and salvage value; both are released by the end of the fifth year.

Setting up $F_B = F_{or}$, we have one equation involving *i* as the only unknown, which could be calculated by trial and error. The value of *i* is found to be 0.207—that is, the D.C.F.R. = 20.7%.

Second Approach: Using the Discounting Technique

Our objective here is to discount the annual cash flow to present values using an assumed value of *i*. The correct *i* is the one that makes the sum of the discounted cash flow equal to the present value of capital investment, *P*. The solution involves using the following equation:

$$P = \sum_{y=1}^{5} p_y$$

where

$$p_y = (\text{annual cash flow})_y \ d_y = (\text{A.C.F.})_y \left(\frac{1}{1+i}\right)^y$$

for the year *y*, between 1 and 5.

Another important criterion that can be used in order to arrive at the correct value of i in the discounting of the cash flow is given by the following relationship: D.C.F.R. is the value that makes P.V.I. = 1, where P.V.I. stands for the present value index and is defined by:

$$P.V.I. = \frac{\text{sum of discounted cash flow (present value)}}{\text{initial capital investment}}$$
(6.4)

The solution of this example applying the discount factor is illustrated in Table 6.4.

If the annual cash flow has been constant from year to year, say A \$/yr, then the following can be applied:

$$A\left[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \dots + \frac{1}{(1+i)^n}\right] = P$$
(6.5)

Multiplying both sides of Equation (6.5) by (1 + i)n, we get:

$$A[(1+i)^{n-1} + (1+i)^{n-2} + \dots + 1] = P(1+i)^n$$
(6.6)

The sum of the geometric series in the left-hand side is given by:

$$\frac{(1+i)^n - 1}{i}$$

Hence, Equation (6.6) can be rewritten in the form:

$$P(1+i)^{n} = A \frac{(1+i)^{n} - 1}{i}$$
(6.7)

It is interesting to point out that this equation is equivalent to Equation (6.3); that is, the future worth of *P*, if invested in the bank, is given by:

$$F_B = P(1+i)^n$$

The future worth of the annual cash flow received from oil investment (*A*), if compounded in a sinking-fund deposit, is given by:

$$F_o = A \frac{(1+i)^n - 1}{i}$$

Now, Equation (6.7) can be used to calculate directly the D.C.F.R. by trial and error knowing the values of *A*, *P*, and *n*.

The D.C.F.R. thus represents the maximum interest rate at which money could be borrowed to finance an oil project.

		i =	15%	i = 1	20%	<i>i</i> = <i>i</i>	25%	i = 2	0.7%
V ()			Present		Present		Present	į	Present
IEAL (y)	Cash FIUW	uy	value (\$)	uy	value (\$)	uy	(¢) antra	uy	ره) value
0	110,000								
1	30,000	0.8696	26,088	0.8333	24,999	0.8000	24,000	0.8290	24,870
2	31,000	0.7561	23,439	0.6944	21,526	0.6400	19,840	0.6870	21,297
3	36,000	0.6575	23,670	0.5787	20,833	0.5120	18,432	0.5700	20,520
4	40,000	0.5718	22,872	0.4823	19,292	0.4096	16,384	0.4720	18,880
5	43,000	0.4971	21,375	0.4019	17,282	0.3277	14,091	0.3910	16,813
	20,000								
	Total		117,444		103,932		92,747		102,380
	P.V.I.		1.07		0.94		0.84		0.93

 TABLE 6.4

 D.C.F.R. for Investment in Lease of Oil Wells
6.2.4 Net Present Value (N.P.V.)

The D.C.F.R. method is based on finding the interest rate that satisfied the conditions implied by the method. Here we provide a value for *i* that is an acceptable rate of return on the investment and then calculate the discounted value (present value) of the cash flow using this *i*. The net present value is then given by:

N.P.V. = (present value of cash flow discounted at a given i)

(6.8)

- capital investment

Example 6.5

Calculate the N.P.V. of the cash flow for the oil lease described in Example 6.4, if money is worth 15%.

SOLUTION

At i = 0.15, the annual cash flow is discounted. The present value of the sum of the cash flows = \$127,000. The N.P.V. is directly calculated using Equation (6.8):

N.P.V. = 127,000 - 110,000

That is, the oil lease can generate \$17,000 (evaluated at today's dollar value) over and above the totally recovered capital investment. The solution is illustrated in Table 7.5.

6.3 Comments on the Techniques of Economic Analysis

All four methods described above determine the return on investment or the attractiveness of a project.

To evaluate whether a project, or a proposal on a project for the future, is yielding, or will yield, a good or bad return, the R.O.I. must be compared to a standard acceptable level of profit which the oil company wishes to maintain. The internal cutoff rate (or breakeven point for return) is the cost of capital, which is the rate of borrowing money at the time of use of these measures for calculating return on investment. There is no precise agreement on how oil management calculates cost of capital, but it should include both the cost of borrowed funds and the cost of equity financing (when applicable):

1. As mentioned previously, the R.O.I. and P.P. are economic indicators to be used for rough and quick preliminary analyses. The R.O.I. method does not include the time value of money and involves some approximation for estimating average income or cash flow. The P.P., on the other hand, ignores the useful life of an asset (later years of project life) and does not consider the working capital.

2. The D.C.F.R. and N.P.V. are regarded as the most generally acceptable economic indexes to be used in the oil industry. They take into account the following factors:

Cash flows and their magnitude

Lifetime of project

Time value of money

Although the D.C.F.R. involves a trial-and-error calculation, computers can be easily used in this regard. The D.C.F.R. is characterized by the following:

It gives no indication of the cash value.

It measures the efficiency of utilizing a capital investment.

It does not indicate the magnitude of the profits.

It is recommended for projects where the supply of capital is restricted and capital funds must be rationed to selected projects.

The N.P.V. method, on the other hand, is considered to measure "profit." The values reported by the N.P.V. yield the direct cash measure of the success of a project; hence they are additive (compare with D.C.F.R.).

- 3. Any of the methods described in this chapter and proposed for economic evaluation in oil projects should be used with discretion and with due regard for its merits and demerits. Each index provides limited knowledge that is helpful in making project decisions. No major investment decision should be totally based on a single criterion. A more careful study should be considered for oil projects ending in different conclusions as a result of using different economic indicators.
- 4. Other important factors to consider in economic evaluation are discussed next. Every oil company has to consider that certain investments will not yield a "measurable profit," because some investments may be needed to improve employee or community goodwill or to meet legal requirements of the government under which the oil operations are located. For example, investments in equipment to reduce air or water pollutants and investments in the social well-being of the community may not contribute dollars to equity of a company. These are examples of those investments that will not yield a measurable profit. And oil companies must face some of these "opportunities," especially when their operations are in countries other than that in which their main administrative offices are located.

An efficient oil company is aware of this type of investment and makes plans in advance for these expenditures. Oil management must, therefore, increase its return on those investments yielding measurable profits accordingly, so that the portfolios of profit and nonprofit investments taken together yield a sufficient overall return. For example, suppose that an oil company has calculated its "needed" return (or cost of capital for owners of the oil company) to be approximately 15%. But 25% of its investments are nonprofit, or "necessity," projects. This means that 75% of its investments are "profitable" ones. To cover the 25% that are nonprofit investments, the returns on the 75% that are profitable will have to be approximately 20% ($15\% \div 75\%$). Thus, oil companies need not only appraise all potential investments individually but also constantly view the position of their portfolios of profit and nonprofit investments taken together.

6.4 Model Examples

Example 6.6

If an oil company expects a cash flow of \$800,000 by the end of 10 years, and 10% is the current interest rate on money, calculate the N.P.V. of this venture.

SOLUTION

No capital investment is involved here, so the problem is simply a discounting procedure.

The present value of the cash flow

= 800,000(1 + 0.1) - 10= \$308,000

Example 6.7

Assume that a distillation unit with an initial cost of \$200,000 is expected to have a useful life of 10 years, with a salvage value of \$10,000 at the end of its life. Also, it is expected to generate a net cash flow above maintenance and expenses amounting to \$50,000 each year. Assuming a selected discount rate of 10%, calculate the N.P.V.

SOLUTION

The present value of the annual cash flow can be found using Equation (6.7):

$$P = A \frac{(1+i)^n - 1}{i(1+i)^n}$$

= 50,000 $\frac{(1.1)^{10} - 1}{0.1(1.1)^{10}}$
= 50,000(6.144) = \$307,25

where this factor is readily obtained from tables found in Appendix A. Calculations are given in Table 6.6.

If alternative investment proposals are to be considered, the above calculations can be made for each proposal to discover which alternative is the most promising for investment in terms of present value. This will be the topic of Chapter 7.

The selection of 10% as the discount rate factor is arbitrary. If money is borrowed for investment, the cost of the loan is usually the discount rate, or sometimes the assumed cost of retained income if money used in the investment is from one's own internal funds and is not borrowed. Adjustment for the time value of money requires the selection of a discount rate. In the above example, at the rate of 10%, the present value of the future stream of cash (cash flow) of \$50,000 annually for 10 years is \$307,250. On the present value of future cash flow, it is obvious that a lower discount rate generates a higher present value; also, a higher discount rate generates a lower present value.

Example 6.8

A feasibility study carried out for an oil company indicated that it is possible to invest \$1 million in either one of two projects. Anticipated cash flows generated by the two projects over the useful lifetime are given in Table 6.5.

- 1. Give your recommendations of which project you choose based on the N.P.V. Use selected values for the discount interest rate (more than one).
- 2. Compute the D.C.F.R. for each project.

SOLUTION

For (1), calculation is done for three different discount interest rates, 8%, 10%, and 12%, as shown in Table 6.6. In addition, a graphic plot is presented (Figure 6.4) for the change of the discounted value (present value) of the cash flows for both projects with the discount rate.

In summarizing the results of Table 6.6, if the cash flows of project 1 and project 2 are discounted at 8%, project 2 is preferable; if the cash flows are discounted at 10%, project 2 is preferred to project 1 because the present value of project 2 is almost \$14,000 more; and if the cash flows are discounted at 12%, project 1 is slightly preferable to project 2 and will continue to be preferable to project 2 as discount rates go higher than 12%.

Therefore, as the example shows, the choice between the two projects depends on the discount rate used. Usually, the oil company's cost of capital for investing in the project will determine which project is selected.

Figure 6.4, on the other hand, gives the present value curves for both project 1 and project 2 resulting from the three discount rates used. The "point of indifference" appears to be between 10% and 12%. Before this point, project 2 has the more favorable present value; after this point, project 1 is favored. As discount rates become higher past the "point of indifference," project 1 will continue to be more desirable for investment purposes. From

		1	= 5%	i	= 20%	i	= 25%	<i>i</i> =	20.7%
Year (y)	Cash Flow	dy	Present Value (\$)	dy	Present Value (\$)	dy	Present Value (\$)	dy	Present Value (\$)
0	110,000								
1	30,000	0.8696	26,088	0.8333	24,999	0.8000	24,000	0.8290	24,870
5	31,000	0.7561	23,439	0.6944	21,526	0.6400	19,840	0.6870	21,297
3	36,000	0.6575	23,670	0.5787	20,833	0.5120	18,432	0.5700	20,520
4	40,000	0.5718	22,872	0.4823	19,292	0.4096	16,384	0.4720	18,880
5	43,000	0.4971	21,375	0.4019	17,282	0.3277	14,091	0.3910	16,813
	20,000								
	Total		117,444		103,932		92,747		102,380
	P.V.I.		1.07		0.94		0.84		0.93
	N.P.V.		\$7,444.40		-\$6,067.70		-\$17,252.90		-\$7,620.00

	e of Oil Wel
	ı a Lease
	Investment in
TABLE 6.5	D.C.F.R. for

TABLE 6.6

Distillation unit cost	\$200,000	
Useful life/year	10	
Salvage value	\$10,000	
Net cash flow each year	\$50,000	
D.C.F.R.	\$10	
Present value of cash flow of \$50,000 annually, for 10 years at 10%	\$307,250	
Present value of cash flows for 10 years, Minus original investment of \$200,000	\$107,250	
Present value of \$10,000 salvage value to be received at the end of years at 10%	\$3,860	
Total value of net cash receipts plus present value	\$111,110	

the data in Table 6.7, it can be seen that at a discount of 12%, the present value of cash flow from project 2 is \$1 million; and at a discount of over 12%, the present value of cash flow gives us the discount rated amount of under \$1 million. This analysis of the present value of cash flow gives us the discount rate at which anticipated cash flow equals the initial investment, which is the D.C.F.R. For project 2, it is about 12%; for project 1, it is about 13%.





Statement of Example 6.8					
Year	Project 1	Project 2			
1	400,000	100,000			
2	320,000	200,000			
υ	200,000	300,000			
4	300,000	400,000			
ъ	100,000	500,000			
Total anticipated cash flow	1,320,000	1,500,000			
		At 8% Discour	ıt		
Year	Project 1			Project 2	
	Discount Factor for 8%	Cash Flow	Discounted Value (col. 1 * col. 2)	Cash Flow	Discounted Value (col. 1 * col. 4)
	1	6	3	4	υ
1	0.926	400,000	370,000	100,000	92,600
2	0.856	320,000	273,920	200,000	171,200
3	0.794	200,000	158,800	300,000	238,200
4	0.735	300,000	220,500	400,000	294,000
D	0.681	100,000	68,100	500,000	340,500
			\$1,091,723		\$1,136,505
					(Continued)

TABLE 6.7

Discount	
At10%	

Year	Projec	ct 1		Project 2	
	Discount Factor for 10%	Cash Flow	Discounted Value (col. 1 * col. 2)	Cash Flow	Discounted Value (col. 1 * col. 4)
	1	2	e	4	5
1	0.909	400,000	363,600	100,000	006'06
2	0.826	320,000	264,320	200,000	165,200
ę	0.751	200,000	150,200	300,000	225,300
4	0.683	300,000	204,900	400,000	273,200
сл	0.621	100,000	62,100	500,000	310,500
			\$1,045,123		\$1,065,105
		At 12% Discou	nt		
Year	Projec	ct 1		Project 2	
	Discount Factor for 12%	Cash Flow	Discounted Value (col. 1 * col. 2)	Cash Flow	Discounted Value (col. 1 * col. 4)
	1	2	3	4	5
1	0.893	400,000	357,200	100,000	89,300
2	0.797	320,000	255,000	200,000	159,400
3	0.712	200,000	142,000	300,000	213,600
4	0.636	300,000	190,800	400,000	254,400
5	0.567	100,000	56,700	500,000	283,500
			\$1,002,143		\$1,000,205

Notation and Nomenclature

- D.C.F.R., Discounted cash flow rate of return (percentage), defined by Equation (6.3)
- $d_{y'}$ Discount factor, $(1 + i)^{-n}$, for the year *y*

 F_{B} , Future worth of an investment if deposited in the bank

 F_{o} , Future worth of compounded cash flows, generated from oil project

N.P.V., Net present value (\$)

P, Present value of an asset equals sum of discounted cash flows and is given by: $P = \text{sum of } p_y$ over the years, y = 1 to y = n

P.P., Payout period (years), defined by Equation (6.2)

- P.V.I., Present value index (dimensionless), defined by Equation (6.4)
- R.O.I., Return on investment, defined as the annual rate of return (percentage) by Equations (6.1a) and (6.1b).

y, Designates a year within the lifetime *n*

7

Analysis of Alternative Selections and Replacements

Khaled Zohdy

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The aim of this chapter is to enable the selection of alternatives, or decisions on replacements for which capital might be required during a period of time. Examples are cited for many engineering projects, particularly in the oil industry. Fundamental aspects of studies are presented for:

- 1. The selection of the most economical alternative among different equipment, different technical processes, or different engineering systems that all do the same job. Emphasis is on mutually exclusive choices.
- 2. Decision on replacement or retirement of an asset or equipment because of changes in service requirements, obsolescence, improved (more efficient) alternative, or other causes.

7.1 Introduction

Decisions involve a choice among a number of possible courses of action. Making a decision should be a simple matter, provided that the problem is clearly stated and (in the field we are addressing) the economic approach is well defined. Many examples can be cited in the oil industry where management, engineers, geologists, and others have to make a choice among alternative projects. The choice can assume many different aspects, for example, the choice among alternative processes proposed for enhanced oil recovery in oil fields, among alternative methods of cooling process streams in gas plants, or among alternative designs of heat exchangers, waste-heat boilers, pumps, or any piece of equipment.

As an example, an oil company is offered a lease of a group of oil wells in which primary production is nearing completion, and the major condition of this offer is to undertake a secondary recovery project (water injection) by the end of the fifth year. The capital investment of this project is estimated to be \$650,000. In return, the revenue in the form of cash flow realized from this lease is as follows:

\$50,000/year for the first 4 years

\$100,000/year for the first 4 years from the 6th to 20th years

A comparison has to be made between the two alternatives: To invest or not to invest? In other words, should the project be accepted or not?

Such a situation could be handled by using the annual cost/present worth economic approach as will be explained later.

As illustrated in Figure 7.1, economic alternatives can be classified into two main categories:

- 1. To choose among different ways to invest money not necessarily to accomplish the same job, in which case the decision is influenced by management rather than by technical people.
- 2. To choose among alternative assets or equipment doing the same job, where mutually exclusive choices are considered and the decision is made by technical people. Mutually exclusive projects imply that when two alternatives are compared, one project or the other is selected (but not both).



FIGURE 7.1

Categories and domain of economic alternatives.

In this chapter, consideration is given to problems that fall under category 1 only. Key consideration should be given to the fact that many cases of alternative analysis can be handled with the "differential technique" or finding the "rate of return on the extra investment" for the difference between two alternate investments.

The following methods are recommended for choosing between alternatives:

- 1. Differential approach (Δ approach) or return on extra investment (R.O.E.I.)
- 2. Total equivalent annual cost (T.E.A.C.)/present value method
- 3. Total capitalized method

In addition, it is important to identify the problem at hand as one of two types:

- 1. *Profit or income expansion*: where revenues (cash flows) are generated, and maximization of the profit is required
- 2. *Cost reduction*: where no cash flows are given; instead expenses are known and reduction in costs is the criterion

Replacement analysis, on the other hand, can be considered some sort of alternative analysis for investment tied up with an old asset versus an additional or a replacement investment. This situation is encountered to replace worn, inadequate, or obsolete equipment and physical assets.

7.2 Differential Approach (Δ Approach), or Return on Extra Investment (R.O.E.I.)

The differential approach is a concept that could be applied for selection among alternatives for a group of equipment, plants, processes, or oil-related venture projects. The principle of minimum capital investment is applied in this method in the following sense: For a set of alternatives needed for a given job and doing the same function, choose the minimum investment as the *base plan*.

The differential approach to be used as a criterion for selecting alternatives is summarized by the following steps:

- 1. Select the minimum capital investment (C.I.) as our base plan, compute Δ C.I. (difference in capital investment) for the alternatives.
- 2. Compute Δ profit (difference in cash income) for the alternatives, for the income-expansion problem, and Δ saving (difference in annual costs) for the alternatives, for the cost-reduction problem.

3. Calculate the rate of return on extra investment (R.O.E.I.) as follows:

$$\Delta \text{C.I.} \rightarrow \text{profit; R.O.E.I.} = \left(\frac{\Delta \text{profit}}{\Delta \text{C.I.}}\right) 100$$

$$\Delta \text{C.I.} \rightarrow \text{saving; R.O.E.I.} = \left(\frac{\Delta \text{saving}}{\Delta \text{C.I.}}\right) 100$$
(7.1)

4. Check to see that the preferred choice has an R.O.E.I. greater than a minimum value prescribed by management.

For alternatives involving small increments in capital investment, the best (most economical) alternative is arrived at by either graphic or analytical solutions. The following solved examples illustrate these principles.

Example 7.1

In the alkanolamine sweetening process of natural gas, two types of coolers have been suggested for the amine solvent: type A and type B. Using the data given next, recommend which alternative should be used if both types are acceptable technically. The minimum rate of return on money invested is 15% and the economic lifetime is 10 years for the coolers.

SOLUTION

Consider straight-line depreciation of 10% of C.I. The problem is a cost-reduction type.

	Tuno A	Type B	
	Type A	Type D	
Capital investment (CI)	10,000	15,000	
<i>n</i> , years	10	10	
Average depreciation = CI/n	1000	1500	
Average operational cost	3000	1500	
Total annual cost = Average depreciation + Average operational cost	4000	3000	
Average rate of return (given)	0.15	0.15	
Difference in CI			5000
Difference in annual cost (saving)			1000
Annual percentage saving = difference in annual cost (saving)/difference in CI			20%

Rate of return on the extra capital is greater than 15%.

Therefore, Type B is recommended.

Assuming that *n* for type B changes between 5 to 15 years, plot the annual % saving versus *n*, as shown in Figure 7.2.



FIGURE 7.2 Change of A% saving versus lifetime of type B.

Example 7.2

Instead of flaring the associated natural gas separated along with crude oil, it was decided to recover the lost heat by using the waste-heat recovery system (W.H.R.S.). For *pilot test runs,* four designs were offered; each has a lifetime of 5 years. The savings and costs associated with each are as follows:

	Type 1	Type 2	Type 3	Type 4
Capital investment(CI)	10,000	16,000	20,000	26,000
<i>n</i> , years	5	5	5	5
Average depreciation = CI/n	2000	3200	4000	5200
Average operational cost	100	100	100	100
Total annual cost = average depreciation + average operational cost	2100	3300	4100	5300
Revenue (income) \$/yr	4100	6000	6900	8850
Annual profit	2000	2700	2800	3550
R.O.I.	20.0%	16.9%	14.0%	13.7%

All four designs seem to be acceptable as far as the minimum annual rate of return (R.O.I.), exceeding 10% (required by management). Which design is to be recommended?

SOLUTION

Using incremental comparison:

	1	2	3	4
First comparing 1 to 2	Acceptable as a basis	11.7%	_	_
Second comparing 2 to 3	_	Basis	2.5%	_
Third comparing 2 to 4	—	Basis	—	8.5%

Conclusion

Design 2 is recommended; it gives more profit than design 1 while return on extra investment (R.O.E.I.) is 11.7%, which is >10% (minimum). Figures 7.3 and 7.4 are bar charts to illustrate the solution of the problem.



FIGURE 7.3

Change of ROI% savings versus different types.

Example 7.3

Insulation thickness is important for heat exchangers in the oil industry. One situation was encountered in the sulfur recovery plant from hydrogen sulfide gas (H₂S) (which has to be removed from natural gas). A heat exchanger was designed and recommendation was made for four possible thicknesses of insulation. The costs and savings related to these cases are as follows. Which one is recommended for 15% minimum R.O.I.?

SOLUTION

For 15% minimum R.O.I., calculations indicate that all four proposals are acceptable, since they generate R.O.I greater than 15%, each. Now, we can apply the differential approach as indicated above. However, let us use the graphic analysis technique, since the problem involves small-investment increments. Referring to Figure 7.5, the annual savings/C.I. curve is drawn as shown using the above data. As can be seen, by increasing the C.I., the annual savings are increased until we hit the optimum point, *M*, which represents the maximum savings. Then, by drawing our tangent line at *P*, we can achieve an R.O.E.I. of about 17% when using C.I. of nearly \$1,600, or an insulation of 2-inch thickness.



FIGURE 7.4 Incremental comparison versus different types.



FIGURE 7.5 Differential solution.

The return on extra investment (R.O.E.I.) or differential method has one major drawback if applied to alternatives with different economic lifetimes. This puts a constraint on using it for these situations, which can be handled by other methods to be discussed next.

Parameter	1-Inch Insulation	2-Inch Insulation	3-Inch Insulation	4-Inch Insulation
Cost of insulation (\$)	1,200	1,600	1,800	1,870
Savings (Btu/hr)	300,000	350,000	370,000	380,000
Value of savings (\$/yr) ^a	648	756	799	821
Annual depreciation cost (\$/yr) ^b	120	160	180	187
Annual profit (\$)	528	596	619	634
R.O.I.	44.0%	37.3%	34.4%	33.9%

^a Based on \$0.3 per million Btu of the heat recovered and 300 working days per year.

^b Based on 10-year lifetime.

	1	2	3	4
First comparing 1 to 2	Acceptable as a basis	17.0%	_	_
Second comparing 2 to 3	_	Basis	11.5%	_
Third comparing 2 to 4	_	Basis	—	14.1%



FIGURE 7.6

Change of ROI% savings versus different types.

Conclusion

Design 2 is recommended; it gives more profit than design 1 while return on extra investment (R.O.E.I.) is 17%, which is greater than 15% (minimum).

Figures 7.6 and 7.7 are graphical plots to illustrate the results obtained in solving this example.

7.3 Total Equivalent Annual Cost (T.E.A.C.)/ Present Value Method

In this method, all costs incurred in buying, installing, operating, and maintaining an asset are put on the some *datum*—that is, on *annual basis*. Generally, the annual equivalent costs are brought to the present value for all alternatives.



FIGURE 7.7 Change of T.E.A.C. versus different types of pumps.

Specifically, the T.E.A.C. is the sum of the annual cost of capital recovery (initial capital plus interest on it) and other annual operating costs. (Remember that depreciation costs cannot be included with the annual operating costs. They are taken care of in the cost of capital recovery.)

T.E.A.C. =
$$A_r$$
 + other annual operating costs (7.2)

where

$$A_r = P\left(\frac{i(1+i)^n}{(1+i)^n - 1}\right)$$
(4.26)

Example 7.4

Recommend which arrangement to select out of the following two cases, where energy saving is required by using higher capital investment.

	Pump with Control Discharge Valve (I)	Pump with a Variable Speed Drive (II)
C.I. (\$)	13,000	17,000
Annual cost of energy for pumping (\$)	6,000	2,800
Annual maintenance costs (\$)	1,500	3,000
Lifetime (yr)	10	10

Assume i = 10%, and the salvage value is negligible.

SOLUTION

For system I:
$$A_r = 13,000 \left[\frac{0.1(1.1)^{10}}{(1.1)^{10} - 1} \right]$$

= $\frac{\$2,116}{10}$
For system II: $A_r = 17,000 \left[\frac{0.1(1.1)^{10}}{(1.1)^{10} - 1} \right]$

= \$2,767

T.E.A.C. for I = 2,116 + 6,000 + 1,500

T.E.A.C. for II: = 2,767 + 2,800 + 3,000

	Pump with Control Discharge Valve (I)	Pump with a Variable Speed Drive (II)
Capital investment(CI)	13,000	17,000
Lifetime (yr)	10	10
Annual maintenance costs (\$)	1500	3000
Annual cost of energy for pumping (\$)	6000	2800
A_r	2116	2767
T.E.A.C.	9616	8567

Assume i = 10%, and the salvage value is negligible. Which design is to be recommended?

System II is recommended, since T.E.A.C. is less than for system I. Figure 7.8 is a graphical plot to illustrate this result.

Example 7.5

GIVEN

Consider two possibilities relative to the purchase of a heat exchanger for an oil refinery to replace an older model for which annual costs are running around \$20,950. Other details are as follows:

Purchase possibility A is a heat exchanger constructed with materials of steel and copper. Its investment cost is \$15,000. Its economic service life is estimated to be 10 years, and salvage value at the end of the 10th year is estimated at \$500. Annual labor,





maintenance, repairs, and operational expenses are estimated at \$11,500; other annual direct costs are 4% of the investment cost of \$15,000, or \$600, when operating under optimum conditions.

Purchase possibility B is a stainless steel heat exchanger with an investment value of \$40,000. Its economic life is also regarded as 10 years, with a scrap value of \$1,000 at the end of the 10th year. Annual labor, maintenance, repairs, and operational expenses are estimated at \$4,000; other annual direct costs are 7% of the investment cost of \$40,000, or \$2,800, when operating under optimum conditions.

The current cost of capital is 8%.

FIND

Using the annual cost method, determine which purchase possibility would be more economical with respect to annual costs.

SOLUTION

Purchase Possibility A with Capital Recovery, Formula "Find A, Given P"		Purchase Possibility B with Capital Recovery, Formula "Find A, Given P"		
(Original cost – salvage value) (recovery factor) + (salvage value) (interest rate)		(Original cost – salvage value) (recovery factor) + (salvage value) (interest rate)		
(\$15,000 - \$500)(0.1490) + (\$500)(0.08) = 2,201 capital recovery of original cost and salvage value		(\$40,000 - \$1,000)(0.1490) + (\$1,000) (0.08) = \$5,891 capital recovery of original cost and salvage value		
Summary of annual costs with capital recovery		Summary of annual costs capital recovery	with	
Recovery of capital	\$ 2,201	Recovery of capital	\$ 5,891	
Annual costs, maintenance, repairs	11,500	Annual costs, maintenance, repairs	4,000	
Annual costs, optimum 600 conditions		Annual costs, optimum conditions	2,800	
Total annual costs\$14,301		Total annual costs	\$12,691	

With a potential savings in annual cost of \$1,610 (\$14,301 – \$12,691) in favor of the stainless steel heat exchanger, purchase possibility B appears to be the more feasible "buy" according to the annual cost method. (Only differences in costs, with cost items common to both purchase possibilities, were used.) Furthermore, the salvage value of the stainless steel exchanger (\$1,000) is \$500 more than for the steel–copper exchanger.

The annual cost method is used where the same costs for each alternative recur annually almost in the same manner. For a series of costs that are non-uniform, an average annual cost equivalent might be calculated. For alternatives with different lifetimes, the time period for comparison might be that of the alternative with the shortest life. Whereas the annual cost method does not give the relative amounts of capital, the present value method does. The present value method reduces all costs to equivalent capital at a given date.

	A: (Steel and Copper) Heat Exchanger	B: Stainless Steel Heat Exchanger
Capital investment cost (CI)	15,000	40,000
Lifetime (yr)	10	10
Salvage value	500	1000
Annual labor, maintenance, repairs, and operational expenses costs (\$)	11,500	4000
Annual direct costs (\$)	600	2800
The current cost of capital is 8%	1200	3200
Capital recovery	2201	5891
Total annual costs	14,301	12,691

SUMMARY

Purchase possibility B appears to be the more feasible "buy" according to the annual cost method. (Only differences in costs, with cost items common to both purchase possibilities, were used.) Furthermore, the salvage value of the stainless steel exchanger (\$1,000) is \$500 more than for the steel–copper exchanger.

The graphical plot in Figure 7.8 illustrates the solution of this example.

Example 7.6

GIVEN

Assume the same two heat exchangers given in Example 7.5, with the same annual costs, economic lives, salvage values, and investments, and with the cost of capital once again 8%.

WANTED

Compare the two alternatives using the present worth values for each of the possibilities, as well as total equivalent capital at the "present" time of consideration of purchase of heat exchangers.

SOLUTION

Using the present value method, a series of known uniform annual costs are reduced to an equivalent present value. This allows one to estimate the dollar value at the present time that is equivalent to the amount of annual costs for some fixed years of service by two alternatives. But uniform annual costs must first be determined, and this is what the present value method does. (The annual cost method does not determine uniform annual costs.)

Now, for each of the possibilities, the present values of installations and the salvage values must be added and deducted, respectively, to current value of annual costs for 10 years in order to get total equivalent capital requirements.

The following calculations are carried out to find the equivalent capital at 8%.

		Purchase Possibility A	Purchase Possibility B
1.	Present value of original (initial) costs	\$15,000	\$40,000
2.	Present value of salvage value; formula "Find <i>P</i> , Given <i>F</i> ," or factor	\$500 × 0.4632 = 232	\$1,000 × 0.4632 = 463
3.	Present value of annual costs: (total costs) × factor of formula "Find <i>P</i> , Given <i>A</i> "	\$12,100 × 6.710 = \$81,191	\$6,800 × 6.710 = \$45,628
4.	(1) - (2) + (3)	\$95,959	\$85,165

A comparison of the calculations for equivalent capital involved, \$95,959 for possibility A and \$85,165 for possibility B for the present time on an economic basis, indicates \$10,794 less favoring possibility B. In other words, a savings of \$1,610 in annual costs, as given by the annual cost method, favoring possibility B is reflected in a \$10,794 reduction in equivalent present value of possibility B when annual costs are uniform and determined with the use of total "present" equivalent capital at 8% of the present value method.

Under conditions of low interest rates, the present or current value of possibility B, which is \$40,000, can still be less than the current worth of \$15,000 for possibility A. Thus, the interest rate is important in order to determine the present value. Lower interest rates, such as say 5% instead of 8%, favor even more the use of higher initial investments, in this case the \$40,000 stainless steel heat exchanger rather than the \$15,000 steel-copper exchanger, because the relative cost for the use of money is lower. Example 7.7 confirms these results. A summary of the solution using Excel is provided along with a graphical chart as shown in Figure 7.9.

	A: (Steel and Copper) Heat Exchanger	B: Stainless Steel Heat Exchanger
Capital investment cost (CI)	15,000	40,000
Lifetime (yr)	10	10
Salvage value * 0.4632	232	463
Annual labor, maintenance, repairs, and operational expenses costs (\$)	11,500	4000
Annual direct costs (\$)	600	2800
Total annual costs	12,100	6800
Present value of annual costs	81,191	45,628
Total "present" equivalent capital at 8%	95,959	85,165

Summary



FIGURE 7.9

Change of total annual costs for different types of H.E.

Example 7.7

Compare the relative annual costs and current present values of the two alternatives in Examples 7.5 and 7.6 for 10 years of service if money is worth 5% instead of 8%.

SOLUTION

(a) For the annual cost method at 5%:

	Purchase Possibility A	Purchase Possibility B
Annual Costs		
Capital recovery = $$14,500 \times 0.1295$ ("Find <i>A</i> , Given <i>P</i> ") + (0.05)(\$500) =	\$ 1,903	\$39,000 × 0.1295 + (0.05)(\$ 1,000) = 5,101
Labor, maintenance, etc.	11,500	4,000
Other direct costs	600	2,800
Total annual costs	\$13,903	\$11,901

Compared to the costs when the interest rate is 8% (see Example 7.5), total annual costs for each possibility are lower when the interest rate is 5%. But the *difference* in annual costs is greater when the interest rate is lower. At 8% the difference is \$1,610 less in favor of possibility B, whereas at 5% it is \$2,002 in favor of possibility B. Thus, lower costs of borrowing favor alternatives with large investment amounts more than alternatives with lower investment amounts.

(b) For the present value (present worth) at 5%:

	Purchase Possibility A	Purchase Possibility B
Present worth of original (initial) costs	\$15,000	\$40,000
Present worth of salvage value ("Find P, Given F")	\$500 × 0.6139 = 307	\$1,000 × 0.6139 = 614
Present worth of annual costs ("Find P, Given A")	\$12,100 × 7.722 = 93,436	\$6,800 × 7.722 = 52,509
Total "present" equivalent capital at 5%	\$108,129	\$91,895

At 5%, the equivalent capital for purchase possibility B is \$16,234 (\$108,129 – \$91,895) less than purchase possibility A. With lower interest rates, differences in equivalent capital are greater: \$16,234 between alternatives at 5% and \$10,794 between alternates at 8%. However, total present equivalent capital amounts are greater with lower interest rates: Totals at 5% are \$108,129 and \$91,895 for possibilities A and B, respectively, and at 8% are \$95,959 and \$85,165.

A comparison of these results and those obtained when money is worth 8% shows that (a) the time-money series is equivalent to larger capital requirements, and (b) the difference in equivalent present value is greater in favor of purchase possibility B than it is for A when money is worth only 5%.

7.4 Total Capitalized Costs (T.C.C.)

T.C.C. was defined in Chapter 4 as the total accumulated sum of money that provides the capital cost of the new equipment, C_v and guarantees a continuous replacement of the asset by the end of its economic lifetime.

The value of T.C.C. (called *K*) is computed using Equation 4.22:

$$K = V_s + \left(\frac{C_R (1+i)^n}{(1+i)^n - 1}\right) + \frac{\text{annual operating expenses}}{i}$$
(4.22)

The capitalized cost is recognized as some form of perpetuity. The method is highly recommended for comparing alternatives having different lifetimes. The alternative having the least value of *K* is the one to be selected.

Example 7.8

The overhead condenser in a stabilization unit of a natural gasoline plant has to be made of corrosion-resistant material. Two types are offered; both have the same capacity (surface area); however, the costs are different because of different alloying materials:

	Condenser A	Condenser B
C.I. (\$)	23,000	39,000
n (years)	4	7

If money can be invested at 8%, which condenser would you recommend based on the T.C.C.?

SOLUTION

$$K_A = 23,000 \left(\frac{(1.08)^4}{(1.08)^4 - 1} \right)$$

= \$86,000
$$K_B = 39,000 \left(\frac{(1.08)^7}{(1.08)^7 - 1} \right)$$

= \$93,000

Therefore, condenser type A is selected (lower *K*).

Example 7.9

Solve Example 7.5 using the capitalized cost technique for 8% and 5% annual interest rates.

SOLUTION

Two methods are presented:

1. Using the relationship given by Equation (5.22), (direct application):

(a) For i = 8%:

	Purchase Possibility A	Purchase Possibility B
n (year)	10	10
C_{R} (\$)	14,500	39,000
V _s (\$)	500	1,000
Total operating cost (\$/yr)	12,100	6,800
$K_A(\$) = 500 + 14,500(1.8629) + \frac{12,100}{0.08}$	$K_B(\$) = 1000 + 39,000(1.8629) + \frac{6800}{0.08}$	
= \$178,762	= \$158,653	
(b) For $i = 5\%$:		
$K_A = 500 + 14,500(2.59) + \frac{12,100}{0.05}$	$K_b = 100 + 3$	$39,000(2.59) + \frac{6800}{0.05}$
= \$280,055	= \$238,0	010

2. Using step-by-step procedure (detailed):

(a) Capital requirements through capitalization, *with interest at* 8%, are as follows:

	Purchase Possibility A	Purchase Possibility B
Total annual costs:		
Net capital invested factor of formula ("Find <i>A</i> , Given <i>F</i> ")	\$14,500 × 0.06903 = \$1,001	\$39,000 × 0.06903 = \$2,692
Annual labor, maintenance, operational costs, etc.	11,500	4,000
Other direct annual costs	600	2,800
Total annual costs to be capitalized	\$13,101	9,492
Capitalization of annual costs	\$13,101/0.08 = 163,763	\$9,492/0.08 = 118,650
Initial costs of annual costs	15,000	40,000
Total capitalized cost when money is worth 8%	\$178,763	\$158,650

(b) Capital requirements through capitalization, with *interest at* 5%, are as follows:

	Purchase Possibility A	Purchase Possibility B
Total annual costs:		
Net capital invested factor of formula ("Find <i>A</i> , Given <i>F</i> ")	\$14,500 × 0.0795 = \$1,153	\$39,000 × 0.0795 = \$3,100
Annual labor, maintenance, operational costs, etc.	1,500	4,000
Other direct annual costs	600	2,800
Total annual cost to be capitalized	\$13,253	\$9 <i>,</i> 900
Capitalization of annual costs	\$13,253/0.05 = 265,060	\$9,900/0.05 = 198,000
Initial costs of investment	15,000	40,000
Total capitalized cost when money is worth 5%	280,060	\$238,000

It is clear that both the direct and detailed methods give the same final answer; however, one would be reluctant to use the latter approach.

At the lower interest rate of 5%, the capitalized cost is \$42,060 less for possibility B (\$280,060 – \$238,000). The results illustrate the peculiar effect of the interest rate and emphasize the potential difficulties in comparing alternates on either a present value or a capitalized cost basis. When cost of capital is high, total capitalized costs become lower, but differences between capitalized costs of higher and lower investment amounts favor higher investments more when cost of capital (interest rate) is lower.

The interest rate is the determining factor, although the relative size of such individual items as initial costs, annual labor costs, annual material, repairs, maintenance, and other costs, when compared to capital recovery costs, can affect total equivalent capital involved.

The important point is that the interest based on the going value of money is always lower than the rate for a venture involving a risk. The engineer using the going rate for interest will bias his comparisons in favor of the alternative equivalent to oil capital requirements. Because of this, the annual cost method is preferred, but the service lives of the alternatives should be equal, and annual costs of alternatives should be uniform. When different service lives are involved, or where non-uniform annual expenditures must be compared for alternatives, it is better to use the present value method and put all costs on a comparable basis in order to get accurate results and avoid "distortions" of costs.

7.5 Replacement Analysis

In the oil industry, the usual experience is that assets are retired while they are still physically capable of continuing to render their service in the oil field, in transportation systems, or in the refining operations. The question is: How can we make the decision to replace an asset?

The decision to make such replacement should generally be made on the grounds of economy along with engineering fundamentals applicable to oil operations. That is, replacing a worn, obsolete, or inadequate asset can be translated into the language of economics.

Reasons behind a replacement can be defined as *a must*, that is, we have to replace, otherwise the operation will come to a halt, or *optional* in which case the asset is functioning, but there is a need for a more efficient or modern type. Such a classification is illustrated in Figure 7.10.



FIGURE 7.10 Replacement analysis.

The principles governing replacement are best explained by using the word "defender" to stand for the old asset, and the word "challenger" to identify the possible new candidate that will make the replacement. In order to utilize the challenger/defender analogy for replacement comparison, the following factors must be considered (Valle-Riestra, 1983):

- All input/output of cash flows associated with the asset have to be known or estimated. This applies in particular to maintenance and operating costs of both defender and challenger.
- Cost estimation of the value of the defender (market value/book value) must be made.
- Methods recommended earlier for the comparison of alternatives such as total equivalent annual cost (T.E.A.C.), present worth, or Δ approach could be applied. In other words, no new techniques are provided. Tax obligations or credits should be considered.

Example 7.10

A tank farm is receiving crude oil through a pipeline. Periodic measurements of the crude oil level are made. The annual labor cost for the manual operation is estimated to be \$50,000. However, if an automated level-measuring system is installed, it will cost \$150,000. Maintenance and operating expenses of the system are \$15,000 and \$5,000, respectively. The system will be operated for 5 years.

Should the automated level-measuring system be installed? Assume that the interest rate is 10%.

SOLUTION

Two alternatives must be compared:

Alternative 1: Manual operation

Annual cost = \$50,000

Alternative 2: Automated level-measuring system

Annual cost = capital recovery cost + operating maintenance

$$= 150,000 \left[\frac{0.1(1.1)^5}{(1.1)^5} \right] + 15,000 + 5,000$$
$$= 39,570 + 20,000 = \$59,570$$

The manual operation, alternative 1, is less expensive.

Example 7.11

An oil company has an existing steam-generation unit. Its cost when new is \$30,000, its lifetime is 10 years, and it has a salvage value of zero.

The annual operating cost is \$22,000. After it has been in use for 5 years, the estimated book value of the unit is found to be \$6,000. The remaining lifetime now is only 3 years.

It has been proposed to replace this unit by another new one. Its cost is \$40,000, lifetime 10 years, operating costs \$15,000/yr, and zero salvage value. Should we continue using this unit or go for the replacement?

The company requires 10% R.O.I.

SOLUTION

Old Unit	Replacement
V _o (\$): 30,000	\$40,000
<i>n</i> : 10 years	<i>n</i> : 10 years
Operating: \$22,000/yr After 5 years of use	Operating: \$15,000/yr
V ₅ = \$6,000	
3 years are left only	
$V_s = 0$	
Now, take these 3 years for comparison:	
$d = \frac{6000}{3} = \$2,000/yr$	$d = \frac{40,000}{10} = 4,000$
Operating costs = \$22,000/yr	Operating costs = 15,000
Total cost = \$24,000/yr	Total cost = \$19,000

Therefore, savings = 24,000 – 19,000 = \$5,000/yr. If replacement takes place, R.O.E.I. = (5,000/\$34,000)100 = 14.7%.

Example 7.12

Consider a control valve that becomes obsolete 3 years before it has been fully depreciated. When fully depreciated, the valve will have a salvage value of \$400, but at this time (3 years before), it has a trade-in (or resale) value of \$1,000. If the book value (original cost – total depreciation to date) is \$760, there is a favorable "bonus" to management of \$240 in trade-in.

But the bonus of \$240 is irrelevant as a sunk cost. If a minimum rate of return is assumed as 10% before taxes, the question is whether the obsolete control valve with 3 years to go before being fully depreciated should be replaced now by a new valve. Calculations are needed to compare the old valve with a new valve, which would cost \$5,000 and have an eventual salvage value of \$500 and a service life of 10 years.

SOLUTION

Annual Cost of Old Valve	
Capital recovery costs (760)(0.40211) 10% for 3 years + 0.10 × \$400	= \$346.00
Operating and maintenance costs (estimated)	= \$1,820.00
Total annual cost of old valve	= \$2,166.00
Tentative Annual Cost of New Valve	
Capital recovery costs (\$4,500)(0.16275) 10% for 10 years + 0.10 × \$500	\$782.00
Operating and maintenance cost (estimated)	= \$1,000.00
Total annual cost of new valve	= \$1,782.00

By comparing the old control valve with the new valve, we can see that purchasing the new valve now would mean an annual savings of \$384 or (\$2,166 - \$1,782). If the old valve is depreciated out, only the salvage value of \$400 could be allowed on capital recovery.

Risk, Uncertainty, and Decision Analysis

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This chapter is devoted to the introduction of the decision analysis framework approach to problem solving. The concept of decision analysis and procedures associated with decision making under certainty, risk, or uncertainty and sequential decisions are introduced. The analysis is further extended to show how decision trees can be used to analyze a decision under uncertainty. The notions of expected value of perfect information and imperfect information are presented in order to assist the decision maker in developing an optimal decision strategy. This applies to many oil engineering operations. The systematic use of information to determine how often specified events may occur and the magnitude of their likely consequences is detailed as well.

8.1 Introduction

The oil and gas industry epitomizes investment decision making under conditions of risk and uncertainty, and hence was one of the first industries to apply decision analysis. Decision analysis provides a framework for analyzing a wide variety of problems encountered in engineering and management. It is a methodology used to determine optimal strategies when a decision maker is faced with uncertain decision alternatives. However, risk analysis will not eliminate risk in the decision-making process.

Some important applications involving risk and economic analysis in oil operations may include:

- Reserve quantification
- Reservoir characteristics
- Recovery factors
- Expected production
- Operations schedule

In the study of risk and economic analysis, the following tools are normally used:

- Monte Carlo simulation
- Decision trees
- Commercial software
- Engineering economy
- Economic indicators
- Database

8.2 Decision Analysis

The rational methodology for conceptualizing, analyzing, and solving problems that require a decision is an approach referred to as *decision analysis*. The first step in the decision analysis approach for a given situation is to identify the alternatives that may be considered by the decision maker. The second step is to identify future events that might occur. These future events, which are not under the control of the decision maker, are referred to as the *states of nature*. The payoff, which is the outcome resulting from making a certain decision, and the probability of occurrence of a particular state of nature should be estimated. This information is organized in what is called a payoff or a decision table. Decision analysis using decision tables is discussed here, followed by classification of decision situations.

8.2.1 Decision Analysis Using Decision Tables

Decision tables are a precise yet compact way to model complicated logic (Wets et al., 1996). Decision tables, like flowcharts and if-then-else and switch-case statements, associate conditions with actions to perform but in many cases do so in a more elegant way.

Decision tables typically contain four elements:

- 1. Courses of action or decision alternatives
- 2. States of nature
- 3. Probabilities of the states of nature
- 4. Payoffs

Identification of the decision alternatives A_i , state of nature S_j , and determination of the payoff values V_{ij} associated with each decision alternative *i* and state of nature *j* with probability P_j are organized in a decision table. Table 8.1 represents the general structure for a payoff or a decision table.

8.2.2 Classification of Decision Situations

The classification scheme for decision-making situations is based on the knowledge the decision maker has about the states of nature. It is reasonable to assume in many decision-making situations that only one state of

TABLE 8.1

Alternative Cour of Action	ses	States	of Natur	re
	P_1	P_2		P_n
	S_1	S_2		S_n
A_1	V_{11}	V_{12}		V_{1n}
A_2	V_{21}	V_{22}		V_{2n}
A_m	V_{m1}	V_{m2}		V_{mn}

General Structure of a Decision Table

nature is relevant. In this case, this single state of nature will occur with certainty (i.e., with probability = 1). This kind of situation is termed a *decision under assumed certainty*. A decision situation is called a *decision under risk* when the decision maker considers several states of nature, and the probabilities of their occurrence are explicitly stated. A decision situation where several states are possible and sufficient information is not available to assign probability values to their occurrence is termed a *decision under uncertainty*.

In summary, decision situations can be classified as follows (Ben-Haim, 2001):

- 1. Decision making under certainty, where complete information is assumed or available
- 2. Decision making under risk, where partial information is known
- 3. Decision making under uncertainty, where limited information is available

8.3 Decision Making under Certainty

In decision making under certainty, it is assumed that complete information is available so that the decision maker knows exactly what the outcome of each course of action will be. Such situations are also termed *deterministic*. The decision table describing certainty is composed of a single column, since only one state of nature is assumed to occur. That is, only one possible payoff is associated with each decision alternative. The optimal decision is the one corresponding to the best payoff in the column.

To summarize, decision making under certainty involves the following steps:

- 1. Determine the alternative courses of action.
- 2. Calculate the payoff values, one for each course of action.
- 3. Select the one with the best payoff (the largest profit or the smallest cost), either by complete enumeration or by the use of an analytical model.

8.3.1 Complete Enumeration

Complete enumeration means examining every payoff, one at a time, comparing the payoffs to each other, and discarding inferior solutions. The process continues until all payoffs are examined.

Example 8.1

Suppose an oil company would like to assign three drilling rigs to drill oil wells at three different stratigraphic locations in a manner that will TIDIFOO

TABLE 8.2			
Drilling Times in Days for Three Different Oil Wells			
Well Number	1	2	3
Rig Number			
A	30	70	40
В	40	60	60
С	30	80	50

minimize total drilling time. The drilling times in days are presented in Table 8.2.

SOLUTION

By complete enumeration as shown in the solution given in Table 8.3, all the alternatives are listed. It is clear that alternative number 5 is the best choice since the total drilling time is the minimum.

8.3.2 Computation with Analytical Models

The complete enumeration method is an effective approach in many decisionmaking situations. However, there are cases in which the number of possible combinations becomes fairly large, and complete enumeration becomes quite complicated and time consuming. In such situations, analytical models (AspenTech Software, 2011) such as linear programming are more effective than complete enumeration. Linear programming and other optimization techniques are discussed in Chapter 10.

8.4 Decision under Risk

Decision situations in which the chance (probability) of occurrence of each state of nature is known or can be estimated are defined as decisions made under risk (Macmillan, 2000). In such cases the decision maker can assess

TAB	LE 8	3.3
-----	------	-----

Complete Enumeration Solution

Alternative	Assignment	Total Drilling Time
1	A-1, B-2, C-3	30 + 60 + 50 = 140
2	A-1, B-3, C-2	30 + 60 + 80 = 170
3	A-2, B-1, C-3	70 + 40 + 50 = 160
4	A-2, B-3, C-1	70 + 60 + 30 = 160
5	A-3, B-2, C-1	$40+60+30=130 \leftarrow$
6	A-3, B-1, C-2	40 + 40 + 80 = 160
the degree of risk that he or she is taking in terms of probability distributions. The following sections present solution approaches to decision making under risk.

8.4.1 Expected Value Criterion

The most accepted solution approach to decision making under risk is the use of expected value (mean or average) as a criterion of choice. The expected payoff of an alternative is the sum of all possible payoffs of that alternative, weighted by the probabilities of those payoffs occurring. Mathematically, the expected value is expressed as follows:

$$EV(d_i) = \sum_j P_j V_{ij} \tag{8.1}$$

where

 $EV(d_i)$ = expected value of alternative *i*

 P_j = the probability that state of nature *j* will occur

 V_{ij} = the payoff resulting from the selection of

alternative *i* under the state of nature *j*

If the problem is one of maximization, the highest expected payoff is selected using complete enumeration. In the case of minimization, the alternative with the lowest expected payoff is sought. When the payoffs are expressed in dollars, the expected payoff is called the expected monetary value, or the EMV criterion.

8.4.2 Expected Value-Variance Criterion

The expected value criterion is suitable mainly for making long-run decisions. For short-run decisions it is desirable to have, in addition to the expected value, a measure of the dispersion of probability distribution. The variance of a probability distribution provides such a measure. If P(x) denotes the probability of experiencing a particular payoff, then the expected payoff and the variance of the payoff can be expressed as follows:

Expected payoff value = $EV(x) = \Sigma x P(x)$ (8.2)

Variance of payoff =
$$\Sigma[x - EV(x)]^2 P(x)$$
 (8.3)

The use of the variance in decision making is illustrated by Examples 8.2 and 8.3.

Example 8.2

Consider an investment of a company, engaged in oil field services, of \$10,000 over a 4-year period that returns R_t at the end of year t, with R_t being a statistically independent random variable. The following probability distribution is assumed for R_t .

R _t	Probability
\$2,000	0.10
\$3,000	0.20
\$4,000	0.30
\$5,000	0.40

SOLUTION

The expected value of the return in a given year is given by:

$$EV(return) = 2000(0.10) + 3000(0.20) + 4000(0.30) + 5000(0.40)$$

```
=4000
```

The variance of an annual return is determined as follows:

Variance (return) =
$$(2000 - 4000)^2(0.10) + (3000 - 4000)^2(0.20)$$

+ $(4000 - 4000)^2(0.30) + (5000 - 4000)^2(0.40)$
= 1,000,000

It is to the advantage of the decision maker to use both the expected value and the variance to develop a criterion that maximizes the expected profit and at the same time minimizes the variance of the profit. The criterion is as follows:

Maximize
$$EV(x) - \omega Var(x)$$
 (8.4)

where *x* is a random variable representing profit, and ω is a weighing factor that indicates the importance of Var(*x*) relative to EV(*x*)

In case *x* represents cost, then the above criterion should be as follows:

Minimize
$$EV(x) + \omega Var(x)$$
 (8.5)

Example 8.3

An oil firm has four alternatives from which one is to be selected. The probability distributions describing the likelihood of occurrence of the present worth of cash flow amounts, expected values, and variance for each alternative are given in Table 8.4.

Present Worth of Cash Flow (\$1,000)							
Alternatives	-\$40	10	60	110	160	EV	Var
A1	0.2	0.2	0.2	0.2	0.2	60	5 * 10 ⁹
A2	0.1	0.2	0.4	0.2	0.1	60	3 * 109
A3	0.0	0.4	0.3	0.2	0.1	60	$2.5*10^9 \leftarrow$
A4	0.1	0.2	0.3	0.3	0.1	65	3.85 * 10 ⁹

Probability Distributions, Expected Values, and Variances of Present Worth Amounts for Four Alternatives

SOLUTION

For any given alternative, the decision maker wishes to maximize the expected value and at the same time to minimize the variance of the present worth of the cash flow. If equal weights to the expected value and variance are given, then the values of the expected value-variance criterion will be as computed in the last column of Table 8.5.

Based on the expected value-variance criterion, alternative A3 should be selected.

8.5 Decision Making under Uncertainty

In decision making under uncertainty, the decision maker considers situations in which several outcomes are possible for each course of action (Lawrence and Lawrence, 2000; Taghavifard et al., 2009). However, in contrast to the risk situation, the decision maker does not know or cannot estimate the probability of occurrence of the possible states of nature. In such cases, the decision maker might prefer to select a decision criterion that does not require any knowledge of the probabilities of states of nature. The most popular criteria available for these cases are:

- 1. Laplace
- 2. Maximin and Minimax

TABLE 8.5

Values of the Expected Value-Variance Criterion for Four Alternatives

Alternatives	EV	Var	EV(x) - W * Var(x)
A1	60,000	5 * 10 ⁹	-4.999 * 109
A2	60,000	3 * 109	-2.999 * 10 ⁹
A3	60,000	2.5 * 109	-2.499 * 109
A4	65,000	$3.85 * 10^9$	$-3.849 * 10^9$

Available Alternatives to the ABC Company

Alternatives	Description
A1	ABC Company will serve as a project manager, with all the work to be subcontracted
A2	ABC Company is to subcontract the design but to do the construction
A3	ABC Company is to subcontract the construction but to do the design
A4	ABC Company is to do both the design and the construction
A5	ABC Company is to bid jointly with another company that has more capability and experience

- 3. Maximax and Minimin
- 4. Minimax Regret
- 5. Hurwicz

Example 8.4

The ABC Engineering and Construction Company has the opportunity to bid on two contracts from an oil company. The first contract, *X*, is to design and construct a deethanizer unit at the oil company's refinery. The second contract, *Y*, is to design and construct a liquified petroleum gas (LPG) plant. The ABC company may be awarded either contract *X* or contract *Y* or both. Thus, there are three possible outcomes or states of nature.

The ABC Company has five alternatives to consider for these contracts, as presented in Table 8.6.

Suppose the present values in thousands of dollars for all the alternatives are as exhibited in the payoff matrix of Table 8.7.

Before proceeding, the payoff matrix should be examined for dominance. The dominance principle is described as follows. Given several alternatives, if one is always preferred, no matter which state occurs, the preferred alternative is said to dominate the others, and the dominated alternative (or alternatives) can be deleted from further consideration.

TABLE 8.7

Payoff Matrix for Profit in Thousands of Dollars for the ABC Company

		State of Nature	
Alternatives	X	Ŷ	X and Y
A1	-4,000	1,000	2,000
A2	1,000	1,000	4,000
A3	-2,000	1,500	6,000
A4	0	2,000	5,000
A5	1,000	3,000	2,000

1 2			
Alternatives	X	Ŷ	X and Y
A2	1,000	1,000	4,000
A3	-2,000	1,500	6,000
A4	0	2,000	5,000
A5	1,000	3,000	2,000

Reduced Payoff Matrix for Profit in Thousands of Dollars for the ABC Company

In Table 8.7, alternative Al should be discarded since it is dominated by all other alternatives. Therefore, the payoff matrix can be reduced to the form shown in Table 8.8.

The management of the ABC Company cannot agree on the probabilities of the states of nature. The problem is to determine which alternative to choose among the four mentioned above in order to maximize the present value of profits. The decision-making criteria presented in the following sections will assist the ABC Company in the selection of one of the four remaining alternatives.

8.5.1 Laplace Criterion

The user of this criterion assumes that all states of nature are equally likely to occur. Thus, equal probabilities are assigned to each. Therefore, the probability of occurrence of each future state of nature is 1/n, where *n* is the number of possible states of nature. The expected values are then computed and the best alternative with the highest expected payoff is selected.

For the case of the ABC Company, the computation of the expected payoff in thousands of dollars is shown in Table 8.9. It is clear from Table 8.9 that A4, with an expected payoff of \$2,333, is the best alternative and would be selected according to the Laplace criterion. However, the assumption of equal probabilities is considered a major deficiency of this criterion since there is no base to assume the probabilities are all equal.

Computation of Expected Payoff for the ABC Company			
Alternative Average Payoff			
A2	(1,000 + 1,000 + 4,000)/3	= 2,000	
A3	(-2,000 + 1,500 + 6,000)/3	= 1,833	
A4	(0 + 2,000 + 5,000)/3	= 2,333	
A5	(1,000 + 3,000 + 2,000)/3	= 2,000	

8.5.2 Maximin and Minimax Criteria

The user of these two criteria is completely pessimistic; the decision maker assumes that the worst will happen, no matter which alternative is selected. To provide protection the decision maker should select the alternative that will give as large a payoff as possible under this pessimistic assumption. The decision maker maximizes the minimum payoffs, and therefore this criterion is known as *maximin*. If P_{ij} is used to represent the payoff for the *i*th alternative and the *j*th state of nature, then the required computation is max_i(min_i P_{ij}).

In the case of cost minimization, the decision maker minimizes the maximum possible cost. In this case the criterion is called *minimax*. As before, if P_{ij} represents the payoff for the *i*th alternative and the *j*th state of nature, then the required computation is min_i(max_i P_{ij}).

For the case of the ABC Company the application of the maximin criterion is illustrated in Table 8.10.

Application of the maximin criterion requires the selection of the minimum value for each row as shown in Table 8.10. The maximum of the minimum payoff value is selected. For the ABC Company either alternative A2 or A5 can be chosen.

Application of the minimax criterion to the ABC Company requires the use of cost data for each alternative under each state of nature. The decision would be to select the maximum cost for each alternative. The decision that results in the minimum of these costs would be selected.

8.5.3 Maximax and Minimin Criterion

An optimistic decision maker assumes that the very best outcome will occur and selects the alternative with the best possible payoff. If P_{ij} represents the payoff for the *i*th alternative and the *j*th state of nature, the required computation is max_i(max_i P_{ij}).

The decision maker seeks the best possible payoff for each alternative. These values are placed in a new column to the right of the decision table. The alternative with the best payoff in this newly added column is selected.

TABLE 8.10

Payoff Matrix for Profit in Thousands of Dollars for the ABC Company by the Maximin Criterion

Alternatives	X	Ŷ	X and Y	$Min_j P_{jj}$	Maximum of Minimum
A2	1,000	1,000	4,000	1,000	\leftarrow
A3	-2,000	1,500	6,000	-2,000	
A4	0	2,000	5,000	0	
A5	1,000	3,000	2,000	1,000	\leftarrow

		State of Nature			Maximum of
Alternatives	X	Ŷ	X and Y	$max_{j}P_{ij}$	Maximum
A2	1,000	1,000	4,000	4,000	,
A3	-2,000	1,500	6,000	6,000	\rightarrow
A4	0,000	2,000	5,000	5,000	
A5	1,000	3,000	2,000	3,000	

Payoff Matrix for Profit in Thousands of Dollars for the ABC Company by the Maximax Criterion

This procedure is illustrated in Table 8.11. As shown in the table, alternative A3 should be selected.

If Table 8.11 contains costs instead of profits as the payoffs, the optimistic decision maker will select as best the lowest cost for each alternative. The minimum value of the lowest cost is selected. This decision-making approach is known as *minimin criterion*.

8.5.4 Minimax Regret Criterion

The concept of regret is equivalent to the determination of opportunity loss. Both concepts represent the important economic concept of opportunity cost, which is the magnitude of the loss incurred by not selecting the best alternative. Under this criterion, a matrix consisting of regret values is first developed. The alternative associated with the minimum regret value is chosen from the set of maximum regret values. The procedure can be summarized in the following three steps:

- 1. Develop a regret (opportunity loss) table. Within each payoff column, each payoff is subtracted from the largest payoff value in the column.
- 2. Obtain the largest (worst) regret value for each alternative. Put the obtained values in a newly formed column.
- 3. Select the lowest regret value from the newly formed column.

Application of the minimax regret criterion to the ABC Company is illustrated in Table 8.12. As indicated in Table 8.12, the minimum largest regret, alternative A4, is selected. A decision maker who uses the minimax regret criterion will make that decision which will result in the least possible opportunity loss.

8.5.5 Hurwicz Criterion

Most decision makers are not completely optimistic or completely pessimistic. Therefore, Hurwicz suggested that a degree of optimism (α) be measured

State of Nature				
Alternatives	X	Y	X and Y	Largest Regret
A2	1 - 1 = 0	3 – 1 = 2	6 - 4 = 2	2
A3	1 - (-2) = 3	3 - 1.5 = 1.5	6 - 6 = 2	3
A4	1 - 0 = 1	3 - 2 = 1	6 - 5 = 1	$1 \leftarrow$
A5	1 - 1 = 0	3 - 3 = 0	6 - 2 = 4	4

TABLE	8.12
-------	------

Payoff Regret Matrix in Millions of Dollars for the ABC Company

on a zero to one scale, where 0 indicates complete pessimism and 1 indicates complete optimism. In the case of maximization, the best alternative is the one with the highest weighted value, where the weighted value, WV, for each *i* alternative is expressed by:

$$WV_i = (1 - \alpha)(\min_j P_{ij}) + \alpha(\max_j P_{ij})$$

The application of the Hurwicz criterion to the ABC Company is illustrated below with $\infty = 0.2$.

$$wv(A2) = (1,000)(0.8) + (4,000)(0.2) = 1,600$$
$$wv(A3) = (12,000)(0.8) + (6,000)(0.2) = -400$$
$$wv(A4) = (0)(0.8) + (5,000)(0.2) = 1,000$$
$$wv(A5) = (1,000)(0.8) + (3,000)(0.2) = 1,400$$

As shown in the above calculation, alternative A2 should be selected according to the Hurwicz criterion.

In the case of minimization when dealing with cost data, the alternative with the lowest Hurwicz should be selected. The major difficulty in using the Hurwicz criterion is the measurement of α .

8.5.6 Summary of Criteria Results

The decisions made to the case of ABC Company for each decision criterion are summarized as follows:

Criterion	Decision
Laplace	A4
Maximin	A2 or A5
Maximax	A3
Minimax regret	A4
Hurwicz	A2

The decision criteria often can result in a mix of decisions, with no one decision being selected more than the others. The criterion or collection of criteria used and the resulting decision depend on the characteristics and philosophy of the decision maker. For example, if the decision maker for the ABC Company is extremely optimistic, he or she may choose alternative A3 even though this alternative has been selected only once by the different criteria.

8.6 Sequential Decisions

Decision-making processes involving a series of sequential or multi-period events become too cumbersome to analyze using decision tables. Therefore, the technique that was developed to handle these cases is called *decision trees* (Deng et al., 2011) which is basically a graphic representation of the decision-making process.

8.6.1 Decision Trees

A decision tree is composed of the following:

- 1. *Decision nodes and alternatives:* At a decision point or node, the decision maker must select one alternative course of action from a finite number of available ones. A decision node is usually designated by a square. Decision alternatives are represented by branches or arcs originating out of the right side of the decision node. If a cost is associated with the alternative, it is written along the branch. An alternative not selected is pruned, and designated by the symbol //. Each alternative branch may result in a payoff, in other decision nodes, or in a chance node.
- 2. *Chance nodes and states of nature:* A chance node indicates that a chance event is expected at this point in the decision-making process; that is, one of a finite number of states of nature is expected to occur. A chance node is designated by a circle. The states of nature are shown on the tree as branches originating from the chance nodes. Since decision trees depict decision making under risk, the assumed probabilities of the states of nature are written above the branches. Each state of nature may be followed by a payoff, a decision node, or another chance node.

The process of constructing a decision tree may be divided conceptually into three steps:

- 1. Build a decision tree that includes all decision nodes, chance nodes, and originating arcs, arranged in chronological order.
- 2. Determine the probabilities of the state of nature on the arcs.
- 3. Include the conditional payoffs on the decision tree.

After the decision tree is constructed, the decision about which alternative should be undertaken can be made. The solution process starts with segments ending in the final payoffs, at the right side of the tree, and continues to the left, segment by segment in the reverse order from which the tree was drawn. This technique is known as the *rollback procedure*. The technique can be summarized by the following two rules:

- If the node is a chance node, calculate the expected value of all the states of nature emerging from the chance node (multiply the payoff values by their corresponding probabilities and sum up the results). The expected values are then written above the chance node inside rectangles. These expected values are considered as payoffs for the next branch to the left.
- 2. If the node is a decision node, the payoffs computed for each alternative are compared, and the best one is selected.

The decision maker must select one alternative at each decision node and discard (prune) all other alternatives. The computation process continues from the right to the left. Eliminating some alternatives slowly reduces the size of the decision tree until only one alternative remains at the last decision node on the left side of the tree.

Example 8.5

An oil drilling company is considering bidding on a \$110 million contract for drilling oil wells. The company estimates that it has a 60% chance of winning the contract at this bid. If the company wins the contract, it will have three alternatives: (1) to drill the oil wells using the company's existing facilities, (2) to drill the oil wells using new facilities, and (3) to subcontract the drilling to a number of smaller companies. The results from these alternatives are given as follows:

Outcomes	Probability	Profit (\$ million)				
1. Using existing facilities:						
Success	0.30	600				
Moderate	0.60	300				
Failure	0.10	-100				
2. Using new facilities:						
Success	0.50	300				
Moderate	0.30	200				
Failure	0.20	-40				
3. Subcontract:						
Moderate	1.00	250				

The cost of preparing the contract proposal is \$2 million. If the company does not make a bid, it will invest in an alternative venture with a



FIGURE 8.1

The general structure of a decision tree.

guaranteed profit of \$30 million. Construct a sequential decision tree for this decision situation, and determine if the company should make a bid.

SOLUTION

The oil company should make the bid because this will result in an expected payoff value of \$143.2 million. The problem is solved using an academic version of Microsoft Excel Add-in, TreePlan Software. To construct a decision tree with TreePlan, go to the Tools menu and choose Decision Tree, which brings up the TreePlan as shown in Figure 8.1

The dialogue boxes used by TreePlan for constructing a decision tree are shown in Figure 8.2. The dialogue boxes enable us to add decision nodes, state of nature nodes, decision alternative branches, state of nature branches, probabilities, payoffs, and all other tree parameters.

8.6.2 The Value of Perfect Information

The decision maker faces two decisions when perfect information is involved. First, if perfect information is available, which alternative should be selected? Second, should the perfect information be acquired? The second decision is based on comparison of the benefits of perfect information with its cost.

Example 8.6

An investment company is considering three different investment alternatives: (a) investing in bonds, (b) investing in stocks, or (c) investing in certificates of deposit, CDs. There are three states of nature for the economy: (a) growth, with 50% probability; (b) depression, with 30%



FIGURE 8.2

TreePlan dialogue boxes.

probability; and (c) inflation, with 20% probability. The rate of return on the three investment alternatives under the three states of the economy is given in Table 8.13.

SOLUTION

The expected value for each decision alternative is computed in the last column of Table 8.13. The decision maker who considers the expected value as his or her criterion will select alternative D1, to invest in bonds, as the best decision. If the decision maker decides on obtaining information concerning the future state of the economy from a research firm, the decision maker can select an investment alternative based on complete information. If the research firm predicts growth, then the best investment alternative is D2, investment in stocks. If depression is predicted by the research firm, D3, investment in CDs will be the best decision investment alternative. If the research firm predicts inflation, then the best decision alternative will be D3, investment in CDs. Thus, the expected rate of return will be:

(0.5)(15) + (0.3)(6.5) + (0.2)(6.5) = 10.75

Kate of Keturn on the Three Investments										
Probability	0.5	0.3	0.2							
States of Nature	Growth	Depression	Inflation	Expected Value						
Alternatives										
D1 Bonds	12	6	3	8.4						
D2 Stocks	15	3	-2	8.0						
D3 CDs	6.5	6.5	6.5	6.5						

Rate of Return on the Three Investments

If the expected rate of return value with perfect information (10.75) is compared with the value of the imperfect information (8.4), an improvement of (10.75 - 8.4) 2.35 is observed. The difference of 2.35 is called the *expected value of perfect information* (EVPI) and is used to answer the question of whether or not the perfect information should be acquired. The mathematical expression for calculating the expected value of perfect information is:

$$EVPI = \sum P_j V_{ij}$$

where P_j is the probability of state of nature *j*, and V_{ij} is the payoff when action d_i is taken and state of nature *j* occurs.

In the case of minimizing:

EVPI = (expected cost without perfect information)

- (expected cost with perfect information)

The EVPI is the maximum amount that would be paid to gain information that would result in a better decision than the decision made without perfect information.

In summary, in order to determine whether or not to purchase perfect information, one should:

- 1. Compute the expected payoff without perfect information and select the best alternative.
- 2. Compute the expected payoff with perfect information.
- 3. Compute the EVPI by subtracting the obtained value in (1) from the value obtained in (2), reverse order for minimization payoff values.
- 4. If the difference is larger than the cost of the information, it should be purchased.

8.6.3 The Value of Imperfect Information

In the previous section the concept of the expected value of perfect information was discussed. In most real-world situations, information is incomplete and not perfectly reliable. If perfect information regarding which state of nature will occur in the future can be obtained, the decision maker in general can make a better decision. Although perfect information is rare, it is often possible to gain additional (imperfect) information that will improve the decision-making process. Gaining the additional information will necessitate revising the probabilities of the states of nature by applying the Bayesian theorem.

9

Break-Even and Sensitivity Analysis

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9.4	Nonli	near Break-Even Analysis	

In this chapter linear break-even analysis is introduced. The concept will be extended to two and to multiple alternatives. The graphic solution of the break-even analysis is also presented. The extension of break-even analysis is further considered to cover nonlinear analysis. Finally, there is a brief discussion about the sensitivity by break-even analysis. Examples are cited for the petroleum industry.

9.1 Introduction

Break-even analysis is a means of identifying the value of a particular project variable that causes the project to exactly break even. In other words, the purpose of break-even analysis is to determine the number of units of a product to produce that will equate total revenue with total cost. At this point, referred to as the break-even point (BEP), profit is zero. It is also defined in terms of finding the values of particular variables that give the project a break-even net present value (NPV) of zero (Cafferky and Wentworth, 2010). As such, the BEP is a point of reference in determining the number of units needed to ensure a specific profit.

Sensitivity analysis, on the other hand, is a means of identifying the project variables that, when varied, have the greatest effect on project acceptability.

The variables having significant impact on the NPV are known as *sensitive variables*.

9.2 Linear Break-Even Analysis

Many decision problems involve a determination of the minimum volume or quantity of a good or service that must be produced or provided in order for revenues to cover the cost of the product or service. At the point where revenues equal costs, the firm will just break even on the product or service. At volumes beyond the break-even points, the firm will realize a profit. This area of decision making is often referred to as cost-volume-profit analysis or break-even analysis (Tisdell, 2004). The break-even point as such gives the decision maker a point of reference in determining how many units will be needed to ensure a profit. Breakeven analysis is concerned with answering three important and basic economic questions:

- 1. What is the minimum level of activity that can be operated at?
- 2. What is the level of activity that will cover the cost?
- 3. At what level of activity will maximum profit be achieved?

9.2.1 Components of Break-Even Analysis

There are three main components of break-even analysis: volume, cost, and profit. Each of these three components is a function of several other components. These components are analyzed as follows:

1. Volume

Volume is the level of production and can be expressed as the number of units produced and sold. It can also be expressed in monetary terms or as a percentage of total capacity available.

2. Costs

Costs are usually divided into two components: fixed and variable. Fixed costs are generally independent of the volume of units produced. Some individual costs that might be incorporated into the fixed cost include rent on plant and equipment, insurance, advertising, staff salaries, depreciation, heat and light, and janitorial services.

Variable costs are the expenses that can be attributed directly to the production of an individual unit of product or service. Examples of this type of cost are raw materials, direct labor expenses, packaging and shipping, sales commissions, and maintenance costs. Total variable costs are a function of the volume and the variable cost per unit. The total cost is the sum of fixed costs and total variable costs.

3. Profit

Profit is the difference between total revenue and total costs. Total revenue is the volume multiplied by the price per unit.

9.2.2 Mathematical Solution

Total revenue, as defined above, may be computed as the product of the variable quantity and the fixed price. This can be expressed mathematically as

$$TR = (P)(Q) \tag{9.1}$$

where *P* is the price, and *Q* is the quantity or volume.

Total cost is the sum of the fixed and variable costs:

$$TC = FC + TVC \tag{9.2}$$

where *TVC* is the total variable cost and is given by:

$$TVC = (VC)(Q)$$

$$FC = \text{fixed cost}$$

$$TC = \text{total cost}$$

$$VC = \text{variable cost per unit}$$
(9.3)

The profit is equal to the excess of revenue beyond total cost: PR = TR - TC:

$$PR = TR - TC \tag{9.4}$$

where *PR* is the profit.

Since break-even occurs where there is neither profit nor loss, then *P* must be set equal to zero at the break-even point. Mathematically, this can be expressed as:

$$0 = TR - TC \text{ or } TR = TC \tag{9.5}$$

Using Equations (9.1), (9.2), and (9.3) and the above result given by Equation (9.5), the following relationship is obtained:

$$QBEP = (FC)/(P - VC) \tag{9.6}$$

In other words, the break-even quantity can be obtained by dividing the total fixed cost by the difference between the unit price and the unit variable cost.

Example 9.1

A refiner determines that the total cost of producing *Q* barrels of gasoline per day is given by

$$TQ = 4,000 + 2Q$$

The revenue (in thousands of dollars) from selling Q barrels of gasoline per day is

$$TR = 4Q$$

- 1. Find the break-even point.
- 2. At the break-even point, what are the cost and revenue?
- 3. Find the break-even point graphically.
- 4. How many barrels of gasoline must be produced and sold in order to earn a profit of \$100,000?

SOLUTION

1. The break-even point occurs when total revenue equals total cost.

$$TC = TR$$

$$4000 + 2Q = 4Q$$

$$2Q = 4,000$$

$$Q = 2,000$$

Thus, the break-even point is 2,000 barrels.

2. Substituting the break-even value (Q = 2,000) in the cost and revenue equations yields

$$TC = 4,000 + 2Q$$

= 4,000 + (2)(2,000) = 8,000
or $TR = 4Q$
= (4)(2,000) = 8,000



FIGURE 9.1 Break-even point (BEP) graph.

3. Since both the total cost and total revenue equations are linear equations, they can be represented by two straight lines, as shown in Figure 9.1.

As shown in Figure 9.1, the break-even point occurs at the intersection of the two lines where Q = 2,000 and TC = TR = \$8,000. If *TR* is greater than *TC*, a profit will be realized. On the other hand, a loss will be realized if *TR* is less than *TC*.

4. The profit can be calculated by finding the difference between the total revenue and total cost.

PR = TR - TCPR = 4Q - (4000 + 2Q)100,000 = 4Q - 4,000 - 2Q100,000 = 2Q - 4,0002Q = 104,000Q = 104,000/2 = 50,200

Thus, the refiner has to produce and sell 50,200 barrels of gasoline per day in order to make a profit of \$100,000.

9.3 Extended Break-Even Analysis

In many engineering economic analyses, the cost of an alternative may be a function of a single variable. The break-even analysis can be applied to two or more alternatives. When two or more alternatives are a function of the same variable, it may be desirable to find the value of the variable that will result in equal cost for the alternatives considered. In the following sections break-even analysis with two or more alternatives will be presented.

9.3.1 Using Two Alternatives

The typical problem in break-even analysis involving two cost alternatives is approached mathematically by equating the total costs of the two alternatives expressed as a function of a common independent variable. This can be illustrated as follows:

TC1 = f1(x)

and

TC2 = f2(x)

where

TC1 = total cost per time period for alternative 1

TC2 = total cost per time period for alternative 2

x = the common independent variable affecting alternatives 1 and 2

To determine the value of *x* that will yield a break-even situation between the two alternatives, the two cost functions are set equal:

$$TC1 = TC2 \tag{9.7}$$

Example 9.2

Using a special type of machine, a company produces a high-pressure oil valve to be used in the hydrocarbon industry. The company's fixed cost on the valve is \$20,000 per month. The variable cost of production per valve is \$900.

The company is considering replacing the old machine with a new one. The new machine costs \$1,200,000. Assuming a 10-year straight-line depreciation period, the monthly depreciation cost of the new machine will be \$10,000. Other fixed cost allocated to this product is \$18,000. However, because this machine will result in less scrappage and waste of

raw materials and will require less operator time to produce a valve, it will reduce the variable cost of the valve to \$500 per valve. Above what volume of production will the new machine be better than the old machine?

SOLUTION

At the break-even point the total costs, *TC*, of the two machines should be equal. Thus:

$$TC1 = TC2$$

20,000 + 900Q = 28,000 + 500Q
 $400Q = 8,000$
 $Q = 20$ units

The problem is illustrated graphically in Figure 9.2.

Thus, if the company plans to produce 20 valves or more during the month, the new machine will be superior. At volumes below 20 units per month, the old machine is more cost efficient.

9.3.2 Using Multiple Alternatives

In the above discussion, break-even analysis has been used where only two alternatives confront the decision maker. This section extends the breakeven analysis to multiple alternatives.



FIGURE 9.2 Break-even analysis for Example 9.2.

Example 9.3

Three different 15-horsepower electric motors are being considered for purchase by an oil company. Motor *X* sells for \$6,000 and has an efficiency rating of 90% (alternative A); motor *Y* sells for \$4,000 and has a rating of 85% (alternative B); and motor *Z* sells for \$3,000 and is rated to be 80% efficient (alternative C). The cost of electricity is \$0.20/kilowatt. An 8-year planning horizon is used (n = 8), and zero salvage values are assumed for all three motors. An annual rate of return, *i*, of 25% is to be used. Determine the range of values for annual usage of the motor (in hours) that will lead to the preference of each motor. (Note that 0.746 kilowatts = 1 horsepower.)

SOLUTION

First, it is necessary to define the common variable between the alternatives and state its dimensional unit. Second, the equivalent uniform annual cost (EUAC) or the present worth (PW) analysis should be used to express the total cost of each alternative as a function of the defined variable. Then, equate the cost equation of each of the two alternatives and solve for the break-even value of the variable.

Let *x* be annual usage in hours and *CRF* be the capital recovery factor given by

$$i(1+i)^n / (1+i)^n - 1$$

The annual electricity cost for 100% efficient motor = ((15 HP)(0.746 kw/hp)(0.20/kw hr)(x hr/year))/efficiency = (2.238)(x)/year.

EUAC for alternative
$$A = $6,000(CRF) + 2.238(x)/0.9$$

= 6,000(0.3004) + 2.487(x)
= 1,802.40 + 2.487(x)

EUAC for alternative B = \$4,000(CRF) + 2.238(x)/0.85

$$= 4,000(0.3004) + 2.633(x)$$

$$= 1,201.6 + 2.633(x)$$

EUAC for alternative
$$C = $3,000(CRF) + 2.238(x)/0.80$$

$$= 3,000(0.3004) + 2.798(x)$$

$$=901.20 + 2.798(x)$$

Let

EUAC for alternative
$$A =$$
 EUAC for alternative B
1,802.40 + 2.487(x) = 1,201.60 + 2.633(x)
 $x =$ 4,115

Let

EUAC for alternative
$$A =$$
 EUAC for alternative C
1,802.40 + 2.487(x) = 901.20 + 2.798(x)
 $x = 2,898$

Let

EUAC for alternative
$$B$$
 = EUAC for alternative C
1,201.60 + 2.633(x) = 901.20 + 2.798(x)
 x = 1,821

Figure 9.3 illustrates the graphic solution for the break-even analysis for the three alternatives. If the anticipated annual usage is below the break-even value of x = 1,821, select alternative C. If $1,821 \le x \le 4,115$, then select alternative B, and if $x \ge 4,115$, then select alternative A.



FIGURE 9.3 Break-even analysis for Example 9.3.

9.3.3 Graphic Solution

Sometimes the relationship between the dependent and the independent variables is not continuous and therefore cannot be expressed in mathematical terms. Other times, the relationship may be complex, and it will be time consuming to develop a mathematical formula. In such cases a graphic solution may be used to determine the break-even point.

Example 9.4

Two packaging methods A and B for automotive lubricants are under consideration by an oil company. Method A uses a special carton, while method B uses regular materials. Figure 9.4 shows the package cost for the two methods as a function of the volume of the merchandise. By examining the graph, it is apparent that a relationship between volume and cost is not a simple one. Method A should be used for the shipment whose volume is less than 3,500 cubic inches, while shipments with larger volume should use method B.

9.4 Nonlinear Break-Even Analysis

In practical applications of break-even analysis, linear relationships are generally assumed in order to simplify the analysis. Linear break-even analysis is based on a constant selling price. This assumption is not always valid. Prices may not be stable but may have to be continuously lowered



FIGURE 9.4 Package cost of automotive lubricants as a function of the volume of merchandise.

with higher volume as the market becomes saturated. Variable costs may also fluctuate continuously with increasing volume. There are unlimited variations in the ways revenue and cost can behave in a nonlinear setting (Lapašinskaitė and Boguslauskas, 2006). In summary, nonlinear break-even analysis is appealing for two reasons: (1) it seems reasonable to expect that in many cases increased sales can be achieved only if prices are reduced; and (2) the cost function of the average variable cost falls over some range of output and then begins to rise. The situation where variable cost decreases as economies of scale are realized and then increases as production capacity approaches the maximum, while total revenue increases at a decreasing rate as price is lowered in order for the sale to reach the market potential is illustrated in Figure 9.5. Solved Example 9.5 is an application for the method.

Example 9.5

The oil refining company sells its product for a fixed price of 359 per box. Total cost (*TC*) of production varies according to the following equation:

$$TC = 2Q^2 - Q + 6,400$$

where *Q* is the production quantity. Find the break-even point.



FIGURE 9.5 The case of break-even with nonlinear revenues and cost.

SOLUTION

Figure 9.6 shows the curves of the total cost and the total revenue (*TR*), where *TR* is a straight line given by the following equation:

TR = 359Q

and the *TC* is the curve given by the *TC* equation.

As shown in Figure 9.6, there are two break-even points. Because the nature of the revenue and cost curves is not very complex, it is possible to calculate these two points exactly rather than estimate them from the graph.

At the two break-even points, total revenue must be equal to total cost:

$$TR = TC$$

$$359Q = 2Q^{2} - Q + 6,400$$

$$2Q^{2} - 360 + 6,400 = 0$$

$$(2Q - 40)(Q - 160) = 0$$

$$Q = 20 \text{ or } Q = 160$$

Therefore, the company will make a profit if it produces between 20 and 160 units. The largest profit will occur halfway between the two break-even points (i.e., Q = 90 units). This will not always be true. The



FIGURE 9.6 Nonlinear break-even analysis.

method of calculus, which will be discussed in Chapter 11, can be used to determine the optimal Q which will yield the maximum profit.

Sensitivity Analysis

In many cases simple break-even analysis is not feasible because several factors may vary simultaneously as the single variable is varied. In such instances, it is helpful to determine how sensitive the situation is to several variables so that proper weight may be assigned to them (Jovanovic, 1999). In general, sensitivity analysis is used to analyze the effects of changes or making errors in estimating parameter values. Sometimes sensitivity analysis is more specifically defined to mean the relative magnitude of the change in one or more coefficients of the variables that will reverse a decision among alternatives. Sensitivity analysis permits a determination of how sensitive the final results are to changes in the values of the estimates.

Example 9.6

Let us consider the refiner who wants to determine the total cost of producing *Q* barrels of gasoline per day, which is given by:

$$TC = 4,000 + 2Q$$

and the total revenue (in thousands of dollars) from selling Q barrels of gasoline per day is expressed by:

TR = 4Q

The value of \$4 in the *TR* equation is the price of selling a barrel of gasoline. In most instances the price is not known with certainty. Let us assume that \$4 is the most likely price, and there is a pessimistic price of \$3 and another price of \$5, which is considered to be optimistic.

SOLUTION

The break-even quantities under the pessimistic, the most likely, and the optimistic prices are shown in Table 9.1. The use of the three price estimates results in providing three different estimates for the breakeven quantities. Three estimates for parameters other than the price could also be considered to provide sensitivity analysis for the quantity or other parameters such as total cost or total revenue.

Example 9.7

Consider a pipe manufacturer who produces various types of pipes, oil, water, and gas. Pertinent data about selling price, variable cost, and fixed cost for the next planning period are given in Table 9.2.

TABLE 9.1

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	Pessimistic	Most Likely	Optimistic
Break-even quantity	4,000	2,000	1,333

1	Selling Price	Variable Cost					
Product (pipes)	Per Meter (\$100s)	Per Meter (\$100s)	Fixed Cost (\$100s)				
Oil	10	5	30000				
Gas	7.5	3.6	50000				
Water	5	2	20000				

TABLE 9.2

Data for Pipe Manufacturing

The fixed cost of each of these activities is considerable and has to be paid regardless of the quantities to be produced. The high fixed costs include the cost of design modification, mold reconstruction, and quality assurance and testing.

For the next planning period the manufacturer has contracted to produce 700 meters of oil pipes and 400 meters of gas pipe. The marketing research group has advised that the demand for water pipe is at most 300 meters. The manufacturer is interested in how much to sell to break even.

SOLUTION

This problem has three products as well as previous commitments and restrictions. To obtain an expression of the break-even in terms of the production quantities, the following decision variables are defined:

X1 = meters of oil pipe to produce

X2 = meters of gas pipe to produce

X3 = meters of water pipe to produce

The break-even expression is:

10 X1 + 7.5 X2 + 5 X3 = 5 X1 + 3.6 X2 + 2 X3 + 100000, or 5 X1 + 3.9 X2 + 3 X3 = 100000

This expression will be used as a constraint for a problem that has an objective of minimizing total variable cost. Thus, the objective function is:

Minimize 5 X1 + 3.6 X2 + 2 X3

The complete model reflecting the break-even constraint, as well as the pre-established requirements and limits on demand, is as follows:

Minimize 5 X1 + 3.6 X2 + 2 X3

subject to:

5 X1 + 3.9 X2 + 3 X3 = 100000 $X1 \ge 700$ $X2 \ge 400$ $X3 \le 300$ $X1, X2, X3 \ge 0$

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FIGURE 9.7 (See Color Insert)

Solution of Example 9.7 by Microsoft Excel Solver.

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FIGURE 9.8 (See Color Insert)

Solution of Example 9.7 by Microsoft Excel Solver (final).

The setup of the Microsoft Excel[®] Solver for the pipe manufacturing example is shown in Figure 9.7. The final answer as reported by the Solver is presented in Figure 9.8. The solution procedure for the above example is further discussed in Chapter 10.

10

Optimization Techniques

Jamal A. Al-Zayer Taqi N. Al-Faraj Mohamed H. Abdel-Aal

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Optimization is a mathematical discipline that concerns the finding of minima and maxima of functions, subject to constraints. In other words, finding an alternative with the most cost-effective or highest achievable performance under the given constraints, by maximizing desired factors and minimizing undesired ones.

This chapter introduces several optimization techniques that are widely used in physical sciences, various fields of engineering and applied sciences, and in management sciences and economics. First, classical optimization methods based on differential calculus are surveyed and demonstrated by empirical examples. Linear programming and some of its basic models are introduced next. Since not all optimization problems are linear, the technique of nonlinear programming is presented and discussed in the last section.

10.1 Introduction

The general scope of an optimization problem is to determine values of the independent variables which give the greatest possible numerical value (maximization) or least possible value (minimization) of a mathematical function.

Process optimization, on the other hand, is the discipline of adjusting a process so as to optimize some specified set of parameters without violating some constraint. The most common goals are minimizing cost and maximizing throughput and efficiency. Process optimization finds many technical applications in the oil sector. This may include equipment sizing, operating procedure, or control optimization. Examples are cited in the chapter.

Classical optimization methods based on differential calculus are surveyed first and demonstrated by empirical examples. The general strategy followed in this method is to establish a partial derivative of the dependent variable from which the absolute conditions are determined. However, many real problems may involve optimum conditions that exist at boundary conditions rather than a true maximum or minimum (to allow for the partial derivative to be equal to zero). This is the case covered using linear programming. It is considered to be the most widely used optimization technique for modeling physical, economic, engineering, and business problems. Linear programming is further explained by surveying some of its basic models. Since not all optimization problems are linear, the techniques of nonlinear programming are presented and discussed in the last section.

10.2 Differential Calculus

The formal procedure for locating an extremum of the design objective function using calculus is as follows. We merely differentiate the function U(d), the objective function, with respect to d, the design variable. Then, set the derivative equal to zero, and solve for d^* .

In review, the necessary and sufficient conditions for locating an extremum in the region a < d < b using calculus, are the following:

U(d) must be continuous for a < d < b.

U(d) must be differential for the same domain.

The first derivative must vanish at some point, d^* , where $a < d^* < b$.

At least one higher derivative must not vanish at d^* .

The first non-vanishing derivative must be an even one.

If this non-vanishing derivative is *positive*, the extremum is a *minimum*, and if it is *negative*, the extremum is a *maximum*.

Differential calculus focuses on rates of change in analyzing the situation. Graphically, differential calculus solves the following problem: given a function whose graph is a smooth curve and given a point in the domain of the function, what is the slope of the line tangent to the curve at this point? With linear functions the slope is constant over the domain of the function. The slope provides an exact measure of the rate of change in the value of y with respect to a change in the value of x. Mathematically, the slope is defined as:

slope = m =
$$\Delta y / \Delta x$$

= $(y_2 - y_1) / (Y_2 - y_1)$

With nonlinear functions the rate of change in the value of y with respect to a change in x is not constant. However, one way of describing nonlinear functions is by the average rate of change over some interval. Graphically the average rate of change of a nonlinear function is represented by a secant line. This is illustrated in Figure 10.1. The instantaneous rate of change of a smooth continuous function can be represented geometrically by the slope of the tangent line drawn at the point of interest. The exact tangent line can be



FIGURE 10.1 Average rate of change for a nonlinear function.

determined by finding the limit of the different quotient as delta *x* approaches a value of zero or finding the derivative as defined by the equation:

$$f'(x) = dy/dx = \lim_{\Delta x \to 0} \Delta y/\Delta x$$

10.2.1 Differentiation Rules

The process of finding a derivative is called *differentiation*. The basic formulas or rules for differentiation are presented below. Proofs are omitted here but they can be found in any introductory calculus textbook.

- 1. If f(x) = c, where *c* is any constant, then f'(x) = 0.
- 2. If $f(x) = x^n$, where *n* is a real number, $f'(x) = nx^{n-1}$.
- 3. If $f(x) = c \cdot g(x)$, where *c* is a constant and g(x) is a differentiable function, $f'(x) = c \cdot g'(x)$.
- 4. If f(x) = u(x) + v(x), where u(x) and v(x) are differentiable; f'(x) = u'(x) + v'(x).
- 5. If f(x) = u(x). v(x), where u(x) and v(x) are differentiable, then $f'(x) = u'(x) \cdot v(x) + v'(x) \cdot u(x)$.
- 6. If f(x) = u(x)/v(x), where u(x) and v(x) are differentiable and $v(x) \neq 0$, then $f'(x) = v \cdot (x) \cdot u'(x) - u(x) \cdot v'(x)/(v(x))^2$.
- 7. If $f(x) = [u(x)]^n$, where u(x) is a differentiable function, then $f'(x) = n \cdot [u(x)]^{n-1} \cdot u'(x)$.
- 8. If $f(x) = e^{u(x)}$, where u(x) is differentiable, then f'(x) = u'(x). $e^{u(x)}$.
- 9. If $f(x) = \ln[u(x)]$, where u(x) is differentiable, then f'(x) = u'(x)/u(x).

Illustrative Example 10.1

If
$$f(x) = -3/x = -3x^{-1}$$

then $f'(x) = (-3)(-1)x^{-1-1} = 3/x^2$

Illustrative Example 10.2

If
$$f(x) = (x^2 - 5)(x - x^3)$$

Let $f'(x) = u'(x) \cdot v(x) + v'(x) \cdot u(x)$
 $= (2x)(x - x^3) + (1 - 3x^2)(x^2 - 5)$
 $= 2x^2 - 2x^4 + x^2 - 5 - 3x^4 + 15x^2$
 $= -5x^4 + 18x^2 - 5$

Illustrative Example 10.3

If
$$f(x) = 3x^2 - 5/1 - 3^3$$

then $f'(x) = (1 - x^3)(6x) - (3x^2 - 5)(-3x^2)/(1 - x^3)^2$
 $= 6x - 6x^4 + 9x^4 - 15x^2/(1 - x^3)^2$
 $= 3x^4 - 15x^2 + 6x/(1 - x^3)^2$

Illustrative Example 10.4

If
$$f(x) = 7x^4 - 5x - 9$$

= $(7x^4 - 5x - 9)^{1/2}$
then $f'(x) = 1/2(7x^4 - 5x - 9)^{1/2-1}(28x^3 - 5)$
= $(14x^3 - 5/2)(7x^4 - 5x - 9)^{-1/2}$
= $(14x^3 - 5/2)/(7x^4 - 5x - 9)^{1/2}$

Illustrative Example 10.5

If
$$f(x) = (3x/(1-x^2))^5$$

then $f'(x) = 5(3x/(1-x^2))^4[(1-x^2)(3) - (3x)(-2x)] / (1-x^2)^2$
 $= 5(3x/(1-x^2))^4(3-3x^2+6x^2) / [(1-x^2)^2]$
 $= 5(3x/(1-x^2))^4[(2+3x^2) / (1-x^2)^2]$

Illustrative Example 10.6

If
$$f(x) = e^x$$

then $f'(x) = 1 \cdot e^x = e^x$

Note that the function and its derivative are exactly the same.

$$f(x) = f'(x) = e^x$$

Illustrative Example 10.7

If
$$f(x) = In x$$

then $f'(x) = 1/x$
Illustrative Example 10.8

If
$$f(x) = \ln(5x^2 - 2x + 1)$$

Let $u(x) = 5x^2 - 2x + 1$
then $f(x') = (10x - 2) / (5x^2 - 2x + 1)$

10.2.2 Application of Differentiation to Optimization

Many practical problems are reduced to either maximizing or minimizing some quantity: An engineer wants to maximize the rate of oil production, a businessman wants to minimize the costs and maximize the profit, a student wants to maximize his grades in all courses, and so forth. In this section, certain types of maximization and minimization problems will be solved using the rules of differentiation.

Example 10.1

The management of an oil company wants to fence in a rectangular shape a gas oil separation plant located on the seashore but will not fence the seashore side. If there are 10,000 m of fence to work with, what is the maximum area that can be enclosed?

SOLUTION

Let the lengths of the sides of the rectangular area be *x* and *y*. And let the area be *A*. Therefore:

$$A = xy \tag{10.1}$$

Since there are 10,000 m of fence to work with, the following relationship exists:

$$10,000 = 2x + y \text{ or}$$
(10.2)
$$y = 10,000 - 2x$$

Thus Equation (11.1) can be rewritten as

$$A = -x(10,000 - 2x)$$

$$= 10,000x - 2x^{2}$$
(10.3)

The variable x in this formula represents a length; therefore, it must be i = 0. Moreover, there are only 10,000 m of fence available; therefore, the 10,000 m of fence cannot all be used on the two sides of length x. Thus

$$2x \le 10,000 \text{ or}$$
$$x \le 5,000$$

With these physical restrictions the problem can be formulated to maximize the value of

$$A = 10,000x - 2x^2,$$

where $0 \le x \le 5,000$.

Taking the first derivative of Equation (10.3) yields

$$dA/dx = 10,000 - 4x \tag{10.4}$$

Setting the derivative (Equation 10.4) equal to zero gives the value of x that maximizes the area.

$$10,000 - 4x = 0$$

 $10,000 = 4x$
 $x = 2,500$

Since the value of x is within the physical restriction, then it is accepted. Substituting the value of x into Equation (11.2) gives

$$y = 10,000 - 2(2,500)$$
$$y = 5,000$$

Therefore, the length of the fenced area should be 5,000 meters and the two sides should be 2,500 meters each.

Example 10.2

The marketing manager of an oil company knows that the demand for the oil varies with its charged price. The company has determined that annual total revenue R (in thousands of dollars) is a function of the price p (in dollars). Specifically,

$$R = f(p) = -50p^2 + 500p$$

Determine the price that should be charged in order to maximize total revenue. What is the maximum value of annual total revenue?

SOLUTION

The revenue function is quadratic and its graph is a parabola that is concave downward as shown in Figure 10.2.



FIGURE 10.2 Quadratic revenue function.

As shown in the figure, the maximum value of *R* will occur at the vertex. The first derivative of the revenue function is:

$$f'(p) = 100p + 500$$
 by setting $f'(p) = 0$,
$$-100p = -500$$

$$p = 5$$

There is one critical point on the graph of f(p) and it occurs when p = 5. Therefore, a relative maximum occurs when the company charges \$5 per unit.

Example 10.3

Automobile hydrocarbon emission rate R(x) in milligrams per minute is found to be related to the speed x in kilometers per hour by

$$R(x) = xe^{-x/50}$$

Concerned drivers would like to avoid driving at the speed that gives the maximum emission rate. Find that speed.

SOLUTION

The maximum speed will occur when the derivative is set equal to zero.

$$R(x) = x[(-1/50)e^{-x/50}] + e^{-x/50}$$
$$= e^{-x/50}[1 - (1/30)x]$$

Setting R'(x) = 0 and dividing by $e^{-x/50}$ yields

1 - (1/50)x = 0

which gives x = 50 as the only critical value.

10.3 Linear Programming (LP)

In the previous case presented for finding the optimum conditions, it was established to find the partial derivative of the independent variable. This will lead to the determination of the absolute optimum conditions. The procedure is based on the assumption that an absolute maximum or minimum occurs within attainable operating limits. This is true for relatively simple problems, in which limiting constraints are not exceeded. Practically speaking, we often encounter industrial problems where optimum values may exist at a boundary or limiting condition rather than at a true maximum or minimum point (as we have seen with differential calculus).

Linear programming is a technique used to solve maximization or minimization problems where constraints are imposed on the decision maker. Typically linear programming deals with the problem of allocating limited resources among competing activities in the best possible way. Many problems of constrained optimization arise in engineering, business, and economics. For example, an oil company has a specified quantity of crude oil and fixed refinery capacity. It can produce gasoline of different octane ratings, diesel fuel, heating oil, kerosene, or lubricants. Given its crude oil supplies and refinery capacity, what mix of outputs should this company produce? The LP approach is to consider a system as decomposable into a number of elementary functions called *activities*. The different activities in which a system can engage constitute its technology. The activities must be combined in such a way to satisfy the system constraints and attain a stated objective as well as possible. The general mathematical model structure for an LP problem is as follows:

$$\min/\max Z = C_1 X_1 + C_2 X_2 + \dots + C_N X_N$$

Subject to:

$$A_{11}X_{1} + A_{12}X_{2} + \dots + A_{1N}X_{N}(\stackrel{\leq}{=})B_{1}$$

$$A_{21}X_{1} + A_{22}X_{2} + \dots + A_{2N}X_{N}(\stackrel{\leq}{=})B_{2}$$

$$A_{31}X_{1} + A_{32}X_{2} + \dots + A_{3N}X_{N}(\stackrel{\leq}{=})B_{3}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$A_{M1}X_{1} + A_{M2}X_{2} + \dots + A_{MN}X_{N}(\stackrel{\leq}{=})B_{N}$$

$$All X_{II} \text{ nonnegative}$$

where the parameters are as follows:

- 1. The *C*'s represent economic conditions.
- 2. The *A*'s represent technological conditions.
- 3. The *B*'s represent availability/requirement of resources.

In a more compact form, the above can be written as:

$$\max \operatorname{or} \min Z = \sum_{j=1}^{N} C_j X_j$$

Subject to:

$$\sum_{j=1}^{N} A_{ij} X_{j} (\stackrel{\leq}{=}) B_{i} (i = 1, \dots, m)$$
$$x_{j} \ge 0 \ (j = 1, \dots, N)$$

Linear programming constraints are of three types:

Type 1: (≤) Establish a maximal availability of resources.

Type 2: (\geq) Establish a minimal requirement of resources.

Type 3: (=) Establish an exact amount of resources.

10.3.1 Assumptions of Linearity

Linear programming models must satisfy certain assumptions of proportionality, additivity, divisibility, nonnegativity, and certainty. Proportionality means output and the usage of each resource are directly proportional to the level of each activity. Additivity implies a reference to the addition of elements or components. The rates of use of resources are assumed not to be affected by changes in the levels of other activities. The assumption of divisibility assumes activity units can be divided into any fractional levels, so that noninteger values for the decision variable are permissible. The parameters and constants in the LP model are assumed to be known and fixed, while negative quantities are not permitted.

10.3.2 Formulation and Solution of LP Models

Linear programming models are formulated in three steps:

- 1. Identification of the decision variables. The decision variables are the unknown quantities to be determined. These variables are usually represented by mathematical symbols, and they reflect the levels of activities. For example, an electrical manufacturing company needs to determine the number of radios (x_1), the number of TV sets (x_2), and the number of stereos (x_3) to produce.
- 2. Development of the objective function as a linear mathematical function of the decision variables. The objective function always consists of either maximizing or minimizing some value: for example, maximizing the profit or minimizing the cost of production.
- 3. Definition of the model constraints as equalities and inequalities of linear functions of the decision variables. The model constraints represent the restrictions placed on the firm by the operating environment. They can be in the form of limited resources, requirements, or restrictive guidelines.

These steps will be illustrated by the following LP example formulation.

Example 10.4

The production manager of an oil refinery must decide on the optimal mix of two blending processes, of which the inputs and outputs per production run are as follows:

	Input (B	arrels)	Output (Barrels)					
Process	Light Crude	Heavy Crude	Regular Gasoline	Premium Gasoline				
1	500	300	500	800				
2	400	500	400	400				

The maximum amounts available of light crude and heavy crude are 20,000 and 15,000 barrels, respectively. Market requirements show at least 10,000 barrels of regular gasoline and at least 8,000 barrels of premium gasoline must be produced. The profits per production run from process 1 and process 2 are \$3 and \$4, respectively.

SOLUTION

1. Identification of decision variables.

Let x_1 = number of production runs of process 1

 x_2 = number of production runs of process 2

Objective function. The objective in this problem is to maximize the profit of the operation. The objective function can be expressed mathematically as follows:

Maximize $Z = 3x_1 + 4x_2$

- 3. Constraints. There are three types of constraints for this problem besides the nonnegativity constraints.
 - a. Limitations on the availability of the light and heavy crudes.

 $500x_1 + 400x_2 \le 20,000$ $300x_1 + 500x_2 \le 15,000$

b. Market requirements for the sale of the two types of gasoline:

 $500x_1 + 400x_2 \ge 10,000$ $800x_1 + 400x_2 \ge 8,000$

c. Nonnegativity constraints:

$$x_1 \ge 0$$

 $x_2 \ge 0$

SOLUTION

Linear programming problems with only two decision variables can be solved graphically. Plotting the linear equations of the constraints will form a convex hull or a solution space, which is called in LP terminology the *feasible region*. The optimal solution, if there is one, will occur on an extreme point or points of the convex hull, and it can be found by plotting isoquant lines of the objective function. The graphic solution of the problem is given in Figure 10.3.

This linear programming problem can also be solved algebraically using the simplex method, which is usually tedious, time consuming, and needs many pivoting iterations. For this reason, especially if the problem has more than two variables and many constraints, computer programs are preferred to be used for solving such problems. Due to the popularity of using linear programming and other optimization techniques in decision-making analysis, many commercial and educational mathematical programming computer packages are available and used extensively to solve large-scale linear programming problems as well as small ones. Among these packages are Linear and Interactive Discrete Optimizer (LINDO), Quantitative Systems for Business (QSB), and Statistical Analysis System/Operations Research (SAS/OR).

The algebraic solution of the above linear programming problem using the QSB computer package is presented in Table 10.1. Additionally, the LP problem can easily be solved using the Microsoft[®] Excel[®] Solver. The setup of the LP problem along with the Solver reports are given in Tables 10.1A and 10.1B.



FIGURE 10.3 Graphic solution of the oil refinery problem.

Varia	bles			Varia	bles		
Number	Names	Solution	Opportunity Cost	Number	Names	Solution	Opportunity Cost
1	X1	+30.769230	0	5	S3	+10,000.000	0
2	X2	+11.538464	0	6	A3	0	0
3	S1	0	+0.00230769	7	S4	+21,230.763	0
4	S2	0	+0.00616385	8	A4	0	0

TABLE 10.1

Computer Solution Use for the Oil Refinery Problem Using QSB

Note: Maximum value of the OBJ = 138.4615, L = 4.

10.3.3 Applications of Linear Programming

10.3.3.1 The Transportation Model

The transportation model received its name because it arises very naturally in the context of determining optimal transporting patterns. The transportation model seeks the determination of a transportation plan of a single commodity from a number of sources to a number of destinations. The data of the model include:

- 1. Level of supply at each source and amount of demand at each destination
- 2. Unit transportation cost of the commodity from each source to each destination

The objective of the model is to determine the amount to be transported from each source to each destination such that the total transportation cost is minimized.

The transportation problem can be stated as follows: Suppose there are *m* sources (origins) $0_1, 0_2, ..., 0_m$ for a commodity, with a_1 units of supply at 0_1 ,

TABLE 10.A

Microsoft Excel Solver Solution for the Oil Refinery Problem

Decision variables	X1	X2				
Solution	30.76923	11.53846	Value			
Objective function coefficients	3	4	138.4615385			Slack/ surplus
Constraint-1	500	400	20000	\leq	20000	0
Constraint-2	300	500	15000	\leq	15000	0
Constraint-3	500	400	20000	≥	10000	10000
Constraint-4	800	400	29230.76923	≥	8000	21230.76923

TABLE 10.1B

Microsoft Excel Solver Answer Report for Example 10.8 Microsoft Excel 12.0 Answer Report Worksheet: [Book1]Sheet1 Report Created: 4/15/2012 11:58:51 AM Target Cell (Max)

			Original			
	Cell	Name	Value	Final Value		
	\$D\$3	Objective function coefficients value	0	138.4615385		
Adjustable Cells						
	\$B\$2	Solution X1	0	30.76923077		
	\$C\$2	Solution X2	0	11.53846154		
Constraints						
	Cell	Name	Cell Value	Formula	Status	Slack
	\$D\$4	Constralnt-1 value	20000	\$D\$4< = \$F\$4	Binding	0
	\$D\$5	Constraint-2 value	15000	\$D\$5< = \$F\$5	Binding	0
	\$D\$6	Constraint-3 value	20000	D\$6> = F\$6	Not binding	10000
	\$D\$7	Constraint-4 value	29230.76923	\$D\$7> = \$F\$7	Not binding	21230.76923

and *n* destinations, D_1 , D_2 , ..., D_n for the commodity, with a demand *b* at D_j . If C_{ij} is the unit cost of transporting from 0_i to D_j , find the optimal solution that will minimize the total transportation cost. The problem can be stated mathematically as follows:

$$\text{Minimize} \sum_{\substack{i \in I \\ j \in j}} C_{ij} X_{ij}$$

subject to:

$$\sum_{j \in j} X_{ij} = a_i, \quad i \in T = \{1, 2, \dots, m\}$$
$$X_{ij} = b_j, \quad j \in J = \{1, 2, \dots, n\}$$
$$X_i \ge 0, \quad i \in I, j \in J$$

where X_{ij} represents the amount shipped from 0_i to D_j . The assumptions that total supply is equal to demand and each amount of supply and demand is nonnegative are necessary conditions for the existence of a feasible solution.

Example 10.5

A contractor must assign workers in an oil field to four work sites each day. The travel time in minutes between each dispatch location and work site is shown on the directed areas of the network diagram. In order to maximize the number of productive work hours per day of each worker, the contractor wishes to minimize the total worker travel time. Travel time is considered to be unproductive work time. The number of workers dispatched from locations 1 and 2 is 30 and 50 workers, s1 = 30, and s2 = 50, respectively. The numbers of workers required at each work site 3, 4, 5, and 6 are 20, 20, 30, and 10, respectively. They are shown on the network diagram (Figure 10.4) as $d_3 = 20$, $d_4 = 20$, $d_5 = 30$, and $d_6 = 10$. Formulate a mathematical model to minimize total travel time.

SOLUTION PROBLEM FORMULATION

Let X_{ij} represent the number of workers dispatched from location *i* to work site *j*.

The objective function is as follows:

Min $40X_{13} + 20X_{14} + 20X_{15} + 50X_{16} + 20X_{23} + 50X_{24} + 10X_{25}$

 $+ 60X_{26}$



FIGURE 10.4 Network diagram for the transportation problem.

and the constraints set is as follows:

$$X_{13} + X_{14} + X_{15} + X_{16} = 30$$

$$X_{23} + X_{24} + X_{25} + X_{26} = 50$$

$$X_{13} + X_{23} = 20$$

$$X_{14} + X_{24} = 20$$

$$X_{15} + X_{25} = 30$$

$$X_{16} + X_{26} = 10$$

$$X_i \ge 0 \text{ where } i = 1, 2$$

$$j = 3, 4, 5, 6$$

The solution of the above problem as given by Excel Solver in Figure 10.4A is to dispatch 20 and 10 workers from location 1 to sites 2 and 6, respectively. Location 2 would dispatch 20 and 30 workers to sites 3 and 5, respectively. The total traveling time will be 1600 minutes.

10.3.3.2 The Assignment Model

Basically, the assignment model can be stated as follows: Given *n* individuals and *n* jobs, each individual is to be assigned to only one job and each job is to be performed by only one individual, in such a way that the total cost is to be minimized. The assignment problem can be described mathematically as follows:

Let
$$X_{ij} = \begin{cases} 1, & \text{if individual is assigned to job j} \\ 0, & \text{otherwise} \end{cases}$$
 (10.5)

The objective function of the assignment model is to minimize the total cost of assignment.

Minimize
$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1, \ 2, \ \dots, \ n$$
(10.6)

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad j = 1, \ 2, \ \dots, \ n \tag{10.7}$$

where c_{ii} is the cost of assigning the *i*th individual to the *j*th job.

Constraint (2) ensures that each individual is assigned to exactly one job, while constraint (3) ensures that each job is covered by one individual. The problem as stated above is an integer linear programming problem. However, if constraint (1) is replaced with the set of nonnegativity restriction:

$$x_{ij} \ge 0$$
 $i = 1, 2, ..., n$
 $j = 1, 1, ..., n$

The assignment problem becomes merely a special case of the transportation problem in which m = n and $a_i = 1$, i = 1, 2, ..., n; $b_i = 1, j = 1, 2, ..., n$.

Example 10.6

To expand on the linear programming approach in solving assignment problems, let us consider the following example:

		Client	
	Α	В	С
Project Leader	(Estir	nated Completion Tin	ne, Days)
Tom	10	15	9
Mary	9	18	5
Jack	6	14	3

We would like to assign project leaders such that the total number of days required to complete all three projects is minimized. Let us begin by defining the following decision variables:

$$x_{ij} = \begin{cases} 1 & \text{if project leader i is assigned to client } j, \ i = 1, \ 2, \ \dots, \ 3 \\ 0 & \text{otherwise} \end{cases}$$

Using the above decision variables, the objective function calling for the minimization of total days of labor can be written as:

Min
$$10x_{11} + 15x_{12} + 9x_{13} + 9x_{21} + 18x_{22} + 5x_{23} + 6x_{31} + 14x_{32} + 3x_{33}$$

The constraints affecting this problem are that all clients must receive exactly one project leader and that the project leaders cannot be assigned to more than one client. The first condition is satisfied by the following linear constraints:

$$x_{11} + x_{21} + x_{31} = 1$$
 Client A
$$x_{12} + x_{22} + x_{32} = 1$$
 Client B
$$x_{13} + x_{23} + x_{33} = 1$$
 Client C

The second condition is reflected in the following constraints:

 $x_{11} + x_{12} + x_{13} = 1$ For the case of Tom $x_{21} + x_{22} + x_{23} = 1$ For the case of Mary $x_{31} + x_{32} + x_{33} = 1$ For the case of Jack $x_{ij} = 0 \text{ or } 1$

The above 0–1 linear programming problem can be solved using any linear programming software or the Microsoft Excel Solver.

The solution of the assignment problem is given by Excel Solver in Figure 10.5. Tom is assigned to client B while Mary and Jack are assigned to clients C and A, respectively. The total completion time is 26 days.

Figure 11.5 - Microsoft Excel																			
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4	TOM	0	1	0	1		_		-										н
5	MARY	0	0	$\leftarrow 1$	1		Optimia	lassignment											ч
6	JACK	1	0	0	1		decision	15											-1
7		1	1	1															4
8																			4
9	Project Leader\Client	A	B	C															-1
10	TOM	10	15	9	_														ч
11	MARY	9	18	5															4
12	JACK	6	14	3			Totalcon	npletion time											ч
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FIGURE 10.5

Setup and solution for the assignment Example 10.6.

10.3.3.3 The Network Models

A network is an arrangement of paths or arcs connected at various nodes through which one or more items move from one node to another. Many decision-making problems can be described as networks. Some obvious examples are concerned with traffic and the shipment of goods. Other examples include production planning, capital budgeting, machine replacement, and project scheduling.

There are several different types of network models. Network flow models concentrate on the shortest route problem, the minimal spanning tree problem, and the maximal flow problem. Project Evaluation and Review Technique (PERT) and the Critical Path Method (CPM) are two other network models used extensively for project management analysis. Although there are special algorithms to solve the different types of network models, many of them can be formulated and solved as linear programming problems.

In this subsection the formulation of the maximal flow network model as a linear programming problem will be demonstrated.

Example 10.7

Determine the capacity of the pipeline network system shown in Figure 10.6. The flow capacity in million gallons per day is indicated on each directed arc. The objective is to maximize the flow that enters the source and exits the sink node. Formulate this maximal flow problem as a linear programming model.

Figure 11.6A Pipeline - Microsoft Excel																								
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FIGURE 10.6

A setup and optimal solution for the pipeline system flow, Example 10.7.

PROBLEM FORMULATION

Let *X_j* represent the amount of flow on arc *j*. *f* represents the amount of flow entering and leaving the pipeline network system. The objective function and constraints are as follows:

Maximize
$$x_1 + x_2 + x_3$$

Subject to the constraints:

$$x_{1} + x_{2} + x_{3} = f$$

-x₁ - x₅ + x₆ = 0
-x₂ - x₄ + x₅ + x₇ = 0
-x₃ + x₄ + x₈ = 0
-x₆ - x₇ - x₈ = -f

along with arc capacity constraints:

$$0 \le x_{1} \le 5$$

$$0 \le x_{2} \le 2$$

$$0 \le x_{3} \le 4$$

$$0 \le x_{4} \le 4$$

$$0 \le x_{5} \le 3$$

$$0 \le x_{6} \le 6$$

$$0 \le x_{7} \le 2$$

$$0 \le x_{8} \le 3$$

The above problem can be solved using Microsoft Excel Solver. The setup and the optimal solution of the pipeline flow system are shown in Figure 10.6A. The maximum flow capacity in the pipeline system is 11 million gallons per day.

10.4 Nonlinear Programming

The efficient methods of solution and the ease of linear programming formulation made linear programming very popular among optimization techniques. However, not all optimization problems consist solely of linear relationships. Some problems encompass an objective function and constraints that are not linear. Such problems are classified as nonlinear programming, which is the topic of this section.

10.4.1 Constrained and Unconstrained Optimization

Consider a profit function given by

$$f(x) = Z = px = FC-VC(x)$$

where *x* is the number of units produced or sold, *P* is price, *FC* is fixed cost, *VC* is the variable cost, and the demand function that depends on the selling price is expressed by

$$x = 2,000 - 50p$$

By substituting the demand function into the profit equation, the following nonlinear function is obtained:

$$Z = 2,000p - 50p^2 - FC - 2,000VC + (VC)(50p)$$

When fixed cost (*FC*) equal to \$3,000 and variable cost (*VC*) equal to \$20 are substituted into the above profit function:

$$Z = -50p^2 + 3,000p - 43,000$$

Setting the derivative of the above profit function equal to zero, the price that gives the maximum profit can be found. For this particular profit function a price of \$30 gives the maximum profit of \$2,000.

This type of nonlinear programming model is referred to as an *unconstrained optimization*. It consists of a single nonlinear objective function and no constraints. If, however, one or more linear or nonlinear constraints are added to the nonlinear objective function, the model is referred to as a constrained nonlinear optimization model.

Constrained optimization problems can be handled in several ways. The substitution method and the Lagrangian are two of the most commonly used solution techniques for solving simple nonlinear optimization problems.

10.4.2 The Substitution Method

The substitution method or procedure converts the problem to one of unconstrained optimization. This is illustrated in Example 10.8.

Example 10.8

Consider the following constrained nonlinear optimization problem:

Minimize
$$TC = 3x^2 + 6y^2 - xy$$
 subject to $x + y = 20$.

Determine the values of *x* and *y* which will result in the least cost.

SOLUTION

From the constrained equation

x = 20 - y

Substituting the value of *x* in the objective function yields the following result:

$$TC = 3(20 - y)^{2} + 6y^{2} - (20 - y)y$$
$$TC = 3(400 - 40y + y^{2}) + 6y^{2} - 20y + y^{2}$$
$$TC = 1,200 - 140y + 10y^{2}$$

Setting the derivative of the above total cost function equal to zero and solving for the value of *y* will give the following result:

$$dTC/dy = -140 + 20y = 0$$
$$20y = 140$$
$$y = 7$$

Substituting the value of *y* into the constraint equation will give the value of *x*:

$$x = 20 - 7$$
$$x = 13$$

Substituting the values of *x* and *y* in the objective function will yield the least total cost, which is \$710.

10.4.3 The Method of Lagrangian Multipliers

The Lagrangian technique for solving constrained nonlinear problems is a procedure for optimizing a function that combines the original objective function and the constraint conditions. In this method, the constraints as multiples of a Lagrangian multiplier, λ , are subtracted from the objective function. The combined equation is called the Lagrangian function. To demonstrate this method, consider the above nonlinear problem (Example 10.8).

Minimize
$$TC = 3x^2 + 6y^2 - xy$$
 subject to $x + y = 20$

Rearranging the constraint to bring all the terms to the left of the equal sign, the following is obtained:

$$x + y - 20 = 0$$

Multiplying this form of the constraint by λ , the Lagrangian multiplier, and adding (subtracting in case of maximization) the result to the original objective function will yield the Lagrangian function:

$$L = 3x^{2} + 6y^{2} - xy + \lambda(x + y - 20)$$

The Lagrangian function can be treated as an unconstrained minimization problem. The partial derivative of the Lagrangian function with respect to each of the three unknown variables x, y, and λ needs to be determined. These are as follows:

$$\partial \text{LTC} / \partial X = 6y - Y + \lambda$$

 $\partial \text{LTC} / \partial Y = 12Y - X + \lambda$
 $\partial \text{LTC} / \partial \lambda = X + Y - 20$

Setting the above equations equal to zero will result in a system of three equations and three unknowns:

$$6\mathbf{x} - \mathbf{y} + \lambda = 0 \tag{10.8}$$

$$-x + 12 Y + \lambda = 0 \tag{10.9}$$

$$x + Y - 20 = 0 \tag{10.10}$$

Equation (10.10) is the constraint condition imposed on the original optimization problem. Solving the equations simultaneously will determine the values of x, y, and λ . These values are as follows:

$$x = 13$$
$$y = 7$$
$$\lambda = -71$$

Α	В	С	D	E	F	G	Н	Ι
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FIGURE 10.7 Solver Example 10.8, Setup.

The Lagrangian multiplier, λ , has an important economic interpretation. It indicates the marginal effect on the original objective function of implementing the constraint requirement by one unit. Here λ can be interpreted as the marginal reduction in total cost that would result if only 19 instead of 20 units of combined output were required. Although the Lagrangian method is more flexible than the substitution method, it can solve only small problems. As the problem size expands, computerized approaches should be used. The above example is solved by Microsoft Excel Solver. Figure 10.7 shows the example setup, while Figure 10.8 presents the Solver parameters.

The Solver program generates two reports: an answer report and a sensitivity analysis report. The value of the Lagrangian multiplier as reported in the sensitivity report is 71 which indicates the amount of change in total cost that would result from a unit change in the combined output.

Example 11.8 - Microsoft Excel																				
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FIGURE 10.8

Solver parameters and solution for the nonlinear Example 10.8.

Section 3

Applications and Case Studies

Section 3 represents a major change in this revised edition. It includes ten chapters covering the three main operations in the oil and associated gas industry from prospects to finished products. It covers:

- Upstream operations: **Sub-Subsurface** (Chapter 11, Exploration and Drilling; Chapter 12, Reserves and Reserve Estimate; and Chapter 13, Production).
- Middle stream operations: **Surface** (Chapter 14, Gas-Oil Separation; Chapter 15, Crude Oil Treatment; and Chapter 16, Gas Treatment and Conditioning).
- Downstream operations: **Refining/Processing** (Chapter 17, Crude Oil Refining: Physical Separation; Chapter 18, Crude Oil Refining: Chemical Conversion; Chapter 19, Natural Gas Processing; and Chapter 20, Oil and Gas Transportation).

The chapters on middle stream operations, known as surface petroleum operations (SPO) and natural gas processing and fractionation, are new to this edition.

As in the second edition, the primary aim of Section 3 is to illustrate how economic analysis is applied to solve engineering problems in different facets of the oil industry. Addressing relevant problems involving oil-engineering decisions is our main focus. Case histories and actual calculations for oil operations and gas processing plants are presented. Many real-world examples are documented specially for Middle East operations and others.

For each chapter in Section 3, the technical aspects are described first, followed by case studies and applications relevant to specific problems.

11

Exploration and Drilling

Hussein K. Abdel-Aal

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Hydrocarbon exploration is a high-risk investment, and risk assessment is paramount for successful exploration portfolio management. Virtually every oil field decision is founded on profitability. With no control of oil and gas prices, and facing steadily rising costs and declining reserves, companies' basic decisions are based on constantly moving targets. Simply put, a producing oil and gas property is a series of cash payments projected in the future.

Technology aspects covered in this chapter deal with the very first activity in finding oil. Methods used for search of oil or oil exploration are discussed followed by types of drilled wells, their numbers, and spacing. The use of economic balance and binomial expansion is proposed to solve relevant problems. The cost of finding oil and the size of capital expenditures in oil fields are considered as well.

Examples of optimization of the number of wells to drill, the cumulative binomial probability of success in drilling wells, and many others are presented.

11.1 Technology Aspects

11.1.1 Introduction

Knowledge of the basic principles as well as some of the common terms and concepts encountered in the oil fields is helpful for complete understanding of the subject. Geological formations, origin and accumulation of petroleum, oil reservoirs and their classification, petroleum prospecting practices, drilling and development operations, and many others are important in our engineering economics discussion.

Since our purpose here is not an explanation of the technical operations in petroleum production, we highlight only the topics pertinent to the economic appraisal or valuation of an oil property. The *oil property* as defined is meant to include any property with underground accumulations of liquid or gaseous hydrocarbons that might be produced at a profit.

Additional background materials on oil production methods and the estimation of recoverable oil reserves are given in Chapter 12.

11.1.2 The Search for Oil: Exploration

The first prerequisite to satisfying man's requirements for refined petroleum products is to find crude oil. Oil searchers, like farmers and fishermen, are in a contest with nature to provide the products to meet human needs. They are all trying to harvest a crop.

But the oil searcher has one problem that the farmer does not have. Before the oil man can harvest his crop, he has to find it. Even the fisherman's problem is not as difficult, since locating a school of fish is simple compared to finding an oil field. The oil searcher is really a kind of detective. His hunt for new fields is a search that never ends; the needle in the haystack could not be harder to find than oil in previously untested territories.

Today, petroleum prospecting and hence its discovery are credited to what is called *subsurface study*. This includes:

The use of geophysical instruments

Cuttings made by the bit as the well is drilled

Core samples collected from the well

Special graphs called *logs*, generated by running some tools into the oil wells during the drilling operations

The net result of these studies is the preparation of different kinds of geological maps that show the changes in the shape of subsurface structures with depth. The geophysical techniques encompass three methods:

- 1. Seismic
- 2. Magnetic
- 3. Gravitational

Each of these techniques utilizes the principles of physical forces and the properties of the earth. For example, in the seismic method, creation of artificial earthquake waves is established by firing explosives into holes. The rates of travel of these waves are analyzed by echo sounding techniques. The most recently invented instruments are reflection seismographs, gravimeters, and airborne magneto-meters. Such devices enable geophysicists to explore not only the surface and the subsurface conditions of the earth searching for oil, but the lunar surface and depths as well. These sophisticated lunar experiments monitor the earth's magnetic and gravitational properties from space.

Stratigraphy, on the other hand, involves drilling a well to obtain stratigraphic correlation and information. Complete sections of the well formations are exposed and rock samples are taken while the drilling operation is in progress. Success in finding oil will depend to a large degree on the accuracy of *well logging*. Several kinds of well logs exist. The most commonly used are:

- 1. Drillers logs
- 2. Sample logs
- 3. Electric logs
- 4. Radioactivity logs
- 5. Acoustic logs

Once the data are collected using core samples and wire-line logs of various kinds, *contour maps* are prepared. A contour map consists of a number of contours, or lines, on which every point of a given area is at the same elevation above or below sea level. These lines must be at regular depth intervals to enable geologists to depict three-dimensional shapes.

Other means of exploring for oil include detailed ground geological surveys aided by preliminary results of aerial photography and photogeological work.

11.1.3 Oil Reservoirs and Classification

The two most important prerequisites for an oil accumulation to occur are:

- 1. A trap that acts as a barrier to fluid flow
- 2. A porous and permeable bed or reservoir rock

Each geological formation, irrespective of age or composition, must process these physical properties of *porosity* and *permeability* in order to be described as a *reservoir rock*.

Some of the reservoir-rock characteristics are as follows:

Although porosity and permeability are important as individual parameters, neither is of value in the absence of the other.

- The reservoir is judged by its thickness and porosity, that is, by the abundance of inter-connected voids, which provide passages for the fluids to flow.
- Flow capacity or permeability depends on porosity to some extent, but porosity does not depend on permeability. In other words, reservoir rocks of high porosity are not necessarily of high permeability, and those of low porosity are not necessarily of low permeability. Generally speaking, *sandstone* reservoirs are more porous than *limestone*.

11.1.3.1 Definitions

A *reservoir* may be defined as any body of underground rocks with a continuously connected system of void spaces filled with hydrocarbon fluids which can move toward wells—drilled into the rocks—under the influence of either natural or artificial driving forces. If the volume of the hydrocarbons produced by the wells is sufficient to permit an *economic recovery*, then the accumulation is known as a commercial reservoir and is usually referred to as a *proven reserve*.

Reservoirs, on the other hand, could be described as a "resource base," which is the sum total of crude oil, natural gas, and natural gas liquids in the ground within an identified geographic area. The reservoir thus includes all stocks, including some stocks that are unrecoverable and therefore not included in "proven reserves."

Proven reserves refer to the reserve stocks of immediate or short-term economic feasibility of extraction; therefore, stocks that are known to exist but cannot profitably be extracted are excluded from reserves. The cost limits, or as far as one can go on profitably employing these reserves, are those costs consistent with the taking of "normal" risk and commercial production.

The void spaces of proven reservoirs normally contain some *interstitial water* (or connate water) along with the hydrocarbons. Since most of this water is held in space by some sort of capillary forces, reservoir rocks turn out to be saturated with the *three* reservoir fluids: oil (liquid), gas, and water.



FIGURE 11.1 Different stages in well drilling.

An *oil field* consists of all "pools" or reservoirs underlying a continuous geographic area, with no large enclosed subareas being considered unproductive.

11.1.4 The Role of Drilled Wells in Development

All the activity described above for oil exploration leads only to an evaluation of the probability that oil is in a particular location. Once it seems probable that there really is oil, wells must be drilled. Reservoirs and oil fields are discovered only by drilling to sufficient depths to verify what was recommended by an exploration team. Wildcat wells, exploratory wells, or test wells are drilled first for probing purposes. An unsuccessful wildcat well is called a *dry hole*. A successful wildcat well is called a *discovery well*. Subsequent wells drilled into proven reservoirs for production purposes are called *development wells*. The drilling of test wells is the most costly single operation in oil exploration. This is discussed further in Section 11.1.8.

One exploratory well alone does not indicate extensive oil accumulation. Other wells, carefully located near the well where oil has been discovered, are drilled to discover if there is a reservoir in the area and approximately how much is available and can be recovered. To do this, first reliable information must be obtained as to the quantity of oil (and gas) that is recoverable, so an economic and proper size and type of surface crude oil production plant can be set up. Second, the characteristics of the oil itself and the nature and amount of oil in the reservoir should be determined from the samples of the reservoir. The raising of oil to the ground surface and then the handling of the oil at ground surface will depend to a great extent on the nature of the oil itself and its associated gas. Crude oil can range from very heavy viscous oil, almost a tar, with little or no gas dissolved in it and under very low pressure, to an extremely light, straw-colored oil with a considerable volume of gas, known as a condensate-type crude. The condensate-type crude is more likely to be found at great depths. Under conditions of high pressure and temperature that exist at deep levels, the crude is usually in the gaseous stage. Between the extremes of a heavy viscous oil and a very light oil, there is an infinite variety of crude oil. The manner of producing these crudes is decided only after examining samples that show their characteristics and physical attributes.

Intelligent wells are increasing in popularity. These contain permanent monitoring sensors that measure pressure, temperature, and flow and telemeter these data to the surface. More importantly, these wells contain surface-adjustable downhole flow-control devices, so, based on the dynamic production information from all the wells in the reservoir, flow rates can be optimized without having to perform a costly intervention.

11.1.5 Number of Wells and Well Spacing

The location as well as the number of wells drilled into a proven reservoir raise questions such as, "How many wells should we drill in the reservoir?"; "How close should the wells be?"; and "How many wells do we need before we can lay pipelines economically?" Usually, use of the economic balance will provide answers to this type of question.

11.1.6 Drilling Operations

There are two methods of drilling a well: the cable tool and the rotary methods. No matter which method is used, a derrick is necessary to support the drilling equipment.

Cable tool drilling is the older method of drilling. In this method a hole is punched into the earth by repeatedly lifting and dropping a heavy cutting tool, a bit, hung from a cable. Today, however, practically all wells are drilled by the rotary method.

Rotary drilling bores a hole into the earth much as a carpenter bores a hole into a piece of wood with a brace and bit. In the middle of the derrick floor there is a horizontal steel turntable that is rotated by machinery. This rotary table grips and turns a pipe extending through it downward into the earth. At the lower end of the pipe, a bit is fastened to it.

As the drill chews its way farther and farther down, more drill pipe is attached to it at the upper end. As section after section of drill is added, the drill pipe becomes almost as flexible as a thin steel rod. Controlling the drill pipe under such conditions and keeping the hole straight at the same time is difficult and requires great skill in drilling.

During the drilling, a mixture of water, special clays, and chemicals, known as drilling mud, is pumped down through the hollow drill pipe and circulated back to the surface in the space between the outside of the pipe and the walls of the pipe. This drilling mud serves several purposes, including lubricating and cooling the bit and flushing rock cuttings to the surface.

As the drilling hole is deepened, it is lined with successive lengths of steel pipe, called *casings*. Each string of casing slides down inside the previous one and extends all the way to the surface. Cement is pumped between these successive strings of casing and seals against any leakage of oil, gas, or water.

To achieve large annual additions to reserves and to output, the rate of drilling must be stepped up sharply. Barrels added per foot drilled are one of the best indicators of the results of drilling effort. This measure should not show a decline. A projection of the trend of barrels added per foot of drilling should be established for oil companies engaged in production.

11.1.7 Factors Affecting Penetration in Drilling

Studies made by experts from drilling and equivalent companies indicate that there is a positive effect of weight and speed of rotation on penetration rate, or feet per hour of drilling. This is true whether toothed or carbidestudded bits are used.

The proper penetration rate of weight on bit rotary speed and hydraulic horsepower can be plotted on a graph to determine optimum drilling at minimum drilling cost. Thus, the penetration rate of a bit varies with weight on bit, rate of rotation, and hydraulic horsepower.

11.1.8 Costs of Drilling

An increase in depth increases drilling costs. Costs increase exponentially with depth, even for a "normal," trouble-free well. An increase in depth can also increase the chances of mechanical problems, which adds to the cost of drilling.

Increased depth also reduces available information about potential reservoirs about quality of crude oil and quantity available (proven reserves). Risks increase with uncertainties about reservoir quantity and quality available.

Costs of drilling depend on the kind of oil and what potential energy the oil possesses by virtue of its initial pressure in its reservoir, and by the amount of dissolved gas it may contain. In many cases the crude may have enough potential energy to permit a well to flow large quantities of oil to the surface without any artificial assistance, such as use of gas or water injection. (This is quite prevalent in oil wells in the Middle East.) But when oil cannot flow unaided, or when the pressure in the reservoir has decreased to a pressure that is too low to be economical, costly mechanisms that lift oil to the ground surface must be employed. Low pressure in the reservoir and low gas content generally go together. This kind of crude, therefore, must be handled in a different manner.

The daily rates of offshore drilling rigs vary by their capability and market availability. With deep-water, drilling rig rates of around \$420,000/day were reported in 2010. A high-pressure, high-temperature well of duration 100 days can cost about \$30 million.

Onshore wells can be considerably cheaper, particularly if the field is at a shallow depth, where costs range from less than \$1 million to \$15 million for deep and difficult wells.

Statistical information for the period 2002 to 2007 on the costs of crude oil and natural gas well drills are reported by U.S. Energy Information as follows:

	GraphClear	2002	2003	2004	2005	2006	2007	View History
Thousand Dollars per Well								
All (real*)		1,011.9	1,127.4	1,528.5	1,522.3	1,801.3	3,481.8	1960-2007
All (nominal)		1,054.2	1,199.5	1,673.1	1,720.7	2,101.7	4,171.7	1960–2007
Crude oil (nominal)		882.8	1,037.3	1,441.8	1,920.4	2,238.6	4,000.4	1960–2007
Natural gas (nominal)		991.9	1,106.0	1,716.4	1,497.6	1,936.2	3,906.9	1960–2007
Dry holes (nominal)		1,673.4	2,065.1	1,977.3	2,392.9	2,664.6	6,131.2	1960–2007
Dollars per Foot								
All (real*)		187.46	203.25	267.28	271.16	324.00	574.46	1960-2007
All (nominal)		195.31	216.27	292.57	306.50	378.03	688.30	1960–2007
Crude oil (nominal)		194.55	221.13	298.45	314.36	402.45	717.13	1960–2007
Natural gas (nominal)		175.78	189.95	284.78	280.03	348.36	604.06	1960–2007
Dry holes (nominal)		284.17	345.94	327.91	429.92	479.33	1,132.09	1960–2007

Source: U.S. EIA: Annual Energy Outlook 2013, Release Dates: April 15, May 2, 2013.

11.2 Economic Evaluation and Application

11.2.1 Economic Balance in Oil Fields (Optimization)

The recovery of oil from underground, or offshore, reservoirs is a good application of the principle of economic balance. The problem is one of determining the optimum number of wells to drill, and the accurate spacing of these wells, to get maximum profit. The following considerations highlight the subject.

The greater the number of wells, the larger will be the ultimate recovery, provided that the recovery rate does not exceed the "most efficient engineering rate." However, the most efficient engineering rate (economic balance) does not necessarily mean the optimum rate for maximum profits. Economic balance, therefore, consists of a balance of greater fixed costs for a larger number of wells drilled plus usually higher operating costs for higher production rates against greater ultimate recovery from the larger number of wells.

Thus the principle of economic balance in the oil fields is to drill as many wells as possible and needed within fixed costs and operating cost limits relative to the greatest ultimate recovery in terms of the realizable value (sales value) for the recovery. There is an upper limit to the number of wells that can be drilled, however, because of technical considerations. In other words, greater fixed costs plus higher operating costs must be considered when increasing the number of wells to be drilled in an attempt to obtain a greater ultimate recovery of oil.

Upon discovery of large enough reserves for commercial drilling, the concept of well spacing becomes important to the oil engineer. The characteristics of reservoirs largely control the well-spacing pattern. For example, reservoirs with thick or multiple zones of oil will usually require more wells, and possibly closer spacing between wells, to take advantage of natural drainage (gravity flow) at its maximum than those reservoirs with thin crude oil composition located in single zones. Furthermore, porous reservoirs will produce more barrels of oil than "tight" reservoirs.

Other factors of a technical nature which should be considered in the spacing of wells, besides thickness versus thinness of the crude itself and the multiple zones versus single zones, include depth to the productive zones of the oil, viscosity of the oil, gravity of the oil, reservoir pressures, and reservoir properties. Therefore, in well spacing, economics of anticipated recoveries based on thickness of oil and saturation of the pay zone become important. Obviously, the greater the number of wells drilled in a single reservoir, the greater will be the ultimate recovery per surface area of oil or gas.

There is a practical limit to the number of wells, and hence the spacing of wells, that can be drilled, however, which is controlled by the cost of drilling and operation. This limit to the number of wells to be drilled is based

on estimated ultimate recovery, in barrels of oil, from each well. Since depth is the principal factor governing drilling costs, depth has a bearing on the problem of well spacing.

There is no hard and fast rule on spacing of wells; the technical and nontechnical factors relative to the oil reservoir must be considered separately. Oil wells drilled in the United States are widely spaced and located at the centers of 40-acre tracts or at like ends of 80-acre tracts. For gas wells, on the other hand, spacing ranges between 160 and 640 acres per well. The acreage assigned to each development well is known as a *drilling unit* prior to completion of the well and as a *production unit* upon successful completion.

Usually, the greater the depth to reach productive zones of oil, the wider the spacing of wells. Since viscous oils do not possess the mobility of ready passage through reservoirs, as lighter, less viscous oils do, closer spacing of wells is usually needed with oils of heavy viscosity properties in order to effect maximum efficient drainage. In the case of gravity, the lighter-gravity oils (with the higher API) contain more dissolved gases, have more mobility, and are less viscous than the lower-gravity oils, and so will require fewer wells and wider spacing to effect maximum efficient drainage. Reservoirs with high pressures, particularly if pressures are maintained by some recycling operations such as use of water, gas, or air, offer higher recovery per well. Thus wider spacing can be employed in reservoirs with high pressures.

Such reservoir properties as porosity, the ability to contain fluids, and permeability influence well spacing. Porous and permeable reservoirs that allow fluids such as oil to flow through the reservoir to the well bore, mean that reservoirs can be effectively drained, so fewer wells with wide spacing are suitable under such conditions. Closer spacing of wells is necessary when "tight" reservoirs, with low porosity and permeability, are involved.

Some nontechnical factors also affect well spacing. These include, for instance, the rate of production desired because of terms of the oil lease, market price of crude, market demand, etc. Also, proration laws of a government can dictate the amount of oil or gas an oil company can produce. When this is the case, the number of wells drilled and the spacing may be affected. Where the rate of payout desired is lengthened and deferment of income over a wide period because of income tax problems is the objective, the number of wells drilled may be cut back. Thus spacing will tend to be wider under such conditions. Conversely, where the rate of payout desired is for a short period, more wells should be drilled with closer spacing.

Example 11.1

The following simple example offers two alternatives relative to the number of wells to be drilled and spaced in a reservoir involving the following information:

	Alternative 1: Drill Two Wells	Alternative 2: Drill Six Wells
Total capital investment (\$)	3,800,000	8,400,000
Annual operating costs	560,000	1,800,000
Total production (bbl/day)	20,000	100,000

REQUIRED

- (a) Determine the spacing between wells.
- (b) Which alternative do you recommend: the wider spacing between two wells or the closer spacing between six wells?

SOLUTION

(a) Let us establish the following table using some common basis:

	Alternative 1	Alternative 2
1. Capital investment/well (\$)	1,900,000	1,400,000
2. Annual operating cost/well (\$)	280,000	300,000
3. Capitalized cost of item (2) using interest rate of 10%	2,800,000	3,000,000
4. Sum of items (1) + (3)	4,700,000	4,400,000
5. Production bbl/(day)(well)	10,000	16,667

Spacing is calculated on the assumption that a producing well is located on an area of *1 acre*. Hence, daily oil production is reported on the basis of bbl/(well)(acre).

Income is reported by assigning an arbitrary value for the drilled oil equal to 33% of the well-head value of produced oil.

For 1 *day* of production, and taking *one well* as a basis for our calculation, we obtain:

Spacing between wells is given by :
$$\frac{\text{Capital investment ($)}}{\text{Revenue ($/acre)}}$$
 (11.1)

For alternative 1, spacing = 17 acres For alternative 2, spacing = 10 acres

Thus a spacing of 17 acres between two wells is recommended for alternative 1, while 10 acres is to be used as spacing for the six-well alternative.

(b) Although operating costs are greater in total and on a per-well basis with six wells, total production is greater and hence total revenues earned, including profits, will be greater. Furthermore, the payout period favors the six-well alternative over the payout period of the alternative on two wells, since more overall production of six wells will increase total revenues received, sufficient to return investment more quickly.

Finally, capital investment per barrel produced per day favors alternative 2. Capital investment per barrel per day with six wells drilled is \$84, whereas capital investment per barrel per day with two wells drilled is \$190.

Obviously, Alternative 2, or six wells, is the selection, assuming everything else favors this alternative, including reservoir pressures, no limit on production, favorable permeability and porosity features, etc.

Example 11.2

Explorers for crude oil try to determine how often success will be gained from a given program of *N* well (wells drilled). "What are the odds of success?" a company might ask. A company drilling, say, 20 or 30 wells per year might want to know the odds of making one, two, three, or five discoveries, with discovery meaning simply a producing well and not profitability of the well. How much oil there is, is not part of discovery but comes under field size distribution. To find these odds of success to total wells drilled, a mathematical technique called binomial (two numbers) expansion is used.

For simplicity, assume that each well in the program has the same chance of success with an assumed 10% success rate. Oil explorers know that some prospects have better "odds" or chances of success than others. For most exploration programs, we can assume an "average success" rate with reasonable safety.

F indicates probability of failure (a dry hole), and *S* indicates probability of success.

For one well (one outcome) F + S = 1.00, or we can write $F + S (F + S)^1$. For two wells, there are four possible outcomes, FF + FS + SF + SS = 1.00; and, of course, FS + SF can be written 2*FS*. Then $F^2 + 2FS = S^2 = 1.00$.

Now, if you remember your algebra, $F^2 + 2FS + S^2$ is the product of (F + S)(F + S) and can be written as $(F + S)^2$. So $F^2 + 2FS + S^2 = (F + S)^2$. The left half of this equation is the expansion of the binomial (F + S) to $(F + S)^2$.

Now we can set up a cumulative binomial probability table, as shown next, with an assumed 10% success rate, for any larger number of wells to be drilled, and we will get some probabilities of success in number of discoveries to total number of wells drilled.

From Table 11.1, a graph can be drawn, as shown in Figure 11.2, to illustrate tables of cumulative binomial probabilities. This graph provides the following information:

- 1. At least one discovery or more is 88% (or 88 chances of success in a total of 100 chances), or with 4.4 chances of *S* in five chances.
- 2. At least two discoveries is 60% (or 60 chances of success in 100 total chances), or 3 in 5 chances.
- 3. At least three discoveries is 30% (30 chances of success in 100 chances), or about 1.5 in 5 chances.
- 4. At least four discoveries is 13% (13 chances of success in 100 chances), or about 1 in 8 chances.

Number of Wells Drilled	Number of Discoveries	Probability Success in Number of Discoveries (%)	Odds of Success
10	1	60	1 in 10
10	2	26	1 in 5
10	3	15	3 in 10
20	1	80	1 in 20
20	2	61	1 in 10
20	3	50	3 in 20
20	4	25	1 in 5
20	5	10	1 in 4
30	1	90	1 in 30
30	2	73	1 in 15
30	3	70	1 in 10

TABLE 11.1

Cumulative Binomial Probability (Using a 10% Success Rate)

The chance of drilling any number of dry holes in succession, like the chance of one dry hole "in succession," is 1.00 - 0.10, or 0.90 (90%). For additional wells, the probabilities are as follows:

2 dry holes in succession = 81%, or 4 in 5 chances

5 dry holes in succession = 69%, or 3 in 5 chances

10 dry holes in succession = 35%, or 1 in 3 chances

20 dry holes in succession = 12%, or 1 in 8 chances



FIGURE 11.2 Cumulative binomial probability, assuming 10% success.
Thus, even with a 10% success rate, even in drilling 20 holes, we still face a 12% chance that all holes will be dry.

The employment of such a table and graph is a possibility for explorers for crude oil in their efforts to predict success and failure, or discoveries to dry holes. It can also be useful to oil engineers in estimating probabilities, or odds of success.

Using the binomial distribution to find the probability of an exact number of successes (discovery wells) in several trials (number of wells to be drilled), the following relation could be applied:

$$p(x) = \binom{N}{x} p^N q^{N-x} \tag{11.2}$$

$$=C_x^N p^N q^{N-x} \tag{11.3}$$

where

p(x) = probability of obtaining exactly *x* successes in *N* trials

N = size of the sample, or number of trials of an event

x = number of successes, or favorable outcomes within the *N* trials

p =probability of success

q = 1 - p = probability of failure

 $\binom{N}{x} = C_x^N$ = number of combinations in which *N* objects can be displayed as groups of size *x*, where the order within the individual groups is unimportant

The mean, variance, and standard deviation of the binomial are given by:

$$m = NP$$

$$\sigma^{2} = Npq$$

$$\sigma = (Npq)^{1/2}$$

Example 11.3

As an example, the probability of obtaining zero heads when a coin is tossed five times is calculated as follows, using Equations (11.2) and (11.3):

$$p(x) = \binom{N}{x} p^{x} q^{N-x}$$
$$p(0) = \binom{5}{0} (0.5)^{\circ} (1-0.5)^{5-0} = (1)(1)(0.5)^{5}$$

Roughly, the probability is 3 of 100 times. That is, where successive tosses were gathered into groups of five tosses in each group, out of 100 such groups, about three would contain no heads.

Example 11.4

Ten wells are to be drilled. The probability of success is taken to be 0.15. What is the probability of there being more than two successful wells?

SOLUTION

The answer to this can be found in one of two ways: (1) the individual probabilities of 3, 4, 5, 6, 7, 8, 9, and 10 successes can be calculated and added together, or (2) the individual probabilities of 0, 1, and 2 successes can be added together and then subtracted from 1 to obtain the same answer. The second method is shorter and is given as follows:

$$p(x) = \binom{N}{x} p^{x} q^{N-x}$$

$$p(0) = \binom{10}{0} (0.15)^{\circ} (0.85)^{10} = 0.1969$$

$$p(1) = \binom{10}{1} (0.15)^{1} (0.85)^{9} = 0.3474$$

$$p(2) = \binom{10}{2} (0.15)^{2} (0.85)^{8} = \underline{0.2759}$$

$$0.8202$$

p(more than 2 producers) = 1 - 0.8202 = 0.1798

Hence, probability is approximately 18%.

Most oil companies are not concerned with how far down drilling proceeds, but with how high the cost will be to get that deep and what the cost will be to go, say, another 100 ft or more. Marginal costs are some direct function of depth. If, then, we let Y be those costs that vary with depth, but no overhead costs, and let X be depth itself, a formula can then be written as

$$\frac{dY}{dX} = C(X)$$
, the cost per foot (11.4)

Thus, depth affects marginal costs. For example, the rise of temperature with depth, among other things, increases the probability that a drilling bit will have to be replaced an additional time in a well drilled an additional 100 ft, because mechanical energy is lost as the drilling process continues. But also, some costs, such as the costs of additional "mud materials," needed to drill a deeper well may actually increase rather slowly in relation to increase in depth, thus giving a decreasing marginal cost in relation to depth.

The one factor that may most affect the costs of drilling is the average footage drilled per hookup. As more information on drilling tendencies in any one oil field becomes available, the number of changes in drilling hookup is reduced, and the speed of the drilling operation is increased. Also, feet per hour at the bottom of the well, combined with the amount of time spent at the bottom, is perhaps the best measure of the relative efficiency and speed of a drilling operation in a particular oil well and for a given amount of controlled footage.

In sum, costs of drilling increase because of the following, usually in some combination:

- 1. A poorly designed casing program
- 2. An inadequate rig or incompetent personnel on the test drill
- 3. Poor selection of proper drilling bits for the formations to be penetrated
- 4. Insufficient drilling bit weight for maximum penetration (economic balance here relative)

Once the oil has been explored, developed, and produced, all costs involved in getting the oil to the surface, where it becomes a commodity as it is piped in gathering lines to central points for gas separation, are called the cost of oil field operation. The basic question, "What does oil cost to find, to develop, and to ready for commercial production?" would be comparably simple to answer if, during a short period of time—say 1 to 3 years—an oil company could start in the oil-producing business, discover say 10 million bbl of oil, develop that 10 million bbl, and finally produce the 10 million bbl of crude. The cost of drilling, developing, and producing could then simply be found by dividing the total amount spent for exploratory, developing, and producing effort by 10 million bbl, which would give a cost per barrel of crude.

But this is just "grocery store accounting." Actual accounting for costs in the oil-producing industry is not that simple. When a company searches for oil, it may spend several years and millions of dollars on exploration and development before any substantial, and commercially feasible, amount of oil is located. In development alone, a company may work for several years and spend many dollars developing the oil reservoir which it is to produce over an even greater number of years; and also, all this time, the process is constantly repeating itself as more oil is being discovered, more oil is being developed, and more oil is being produced.

11.3 Conclusions

An oil company's success is measured by its ability to discover reserves. In its search for oil, it spends substantial amounts of money in many different ventures in widely scattered areas. The oil company does this knowing that many of these ventures will be nonproductive and will eventually be abandoned. On the other hand, the oil company recognizes that successes in other areas must be large enough to recoup all money spent in order to break even or to provide a profit. Thus, the true assets are the oil reserves, and these costs are capitalized, but the costs of nonproductive exploration activities and of dry holes are also a necessary part of the full cost of finding and developing these oil reserves.

11.4 Glossary

The following are some expressions and definitions used in cost terminology and reserves reporting, which are used here as well as in the following chapters.

- **Finding oil:** includes exploration (search) for oil, development of successful exploration discoveries, including the drilling of wells, and finally, the drilling and preparing of oil for commercial production, including the laying of gathering pipelines and pump installation for the movement of oil to central points for gas separation
- Finding and development costs: used by securities analysts to measure and compare petroleum company performances in acquiring reserves
- **Exploration costs:** expense and capital costs to identify areas that may warrant examination (includes geophysical, geological, property retention costs, dry hole expenses, exploration drilling)
- **Development costs:** expense and capital costs incurred to bring on-stream a producing property (includes development well drilling and equipment, enhanced recovery and extraction, and treatment facilities)
- **Discoveries:** newly found proven reserves, including production sharing type reserves, which may or may not be included (booked) in annual reserve estimates
- Extensions: additions to existing fields, normally booked in the same year
- **Punchback:** deepening to new horizons or completing back to shallower horizons, the reserves of which may or may not be booked
- **Revisions:** additions or deletions to previous reserve estimates based on updated information on production and ultimate recovery
- **Improved recovery:** additions to reserves due to secondary and tertiary recovery, booked when production commences
- Purchase of reserves in place: proven reserves purchased from outside companies
- **Property acquisition costs:** those costs incurred to purchase or lease proven or unproven reserve properties, capitalized when incurred

Reserves and Reserve Estimate

K.A. Al-Fusail (Deceased)*

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Reserve estimation is one of the most essential tasks in the petroleum industry. The total estimated amount of oil in an oil reservoir, including both producible and non-producible oil, is called "oil in place." Practically speaking, because of reservoir characteristics and limitations in petroleum extraction technologies, only a fraction of this oil can be brought to the surface, and it is only this producible fraction that is considered to be reserve.

An oil evaluation study has as its primary purpose the determination of the value of oil in place. Such evaluation includes estimates of reserves. Methods most commonly used to estimate the reserve of recoverable hydrocarbons are here, including volumetric, material balance, and decline curve methods. The role of economic evaluation for oil properties is illustrated in the five examples at the end of the chapter.

^{*} This chapter was originally written by K.A. al-Fusail in the second edition of this book, and has been updated and revised for this edition by Hussein Abdel-Aal.

12.1 Technology Aspects

12.1.1 Introduction

Evaluation of an oil property depends on the development of the underground accumulation of hydrocarbons and the amount of money that will be received from selling the produced hydrocarbons. Such evaluation includes estimate of reserves, estimate of gross income, estimate of net income after taxes and production costs, and calculation of present worth value of the property.

Development of an oil or gas reservoir depends on the producible amount of hydrocarbons. This amount is called "reserves." The "proved reserve" is the form of reserve that is recoverable by the force of natural energy existing in the reservoir or by secondary processes. The "probable reserve" is the reserve that has not been proved by production at a commercial flow rate.

The methods most commonly used to estimate the reserve of recoverable hydrocarbons are:

- 1. Volumetric
- 2. Material balance
- 3. Decline curve

Each of these methods is discussed in turn.

12.1.2 Volumetric Methods

Estimation of reserve is performed by an equation that is not complicated to use, provided the required data are available. The data include the area of the production zone (*A*), the formation thickness (*h*), the porosity (φ), and the initial water saturation (S_{wi}). The equation has the form:

$$N = \frac{7758 \ Ah\phi(1 - S_{wi})}{B_{oi}}$$
(12.1)

where

N = bbls of initial oil in place at surface temperature and pressure condition, which is called stock tank

 B_{oj} = initial oil formation volume factor, which is defined as bbl at reservoir condition (rb), divided by bbl at surface condition (STB)

Once the recovery factor is known, then the amount of recoverable oil can be figured out. The bulk volume of the reservoir can be calculated using subsurface and isopachous maps. The isopachous map consists of isopach lines that connect points of formations having equal thickness. The areas lying between the isopach lines of the entire reservoir under consideration are used to calculate the volume contained in it.

Simpson's rule, trapezoidal rule, and pyramidal rule are normally used to determine the reservoir bulk volume (V_B). Simpson's rule provides the following equation:

$$V_{B} = h / 3(A_{o} + 4A_{1} + 2A_{2} + 4A_{3} + 2A_{n-2} + 4A_{n-1} + 4A_{n-1} + A_{n}) + t_{n}A_{n}$$
(12.2)

where

h = interval between the isopach lines in ft

 B_{o} = area in acres enclosed by successive isopach lines in acres

 A_1, A_2, A_3, A_n = areas enclosed by successive isopach lines in acres

 $t_{\rm n}$ = average thickness above the top

Trapezoidal rule provides the following equation:

$$V_B = h/2(A_o + 2A_1 + 2A_2 + \cdots 2A_{n-1} + A_n) + T_n A_n$$
(12.3)

Pyramidal rule has the form:

$$V_B = h / 3(A_n + A_{n+1} + \sqrt{A_n A_{n+1}})$$
(12.4)

This equation calculates the reservoir bulk volume between any two successive areas (ΔV_B), and the total reservoir bulk volume is the summation of all the calculated bulk volumes.

The accuracy of trapezoidal rule and pyramidal rule depends on the ratio of the successive areas. If the ratio of the areas is smaller than 0.5, the pyramidal rule is used; otherwise the trapezoidal rule is used.

The formula as provided in Equation (12.1) can be applied to calculate free gas in a gas reservoir as given below:

$$G = 43560 \ V_B \ \phi(1 - S_w) \ / \ B_g \tag{12.5}$$

where

G = gas in place

 B_g = gas formation volume factor

 V_B = reservoir bulk volume

 S_w = connate water

12.1.3 Material Balance Equation

The material balance equation accounts for the fluids that leave, enter, or accumulate in the reservoir at any time. The oil reservoir is classified as an undersaturated or saturated reservoir based on the reservoir pressure. A reservoir with pressure higher than the bubble point pressure is considered to be an *undersaturated* reservoir. The material balance for such a reservoir, with

the assumption that the oil is produced by the fluid expansion only and the reservoir is constant, is derived below:

Assume that the initial production, P_i , dropped to P due to N_p STB produced. Then,

Initial volume = NB_{oi} bbl at the reservoir condition, rb

Final volume = $(N - N_p)B_o$ bbl at the reservoir condition, rb

Since the reservoir volume is constant, then:

Initial volume = Final volume

$$NB_{oi} = (N - N_p)B_{oi}$$

$$N = N_p B_o / (B_o - B_{oi})$$
(12.6)

A reservoir with pressure lower than the bubble point pressure will cause gas to form, resulting in a free gas phase. Such a reservoir is called a *saturated* reservoir. The derivation of material balance equation for this case is given next:

The initial volume = NB_{oi}

Final volume = remaining oil + free gas

 $= (N - N_p)B_o + G_f B_g$

 G_f = initial gas – remaining gas – produced gas

$$= NRsi - (N - N_p)R_s - N_pR_p$$

Assume the reservoir volume is constant, then:

Initial volume = Final volume

$$NB_{oi} = (N - N_p)B_o + (NR_{si} - (N - N_p)R_s - N_pR_p)B_p$$

= $N(B_o + B_g(R_{si} - R_s) - N_pR_p)B_p$
= $N(B_o + B_g(R_{si} - R_s) - N_p(B_o + B_g(R_p - R_s))$ (12.7)
 $N_p(B_o + B_g(R_p - R_s)) = N(B_o + B_{oi} + B_g(R_{si} - R_s))$
 $N = N_p(B_o + B_g(R_p - R_s)) / B_o - B_{oi} + B_g(R_{si} - R_s)$

where

N = oil in place, rb $N_p = \text{oil produced, STB}$ $B_o = \text{formation volume factor, rb/STB}$ $B_{oi} = \text{initial formation volume factor, rb/STB}$ $B_g = \text{gas formation volume factor, rb/STB}$ $R_{si} = \text{initial gas in solution, SCF/STB}$ $R_s = \text{gas in solution at a pressure lower than } P_i$ $R_p = \text{cumulative gas-oil ratio}$

If the reservoir has a *gas cap* at the time of discovery, then the material balance equation will have the form:

$$N = N_p (B_o + B_g (R_p - R_s)/B_o - B_{oi} + B_g (R_{si} - R_s) + m B_{oi} (B_g/B_{gi} - 1)$$
(12.8)

where

m = volume of free gas/oil volume = $G_f B_{gi} / NB_{oi}$

If the reservoir is *under water drive*, the water influx as well as the water production needs to be added to the material balance. Then, Equations (12.7) and (12.8) become:

$$N = N_p (B_o + B_g (R_p - R_s)) - W_e + B_w W_p / B_o - B_{oi} + B_g (R_{si} - R_s)$$
(12.9)

$$N = N_p (B_o + B_g (R_p - R_s)) - W_e + B_w W_p / B_o - B_{oi} + B_g (R_{si} - R_s) + m B_{oi} (B_g / B_{gi} - 1)$$
(12.10)

All these terms, except $N_{p'} R_{p'} W_{e'} W_{p'}$ are functions of pressure and also are properties of the fluids. These data should be measured in the laboratory. R_p depends on the production history. It is the quotient of both the gas produced (G_p) and the oil produced (N_p) . A water influx can be calculated by using different methods depending on the flowing conditions. The boundary pressure as well as the time are used to calculate the water influx. The value of *m* is determined from the log data which provide the gas-oil and oil-water contacts and also from the core data. Therefore, the accuracy of the calculated oil in place depends upon how accurately we take these measurements for such calculations.

12.1.4 Material Balance Equation for Gas Reservoir

(a) No water drive: If the reservoir volume stays constant and G_p, gas produced during a time t, and B_{gi} drop to B_g, then material balance is given by Equation (12.11) as follows: Initial volume = Final volume

$$GB_{gi} = B_g (G - G_p)$$

= $B_g g - G_g G_p$ (12.11)
 $G_p B_g = G(B_g - B_{gi})$
 $G = G_p B_g / B_g - B_{gi}$

(b) *With water drive:* The material balance:

$$G = G_p B_g - W_{ad} + W_p B_w / B_g - B_{gi}$$
(12.12)

If the measured data are accurate, the calculated gas in place will always be accurate. In Equation (12.12), the water influx can be found using the pressure drop during the production history with other parameters.

12.1.5 Material Balance Equation, Straight-Line Concept

The material balance equation given by Equation (12.10) may be expressed as a straight-line equation which will have the form:

$$F = NE_o + N_m E_o + W_e \tag{12.13}$$

where

$$\begin{split} F &= N_{p}(B_{o} + B_{g}(R_{p} - R_{s})) + W_{p} + W_{w}, \ rb \\ E_{o} &= [B_{o} - B_{oi} + B_{g}(R_{si} - R_{s})]rb/STB \\ E_{g} &= B_{oi}[(B_{g}/B_{gi}) - 1)]rd/TB \end{split}$$

F represents the total underground withdrawal, while E_o denotes the oil expansion and the expansion of associated gas, while E_g represents the gas cap expansion.

Equation (12.13) includes all the drive mechanisms. If any one of these mechanisms is not acting in the reservoir, then the term representing such a mechanism must be deleted from the equation.

(a) No water drive, no original gas cap:

$$W_c = O \text{ and } m = O$$

$$F = NE_o$$
(12.14)

A plot of *F* versus E_o gives a straight line passing through the origin with a slope of *N* (initial oil in place).

(b) No water drive ($W_c = 0$). Equation (12.13) will be reduced to:

$$F = N(E_o + mE_g) \tag{12.15}$$

Again plotting *F* versus $(E_o + mE_g)$ yields a straight line passing through the origin with a slope of *N*.

(c) *No water drive and* m *is not known*. Equation (12.15) can be written differently:

$$F/E_o = N + mNE_g/E_o \tag{12.16}$$

A plot of F/E_o versus E_g/E_o should result in a straight line with the intercept of *N* with *Y*-axis. The value of *m* can be known from the slope.

(d) For water drive reservoir, m = 0, Equation (12.13) will have the form:

$$F = NE_o + W_e$$

Divide by E_0 :

$$F/E_o = N + W_e/E_o$$
 (12.17)

A plot of F/E_o versus W_e/E_o should give a straight line with N being the Y intercept providing the calculated water influx is correct.

The same concept can be applied to the gas reservoir to express the gas material balance equation as a straight line. Equation (12.12) becomes:

$$G_p B_g = G E_g$$
(12.18)
where $E_g = B_g - B_{gi}$

Plotting G_pB_g versus E_g should give a straight line with *G* being the slope. If the reservoir is under water drive, Equation (12.12) can be written as:

$$GE_g = G_p B_g - W_e + W_p$$
$$GE_g = G_p B_g + W_p - W_e$$
$$W_e + GE_g = G_p B_g + W_p$$

Divide by E_g :

$$W_e/E_g + G = G_p B_g + W_p/E_g$$
 (12.19)

A plot of $G_pB_g + W_p/E_g$ versus W_e/E_g should result in a straight line with *G* being the *Y* intercept.

Using the straight-line technique to estimate oil or gas reserves will minimize the error in the calculated reserve because a number of data will be used for the reserve estimation, and the error in the data will be averaged.

The gas in place can be estimated by another approach that requires plotting P/z versus cumulative gas production for a volumetric reservoir. Such a plot results in a straight line with *G* being the *X*-axis intercept. Estimation of gas reserve using early production data may result in error by as much as a factor of 2. Therefore, this method should be used only when the cumulative gas production reaches a stage of about 20% of the gas in place.

12.1.6 Decline Curve Methods

Predicting the reserve using decline curve methods requires production rates of all the wells. The production rate generally declines with time, reaching an end point that is referred to as the economic limit. The economic limit is a production rate at which the income will just meet the direct operating cost of a well or a certain field. Typical decline curve analysis consists of plotting production rate versus time and trying to fit the obtained data into a straight line or other forms that can be extrapolated up to the economic limit to estimate the reserve on the assumption that all the factors affecting the well performance have exactly the same effect in the future as they had in the past.

The commonly used decline curves are:

- 1. Constant percentage decline
- 2. Hyperbolic decline
- 3. Harmonic decline

12.1.6.1 Constant Percentage Decline

The constant percentage decline is known as the exponential decline and is used more widely than the other forms of decline due to its simplicity. In this case, the decline rate is assumed to be constant during the production time. The decline rate in production rate with time is:

$$D = -\Delta q / (q / \Delta t) \tag{12.20}$$

where

D = decline rate $\Delta q = q_i - q; q_i$ is initial production rate, and q is production at a time (t) Δt = time t required for q_i to decline to qIntegrating Equation (12.20) to get rate-time relation:

$$-\int_{0}^{t} Ddt = \int_{q_{i}}^{q} \frac{dq}{q}$$

$$q = q_{i}e^{-Dt}$$
(12.21)

Integrating Equation (12.21) with respect to time:

$$-\int_0^t q dt = q_i \int_0^t e^{-Dt}$$

or

$$N_p = -q_i / D(1 - e^{-Dt})$$
(12.22)

From Equation (12.22):

 $q/q_i = e^{-Dt}$

Substitute in Equation (12.20), then:

$$N_p = q_i - q / D \tag{12.23}$$

Equation (12.23) can be rearranged as follows:

$$q = q_i - N_p D \tag{12.24}$$

A plot of q versus N_p will result in a straight line. The slope of the line is D, and q_i is the intercept of the Y-axis. Equation (12.21) also yields a straight line if q is plotted against t on semilog paper. The slope of such a plot is D, and the intercept is q_i . The N_p is the cumulative production between any two production rates.

$$q = q_i e^{-Dt}$$
$$-\ln q/q_i = Dt$$

When the decline rate is not constant, then the hyperbolic decline can be assumed, and the decline rate varies according to the following equation:

$$D = D_i (q/q_i)^n \tag{12.25}$$

where

n = decline constant between zero and 1

 D_i = initial decline rate

The general equation for hyperbolic rate decline can be obtained by substituting Equation (12.17) into Equation (12.21) and then integrating the resulting equation. The equation thus finally derived will have the form:

$$q = \frac{q_i}{(1+Dnt)^{1/n}}$$
(12.26)

The cumulative production rate obtained from the hyperbolic decline can be derived as follows:

$$N_p = \int_0^t q dt$$

Equation (12.20) can be written as:

$$\frac{dq}{D} = -\frac{q}{dt}$$

Substitute *D* value from Equation (12.25) in the above equation, and then substitute *q* value in the equation to calculate N_{p} :

$$N_{p} = \int_{q_{1}}^{q_{2}} \frac{dq}{D_{i} \left(\frac{q}{q_{i}}\right)^{n}}$$

$$= \frac{q_{i}^{n}}{D_{i}} \int_{q_{1}}^{q_{2}} \frac{dq}{q^{n}}$$

$$= \frac{q_{i}^{n}}{D_{i}(1-n)} (q_{1}^{1-n} - q_{2}^{1-n})$$
(12.27)

The values of q_i , D_i , and n are assumed to be known and are constant, and thereafter Equation (12.27) can be used without any difficulty. The values of q_i , D_i , and n can be obtained by comparing the actual decline data with a series of curves of hyperbolic type. A plot of q/q_i versus time may fit in one of the curves which gives the values of q_i , D_i , and n.

12.1.6.2 Harmonic Decline

In this curve, when the decline rate is not constant, it decreases as the production rate increases. Such a varying rate in decline is called a *harmonic decline*. It also occurs if the decline constant n of Equation (12.27) is 1. An equation derived for such decline is:

$$q = q_i / 1 + a_i t \tag{12.28}$$

This type of decline may take place in reservoirs where gravity drainage controls the production. Gravity drainage exists in tilted reservoirs where oil production is affected by drainage of oil from upstructure to downstructure which causes segregation of gas and oil in the reservoir. Cumulative production can be obtained by integrating Equation (12.24) with respect to time:

$$N_{p} = \int_{0}^{t} q dt$$

$$N_{p} = q_{i} \int_{0}^{t} \frac{dt}{1 + a_{i}t}$$

$$= q_{i}/a_{i} \ln(1 + a_{i}t)$$
(12.29)

But from Equation (12.25):

$$(1+a_i t) = q/q_i$$

Substitute in Equation (12.25)

$$N_p = (q_i/q_i) \ln(q/q_i)$$
 (12.30)

A graphic harmonic decline analysis can be obtained by writing Equation (12.24) as:

$$\frac{1/q_i(1+a_it) = 1/q}{1/q_i + (a_it)1/q_i = 1/q}$$
(12.31)

Plotting 1/q versus *t* on Cartesian coordinates should result in a straight line, with a_i/q_i being the slope and $1/q_i$ the intercept with 1/q-axis. From the slope a_i can be known. Also Equation (12.30) can be rewritten in a different form:

$$N_{p} = q_{i}/a_{i}(\ln q - \ln q_{i})$$

$$(a_{i}/q_{i})N_{p} = \ln q - \ln q_{i} \qquad (12.32)$$

$$\ln q_{i} + (a_{i}/q_{i})N_{p} = \ln q$$

A plot of *q* versus N_p on semilog paper will result in a straight line with slope being a_i/q_i and intercept q_i . This straight line can be extrapolated into the economic limit to calculate the reserve.

12.1.7 Comparison of the Methods

Comparison of all the predictive methods depends on the data available and the accuracy of these data. Volumetric methods are usually used in the early life of the reservoir, while the material balance equations or the decline curve methods can be used when enough data are collected. However, material balance equation techniques depend on many measurements, such as $B_{or} B_{-gr} R_{sr} R_{pr}$ and total production; hence more error is anticipated in the calculated reserves. The error in the calculated reserve by the decline curve is less than with other methods.

12.2 Economic Evaluation and Application

Evaluation of an oil property is concerned with its money value; i.e. its profitability. The profitability depends on the *development* of underground accumulations of hydrocarbons and on the sale value of the hydrocarbons, which helps to estimate the present worth value of such property at any time under certain specified conditions. The gross income of hydrocarbon sales depends on the current prices of oil and gas and the predicted economic conditions. The net profit is related to all expenses that are deducted from the gross income, such as operating cost, which includes the expenses required to produce the hydrocarbon and to maintain the reservoir, taxes, and royalty when applicable.

The following applications and case studies illustrate the role of economic evaluation for an oil property.

Example 12.1 Given the following data:

Area = 1,200 acres Formation thickness = 20 ft Average porosity = 20% Connate water = 25% Formation volume factor = 1.3 rb/STB Initial gas in solution (Rsi) = 650 SCF/STB

- (a) Calculate the oil in place.
- (b) Calculate the total gas in solution.

SOLUTION

Part (a):

$$\begin{split} N &= 7758 \ Ah\phi(1-S_w)B_{oi} \\ &= 7758 \times 1200 \times 20 \times .2(1-.25)/1.2 \\ &= 24,274,000 \ STB \end{split}$$

Then, oil in place = <u>24,274,000</u> STB

Part (b)

Total gas in solution: = (oil in-place)(initial gas in solution) = (N)(Rsi) = $24,274,000 \times 650$ = 15.78×10^9 SCF

Example 12.2

An oil reservoir has a gas cap at the time of discovery. The size of this gas cap is not known. The production data and the fluid properties are given as a function of pressure in Table 12.1.

- (a) Calculate the oil in place using the material balance equation as a straight line.
- (b) Use the material balance equation itself.

SOLUTION

Since the production was due to gas cap expansion and the gas cap size is not known, the following equation can be used:

$$F/E_o = N + mN E_o/E_o \tag{12.33}$$

All the calculations are given in Table 12.2.

TABLE 12.1

P, psi	N _p , STB	B _o , rb/STB	R _s , SCF/STB	B _g , rb/SCF	R _p , SCF/STB
3200	0	1.35	520	0.000932	0
2950	2.50×10^8	1.345	444	0.00095	950
1800	3.37×10^{8}	1.34	435	0.000995	1,000
2765	4.95×10^8	1.32	410	0.0011	1,150
2500	6.62×10^{8}	1.308	395	0.00123	1,280

Data for Example 12.2

F	E _o	E_g	F/E _o	E_g/E_o		
4.57×10^{8}	0.0672	0.0255	6.8×10^{9}	0.379		
$6~407 \times 10^8$	0.0745	0.09125	8.6×10^9	1.22		
10.56×10^8	0.091	0.238	11.6×10^{9}	2.615		
15.87×10^8	0.1118	0.4182	14.1×10^9	3.743		
	F 4.57 × 10 ⁸ 6 407 × 10 ⁸ 10.56 × 10 ⁸ 15.87 × 10 ⁸	F E_o 4.57 × 10 ⁸ 0.0672 6 407 × 10 ⁸ 0.0745 10.56 × 10 ⁸ 0.091 15.87 × 10 ⁸ 0.1118	F E_o E_g 4.57 × 10 ⁸ 0.0672 0.0255 6 407 × 10 ⁸ 0.0745 0.09125 10.56 × 10 ⁸ 0.091 0.238 15.87 × 10 ⁸ 0.1118 0.4182	F E_o E_g F/E_o 4.57 × 10 ⁸ 0.0672 0.0255 6.8×10^9 6 407 × 10 ⁸ 0.0745 0.09125 8.6×10^9 10.56 × 10 ⁸ 0.091 0.238 11.6×10^9 15.87 × 10 ⁸ 0.1118 0.4182 14.1×10^9		

TABLE 12.2

Colution	for	Evenable	12.2
Solution	TOL	Example	12.2

Plotting F/E_o against E_g/E_o as shown in Figure 12.1 yields a straight line. The values of the intercept and the slope are given as follows:

 $Y \text{ intercept} = 5.9 \times 109$ $Slope = 2.2093 \times 109$

From Equation (12.20), the Y intercept is N and the slope is mN, then:

 $m = \text{slope/N} = 2.2093 \times 10^9 / 5.9 \times 10^9$ = 0.3745

Now *m* is known, the material balance equation can be used to calculate initial oil in place. Equation (12.10) will be used:

N(At, P = 2950 psi) = 1.084×10^{10} STB N(AT, P = 2500 psi) = 5.803×10^{9} STB

Since m is known, Equation (12.15) can be used to determine the oil in place N. The calculation is shown in Table 12.3.

Example 12.3

For application of the constant decline curve, the following production history for a well is given:

Year	B/day
1	9,600
2	7,200
3	6,700
4	5,700
5	5,200
6	4,650
7	4,300
8	3,800

- (a) Estimate the remaining life of this field if the economic limit is 800 B/D.
- (b) What is the recoverable oil as of year 8?
- (c) What is the net income if the price of oil is assumed to be \$85/bbl?



FIGURE 12.1 Solution of Example 12.2.

SOLUTION

Since the decline rate follows the constant percentage decline, then a plot of *q* versus time on semi-log is recommended and gives a straight line. The slope of the line represents the decline rate, *D*.

$$D = -(2.3) \frac{\log 3,800 - \log 7,300}{(8-2) \times 12} = 0.02086/\text{month}$$

(a) Using Equation (12.21), the revising number of years can be calculated as follows:

> $q = q_i e^{-Dt}$ $-\ln q / q_i = Dt$

or

$$t = \frac{\ln q/q_i}{D} = \frac{-\ln (800/3, 800)}{0.02086}$$

= 74.695 month
= 6.225 year

TABLE 12.3

Data to Determine Oil in Place, N					
$P F E_o + E_g$					
2950	4.57×10^{8}	0.07675			
2800	6.407×10^8	0.1087			
2650	10.56×10^8	0.180			
2500	15.87×10^8	0.2684			

(b)

Recoverable oil =
$$q_1 - q_2/D$$

$$=\frac{3,800-800}{0.02086}$$
$$=143,816 \text{ bb1}$$

- (c) Total income = 143,816 × 85 = \$12,224,360
- If the operating expenses are taken to be \$38/bbl, then the gross income = \$6,759,352.
- If this gross income is to be taxed at 46%, the net profit=\$3,650,000

Example 12.4

Use the calculated oil in place in Example 12.1 assuming the following values:

Sale value of the oil	= \$85/bbl	
Operating costs	= \$47/bbl	
Calculated oil in place	= 24,274,000 STB	
Gross income	= (Oil in place) × Price	
= 24,274,000 bbl × 85 \$/bbl =	20.6×10^{8}	
Production taxes	$=20.6\times10^8\times0.046$	$=$ \$0.9476 \times 10 ⁸
Operating costs	= 24,274,000 × 47	$=\$11.4\times10^8$
Net income	= Gross income – (production costs + operating costs) = $20.6 \times 10^8 - (0.9476 +$	
	$(11.4) \times 10^8$	
	$= 8.2524 \times 10^{8}$	

This calculation excludes any capital expenditure that may be justified in the future. Also, the calculation is based on today's oil price, which may change in the future.

Example 12.5

A similar calculation can be done for Example 12.2 assuming the oil price, operating cost, and production taxes are the same as used in the previous calculations.

SOLUTION

Gross income	= oil in place x price
	$= 5.9 \times 109 \times 85$
	$=$ \$500 \times 109
Production taxes	$= \$500 \times 109 \times 0.046$
	= \$23 × 109

Operating costs	$= 5.9 \times 109 \times 47$
	$=$ \$277 \times 109
Net Income	= gross income – (production taxes + operating costs) = \$ 500 × 109–300 × 109
	= \$ 200 × 109

Again, this net income excludes any capital expenditures that may be needed in the future. Other taxes that may be applicable are not combined.

13

Production Operations

Mohamed A. Aggour Hussein K. Abdel-Aal

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Oil exploration, drilling, and property evaluation have been treated in previous chapters. In this chapter, the various operations associated with the production of oil and natural gas are presented. Production is the operation that brings hydrocarbons to the surface and prepares them for processing.

As part of subsurface operations, this chapter covers completion and workover operations and production methods (natural flow and artificial lift). Surface petroleum operations including gas-oil separation, crude oil treatment (dehydration, desalting, and stabilization), and gas treatment and conditioning are treated in consecutive separate chapters. Following the introduction of each major production operation, economic-based decisions are presented. Examples (case studies) illustrating the economic analysis in this strategic phase of the oil operations are presented at the end of the chapter.

13.1 Technology Aspects

13.1.1 Introduction

Petroleum production engineering covers the widest scope of engineering/ operations in the petroleum industry. It starts with the selection, design, and installation of well completion and ends with the delivery of the useful fluids (i.e., oil and natural gas) to the customer. Between the two ends lie a large number of engineering activities and operations. For example, the design and installation of the well tubing and surface flowline, the workover operations that keep the well at its best producing conditions, the selection and design of the oil/gas production method, and the design, installation, and operation of the surface separation and treatment facilities are all the responsibility of the petroleum production engineer.

The economics of most of the above-mentioned operations have to be evaluated before they are executed. In some cases, several technically viable alternatives exist for executing a particular operation. In such cases, the decision to select one alternative over the others would be based entirely on economic evaluation of the various alternatives.

In the following sections, brief descriptions of the various major production operations are presented along with examples of the economic evaluation of some operations.

13.1.2 Well Completions

After a well has been drilled, it must be completed before oil and gas production can begin. The first step in this process is installing casing pipe in the well.

Oil and gas wells usually require four concentric strings of pipe: conductor pipe, surface casing, intermediate casing, and production casing. The production casing or oil string is the final casing for most wells. The production casing completely seals off the producing formation from water aquifers.

The production casing runs to the bottom of the hole or stops just above the production zone. Usually, the casing runs to the bottom of the hole. In this situation the casing and cement seal off the reservoir and prevent fluids from leaving. In this case the casing must be perforated to allow liquids to flow into the well. This is a perforated completion. Most wells are completed by using a perforated completion. Perforating is the process of piercing the casing wall and the cement behind it to provide openings through which formation fluids may enter the wellbore.

13.1.2.1 Factors Influencing Well Completion Design

While safety and cost are of prime importance in selecting and designing a well completion, the engineer has to consider the following factors in finalizing the completion design:

Type of reservoir and drive mechanisms Rock and fluid properties Need for artificial lift Future needs for stimulation and workover Future needs for enhanced recovery methods Normally, the technical factors are first considered to determine possible completion designs; then the economic aspects are considered to select the most economical design.

13.1.3 Tubing and Packers

After cementing the production casing, the completion crew runs a final string of pipe called the tubing. The well fluids flow from the reservoir to the surface through the tubing. Tubing is smaller in diameter than casing—the outside diameter ranges from about 1 to 4-1/2 inches.

A packer is a ring made of metal and rubber that fits around the tubing. It provides a secure seal between everything above and below where it is set. It keeps well fluids and pressure away from the casing above it. Since the packer seals off the space between the tubing and the casing, it forces the formation fluids into and up the tubing.

13.1.4 Sizing Production Tubing

The starting point in a completion design is determination of the production tubing (conduit) size. This is extremely important as it affects the entire drilling program and the cost of the project.

To determine the size of the tubing, the engineer has to conduct what is known as *well performance analysis*. This analysis requires the study of two relationships:

• The first relationship describes the flow of fluids from the formation into the wellbore; it is called the *inflow performance relation* (IPR). The IPR is represented, normally, as the relationship between the bottom-hole flowing pressure (P_{wf}) and the flow (production) rate (q). Depending on the type of reservoir and the driving mechanism, the IPR may be linear or nonlinear, as illustrated in Figure 13.1. When



Production Rate (q)

FIGURE 13.1 Inflow performance relation (IPR).



FIGURE 13.2

Outflow (vertical flow) performance.

the IPR is linear, it can be represented with what is called the *productivity index* (PI), which is the inverse of the slope of the IPR.

• The second relationship describes the relation between the flow rate of fluids and the pressure drop in the production tubing. It is called the *outflow performance* or the tubing *multiphase* flow performance. Several *multiphase* flow correlations exist for determining the relationship between flow rate and pressure drop in a well tubing. For a fixed wellhead pressure, the relationship between P_{wf} and q is as illustrated in Figure 13.2.

The interaction of the two relationships would provide several solutions, as shown in Figure 13.3. That is, several tubing sizes could be used, but each would yield a different production rate. Normally, higher production rates are obtained using larger tubing sizes; this means higher drilling and completion costs. The final selection of the tubing size should therefore be based on economic analysis of the various alternatives, as illustrated in Example 13.1.

13.1.5 Workover Operations

Workovers refer to any operation performed on the well after its initial completion. Workover operations are conducted either to remedy specific problems developed during the completion or production operations, or to enhance the well productivity. Following are brief descriptions of some of the common workover operations.

13.1.6 Production Methods

Production method refers to the way in which the well fluids are delivered to the surface. Ideally, wells should be produced to deliver the fluids to the



FIGURE 13.3 IPR and outflow performances for different performances.

surface with a wellhead pressure sufficient to force the fluid flow through all surface facilities. There are two ways a well may be produced; these are described below.

13.1.6.1 Natural Flow

A well is said to be produced naturally if it only utilizes the naturally stored energy (i.e., reservoir pressure) to lift the fluids to the surface. Most wells start their lives with natural flow. With time, the reservoir energy (pressure) is depleted, resulting in reduced production rates or reduced wellhead pressure, or both. When this occurs, artificial lift may be implemented.

13.1.6.2 Artificial Lift

Artificial lift refers to the use of external means to help lift the well fluids from the bottom of the well to the surface. Essentially, artificial lifting enables well production at lower bottom-hole pressures. It may be applied on a flowing well to increase its production in order either to meet market demands or to make the project economics more attractive. Artificial lifting is mostly applied, however, to wells that otherwise would not produce at all or would produce below the economic limit of operation.

There are four types of major artificial lifting systems in commercial use. These are sucker rod pumping (SRP), gas lift (GL), electric submersible pumping (ESP), and subsurface hydraulic pumping (SHP). Normally, each method will be more suitable for a specific set of well and reservoir conditions. In some cases, however, we may find that more than one method of artificial lift can be used on a specific well, considering all the technical conditions. In such cases, the selection should be based on economic evaluations of the applicable methods. In addition to the capital cost (initial investment) consideration, operating cost of the various methods should be of prime importance, particularly for long-life projects. For example, the highest capital cost system (gas lift using integral compressors) has the lowest operating cost, which would probably make that system more attractive than the others.

As mentioned, many flowing wells may be placed on gas lift to increase their production for economic gains. A case study after Regnault is presented in Example 13.2.

13.2 Economic Evaluation and Application

Example 13.1 (Case Study)

Determination of tubing size: Determine whether to use 3 in. or 4 in. tubing to complete a well for which the following data and assumptions are given.

TECHNICAL DATA

Well location: offshore
Depth: 8000 ft
Wellhead pressure: 80 psi
Initial reservoir pressure: 3,000 psi
Expected pressure decline: 250 psi drop every year until it reaches 2000 psi, at which time a water injection operation will maintain a constant pressure
Productivity index = 10 BPD/psi (assumed constant)
Production: all oil
Produced gas/oil ratio: 600 scf/bbl

ECONOMIC DATA

Average price of oil: \$80.0/bbl Average operating cost: \$35/bbl Difference in costs of drilling and completing with 4 in. and 3 in. tubings = \$6,520,000 Annual discount rate of money is 14%

SOLUTION

For the purpose of illustration, we will perform the analysis over a period of 5 years only. The same procedure is used for a detailed analysis over the life of the well

We first determine the IPR curves for reservoir pressures of 3,000, 2,750, 2,500, 2,250, and 2,000 psi, and productivity index of 10 BPD/psi. This produces the linear IPR shown by the five parallel straight lines in Figure 13.4.

Using a constant wellhead pressure of 80 psi, we assume several rates and determine the corresponding Pwf (bottom-hole flowing pressure) from vertical multiphase flow correlations for the 3 in. and 4 in. tubings. The outflow performance relations for the two tubings are shown in Figure 13.4.

The intersections of each outflow performance curve with the five IPRs provide the maximum possible production rate with the specific tubing for the first 5 years.

A comparison of the rates obtained with each tubing along with the extra production and income resulting from using the 4 in. over the 3 in. tubing is shown in Table 13.1. It is evident from Table 13.1 that we should select the 4 in. completion since the additional income gained from increased production would cover the additional cost in only a fraction of a year. Although the choice here is straightforward, we will proceed with the calculations of some economic measures for the purpose of illustration.

Calculations for the Pay Out Period (P.O.P.)

Average BPD = 3,000 Average yearly production = $3000 \times 300 \text{ days}/y = 900,000 \text{ bbl}$ Income before depreciation costs = $900,000 \times (80 - 35) = $40,500,000$ Net income = $40.5 \times 10^6 - 1.3 \times 10^6$ = 39.2×10^6



FIGURE 13.4 Flowing bottom-hole pressure versus production.

	Production	nRate, BPD		
Year	3 in.	4 in.	Average Rate Increase, BPD	Net Income, \$10 ⁶
1	5,750	9,250	3,500	47.25
2	5,200	8,650	3,450	46.57
3	4,600	7,720	3,120	42.12
4	4,000	6,700	2,700	36.45
5	3,350	5,600	2,250	31.72

TABLE 13.1

Comparison of Rates and Income for 3 in. and 4 in. Completions

where the capital of 6.5×10^6 is depreciated over the lifetime of 5 years.

P.O.P = Depreciable capital investment/average annual cash flow

 $= 6.5 \times 10^{6}/39.2 \times 10^{6}$

= 0.165 year

= 2 month

CONCLUSIONS

For our present example, a payout period of 2 months is found, indicating that the choice of the 4 in. completion is economically attractive.

Example 13.2 (Case Study)

Economic evaluation of a gas lift well: Perform an economic analysis of placing a well on gas lift given the following data:

Well depth = 8,000 ft
Reservoir pressure (PR) = 2,400 psi and decreases 100 psi for each 200,000 bbl of oil recovery
Productivity index = 4 BPD/psi (initially) and then changes as 0.00143 PR
Wellhead pressure = 120 psi (constant)
Injection gas pressure = 900 psi (from a central station)
Tubing size = 2.5 in
Oil price = \$80.00/bbl
Injection cost = \$0.5/MSCF
Production cost = \$2.5/bbl
Maintenance cost = \$1.0/bbl
Pulling the well = 600,000
New equipment = 415,000

SOLUTION

Based on the data given in Table 13.2, calculations are carried out as presented in Table 13.3.

The payout period, POP is calculated using the average annual cash flow over the 5-year period:

P.O.P = Depreciable Capital Investment/Average Annual Cash Flow

= 1.095 x 106 (\$)/5.54 x 106 (\$ per year)

= 0.1977 years

= 2.37 month

TABLE 13.2

	Average Rate, BPD		Increased Production		
Year	Natural Flow	Gas Lift	Average Rate, BPD	Yearly bbl	Injection Gas, MMSCF/Year
0-1	1,450	1,600	150	54,750	666
1–2	1,100	1,320	220	80,300	622
2–3	850	1,080	230	83,950	578
3–4	675	880	205	74,825	538
4–5	540	700	160	58,400	490

Comparison of Natural Flow and Gas Lift Wells

TABLE 13.3

Results of Calculations for Placing the Wells on Gas Lift

Year	Annual Gross Revenue × 10 ⁶	Injection Costs × 10 ²	(Prod. + Maint. Costs) × 10 ⁶	Annual Net Revenue × 10 ⁶	Net Cash × 10 ⁶
0	_		_	_	-1.095
1	4.38	333	0.137	4.24	4.24
2	6.42	311	0.200	6.22	6.22
3	6.71	289	0.209	6.50	6.50
4	5.98	259	0.187	5.79	5.79
5	4.67	245	0.146	4.52	4.52

The return on investment, on the other hand, is 500%.

Example 13.3 (Case Study)

During field operations, the manager in charge is considering the purchase and the installation of a new pump that will deliver crude oil at a faster rate than the existing one.

The purchase and the installation of the new pump will require an immediate layout of \$15,000. This pump however, will recover the costs by the end of one year.

The relevant cash flows for the case study are established as given in Table 13.4.

If the oil company requires 10% minimum annual rate of return on money invested, which alternative should be chosen?

TABLE 13.4

Data for Example 13.3

		Year	
	0	1	2
Install new (larger pump)	-15,000	19,000	0
Operate existing (old pump)	0	95,000	95,000

Data for Ex	ample 13.5			
		Year		Net Present Worth @ 10%
0	1–4	5	6–20	
0	\$50,000	-\$650,000	\$100,000	\$ 227,000

TABLE 13.5

SOLUTION

The present worth method is applied in solving this problem (see chapter 6). Calculate the present worth for both alternatives, where:

Present worth = Present values of cash flows, discounted at 10% – Initial capital Investment

- (a) For the new pump: P.V. = (190,000)/1.1 = \$ 172,727 Present W = 172,727 - 15,000 = \$ 157,727
- (b) For the old pump:
 P.V. = (95,000)/(1.1 + (95,000)/(1.1)2
 = 78,512 + 86,363
 = \$164,875

Based on the above results, keep the old pump. It gives higher present value.

Example 13.4 (Case Study)

The XYZ oil production company was offered a lease deal for oil wells on which the primary reserves are close to exhaustion. The major condition of the deal is to carry out secondary recovery operation using waterflood at the end of the five years. No immediate payment by the XYZ Company is required. The relevant cash flows are estimated as given in Table 13.5

The decision to be made: Should the lease and the secondary flood proposal be accepted?

Justify your answer, and check the present worth value.

SOLUTION

The fact that the proposal at hand gives a **positive** present worth, makes it a viable one. The project should be undertaken.

Next, calculation is carried out to check the present worth reported above in the table.

The cash flows are discounted to present values, at 10%. Using the compound interest factors listed in Appendix B, the following results are obtained:

The discounted values = 50,000 (3.1698) - 650,000 (0.5645) + 100,000 (4.7227)= 158,490 - 403,585 + 472,270= \$227,175

14

Gas-Oil Separation

Hussein K. Abdel-Aal

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	0	

Well effluents flowing from producing wells are usually identified as turbulent, high-velocity mixtures of gases, oil, and salt water. As these streams flow, reaching the surface, they undergo continuous reduction in temperature and pressure, forming a two-phase fluid flow: gas and liquid. The gathered fluids emerge as a mixture of crude oil and gas that is partly free and partly in solution. They must be separated into their main physical components, namely, oil, water, and natural gas. The separation system performs this function, which is usually made up of a free water knock-out (FWKO), flow line heater, and gas-oil (two-phase) separators, or gas-oilwater (three-phase) separators. Gas-oil separators work on the principle that the three components have different densities, which allows them to stratify when moving slowly with gas on top, water on the bottom, and oil in the middle.

The physical separation of these three phases is carried out using what is called stage separation, in which a series of separators operating at consecutively reduced pressures are used. The purpose of stage separation is to obtain maximum recovery of liquid hydrocarbons from the fluids coming from the wellheads and to provide maximum stabilization of both the liquid and gas effluents. Two case studies are presented at the end of the chapter: "Optimum Separating Pressure for Three-Stage Separators" and "Causes of Tight Emulsions in Gas Oil Separation Plants."

14.1 Technology Aspects

14.1.1 The Separation Process

The process involved in a gas-oil separator encompasses two main stages in order to free oil from gas. These are recognized as flash separation of the gas-oil mixture followed by oil recovery.

14.1.1.1 Flash Separation

In order to understand the theory underlying the separation of well-effluents of hydrocarbon mixtures, it is assumed that such mixtures contain essentially three main groups of hydrocarbons:

- Light group, which consists of methane(CH₄) and ethane (C₂H₆)
- Intermediate group, which consists of two subgroups: propane (C_3H_8) /butane (C_4H_{10}) and pentane (C_5H_{10}) /hexane (C_6H_{12})
- Heavy group, which is the bulk of crude oil and is identified as C_7H_{14+} . Constituents of crude oil and natural gas are illustrated in Table 14.1.

Our objective in separating the gas-oil mixture is twofold:

- (a) To get rid of all C_1 and C_2 , i.e., light gases
- (b) To save the heavy-group components as our liquid product

To accomplish these objectives, we unavoidably lose part of the intermediate group in the gas stream, whose heavier components ($C_{5/}C_6$) would definitely belong to the oil product.

The problem of separating gases in general from crude oil in the wellfluid effluents breaks down to the well-known problem of flashing a feed mixture into two streams: vapor and liquid. This takes place using a flashing column (a vessel without trays). Gases liberated from the oil are kept in intimate contact. As a result, thermodynamic equilibrium is established between the two phases. This is the basis of flash calculations, which are carried out to make material balance calculations for the flashing streams.

Constituents o	f Crude Oil and As	ssociated Gases ^a				
Hydrocarbons	Identification of the Constituents		(i) I ₁	n the Field Stre	ams	(ii) As Commercial Products
Name	Formula	Normal B.P. (°F)	Liquid Phase (at Normal Conditions)	Two Phases	Gaseous Phase (and Liquefied Gases)	
Methane	CH_4	-259			•	Natural gas
Ethane	C_2H_8	-128			LNG Dry Gas	Natural gas
Propane	C_3H_8	-44				Natural gas, propane
Isobutane	$i-C_4H_{10}$	+11		F Field		Natural gas, butane
<i>n</i> -Butane	$n-C_4H_{10}$	31	Stock Tank	Separator	•	Natural gas, motor fuel, butane
Pentanes	C_5H_{12}	06	Condensate Natural	Gas	NGL	Natural gas, motor fuel
Hexane	C_6H_{14}	145	Gasoline	Gas	* *	Natural gas, motor fuel
Heptane	C_7H_{16}	195	Debutanized	Condensate	•	Natural gas, motor fuel
Octane	C_8H_{18}	245	Condensate	Well Effluent		Natural gas, motor fuel
Decanes	$C_{10}H_{22}$	345	•			Motor fuel
Tetradecane	$C_{14}H_{30}$	490	Stock Tank			Kerosene, light furnace oil
Hexadecane	$C_{16}H_{34}$	549	Crude Oil	•		Mineral seal oil, furnace oil
				Crude Oil Well		
Triacontane	$C_{30}H_{62}$	855		Effluent		Light lubricating oil, heavy fuel oil
Tetracontane	$C_{40}H_{62}$	1012		T		Lubricating oil, heavy fuel oil
Asphaltene	$C_{80}H_{162}$	1200		•		Asphalt, road oil, bunker fuel oil
Note: aLPG, liqu	lefied petroleum gase	s; NGL, natural gas	liquids (normally C_{3^+});	LNG, liquefied	natural gas.	

Ċ < 1 5 -J. TABLE 14.1 4

Gas-Oil Separation
14.1.1.2 Oil Recovery

Once flashing takes place, our concern is next on recovering the crude oil. The effective method used implies two consecutive steps:

(a) *To remove oil from gas:* Here we are primarily concerned with recovering as much oil as we can from the gas stream. Density difference or gravity differential between oil and gas is the first means to accomplish separation at this stage. At the separator's operating condition of high pressure, this difference in density becomes large (gas law); and the oil is about eight times as dense as the gas. This could be a sufficient driving force for the oil particles to settle down and separate. This is true for large size separator, with a diameter of 100 microns or more. For separators with smaller diameters, mist extractors are needed.

Other means of separation include change of velocity of incoming flow, impingement, and the action of centrifugal force. These methods would imply the addition of some specific designs for the separator to provide the desired method for achieving separation.

(b) To remove gas from "locked" oil: The objective here is to recover and collect any non-solution gas that may be entrained or "locked" in the oil. The recommended methods are settling, agitation, and applying heat chemicals.

14.1.2 Functional Components of a Gas-Oil Separator and Control Devices

Regardless of their configurations, gas-oil separators usually consist of four functional sections:

- *Section A:* Initial separation takes place in this section at the inlet of the separator. It is used to collect the entering fluid.
- *Section B:* This is designated as the gravity settling section through which the gas velocity is substantially reduced allowing for the oil droplets to fall and separate.
- *Section C:* This is known as the mist extraction section. It contains woven-wire mesh pad, which is capable of removing many fine droplets from the gas stream.
- *Section D:* This is the final component in a gas-oil separator. Its main function is to collect the liquid recovered from the gas before it is discharged from the separator.

In addition to these main components, gas-oil separators normally include the following control devices:

- An oil level controlling system that consists of oil level controller (OLC) plus an automatic diaphragm motor-valve on the oil outlet. In the case of a three-phase separator, an additional system is required for the oil-water interface. Thus a liquid level controller plus a water discharge control valve are needed.
- An automatic back-pressure valve on the gas stream, leaving the gas-oil vessel to maintain a fixed pressure inside.
- Pressure relief devices.

14.1.3 Methods and Equipment Used in Separation

In the separator, crude oil separates out, settles, and collects in the lower part of the vessel. The gas lighter than oil fills the upper part of the separator. Crude oil with high gas-oil ratio (GOR) must be admitted to two or three stages, as indicated in Figure 14.1. Movement of crude oil from one separator to the next takes place under the driving force of the flowing pressure. Pumps are needed for the final trip to transfer the oil to its storage tank.

The essential characteristics of a gas-oil separator are to cause a decrease in the flow velocity, permitting separation of gas and liquid by gravity and to operate at a temperature above the hydrate point of the flowing gas.

The conventional method using multistage flash separators is recommended for relatively high-pressure high-GOR fluids. Separation takes



FIGURE 14.1 Flow of crude oil from oil well through GOSP.

place in a stage known as flash distillation (unit operation). The number of stages is a strong function of the API gravity of oil, GOR, and flowing pressure. Based on configuration, three types of separators are known: horizontal, vertical, and spherical. It is most common to see large horizontal gas-oil separators used in processing well fluids in the Middle East, with three or more separators.

The need for what is called "modern GOSP" may arise as the water content of the produced crude increases. The function of such a setup is multipurpose: it will separate the hydrocarbon gases from oil; it will remove water from crude oil; and it will reduce salt content to acceptable limits. Three-phase separators are common in many fields in the Middle East. Figure 14.2 illustrates the function of a modern GOSP.

14.1.4 Design Equations for Sizing Gas-Oil Separators

Before presenting the design equations, we first present some basic fundamentals and assumptions relevant to the sizing of gas-oil separators.

Fundamentals:

- The difference in densities between the liquid and gas is taken as a basis for calculating the gas capacity.
- In the gravity settling section, liquid drops will settle at a velocity determined by equating the gravity force acting on the drop with the drag force caused by its motion relative to the gas phase.
- A normal retention time to allow for the gases to separate from oil is considered to be between *30 seconds and 3 minutes*. Normally retention time is defined as the residence time or the time for a molecule of liquid to be retained in the vessel.



FIGURE 14.2 Functions of modern GOSP.

Retention time = Volume of vessel/liquid flow rate

- For vertical separators, liquid particles (oil) separate by settling downward against up-flowing gas stream, while for horizontal ones liquid particles assume a trajectory-like path, while it flows through the vessel.
- For vertical separators, the gas capacity is proportional to the crosssectional area of a separator, while for a horizontal one the gas capacity is proportional to the area available for disengagement. The volume of accumulation of either type will be the determining factor for the liquid capacity.

Assumptions:

- No oil foaming takes place during the gas-oil separation (otherwise retention time should be increased to 5 to 20 minutes).
- The cloud point of the oil and hydrate point of the gas are below the operating temperature of 60°F.
- The smallest separable liquid drops are spherical ones having diameter of 100 microns.
- Liquid carry-over with separated gas does not exceed 0.10 gallon/ MMSCF.

Sizing of gas-oil separators requires the calculation of two parameters:

- The oil capacity a separator can handle
- The gas capacity to be processed by a separator

The equations needed to calculate the oil capacity and gas capacity are as follows:

Rated oil capacity, q = [50.54 d2 L]/t bbl/day (14.1)

where *d* is inside diameter of the vessel in ft, *L* is the shell height in ft, and *t* is the retention time in minutes.

Gas capacity,
$$Q = 86400[C1C2C3/z]$$
. A SCF/day (14.2)

where $C_1 = [P_f/T_f]$. [520/14.7]; C_2 is the difference in densities of oil and gas/ density of gas; C_3 is the separation coefficient of the vessel with typical values of 0.167 and 0.5 for vertical and horizontal separators, respectively; z is the gas compressibility factor; P_f and T_f designate the flowing pressure and flowing temperature, respectively.

Equation (14.1) is applicable for horizontal separators, while Equation (14.2) applies for both horizontal and vertical separators, depending on the value of *A*. For horizontal, $A = \frac{1}{2}$ the cross section area, while for vertical, A = the entire cross section = $\frac{1}{4}D^2$.

Equation (14.2) relates the gas capacity of gas-oil separator, *Q*, to the corresponding cross-sectional area, *A*. This enables finding the diameter of a separator needed to handle a given input of a gas flow rate.

14.2 Economic Evaluation and Application

14.2.1 Process Economics and Design Parameters

As we have seen above, gas-oil separation plants are needed for environmental reasons. It is not appropriate to burn off the gases associated with crude oil. The economic reasons for processing and treating the produced crude are obvious. Recovering associated gases prevents wasting a natural resource, which was originally flared off. There are other economic reasons for using GOSP. Removing contaminants from the crude, such as salt and hydrogen sulfide, protects plants from corrosion damage.

During crude-oil processing at the GOSP, one of the most important variables that determines the efficiency of oil/water/gas separation is the tightness of the incoming emulsion. The tighter the emulsion, the higher is the dosage of demulsifier needed to break them. The performance of the GOSP is closely tied to the characteristics of the feed emulsions.

Another aspect of GOSP performance is related to the process facilities (hardware) and process variables. The hardware includes the number and type of separators, dehydrators, and desalters, water/oil separators (WOSs), and other hardware at the GOSP. Process variables include oil and water-flow rates, temperatures, water cuts, and GOSP operating conditions. A higher residence time of fluids in the GOSP will generally lead to better separation and better performance, all other variables being constant. In addition to the residence time, process retrofits in the vessels also tend to enhance performance.

Usually it is most economical to use three to four stages of separation for the hydrocarbon mixture. Five or six may pay out under favorable conditions, when, for example, the incoming wellhead fluid is found at very high pressure. However, the increase in liquid yield with the addition of new stages *is not linear*. For instance, the increase in liquids gained by adding one stage to a single-stage system is likely to be substantial. However, adding one stage to a three- or four-stage system is not as likely to produce any major significant gain. In general, it has been found that a three-stage separating system is the most cost effective.

The following parameters are detrimental in evaluating the performance and the economics of GOSP:

- 1. Optimum separation conditions: separator pressure and temperature
- 2. Compositions of the separated gas and oil phases
- 3. Oil formation volume factor

- 4. Product gas-oil ratio
- 5. API gravity of the stock tank oil

Case Study 14.1

Optimum Separating Pressure for Three-Stage Separators

OBJECTIVE

Optimizing the gas-oil separation facility in order to find the optimal conditions of pressure and temperature under which we would get the most economical profit from the operation.

PROCESS

In the case at hand, it is assumed that we have three separators: high, intermediate-, and low-pressure separators. It is the pressure of the second stage (intermediate) that could freely be changed and optimized. The pressure in the first separator (high pressure), on the other hand, is usually kept fixed either to match the requirement of a certain pressure gas injection facility or to meet a sales obligation through a pipeline, or it is the flow conditions of the incoming feed line. Similarly, the pressure in the third separator (low pressure) is fixed; usually it is the last stage functioning as the storage tank.

The optimum pressure is defined as the one that gives the desired separation of gases from crude oil, with the maximum recovery of oil in the stock tank. Under these conditions, we should have minimum gas/ oil ratio.

If *R* designates the recovery of the oil and is defined as R = O/G of oil per SCF gas, then the optimum operating pressure in the second stage (P_2)_O should be the value that makes *R* maximum; or 1/R is minimum.

APPROACH

The method depends on using a pilot unit to do experimental runs, in which the pressure in the second stage is to be changed from run to run. A sample of the gases leaving the three separators is to be analyzed for the content of some key component, say C_5^+ . It is established, therefore, to minimize the loss of C_5^+ in the gas stream separated from the crude oil.

The experimental runs will look as follows:

Run Number	P_2 [psi]	$(G/O)_2[scf/bbl]$	$(G/O)_3$ [scf/bbl]
1	_	_	_
2		_	_

The change in (*G*/*O*) for both separators with P_2 is plotted as shown in Figure 14.3. It is seen that with the increase in $P_{2'}$ (*G*/*O*)₂ decreases indicating more condensation of heavier hydrocarbons. On the other hand, increasing P_2 will increase (*G*/*O*)₃, because the pressure difference



FIGURE 14.3 Variation of (*G*/*O*) with *P* (G, gas quantity; CF, O oil quantity, bbl).

between stages 2 and 3 will increase causing more hydrocarbons to vaporize from stage 3. The cumulative sum of $(G/O)_2$ plus $(G/O)_3$, named $(G/O)_T$ is plotted against P_2 .

It is concluded that the value of $(P_2)_O$ corresponds to the minimum $(G/O)_T$. This minimum $(G/O)_T$ leads to 1/R or $(O/G)_T$, the maximum oil recovery, bbl per SCF of gas separated.

CONCLUSION

This optimization approach would lead us to calculate the value of oil revenue for the system by simply using the following formula:

Target Profit from Oil Sales day = (fbbl) [price of oil] $(O/G)_T \times [bbl oil/SCF](Q)[SCF/day]$

Case Study 14.2: Causes of Tight Emulsions in Gas-Oil Separation Plants

OBJECTIVE

To evaluate the relative performance of de-emulsifiers and to optimize their usage in GOSPs while meeting crude and water specifications.

PROCESS

Formation of emulsions during oil production is a costly problem, both in terms of production losses and chemical costs. In these days of high oil prices and the need to reduce production costs, there is an economic necessity to control, optimize, or eliminate the problem by maximizing oil-water separation.

The giant Ghawar field in Saudi Arabia has several wet crude handling facilities referred to as gas-oil separating plants (GOSPs), located at Mubarraz area. These GOSPs process Arabian Light crude, and their primary function is to separate oil, water, and gas.

Analysis of crude oils from wells in Ghawar indicates that these oils are produced in the form of tight water-in-oil emulsions. Tight or strong emulsions are difficult to separate and cause production and operational problems. These problems have led, at times, to an increase in de-emulsifier usage, production of off-spec crude, and occasionally caused equipment upsets in the GOSP.

The main causes of emulsion problems are (1) the presence of asphaltenes and fine solids in the crude, (2) lower temperatures in the wintertime, and (3) an increase in water production.

APPROACH

In this case study of tight emulsions, once you have collected all the positive and negative factors and have quantified them, you can put them together into an accurate cost-benefit analysis.

On the cost side, one can envisage the following:

- Cost of de-emulsifier
- Addition of asphaltenes dispersants, and surfactants to the crude oil.
- Using elaborate techniques to quantify the oil-water separation process, such as ESI, Emulsion Separation Index (method developed by Saudi Aramco).

On the benefit side, we get:

- A reduction in the quantity of de-emulsifiers used
- Less production losses
- Less operation problems
- An increase in oil revenue
- Fast rate of separation in the GOSP, which gives less residence time, thus reducing the diameter of the separator

15

Crude Oil Treatment: Dehydration, Desalting, and Stabilization

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Oil leaving the gas-oil separators may or may not meet the purchaser's specifications. As presented in Chapter 14, associated gas and most of the free water in the well stream are removed in the separators. The free water separated is normally limited to water droplets of 500 µm and larger. Oil stream leaving the separators would normally contain water droplets of smaller size along with water emulsified in the crude oil. This chapter deals first with the dehydration stage of crude oil to free it from the emulsified water. Depending on the original water content of the oil as well as its salinity, oil field treatment could produce oil with a remnant water content of 0.2 to 0.5 of 1%.

The next stage in the treatment process of crude oil is desalting. The removal of salts found in the form of what is termed *remnant brine* is carried out in the desalting process. This reduces the salt content in the crude oil

to the acceptable limits of 15 to 20 PTB (pounds per thousand barrel). After treating the oil by the dehydration and the desalting process, the possibility of stabilizing the crude oil and sweetening exists in the case of sour oil. This is covered in the final part of this chapter.

Case studies presented in this chapter are "Static Mixer Improves Desalting Efficiency" and "Upgrading Quality of Crude Oil by Using a Desalting Unit."

15.1 Technology Aspects

15.1.1 Dehydration of Crude Oil

Dewatering, or dehydration followed by desalting of crude oil upstream of crude distillation unit, is considered a key process operation for the removal of saline water, salts, and other contaminants from crude oil before it reaches any major unit operation. As stated above, dehydration of crude oil is simply to free it from the emulsified water.

15.1.1.1 Emulsion Formation

Crude oil emulsions form when oil and saline water (brine) come into contact with each other, when there is sufficient mixing, and when an emulsifying agent or emulsifier is present. The amount of mixing and the presence of emulsifier are critical for the formation of an emulsion. During crude oil production, there are several sources of mixing, often referred to as the amount of shear, including flow through reservoir rock, flow through tubing, and flow lines to reach the surface equipment.

The presence, amount, and nature of the emulsifier agent determines, to a large extent, the type and tightness of an emulsion.

Produced oil field water-in-oil emulsions contain oil, water, and an emulsifying agent. Emulsifiers stabilize emulsions and include surface-active agents and finely divided solids. Figure 15.1 depicts water-in-oil emulsion.

15.1.1.2 Emulsion Treatment

The resolution of emulsified oil follows a three-step procedure:

1. Reduction or rupture of the stabilizing films surrounding the water droplets. This step is called the *de-stabilization process*, and can be effectively carried out by adding chemicals and heating the emulsified oil.



FIGURE 15.1 (See Color Insert) Water-in-oil emulsion.

- 2. Coalescence of the liberated water droplets occurs, forming larger drops of water. This process is enhanced by electric field and heating. It is also a function of residence time in the vessel.
- 3. Gravitational settling with subsequent separation of water drops from oil (time element).

Water-oil emulsions are resolved by treatment equipment utilizing a combination of any of the following dehydration aids:

- Heating
- Chemical treatment
- Electrical field

Some emulsions can be broken with either chemical and time or heat and time. Time is the one indispensible variable or element. It is the element that determines the size of the equipment, which in turn determines its cost.

15.1.1.3 Heating

The most pronounced effect is the reduction of oil viscosity. Other advantages are also contributed to heat, including:

- (a) An increase in the difference in specific gravity between oil and water
- (b) An increase in the droplet size as demonstrated by its molecular movement which enhances coalescence
- (c) Help in de-stabilization of the emulsifying film

Some of the disadvantages of heat compared to other methods are:

- (a) Loss of valuable hydrocarbons
- (b) Consumption of fuels for heaters
- (c) Costly equipment
- (d) Gases liberated during heating adds additional problems in handling and cause safety hazards.

Field heaters are of two types:

Direct, in which the crude oil is passed through a coil exposed to the hot gases used as a fuel

Indirect, in which water is used as a transfer medium for heat from hot flue gases to the oil to be heated and immersed in the water.

Both methods are illustrated in Figure 15.2. Examples of some industrial field heaters are line heaters, wash tanks, and gun-barrel treaters.

15.1.1.4 Chemical Treating

Chemical additives function to break crude oil emulsions by adding agents comprising high molecular weight polymers adsorbed at the water-oil interface. These chemicals (called de-emulsifiers) can either rupture the film or displace the stabilizers due to reduction in surface tension on the inside of the film. They are complex organic compounds with surface active characteristics such as sulfonates, polyglycol esters, polyamine compounds, and many others. They are usually added using a small chemical pump up-stream of



FIGURE 15.2 Methods of heating oil emulsions.

the choke. Dosage is estimated to be about 1 quart of the chemical for each 100 barrels of oil.

The principle of breaking oil-water emulsions using electric current, which is known as electro-static separation, is discussed in the following section.

15.1.2 Desalting of Crude Oil

15.1.2.1 Introduction

The removal of salt from crude oil for refinery feed stocks is required by most of the refiners, particularly if the salt content exceeds the range of 15 to 20 PTB. Values for the salt content of some typical crude oils could be as low as 8 to 10 PTB for the Middle East, while the value could reach a high above 70 PTB for Oklahoma. Crude oil arriving from oil fields generally contains 1% or more of saline water and organic salts. The salinity of the water could be in the range of 15,000 to 30,000 ppm or even much higher. Part of the salts contained in the crude oil, particularly magnesium chloride, are hydrolyzed at temperatures above 120°C. Upon hydrolysis, the chlorides get converted into hydrochloric acid and corrode the distillation column's overhead and the condensers.

The most economical place for desalting is the refinery. However, in many situations, when marketing or pipeline requirements are imposed, field treatments are applied. The principles stay the same, using unit-operations fundamentals.

Salt in crude oil is in most cases found dissolved in the remnant water within the oil. The amount of salt found in crude oil depends on two factors:

The quantity of remnant water that is left in oil after normal dehydration The salinity or the initial concentration of salt in the source of this water

Salt content in oil is a function of both the quantity of remnant water found in oil and the concentration of salt in it. One has to make the economic compromise of using both approaches for reducing the salt content of crude oil. Economically, there is a limit on reducing the salinity by lowering the quantity of remnant water, by dehydration only. The other alternative is to substantially decrease the salt content of the remnant water by mixing it with water with a much lower concentration of salts in it. This is what we accomplish in the desalting of crude oil.

15.1.2.2 Description of Desalting Process

The desalting process involves basically two steps, as given in Figure 15.3. The first is adding fresh water to the crude oil, to be thoroughly mixed. This is followed by a separation or dehydration step. In other words, the process is like "washing" the salty crude oil with water followed by separating the water phase from crude oil.

The mixing step in the desalting step is normally accomplished by pumping the crude oil and wash water, each separately through a mixing device



FIGURE 15.3

The basic concept of the desalting operation of crude oil.

(could be throttling valve or orifice plate mixers). In the electrical desalting process, a high potential field (16,500–33,000 volts) is applied across the settling vessel to help the coalescence of water drops. In the process, 2% to 5% by volume of water is emulsified in the untreated crude oil and heated to a temperature of 180 to 300°F. In the desalting process, it is a common practice to apply pressure to suppress losses of hydrocarbons from the oil. The pressure used is normally in the range of 50 to 250 psi.

The steps involved in the desalting of crude oil with preliminary dehydration are as follows:

- 1. Adding a de-emulsifier to the feed oil to enhance breaking emulsions.
- 2. Pumping the feed oil through heat exchangers to heat it to 200 to 300°F to enhance separation of the water from oil.
- 3. Adding wash water to the feed, ensuring thorough and effective mixing.
- 4. Allowing the emulsion that is formed between wash water and remnant water in oil to settle in the desalter, subjected to a high-voltage electric field. This will help in the separation of the two phases.

- 5. Removing effluent water and contaminants from the desalter.
- 6. Obtaining "dry" oil from the top to be shipped to destination.

15.1.3 Stabilization and Sweetening of Sour Crude Oil

15.1.3.1 Introduction

Previous discussions have dealt with the separation of water and the removal of salts from the liquid phase comprising crude oil. Our objective here is to present methods for stabilizing the crude oil relative to specified vapor pressure and allowable concentration of hydrogen sulfide (H_2S). Some produced crude oils contain hydrogen sulfide and other sulfur products. When it contains more than 400 ppm of H_2S gas, the oil is classified as sour crude. Sour crude oils present serious safety and corrosion problems. In such cases, another treatment, known as the sweetening process, is needed to remove hydrogen sulfide or reduce its content to acceptable limits.

In addition to this, maximization of yield of production by minimizing the loss of valuable lighter hydrocarbons should also be a target. The series of hydrocarbons distributed between the gas phase and liquid phase has a wide spectrum as was shown in Table 14.1. Cuts can be identified as finished products, depending on the individual hydrocarbons that are included.

Dual operation of stabilization and sweetening of crude oil targets the above objectives. Retention of too many light ends in the presence of hydrogen sulfide can cause many problems. Refiners and shipping tankers impose restrictions on crude oil to have a vapor pressure of 5 to 20 RVP (Reid vapor pressure) and a maximum of 10 to 100 ppmw (parts per million by weight) of hydrogen sulfide. This dual operation will also lead to an increase in the API gravity of the oil, an advantage in its sales value.

H ₂ S Concentration	Standard	Health Effect
15 ppm	TLV-STEL ^a	A small percentage of workers may experience eye irritation
300 ppm	IDLH ^b	Maximum concentration from which one could escape within 30 minutes without a respirator
700 ppm		Quick loss of consciousness, breathing will stop, and death will result if not rescued promptly

The environmental effect from exposure to H_2S , as well as some exposure standards as reported in the oil industry, are as follows:

^a Threshold-limit value for 15 minutes. Short-term exposure limit.

^b Generally recognized "Immediately Dangerous to Life and Health" concentration.



FIGURE 15.4 Typical trayed stabilizer.

15.1.3.2 Process Description

The total pressure exerted by the crude oil is contributed to by the partial pressure of low boiling compounds that may be present in small quantities. Examples are methane and hydrogen sulfide. The maximum volume of hydrocarbon liquid that is stable under stock tank conditions can be obtained by using what is known as a *trayed stabilizer*. This is a fractionating column, but with no reflux pumps and no condensers. Cold feed is introduced to the top plate of the column. This provides internal reflux, where the falling liquid contacts the warm vapors rising from the bottom of the column. The rising vapors strip the lighter ends from the crude, while the crude absorbs and dissolves some of the heavy ends from the vapors. A flow diagram for the process is given in Figure 15.4. Stabilization generally increases the recovery of stock tank of crude by 3% to 7% over simple stage stabilization or separation

15.2 Economic Evaluation and Application

Case 15.1: Static Mixer Improves Desalting Efficiency^{*} Objective

To study the economic feasibility of replacing a typical globe-type mix valve by a static mixer in a crude oil desalter.

⁺ From *Chemical Online Newsletter*, October 13, 2000, and Linga, H., Al-Qahtani, F.A., and Al-Qahtani, S.N., New Mixer Optimizes Crude Desalting Plant, SPE 124823, paper presented at the 2009 SPE Annual Technical Conference and Exhibition in New Orleans, LA, 2009.

APPROACH

The case could be handled using the method presented in Chapter 7. In this method, all costs incurred in buying, installing, operating, and maintaining an asset are put on an annual basis. Selection is then based on what we call the "differential approach," or the return on extra investment.

PROCESS DESCRIPTION

The mixer was installed at a 150,000 bbl/d crude distillation unit's desalter. Crude at this refinery is a mixture of local production and imports from Indonesia and Alaska. The crude oil and water are then simultaneously mixed though two-by-two division, cross-current mixing, and back-mixing, which improves turbulence and increases mixing efficiency without requiring high fluid shear velocities.

Desalter Performance ^a			
	Salt In ^b	Salt Out ^b	% Removal
Mix Valve (Globe Valve)			
90,000 b/d 22° API Crude	42	4.4	89%
14° API Crude §		_	_
Static Mixer (New Mixer)			
90,000 b/d 22° API Crude	41	1.6	96%
45,000 b/d 14º Crude	43	1/4	97%

^a Desalter mix valve and static mixer are designed for full design crude unit feed rate of 150 MBPD.

^b PTB.

Merits of the Static Mixer

This table summarizes the main performance of the static mixer as compared to the globe valve.

CONCLUSIONS

The modified desalter system has operated well on 14° and 22° API naphthenic crudes, with less than 5% oil in the effluent water. At the same time, the mixer has helped reduce emulsions formed by too much pressure drop created by the mix valve. With less oil carry under, less fuel is consumed from having to reheat recycled oil up to 300°F before it reenters the crude unit. Salt removal also increased as a result of using the static mixer (see table). Depending on the type of crude oil, the refiner has been able to remove between 5% and 10% more salt than by the mix valve method. With less salt carried over out of desalter, less corrosive HCI will be generated in the crude unit furnaces. This will require less ammonia to neutralize the atmospheric column overhead stream. Also, pressure drop due to the mixing device was decreased from 10 psi to 1.5 psi.

The payout period of the new mixer was calculated and found to be 1 year. In other words, the mixer will pay for itself in its first year of operation with combined savings of \$4,000/year in power consumption and chemical costs and \$1,000/year in fuel costs.

Case 15.2: Upgrading the Quality of Crude Oil by Using a Desalting Unit

OBJECTIVE

Evaluation of the economic feasibility of a desalting unit.

APPROACH

Calculation of the return on investment (ROI) and pay-out period (POP).

PROCESS DESCRIPTION

The following results were obtained from field desalting of a crude oil in the Middle East using one stage (Abdel-Aal, 1998):

- Crude oil flow rate (feed) = 120,000 BPD
- BS&W, vol% of feed = 1.6
- Salt content of feed (PTB) = 900
- Water of dilution, vol.% = 2
- Salt content of desalted oil (PTB) = 46

Eventually, for this type of crude oil, a two-stage desalting unit should be applied to bring the salt content in the final product to 15 to 20 PTB. This upgrading process is to be investigated along the following guidelines.

GIVEN

- The upgrading of crude oil to an acceptable PTB could realize a savings of 0.1 \$/bbl in the shipping costs of the oil.
- The crude oil desalting unit has a design capacity of 120,000 bbl/day.
- The capital investment is estimated to be \$5 million, service life is 10 years, and operating factor is 0.95.
- The total annual operating expenses are \$10/1000 bbl, and the annual maintenance expenses are 10% of the capital investment.

FIND

- (a) The return on investment, ROI
- (b) The payout period, POP

SOLUTION

Annual savings in shipping costs of upgraded crude oil = \$4.1610 $\times 10^6$

Total annual expenses incurred by installing the desalting unit = $\$1.4161 \times 10^6$

Net savings = 2.7449×10^{6}

ROI = net savings/capital investment = 55%

POP (number of years to recover the capital investment) = 1.8 years

16

Gas Treatment and Conditioning

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H.H. Redhwi

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Natural gas is valuable both as a clean source of energy and as a chemical feedstock. Before reaching the customer, it has to go through several processing steps. These steps are necessary partly to be able to transport the gas over long distances and partly for the recovery of valuable components contained in the gas.

Natural gas associated with oil production or produced from gas fields generally contains undesirable components such as H_2S , CO_2 , N_2 , and water vapor. In this chapter, natural gas conditioning is detailed. This includes the removal of undesirable components before the gas can be sold in the market. Specifically, the gas contents of H_2S , CO_2 , and water vapor must be removed or reduced to acceptable concentrations. N_2 , on the other hand, may be removed if it is justifiable. Gas compression is usually needed after these treatment processes. Dehydration and gas treatment are presented in this chapter. Sweetening of natural gas almost always precedes dehydration and other gas plant processes carried out for the separation of natural gas liquid (NGL). Dehydration is usually required before the gas can be sold for pipeline marketing and is a necessary step in the recovery of NGL from natural gas.

Gas processing is made up of two operations: NGL recovery, and separation from the bulk of gas and its subsequent fractionation into desired products. This is the subject of Chapter 19.

The case studies at the end of the chapter are "Utilization of Natural Gas Recovered from Gas Plant," "How to Control the CO₂ Specs in the Sweet Gas," and "Non-Catalytic Partial Oxidation (NCPO) of Sour Natural Gas."

16.1 Technology Aspects

16.1.1 Overview of Gas Field Processing

In its broad scope, gas field processing (G.F.P.) includes dehydration, acidic gas removal (H_2S and CO_2), and the separation and fractionation of liquid hydrocarbons (NGL). Sweetening of natural gas almost always precedes dehydration and other gas plant processes carried out for the separation of NGL. Dehydration, on the other hand, is usually required before the gas can be sold for pipeline marketing, and it is a necessary step in the recovery of NGL from natural gas.

A system involving G.F.P. can be divided into two main stages:

- 1. Stage I, known as gas treatment or gas conditioning which is covered in this chapter
- 2. Stage II, known as gas processing, covered in Chapter 19

The gas treatment operations carried out in stage I involve the removal of gas contaminants (acidic gases), followed by the separation of water vapor (dehydration). Gas processing, stage II, comprises two operations: NGL recovery and separation from the bulk of gas and its subsequent fractionation into desired product.

Gas field processing in general is carried out for two main objectives:

- 1. The need to remove impurities from natural gas
- 2. The desirability of increasing liquid recovery above that obtained by conventional gas processing

Natural gas field processing and the removal of various components from it tend to involve the most complex and expensive processes. A sour gas leaving a gas-oil separation plant (GOSP) might require first the use of an amine unit (MEA) to remove the acidic gases, a glycol unit (TEG) to dehydrate it, and a gas compressor to compress it before it can be sold.

16.1.2 Effect of Impurities (Water Vapor, H₂S/CO₂) and Liquid Hydrocarbons Found in Natural Gas

The effect that each of these components has on the gas industry, as the end user, is briefly outlined:

- 1. Water vapor: This is a common impurity. It is not objectionable as such. If it condenses to liquid, it accelerates corrosion in the presence of H₂S gas. If it leads to the formation of solid hydrates (made up of water and hydrocarbons), it will plug valves and fittings in the pipe.
- 2. H_2S/CO_2 : Both gases are harmful, especially H_2S , which is toxic if burned to give SO_2 and SO_3 , which are nuisances to consumers. Both gases are corrosive in the presence of water. In addition, CO_2 contributes a lower heating value to the gas.
- 3. Liquid hydrocarbons: The presence of liquid hydrocarbons is undesirable in gas that is used as a fuel. The liquid form is objectionable for burners designed for gas fuels. In the case of pipelines, handling two-phase flow (gas and liquid) is undesirable.

16.1.3 Sour Gas Treating

16.1.3.1 Selection of Gas-Sweetening Process

The key parameters to be considered in the selection of a given sweetening process include the following:

- 1. Type of impurities to be removed (H₂S and mercaptans)
- 2. Inlet and outlet acid gas concentrations
- 3. Gas flow rate, temperature, and pressure
- 4. Feasibility of sulfur recovery
- 5. Acid gas selectivity required
- 6. Presence of heavy aromatic in the gas
- 7. Well location
- 8. Environmental consideration
- 9. Relative economics

Generic and specialty solvents are divided into three different categories to achieve sales gas specifications:

- 1. Chemical solvents
- 2. Physical solvents
- 3. Physical-chemical (hybrid) solvents

The selection of the proper gas-sweetening process depends on the sulfur content in the feed and the desired product as illustrated in Figure 16.1. Several commercial processes are available, as shown in Figure 16.2.

16.1.3.2 Amine Processes

Amine gas sweetening is a proven technology that removes H_2S and CO_2 from natural gas and liquid hydrocarbon streams through absorption and chemical reaction. Aqueous solutions of alkanolamines are the most widely used for sweetening natural gas. Each of the amines offers distinct advantages to specific treating problems:

- (a) *MEA, Monoethanolamine:* Used in low-pressure natural gas treatment applications requiring stringent outlet gas specifications.
- (b) *MDEA*, *Methyldiethanolamine*: Has a higher affinity for H₂S than CO₂ which allows some CO₂ "slip" while retaining H₂S removal capabilities.
- (c) *DEA*, *Diethanolamine*: Used in medium- to high-pressure treating and does not require reclaiming, as do MEA and DGA systems.
- (d) *Formulated (specialty) solvent:* A variety of blended or specialty solvents are available on the market.



FIGURE 16.1 Selection of gas-sweetening processes.



FIGURE 16.2 Classification of gas-sweetening processes.

A typical amine process is shown in Figure 16.3. The acid gas is fed into a scrubber to remove entrained water and liquid hydrocarbons. The gas then enters the bottom of the absorption tower, which is either a tray (for high flow rates) or packed (for lower flow rate). The sweet gas exits at the top of tower. The regenerated amine (lean amine) enters at the top of this tower, and the two streams are contacted countercurrently. In this tower, CO_2 and H_2S are absorbed with the chemical reaction into the amine phase. The exit amine solution, loaded with CO_2 and H_2S , is called *rich amine*. This stream is flashed, filtered, and then fed to the top of a stripper to recover the amine, and acid gases (CO_2 and H_2S) are stripped and exit at the top of the tower. The refluxed water helps in steam stripping the rich amine solution. The regenerated amine (lean amine) is recycled back to the top of the absorption tower.

16.1.4 Gas Dehydration

16.1.4.1 Introduction

Natural gas usually contains significant quantities of water vapor. Changes in temperature and pressure condense this vapor altering the physical state from gas to liquid to solid. This water must be removed in order to protect the system from corrosion and hydrate formation. The wet inlet gas temperature and supply pressures are the most important factors in the accurate design of a gas dehydration system. Without this basic information the sizing of an adequate dehydrator is impossible.



FIGURE 16.3 Flowsheet for the amine process.

Natural gas dehydration is defined as the process of removing water vapor from the gas stream to lower the dew point of the gas. There are three basic reasons for the dehydration process:

- 1. To prevent hydrate formation. Hydrates are solids formed by the physical combination of water and other small molecules of hydrocarbons. They are icy hydrocarbon compounds of about 10% hydrocarbons and 90% water.
- 2. To avoid corrosion problems. Corrosion often occurs when liquid water is present along with acidic gases, which tend to dissolve and disassociate in the water phase, forming acidic solutions.
- 3. To avoid side reactions, foaming, or catalyst deactivation during downstream processing in many commercial hydrocarbon processes.

16.1.4.2 Prediction of Hydrate Formation

Methods for determining the operating conditions leading to hydrate formation are essential in handling natural gas. In particular, we should be able to find:

- 1. Hydrate formation temperature for a given pressure
- 2. Hydrate formation pressure for a given temperature
- 3. Amount of water vapor that saturates the gas at a given pressure and temperature (i.e., at the dew point)

At any specified pressure, the temperature at which the gas is saturated with water vapor is defined as the "dew point.' Cooling of the gas in a flow line due to heat loss can cause the gas temperature to drop below the hydrate formation temperature. Elaborate discussion of both approximate methods and analytical methods is presented by Abdel-Aal et al (2003).

16.1.4.3 Methods Used to Inhibit Hydrate Formation

Hydrate formation in natural gas is promoted by high-pressure, low-temperature conditions and the presence of liquid water. Therefore, hydrates can be prevented by adopting one (or more than one) of the following procedures:

- 1. Raising the system temperature or lowering the system pressure (temperature/pressure control)
- 2. Injecting a chemical such as methanol or glycol to depress the freezing point of liquid water (chemical injection)
- 3. Removing water vapor from the gas (liquid-water drop out); in other words depressing the dew point by dehydration.

16.1.4.4 Dehydration Methods

The most common dehydration methods used for natural gas processing are the following:

- 1. Absorption, using the liquid desiccants (e.g., glycols and methanol)
- 2. Adsorption, using solid desiccants (e.g., alumina and silica gel)
- 3. Cooling/condensation below the dew point, by expansion or refrigeration

This is in addition to the hydrate inhibition procedures described earlier. The various dehydration methods are shown in Figure 16.4.

16.1.4.5 Dehydration Using Absorption System

The absorption process is shown schematically in Figure 16.5. The wet natural gas enters the absorption column (glycol contactor) near its bottom and flows upward through the bottom tray to the top tray and out at the top of the column. Usually six to eight trays are used. Lean (dry) glycol is fed at the top of the column, and it flows down from tray to tray, absorbing water vapor from the natural gas. The rich (wet) glycol leaves from the bottom of the column to the glycol regeneration unit. The dry natural gas passes through mist mesh to the sales line. The rich glycol is preheated in heat exchangers, using the hot lean glycol, before it enters the still column of the glycol reboiler. This



FIGURE 16.4 Gas dehydration methods.







FIGURE 16.6

A solid desiccant unit for natural gas dehydration.

cools down the lean glycol to the desired temperature and saves the energy required for heating the rich glycol in the reboiler.

16.1.4.6 Dehydration Using Adsorption (Solid-Bed Dehydration)

When very low dew points are required, solid-bed dehydration becomes the logical choice. This method is based on fixed-bed adsorption of water vapor by a selected desiccant. A number of solid desiccants could be used, such as silica gel, activated alumina, or molecular sieves. The selection of these solids depends on economics. The most important property is the capacity of the desiccant, which determines the loading design expressed as the percentage of water to be adsorbed by the bed. The capacity decreases as temperature increases. Figure 16.6 represents solid bed dehydration.

16.2 Economic Evaluation and Application

Case Study 16.1: Utilization of Natural Gas Recovered from Gas Plant

OBJECTIVE

To investigate the economics of utilizing natural gas as a fuel for heating crude oil.

PROCESS

Natural gas is recovered from GOSP using an absorber de-ethanizer system, along with an amine treating unit and a gas dryer to have available desulfurized gas that can be used or sold as a fuel gas.

GIVEN

The total cost for the recovery of this gas is estimated to be 0.75/MCF. It has been suggested to use this gas as a fuel for heating 5000 bbl/day of 40° API crude oil from 80°F to 250°F.

FIND

- 1. The cost of heating the crude oil using this gas.
- 2. Compare it with the cost of heating fuel oil at \$2.2/MM Btu.
- 3. Do you recommend change in operation to use the fuel gas as a heating fuel instead of using the fuel oil?

SOLUTION

The heat duty required is calculated using the well-known equation: $Q = mcp\Delta T = 127.7$ MM Btu/day.

Assuming the heating value of the gas is 960 Btu/ft³ and the heat efficiency is 60%; then the fuel gas consumption will be 221700 ft³/day.

The cost of using this fuel gas for heating = $2217000 \text{ ft}^3/\text{day} \times \$0.75/\text{MCF} = \$166.28/\text{day}$

The cost of using the fuel oil for heating = [127.7 MM Btu × \$2.2/MM Btu]/0.6 = \$468.23/day

A daily savings in the cost of fuel of about \$300 is realized if the change to fuel gas takes place. One has to consider other economic factors in making this analysis. The capital cost involved in changing the burner system has to be considered.

Case 16.2: How to Control the CO₂ Specs in Sweet Gas (Discussion)

Methyl diethanolamine (MDEA) has become the amine molecule chosen to remove hydrogen sulfide, carbon dioxide, and other contaminants from hydrocarbon streams. Amine formulations based on MDEA can significantly reduce the costs of acid gas treating. Under the right circumstances, MDEA-based solutions can boost plant capacity, lower energy requirements, or reduce the capital required.

The ultimate goal of amine sweetening is to produce specification quality product as economically as possible. Amine technology has produced selective absorbents that remove H_2S in the presence of CO_2 . The use of selective amines results in:

- Lower circulation rates
- Reduction in reboiler sizes and duties, while meeting the H₂S specification. Unfortunately, many operators now are exceeding

the CO_2 specification in their sweet gas streams due to changes in inlet composition or increased throughput. Achieving specifications within the constraints of the process equipment is most cost effective and desirable.

In general, if the objective is to slip as much CO₂ as possible, the engineer should consider using the most selective amine at the lowest concentration and circulation rate with the fewest number of equilibrium stages in the absorber to achieve the H₂S specification. Cold absorber temperatures tend to increase the CO_2 slip and enhance H_2S pickup. If the objective is to achieve a certain CO₂ concentration, then the problem is more complicated. Variables to consider include increasing the amine concentration and using mixtures of amines. However, equipment size may have to be reevaluated. Increasing the lean amine temperature increases CO₂ pickup for the selective amines to a point. The maximum temperature depends on amine concentration, inlet gas composition, and loading. Higher lean amine temperature also increases water and amine losses and decreases H₂S pickup. Alternatively, solvents that are designed for carbon dioxide removal are also available. For example, DOW's Specialty Amines cover the full range from the maximum carbon dioxide slip, to nearly complete carbon dioxide removal.

Case 16.3: Non-Catalytic Partial Oxidation (NCPO) of Sour Natural Gas (Discussion)

In order to exclude the costly DGA treatment of sour natural gas, the NCPO approach proposed by Abdel-Aal and Shalabi (1996) is recommended to produce synthesis gas from sour gas by direct partial oxidation, as illustrated in Figure 16.7.

Currently, synthesis gas is produced by steam reforming of sweet natural gas. This is a catalytic process in which the feed gas has to be sulfur free to avoid catalyst poisoning. As a result, acidic gas removal is a prerequisite for the steam-reforming process, as shown in Figure 16.8. H_2S is separated from the natural gas by one of the physiochemical separation methods. The separation process is expensive and involves the use of



FIGURE 16.7

Non-catalytic partial oxidation of sour natural gas.



FIGURE 16.8

Current technology to produce synthesis gas from sour natural gas.

amine solvents. The chemisorption of acidic gas into the solvents is followed by regeneration of these solvents. Although the bulk production of synthesis gas is done via catalyzed steam reforming of sweet natural gas, non-catalyzed partial oxidation of sour natural gas with appropriate conditions may prove to be more attractive.

17

Crude Oil Refining: Physical Separation

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Refineries rely on four major chemical processing operations in addition to the backbone physical operation of fractional distillation, in order to alter the ratios of the different fractions. These are normally called the Five Pillars of petroleum refining:

Pillar 1: Fractional distillationPillar 2: CrackingPillar 3: Unification (alkylation)

Pillar 4: Alteration (catalytic reforming) Pillar 5: Hydroprocessing

The basic aspects of current refining operations involving physical separation are presented in this chapter along with the application of economic techniques and analysis to many problems encountered in the petroleum industry.

The physical separation of crude oil into valuable products (cuts) is highlighted. Crude oil separation is accomplished in two consecutive steps: first by fractionating the total crude oil at essentially atmospheric pressure, and then by feeding the bottom residue from the atmospheric tower to a second fractionator operating at high vacuum. Types of oil refineries and their classifications are given.

Economic analysis is presented for the refining operations in various ways to determine the most economical refining scheme to find out, for example, whether to use new or existing equipment. Economic balance in design and in yield and recovery is explained with and case studies and solved examples are given.

17.1 Technology Aspects

17.1.1 Introduction

Petroleum is of little use when it first comes from the ground. It is a raw material, much as newly fallen trees are raw materials for furniture, construction, etc. Thus crude oil must be put through a series of processes to be converted into the hundreds of finished oil products derived from it. These processes, collectively, are known as refining. However, by today's technological standards, the term *refining* is a misnomer. In the early petroleum industry, the refining process involved nothing more than the use of a crude still (pipe still) that produced useful oil products by physical separation only.

Currently the expression *crude oil processing* is more appropriate, since more than 85% of petroleum products are produced by processes involving chemical changes along with the basic physical separation.

The first step in refining is distillation. This step roughly separates the molecules in crude according to their size and weight. The process is analogous to taking a barrel of gravel containing stones of many different sizes and running the gravel through a series of screens to sift out first the small stones, next those slightly larger, and so on up to the very largest stones. As applied to crude oil, the distillation process "sifts out" progressively such components as gas, gasoline, kerosene, home heating oil, lubricating oils, heavy fuel oils, and asphalt.



FIGURE 17.1

Flow diagram of the physical and chemical processes of crude oil refining.

Distillation is a physical process. It can separate crude into various cuts, but it cannot produce more of a particular cut than existed in the original crude. Unfortunately, too, consumers' demands for different oil products do not necessarily parallel the natural proportions of the crude. For example, if we had to depend on the amount of gasoline naturally present in crude (about 20%), we would not have enough to run all the automobiles presently on the road. This leads us to the next step in refining, which involves chemical conversion; the topic of Chapter 18. A schematic layout of both physical operations and chemical conversion processes is presented in Figure 17.1.

17.1.2 Distillation of Crude Oil: Overview

17.1.2.1 Fractional Distillation: Pillar 1

Crude oil entering the refinery contains all the materials that will leave the refinery as finished products, which need only to be separated from other compounds found in the crude oil. This is accomplished using distillation units. Other products would have resulted from what is known as chemical conversion processes (C.C.P.s) which take place in processing units *downstream* of the distillation units.

The various components found in crude oil have different sizes, weights, and boiling temperatures; therefore, the first step is to separate these components. Because they have different boiling temperatures, they can be
separated easily by *fractional distillation*. The steps involved in the fractional distillation of crude oil are as follows:

- Oil contains a complex mixture of hydrocarbons. The first step in obtaining something of value from crude oil is dehydration and desalting (when needed).
- Oil is then heated in a furnace and sent to a distillation column that is filled with plates or trays, operating at atmospheric pressure (*atmospheric distillation*). Heat is added to the bottom of the column (in the form of *open steam*) and removed at the top of the column (*condenser*).
- Separation of the oil into fractions takes place based on differences in the boiling points of the hydrocarbons. The bottom fraction (*residue*) is sent to another column operating at a pressure of 75 mm Hg (one tenth of an atmosphere) (*vacuum distillation*). In this column, operating at lower temperature can separate the heaviest faction without thermal degradation. You will notice that whereas *atmospheric* columns are thin and tall, *vacuum* columns are thick and short.

17.1.2.2 Operating Pressure

The primary physical separation process, which is used is almost every stage while processing the crude oil, is fractional distillation, as explained above. The distillation operation can take place at atmospheric pressure, under vacuum, or under high operating pressure. The three operations are common in the oil refining industry. For example, crude oil fractionation is always accomplished at atmospheric pressure (slightly higher), topped crude oil (fuel oil residue) is distilled under vacuum, while the *stabilization* of straightrun gasoline utilizes high-pressure fractionators or stabilizers. A comparison between these three systems of fractionation is shown in Table 17.1, which shows the technical merits and economic implications of each system.

17.1.3 Crude Oil Desalting

Crude oil, once produced, undergoes a series of field treatment operations called *surface operations* before it is subjected to distillation. Basically, this involves gas-oil separation, crude oil dehydration to separate suspended and emulsified water, and desalting to reduce the salt content to allowable limits. These operations were covered fully in previous chapters and are briefly illustrated in Figure 17.2. The *economic impact* of these treatment steps on the quality and hence the price of the produced crude oils is great. Most pronounced is the salt content of the oil.

High-salt-content oils (greater than 15 to 20 lb of salt expressed as NaCl/1000 bbl of oil, PTB) have to be desalted in order to avoid or minimize the fouling effect and the corrosion caused by the salt. Chloride salts can deposit on

TABLE 17.1

Operation Features	Atmospheric Distillation	Vacuum Distillation	Pressure Distillation
Application	Fractionation of crude oils	Fractionation of heavy residues (fuel oil)	Fractionation and/or separation of light hydrocarbons
Justification	Always, work near atmospheric pressure	To avoid thermal decomposition	To allow condensation of the overhead stream using cooling water
Extra equipment (as compared with atmospheric distillation)		Steam jet ejectors and condensers to produce and maintain vacuum	Stronger thickness for the vessel shell
Extra design features (as compared with atmospheric distillation)		Larger diameter because of higher vapor flow rate	Increased number of trays (N) because separation becomes more difficult; increased reflux ratio

Types and Features of Distillation Operations

equipment surfaces, causing fouling, or can decompose, forming hydrochloric acids, while they attack the vessels and pipes during the distillation process. The removal of these salts is aimed at making an economical operating cycle in the refining process of crude oil. The reduction of salt content of 5 PTB is feasible. Even with this low salt content, the processing of 25,000 bbl/ day of crude oil could result in an amount of HCl equal to 65 lb/day.

Another advantage of the desalting process of crude oil is removal of some of the metals found in the oil, such as vanadium and nickel, whose presence



FIGURE 17.2

Outline of the operations carried out to treat crude oil before distillation.

causes poisoning of the catalysts used in the chemical conversion processes for crude oil fractions.

17.1.4 Separation of Crude Oil: Heavy on the Bottom, Light on the Top

Once crude oil is heated in the furnace, the resulting liquids and vapors are discharged into the distillation towers, where they separate into components or *fractions* according to weight and boiling point. The lightest fractions, including gasoline and liquid petroleum gas (LPG), vaporize and rise to the top of the tower, where they condense back to liquids. Mediumweight liquids, including kerosene and diesel oil distillates, stay in the middle. Heavier liquids, called gas oils, separate lower down, while the heaviest fractions with the highest boiling points settle at the bottom. These tarlike fractions, called *residuum*, are literally the "bottom of the barrel." This normally takes place in what is named a *topping* plant, shown in Figure 17.3. A schematic presentation of the distillation process of crude oil is shown in Figure 17.4.

17.1.5 Distillation Schemes

In the modern distillation process, after being desalted crude oil is run through a series of heat exchangers, where it is preheated to about 500° F. Its temperature is then raised to the appropriate flashing temp ($700-750^{\circ}$ F)



FIGURE 17.3 A flow diagram for a topping plant.



FIGURE 17.4 The distillation process of crude oil.

by direct heating in a furnace before it enters the bottom of the atmospheric fractionating or topping column.

Various components of the hydrocarbons found in the crude oil have different boiling points; that is, they change from liquid to vapor, or condense back from vapor to liquid, at different temperatures. By taking advantage of this fact, it is possible to operate the oil into different fractions, or cuts.

The main cuts produced by the atmospheric fractionator are:

- Overhead product, known as gasoline or naphtha, which is normally charged to a stabilizing column to remove the light hydrocarbons (propane and butanes); that is, distillation under pressure
- Side-stream products such as kerosene and gas oil, which are introduced to side-stream strippers to control the flash point of such product; that is, stripping operation
- Bottom product, known as fuel oil or reduced crude oil, which is further distilled using *vacuum* distillation units

Figure 17.5 illustrates a distillation scheme for crude oil that involves all types of distillation operations: atmospheric, under vacuum and under pressure, and the stripping operations that take place inside the side strippers and at the bottoms of the atmospheric column and the vacuum tower.





17.2 Economic Evaluation and Application

17.2.1 Types of Refineries and Economic Analysis

Depending on the type of crude oil used, the processes selected, and the products needed, as well as the economic considerations involved, refineries can have different classifications, as shown in Figure 17.6. The products that dictate the design of a *fuel* refinery or conventional refinery are relatively few in number but are produced in large quantities, such as gasoline, jet fuels, and diesel fuels. The number of products, however, increases with the degree of complexity of a fuel refinery, which varies from simple to complex or to fully integrated.

A simple refinery consists mainly of a crude oil atmospheric distillation unit, stabilization splitter unit, catalytic reforming plant, and product-treating facilities. Products are limited: LPG, gasoline, kerosene, gas oil, diesel oil, and fuel oil. A complex refinery will employ additional physical separation units (such as vacuum distillation) and a number of chemical conversion processes, including hydrocatalytic cracking, polymerization, alkylation,



FIGURE 17.6 Classifications of refineries.

and others. The fully integrated refinery will provide other processes and operations necessary to produce practically all types of petroleum products, including lubrication oils, waxes, asphalts, and many others.

A *chemical* refinery, on the other hand, is a special case of the conventional oil refinery in which the emphasis is on manufacture of olefins and aromatics from crude oil. A chemical refinery can be defined as one that includes an olefin complex for the pyrolysis of petroleum fractions (for example C_2H_6 to C_2H_4). It must not produce motor gasolines; that is, it is a non-fuel-producing refinery. In other words, the purpose of chemical refining is to convert the whole crude oil directly into chemical feedstocks. An example is the heavy oil cracking (HOC) process, in which the atmospheric residuum is catalytically cracked directly into lighter products. Chemical refining is an economically attractive venture for large chemical companies that can penetrate the market by selling large quantities of olefins and aromatics.

Economic analysis is used in refining to determine the most economical refining operations, to determine whether to use new or existing equipment, etc. Economic analysis, including cost analysis, is complicated in a refinery because an operation in a refinery with lower operating costs is not necessarily the most desirable procedure, and similarly, an operation giving higher yields, or production rates, is not necessarily a more economical one. A highest yield with lowest cost is what the refiner would like to achieve.

Economic analysis is further complicated by the fact that several hundred different products may be produced from one basic raw material, crude oil. There are also other complications. The basic crude may consist of a number of different crudes that have considerably different characteristics and different selling prices (to independent refiners only). Furthermore, it is becoming increasingly more difficult for refiners to determine which products are prime products and which are by-products. The economic analyst faces the problem of establishing reasonable differentials between the costs or values of various products and raw materials consistent with the amounts of one product that can be produced from another product or from crude oil.

Economic analysis helps a refiner determine whether he can meet the competition. Application of the results of economic analysis by a petroleum refiner will result in an improved competitive position in the industry and may result in increased profits.

The basic tools used in the economic study of a refinery are operating costs of existing unit operations in a refinery. These can be obtained from normal accounting records. Direct operating costs that are controllable consist of direct supervision, operating labor, maintenance and repair labor, plus materials, chemicals, fuel utilities, auxiliary services, royalties, and employee benefits.

There are also costs of an auxiliary operating nature, such as overhead and burden items, that are generally not controllable. These include depreciation, taxes, administrative and general expenses, and other items not charged directly to operation units.

17.2.2 Economic Balance

Economic balance in refining operations means that costs are balanced with revenue, inputs with outputs, and crudes with refined products. The object is to find the combination of least cost with the greatest contribution.

There are two significant corollaries of great significance to the oil refiner which follow from the principle of diminishing productivity: the principle of variable proportion, and the principle of least-cost combination.

The principle of variable proportion enters into all decisions relative to combining economic factors (inputs) for full production. In chemistry, we know that elements combine in definite proportions. For instance, the combination of two atoms of hydrogen with one atom of oxygen will produce one molecule of water: $H_2 + O \rightarrow H_2O$. No other combination of hydrogen atoms and oxygen atoms will produce water. What is true in this instance is also true in all other chemical combinations, and in oil production as well. In other words, a law of definite proportions governs the combination of the various chemical elements and the various factors of production, such as amount of labor, materials consumed, and capital in a plant investment.

Economic balance applies to both physical operations (unit operations) and chemical conversion processes. It may involve a design problem or address a processing operation or a separation step. In other words, economic balance may refer to the period before installation of equipment, in which case it consists of a study of costs and values received on *design* of equipment, or the period after installation of equipment, in which case it is a study of costs and values received on *processing operations*. The latter means, on one hand, economic balancing of costs against optimum yield or optimum recovery, and on the other hand, elimination of as much waste as possible.

17.2.2.1 Economic Balance in Design

Design of equipment for process operations is complex because of the many variables involved and the fact that broad generalizations about these variables cannot be made. Economic balance is not discussed in detail here, as much of it is beyond the scope of this book. A number of cases of economic balance in design, however, will be discussed.

- Economic balance in evaporation is a problem of determining the most economical number of effects to use in a multiple-effect evaporation operation. There is economy in increasing the amount of steam used because direct costs are reduced, but at the same time there is an increase in fixed costs when an increasing number of effects are used. So selection of which number of effects will balance direct costs is desirable.
- Economic balance in vessel design may involve specific design problems, such as heating and cooling, catalyst distribution, design of pressure vessels for minimum cost, etc.
- Economic balance in fluid flow involves the study of costs in which such direct costs as power costs for pressure drop and repairs and fixed costs of pipe, fittings, and installation are related to pipe size. For example, power costs decrease as pipe size increases, and total costs are at a minimum point at some optimum pipe size.
- Economic balance in heat transfer requires an understanding of how fixed costs vary, with a selected common variable used as a basis for analysis. Variable costs must also be related to this same variable. Thus both fixed costs and variable costs are required for economic balance.

In any study of design or operations, only the variable cost, often referred to as direct costs which is affected by variations in operation, is included.

The following case study stresses the role of economic balance in design in many applications throughout the processing of crude oil, which may involve the transfer of material, heat, or mass with or without chemical conversions.

Case Study 17.1: Optimum Reflux Ratio

In designing a bubble plate distillation column, the design engineer must calculate:

- 1. Number of plates
- 2. Optimum reflux ratio
- 3. Diameter of the column

It is well established that if the reflux ratio is increased from its minimum value, R_m , the number of plates would be decreased to attain the same

desired separation. This means lower fixed costs for the column. The other extreme limit for the reflux could be reached by further increase in R with corresponding decrease in the number of trays until the total reflux, R_t , is reached (case of minimum number of trays, N_m). Attention is now directed to the effect on the diameter of the column of increasing the reflux ratio, that is, increasing vapor load

As *R* increases, the vapor load inside the column increases; consequently, the diameter of the column must be increased to attain the same vapor velocity. A point is reached where the increase in column diameter is more rapid than the decrease in the number of trays. Hence the only way to determine the optimum conditions of reflux ratio that will result in the right number of trays for the corresponding column diameter is to use economic balance. For different variable reflux ratios, the corresponding annual fixed costs and operating costs must be combined and plotted versus the reflux ratio.

Annual fixed costs are defined as the annual depreciation costs for the column, the reboiler, and the condenser, where the cost of a column for a given diameter equals the cost per plate of this particular diameter times the number of plates. Therefore, the operating cost equals the cost of the steam plus the cost of cooling water. Figure 17.7 illustrates how we obtain the optimum reflux ratio (a design parameter) by minimizing the total annual costs of the distillation column.



FIGURE 17.7

A plot of the total annual costs versus the reflux ratio. (From Peters, Max, and Timmerhaus, Klaus, *Plant Design and Economics for Chemical Engineers*, McGraw-Hill, New York, 1981. With permission.)

17.2.2.2 Economic Balance in Yield and Recovery

Principles of economic balance must be applied to different processes in the oil refinery for the purpose of determining how variations in yield, as affected by design or operation, will produce maximum profit. The effect of changing the crude feed and refined oil product compositions on the overall profit for a refinery process can best be illustrated in most cases as follows.

A typical study of economic balance in yield and recovery reveals that obtaining a higher-grade product from a fixed amount of given feed means an increase in variable costs because of costs of increased processing. The final refined oil product has a higher value, but for some product grades the costs may equal the selling price, with the result that it becomes uneconomical to exceed that particular specification. At some optimum grade of a product, however, a maximum gross profit, or difference between the sales dollars curve and the total costs curve, may be obtained per barrel of pure material (crude) in the feedstock.

In general, capacity is reduced as grade is increased, with the result that the maximum profit per barrel of pure material (crude) may not correspond to the maximum annual profit. Although graphic analysis is the best procedure to use for such problems, there are also some useful mathematical relations. For example, if *D* is total refined product, *F* is total feed (crude), and *Y* is a conversion factor relating feed (crude) and product (refined), then, under physical operations,

Y = D, bbl of total refined product/F bbl of total crude feed

Y = output/input

or recovery in percent form. Also, if fixed costs are constant for a given process, then fixed costs will be constant for a given value of *F* or total feed (crude). However, as is usually the case, equipment costs will be higher for a higher-grade product, with the result that the annual fixed cost per unit of refined product increases.

For a given crude feed rate, raw material costs are constant but refinery processing costs usually increase for a higher-grade product to give a variable cost curve that also increases. The value of the finished product, like that of fixed costs and variable costs per unit of refined oil, will vary with the grade of product.

Figure 17.8 is a typical economic chart with curves illustrating economic balance curves in a refinery. Recovery, or ratio of output to input, in the oil refinery is greater than recovery in the oil fields. To make a profit the refiner must stick to the product grades marked between A and B, shown in Figure 17.8.

or



FIGURE 17.8

Economic level of refined oil production from a given feed of crude oil.

Case Study 17.2: Crude Oil Desalter

The salt content of a Middle-Eastern crude oil (API gravity 24.2) was found to be 60 PTB. In order to ship and market this oil, it is necessary to install a desalting unit in the field, which will reduce the salt content to 15 PTB. This upgrading in the quality of oil in terms of an acceptable PTB could realize a possible saving of 0.1 \$/bbl in the shipping cost of the oil: Assume the following:

The crude oil desalter has a design capacity of 120,000 bbl/day.

- The current capital investment of the desalting unit is estimated to be \$3 million plus another \$2 million for storage tanks and other facilities.
- Service life of equipment is 10 years with negligible salvage value, while the operating factor = 0.95.
- The total operating expenses of the desalter are estimated to be \$10/1000 bbl.
- The annual maintenance expenses are 10% of the total capital investment. Evaluate the economic merits of the desalter by calculating the ROI and payout period (P.P.).

SOLUTION

The total annual cost is the sum of the annual operating expenses plus annual depreciation costs. Assuming straight-line depreciation:

d, the annual depreciation cost = $5 \times 106/10 = 0.5 \times 106$ \$/year (17.1)

The annual operating expenses = (10/1,000)(120,000)(365)(0.95)

$$= 0.416 \times 106$$
 (17.2)

The annual maintenance	$= (0.1)(5 \times 106)$	
	= 0.5 × 106 \$/year	(17.3)
Total annual costs	= (1) + (2) + (3)	
Annual savings	= (0.1)(120,000)(365)(0.95)	
	= 4.161 × 106 \$/year	
Net Savings	$= (4.161 - 1.4161) \times 106$	
	= 2.7449 × 106 \$/year	
R.O.I	= 54.9%	

17.3 Conclusions

17.3.1 Refining Costs

Refining costs include variable and direct refining costs, fixed charges, overhead expenses, and general expenses:

- Direct refining costs include costs of utilities, costs of fuel, operating labor and supervision, maintenance, and repair costs (the latter is usually estimated to be 8% of the capital investment).
- Fixed charges include depreciation costs, local taxes, and insurance (taxes and insurance costs are estimated to be about 3% of the capital investment).
- Depreciation costs account for the charges associated with the amortizable investment. One method is to calculate it on fixed annuities on the basis of amortization of 15 years and a certain interest rate, say 8%, on capital outlay. (See Chapter 5 for methods of calculating depreciation.)
- Overhead expenses account for employee benefits, medical service, etc., and are estimated to be 50% to 75% of the operating labor and supervision.
- General expenses account for administration, sales, and research expenses, estimated to be about 10% of sales.

How much fuel does a refinery use? It is estimated that the amount of heat needed in the processing of oil varies between 555,000 and 700,000 Btu/bbl of crude.

If crude oil has a heating value of 6 million Btu/bbl, the above figures indicate the equivalent of 9.2% to 11.7% of the crude oil. The heat requirements

include the burning of coke from catalyst as well as such common fuels as refinery gas, natural gas, residual fuel oil (pitch), and acid sludge and coal.

Fuel oil	13.7%
Coal	1.2%
Natural gas	37.4%
Refinery gas	43.0%
Coke (catalyst)	4.2%
Acid sludge	0.5%
Total	100%

Fuel is used for:

- 1. Sensible heat and latent heat in direct-fired heaters
- 2. Generating steam and electricity
- 3. Gas and diesel engines
- 4. Heating catalyst or solid material in the catalytic cracking process

Not more than 65% of the heat, or about 4,500,000 Btu, actually enters the barrel of oil, the remaining 35%, or about 1,575,000 Btu, of heat goes up the stack and out or remains as chemical energy in carbon monoxide (in regenerator flue gas).

17.3.2 Profitability Analysis

Methods described in Chapter 6, which include annual rate of return, payment period, discounted cash flow rate of return, and net present value, can be applied to examine the profitability of the proposed refining operations (both physical and chemical ones) that take place in a refinery.

17.3.3 Cost and Economic Analysis of Refining Operations

There are certain costs in the oil industry which are approximately the same regardless of the amount of refined products produced. These costs are known as *overhead*, or *fixed costs*. Some of these costs are interest, pensions, taxes, depreciation and depletion, payroll, goods and services purchased from others, etc.

The composition of these costs will differ in some respect for different oil companies. Also, oil refiners may have different percentages for the components of their fixed costs depending on the degree of integration and their capital structure. However, regardless of their composition, such costs are relatively large for major producers in the petroleum industry and, with typical low operating rates, are a substantial percentage of total costs. Costs over and above fixed costs represent *additional costs*, incidental to the production of each additional barrel of refined oil products (marginal costs), assuming the oil refinery is in operation. The addition to total costs arising from the production of each additional barrel of refined oil products is the same regardless of the operating rate at which the additional output is obtained, as long as the other factors affecting costs remain constant. This phenomenon of constant additional costs covers a range of output from 20% of capacity to about 90% of physical limit of output. As the physical limit of capacity, or 100%, is reached, the equipment becomes overtaxed and for various reasons operates less efficiently and at greater cost. In such cases, additional costs incidental to production (marginal costs) of an additional unit of output cease to be constant and probably rise sharply, so the basic economic law of diminishing returns makes further production uneconomical.

18

Crude Oil Refining: Chemical Conversion

Abdullah M. Aitani

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This chapter discusses the various aspects of crude oil refining as a primary source of fuel and as a feedstock for petrochemicals. The main objective of chemical conversion in oil refining is to convert crude oils of various origins into valuable products having the qualities and quantities demanded by the market. Various refining processes based on chemical conversion such as thermal and catalytic processes as well as general properties of refined products are briefly reviewed. The refining industry currently faces several challenges related to increasing demand for transportation fuels, stringent specifications of these products, crude oil availability, reduction of carbon emissions, and renewable fuels.

The following engineering problems involving economic analysis "Refinery Cost and Profitability," "Integration and Environmental Issues," and "Refinery FCC Revamps" are presented in the form of case studies.

18.1 Technology Aspects

18.1.1 Introduction

18.1.1.1 Overview

The processing of crude oil utilizes chemicals, catalysts, heat, and pressure to separate and combine the basic types of hydrocarbon molecules into groups of similar molecules. Petroleum refining has evolved continuously in response to changing demands for better and different products. The trend in demand has also been accompanied by continuous improvement in product quality such as octane number for gasoline, cetane number for diesel and sulfur content of all products. Over the last 25 years, the "API gravity of processed crude oils has been decreasing while average sulfur content has been increasing. Today's refinery utilizes an array of various catalytic and non-catalytic processes to meet new product specifications and to convert less desirable fractions into more valuable liquid fuels, petrochemical feedstock, and electricity. The refinery has shifted from only performing physical separations to something close to a chemical plant.

In 2011, a total of 655 refineries were operating worldwide, distributed in 116 countries, with a total processing capacity of 88.1 million barrels per day (b/d). Their annual throughput was about 75 million b/d, for an average capacity utilization of 85%. Table 18.1 presents an overview of regional refining operations worldwide. Asia has become the major refining hub, with about 28% of world refining capacity, followed by North America at 24% and Western Europe at 16% (True and Koottungal, 2011). As shown in Figure 18.1, the number of world refineries during the period 2002 to 2012 dropped by 65 refineries, and overall capacity increased by 6 million b/d. While most refineries in North America are configured to maximize gasoline production, refineries in Europe and Asia are configured to maximize diesel and jet fuel production.

TABLE 18.1

Region	Number	Million b/d				Coke	
	of Refineries	Crude Distillation	Catalytic Reforming	Catalytic Cracking	Catalytic Hydrocracking	Catalytic Hydrotreating	1000 tons/d
North America	148	21.3	4.1	6.5	1.9	16.4	134.7
South America	66	6.6	0.5	1.3	0.2	1.9	24.6
Western Europe	99	14.4	2.1	2.2	1.2	10.1	12.6
Eastern Europe	89	10.4	1.5	0.9	0.3	4.3	12.5
Asia	164	24.9	2.3	3.2	1.3	10.2	20.3
Mideast	44	7.3	0.8	0.4	0.5	2.0	3.3
Africa	45	3.2	0.5	0.2	0.06	0.8	1.8
Total	655	88.1	11.5	14.7	5.5	45.7	209.8

Regional Outlook of World Refining Operations

Source: True, W.R., and Koottungal, L., Global Capacity Growth Reverses; Asian, Mideast Refineries Progress, *Oil Gas Journal*, December 5, 2011. With permission.

18.1.1.2 Refinery Configuration

Refineries are considered manufacturing plants for various types of transport fuels and some chemical feedstocks. The refining industry has witnessed technological improvements over the last century, mainly related to new catalysts, reactors, gas compressors, heat exchangers, product treatment, heat and chemical integration, unit control, logistics, and many more. As a result, refineries



FIGURE 18.1

Worldwide refining: capacity and number of refineries. (From True, W.R., and Koottungal, L., Global Capacity Growth Reverses; Asian, Mideast Refineries Progress, *Oil Gas Journal*, December 5, 2011. With permission.)



FIGURE 18.2

A typical high-conversion refinery. (From Aitani, A., Oil Refining and Products, In: *Encyclopedia of Energy*, C.J. Cleveland, Ed., Elsevier, Amsterdam, New York, vol. 4, pp. 715–729, 2004. With permission.)

have become larger, more selective, more integrated, and more energy-efficient. Thereby, refining costs have decreased, and consequently world consumption of refined products has increased drastically, mainly in Asian countries. In general, the refining industry has been characterized as a high-volume, low-profit margin industry. However, despite all technological improvements the refining industry is still looking for more efficient processes and catalysts.

Figure 18.2 presents a schematic diagram of a typical high-conversion refinery, showing various processing units. These units range from relatively simple crude oil distillation to the more complex ones: vacuum distillation, hydrotreating, catalytic reforming, catalytic cracking, hydrocracking, alkylation, and isomerization. In 2011, with respect to crude distillation, catalytic hydrotreating represented 52% followed by vacuum distillation capacity at

33%, catalytic cracking at 17% and catalytic reforming at 13%. In general, the chemical conversion and treatment processes in a modern refinery can be grouped as follows (Aitani, 2004):

- *Light oil processing* prepares light distillates through rearrangement of molecules using isomerization and catalytic reforming or combination processes such as alkylation and polymerization.
- *Heavy oil processing* changes the size or structure of hydrocarbon molecules through thermal or catalytic cracking processes.
- *Treatment processes* involve a variety and combination of processes including hydrotreating, drying, solvent refining, and sweetening.

18.1.2 Crude Oil and Refined Products

18.1.2.1 Type and Composition of Crude Oils

Crude oil is a mixture of hydrocarbon compounds such as paraffins, naphthenes, and aromatics plus small amounts of organic compounds of sulfur, oxygen, and nitrogen, in addition to small amounts of metallic compounds of vanadium, nickel, and sodium. Although the concentration of nonhydrocarbon compounds is very small, their influence on catalytic petroleum processing is large. Elemental composition of crude oils depends on the type and origin of crude. However, these elements vary within narrow limits. The proportions of these elements in typical crude are 84.5% carbon, 13% hydrogen, 1–3% sulfur, and less than 1% each of nitrogen, oxygen, metals, and salts. Physical properties of crude oils vary within a wide range.

Table 18.2 presents typical product yields from U.S. Gulf Coast refineries processing various types of crude oils where the main objective is to maximize gasoline. Crude oils can be classified in many different ways, generally based on their density or API gravity, sulfur content, and hydrocarbon composition (the higher the API gravity, the lighter the crude). Condensate ranks highest, with densities reaching more than 50° API, while densities of heavy crudes may reach as low as 10° API. In general, refinery crude base stocks consist of mixtures of two or more different crude oils. Currently, there are more than 600 different commercial crude oils traded worldwide. In 2011, world crude oil supply was about 90 million b/d of which 35% was

TABLE 18.2

J 1 ()		0			
Major Refined Product	Boiling Point,°C	West Texas Intermediate	Arabian Light	Arabian Heavy	Nigerian Bonny Light
Gasoline	10-200	48.1	38.9	36.8	44.9
Kerosene/jet	200-260	8.1	8.2	6.7	7.8
Diesel	260-345	30.9	24.7	9.7	39.6
Fuel oil	345+	9.8	23.7	41.6	4.5

Typical Yields (%) of Refineries Processing Selected Crude Oils-U.S. Gulf Coast

produced by OPEC countries. The demand for crude oil is projected to reach 110 million b/d with major increase in Asian countries for transportation fuels, mainly diesel (OPEC, 2011).

18.1.2.2 Refined Products

Major oil products are mainly transportation fuels that represent more than 70% of total refined products worldwide. Gasoline, diesel, and jet fuel are the major refined products having complex mixtures of hydrocarbons that include paraffins, naphthenes, and aromatics (which give fuel its unique odor). Table 18.3 presents a list of refined products with their properties and uses. About 80–85 vol% of the refined products produced in a medium-type conversion refinery (processing Arabian Light crude oil) are lighter than the boiling temperature of 345°C compared to the 55 vol% existing in the crude oil. Almost half of the products are gasoline and lighter distillates. The demand for transportation fuels and petrochemical feedstocks has been increasing steadily compared to the decreasing demand for heating fuels and residual fuel oil that are being replaced by natural gas.

A major driver of future refining capacity requirements and economics is the level and quality of product demand. Oil demand developments in specific sectors determine to a great extent the current and future demand structure in respect to the product slate. Figure 18.3 compares world demand for refined products between 2010 and 2035, reflecting the continuing importance of transportation fuels as well as the continuing shift to middle distillates and light products. Out of 23 million b/d of additional demand expected by 2035, compared to the 2010 level, around 57% is for middle distillates (gas oil/diesel and jet/kerosene) and another 40% is for gasoline and naphtha (OPEC, 2011). The growing petrochemical industry provides momentum for naphtha demand, while residual fuel oil is set to decline in all key sectors of its consumption.

With respect to refinery product slate, middle distillates will not only record the biggest volume rise, they will also increase their share in the overall slate from 36% in 2010, to 41% by 2035. The share of light products (LPG, naphtha, and gasoline) will remain at about 43% in 2035. In contrast, the share of the heavy end of the refined barrel will decrease by around 6%, from 22% in 2010 to 16% by 2035. Needless to say, these structural changes cannot be achieved by simply increasing refinery crude runs. They require investments to change the configuration of the global refining system.

18.1.3 Light Oil Processing

18.1.3.1 Catalytic Hydrotreating

Catalytic hydrotreating is used to remove about 90% of contaminants such as nitrogen, sulfur, oxygen, and metals from liquid petroleum fractions. These

TABLE 18.3

Refined Product	Source, Properties, and Uses
LPG	Liquefied petroleum gas (LPG) consists of propane and butanes. It is used as fuel and in the manufacture of olefins. Butanes are also used in the manufacture of ethers and to adjust the vapor pressure of gasoline. LPG is also used in transportation and domestic and household applications.
Naphtha	BTX aromatics from naphtha reforming are the main petrochemical feedstocks derived from refinery. These products are the basis for integrating refining and petrochemical operations. Benzene and xylenes are precursors for many valuable chemicals and intermediates such as styrene and polyesters.
Gasoline	Mixture of hydrocarbons made up of different refinery streams mainly straight-run naphtha, isomerized C_5/C_6 paraffins, reformate, hydrocracking, FCC gasoline, oligomerate, alkylate, and ethers. The important qualities for gasoline are octane number, volatility, vapor pressure, and sulfur content.
Kerosene/jet	Middle-distillate product used for jets and in cooking and heating (kerosene). When used as a jet fuel, some of the critical qualities are freeze point, flash point, and smoke point. Kerosene is also used for lighting, heating, solvents, blending into diesel fuel, and paraffins dehydrogenated for use in detergents.
Diesel/heating oil	A blend from atmospheric distillation, hydrocracking, FCC light cycle oil, and some products obtained from visbreaking and coking. Its main property for automotive engine combustion is cetane number. Sulfur reduction and cetane improvement are heavily investigated to produce a clean diesel.
Residual fuel/bunkers	The least valuable refined product, selling at a price below that of crude oil. Many marine vessels, power plants, commercial buildings, and industrial facilities use residual fuels or combinations with distillate fuels for heating. Two most critical properties are viscosity and low sulfur content.
Lubes/wax	Vacuum distillation is the main source for lubes. Antioxidants and viscosity improvers are added to provide properties required for motor oils, industrial greases, lubricants, and cutting oils. The most critical quality is a high viscosity index.
Petcoke/asphalt	Petcoke (petroleum coke) has a variety of uses from electrodes to charcoal briquettes. Bitumen or asphalt is a semisolid material produced from vacuum distillation. It is classified into various commercial grades and is mainly used for paving roads and roofing materials.

Properties and Uses of Refined Products

contaminants can have detrimental effects on the equipment and the quality of the finished product. Hydrotreating for sulfur or nitrogen removal is called hydrodesulfurization (HDS) or hydrodenitrogenation (HDN), respectively. World capacity for all types of hydrotreating currently stands at about 45.7 million b/d % (True and Koottungal, 2011). Hydrotreating is used to



^{**} Includes bitumen, lubricants, waxes, still gas, coke, sulphur, direct use of crude oil, etc.

FIGURE 18.3 (See Color Insert)

Worldwide demand for refined products: 2010 and 2035 in million b/d. (From OPEC, Oil Demand by Product, *World Oil Outlook*, Vienna, 2011. With permission.)

pretreat catalytic reformer feeds, saturate aromatics in naphtha, desulfurize kerosene/jet, diesel, distillate aromatics saturation and to pre-treat catalytic cracker feeds. Hydrotreating processes differ depending upon the feedstock available and catalysts used. Mild hydrotreating is used to remove sulfur and saturate olefins. More severe hydrotreating removes nitrogen and additional sulfur and saturates aromatics. In a typical catalytic hydrotreater, the feedstock is mixed with hydrogen, preheated in a fired heater (315–425°C), and then charged under pressure (up to 68 atm) through a fixed-bed catalytic reactor. In the reactor, sulfur and nitrogen compounds in the feed are converted into H_2S and NH_3 . Hydrotreating catalysts contain cobalt or molybde-num oxides supported on alumina and less often nickel and tungsten.

18.1.3.2 Catalytic Naphtha Reforming

The reforming process combines catalyst, hardware, and process to produce high-octane reformate for gasoline blending or BTX (benzene, toluene, and xylene) aromatics for petrochemical feedstocks. Reformers are also the source of much-needed hydrogen for hydroprocessing operations. Naphtha reforming reactions comprise cracking, polymerization, dehydrogenation, and isomerization that take place simultaneously. UOP and Axens are the two major licensors and catalyst suppliers for catalytic naphtha reforming. There is a necessity of hydrotreating the naphtha feed to remove permanent reforming catalyst poisons and to reduce the temporary catalyst poisons to low levels. Currently, there are more than 700 reformers worldwide with a total capacity of about 11.5 million b/d % (True and Koottungal, 2011). About 40% of this capacity is located in North America, followed by 20% each in Western Europe and the Asia-Pacific regions. Reforming processes are generally classified into semi-regenerative, cyclic, and continuous catalyst regenerative (CCR). Most grassroots reformers are designed with continuous catalyst regeneration. CCR is characterized by high catalyst activity with reduced catalyst requirements, more uniform reformate of higher aromatic content, and high hydrogen purity.

18.1.3.3 Isomerization

Isomerization is an intermediate feed preparation-type process. There are more than 230 units worldwide with a processing capacity of 1.7 million b/d of light paraffins. Two types of units exist: C_4 isomerization and C_5/C_6 isomerization. A C_4 unit converts normal butane into isobutane, to provide additional feedstock for alkylation units, whereas a C_5/C_6 unit will isomerize mixtures of C_5/C_6 paraffins, saturate benzene, and remove naphthenes. Isomerization is similar to catalytic reforming in that the hydrocarbon molecules are re-arranged, but unlike catalytic reforming, isomerization just converts normal paraffins to isoparaffins. The greater value of branched paraffins over straight paraffins is a result of their higher octane contribution. The extent of paraffin isomerization is limited by a temperature-dependent thermodynamic equilibrium. For these reactions a more active catalyst permits a lower reaction temperature and that leads to higher equilibrium levels. Isomerization of paraffins takes place under medium pressure (typically 30 bar) in a hydrogen atmosphere.

18.1.3.4 Alkylation

Alkylation is the process that produces gasoline-range compounds from the combination of light C_3 - C_5 olefins (mainly a mixture of propylene and butylene) with isobutene. The highly exothermic reaction is carried out in the presence of a strong acid catalyst, either sulfuric acid or hydrofluoric acid. World alkylation capacity is currently 2.1 million b/d (True and Koottungal, 2011). The alkylate product is composed of a mixture of highoctane, branched-chain paraffinic hydrocarbons. Alkylate is a premium clean gasoline blending with octane number depending upon the type of feedstocks and operating conditions. Research efforts are directed toward the development of environmentally acceptable solid superacids capable of replacing HF and H_2SO_4 .

18.1.3.5 Etherification

Etherification results from the selective reaction of methanol or ethanol to isobutene. The ether products such methyl tertiary butyl ether (MTBE) or

other oxygenates are used as components in gasoline because of their high octane blending value. Refinery capacity of oxygenate units is about 193,000 b/d with almost all units associated with the alkylation process. The exothermic reaction is conducted in liquid phase at 85 to 90°C over a highly acidic ion-exchange polystyrene resin catalyst. The reaction is rapid, and equilibrium is limited under typical reaction conditions. In general, MTBE is the preferred oxygenate because of its low production cost and convenient preparation route relative to other ethers.

18.1.3.6 Polymerization and Dimerization

Catalytic polymerization and dimerization refer to the conversion of FCC light olefins such as ethylene, propylene, and butenes into higher-octane hydrocarbons for gasoline blending. The process combines two or more identical olefin molecules to form a single molecule with the same elements in the same proportions as the original molecules. World capacity of polymerization and dimerization processes is about 195,000 b/d (True and Koottungal, 2011). In the catalytic process, the feedstock is either passed over a solid phosphoric acid catalyst on silica or comes in contact with liquid phosphoric acid, where an exothermic polymeric reaction occurs. Another process uses a homogenous catalyst system of aluminum-alkyl and a nickel coordination complex. The hydrocarbon phase is separated, stabilized, and fractionated into LPG and oligomers or dimers.

18.1.4 Heavy Distillate Processing

18.1.4.1 Fluid Catalytic Cracking (FCC)

Catalytic cracking is the largest refining process for gasoline production, with global capacity of more than 14.4 million b/d (True and Koottungal, 2011). The fluidized catalytic cracking (FCC) process converts heavy feedstocks such as vacuum distillates, residues, and deasphalted oil into lighter products rich in olefins and aromatics. FCC catalysts are typically solid acids of fine particles especially zeolites (synthetic Y-Faujasite) with content generally in the range of 5-20 wt%, while the balance is silica-alumina amorphous matrix. Additives to the FCC catalyst make no more than 5% of the catalyst and are basically used to enhance octane, as metal passivator, as SOx reducing agents, in CO oxidation, to enhance propylene, and to reduce gasoline sulfur. The FCC unit comprises a reaction section, product fractionation, and regeneration section. Typical operating temperatures of the FCC unit are from 500-550°C at low pressures. Hydrocarbon feed temperatures range from 260-425°C, while regenerator exit temperatures for hot catalyst are 650-815°C. Since the FCC unit is a major source of olefins (for downstream alkylation unit or petrochemical feedstock), an unsaturated gas plant is generally considered a part of it.

18.1.4.2 Catalytic Hydrocracking

Catalytic hydrocracking of heavy petroleum cuts is an important process for the production of gasoline, jet fuel, and light gas oils. The world conversion capacity for hydrocracking is about 5.5 million b/d (True and Koottungal, 2011). The process employs high pressure, high temperature, a catalyst, and hydrogen. In contrast to FCC, the advantage of hydrocracking is that middle distillates, jet fuels, and gas oils of very good quality are provided. In general, hydrocracking is more effective in converting gas oils to lighter products, but it is more expensive to operate. Heavy aromatic feedstock is converted into lighter products under a wide range of very high pressures (70–140 atm) and fairly high temperatures (400–820°C) in the presence of hydrogen and special catalysts. Hydrocracking catalysts have bifunctional activity combining an acid function (halogenated aluminas, zeolites) and a hydrogenating function (one or more transition metals, such as Fe, Co, Ni, Ru, Pd, and Pt, or by a combination of Mo and W).

18.1.5 Residual Oil Processing

18.1.5.1 Coking

About 90% of coke production comes from delayed coking. The process is one of the preferred thermal cracking schemes for residue upgrading in many refiners, mainly in the United States. The process provides essentially complete rejection of metals and carbon while providing partial or complete conversion to naphtha and diesel. World capacity of coking units is 4.7 million b/d (about 54% of this capacity is in U.S. refineries), and total coke production is about 172,000 t/d (True and Koottungal, 2011). New cokers are designed to minimize coke and produce a heavy coker gas oil that is catalytically upgraded. The yield slate for a delayed coker can be varied to meet a refiner's objectives through the selection of operating parameters. Coke yield and the conversion of heavy coker gas oil are reduced, as the operating pressure and recycle are reduced and to a lesser extent as temperature is increased.

18.1.5.2 Visbreaking

Visbreaking is a non-catalytic residue mild-conversion process with a world capacity of 3.8 million b/d (True and Koottungal, 2011). The process is designed to reduce the viscosity of atmospheric or vacuum residues by thermal cracking. It produces 15–20% of atmospheric distillates with proportionate reduction in the production of residual fuel oil. Visbreaking reduces the quantity of cutter stock required to meet fuel oil specifications and, depending upon fuel oil sulfur specs, typically reduces the overall quantity of fuel oil produced by 20%. In general, visbreakers are typically used to process to

vacuum residues. The process is available in two schemes: coil cracker and soaker cracker. The coil cracker operates at high temperatures during a short residence time of about 1 minute. The soaker scheme uses a soaking drum at $30-40^{\circ}$ C at about 10-20 residence time.

18.1.5.3 Residue Hydrotreating and RFCC

Refineries that have substantial capacity of coking, visbreaking, or deasphalting are faced with large quantities of visbreaker tar, asphalt, or coke, respectively. These residues have high viscosity and high organic sulfur content (4–7 wt%) with primary consequences reflected in the potential for sulfur emissions and the design requirements for sulfur removal system. Residue hydrotreating is another method for reducing high-sulfur residual fuel oil yields. Atmospheric and vacuum residue desulfurization units are commonly operated to desulfurize the residue as a preparatory measure for feeding low sulfur vacuum gas-oil feed to cracking units (FCC and hydrocrackers), low sulfur residue feed to delayed coker units, and low sulfur fuel oil to power stations. The processing units used for hydrotreating of resids are either a downflow, trickle-phase reactor system (fixed catalyst bed) or a liquid recycle and back-mixing system (ebullating bed). Economics generally tend to limit residue hydrotreating applications to feedstocks containing less than 250 ppm nickel and vanadium.

Residue FCC (RFCC) is a well-established approach for converting a significant portion of the heavier fractions of the crude barrel into a high-octane gasoline blending component. In addition to high gasoline yields, the RFCC unit produces gaseous, distillate, and fuel oil-range products. The RFCC unit's product quality is directly affected by its feedstock quality. In particular, unlike hydrotreating, RFCC redistributes sulfur but does not remove it from the products. Consequently, tightening product specifications have forced refiners to hydrotreat some, or all, of the RFCC's products. Similarly, in the future the SOx emissions from an RFCC may become more of an obstacle for residue conversion projects. For these reasons, a point can be reached where the RFCC's profitability can economically justify hydrotreating the RFCC's feedstock.

18.1.6 Auxiliary and Treating Processes

18.1.6.1 Hydrogen Production

Refiners are experiencing a substantial increase in hydrogen requirement to improve product quality and process heavy sour crudes. Hydrogen plays a critical role in the production of clean fuels, and optimum use of hydrogen can maximize refinery profits. Hydrocracking, distillate hydrotreating, and other hydroprocessing operations for the saturation of aromatics and olefins will accelerate the demand for hydrogen to the turn of the century and beyond. Catalytic naphtha reforming alone is not able to meet refinery hydrogen requirements. A recent survey on world refining indicated that the capacity of supplementary refinery hydrogen, produced mainly by steam reforming process reached 14,160 MMcfd (True and Koottungal, 2011). There is a growing recognition that there will be a significant future shortage of refinery hydrogen supply. Specific hydrogen production units such as steam methane reformers or partial oxidation of heavy residues will have to be built. The refining industry will require a substantial amount of on-purpose hydrogen to meet processing requirements with Asia Pacific and the Middle East representing nearly 40% of global requirements. About two-thirds of incremental refinery hydrogen demand will be for expanding hydrocracking operations.

18.1.6.2 Residue Gasification

The gasification of refinery residues into clean syngas provides an alternative route for the production of hydrogen and the generation of electricity in a combined turbine and steam cycle. Compared to steam-methane reforming, gasification of residues can be a viable process for refinery hydrogen production when the natural gas price is in the range of \$3.75 to 4.00 per million Btu. The largest application of syngas production is in the generation of electricity power by the integrated gasification combined cycle (IGCC) process. Electricity consumption in the modern conversion refinery is increasing, and the need for additional power capacity is quite common, as is the need to replace old capacity. The IGCC plant consists of several steps: gasification section, gas desulfurization, and combined cycle system.

18.1.6.3 Aromatics Extraction

BTX aromatics are high-value petrochemical feedstocks produced by catalytic naphtha reforming and extracted from the reformate stream. Whether or not other aromatics are recovered, it is sometimes necessary to remove benzene from reformate in order to meet mandated specifications on gasoline composition. Aromatics production in refineries reached 1.4 million b/d in 2011 (True and Koottungal, 2011). Most new aromatic complexes are configured to maximize the yield of benzene and paraxylene and sometimes orthoxylene. The solvents used in the extraction of aromatics include dimethylformamide (DMF), formylmorpholine (FM), dimethylsulfoxide (DMSO), sulfolane, and ethylene glycols.

18.1.6.4 Sulfur Recovery

Sulfur recovery converts hydrogen sulfide in sour gases and hydrocarbon streams to elemental sulfur. Total sulfur production in world refineries reached about 84,000 tons/d in 2011 compared to about 28,000 tons/d in 1996 corresponding to a yearly growing recovery rate of 14%. In other words, an average refinery today recovers 1 kg sulfur from one processed barrel of crude oil compared to less than 0.4 kg sulfur recovered in 1996. This indicates the increasing severity of operations to meet stringent environmental requirements. The most widely used sulfur recovery system is the Claus process, which uses both thermal and catalytic-conversion reactions. A typical process produces elemental sulfur by burning hydrogen sulfide under controlled conditions. Knockout pots are used to remove water and hydrocarbons from feed gas streams. The gases are then exposed to a catalyst to recover additional sulfur. Sulfur vapor from burning and conversion is condensed and recovered.

18.2 Economic Evaluation and Application

Case 18.1: Refinery Complexity Index

The concept of refinery complexity was introduced by W. Nelson in the 1960s to quantify the relative cost of processing units that make up a refinery (Kaiser and Gary, 2007). A refinery's complexity index indicates how complex it is in relation to a refinery that performs only crude distillation. Table 18.4 presents a list of complexity factors for refinery

TABLE 18.4

Processing Unit	Complexity Factor
Atmospheric distillation	1.0
Vacuum distillation	2.0
Thermal cracking	3.0
Delayed/fluid coking	6.0
Visbreaking	2.5
Catalytic cracking (FCC)	6.0
Catalytic reforming	5.0
Catalytic hydrocracking	6.0
Catalytic hydrorefining	3.0
Catalytic hydrotreating	2.0
Alkylation	10.0
Aromatics, BTX	15.0
Isomerization	15.0
Polymerization	10.0
Lubes	6.0
Asphalt	1.5
Hydrogen manufacturing, MMscfd	1.0
Oxygenates	10.0

Source: Kaiser, M.J., Gary, J.H., Study Updates Refinery Investment Cost Curves, Oil & Gas Journal, April 23, 2007. With permission.

processes that are used in the calculation of refinery complexity index. A complexity factor of 1 was assigned to the atmospheric distillation unit and expressed the cost of all other units in terms of their cost relative to distillation. For example, if a crude distillation unit of 100,000 b/d capacity costs \$20 million to build, then the unit cost/daily barrel of throughput would be \$200/b/d. If a 20,000 b/d catalytic reforming unit costs \$20 million to build, then the unit cost is \$1000/b/d of throughput and the complexity factor of the catalytic reforming unit would be 1000/200 = 5.

The complexity rating of a refinery is calculated by multiplying the complexity factor of each process by the percentage of crude oil it processes, then totaling these individual factors. This method accounts only for the refinery processing units of the Inside Battery Limits (ISBL) units, and not for off-sites and utilities. As an example, consider the case of a refinery with 400,000 b/d crude capacity and 140,000 b/d vacuum distillation capacity. The throughput of the vacuum tower relative to the crude distillation capacity is 35%. Given a vacuum unit complexity factor of 2, then the contribution of this unit to the overall complexity is 2 × 0.35, or 0.7.

The complexity index can be generalized across any level of aggregation, such as a company, state, country, or region. In general, refineries can be classified as hydroskimming, cracking, and deep conversion, in order of both increasing complexity and cost. Table 18.5 compares the complexity indices of various types of refineries. A high-conversion coking refinery has a complexity index of 9 compared with a topping refinery that has a complexity index of 1. The most complex, deep conversion refinery is able to transform a wide variety of crudes, including the lower-quality heavy sour crudes into the higher-value products (e.g., gasoline, diesel). The ability to meet stringent product specifications, notably ultra low sulfur gasoline and diesel fuel, is also a characteristic of high-complexity refineries.

Case 18.2: Refinery Cost and Profitability

As with any manufacturing plant, refinery costs are mainly associated with refinery construction cost (capital) and refinery operation cost (variable and fixed). In estimating construction cost, data are correlated with variables such as capacity, process units, complexity, location, and type of crude processed (light sweet, heavy sweet, light sour or heavy sour crude). For complexity,

TABLE 18.5

Refinery Type	Process	Complexity
Coking	Coking/resid upgrading to process medium/sour crude oil	9
Cracking	Vacuum distillation and catalytic cracking to process light sour crude oil to produce light and middle distillates	5
Hydroskimming	Atmospheric distillation, naphtha reforming, and desulfurization to process light sweet crude oil to produce gasoline	2
Topping	Separate crude oil into refined products by atmospheric distillation, produce naphtha but no gasoline	1

Complexity Index of Various Refineries

the Nelson complexity index is generally used because it is publicly available on *Oil & Gas Journal*, which also publishes cost indices that can be used to estimate and update these costs using the Nelson-Farrar Cost Index (Kaiser and Gary, 2007). These include data on pumps and compressors, electrical machinery, internal-combustion engines, instruments, heat exchangers, chemical costs, materials components, fuel cost, labor cost, wages, chemical costs, maintenance, etc. One remark on the utilization of complexity factors in estimating cost is that they do not account for the impact of capacity on cost because the complexity factor is capacity-invariant, and trends in complexity factors change slowly (or not at all) over time.

Refiners may undertake capital investment for a variety of reasons, for example, expanding existing or creating new production facilities, implementing new or enhanced technology, or regulatory compliance. Facility expansion and new technology implementation are indicators that the industry expects increasing demand and economic growth. While there hasn't been a new refinery constructed in the last 30 years, there has been an increase in U.S. refining capacity. In the current situation, it is more cost effective to expand a refinery in the United States than it is to build new. API estimates it would cost at least \$24,000 per daily barrel of oil process for a new refinery and \$15,000 per daily barrel of oil process for a new refinery could take at least 5 to 10 years.

Refinery gross profitability (margin) is a measure of the economics of a specific refinery. It is measured as the difference (\$/b) between a refinery's product income (total of barrels for each product multiplied by the price of each product) and the cost of raw materials (crude oil and other chemicals/catalysts). For example, if a refinery receives \$120 from the sale of the products refined from a barrel of crude oil that costs \$100/bbl, then the refinery gross margin is \$20/bbl. The net (cash) margin is equal to the gross margin minus operating costs. Therefore, for this specific refinery that has an operating cost of \$6 per barrel, its net margin is \$14/b. In many occasions, the measure of refinery profitability is complicated by the fact that the refinery produces several hundred different products from a mixture of different crudes that have various characteristics and different selling prices. Furthermore, it is becoming increasingly more difficult for refiners to determine which products are prime products and which are by-products (Abdel-Aal, 1992).

Figure 18.4 presents benchmark refining margins for three major global refining centers: U.S. Gulf Coast (USGC), North West Europe (NWE–Rotterdam), and Singapore. In each case they are based on a single crude oil appropriate for that region and have optimized product yields based on a generic refinery configuration (cracking, hydrocracking, or coking), again appropriate for that region. The margins are on a semi-variable basis (i.e., the margin after all variable costs and fixed energy costs). Refining profitability varies according to competitive market demand for refined products. It may range between –\$2/b to \$20/b or more in refinery markets that have very limited spare capacity. In competitive markets the refinery margins change daily as the market prices of both crude oil and products change (BP, 2012). Under such conditions the refinery revenues (average margin × throughput) over the course of



FIGURE 18.4 (See Color Insert)

Regional refining margins (in U.S. \$/b) for refineries in the U.S. Gulf Coast, North West Europe, and Singapore. (From *BP Statistical Review of World Energy*, June 2012, BP, London. With permission.)

a year must be equal to or exceed its operating costs, depreciation, and taxes, plus a fair return on investment.

Case 18.3: Integration and Environmental Issues

Refining-petrochemical integration is mainly carried out between a refinery and an aromatic complex, or between a refinery and olefins plants (steam crackers). Table 18.6 shows the process streams resulting from refinery integration with aromatics complex (through CCR unit) and steam cracker (Leighton, 2009). While aromatics (paraxylene and benzene) are readily traded, olefins require further processing to

TABLE 18.6

Refinery Integration Interface with Aromatics Complex and Steam Cracker

Aromatics Complex	Refinery Feed Stream		Olefins Steam Cracker	
Hydrogen		Heavy	Hydrogen	
LPG		Naphtha	C4 Raffinate	
C5/C7 Raffinate	Light Naphtha		C5/C7+ Pygas	
C9+ Aromatics	FCC C3=		Pyrolysis Fuel Oil	
Example: Satorn KSA	FCC Dry Gas		Example: PetroRabigh_KSA	
Example: Sucorp, Rorr	Light Ends		Example. Ferrorabigit, Norr	

Source: Leighton, P., Potential of Integrated Facilities—Finding Value Addition, World Refining Association: Petchem Arabia, 4th Annual Meeting, Abu Dhabi, October 2009. With permission.

polyolefins or other derivatives. Further integration issues suggest the utilization of FCC gasoline that is highly aromatic and naphthenic for aromatics production. Moreover, FCC units have long been a source of petrochemical propylene using special process designs and catalysts.

It is strongly believed that refining and petrochemical integration improve refining margins and overall profitability of the integrated venture. Industry experience has shown that refineries that are integrated with petrochemicals had greater savings in investment cost and operating costs. Other drivers for this integration include flexibility in upgrading low-value fuel streams to petrochemical feed and the utilization of hydrogen and C4 raffinate in refinery processing. The integration brings processing synergies that reduce the cost of production of both the fuels and petrochemical products.

Refiners on the other hand are faced with various environmental issues related to the changing specifications of refined products. In many locations, refinery configuration has changed substantially mainly due to the declining quality of crude oil supply and environmental regulations. Refiners are faced with huge investments to meet new stringent specifications for sulfur, aromatics, and olefins content. Gasoline sulfur reduction is centered around the FCC unit employing feed pretreatment or gasoline post-treatment. For diesel fuel, a sulfur content of less than 30 ppm or maybe 15 ppm is needed, as well as an increase in the cetane number and reduction in polyaromatics content. To fulfill all these requirements, refiners have either to revamp existing units or invest in new hydroprocessing and hydrogen production units. However, the need for more hydrogen may itself contribute to an increase of CO₂ emissions which could stand at about 20% of total refineries emission by 2035 (14% in 2005), as natural gas steam reforming should be the dominant technology. In addition the upgrading of extra heavy crude will account for more than 15% of the refineries' emissions in 2035 (4% in 2005).

Most environmental concerns in waste gas are around the emissions of SO_x , NO_x , CO, hydrocarbons, and particulates. The oxides are present in flue gases from furnaces, boilers, and FCC regenerators. Tail gas treatment and selective catalytic reduction (SCR) units are being added to limit SO_2 and NO_x emissions. Water pollutants include oil, phenol, sulfur, ammonia, chlorides, and heavy metals. New biological processes can be used to convert H_2S or SOx from gaseous and aqueous streams. Spent catalysts and sludges are also of concern to refineries in reducing pollution.

Case 18.4: Refinery FCC Revamps

This is a case study of a Gulf coast refinery in which the conversion capability of the existing FCC unit was found to be limiting the refinery economics (Ladjan and Schnaith, 2011). Changing feed quality, combined with feed rate increases, beyond the original design, were limiting the performance of the unit. Further changes in feed quality were proposed to increase the heavy syn-crude percentage processed by the refinery.

A team consisting of refinery personnel, UOP, IAG, and Andrews Consulting was assembled to evaluate the following refinery objectives:

- Increase production of more valuable liquid products
- Address catalyst circulation limits
- Maintain same level of coke yield
- Provide flexibility for future changes in feed quality

An economic analysis was performed based on installed cost estimates from IAG and yield estimates from UOP. The total installed cost estimate for the new regenerated catalyst standpipe, wye section, feed distributors, and upper riser was \$5.9 MM. The unit profitability estimate based on the heavier feed and new yields was \$4.2 MM per year for *a simple payback of less than 15 months.*

The post-revamp operation had an improved conversion and reduced coke yield per expectations. The next table shows the base case compared to the revamped operation (Lacijan and Schnaith, 2011):

	Base Case	Post-Revamp
Feed rate, b/d	48,000	47,450
Feed API	24.4	24.6
UOP K	11.75	11.69
Feed conradson carbon, wt%	0.3	0.2
Feed steam, wt%	2.1	1.3
Cat/oil	6.1	6.1
Yields, wt%		
C ₂ minus	2.9	2.8
C ₃ s	5.5	6.3
C ₄ s	9.6	9.9
Gasoline, 221°C TBP EP	46.0	48.8
LCO, 343°C TBP EP	17.8	18.3
MC Botts	14.7	9.7
Coke	4.4	4.2
Conversion	67.5	72

It has been stated that part of the success of the revamp in this case study was due to a focused team accountable for the goals and execution of the project.

18.3 Concluding Remarks

This chapter focused on major chemical conversion processes used in processing crude oil into useful fuels and petrochemical feedstocks. The refining industry worldwide has gradually adapted to process heavier sour crude supplies and produce ultra low sulfur (ULS) gasoline and diesel fuels. Transportation fuels with ultra low sulfur content are needed to satisfy the demand of the automotive industry in reducing emissions of internal combustion engines. There is an increased demand for alkylate and isomerate streams in gasoline. This will increase its hydrogen content and enhance combustion, thereby reducing the levels of carbon dioxide emissions. To enhance their margins and reduce residual fuel oil production, refiners need to enhance and integrate their business with petrochemicals production and power generation. In the long run, the refinery will not just produce fuels, but also chemicals and electricity. Despite all energy alternatives, crude oil will remain one of the world's primary energy and fuel sources, retaining about 30% of world energy up to 2035.

19

Natural Gas Processing: Recovery, Separation, and Fractionation of NGL (Natural Gas Liquids)

Mazyad Al Khaldi

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Gas field processing is generally is carried out for two main objectives:

- 1. The necessity of removing impurities from gas (the topic of Chapter 16)
- 2. The desirability of increasing liquid recovery above that obtained by conventional separation

Natural gas processing, the topic of this chapter, comprises two consecutive operations: NGL recovery (extraction) and separation from the bulk of gas followed by subsequent fractionation into desired products. The purpose of a fractionator's facility is simply to produce individual finished streams needed for market sales. Fractionation facilities play a significant role in gas plants. A case study involving the optimum recovery of butane using lean oil extraction is presented.

19.1 Technology Aspects

19.1.1 Introduction

As presented in Chapter 16, natural gas field processing and the removal of various components from it tend to involve the most complex and expensive processes. Natural gas leaving the field can have several components that
will require removal before the gas can be sold to a pipeline gas transmission company. All of the H_2S and most of the water vapor, CO_2 , and N_2 must be removed from the gas. Gas compression is often required during these various processing steps.

The condensable hydrocarbons heavier than methane which are recovered from natural gas are called (NGL). Associated gas usually produces a higher percentage of natural gas liquids. It is generally desirable to recover NGL present in gas in appreciable quantities. This normally includes the hydrocarbons known as C_3^+ . In some cases, ethane C_2 can be separated and sold as a petrochemical feed stock. NGL recovery is the first operation in gas processing, as explained in Chapter 16. To recover and separate NGL from a bulk of a gas stream would require a change in phase; that is, a new phase has to be developed for separation to take place by using one of the following:

- 1. An energy-separating agent: examples are refrigeration (cryogenic cooling) for partial or total liquefaction and fractionation.
- 2. A mass-separating agent: examples are adsorption and absorption (using selective hydrocarbons, 100 to 180 molecular weight).

The second operation is concerned with the fractionation of NGL product into specific cuts such as LPG (C_3/C_4) and natural gasoline. The fact that all of the field processes do not occur at or in the vicinity of the production operation does not change the plan of the system of gas processing and separation.

19.1.2 Why Field Processing?

The principal market for natural gas is achieved via transmission lines, which distribute it to different consuming centers, such as industrial, commercial, and domestic. Field processing operations are thus enforced to treat the natural gas in order to meet the requirements and specifications set by the gas transmission companies. The main objective is to simply obtain the natural gas as a main product free from impurities. Field processing units are economically justified by the increased liquid product (NGL) recovery above that obtained by conventional separation. A typical natural gas processing plant is shown in Figure 19.1.

19.1.3 Recovery and Separation of NGL

19.1.3.1 Options of Phase Change

To recover and separate NGL from a bulk of gas stream, a change in phase has to take place. In other words, a new phase has to be developed for separation to occur. Two distinctive options are in practice depending on using energy separating agents (ESAs) or mass separating agents (MSAs).



FIGURE 19.1

Description of a typical natural gas processing plant (source: Wikipedia).

1. Energy Separating Agent

The distillation process best illustrates a change in phase using ESA. To separate, for example, a mixture of alcohol and water, heat is applied. A vapor phase is formed in which alcohol is more concentrated, and then separated by condensation. This case of separation is expressed as follows:

A mixture of liquids + Heat \rightarrow Liquid + Vapor

For the case of NGL separation and recovery in a gas plant, removing heat (by refrigeration) will allow heavier components to condense; hence, a liquid phase is formed. This case is represented as follows:

A mixture of hydrocarbon vapor - Heat \rightarrow Liquid + Vapor

Partial liquefaction is carried out for a specific cut, whereas total liquefaction is done for the whole gas stream.

2. Mass Separating Agent

To separate NGL, a new phase is developed by using either a solid material in contact with the gas stream (adsorption) or a liquid in contact with the gas (absorption).

19.1.4 Parameters Controlling NGL Separation

A change in phase for NGL recovery and separation always involves control of one or more of the following three parameters:

- Operating pressure, P
- Operating temperature, T
- System composition or concentration, *x* and *y*

To obtain the right quantities of specific NGL constituents, a control of the relevant parameters has to be carried out.

First

For separation using ESA, pressure is maintained by direct control. Temperature, on the other hand, is reduced by refrigeration using one of the following techniques:

- (a) Compression refrigeration
- (b) Cryogenic separation; expansion across a turbine
- (c) Cryogenic separation; expansion across a valve

In the cryogenic cooling process to recover NGL, gas is cooled to very low temperature (-100 to -120° F) by adiabatic expansion of the gas mixture by turbo expanders. The water and acid gases are removed before chilling the gas to avoid ice formation. After chilling, the gas is sent to a demethanizer to separate methane from NGL.

Second

For separation using MSA, a control in the composition or the concentration of the hydrocarbons to be recovered (NGL); *y* and *x* are obtained by using adsorption or absorption methods.

Adsorption provides a new surface area, through the solid material, which entrains or adsorbs the components to be recovered and separated as NGL. Thus, the components desired as liquid are deposited on the surface of the selected solid and then regenerated off in a high concentration; hence, their condensation efficiency is enhanced. About 10% to 15% of the feed is recovered as liquid. Adsorption is defined as a concentration (or composition) control process that precedes condensation. Therefore, refrigeration methods may be coupled with adsorption to bring in condensation and liquid recovery.

Absorption, on the other hand, presents a similar function of providing a surface or contact area of the liquid-gas interface. The efficiency of condensation, and hence NGL recovery, is a function of *P*, *T*, gas and oil flow rates, and contact time. Again, absorption could be coupled with refrigeration to enhance condensation.

In the lean oil extraction method, the treated gas is cooled by heat exchange with liquid propane and then washed with a cold hydrocarbon liquid, which dissolves most of the condensable hydrocarbons. The uncondensed gas is dry natural gas and contains mainly methane with small amounts of ethane and other heavier hydrocarbons. The condensed hydrocarbons or natural gas liquids (NGL) are stripped from the rich solvent, which is recycled back to the process.

In summary, proper design of a system implies the use of optimum levels of all operating factors plus the availability of sufficient area of contact for mass and heat transfer between phases.

19.1.5 Fractionation of NGL

Due to their added value, heavier hydrocarbons are often extracted from natural gas and fractionated using several tailor-made processing steps. In general, and in gas plants in particular, fractionating plants have common operating goals:

- Production of on-specification products
- Control of impurities in valuable products (either top or bottom)
- Control in fuel consumption

The goals for the tasks for system design of a fractionating facility are as follows:

- Fundamental knowledge of the process or processes selected to carry out the separation, and in particular, distillation
- Guidelines regarding sequence of separation (i.e., synthesis of separation sequences)

NGL are normally fractionated into three streams:

- An ethane-rich stream used for producing ethylene
- Liquefied petroleum gas (LPG). It is a propane-butane mixture and is important feedstock for olefin plants.
- Natural gasoline

Natural gas liquids may contain significant amounts of cyclohexane.

19.1.6 Shale Gas

Conventional gas reservoirs are areas where gas has been "trapped." After natural gas is formed, the earth's pressure often pushes the gas upward through tiny holes and fractures in rock until it reaches a layer of impermeable rock where the gas becomes trapped. This gas is relatively easy to extract, as it will naturally flow out of the reservoir when a well is drilled. Unconventional gas occurs in formations where the permeability is so low that gas cannot easily flow (e.g., tight sands), or where the gas is tightly adsorbed (attached) to the rock (e.g., coal-bed methane). Gas shales often include both scenarios—the fine-grained rock has low permeability and gas is adsorbed to clay particles. The pore spaces in shales are typically not large enough for even tiny methane molecules to flow through easily. Consequently, gas production in commercial quantities requires fractures to provide permeability.

Shale gas is defined as natural gas from shale formations (i.e., natural gas trapped within shale, fine-grained sedimentary rocks, formations). Shale has low matrix permeability to allow significant fluid flow to the wellbore, therefore commercial production requires mechanically increasing permeability.

Shale gas reserves have been known for a long time, but natural fracture technology used earlier was uneconomical to produce shale gas. Recent developments in horizontal drilling and hydraulic fracturing (called fracking) made it viable. Mitchell energy, a Texas gas company, first achieved economical shale gas fracture in 1998. Shale gas is currently in an evolutionary stage and so far is largely confined to North America. The complete technology and economic factors are yet to mature. Several high-profile shale gas drilling efforts in Europe have already failed.

Shale gas costs more to produce than NG from conventional wells. The high cost is mainly due to the expense of massive hydraulic fracturing treatments required to produce shale gas and horizontal drilling. Drilling a vertical and horizontal well cost about \$1 million and \$4 million, respectively. The huge requirement of water for hydraulic fracking and then the wastewater treatment are major cost inhibitors. Overall, addressing environmental concerns associated with shale gas hugely adds to its cost. Shale gas production may be feasible only in those regions where energy/NG prices are high. The shale gas production cost in the United States is estimated to be between \$4 and \$7 per MMBTU, but it is termed as "foggy economics" since all factors were not considered. Earlier it was thought that shale gas will produce less greenhouse gases, but scientists have recently concluded otherwise and opine that it will accelerate global warming. Shale gas production requires large amounts of water and chemicals added to it to facilitate an underground fracturing process that releases gas. A maximum of 70% of used water is recovered and the rest remains underground which can lead to contamination. Significant use of water for shale gas production may affect the availability of water for other uses and can affect aquatic habitat. The treatment of a large amount of recovered wastewater before re-use or disposal is an important and challenging issue. There is some evidence of groundwater contamination in areas of fracking. The environmental impacts of shale gas production are therefore challenging but still considered to be manageable.

So far shale gas is confined mostly to North America. There is little drilling progress in China, Australia, and Poland. In other countries, it is still in the pilot stages. Canada has huge shale gas reserves but exploration is restricted due to strict environmental regulations and related issues. In the United States, BP predicted NG self-sufficiency and NG share of total energy consumption to double to 40% with 4% anticipated annual growth in shale gas production by 2030. EIA, however, slashed BP shale gas forecast reserves by 41% in January 2012. The energy demand (dominated by oil) will still grow in the next two decades by 39%, but most of the growth in demand will be from Asian countries, especially China and India. In Saudi Arabia, evaluation of shale gas reserves is in progress and production may start in 2020 but low NG price remains a major issue in developing the prospects.

19.2 Economic Evaluation of Selected Problems

Case 19.1: Recovery of Butane Using Lean Oil Extraction

Associated natural gas is passed through an absorption unit to recover heavier hydrocarbons (butane plus), which can be sold for a value of \$7.5/gal. Calculations show that the minimum total cost for the recovery and the extraction of the butanes in the plant is estimated to be \$1.2/gal of *butane* recovered. Other additional costs for processing the absorbing oil used in the recovery are estimated to be \$27/million gal of the lean oil circulated.

The engineering group in the plant developed the following empirical relationship for the rate (R) of the absorber oil used as a function of the rate of butane produced (P):

R, millions of gal/hr = 0.004 P1.3, where *P* is in gal/hr

- 1. Compute the optimum butane recovery P_{o} and the optimum circulating oil rate R_{o} applicable to this plant
- 2. What is the value of *P* at which the process of recovery breaks even?

SOLUTION

Profit = Income - Expenses

Income = 7.5 \$/gal * P gal/hr = 7.5 P \$/hr

Expenses = 1.2 \$/m gal* P gal/hr + 27 \$/m gal* R

= 1.2 P + 27 (0.004 P1.3) = 1.2 P + 0.108 P1.3 \$/hr

Profit = 7.5 P - 1.2 P - 0.108 P^{1.3} = 6.3 P - 0.108 P^{1.3}

 $d/dp(profit) = 6.3 - 1.3(0.108) P^{0.3}$; setting this derivative equal to zero:

 $P_{opt} = [6.3/1.3(0.108)]^{1/0.3} = 320,647$ gal butane/hr

 $R_{opt} = 0.004(320,647)^{1.3} = 57,530$ million gal oil/hr

At the break-even point, profit = 0

6.3 P – 0.108 P^{1.3} = 0; hence, P^{0.3} = 6.3/0.108

 $P_{\rm B} = (6.3/0.108))^{1/03} = 768,777$ gal butane/hr

Case 19.2: The Problem of Finding the Optimum Diameter of an Absorption Tower (Discussion)

The tower must process a gas feed stream at a fixed rate to remove a soluble gas component by absorption in a liquid phase. Here we have the two scenarios:

- Increasing the diameter of the tower lowers the gas velocity in the bed, reducing the pressure drop, hence lowering the pumping costs of the feed gas, but a large-diameter tower is more costly to construct.
- Choosing a smaller diameter will cause flooding inside the column to occur, and liquid is carried up the gas stream, making the tower inoperative.

CONCLUSION

Some balance must be reached between the pumping costs and the construction costs in order to lower the total costs of operation. Also, it is not practical to construct a tower of extremely large diameter because of liquid distribution problems.

Apparently, there are constraints on the tower diameter. Solution is reached by optimization technique in order to minimize the total annual costs of operating the tower as a function of the tower diameter.

Total annual costs of operation = Capital cost of tower, depreciated over lifetime (\$/year) + Annual operating (pumping) costs (\$/year).

Reference to Chapter 10, "Optimization Techniques," is recommended for modeling this problem to get a solution.

20

Oil and Gas Transportation

M.A. Al-Sahlawi

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Oil and gas transportation is an essential component of everyday life. Transportation in this sector is crucial to our economy. Pipelines serve as a "midstream" function in the oil and gas value chains, hooking other parts in the value chain together. Most crude oil is transported by pipelines on land and by tankers across the seas. Railroad tank cars and tank trucks, on the other hand, remain in many parts of the world an important mode of transport for local consumer markets. Moving natural gas, however, requires a network of pipelines from the production wells to the processing plants and to the final consumers.

In this chapter different methods of transportation facilities are described, capital investment costs are reported, and economic and cost analyses are presented, since the cost of transportation is one of the major factors influencing oil and gas prices.

20.1 Introduction

Oil and natural gas are only rarely found near the points at which they are consumed. The main oil and gas deposits are located in emerging or developing countries. Once domestic demand is met, these countries export most of their hydrocarbon production to industrialized regions. Europe, North America, and East Asia have strict energy requirements but are not self-sufficient enough in terms of oil and gas supplies. As a result, transportation of crude oil and its products as well as natural gas is a significant part of the cost buildup from initial exploration and discovery through to the final user.

For the world oil industry, there are four basic kinds of transportation. In rough order of importance, they are:

- Tankers
- Pipelines
- Railroads
- Tank cars and tank trucks

Oil tankers are by a fairly wide margin the cheapest form of transportation on a barrels-per-kilometer basis, with pipelines second, railroad tank cars third, and tank trucks fourth. But which form of transport is technically or economically feasible depends mainly on geographic factors, such as the location of the markets to be served, the size of the market, and what kind of road or railroad facilities are available.

The choice of transportation facilities also depends partly on whether crude oil, natural gas, or oil products are involved. In general, it does not make much difference what kind of crude oil is put into a tanker or a pipeline, since the mixing of one type or another does not significantly affect the refining pattern to which the oil is subjected, but it can make a great deal of difference if oil products, which are often subject to very tight specifications, are mixed a great deal in their transportation systems. This means that product transportation systems have to be segregated to avoid contamination among the various products, which significantly affects the economics involved. With respect to natural gas, it is difficult to transport by tank trucks because of its low density, and it is expensive to transport by pipelines across oceans.

20.2 The Tanker Market

Oceangoing tankers account for the largest amount of worldwide oil movements on a volume basis. In 2005, oil tankers made up 40 percent of the world fleet in terms of deadweight tonnage. This is mainly due to the fact that the main oil producers in the world, and particularly in the Middle East, are far from, and have few if any land connections with, their natural markets. This also applies to Africa and the Far East. The natural markets involved are the United States, Western Europe, Japan, and China. In terms of tanker fleet capacity, Liberia and Western Europe seem to own and operate most of the world tanker fleet, as shown in Table 20.1.

In the beginning of 2011, the world fleet of oil tankers stood at 394 million DWT, as shown in Figure 20.1. The tanker fleet for crude oil constituted 2,240 vessels of 331 million DWT and its capacity is forecasted to grow by 2 percent annually.

Distribut		voria O		er rieet i	by Regi	on: will	ION DW	1, 1970-	-2007	
Region	1970 ª	% of World	1980 ^a	% of World	1988ª	% of World	2003 ^b	% of World	2007 ^b	% of World
United States	09.3	06.13	16.16	4.97	16.53	06.79	18.35	5.30	25.80	7.50
Europe	52.10	34.20	86.56	26.60	48.41	19.90	55.10	15.30	47.90	12.80
Asia (Japan)	15.20	10.10	30.05	09.25	14.68	06.03	51.39	13.90	63.70	16.40
Africa (Liberia)	37.40	24.65	100.27	30.87	56.12	23.10	42.30	11.44	46.40	12.20
Panama	5.50	03.63	12.17	03.75	20.70	08.50	37.80	10.12	52.80	13.10

TABLE 20.1

World

151.72

100

324.80

stribution of World Oil Tanker Fleet by Region: Million DWT 1970–2007 Di

Champness, M., and Jenkins, G., Oil Tanker Databook, Elsevier Applied Science, New York, 1985; World Tanker Fleet Review, John I. Jacobs, London, July–December 1988. WIth permission.

243.72

100

367.85

100

429.50

100

100

^b Compiled from UNCTAD Statistics (Trade), Geneva; Lloyds Register, Fairplay (Fleet ownership), London; U.S. Central Intelligence Agency and U.S. Maritime Administration Statistics; and R.S. Platou Economic Research, Oslo, Norway. With permission.

Over the past 50 years, tanker technology has experienced many changes. During World War II and for some years thereafter, the standard oceangoing tanker was the T-2, with a capacity of some 26,000 deadweight tons (DWT), in this case about 150,000 barrels. Because of the economies of scale and the very large increases in world oil demand between 1945 and 1975, the maximum size of tankers grew from 26,000 to over half a million DWT, a more than 20-fold increase.

Within this size category, there are five main size classes: Panamax, with up to 70 thousand DWT and 0 to 5 million barrels capacity; Aframax ranging from 70 to 120 thousand DWT with 0.750 million barrels capacity; Suezmax ranging



FIGURE 20.1

Oil tanker fleet, million DWT, 1976–2012.



FIGURE 20.2

Composition of the World Long Haul Tanker Fleet.

between 120 and 200 thousand DWT with a capacity of one million barrels; very large crude carrier (VLCC) ranging from 200 to 325 thousand DWT with 2 million barrels capacity; and finally (ULCC) ranging between 320 and 550 thousand DWT and four million barrels capacity. The composition of these vessels in terms of number of ships and capacity is presented in Figure 20.2.

More detailed features of VLCC and ULCC are shown in Table 20.2, while Table 20.3 shows the world oil tanker fleet tonnage of year 2010, by size categories. The noted 30-year increase from 1945 to 1975 in tanker size has apparently about run its course, at least for the medium-term future. There are two basic reasons for this. One is that the economies of scale increase only very slowly above the half-million DWT level. The other is that limitations on harbor depths and port facilities in many areas tend to make inconvenient or impossible the use of even the current large tankers.

A tanker market can be considered a competitive market where the tanker rate is determined by the interaction of supply and demand. The demand for tanker services is inelastic with respect to spot rate (price) and depends on the degree

comparison between videe an	u obce	
Features	VLCC	ULCC
MDWT	160-320	320-550
Draft, ft	65	75
Length, ft	1,145	1,240
Beam, ft	170	225
Cost \$ million		
Single-hutted	100	n/a
Double-hutted	120	n/a
Charter rates, \$ thousand/d	30-40	35–45

TABLE 20.2

Comparison between VLCC and ULCC

Source: Saudi Aramco, Engineering Services, Oil Pipelines: Spreading the Network, AramcoExpats, Dhahran, Saudi Arabia, http://www.aramcoexpats.com/articles/2006/10/oil-pipelines-spreading-the-network/, October 7, 2006. With permission.

TABLE 20.3

Existing Commercial Tanker Fleet, 2010

Group Size, DWT	Number of Vessels	Percent
1,000–2,000	278	14%
2,000-3,000	398	19%
3,000–5,000	677	33%
5,000-8,000	432	21%
8,000 and over	266	13%
Total	2051	100%

Source: United Nations Conference for Trading and Development, UNCTAD, Geneva, 2010. With permission.

of substitution among different modes of oil transportation. It also depends on the cost of transportation, and to a certain extent the elasticity of demand for oil and oil products. On the other hand, the supply responds slowly to short-run increases in demand, and thus can be represented by an inelastic supply curve.

Fluctuations in freight and charter rates are affected by shifts in market supply and demand. Figure 20.3 shows market equilibrium rate and tanker tonnage at point *E* where the demand intersects the supply.

With respect to chartering, tankers are usually chartered either short or long term. Short-term chartering rates are often used as a reference for longterm chartering rates. However, the world-scale system has been used since 1969 as a basis for freight and charter rates. World-scale rates can always be represented in dollars; usually the smaller-size tankers have the lowest dollar rates. For example, spot charter rates in world scale as well as in dollars for the months of February 2009 are given in Table 20.4.

World-scale rates also serve as an indication of the availability of different size tankers at each time world-scale rates are published. Furthermore, types



Tanker Services

FIGURE 20.3 Demand-and-supply relationship for tankers.

Source (Leading)	Discharging (Unloads)	Cargo	Cargo Size 1000 Barrels	World- Scale Rates	Freight Cost, \$/b
Caribbean	New York	Distillate	200	215	2.48
Northern Europe	New York	Distillate	200	158	2.91
Northern Europe	Houston	Crude oil	400	82	2.22
West Africa	Northern Europe	Crude oil	910	71	1.6
West Africa	Houston	Crude oil	910	74	2.27
Arabian Gulf	Houston	Crude oil	1,900	36	2.08
Arabian Gulf	Japan	Crude oil	1,750	48	1.61
Arabian Gulf	Northern Europe	Crude oil	1,900	36	1.51

TABLE 20.4

Freight Rates for Different Types and Sizes of Cargo between the Source and Discharge, February 2009

Source: Average data for February 2009 as published in *Oil & Gas Journal* from Drewery Shipping Consultant Ltd., www.petrostrategies.org. With permission.

and size of cargo with different distance will determine freight rates. The freight rates for most tanker sizes in 2010 performed on average 30 percent higher than in 2009, as shown in Table 20.5. It is noted that freight rates for all vessel types in the first quarter of 2011 have decreased by about 15 percent compared to the same period of 2010 but remained more than 20 percent higher than in first-quarter 2009.

The main loading points are Arabian Gulf, West Africa, the Mediterranean, the Caribbean, and Singapore, while the main discharging points are East of Asia, Southern Africa, North-West Europe, the Mediterranean, the Caribbean, and the East Coast of North America. However, new routes and variations in freight rates have emerged because of political and financial critical events that have affected the tanker market. For example, the current global economic crisis has been reflected in the supply and demand for tankers and has changed the freight rates.

The main reason this variation in tanker rates can be sustained in competitive markets is that many seaports are not deep enough to handle the very large tankers with their scale economies. The largest tankers in the fleet have drafts (the depth of their hulls fully loaded) of as much as 30 m. Dredging many harbors to those depths would be uneconomical, and in some cases the lengths of the ships involved are too great to give them room to maneuver in any but the largest harbors.

There is another important reason for the observed variation in tanker sizes and therefore in their basic costs. Some ships are used only for light oil product movements; these are called "clean" tankers, because it is possible without extensive cleaning to carry motor gasoline on one trip and heating

VICCUITCMathematicAndAn	Vessel Type		2009						201	0									20	1		
Control control Dec Jan Fol April and control contro contro control control control control contro control contro c	VLCC/ULCC	I														% Change						
Athenian	(200,000 dwt+)	Route	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep (Oct	Vov 1	Jec	2009/2010	Jan	Feb	Mar	Apr	May	Jun
Calification Analysis Analysis Analysis Analysis Analysis Analysis of Korea analysis ananysis ananysis		Arabian	56	104	71	84	90	72	95	58	51	48	47	75	61	8.9	48	74	63	50	51	54
Arabian 53 88 76 76 91 53 84 55 50 45 57 50 55 60 49 54 offores adifeturity 3 70 - 57 60 49 49 54 Arabian 35 65 52 56 53 63 43 42 57 63 37 50 35 36 37 Arabian 35 65 52 56 53 63 43 36 52 36 37 36 37 37 37 37 37 35 Califeurye 1		Gulf-Japan																				
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of Korea Arbitan 3 7 3 4 2 4 4 5 6 3 4 3 6 3 4 3 4 3 5 4 3 4 3 5		Gulf-Republic																				
Arabian 34 70 - 57 66 34 42 42 57 68 34 57 6 34 57 5 53 63 53 63 53 63 53 53 65 53 53 63 53 53 64 36 29 37 42 38 33 34 Caribbean/ East Coast of Curibbean/ East Coast of North America 0 0 0 39 35 43 36 37 38 37 39 34 37 39 34 37 34 37 34 34 </td <td></td> <td>of Korea</td> <td></td>		of Korea																				
Culf-Europe Culf-Europe Carbban/ S 55 56 58 53 63 43 36 29 37 49 39 37 31		Arabian	34	70	I	57	99	52	58	42	42	40	34	42	57	6.8	34	37	I	38	38	43
Arabian 35 65 35 56 5 7 4 5 7 4 5 7 4 5 7 4 5 <th< td=""><td></td><td>Gulf-Europe</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></th<>		Gulf-Europe																				
Gulf- Caribbean/ EastCoast of North America Curbbean/ EastCoast of North America S Arabian 0 0 89 80 52 52 53		Arabian	35	65	52	56	58	53	63	48	39	35	30	44	36	2.9	32	37	42	38	37	39
Caribear/ East Coast of North America 0 0 0 9 89 80 52 52 52 66 52 Arabian 0 0 0 89 80 52 56 52 66 52 66 52 53 107 83 84 5 Suannu (100,000-160,000 dw) Mest 77 127 100 104 114 125 110 85 78 66 532 63 75 107 83 84 5 Africa-North- West Europe 73 114 97 98 113 103 73 73 114 66 73 66 73 73 107 83 5 75 75 75 75 75 75 75 75 75 76 75 76 75 76 75 76 75 76 75 75 75 76 75 76 75 76 75 76 75 76 75 76 75 76 75 76 75 76		Gulf-																				
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West Europe West Europe 73 114 97 98 113 103 41.1 60 72 101 79 81 66 Meal 73 114 97 98 112 118 103 73 74 65 78 81.1 60 72 101 79 81 66 Africa- Caribbean/ East Coast of North America A 103 115 10 129 81 7 91 113 36.1 71 82 130 86 80 74 74		Africa-North-																				
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Caribbean/ East Coast of North America Mediterranean- 63 127 103 115 110 129 102 96 84 72 97 101 113 36.1 71 82 130 86 80 74 Mediterranean		Africa-																				
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Mediterranean- 63 127 103 115 110 129 102 96 84 72 97 101 113 36.1 71 82 130 86 80 74 Mediterranean		North America																				
Mediterranean		Mediterranean-	63	127	103	115	110	129	102	96	84	72	97	101	113	36.1	71	82	130	86	80	74
		Mediterranean																				

Clean and Dirty Spot Rates on Major Tanker Routes for Various Sizes of Vessels

TABLE 20.5

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1 2	

Clean and Dirty Spot Rates on Major Tanker Routes for Various Sizes of Vessels (Continued)

Vessel Type		2009						201	0						•			201			
VLCC/ULCC (200.000 dwt+)	Route	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	% Change 2009/2010	Jan	Feb	Mar	Apr	May] un
Aframax (70,000-	-100,0(X) dwt)																				
	North-West	115	137	113	126	116	141	100	108	107	90	103	94	162	40.9	88	97	122	95	66	94
	Europe-North- West Europe																				
	North-West	100	135	117	110	I	153	104	103	115	I	I	85	120	20.0	131	06	135	85	06	84
	Europe-																				
	Caribbean/ East Coast of																				
	North America																				
	Caribbean-	112	173	146	127	123	167	131	137	115	66	98	127	146	30.4	125	98	125	123	104	98
	Caribbean/																				
	East Coast of																				
	North America																				
	Mediterranean-	117	124	95	135	114	160	110	108	107	87	112	92	138	17.9	75	97	122	95	66	94
	Mediterranean																				
	Mediterranean-	108	121	92	119	110	151	102	91	102	111	16.8	86	87	110	115	66	98			
	North-West																				
	Europe																				
	Indonesia-East Asia	95	136	118	116	66	127	114	111	98	92	91	102	111	16.8	88	87	110	115	66	96
Handy Size (less	than 50,000 dwt)																				
	Mediterranean-	120	I	164	130	158	173		146	139	129	132	126	168	40.0	140	116	134	155	138	130
	Mediterranean																				

																		1
116				126					111		124		152				177	
133				139					129		145		171					
139				155					123		142		191				185	
147				174					105		122		190				159	
111				105					98		111		120				135	
134				155					107		119		133				139	
31.5				72.4					12.6		5.8		59.6				22.2	
146				200					125		126		158				193	
121				119					66		120		133				165	
118				117					101		110		135				183	
119				112					130		141		129					
131				138					144		161		135				155	
138				142					124		128		169				161	
145				129					112		123		127				240	
161				163					124		143		119				215	
145				146					106		126		137				143	
139				151					118		124		159				144	
183				181					123		139		139				156	
171				176					140		151		149				145	
111				116					111		121		66				158	
Mediterranean-	Caribbean/	East Coast of	North America	Caribbean-East	Coast of North	America/Gulf	of Mexico	ar	Arabian	Gulf-Japan	Arabian	Gulf-Japan	Caribbean/East	Coast of North	America/Gulf	of Mexico	Singapore-East	Asia
								All Clean Tanke	70,000-80,000	dwt	50,000-60,000	dwt	35,000-60,000	dwt			25,000–35,000	dwt

Source: UN Conference for Trading and Development, UNCTAD, Review of Maritime Transport, Geneva, 2011. With permission.

oil or diesel fuel on the next. This gives such ships considerable flexibility to react to market forces in particular areas, such as the shift in the U.S. market from high motor gasoline demand in the summer to high heating oil demand in the winter. By contrast, crude oil carriers are described as "dirty" ships because of the extensive and expensive scrubbing of their tanks that would be necessary if the vessel were to be cleaned up to carry light oil products.

The distinction between clean and dirty tankers is an important one. Not only does it determine to a large extent the size of ships that can be economically used in particular markets; it also affects the ability of source-oriented refineries to compete with consuming area refineries in the selling of light products. If the latter can use dirty carriers in their cost buildup, and the former are burdened with clean ship costs to move their products to market, the differential freights can have a very important impact on profitability of refinery operations.

Some oil producers, such as Kuwait, achieve some light product transportation cost savings by shipping both crude and light products to such major consuming countries as Japan in the same tanker. Since large ships are divided into several compartments, partly for safety in the event of an accident, and partly to lend stability from sloshing in heavy waves, it is sometimes feasible to permanently commit some of the compartments to clean products and thus to take advantage of the overall large tanker's economics of scale. This is not, however, normal shipping practice.

In addition to ocean-going tankers, there is a substantial waterborne traffic in barges, particularly on the major rivers in industrial countries. Some barges carry crude oil from ocean ports such as Rotterdam up the Rhine River and New Orleans up the Mississippi to inland refineries, but most of such traffic is involved with oil product movements to bulk terminals upstream. Barges offer more flexibility than do pipelines and are less costly where rivers are deep enough to handle them, though on a ton/kilometer basis their generally smaller sizes make them less economical than oceangoing tankers. Some consideration has been given to building very large barges for ocean traffic, but control problems in bad weather have presented enough danger to discourage such a development.

20.3 Tanker Planning and Scheduling

Planning and scheduling are required to ensure continuous supply of oil and gas according to the planned production rates. This means that an optimal solution for a set of complex and interrelated operational problems which involve cargo loading and unloading processes should be reached. Terminal facilities for efficient loading of tankers, especially the larger ones and the supertankers, are needed in major crude-oil shipping ports. Planning ship arrivals and loading these ships within a minimum amount of time, since waiting time involves money expenditures, is the main objective of an export terminal of an oil company. It is most important to keep the tankers moving. The importance of this can best be explained by example.

Example 20.1

Matching Terminal Facilities to Liftings

Assume the following facts relative to the programming of ships through basic terminal facilities for a 160,000-dwt maximum at the terminal:

Liftings are 1,000,000 bbl/day, or 133,333 tons. Production in the oil field is 1,500,000 bbl/day. Tankage capacity is 6,000,000 bbl. Number of berths at the terminal is 1. Number of 100,000 bbl/hr loading systems is 1. Loading of ships is from 40,000 to 100,000 bbl/hr, depending on the size of the tanker, but straight liftings take a maximum of 10 hr.

With the above facts in mind, assume the following steps on arrival and departure from the terminal. (In actual practice, this computation can undoubtedly be performed with the aid of a computer, with which a more precise program could be put together.) Also, assume these steps with "times" in hours for each ship to be loaded.

Queue awaiting cargo	Tankage \rightarrow Lost production when tanks are full Production
Queue awaiting (one berth)	4.5 hr estimated
Mooring	1.0 hr estimated
↓ Deballasting	3.5 hr estimated
↓ Minimum loading average	10.0 hr estimated
↓ Documenting, tests	1.5 hr estimated
↓ Unmooring	0.5 hr estimated

In case no computer is available, the next step after receiving notification of orders is to program a list of tanker arrivals by date for cargo loading, bearing in mind that a minimum amount of awaiting cargo time per tanker is most desirable.

Table 20.6 shows how 45 ships might be programmed for a typical January, using the following data. (Usually this programming is done by computer.)

Date	Time, hr	Ship Size, dwt	Liftings, Tons
Jan. 1	4.5 10.0 2.0	105,000	100,000
Jan. 2	4.5 10.0 $\begin{cases} 3 \text{ hr} \end{cases}$	105,000	33,333
	[7 hr 2.0 4.5		70,000
Jan. 3	10.0 $2.0 \begin{cases} 0.5 \text{ hr} \\ 1.5 \text{ hr} \end{cases}$	65,000	63,333
	4.5 10.0 2.0	125,000	123,333
Jan. 4	4.5 $10.0 \begin{cases} 1.5 \text{ hr} \\ 8.5 \text{ hr} \end{cases}$	70,000	10,000 56,667
Jan. 5	2.0 4.5 10.0 9.0 hr	85,000	76,667
	10.0 hr 2.0 4.5	130.000	7,667
Jan. 6	2.0 4.5 10.0 2.0	90,000	88,333
Jan. 7	$ \begin{array}{c} 4.5 \\ 7.5 \text{ hr} \\ 2.5 \text{ hr} \end{array} $	65,000	45,000 15,000
	2.0 4.5 10.0 2.0	120,000	118,333

TABLE 20.6

Programming Tanker Distribution at Terminal

Programming Tanker Distribution at Terminal

Date	Time, hr	Ship Size, dwt	Liftings, Tons
Jan. 8	[3.0 hr		
	4.5 { 1.5 hr		
	10.0	58,000	55,333
	2.0	,	,
	4.5		
Jan. 9	[6.0 hr	130,000	78,000
	10.0		50.000
	[4.0 ftr		50,000
	2.0		
	4.5		
	10.0	85,000	81,333
	2.0		
Jan. 10	$45 \int 1.5 hr$		
	4.5 3.0 hr		
	10.0	65,000	61,333
	2.0	,	,
	4.5	155,000	72,000
Jan. 11	4.5 hr		
	10.0		80.000
	[0.011		80,000
	2.0		
	4.5		
	10.0	55,000	53,333
Jan. 12	2.0		
	4.5		
	10.0	115,000	114,000
	2.0		
	4.5		
Jan. 13	10.0 $\int 3 hr$	70,000	19,333
	10.0 7 hr		46,667
	2.0		
	4.5		
	10.0	90,000	87,000
			(Continued)

Date	Time, hr	Ship Size, dwt	Liftings, Tons
Jan. 14	$20\int 0.5 \text{ hr}$		
	$\left[1.5 \text{ hr}\right]$		
	10.0	120,000	118,333
	2.0		
Jap 15	4.5	100.000	15 000
Jan. 15	1.5 hr	100,000	15,000
	[8.5 hr		85,000
	2.0		
	4.5		
Jan. 16	$10.0 \int 9 hr$	55,000	46,000
	10.0 1 hr		5,333
	2.0		
	10.0	130,000	128,000
	2.0		
Jan. 17	4.5		
	10.0	60,000	58,333
	4.5		
Jan. 18	7.5 hr	100,000	75,000
	10.0 { 2.5 hr		25,000
			,
	2.0		
	4.5 10.0	110,000	108,333
	2.0		
Jan. 19	$4 = \int 3.0 hr$		
	$\left\{1.5 \text{ hr}\right\}$		
	10.0	105,000	100,000
	2.0	60.000	33 333
Jan. 20	1.0	00,000	55,555
	$10.0 \begin{cases} 0.0 \text{ m} \\ 4.0 \text{ h} \end{cases}$		
	[4.0 nr		22,000

Programming Tanker Distribution at Terminal

Programming Tanker Distribution at Terminal

Date	Time, hr	Ship Size, dwt	Liftings, Tons
	2.0 4.5 10.0 2.0	115,000	111,333
Jan. 21	$4.5 \begin{cases} 1.5 \text{ hr} \\ 3.0 \text{ hr} \end{cases}$	80,000	76,000
Jan. 22	$ 10.0 \\ 2.0 \\ 4.5 \\ 10.0 \\ \begin{cases} 4.5 \text{ hr} \\ 5.5 \text{ hr} \end{cases} $	130,000	57,333 76,000
Ian. 23	2.0 4.5 10.0 2.0	60,000	57,333
,	4.5 10.0 2.0	80,000	76,000
Jan. 24	$10.0 \begin{cases} 3.0 \text{ hr} \\ 7.0 \text{ hr} \end{cases}$	100,000	57,333 40,000
Jan. 25	$2.0 \\ 4.5 \\ 10.0 \\ 2.0 \begin{cases} 0.5 \text{ hr} \\ 1.5 \text{ hr} \end{cases}$	95,000	93,333
	4.5 10.0 2.0 4.5	110,000	105,333
Jan. 26	$10.0 \begin{cases} 1.5 \text{ hr} \\ 8.5 \text{ hr} \end{cases}$	100,000	28,000 68,333
	2.0 4.5		

(Continued)

ie, hr	Ship Size, dwt	Liftings, Tons
9.0 hr	75,000	65,000
1.0 hr		7,333
2.0		
4.5		
0.0	130,000	126,000
2.0		
4.5		
0.0	80,000	76,300
2.0		
4.5 	75.000	57 033
7.5 hr	75,000	57,055
2.5 hr		15,900
2.0		
4.5		
0.0	120,000	117,433
2.0		
3.0 hr		
1.5 hr		
0.0	70,000	67,250
2.0	115 000	< c 00 0
4.5	115,000	66,083
6.0 hr		
4.0 hr		45,920
2.0		
4.5		
0.0	90,000	87,413
2.0		
1.5 hr		
3.0 hr		
	<pre>9.0 hr 1.0 hr 2.0 4.5 0.0 2.0 4.5 0.0 2.0 4.5 7.5 hr 2.5 hr 2.0 4.5 0.0 2.0 4.5 7.5 hr 2.0 4.5 0.0 2.0 1.5 hr 3.0 hr 3.0 hr 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5</pre>	te, nr Ship Size, dwt 9.0 hr 75,000 1.0 hr 2.0 4.5 30,000 2.0 4.5 0.0 130,000 2.0 4.5 4.5 0.0 0.0 80,000 2.0 4.5 4.5 75,000 2.0 4.5 7.5 hr 75,000 2.0 4.5 4.5 120,000 2.0 4.5 3.0 hr 120,000 2.0 4.5 1.5 hr 0.0 2.0 4.5 4.5 115,000 6.0 hr 4.5 4.5 90,000 2.0 4.5 4.5 90,000 2.0 4.5 4.5 90,000 2.0 4.5 4.5 90,000 2.0 4.5 3.0 hr 4.5

Programming Tanker Distribution at Terminal

Preloading time	4.5 hr per tanker
Mooring, 1.0 hr	
Deballasting, 3.5 hr	
Average loading time Postloading time Documenting, 1.5 hr Unmooring, 0.5 hr	10.0 hr per tanker 2.0 hr per tanker

Total average cargo hours per ship 16.5 hr

Efficient programming of tankers reduces average awaiting cargo times in a terminal and minimizes the required investments for production capacity and tankage.

Since in this case production in the oil fields is 1,500,000 bbl/day and liftings at the terminal are 1,000,000 bbl/day, an excess of 500,000 bbl/day will build up each day. A single-point mooring (SPM) investment in the area of, say, \$10 million might be a possibility for the loading of two 200,000-dwt tankers every 3 days, which would take 3,000,000 bbl (100,000 tons × 7.5 bbl × 2) and still leave some capacity available for berth loading in case of emergencies.

The results of programming are summarized in Table 20.7 and the breakdown by size of ships loaded in January is given in Table 20.8, showing a total of 45 different-sized ships loaded during January with an average loading time of 10 hr, using one berth, and a loading system capacity of 100,000 bbl/hr. Liftings per 10 hr totaled 1,000,000 bbl of crude, or 133,333 dwt, and the loading system operated from 40,000 to 100,000 bbl/hr, depending on ship size.

If we assume that total average port time per tanker was 20.5 hr, average awaiting cargo time per tanker would have been 4 hr, since there are 16.5 cargo hours per ship.

Although this example is oversimplified, it does point out the important terminal considerations involved in planning terminal loadings, such as number of barrel liftings, oil production in the field, tanker capacity, number of berths involved, and loading system capacity.

Obviously, any change in one of the terminal factors, such as an added berth or an improved loading system, should bring increases in the other

TABLE 20.7

Programming Result for the Month of January

Total hours in a 31-day month	744
Total pre- and postloading time $(30 \times 6.5 \text{ hr})$	195
Total loading time $(30 \times 10 \text{ hr})$	300
Total waiting time $(30 \times 4 \text{ hr})$	120
Total hours accounted for on 45 ships (16.5 hr \times 45 ships)	732.5

Breakdown by Size of Ships Loaded in January			
Ship Size, dwt	Number Loaded		
40,000–50,000	0		
50,000–60,000	3		
60,000–75,000	10		
75,000–100,000	10		
100,000–150,000	21		
150,000–200,000	1		

TABLE 20.8

factors-otherwise there might be excess idle capacity. Also, any increase in oil production must include a corresponding increase in tank capacity lifting berths, and possibly loading system capacity. Thus, some important ratios that are worthwhile to watch are (1) tank capacity to oil production and to liftings; (2) production capacity to liftings (this ratio decreases as liftings increase), since this ratio affects waiting times; (3) number of berths to liftings; (4) waiting times to number of berths; and (5) waiting times to lifting level.

In an actual terminal situation, optimum average waiting time per ship could be determined by balancing the value of ship time spent awaiting cargoes against corresponding investments in terminal facilities. Thus, if average waiting time per ship is 12 hr at an average cost of \$4,000/hr to a shipper and 400 ships are loaded per year, total added cost to the shipper is $400 \times$ $12 \times $4,000$, or \$19,200,000. If an investment for added terminal per facilities involves, say, \$14,000,000, a savings of \$5,200,00 per year can be made by cutting down on the average waiting time.

An optimum combination of production capacity and tank capacity can also be determined. Investment required for additional production capacity is considered relative to the increase in investment required for equivalent tankage.

As to tanker scheduling, it has to go in parallel with tanker planning since it determines the needed number of different-sized tankers. The number of tankers can be calculated on the basis of the number of ton-days required as follows:

No. of tankers required = no. of tons-days needed divided by ton-days

per year per tanker

where

Total no. of ton-days needed = requirement per year \times no. of days

required by tanker for round trip

and

Ton-days per year per tanker = running time per year for a tanker ×

net weight (dwt) per tanker

Example 20.2

Assume a refinery required 20 million tons of crude oil a year. The tanker used is 270,000 dwt with net weight of 234,375 tons and 320 days running time per year. The travel time (round trip) from the source of supply of crude to the refinery is 30 days. The number of 270,000-dwt tankers needed per year can be found as follows:

Total no. of ton-days needed = 20,000,000 tons $\times 30$ days

= 600,000,000 ton-days

Ton-days per year per each 270,000-dwt tanker = $320 \text{ days} \times 234,375$

tons

= 75,000,000 ton-days

No. of tankers needed = 600,000/75,000,000

= 8 tankers

Determining tonnage requirement is not the only purpose of tanker scheduling; scheduling also involves matching terminal facilities to liftings in order to load the ships as quickly as possible.

20.4 Pipelines

20.4.1 General Review

Pipelines are the second most important form of oil and gas transportation. Their uses are more complex than the uses of tankers, which by their nature only move crude oil or products and gas from or to a rather limited number of points on the oceans or navigable rivers. Pipelines, however, are used for gathering systems in oil fields, for moving the crude oil thus collected to refineries or marine terminals, and often for moving refined products from refineries to local distribution points. They may also be used, like the old Trans-Arabian Pipeline (Tapline), to avoid long ocean voyages, or as in the Saudi lines to Yanbu on the Red Sea to avoid the possible strategic danger of closing the Straits of Hormuz.

Pipelines often cross national borders, and this can pose strategic and political problems. Saudi Arabia's Tapline has been closed for many years on political grounds, as have the Iraqi pipelines to Banias and Tripoli on the Mediterranean. In Western Europe there has been a persistent fear that the large natural gas pipelines from Russia may be used as a bargaining device in Russia-EU relations. For example, in 2009 Russian state-owned gas company Gaspron cut off gas supplies to Ukraine, the supplying point to Europe, in an attempt to propose a new pricing system that affected the supply to Europe. Transit fees, that is the charges that are levied by countries through which a pipeline passes, may be increased arbitrarily, thus altering the pipeline's economics substantially. Such factors as these must be taken very carefully into account when planning major pipeline investments. The within and between-countries pipelines in the world are shown in Figure 20.4. The United States presents 40 percent of the world's pipeline network, followed by Russia with 12 percent.

A word about natural gas pipelines may be appropriate here. In general, because of its chemical and physical properties and its low energy content per unit volume, natural gas can only be shipped by tanker or other surface transportation at very high cost because of the need to compress it at very high pressures or to cool it until it becomes a liquid as in liquefied natural gas (LNG). This means that almost all natural gas moves by pipelines, which generally confines such movements to contiguous land masses. (As in most rules, there are exceptions. Natural gas is delivered to Southern Europe by undersea pipelines from North Africa.) Within the industrialized nations of North America and Western Europe, natural gas pipelines move much larger quantities of energy than do oil pipelines. The main gas pipeline from Russia to Western Europe is 2 m in diameter, by far the largest size of any pipeline that has ever been built. Table 20.9 lists examples of recently constructed pipelines in different parts of the world.



FIGURE 20.4 World pipeline network.

TABLE 20.9

Name Length Cost Completion Capacity Remarks Oil (O) or Gas(G) Kms US\$bn Est. Year West to East G 4200 16.9 2004 18 bcm/yTarim-Shanghai Siberia-Pacific O 4200 18 Plan[i] 80 mtpy Taishet to Nakhodha Siberia-Korea G 4000 12 Plan 1.91 tcm Kovykta via PRC Sakhlain I O+G 12 2005 10 mbpd 220 Sakhalin II O+G 1670 10 2007 12 mtpy Asian Gas Grid 5000 6 2008 ? Indonesia-China G[ii] Kazakhstan O 3000 9.5 2011 25 Mt/y to Xinjiang TransSahara G 6000 7.5 Feas. 500 MMscfd Spain, WB TransAsean G 7 ? Thailand. 4500 2020 Philippines Iran-India G 2660 6 2010? 180 Mcm/d Via Kashmir Yamal-Nenets G Siberia, Belarus, 4000 6 2004 1 tcmy Poland, Germany Chad-Cameroon O 1070 4.2 2003 225 Kb/d Tangguh BP, G 5 2008 7 Mt/y Bintuni[iii] W. Papua LNG Blue Stream G 1220 3.4 2002 16 b.cu.m/ Russia, under Black Sea/ yr Ankara BTC O+G 1760 3 2005? 1 mbpd PNG/Australia G 3 Highlands' 3600 Plan 600 mcm/yKatubu Kazakhstan O Chev 1400 2.7 2001 600 Kb/d Black Sea Bolivia-Brazil G 3150 2.1 1999 30 Mcum.d Peru: Camisea G 715 2 2004 450 mcfd 2 1.2 md TransAfghan G Plan ADB, WB? 1800 Ex-Unocal Tsaidam G 1.9? 950 1999? To Gansu Xinjiang p 1.2 2006? 10 Mtpy To Lanzhou Sudan O 1.0?1999 0.240 mbpd 1600 Ecuador OCP O 503 1.5 850 kbpd 2003 Mozambique O 665 1.3 2004 Burma Yadana G 560 1.2 1998 525 mcfpd WAGP G 1033 0.6 2006 200+ mcfpd Iran/TurkeyG 2530 0.12001 1.5 Mcm/d Imports from Tanzama O 1710 0.1 1995? 8 Mb/d Dares-Salaam

Example of Recent Pipelines in the World

Source: U.S. Department of Energy, International Energy Outlook, 2005, Oil & Gas Journal. With permission.

Pipelines used for oil can be converted to natural gas, and vice versa, if the basic economic or strategic considerations make it appropriate. Similarly, if supply/demand conditions change, the direction of flow through pipelines can be reversed simply by turning around the pumping stations along the pipeline routes.

Unlike tankers, oil pipelines in the industrial world tend to be devoted to oil product movements, particularly for the light ends of the oil barrel. (Heavy fuel oil, in most climates, needs to be heated to flow efficiently, and consequently is not put into oil product pipelines.) The light ends of the barrel include motor gasoline, airline jet fuel, diesel fuel, home heating oil, and such blending components as gas oil and naphtha. They move through the pipes in batches, that is, blocs of a given volume, that are diverted into storage at various delivery points along the line. Given the current sophistication of pipeline technology, mixing of the products during transportation is minimal, so that even the tight product specifications that are required of airline jet fuel and motor gasoline are not compromised in the delivery of such products to their ultimate users.

Pipeline sizes (the inside diameter of the pipe) as well as the lengths involved are determined by market economics. Depending on what products are being delivered, and the market volumes demanded, pipelines vary in size from less than 10 cm to about 2 m. Again for reasons of economies of scale, pipeline systems tend to be overbuilt relative to current demand levels. Saudi Arabia, for example, has export capability through its pipelines and the ocean terminals with which they connect of some 15 million barrels per day of crude oil, even through the country's maximum historical production was about 12 million barrels in early 1980s during the Iraq/Iran War.

Market demand growth can, of course, outstrip a pipeline's basic ability to handle the demanded volumes. The first way to solve this problem is to increase the speed with which the oil passes along the line by adding pumping stations. But since pipeline friction increases geometrically with the speed of flow, at some point it becomes economical to add more pipes. This process is called "looping," and it consists of laying another pipeline alongside the existing one. Doing so involves several economic advantages: ease of access via existing roads, the ability to perform regular pipeline inspections of all the pipelines simultaneously, added flexibility if one of the pipelines is damaged, and the common use of pumping stations, to mention only a few.

In summary, pipelines serve a vital function in the transportation of both oil and natural gas. How pipelines fit into the world energy system is primarily a function of technical economics but also involves strategic and political considerations that go beyond simple pipeline economies.

20.4.2 Pipeline Economics

Economics of scale are the major element in pipeline economies. From a theoretical point of view, doubling the pipeline diameter will tend to increase



FIGURE 20.5

Cost throughput curves for different diameter pipes (after Stephenson).

the amount delivered by more than fourfold in a given period of time—other factors remaining constant. This implies that total cost might double while the cost per unit delivered would decline. In planning a pipeline system, a cost-throughput graphic relationship for different pipe diameters is shown in Figure 20.5.

Each curve in the figure shows the possible combinations between total cost per MCF delivered and MCF/diameter throughput for a certain diameter. For example, at throughput Q_1 , diameter D_2 gives the minimum total cost; however, for diameter D_2 , costs would be lower if throughput was increased to Q_2 .

At the construction and operation stages, pipeline economics involve two cost elements: initial capital cost and operating cost. Pipeline construction in costs constituted about 40% of total investment. Figure 20.6 shows total average construction cost per mile for natural gas pipelines with diameters



FIGURE 20.6 Total average construction costs (2005). (From Oil & Gas Journal, Databook).

ranging from 8 to 36 in. These costs vary among diameter classifications and are affected by geographic location, terrain, and pipe length. It has been noted that the cost per mile for a given diameter is lower where the pipeline is longer.

The major cost components of pipeline construction are material, labor, right-of-way (ROW) damages, and miscellaneous. In most cases, material and labor account for more than 65% of construction cost as shown in Figure 20.7. The investment distribution of constructing pipelines for both crude and oil product are similar.

Table 20.10 lists the various cost items that make up the total capital investment for crude oil and oil products investment distribution pipelines as gathered from U.S. major oil pipeline companies in 2005. Capital cost investment totaled \$2.62 billion for pipelines with 30 in. diameter while 12 in. pipelines total investment is the lowest with 0.32 billion.

As to the pipeline operating costs, they seem to vary among different sizes, uses, and locations. For example, total operating costs for the U.S. interstate natural gas pipelines were estimated by natural gas companies to be \$3.41 billion in 1985. A major part of operating costs is the cost of pipeline power consumption.



FIGURE 20.7 Oil pipeline investment distribution.

Size	Year	ROW	Material	Labor	Misc.	Total
8 in.	2004	239,860	84,651	599,280	591,276	1,515,065
	2000	20,099	51,065	385.845	137,789	594,479
12 in.	2004	595,684	212,495	1,740,0003	691,419	320,361
	2000	30,721	83,069	264,461	163,663	541,849
16 in.	2005	88,312	144,768	238,056	181,419	652,555
	2000	132,500	121,675	442,903	259,815	988,143
20 in.	2005	28,799	191,553	385,889	187,486	793,927
	2000	175,788	227,202	506,423	318,035	1,227,447
24 in.	2005	99,492	324,099	553,603	289,991	1,267,185
	2000	119,147	238,555	461,141	327,696	1,146,538
30 in.	2005	108,418	580,031	1,296,165	639,103	2,623,718
	2000	138,324	389,249	639,270	463,670	1,630,514
36 in.	2005	161,665	819,178	929,436	633,630	2,543,909
	2000	195,848	454,764	779,527	442,122	1,874,260

TABLE 20.10

Total Capital Investment in Liquid Pipelines, per Mile, 2000–2005

Source: Oil & Gas Journal, Databook, 2006. With permission.

Figure 20.8 explains the relationship between relative fuel cost and pipe diameter for different horsepowers where throughput and length of pipe are given. It is noted that the power cost per extra unit of throughput decreases as pipeline diameter increases.

Costs-per-mile figures may reveal more about cost trends of pipelines than aggregate costs. For gas projects in the United States the average land



FIGURE 20.8 Horsepower and fuel cost versus pipe diameter.

cost of pipelines was about \$1,312,164/mile for 2000–2001, compared with \$898,900/mile for 1995–1996. For offshore projects, the 2000–2001 figure was \$2,578,413/mile compared with \$1,611,818/mile for 1995–1996.

For many projects surveyed in the United States for the 1995 to 2001 period, breakdowns of the total cost per mile for land and offshore pipeline construction are shown in Table 20.11.

20.4.3 Piping and the Oil Fields

The value of a pipeline is in its economy of operation and in its consistency of operation. Today, there is great diversity in size of pipe used to carry crude oil, refined oil products, and natural gas ranging from 6 in. to as much as 36 in., and in some cases in the Middle East, even 48 in. piping. Lines are single or multiple, laid on top of the surface or buried in the ground, with booster pumps spaced anywhere from approximately every 25 miles to as much as 200 miles apart.

Pipeline costs vary, of course, with capacity, the character of the terrain which the lines will traverse, and the type of product the line is intended to carry, that is, its function. In general, there are three types of pipeline:

- 1. Those that run from the oil field to loading ports and are complementary to ocean transport. Without these, there would be no transport by tankers at all, so they are not competitive with transport by tankers.
- 2. Those long-distance pipelines that naturally shorten the alternative sea route. They can be competitive with ocean transport tankers if tanker rates are high. But in times of low tanker rates, such pipelines are not competitive with transport by tankers. A good example of this type of pipeline is Tapline, the 1,100-mile pipeline from Ras

TABLE 20.11

Estimated Pipeline Construction per Mile (Onshore)

	1995–1996	2000–2001	% Change
Land			
Material	\$274,210 (31%)	\$279,565 (21%)	2%
Labor	\$422,610 (47%)	\$571,719 (44%)	35%
Miscellaneous	\$154,012 (17%)	\$344,273 (26%)	124%
ROW and damages	\$48,075 (5%)	\$120,607 (9%)	151%
Total	\$898,907	\$1,316,164	38%
Offshore			
Material	\$684,604 (42%)	\$413,995 (16%)	-40%
Labor	\$527,619 (33%)	\$1,537,249 (60%)	191%
Miscellaneous	\$396,394 (25%)	\$510,271 (20%)	29%
ROW and damages	\$3,201 (0%)	\$116,898 (4%)	3552%
Total	\$1,611,818	\$2,578,413	60%

Source: Oil & Gas Journal, Pipeline Economics Survey, various issues. With permission.

Tanura in Saudi Arabia through four countries to Sidon, Lebanon. Transport by Tapline saves approximately 3,300 miles each way of ocean transport, and also saved Suez tolls when the Suez Canal was open. At this writing the Suez Canal has just reopened.

3. Those pipelines that transport oil from ports of discharge to inland refineries located in industrial areas, remote from a seaport. They can be competitive with domestic railroad and motor carriers. Examples of this type of pipeline are the pipelines of Rotterdam on the Rhine and Wilhelmshaven on the Ruhr.

Figure 20.9 illustrates the transport of oil by pipelines which run into millions of pipe feet and tonnage per oil field, as well as per refinery. From each individual wellhead in an oil field, the crude oil is collected in small-diameter gathering pipelines, which then converge on a collecting center. At the collecting center, the crude oil passes through gas separators, where gas is "liberated" from the crude oil. Usually, there are a number of collecting centers in different parts of the oil field.

From the collecting center, pipes of extremely large diameter lead the crude oil to a tank farm, a center or group of large circular enclosed storage tanks. From here, the crude is conveyed either to a refinery or to storage tanks at terminals for overseas delivery by sea tankers or long-distance pipeline.



FIGURE 20.9 Transport of oil by pipelines.
Large-diameter pipe is used where volume is large, where it is practical, and where long distances are involved, for the greater the diameter of the pipe, the less is the fall in pressure and thus the fewer pumping stations required. For example, the East-West pipeline of Saudi Aramco which is known as the Petroline is presented in Figure 20.10. The 1200 km and 48 in. pipeline transports nearly 50 percent of Aramco's total crude oil output to Saudi refineries on the Red Sea and more than 2.3 mbd crude export via Yanbu terminal.

Estimated average pipeline investment for any amount of piping involves millions of dollars. Size of pipe in diameter, length of the line in distances of miles and feet traveled, and type of pipe used all contribute to total investment in pipelines.

The following example illustrates how the immense costs of a pipeline could be recovered quickly by pumping crude oil.

Example 20.3

If the investment cost of pipeline in flat terrain is taken to be \$900,000/ mile and the pipeline is 1,000 miles, while the rate of pumping crude oil is assumed to be 500,000 bbl/day, calculate the total capital investment of the pipeline and compare this figure with the gross revenue per year received by selling the oil at \$80/bbl.

The capital investment of the pipeline = 900,000 \$/mile \times 1000 miles = \$9 \times 108 The annual revenue of sales (gross) = 500,000 bbl/day \times 350 day/yr \times 80 \$/bbl = \$ 1.4 \times 1010



FIGURE 20.10 East-West of Saudi Aramco.

Diameter, in.	Useful Range, Million Tons/Year ^a	Usual Pump Station Spacing, Miles
6	0.4–0.7	30–80
8	0.7–1.3	
10	1.3–2.5	40-100
12	2.0-4.1	
16	4.1-8.0	
20	7.0–13.0	
24	12.0-18.0	60–200
30	15.0-25.0	
36	20.0-40.0	

TABLE 20.12

Crude	Oil	Pipeline	Capacities
-------	-----	----------	------------

^aForty million tons or 300 million bbl.

As far as the crude oil pipeline capacities are concerned, each pipeline must be considered an individual problem. Generally speaking, the economic capacity of each of the various diameters of pipelines as well as the usual spacing between pump stations (booster pumps) lies between the limits given in Table 20.12.

When moving oil and oil products, such operating costs as the following, based on a per-ton mile basis, will be important:

- 1. Construction costs of pipeline and equipment
- 2. Amortization of investment
- 3. Interest on invested capital
- 4. Energy costs for operating pumping stations, etc.
- 5. Personnel and maintenance costs
- 6. Royalties to governments of countries crossed by the pipeline

Finally, these large sizes of pipe are costly to ship because the space they occupy relative to their weight is high, and therefore freight costs are increased. To reduce freight costs, it has become the practice today to design these large pipelines for equal quantities of two slightly different sizes of pipe, so that they can be "nested" for shipment; for example, one length of 20" pipe is placed inside each length of 22" pipe.

20.5 Economic Balance in Piping and Optimum Pipe Diameter

When pumping of a specified quantity of oil over a given distance is to be undertaken, a decision has to be made as to (1) whether to use a large-diameter pipe with a small pressure drop, or (2) whether to use a smaller-diameter pipe with a greater pressure drop. The first alternative involves a higher capital cost with lower running costs; the second, a lower capital cost with higher running costs specifically because of the need for more pumps. Therefore, it is necessary to arrive at an economic balance between the two alternatives. Unfortunately, there are no hard and fast rules or formulas to use; every case is different. Costs of actual pumping equipment undoubtedly must be considered, but the area in which the pipes will "run" is also important. For instance, to obtain the same pumping effort in the desert as opposed to a populated area could involve much higher costs in the form of providing outside services and even creating a small, self-contained township. In the flow of oil in pipes, the fixed charges are the cost of the pipe, all fittings, and installation. All these fixed costs can be related to pipe size to give an approximate mathematical expression for the sum of the fixed charges.

In the same way, direct costs, or variable costs, comprising mostly the costs of power for pressure drop plus costs of minor items such as repairs and maintenance, can be related to pipe size. For a given flow, the power cost decreases as the pipe size increases. Thus direct costs decrease with pipe size. And total costs, which include fixed charges, reach a minimum at some optimum pipe size. This factor can be expressed roughly in a series of simplified equations that express relations in terms of weight rate of flow and fluid density, then weight (or mass) rate of flow and annual cost per foot for most cases of turbulent flow.

To summarize, in choosing the inside diameter of pipe to be used, either in the oil field or in a refinery, selection should generally be based on costs of piping versus costs of pumping. Small-diameter pipe, which usually involves quicker drops in pressure than large-diameter pipe and therefore must be supplemented with more pumping equipment when laid for long distances, costs less than large-diameter pipe, but cost of pumping can add considerably to total cost of transferring a given amount of oil. Conversely, large-diameter pipe will have a fixed capital charge, even though pumping costs are minimized since natural pressure drops are less than with smalldiameter pipe. Thus, an economic balance is desirable.

Example 20.4

This is an example of the principle of economic balance as applied to piping involving two alternatives. One alternative is the use of a largediameter pipe with a small pressure drop; the other alternative is a small-diameter pipe with a greater pressure drop and more pumps. Pumps and pump room installation are considered part of the investment in pipelines.

Assume that the requirement is to transfer 100,000 bbl/day of crude oil for a distance of 200 miles by pipe. In order to arrive at the optimum conditions where total annual costs will be minimized; the fixed costs, or installation costs, and corresponding operating costs for the pipeline for different diameters must be determined and the optimization technique then applied. This is illustrated as follows:

First: Calculate the fixed charges (installation costs) of piping and pumps and their installation. For a distance of 200 miles and for such a quantity of oil, 100,000 bbl/day, the number of pump stations varies between two and three.

In order to convert the total fixed costs to an annual basis, a payout time has to be assumed. This is taken to be 5 years, plus 5% annual maintenance. Therefore, the annual "fixed charges" are 0.20 + 0.05 = 0.25% of the total fixed costs.

Second: Operating expenses should include the following:

- 1. Labor, supervision, and salaries
- 2. Electrical power consumed

Using the above data and taking into consideration the pressure drop (PD) for each diameter of pump, one can estimate the number of stations needed and the brake horsepower used in pumping the oil. The ultimate solution leading to the optimum diameter is found from the graph shown in Figure 20.11.

Mathematically speaking, one can obtain the economic pipe/diameter for a pipeline using the optimization techniques described earlier in Chapter 10.

Example 20.5

This example illustrates determination of the optimum pipe (D_{opt}) through optimization of the total annual cost. Assume the following formulas:

Annual operating cost = $F_1(1/D_{vive})$

Annual fixed costs = $F_2(D_{pipe})$

where F_1 and F_2 are some defined functions of the diameter D of the pipe. The total annual costs for transferring oil will be equal to $F_1 1/D_{vive} + F_2 D_{pive}$.



FIGURE 20.11 Optimum pipe diameter.

The optimum economic diameter of the pipeline is reached when the total annual costs are at the minimum—that is, taking the derivative of the total annual cost w.r.t. the pipe diameter, *D*.

Therefore,

$$d/dD_{pipe}[(\text{total costs}) = d/dD((F_1 \ 1/D_{pipe}) + F2(D))]$$

and letting this product equal zero, solving for the value of $D = D_{out}$.

To illustrate the principle of $D = D_{opt}$ in a simplified manner, take F_1 and F_2 as linear functions of some constants:

$$F_1(1/d) = a/D + b$$
 and $F_2(D) = cD + d$

where *a*, *b*, *c* are constants to be defined.

The total annual costs = l/D + b + CD + dand

$$d(T.C.)/dD = -a/D^2 + C = 0.$$

This gives

 $a/D^2 = c$

Hence,

$$D_{ont} = (a/c)^{1/2}$$

The exact equation for predicting D_{opt} for turbulent flow for incompressible fluids inside steel pipes of constant diameter is given by the equation:

$$D \text{ opt} = 2.2 \text{ W} 0.45 / \ell 0.32$$

where D > 1''; W are thousands of pounds mass flowing per hour; and ℓ is density, or lb-mass/ft³.

Then, to calculate D_{opt} , if we are considering the transfer of 500,000 bbl/ day of oil of an average API of 33° (with $\ell = 53.70$ lb/ft³) across a distance of 1,000 miles, we have:

First: Calculation for *D*_{opt}:

 $W = 500,000 \text{ bbl/day} \times 300 \text{ lb/bbl} \times 1/24 = 6.25 \text{ lb/hr}$ $D_{opt} = 2.2 (6.25)0.45/(53.7)0.32$ = 31 inches

Therefore 31 inches is the optimum economic pipe diameter in this particular case.

Second: Calculating the cost of pipeline (1000 mile, 31 inch):

Assuming the construction cost of the pipeline is \$900,000/ mile, the total costs will be \$900 million. If a pumping station is needed every 150 miles a total of about six stations for the 1000mile pipeline should be considered, Assuming the cost of each pump station is \$8 million, the total cost of using the pipeline will be:

- = \$ 900 million for the pipe plus \$48 million for 6 stations
- = \$ 948 million
- = \$ 950 million approximately

Then, if crude oil sells for \$80/bbl, it is readily found that about 12 million bbl will have to be moved through the line before the total investment of \$950 million is recovered (\$950,000,000 divided by \$80). This recovery is simplified, since maintenance and repair expenses plus cost of money invested are not considered.

20.6 Railroad Tank Cars

Before the development of pipeline systems, transportation of oil by railroad tank cars was by a wide margin the most important method of moving oil from its point of production through refining and to its point of final consumption. This dominance was initially a function of the fact that railroads were extensively developed in most areas at least a half century before the economic use of motor trucks and the road networks that were established to serve more local markets than could be reached by rail transport. The other factor that contributes to the importance of railroad tank cars in today's markets is that on a ton/kilometer basis, rail transport is generally between two and three times as efficient as oil and oil product movement by truck. This is partly because railroad tank cars are significantly larger than even the biggest tank trucks and thus enjoy greater economies of scale, and partly because each tank truck needs a driver while an entire trainload of perhaps a hundred cars requires only two or three employees. Roadbed costs also tend to be less, and required maintenance is not as expensive as the tank truck alternative requirements.

The relative economies of the three land-based transportation systems pipelines, railroad tank cars, and tank trucks—can be illustrated by the way Iraq moved its crude oil to world markets during the Iraq/Iran War. Being essentially barred from using tankers in the Gulf by Iran's control of the Shatt El-Arab waterway, and with its pipelines to the Mediterranean Sea blocked by political action by Syria, Iraq turned principally to a pipeline across Saudi Arabia to Yanbu on the Red Sea, secondarily to a rail link with Turkey, and finally to the most expensive mode of all, tank trucks by road to Turkey and to the Gulf of Aqaba through Jordan. With the war over, Iraq established limited tanker access through the Gulf, implemented an expansion of its pipelines across Saudi Arabia to Yanbu, discontinued its long-haul truck movements across Jordan, and phased out its truck movements to Turkey, in that order. Railroad tank cars remain in many parts of the world, in industrialized and developing countries, an important mode of transportation. Many small markets do not economically justify building pipelines to serve them but are still large enough and close enough to rail connections to make rail the main method of basic oil product transportation. This means that tank trucks only have to do short hauls to get the oil to its final consumers.

20.7 Tank Trucks

Tank trucks tend to be very much oriented to specific local consumer markets. All gasoline and diesel service stations, for example, are supplied by tank trucks, as are all home heating oil customers. Rail transportation systems are not flexible enough to reach many small or medium-sized consumers of even commercial and industrial oil products. Large fuel users, such as electric utilities or steel plants, are likely to be supplied by individual pipelines from local refineries, barges if they are on the waterfront, and railroad tank cars if they are both not available to water and too far away to justify a product pipeline. Heavy fuel oil is also sometimes too viscous to pump at ambient temperatures and thus requires heated delivery systems, whether pipelines, railroad tank cars, or tank trucks; this involves added capital and operating costs and is a significant factor in heavy fuel oil's competitive position with coal.

Tank trucks, because of their flexibility, are also involved fairly extensively on the crude oil supply side, particularly in North America but also in other countries where field size and flow rates do not justify pipeline gathering systems. Oil from small wells is pumped into small tanks at the well sites; these are regularly emptied and the oil trucked to the nearest refinery, rail connection, or pipeline access point. In the United States, for example, about 3% of total oil production, from well over half of the country's wells, is handled in this fashion.

The inefficiencies of this system, relative to the gathering costs of major oil fields, are such as to make such production barely inframarginal. This was why in the 1985 to 1986 decline in world oil prices about 500 barrels per day of U.S. producing capacity was shut down. Had the transport costs of bringing the output of many wells to market not been so high, it is likely that these cutbacks would have been substantially lower.

A significant exception to the generalization that most final consumers are served by tank trucks is the airline sector. Because of the volumes involved and the need to maintain product purity as well as consistent availability, most airports are served by pipelines from local refineries or distribution points. Again, relative economics are the dominant factor. But in these cases, the importance of assured supply and tight product specifications as to quality are enough to justify a market premium of perhaps U.S. 2 cents per gallon, or 85 cents per barrel. (Final delivery for the last few hundred meters, however, is by tank truck into the aircraft fuel tanks.)

20.8 Environment Impacts

Oil and gas transportation generates serious problems of land and marine pollution in forms of accidents, oil spills, and operational discharge. Monitoring and evaluating such environmental impacts have received great attention from policy and operation sides. The *Exxon Valdez* disaster of 1989 raised the awareness of environmental risks of maritime transport activities. It caused more than 11 million gallons of oil to leak from the vessel which took 3 years to clean and cost \$2.5 billion. Although oil spills are a definite source of marine and coastal pollution, industrial waste remains the major cause of ocean oil pollution. Tanker accidents contribute 5% and tanker operations account for 7%, and other shipping accounts for 14%. However, better operations and improved ship design have reduced the number of large spills. It is noted that the frequency of large spills has declined during the 1990s. Figure 20.12 shows accidental oil spills from tankers.

The improvements in tanker operations and strict regulations have reduced the frequency and amount of oil spills. Tankers used to discharge dirty ballast water (oil mixed with sea water) into the ocean. Now with improved designs of the tankers and legal obligations, many tankers have segregated



Thousand Tonnes

FIGURE 20.12 Accidental oil spills from tankers.

ballast tanks that separate oil from water. The environmental impacts are not restricted to marine settings but also occur on land when it comes to pipeline transportation of oil and gas. Petroleum industry damages to the environment occur at different stages from production to distribution including processing and refining. The effects of the damage are social and economic in terms of cleanup, prevention, and financial compensation in case of social harms.

The oil industry has developed information and techniques for precautions to avoid and manage the consequences of oil spills. As a precaution measure in transporting oil and gas by tankers, the oil industry performs a necessary ship vetting process. Such vetting arrangements will ensure that the tanker is meeting the necessary requirements of safe berthing and loading operation. However, in the event of an oil spill, a series of planned actions will be implemented. These actions start from spill collection and monitoring to cleanup of the sea and shoreline.

20.9 Summary

Moving oil from the wellhead, through the refining process, to the ultimate user of oil products involves a complex blend of oceangoing tankers, river barges, pipelines, and rail and road tank cars. Which form the mix takes in any particular case is a function of both geography and economics, with occasional political and strategic factors thrown in. Economics of scale are often important in determining which set of transport modes will be used. And all require a complex system of infrastructure: terminals, storage tanks, good roads, and railroad tracks and rolling stock. They also need to be flexible to accommodate both market growth and shifting relative product demand. Above all, basic economics are the primary shaper of the way transport systems develop. This is applicable to natural gas transportation from the gathering pipelines system to distribution through pipelines or LNG tankers.

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Appendix A: Conversion Factors

TABLE A.1

Alphabetical Conversion Tables

To Convert From	Do This		
Atmospheres to inches of mercury @32°F (Atm to inHg32	(atm) * 29.9213 = (inHg32)		
Atmospheres to inches of mercury @60°F (Atm to inHg60)	(atm) * 30.0058 = (inHg60)		
Atmospheres to millibars (atm to mb)	(atm) * 1013.25 = (mb)		
Atmospheres to pascals (atm to Pa)	(atm) * 101325 = (Pa)		
Atmospheres to pounds/square inch (atm to lb/in ²)	(atm) * 14.696 = (lb/in ²)		
Centimeters to feet (cm to ft)	(cm) * 0.032808399 = (ft)		
Centimeters to inches (cm to in)	(cm) * 0.39370079 = (in)		
Centimeters to meters (cm to m)	(cm) * 0.01 = (m)		
Centimeters to millimeters (cm to mm)	(cm) * 10 = (mm)		
Degrees to radians (deg to rad)	(deg) * 0.01745329 = (rad)		
Degrees Celsius to degrees Fahrenheit (C to F)	[(C) * 1.8] + 32 = (F)		
Degrees Celsius to degrees Kelvin (C to K)	(C) + 273.15 = (K)		
Degrees Celsius to degrees Rankine (C to R)	[(C) * 1.8] + 491.67 = (R)		
Degrees Fahrenheit to degrees Celsius (F to C)	[(F) - 32)] * 0.555556 = (C)		
Degrees Fahrenheit to degrees Kelvin (F to K)	[(F) * 0.555556] + 255.37 = (K)		
Degrees Fahrenheit to degrees Rankine (F to R)	(F) + 459.67 = (R)		
Degrees Kelvin to degrees Celsius (K to C)	(K) - 273.15 = (C)		
Degrees Kelvin to degrees Fahrenheit (K to F)	[(K) - 255.37] * 1.8 = (F)		
Degrees Kelvin to degrees Rankine (K to R)	(K) * 1.8 = (R)		
Degrees Rankine to degrees Celsius (R to C)	[(R) - 491.67] * 0.555556 = (C)		
Degrees Rankine to degrees Fahrenheit (R to F)	(R) - 459.67 = (F)		
Degrees Rankine to degrees Kelvin (R to K)	(R) * 0.555556 = (K)		
Feet to centimeters (ft to cm)	(ft) * 30.48 = (cm)		
Feet to meters (ft to m)	(ft) * 0.3048 = (ft to m)		
Feet to miles (ft to mi)	(ft) * 0.000189393 = (mi)		
Feet/minute to meters/second (ft/min to m/s)	(ft/min) * 0.00508 = (m/s)		
Feet/minute to miles/hour (ft/min to mph)	(ft/min) * 0.01136363 = (mph)		
Feet/second to kilometers/hour (ft/s to kph)	(ft/s) * 1.09728 = (kph)		
Feet/second to knots (ft/s to kt)	(ft/s) * 0.5924838 = (kt)		
Feet/second to meters/second (ft/s to m/s)	(ft/s) * 0.3048 = (m/s)		
Feet/second to miles/hour (ft/s to mph)	(ft/s) * 0.681818 = (mph)		
Grams/cubic centimeter to pounds/cubic foot (gm/cm ³ to lb/ft ³)	$(gm/cm^3) * 62.427961 = (lb/ft^3)$		

(Continued)

TABLE A.1 (Continued)

Alphabetical Conversion Tables

To Convert From	Do This		
Grams/cubic meter to pounds/cubic foot (gm/m ³ to lb/ft ³)	(gm/m^3) * 0.000062427961 = (lb/ft^3)		
Hectopascals to millibars (hPa to mb)	Nothing, they are equivalent units		
Inches to centimeters (in to cm)	(in) * 2.54 = (cm)		
Inches to millimeters (in to mm)	(in) * 25.4 = (mm)		
Inches of mercury @32°F to atmospheres (inHg32 to atm)	(inHg32) * 0.0334211 = (atm)		
Inches of mercury @32°F to millibars (inHg32 to mb)	(inHg32) * 33.8639 = (mb)		
Inches of mercury @32°F to pounds/square inch (inHg32 to lb/in²)	(inHg32) * 0.49115 = (lb/in ²)		
Inches of mercury @60°F to atmospheres (inHg60 to atm)	(inHg60) * 0.0333269 = (atm)		
Inches of mercury @60°F to millibars (inHg60 to mb)	(inHg60) * 33.7685 = (mb)		
Inches of mercury @60°F to pounds/square inch (inHg60 to lb/in ²)	$(inHg60) * 0.48977 = (lb/in^2)$		
Kilograms/cubic meters to pounds/cubic foot (kg/m ³ to lb/ft ³)	$(kg/m^3) * 0.062427961 = (lb/ft^3)$		
Kilograms/cubic meters to slugs/cubic foot (kg/m ³ to slug/ft ³)	$(kg/m^3) * 0.001940323 = (slug/ft^3)$		
Kilometers to meters (km to m)	(km) * 1000 = (m)		
Kilometers to miles (km to mi)	(km) * 0.62137119 = (mi)		
Kilometers to nautical miles (km to nmi)	(km) * 0.5399568 = (nmi)		
Kilometers/hour to feet/second (kph to ft/s)	(kph) * 0.91134 = (ft/s)		
Kilometers/hour to knots (kph to kt)	(kph) * 0.5399568 = (kt)		
Kilometers/hour to meters/second (kph to m/s)	(kph) * 0.277777 = (m/s)		
Kilometers/hour to miles/hour (kph to mph)	(kph) * 0.62137119 = (mph)		
Kilopascals to millibars (kPa to mb)	(kPa) * 10 = (mb)		
Knots to feet/second (kt to ft/s)	(kt) * 1.6878099 = (ft/s)		
Knots to kilometers/hour (kt to kph)	(kt) * 1.852 = (kph)		
Knots to meters/second (kt to m/s)	(kt) * 0.514444 = (m/s)		
Knots to miles/hour (kt to mph)	(kt) * 1.1507794 = (mph)		
Knots to nautical miles/hour (kt to nmph)	Nothing, they are equivalent units		
Langleys/minute to watts/square meter (ly/min to W/m ²	$(ly/min) * 698.339 = (W/m^2)$		
Watts/square meter to langleys/minute (W/m ² to ly/min)	$(W/m^2) * 0.00143197 = (ly/min)$		
Meters to centimeters (m to cm)	(m) * 100 = (cm)		
Meters to feet (m to ft)	(m) * 3.2808399 = (ft)		
Meters to kilometers (m to km)	(m) * 0.001 = (km)		
Meters to miles (m to mi)	(m) * 0.00062137119 = (mi)		
Meters/second to feet/minute (m/s to ft/min)	(m/s) * 196.85039 = (ft/min)		
Meters/second to feet/second (m/s to ft/s)	(m/s) * 3.2808399 = (ft/s)		

TABLE A.1 (Continued)

Alphabetical Conversion Tables

To Convert From	Do This		
Meters/second to kilometers/hour (m/s to kph)	(m/s) * 3.6 = (kph)		
Meters/second to knots (m/s to kt)	(m/s) * 1.943846 = (kt)		
Meters/second to miles/hour (m/s to mph)	(m/s) * 2.2369363 = (mph)		
Miles to feet (mi to ft)	(mi) * 5280 = (ft)		
Miles to kilometers (mi to km)	(mi) * 1.609344 = (km)		
Miles to meters (mi to m)	(mi) * 1609.344 = (m)		
Miles/hour to feet/minute (mph to ft/min)	(mph) * 88 = (ft/min)		
Miles/hour to feet/second (mph to ft/s)	(mph) * 1.466666 = (ft/s)		
Miles/hour to kilometers/hour (mph to kph)	(mph) * 1.609344 = (kph)		
Miles/hour to knots (mph to kt)	(mph) * 0.86897624 = (kt)		
Miles/hour to meters/second (mph to m/s)	(mph) * 0.44704 = (m/s)		
Millibars to atmospheres (mb to atm)	(mb) * 0.000986923 = (atm)		
Millibars to hectopascals (mb to hPa)	Nothing, they are equivalent units		
Millibars to inches of mercury @32°F (mb to inHg32)	(mb) * 0.02953 = (inHg32)		
Millibars to inches of mercury @60°F (mb to inHg60)	(mb) * 0.02961 = (inHg60)		
Millibars to kilopascals (mb to kPa)	(mb) * 0.1 = (kPa)		
Millibars to millimeters of mercury @32°F (mb to mmHg)	(mb) * 0.75006 = (mmHg)		
Millibars to millimeters of mercury @60°F (mb to mmHg)	(mb) * 0.75218 = (mmHg)		
Millibars to newtons/square meter (mb to N/m^2)	$(mb) * 100 = (N/m^2)$		
Millibars to pascals (mb to Pa)	(mb) * 100 = (Pa)		
Millibars to pounds/square foot (mb to lb/ft ²)	$(mb) * 2.088543 = (lb/ft^2)$		
Millibars to pounds/square inch (mb to lb/in ²)	$(mb) * 0.0145038 = (lb/in^2)$		
Millimeters to centimeters (mm to cm)	(mm) * 0.1 = (cm)		
Millimeters to inches (mm to in)	(mm) * 0.039370078 = (in)		
Millimeters of mercury @32°F to millibars (mmHg to mb)	(mmHg) * 1.33322 = (mb)		
Millimeters of mercury @60°F to millibars (mmHg to mb)	(mmHg) * 1.32947 = (mb)		
Nautical miles to kilometers (nmi to km)	(nmi) * 1.852 = (km)		
Nautical miles to statute miles (nmi to mi)	(nmi) * 1.1507794 = (mi)		
Nautical miles/hour to knots (nmph to kt)	Nothing, they are equivalent units		
Newtons/square meter to millibars (N/m ² to mb)	$(N/m^2) * 0.01 = (mb)$		
Pascals to atmospheres (Pa to atm)	(Pa) * 0.000009869 = (atm)		
Pascals to millibars (Pa to mb)	(Pa) * 0.01 = (mb)		
Pounds/cubic foot to grams/cubic centimeter (lb/ft ³ to gm/cm ³)	$(lb/ft^3) * 0.016018463 = (gm/cm^3)$		
Pounds/cubic foot to grams/cubic meter (lb/ft³ to gm/m³)	$(lb/ft^3) * 16018.46327 = (gm/m^3)$		

TABLE A.1 (CONTINUED)

Alphabetical Conversion Tables

To Convert From	Do This
Pounds/cubic foot to kilograms/cubic meter (lb/ft³ to kg/m³)	$(lb/ft^3) * 16.018463 = (kg/m^3)$
Pounds/square foot to millibars (lb/ft ² to mb)	$(lb/ft^2) * 0.478803 = (mb)$
Pounds/square inch to atmospheres (lb/in² to atm)	$(lb/in^2) * 0.068046 = (atm)$
Pounds/square inch to inches of mercury @32°F (lb/in² to inHg32)	$(lb/in^2) * 2.03602 = (inHg32)$
Pounds/square inch to inches of mercury @60°F (lb/in² to inHg60)	(lb/in ²) * 2.04177 = (inHg60)
Pounds/square inch to millibars (lb/in ² to mb)	$(lb/in^2) * 68.9474483 = (mb)$
Radians to degrees (rad to deg)	(rad) * 57.29577951 = (deg)
Slugs/cubic foot to kilograms/cubic meter (slug/ft³ to kg/m³)	$(slug/ft^3) * 515.378 = (kg/m^3)$
Statute miles to nautical miles (mi to nmi)	(mi) * 0.86897624 = (nmi)

Note: Follow simple formulas to make **conversions** in speed, pressure, and various units; for example, MPH to M/S or C to F.

Source: CSGNetwork, Conversion Factors Table, www.csgnetwork.com/convfactorstable.html.

TABLE A.2

Metric Tons to Barrels (Crude Oil)

	(
Abu Dhabi	7.624	Albania	6.672
Algeria	7.661	Angola	7.206
Argentina	7.196	Australia	7.775
Austria	6.974	Bahrain	7.335
Bolivia	8.086	Brazil	7.315
Brunei	7.334	Bulgaria	7.300
Burma	7.464	Canada	7.428
Chile	7.802	China	7.300
Colombia	7.054	Congo	7.478
Cuba	6.652	Czechoslovakia	6.782
Denmark	7.650	Dubai	7.295
Ecuador	7.580	Egypt	7.240
France	7.287	Gabon	7.245
Germany, West	7.223	Hungary	7.630
India	7.441	Indonesia	7.348
Iran	7.370	Iraq	7.453
Japan	7.352	Italy	6.813
Libya	7.615	Kuwait	7.281
Mexico	7.104	Malaysia	7.709
Morocco	7.602	Mongolia	7.300
Neutral Zone	6.825	Netherlands	6.816
New Zealand	8.043	New Guinea	7.468
Norway	7.444	Nigeria	7.410
Pakistan	7.308	Oman	7.390
Poland	7.419	Peru	7.517
Romania	7.453	Qatar	7.573
Senegal	7.535	Saudi Arabia	7.338
Spain	7.287	Sharjah	7.650
Taiwan	7.419	Syria	6.940
Tunisia	7.709	Trinidad	6.989
United Arab Emirates	7.522	Turkey	7.161
United States	7.418	United Kingdom	7.279
Zaire	7.206	U.S.S.R.	7.350
Venezuela	7.005	Yugoslavia	7.407

TABLE A.3

Metric Tons to Barrels (Products	5)
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Refined Products		Other Products	Other Products	
Aviation gasoline	8.90	Grease	6.30	
Motor gasoline	8.50	Paraffin oil, pure	7.14	
White spirits	8.50	Paraffin wax	7.87	
Kerosene	7.75	Petrolatum	7.87	
Jet fuel	8.00	Asphalt and road oil	6.06	
Distillate gas and diesel oil	7.46	Petroleum coke	5.50	
Residual fuel oil	6.66	Bitumen	6.06	
Lubricating oil	7.00	LPG	11.60	
		Miscellaneous products	7.00	

TABLE A.4

Crude Oil Measure^a

			То			
From	Tons	Long Tons	Barrels	Gallons (Imperial)	Gallons (U.S.)	Tons/ Year
			Mul	tiply by		
Tons (metric)	1	0.984	7.33	256	308	
Long tons	1.016	1	7.45	261	313	
Barrels	0.136	0.134	1	35	42	
Gallons (Imperial)	0.00391	0.0383	0.0286	1	1.201	
Gallons (U.S.)	0.00325	0.00319	0.0238	0.833	1	
Barrels/day						49.8
		1. 1. (•			

^a Based on average Arabian light (33.5 API gravity).

TABLE A.5

Refined	Proc	luct N	leasures
---------	------	--------	----------

To Convert:	Barrels to Tons	Tons to Barrels	Barrels/Day to Tons/Year	Tons/Year to Barrels/Day
		Multiply by		
Motor spirit	0.118	8.45	43.2	0.0232
Kerosene	0.128	7.80	46.8	0.0214
Gas oil/diesel	0.133	7.50	48.7	0.0205
Fuel oil	0.149	6.70	54.5	0.0184

TABLE A.6

Calorific Equivalent

	One Million Tonnes of Oil Approximately Equals
Heat Units	
In Btu's	40×10^{12}
In therms	397×10^6
In teracalories	10,000
Solid Fuels ^a	
In tonnes of coal	1.5×10^{6}
In tonnes of lignite	3×10^{6}
Natural Gas ^b	
In cubic meters	1.111×10^{9}
In cubic feet	39.2×10^{9}

^{*a*} Calorific values of coal and lignite, as produced.

^{*b*} 1 cubic foot = 1,000 Btu; 1 cubic meter = 9,000 Kcal.

TABLE A.7

Natural Gas/LNG/LPG

Natural Gas	LNG	LPG
(One billion cubic meters equals approximately 35.3 × 10 ⁹ cubic feet)	(One million tonnes equals approximately 0.05 TCF [gas])	(One million tonnes equals approximately 11.8 × 10 ⁶ barrels of LPG)
0.89 × 10 ⁶ tonnes of crude oil	1.23 × 10 ⁶ tonnes of crude oil	1.1×10^6 tonnes of crude oil
0.8×10^6 tonnes of LPG	1.1×10^6 tonnes of LPG	1.25×10^9 cubic meters (gas)
0.725×10^6 tonnes of LNG	1.4×10^9 cubic meters (gas)	0.91×10^6 tonnes of LNG
1.35×10^6 tonnes of coal	1.9×10^6 tonnes of coal	1.7×10^6 tonnes of coal
36×10^{12} British Thermal Units (Btu)	$52 \times 10^{12} \text{ Btu}$	47×10^{12} Btu
38 × 1015 joules (38 PJ)	55 PJ	50 PJ

Notes: Tonnes, metric tons; TCF, trillion cubic feet; Mtoe, million tonnes crude oil equivalent; Mtpa, million tonnes per annum; 1 trillion, 1 million million (10¹²); 1 billion, 1 thousand million (10⁹); mmscfd, million cubic feet per day; mmbtu, million British Thermal Units; PJ, petajoules (10¹⁵ joules).

B.1 Compound Interest Tables (Using MS Excel)

Using MS Excel a compound interest table could be established to calculate compound interest factors for different interest rates and time periods. An example is cited next for i = 10% and n = 1 to 50 years.

10.00%								10.00%
п	F/P	P/F	A/F	A/P	F/A	P/A	A/G	P/G
1	1.1000	0.9091	1.0000	1.1000	1.0000	0.9091	0.0000	0.0000
2	1.2100	0.8264	0.4762	0.5762	2.1000	1.7355	0.4762	0.8264
3	1.3310	0.7513	0.3021	0.4021	3.3100	2.4869	0.9366	2.3291
4	1.4641	0.6830	0.2155	0.3155	4.6410	3.1699	1.3812	4.3781
5	1.6105	0.6209	0.1638	0.2638	6.1051	3.7908	1.8101	6.8618
6	1.7716	0.5645	0.1296	0.2296	7.7156	4.3553	2.2236	9.6842
7	1.9487	0.5132	0.1054	0.2054	9.4872	4.8684	2.6216	12.7631
8	2.1436	0.4665	0.0874	0.1874	11.4359	5.3349	3.0045	16.0287
9	2.3579	0.4241	0.0736	0.1736	13.5795	5.7590	3.3724	19.4215
10	2.5937	0.3855	0.0627	0.1627	15.9374	6.1446	3.7255	22.8913
11	2.8531	0.3505	0.0540	0.1540	18.5312	6.4951	4.0641	26.3963
12	3.1384	0.3186	0.0468	0.1468	21.3843	6.8137	4.3884	29.9012
13	3.4523	0.2897	0.0408	0.1408	24.5227	7.1034	4.6988	33.3772
14	3.7975	0.2633	0.0357	0.1357	27.9750	7.3667	4.9955	36.8005
15	4.1772	0.2394	0.0315	0.1315	31.7725	7.6061	5.2789	40.1520
16	4.5950	0.2176	0.0278	0.1278	35.9497	7.8237	5.5493	43.4164
17	5.0545	0.1978	0.0247	0.1247	40.5447	8.0216	5.8071	46.5819
18	5.5599	0.1799	0.0219	0.1219	45.5992	8.2014	6.0526	49.6395
19	6.1159	0.1635	0.0195	0.1195	51.1591	8.3649	6.2861	52.5827
20	6.7275	0.1486	0.0175	0.1175	57.2750	8.5136	6.5081	55.4069
21	7.4002	0.1351	0.0156	0.1156	64.0025	8.6487	6.7189	58.1095
22	8.1403	0.1228	0.0140	0.1140	71.4027	8.7715	6.9189	60.6893
23	8.9543	0.1117	0.0126	0.1126	79.5430	8.8832	7.1085	63.1462
24	9.8497	0.1015	0.0113	0.1113	88.4973	8.9847	7.2881	65.4813
25	10.8347	0.0923	0.0102	0.1102	98.3471	9.0770	7.4580	67.6964
26	11.9182	0.0839	0.0092	0.1092	109.1818	9.1609	7.6186	69.7940

10.00%								10.00%
n	F/P	P/F	A/F	A/P	F/A	P/A	A/G	P/G
27	13.1100	0.0763	0.0083	0.1083	121.0999	9.2372	7.7704	71.7773
28	14.4210	0.0693	0.0075	0.1075	134.2099	9.3066	7.9137	73.6495
29	15.8631	0.0630	0.0067	0.1067	148.6309	9.3696	8.0489	75.4146
30	17.4494	0.0573	0.0061	0.1061	164.4940	9.4269	8.1762	77.0766
31	19.1943	0.0521	0.0055	0.1055	181.9434	9.4790	8.2962	78.6395
32	21.1138	0.0474	0.0050	0.1050	201.1378	9.5264	8.4091	80.1078
33	23.2252	0.0431	0.0045	0.1045	222.2515	9.5694	8.5152	81.4856
34	25.5477	0.0391	0.0041	0.1041	245.4767	9.6086	8.6149	82.7773
35	28.1024	0.0356	0.0037	0.1037	271.0244	9.6442	8.7086	83.9872
40	45.2593	0.0221	0.0023	0.1023	442.5926	9.7791	9.0962	88.9525
45	72.8905	0.0137	0.0014	0.1014	718.9048	9.8628	9.3740	92.4544
50	117.3909	0.0085	0.0009	0.1009	1163.9085	9.9148	9.5704	94.8889

B.2 Standard Compound Interest Tables

1/2//0							
	To find F , given P : $(1 + i)^n$	To find <i>P,</i> given <i>F:</i> 1	To find <i>A,</i> given <i>F:</i> <i>i</i>	To find <i>A,</i> given <i>P</i> : <i>i</i> (1+ <i>i</i>)"	To find F, given A: (1 ± i)" = 1	To find <i>P,</i> given <i>A</i> : (1 + i) ⁿ – 1	
		$(1+i)^{n}$	$(1+i)^n - 1$	$(1+i)^n - 1$	$\frac{i}{i}$	$i(1+i)^n$	
u	$(f/p)_{n}^{1/2}$	$(p/f)_{n}^{1/2}$	$(a/f)_n^{1/2}$	$(a/p)_n^{1/2}$	$(f/a)_{n}^{1/2}$	$(p/a)_{n}^{1/2}$	и
1	1.005	0.9950	1.00000	1.00500	1.000	0.995	Ч
7	1.010	0.9901	0.49875	0.50375	2.005	1.985	2
ю	1.015	0.9851	0.33167	0.33667	3.015	2.970	ю
4	1.020	0.9802	0.24183	0.25313	4.030	3.950	4
Ŋ	1.025	0.9754	0.19801	0.20301	5.050	4.926	IJ
9	1.030	0.9705	0.16460	0.16960	6.076	5.896	9
	1.036	0.9657	0.14073	0.14573	7.106	6.862	7
8	1.041	0.9609	0.12283	0.12783	8.141	7.823	8
6	1.046	0.9561	0.10891	0.11391	9.182	8.779	6
10	1.051	0.9513	0.09777	0.10277	10.288	9.730	10
11	1.056	0.9466	0.08866	0.09366	11.279	10.677	11
12	1.062	0.9419	0.08107	0.08607	12.336	11.619	12
13	1.067	0.9372	0.07464	0.07964	13.397	12.556	13
14	1.072	0.9326	0.06914	0.07414	14.464	13.489	14
15	1.078	0.9279	0.06436	0.06936	15.537	14.417	15
16	1.083	0.9233	0.06019	0.06519	16.614	15.340	16
17	1.088	0.9187	0.05615	0.06151	17.697	16.259	17
18	1.094	0.9141	0.05323	0.05823	18.786	17.173	18
19	1.099	0.9096	0.05030	0.05530	19.880	18.082	19
20	1.105	0.9051	0.04767	0.05267	20.979	18.987	20
21	1.110	0.9006	0.04528	0.05028	22.084	19.888	21
22	1.116	0.8961	0.04311	0.04811	23.194	20.784	22

1/2%

23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	06	95	100
21.676	22.563	23.446	24.324	25.198	26.068	26.933	27.794	28.651	29.503	30.352	31.196	32.035	36.172	40.207	44.143	47.981	51.726	55.377	58.939	62.414	65.802	69.108	72.331	75.476	78.543
24.310	25.432	26.559	27.692	28.830	29.975	31.124	32.280	33.441	34.609	35.782	36.961	38.145	44.159	50.324	56.645	63.126	69.770	76.582	83.566	90.727	98.068	105.594	113.311	121.222	129.334
0.04613	0.04432	0.04265	0.04111	0.03969	0.03836	0.03713	0.03598	0.03490	0.03389	0.03295	0.03206	0.03122	0.02765	0.02487	0.02265	0.02084	0.01933	0.01806	0.01697	0.01602	0.01520	0.01447	0.01383	0.01325	0.01273
0.04113	0.03932	0.03767	0.03611	0.03469	0.03336	0.03213	0.03098	0.02990	0.02889	0.02795	0.02706	0.02622	0.02265	0.01987	0.01765	0.01548	0.01433	0.01306	0.01197	0.01102	0.01020	0.00947	0.00883	0.00825	0.00773
0.8916	0.8872	0.8828	0.8784	0.8740	0.8697	0.8653	0.8610	0.8567	0.8525	0.8482	0.8440	0.8398	0.8191	0.7990	0.7793	0.7601	0.7414	0.7231	0.7053	0.6879	0.6710	0.6545	0.6383	0.6226	0.6073
1.122	1.127	1.133	1.138	1.144	1.150	1.156	1.161	1.167	1.173	1.179	1.185	1.191	1.221	1.252	1.283	1.316	1.349	1.383	1.418	1.454	1.490	1.528	1.567	1.606	1.647
23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	06	95	100

1%							
	To find F,	To find P,	To find A,	To find A,	To find F,	To find P,	
	given <i>P</i> : $(1 + 1)^n$	given <i>r</i> : 1	given F: i	given <i>P</i> : <i>i</i> (1+ <i>i</i>)"	$g_{1} = i N'' = 1$	given A: $(1+i)^n - 1$	
		$(1 + i)^n$	$(1+i)^n - 1$	$(1+i)^n - 1$	$\frac{1}{1-\frac{1}{1-1}}$	$i(1+i)^{n}$	
и	$(f/p)_{n}^{1/2}$	$(p/f)_{n}^{1/2}$	$(a/f)_{n}^{1/2}$	$(a/p)_n^{1/2}$	$(f/a)_{n}^{1/2}$	$(p/a)_{n}^{1/2}$	и
1	1.010	0.9901	1.00000	1.01000	1.000	066.0	1
7	1.020	0.9803	0.49751	0.50751	2.010	1.970	2
ю	1.030	0.9706	0.33002	0.34002	3.030	2.941	ю
4	1.041	0.9610	0.24628	0.25628	4.060	3.902	4
Ŋ	1.051	0.9515	0.19604	0.20604	5.101	4.853	ß
9	1.062	0.9420	0.16255	0.17255	6.152	5.795	9
~	1.072	0.9327	0.13863	0.14863	7.214	6.728	4
8	1.083	0.9235	0.12069	0.13069	8.286	7.652	8
6	1.094	0.9143	0.10674	0.11674	9.369	8.566	6
10	1.105	0.9053	0.09558	0.10558	10.462	9.471	10
11	1.116	0.8963	0.08645	0.09645	11.567	10.368	11
12	1.127	0.8874	0.07885	0.08885	12.683	11.255	12
13	1.138	0.8787	0.07241	0.08241	13.809	12.134	13
14	1.149	0.8700	0.06690	0.07690	14.947	13.004	14
15	1.161	0.8613	0.06212	0.07212	16.097	13.865	15
16	1.173	0.8528	0.05794	0.06794	17.258	14.718	16
17	1.184	0.8444	0.05426	0.06426	18.430	15.562	17
18	1.196	0.8360	0.05098	0.06098	19.615	16.398	18
19	1.208	0.8277	0.04805	0.05805	20.811	17.226	19
20	1.220	0.8195	0.04542	0.05542	22.019	18.046	20
21	1.232	0.8114	0.04303	0.05303	23.239	18.857	21

00	56 23	43 24	23 25	95 26	60 27	16 28	66 29	08 30	42 31	70 32	90 33	03 34	09 35	35 40	95 45	96 50	47 55	55 60	27 65	69 70	87 75	88 80	78 85	61 90	43 95	29 100
7 17.0	5 20.4	3 21.2	3 22.0	5 22.7	1 23.5	9 24.3) 25.0	5 25.8	3 26.5	1 27.2	9 27.9	3 28.7) 29.4	5 32.8	1 36.0	3 39.1	2 42.1) 44.9	7 47.6	5 50.1	3 52.5	2 54.8	9 57.0	3 59.1	4 61.1	1 63.0
74.477	25.716	26.973	28.243	29.526	30.821	32.129	33.450	34.785	36.133	37.494	38.869	40.258	41.660	48.886	56.481	64.463	72.852	81.670	90.937	100.676	110.913	121.672	132.979	144.863	157.354	170.481
98NGN'N	0.04889	0.04707	0.04541	0.04387	0.04245	0.04112	0.03990	0.03875	0.03768	0.03667	0.03573	0.03484	0.03400	0.03046	0.02771	0.02551	0.02373	0.02224	0.02100	0.01993	0.01902	0.01822	0.01752	0.01690	0.01636	0.01587
0.04086	0.03889	0.03707	0.03541	0.03387	0.03245	0.03112	0.02990	0.02875	0.02768	0.02667	0.02573	0.02484	0.02400	0.02046	0.01771	0.01551	0.01373	0.01224	0.01100	0.00993	0.00902	0.00822	0.00752	0.00690	0.00636	0.00587
0.8034	0.7954	0.7876	0.7798	0.7720	0.7644	0.7568	0.7493	0.7419	0.7346	0.7273	0.7201	0.7130	0.7059	0.6717	0.6391	0.6080	0.5785	0.5504	0.5237	0.4983	0.4741	0.4511	0.4292	0.4084	0.3886	0.3697
1.245	1.257	1.270	1.282	1.295	1.308	1.321	1.335	1.348	1.361	1.375	1.391	1.403	1.417	1.489	1.565	1.645	1.729	1.817	1.909	2.007	2.109	2.217	2.330	2.449	2.574	2.705
77	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	06	95	100

Appendix B

$1^{1/2}$ %								
	To find F , given P : $(1 + i)^n$	To find <i>P</i> , given <i>F</i> :	To find <i>A</i> , given <i>F</i> :	To find <i>A</i> , given <i>P</i> :	To find F, given A:	To find <i>P</i> , given <i>A</i> :		
		1	i	$i(1+i)^n$	$(1+i)^n - 1$	$(1+i)^n - 1$		
		$(1 + i)^n$	$(1+i)^n - 1$	$(1+i)^n - 1$	i	$i(1+i)^n$		
u	$(f/p)_n^{1\ 1/2}$	$(p/f)_n^{1\ 1/2}$	$(a/f)_n^{1\ 1/2}$	$(a/p)_n^{1\ 1/2}$	$(f/a)_n^{1\ 1/2}$	$(p/a)_n^{1\ 1/2}$	и	
1	1.015	0.9852	1.00000	1.01500	1.000	0.985	1	
2	1.030	0.9707	0.49629	0.51128	2.015	1.956	2	
Ю	1.046	0.9563	0.32838	0.34338	3.045	2.912	ю	
4	1.061	0.9422	0.24444	0.25944	4.091	3.854	4	
ß	1.077	0.9283	0.19409	0.20909	5.152	4.783	5	
9	1.093	0.9145	0.16053	0.17553	6.230	5.697	9	
~	1.110	0.9010	0.13656	0.15156	7.323	6.598	7	
8	1.126	0.8877	0.11858	0.13358	8.433	7.486	8	
6	1.143	0.8746	0.10461	0.11961	9.559	8.361	6	
10	1.161	0.8617	0.09343	0.10843	10.703	9.222	10	
11	1.178	0.8489	0.08429	0.09930	11.863	10.071	11	
12	1.196	0.8364	0.07668	0.09168	13.041	10.908	12	
13	1.214	0.8240	0.07024	0.08524	14.237	11.732	13	
14	1.232	0.8118	0.06472	0.07972	15.450	12.543	14	
15	1.250	0.7999	0.05994	0.07494	16.682	13.343	15	
16	1.269	0.7880	0.05577	0.07077	17.932	14.131	16	
17	1.288	0.7764	0.05208	0.06708	19.201	14.908	17	
18	1.307	0.7649	0.04881	0.06381	20.489	15.673	18	
19	1.327	0.7536	0.04588	0.06086	21.797	16.426	19	
20	1.347	0.7425	0.04325	0.05825	23.124	17.169	20	

414

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	09	65	70	75	80	85	06	95	100
17.900	19.621	19.331	20.030	20.720	21.399	22.068	22.727	23.376	24.016	24.646	25.267	25.879	26.482	27.076	29.916	32.552	35.000	37.271	39.380	41.338	43.155	44.842	46.407	47.861	49.210	50.462	51.625
24.471	25.838	27.225	28.634	30.063	31.514	32.987	34.481	35.999	37.539	39.102	40.688	42.229	43.933	45.592	54.268	63.614	73.683	84.530	96.215	108.803	122.364	136.973	152.711	169.665	187.930	207.606	228.803
0.05587	0.05370	0.05173	0.04992	0.04826	0.04673	0.04532	0.04400	0.04278	0.04164	0.04057	0.03958	0.03864	0.03776	0.03693	0.03343	0.03072	0.02857	0.02683	0.02539	0.02419	0.02317	0.02230	0.02155	0.02089	0.02032	0.01982	0.01937
0.04087	0.03870	0.03673	0.03492	0.03325	0.03173	0.03032	0.02900	0.02778	0.02664	0.02557	0.02458	0.02364	0.02276	0.02193	0.01834	0.01572	0.01357	0.01183	0.01039	0.00919	0.00817	0.00730	0.00655	0.00589	0.00532	0.00482	0.00437
0.7315	0.7207	0.7100	0.6995	0.6892	0.6790	0.6690	0.6591	0.6494	0.6396	0.6303	0.6210	0.6118	0.6028	0.5939	0.5513	0.5117	0.4750	0.4409	0.4093	0.3799	0.3527	0.3274	0.3039	0.2821	0.2619	0.2431	0.2256
1.367	1.388	1.408	1.430	1.451	1.473	1.495	1.517	1.540	1.563	1.587	1.610	1.634	1.659	1.684	1.814	1.954	2.105	2.268	2.443	2.632	2.835	3.055	3.291	3.545	3.819	4.114	4.432
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	06	95	100

r T F	e T E	Ē	÷	E E	e T	
for the formula F_{i} given P : $(1 + i)^n$	lo tind <i>P</i> , given <i>F</i> :	Io tind A, given F:	given P :	lo tind F, given A:	lo tind P, given A:	
	$\frac{1}{(1+i)^n}$	i $(1+i)^n - 1$	$\frac{i(1+i)^n}{(1+i)^n-1}$	$\frac{(1+i)^n-1}{i}$	$\frac{(1+i)^n-1}{i(1+i)^n}$	
$(f/p)_n^2$	$(p/f)_n^2$	$(a/f)_n^2$	$(a/p)_{n}^{2}$	$(f/a)_n^2$	$(p/a)_n^2$	и
1.020	0.9804	1.00000	1.02000	1.000	0.980	1
1.040	0.9612	0.49505	0.51505	2.020	1.942	2
1.061	0.9423	0.32675	0.34675	3.060	2.884	С
1.082	0.9238	0.24262	0.26262	4.122	3.808	4
1.104	0.9057	0.19216	0.21216	5.204	4.713	IJ
1.126	0.8880	0.15853	0.17853	6.308	5.601	9
1.149	0.8706	0.13451	0.15451	7.434	6.472	~
1.172	0.8535	0.11651	0.13651	8.583	7.325	8
1.195	0.8368	0.10252	0.12252	9.755	8.162	6
1.219	0.8203	0.09133	0.11133	10.950	8.983	10
1.243	0.8043	0.08216	0.10218	12.169	9.787	11
1.268	0.7885	0.07456	0.09456	13.412	10.575	12
1.294	0.7730	0.06812	0.08812	14.680	11.348	13
1.319	0.7579	0.06260	0.08260	15.974	12.106	14
1.346	0.7430	0.05783	0.07783	17.293	12.849	15
1.373	0.7284	0.05365	0.07365	18.639	13.578	16
1.400	0.7142	0.04997	0.06997	20.012	14.292	17
1.428	0.7002	0.04670	0.06670	21.412	14.992	18
1.457	0.6864	0.04378	0.06378	22.841	15.678	19
1.486	0.6730	0.04116	0.06116	24.297	16.351	20

2%

416

1.5	16	0.6598	0.03878	0.05878	25.783	17.011	21
1.5°	46	0.6468	0.03663	0.05663	27.299	17.658	22
1.5.	77	0.6342	0.03467	0.05467	28.845	18.292	23
1.6(38	0.6217	0.03287	0.05287	30.422	18.914	24
1.6	41	0.6095	0.03122	0.05122	32.030	19.523	25
1.6	73	0.5976	0.02970	0.04970	33.671	20.121	26
1.7(07	0.5859	0.02829	0.04829	35.344	20.707	27
1.74	41	0.5744	0.02699	0.04699	37.051	21.281	28
1.7.	76	0.5631	0.02578	0.04578	38.792	21.844	29
1.8	11	0.5521	0.02465	0.04465	40.568	22.396	30
1.8	48	0.5412	0.02360	0.04360	42.379	22.938	31
1.8	85	0.5306	0.02261	0.04261	44.227	23.468	32
1.92	22	0.5202	0.02169	0.04169	46.112	23.989	33
1.9	51	0.5100	0.02082	0.04082	48.034	24.499	34
2.0	00	0.5000	0.02000	0.04000	49.994	24.999	35
2.2(38	0.4529	0.01656	0.03656	60.402	27.355	40
2.4	38	0.4102	0.01391	0.03391	71.893	29.490	45
2.6	92	0.3715	0.01182	0.03182	84.579	31.424	50
2.9	72	0.3365	0.01014	0.03014	98.587	33.175	55
3.2	81	0.3048	0.00877	0.02877	114.052	34.761	60
3.6	23	0.2761	0.00763	0.02763	131.126	36.197	65
4.0	00	0.2500	0.00667	0.02667	149.978	37.499	70
4.4	16	0.2265	0.00586	0.02586	170.792	38.677	75
4.8	75	0.2051	0.00516	0.02516	193.772	39.745	80
5.3	33	0.1858	0.00456	0.02456	219.144	40.711	85
5.9	43	0.1683	0.00405	0.02405	247.157	41.587	06
6.5	52	0.1524	0.00360	0.02360	278.085	42.380	95
7.2	45	0.1380	0.00320	0.02320	312.232	43.098	100
21/2%							
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	To find F , given P : $(1 + i)^n$	To find P , given F :	To find A , given F :	To find A , given P :	To find <i>F</i> , given <i>A</i> :	To find <i>P</i> , given <i>A</i> :	
		1	1	$i(1+i)^n$	$(1+i)^n - 1$	$(1+i)^n-1$	
		$(1+i)^n$	$(1+i)^n - 1$	$(1+i)^n - 1$	1	$i(1+i)^n$	
и	$(f/p)_n^{2\ 1/2}$	$(p/f)_n^{2\ 1/2}$	$(a/f)_n^{2\ 1/2}$	$(a/p)_n^{2\ 1/2}$	$(f/a)_n^{2\ 1/2}$	$(p a)_n^{2\ 1/2}$	и
1	1.025	0.9756	1.00000	1.02500	1.000	0.976	1
2	1.051	0.9518	0.49383	0.51883	2.025	1.927	2
ю	1.077	0.9386	0.32514	0.35014	3.076	2.856	С
4	1.104	09060	0.24082	0.26512	4.153	3.762	4
IJ	1.131	0.8839	0.19025	0.21525	5.256	4.646	IJ
9	1.160	0.8623	0.15655	0.18155	6.388	5.508	9
~	1.189	0.8413	0.13250	0.15750	7.547	6.349	7
8	1.218	0.8207	0.11447	0.13947	8.736	7.170	8
6	1.249	0.8007	0.10046	0.12546	9.955	7.971	6
10	1.280	0.7812	0.08926	0.11426	11.203	8.752	10
11	1.312	0.7621	0.08011	0.10511	12.483	9.514	11
12	1.345	0.7436	0.07249	0.09749	13.796	10.258	12
13	1.379	0.7254	0.06605	0.09105	15.140	10.983	13
14	1.413	0.7077	0.06054	0.08554	16.519	11.691	14
15	1.448	0.6905	0.05577	0.08077	17.932	12.381	15
16	1.485	0.6736	0.05160	0.07660	19.380	13.055	16
17	1.522	0.6572	0.04793	0.07293	20.865	13.712	17
18	1.560	0.6412	0.04407	0.06967	22.386	14.353	18
19	1.599	0.6255	0.04176	0.06676	23.946	14.979	19
20	1.639	0.6103	0.03915	0.06415	25.545	15.589	20

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	90	95	100
16.185	16.765	17.332	17.885	18.424	18.951	19.494	19.965	20.454	20.930	21.395	21.849	22.292	22.724	23.145	25.103	26.833	28.362	29.714	30.909	31.965	32.898	33.723	34.452	35.096	35.666	36.169	36.614
27.183	28.863	30.584	32.349	34.158	36.012	37.912	39.860	41.856	43.903	46.000	48.150	50.354	52.613	54.928	67.403	81.516	97.484	115.551	135.992	159.118	185.284	214.888	248.383	286.279	329.154	377.664	432.549
0.06179	0.05965	0.05770	0.05591	0.05428	0.05277	0.05138	0.05009	0.04689	0.04778	0.04674	0.04577	0.04486	0.04401	0.04321	0.03984	0.03727	0.03526	0.03365	0.03235	0.03128	0.03040	0.02965	0.02903	0.02849	0.02804	0.02765	0.02731
0.03679	0.03465	0.03270	0.03091	0.02928	0.02777	0.02638	0.02509	0.02389	0.02278	0.02174	0.02077	0.01986	0.01901	0.01821	0.01464	0.01227	0.01026	0.00865	0.00735	0.00628	0.00540	0.00465	0.00403	0.00349	0.00304	0.00265	0.00231
0.5954	0.5809	0.5667	0.5529	0.5394	0.5262	0.5134	0.5009	0.4887	0.4767	0.4651	0.4538	0.4427	0.4319	0.4214	0.3724	0.3292	0.2909	0.2572	0.2273	0.2009	0.1776	0.1569	0.1387	0.1226	0.1084	0.0958	0.0846
1.680	1.722	1.765	1.809	1.854	1.900	1.948	1.996	2.046	2.098	2.150	2.204	2.259	2.315	2.373	2.685	3.038	3.437	3.889	4.400	4.978	5.632	6.372	7.210	8.157	9.229	10.442	11.814
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	06	95	100

3%	To find F given $P: (1+i)^n$	To find <i>P</i> , given <i>F</i> : 1	To find A, given F: i		To find <i>A</i> , given <i>P</i> : <i>i</i> (1+ <i>i</i>)"	To find A,To find F,given P:given A: $i(1+i)^n$ $(1+i)^n - 1$
		$\frac{1}{(1+i)^n}$	$\frac{1}{(1+i)^n-1}$	1	$\frac{i(1+i)^n}{(1+i)^n-1}$	$\frac{i(1+i)^n}{(1+i)^n-1} \frac{(1+i)^n-1}{i}$
	$(f/p)_n^3$	$(p/f)_n^3$	$(a/f)_n^3$		$(a/p)_n^3$	$(a/p)_n^3$ $(f/a)_n^3$
	1.030	0.9709	1.00000		1.03000	1.03000 1.000
	1.061	0.9426	0.49261		0.52261	0.52261 2.030
	1.093	0.9151	0.32353		0.35353	0.35353 3.091
	1.126	0.8885	0.23903		0.26903	0.26903 4.184
	1.159	0.8626	0.18835		0.21835	0.21835 5.309
	1.194	0.8375	0.15460		0.18460	0.18460 6.468
	1.230	0.8131	0.13051		0.16051	0.16051 7.662
	1.267	0.7894	0.11246	-	0.14246	0.14246 8.892
	1.305	0.7664	0.09843	-	0.12843	0.12843 10.159
	1.344	0.7441	0.08723	0	0.11723	0.11723 11.464
	1.384	0.7224	0.07808	-	0.10808	0.10808 12.808
	1.426	0.7014	0.07046		0.10046	0.10046 14.192
	1.469	0.6810	0.06403		0.09403	0.09403 15.618
	1.513	0.6611	0.05853		0.08853	0.08853 17.086
	1.558	0.6419	0.05377	-	0.08377	0.08377 18.599
	1.605	0.6232	0.04961	U	0.07961	0.07961 20.157
	1.653	0.6050	0.04595	0	0.07595	0.07595 21.762
	1.702	0.5874	0.04271	_	0.07271	0.07271 23.414
	1.754	0.5703	0.03981		0.06981	0.06981 25.117
	1.806	0.5537	0.03722		0.06722	0.06722 26.870
	1.860	0.5375	0.03487		0.06487	0.06487 28.676

22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	60	95	100
15.937	16.444	16.936	17.413	17.877	18.327	18.764	19.188	19.600	20.000	20.389	20.766	21.132	21.487	23.115	24.519	25.730	26.774	27.676	28.453	29.123	29.702	30.201	30.631	31.002	31.323	31.599
30.537	32.453	34.426	36.459	38.553	40.710	42.931	45.219	47.575	50.003	52.503	55.078	57.730	60.462	75.401	92.720	112.797	136.072	163.053	194.333	230.594	272.631	321.363	377.857	443.349	519.272	607.288
0.06275	0.06081	0.05905	0.05743	0.05594	0.05456	0.05329	0.05211	0.05102	0.05000	0.04905	0.04816	0.04732	0.04654	0.04328	0.04079	0.03887	0.03735	0.03613	0.03515	0.03434	0.03367	0.03311	0.03265	0.03226	0.03193	0.03165
0.03275	0.03081	0.02905	0.02743	0.02594	0.02456	0.02329	0.02211	0.02102	0.02000	0.01905	0.01816	0.01732	0.01654	0.01326	0.01079	0.00887	0.00735	0.00613	0.00515	0.00434	0.00367	0.00311	0.00265	0.00226	0.00193	0.00165
0.5219	0.5067	0.4919	0.4776	0.4637	0.4502	0.4371	0.4243	0.4120	0.4000	0.3883	0.3770	0.3660	0.3554	0.3066	0.2644	0.2281	0.1968	0.1697	0.1464	0.1263	0.1089	0.0940	0.0811	0.0699	0.0603	0.0520
1.916	1.974	2.033	2.094	2.157	2.221	2.288	2.357	2.427	2.500	2.575	2.652	2.732	2.814	3.262	3.782	4.384	5.082	5.892	6.830	7.918	9.179	10.641	12.336	14.300	16.578	19.219
22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	06	95	100

				и	1	2	ю	4	ß	9	7	8	6	10	11	12	13	14	15	16	17	18	19	00
	To find <i>P</i> , given <i>A</i> :	$(1+i)^n - 1$	$i(1+i)^n$	$(p/a)_{n}^{4}$	0.962	1.886	2.775	3.630	4.452	5.242	6.002	6.733	7.435	8.111	8.760	9.385	9.986	10.563	11.118	11.652	12.166	12.659	13.134	13.590
	To find <i>F</i> , given <i>A</i> :	$(1+i)^n - 1$	i	$(f/a)_n^4$	1.000	2.040	3.122	4.246	5.416	6.633	7.898	9.214	10.583	12.006	13.486	15.026	16.627	18.292	20.024	21.825	23.698	25.645	27.671	29.778
	To find <i>A</i> , given <i>P</i> :	$i(1+i)^n$	$(1+i)^n - 1$	$(a/p)_{n}^{4}$	1.04000	0.53020	0.36035	0.27549	0.22463	0.19076	0.16661	0.14853	0.13449	0.12329	0.11415	0.10655	0.10014	0.09467	0.08994	0.08582	0.08220	0.07899	0.07614	0.07358
	To find A , given F :	i	$(1+i)^{n} - 1$	$(a/f)_n^4$	1.00000	0.49020	0.32035	0.23549	018463	0.15076	0.12661	0.10853	0.09449	0.08329	0.07416	0.06655	0.06014	0.05467	0.04994	0.04582	0.04220	0.03899	0.03614	0.03358
	To find <i>P</i> , given <i>F</i> :	1	$(1 + i)^n$	$(p/f)_n^4$	0.9615	0.1246	0.8190	0.8548	0.8219	0.7903	0.7599	0.7307	0.7026	0.6756	0.6496	0.6246	0.6006	0.5775	0.5553	0.5339	0.5134	0.4936	0.4746	0.4564
	To find F , given P : $(1 + i)^n$			$(f/p)_{n}^{4}$	1.040	1.082	1.125	1.170	1.217	1.265	1.316	1.369	1.423	1.480	1.539	1.601	1.665	1.732	1.801	1.873	1.948	2.026	2107	2.191
4%				и	1	7	ю	4	Ŋ	9	4	8	6	10	11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	09	65	70	75	80	85	60	95	100
14.029	14.451	14.857	15.247	15.622	15.983	16.330	16.683	16.984	17.292	17.588	17.874	18.148	18.411	18.665	19.793	20.720	21.482	22.109	22.623	23.047	23.395	23.680	23.915	24.109	24.267	24.398	24.505
31.969	34.248	36.618	39.083	41.646	44.312	47.084	49.968	52.966	58.085	59.328	62.701	68.210	69.858	73.652	95.026	121.029	152.667	191.159	237.991	294.968	364.290	448.631	551.245	676.090	827.983	1012.785	1237.624
0.07128	0.06920	0.06731	0.06559	0.06401	0.06257	0.06124	0.08001	0.05888	0.05783	0.05686	0.05595	0.05510	0.05431	0.05358	0.05052	0.04826	0.04855	0.04523	0.04420	0.04339	0.04275	0.04223	0.04181	0.04140	0.04121	0.04099	0.04081
0.03128	0.02920	0.02731	0.02559	0.02401	0.02257	0.02124	0.02001	0.01888	0.01783	0.01686	0.01595	0.01510	0.01431	0.01358	0.01052	0.00826	0.00655	0.00523	0.00420	0.00339	0.00275	0.00223	0.00181	0.00146	0.00121	0.00099	0.00081
0.4388	0.4220	0.4057	0.3901	0.3751	0.3607	0.3468	0.3335	0.3207	0.3083	0.2965	0.2851	0.2741	0.2636	0.2534	0.2083	0.1712	0.1407	0.1157	0.0951	0.0781	0.0642	0.0528	0.0434	0.0357	0.0293	0.0241	0.0198
2.279	2.370	2.465	2.563	2.666	2.772	2.883	2.999	3.119	3.243	3.373	3.508	3.648	3.794	3.946	4.801	5.841	7.107	8.646	10.520	12.799	15.572	18.945	23.050	28.044	34.119	41.511	50.505
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	90	95	100

d P.	A:	$\frac{1}{(i)^n}$	u s	52 1	59 2 2	3 3	4 4	<u>9</u> 5	76 6	36 7	3 8	9 9	22 10)6 11	33 12	94 13	99 14	30 15	38 16	74 17	90 18	35 19	
To fine	given	$\frac{(1+i)^n}{i(1+i)}$	$(p/a)_n^{\epsilon}$	0.95	1.85	2.72	3.54	4.32	5.07	5.78	6.46	7.10	7.72	8.30	8.86	9.39	9.89	10.38	10.83	11.27	11.69	12.08	
To find F	given A:	$\frac{(1+i)^n-1}{i}$	$(f/a)_n^5$	1.000	2.050	3.153	4.310	5.526	6.802	8.142	9.549	11.027	12.578	14.207	15.917	17.713	19.599	21.579	23.657	25.840	28.132	30.539	
To find A.	given P:	$\frac{i(1+i)^n}{(1+i)^n-1}$	$(a/p)_n^5$	1.05000	0.53780	0.36721	0.28201	0.23097	0.19702	0.17282	0.15472	0.14069	0.12950	0.12039	0.11283	0.10646	0.10102	0.09634	0.09227	0.08870	0.08555	0.08275	
To find A.	given F:	$\frac{i}{(1+i)^n-1}$	$(aff)_n^5$	1.00000	0.48780	0.31721	0.23201	0.18097	0.14702	0.12282	0.10472	0.09069	0.07950	0.07039	0.06283	0.05646	0.05102	0.04634	0.04227	0.03870	0.03555	0.03275	
To find P.	given F:	$\frac{1}{(1+i)^n}$	$(p/f)_n^5$	0.9524	0.9070	0.8638	0.8227	0.7835	0.7462	0.7107	0.6768	0.6446	0.6139	0.5847	0.5568	0.5303	0.5051	0.4810	0.4581	0.4363	0.4155	0.3957	
To find F.	given $P: (1+i)^n$		$(f/p)_n^5$	1.050	1.103	1.158	1.216	1.276	1.340	1.407	1.477	1.551	1.629	1.710	1.796	1.886	1.980	2.079	2.183	2.292	2.407	2.527	
5%			u	-	ы	ю	4	Ŋ	9	~	8	6	10	11	12	13	14	15	16	17	18	19	

22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	60	95	100
13.163	13.489	13.799	14.094	14.375	14.643	14.898	15.141	15.372	15.593	15.803	16.003	16.193	16.374	17.159	17.774	18.256	18.633	18.929	19.161	19.343	19.485	19.596	19.684	19.752	19.806	19.848
38.505	41.430	44.502	47.727	51.113	54.669	58.403	62.323	66.439	70.761	75.299	80.064	85.067	90.320	120.800	159.700	209.348	272.713	353.584	456.798	588.529	756.654	971.229	1245.087	1594.607	2040.694	2610.025
0.07597	0.07414	0.07247	0.07095	0.06956	0.06829	0.06712	0.06605	0.06505	0.06413	0.06328	0.06249	0.06176	0.06107	0.05828	0.05626	0.05478	0.05367	0.05283	0.05219	0.05170	0.05132	0.05103	0.05080	0.05063	0.05049	0.05038
0.02597	0.02414	0.02247	0.02095	0.01956	0.01829	0.01712	0.01605	0.01505	0.01413	0.01328	0.01249	0.01176	0.01107	0.00828	0.00626	0.00478	0.00367	0.00283	0.00219	0.00170	0.00132	0.00103	0.00080	0.00063	0.00049	0.00038
0.3418	0.3256	0.3101	0.2953	0.2812	0.2678	0.2551	0.2429	0.2314	0.2204	0.2099	0.1999	0.1904	0.1813	0.1420	0.1113	0.0872	0.0683	0.0535	0.0419	0.0329	0.0258	0.0202	0.0158	0.0124	0.0097	0.0076
2.925	3.072	3.225	3.386	3.556	3.733	3.920	4.116	4.322	4.538	4.765	5.003	5.253	5.516	7.040	8.985	11.467	14.636	18.679	23.840	30.426	38.833	49.561	63.254	80.730	103.035	131.501
22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	06	95	100

6%	:	;	;		;	;	
	To find F , given P : $(1 + i)^n$	To find <i>P</i> , given <i>F</i> : 1	To find <i>A</i> , given <i>F</i> : <i>i</i>	To find <i>A</i> , given <i>P</i> : <i>i</i> (1 + <i>i</i>)"	To find F , given A : $(1 + i)^n - 1$	To find P, given A: $(1+i)^n - 1$	
		$(1+i)^n$	$(1+i)^n - 1$	$(1+i)^n - 1$	i	$i(1+i)^n$	
и	$(f/p)_n^6$	$(p/f)_n^6$	$(a/f)_n^6$	$(a/p)_n^6$	$(f/a)_n^6$	$(p/a)_n^6$	и
1	1.080	0.9434	1.00000	1.06000	1.000	0.943	1
7	1.124	0.8900	0.48544	0.54544	2.060	1.833	7
З	1.191	0.8396	0.31411	0.37411	3.184	2.673	С
4	1.262	0.7921	0.22859	0.28859	4.375	3.465	4
Ŋ	1.338	0.7473	0.17740	0.23740	5.637	4.212	5
9	1.419	0.7050	0.14336	0.20336	6.975	4.917	9
	1.504	0.6651	0.11914	0.17914	8.394	5.582	4
8	1.594	0.6274	0.10104	0.16104	9.897	6.210	8
6	1.689	0.5919	0.08702	0.14702	11.491	6.802	6
10	1.791	0.5584	0.07587	0.13587	13.181	7.360	10
11	1.898	0.5268	0.08679	0.12679	14.972	7.887	11
12	2.012	0.4970	0.05928	0.11928	16.870	8.384	12
13	2.133	0.4688	0.05296	0.11296	18.882	8.853	13
14	2.261	0.4423	0.04758	0.10756	21.015	9.295	14
15	2.397	0.4173	0.04296	0.10296	23.276	9.712	15
16	2.540	0.3936	0.03895	0.09895	25.673	10.106	16
17	2.693	0.3714	0.03544	0.09544	28.213	10.477	17
18	2.854	0.3503	0.03236	0.09236	30.906	10.828	18
19	3.026	0.3305	0.02962	0.08962	33.760	11.158	19
20	3.207	0.3118	0.02718	0.08718	36.786	11.470	20
21	3.400	0.2942	0.02500	0.08500	39.993	11.764	21

22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	06	95	100
12.042	12.303	12.550	12.783	13.003	13.211	13.406	13.591	13.765	13.929	14.084	14.230	14.368	14.498	15.046	15.456	15.762	15.991	16.161	16.289	16.385	16.456	16.509	16.549	16.579	16.601	16.618
43.392	48.996	50.816	54.865	59.156	63.706	68.528	73.640	79.058	84.802	90.890	97.343	104.184	111.435	154.762	212.744	290.336	394.172	533.128	719.083	967.932	1300.949	1746.600	2342.982	3141.075	4209.104	5638.368
0.08305	0.08126	0.07968	0.07823	0.07690	0.07570	0.07459	0.07358	0.07265	0.07179	0.07100	0.07027	0.06960	0.06897	0.06646	0.06470	0.06344	0.06254	0.06188	0.06139	0.06103	0.06077	0.06057	0.06043	0.06032	0.06024	0.06018
0.02305	0.02128	0.01968	0.01823	0.01690	0.01570	0.01459	0.01358	0.01265	0.01179	0.01100	0.01027	0.00960	0.00897	0.00646	0.00470	0.00344	0.00254	0.00188	0.00139	0.00103	0.00077	0.00057	0.00043	0.00032	0.00024	0.00018
0.2775	0.2618	0.2470	0.2330	0.2198	0.2074	0.1956	0.1846	0.1741	0.1643	0.1550	0.1462	0.1379	0.1301	0.0972	0.0727	0.0543	0.0406	0.0303	0.0227	0.0169	0.0126	0.0095	0.0071	0.0053	0.0039	0.0029
3.604	3.820	4.049	4.292	4.549	4.822	5.112	5.418	5.743	6.088	6.453	6.841	7.251	7.686	10.286	13.765	18.420	24.650	32.988	45.145	59.076	79.057	105.796	141.579	189.465	253.546	339.302
22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	90	95	100

7%							
	To find F, given P: $(1 + i)^n$	To find <i>P</i> , given <i>F</i> :	To find <i>A</i> , given <i>F</i> :	To find <i>A,</i> given <i>P</i> :	To find F, given A:	To find <i>P</i> , given <i>A</i> :	
		$\frac{1}{(1+i)^n}$	i $(1+i)^n - 1$	$\frac{i(1+i)^n}{(1+i)^n-1}$	$\frac{(1+i)^n-1}{i}$	$\frac{(1+i)^n-1}{i(1+i)^n}$	
u	$(f/p)_n^7$	$(plf)_n^7$	$(a/f)_n^7$	$(a/p)_n^7$	$(f/a)_n^7$	$(p/a)_n^7$	u
-	1.070	0.9346	1.00000	1.07000	1.000	0.935	1
2	1.145	0.8734	0.48309	0.55309	2.070	1.808	7
ю	1.225	0.8163	0.31105	0.38105	3.215	2.624	ю
4	1.311	0.7629	0.22523	0.29523	4.440	3.387	4
ß	1.403	0.7130	0.17389	0.24389	5.751	4.100	IJ
9	1.501	0.6663	0.13980	0.20980	7.153	4.767	9
~	1.606	0.6227	0.11555	0.18555	8.654	5.389	7
8	1.718	0.5820	0.09747	0.16747	10.260	5.971	8
6	1.838	0.5439	0.08349	0.15349	11.978	6.515	6
10	1.967	0.5083	0.07238	0.14238	13.816	7024	10
11	2.105	0.4751	0.06336	0.13336	15.784	7.499	11
12	2.252	0.4440	0.05590	0.12590	17.888	7943	12
13	2.410	0.4150	0.04965	0.11965	20.141	8.358	13
14	2.579	0.3878	0.04434	0.11434	22.550	8.745	14
15	2.759	0.3624	0.03979	0.10979	25.129	9.108	15
16	2.952	0.3387	0.03586	0.10586	27.888	9.447	16
17	3.159	0.3166	0.03243	0.10243	30.840	9.763	17
18	3.380	0.2959	0.02941	0.09941	33.999	10.059	18
19	3.617	0.2765	0.02675	0.09675	37.379	10.363	19
20	3.870	0.2584	0.02439	0.09439	40.995	10.594	20
21	4.141	0.2415	0.02229	0.09229	44.865	10.836	21

22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	60	95	100
11.061	11.272	11.469	11.654	11.826	11.987	12.137	12.278	12.409	12.532	12.647	12.754	12.854	12.948	13.332	13.606	13.801	13.940	14.039	14.110	14.160	14.196	14.222	14.240	14.253	14.263	14.269
49.006	53.436	58.177	63.249	68.676	74.484	80.698	87.347	94.461	102.073	110.218	118.923	128.259	138.237	199.635	285.749	406.529	575.929	813.520	1146.755	1614.134	2269.657	3189.063	4478.576	6287.185	8823.854	12381.662
0.09041	0.08871	0.08719	0.08581	0.08456	0.08343	0.08239	0.08145	0.08059	0.07980	0.07907	0.07841	0.07780	0.07723	0.07501	0.07350	0.07246	0.07174	0.07123	0.07087	0.07062	0.07044	0.07031	0.07022	0.07016	0.07011	0.07008
0.02041	0.01871	0.01719	0.01581	0.01456	0.01343	0.01239	0.01145	0.01059	0.00980	0.00907	0.00841	0.00780	0.00723	0.00501	0.00350	0.00246	0.00174	0.00123	0.00087	0.00062	0.00044	0.00031	0.00022	0.00016	0.00011	0.00008
0.2257	0.2109	0.1971	0.1842	0.1722	0.1609	0.1504	0.1406	0.1314	0.1228	0.1147	0.1072	0.1002	0.0937	0.0668	0.0476	0.0339	0.0242	0.0173	0.0123	0.0088	0.0063	0.0045	0.0032	0.0023	0.0016	0.0012
4.430	4.741	5.072	5.427	5.807	6.214	6.649	7.114	7.612	8.145	8.715	9.325	9.978	10.677	14.974	21.002	29.457	41.315	57.946	81.273	113.989	159.876	224.234	314.500	441.103	618.670	867.716
22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	09	65	70	75	80	85	06	95	100

I

8%	: - -	r T F	:		;	: : :	
	To find F, given P: (1 + i)"	To find <i>P</i> , given <i>F</i> :	To find A, given F:	To find A , given P :	To find <i>F,</i> given A:	To find <i>P</i> , given <i>A</i> :	
		$\frac{1}{(1+i)^n}$	$\frac{i}{(1+i)^n-1}$	$\frac{i(1+i)^n}{(1+i)^n-1}$	$\frac{(1+i)^n-1}{i}$	$\frac{(1+i)^n-1}{i(1+i)^n}$	
и	$(f/p)_{n}^{8}$	$(p/f)_n^8$	$(a/f)_n^8$	$(a/p)_n^8$	$(f/a)_n^8$	$(p/a)_n^8$	и
1	1.080	0.9259	1.00000	1.08000	1.000	0.926	1
7	1.166	0.8573	0.48077	0.58077	2.080	1.783	2
ю	1.260	0.7938	0.30803	0.38803	3.246	2.577	3
4	1.360	0.7350	0.22192	0.30192	4.506	3.312	4
Ŋ	1.469	0.6806	0.17046	0.25048	5.867	3.933	IJ
9	1.587	0.6302	0.13832	0.21832	7.336	4.623	9
\sim	1.714	0.5835	0.11207	0.19207	8.823	5.206	7
8	1.851	0.5403	0.09401	0.17401	10.837	5.747	8
6	1.999	0.5002	0.08008	0.16008	12.488	6.247	6
10	2.159	0.4632	0.06903	0.14903	14.487	6.710	10
11	2.332	0.4289	0.06008	0.14008	16.645	7.139	11
12	2.518	0.3971	0.05270	0.13270	18.877	7.536	12
13	2.720	0.3677	0.04652	0.12652	21.495	7.904	13
14	2.937	0.3405	0.04130	0.12130	24.215	8.244	14
15	3.172	0.3152	0.03883	0.11683	27.152	8.559	15
16	3.426	0.2919	0.03298	0.11298	30.324	8.851	16
17	3.700	0.2703	0.02983	0.10963	33.750	9.122	17
18	3.996	0.2502	0.02670	0.10670	37.450	9.372	18
19	4.316	0.2317	0.02413	0.10413	41.446	9.604	19
20	4.661	0.2145	0.02185	0.10185	45.762	9.818	20
21	5.034	0.1987	0.01983	0.09983	50.423	10.017	21

22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	06	95	100
10.201	10.371	10.529	10.675	10.810	10.935	11.051	11.158	11.258	11.350	11.435	11.514	11.587	11.655	11.925	12.108	12.233	12.319	12.377	12.416	12.443	12.461	12.474	12.482	12.488	12.492	12.494
55.457	60.893	66.765	73.106	79.954	87.351	95.339	103.966	113.283	123.348	134.214	145.951	158.627	172.317	259.057	386.506	573.770	848.923	1253.213	1847.248	2720.080	4002.557	5886.935	8655.706	12723.939	18701.507	27484.516
0.09803	0.09842	0.09498	0.09368	0.09251	0.09145	0.09049	0.08962	0.08883	0.08811	0.08745	0.08685	0.08630	0.08580	0.08386	0.08259	0.08174	0.08118	0.08080	0.08054	0.08037	0.08025	0.08017	0.08012	0.08008	0.08005	0.08004
0.01803	0.01642	0.01498	0.01368	0.01251	0.01145	0.01048	0.00962	0.00883	0.00811	0.00745	0.00685	0.00630	0.00580	0.00386	0.00258	0.00174	0.00118	0.00080	0.00054	0.00037	0.00025	0.00017	0.00012	0.00008	0.00005	0.00004
0.1839	0.1703	0.1577	0.1460	0.1352	0.1252	0.1159	0.1073	0.0994	0.0920	0.0852	0.0789	0.0730	0.0676	0.0460	0.0313	0.0213	0.0145	0.0099	0.0067	0.0046	0.0031	0.0021	0.0014	0.0010	0.0007	0.0005
5.437	5.781	6.341	6.848	7.396	7.988	8.627	8.317	10.063	10.868	11.737	12.676	13.690	14.785	21.725	31.920	46.902	68.914	101.257	148.780	218.606	321.205	471.955	693.456	1018.915	1497.121	2199.761
22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	06	95	100

9%							
	To find F , given P : $(1 + i)^n$	To find <i>P</i> , given <i>F</i> :	To find <i>A</i> , given <i>F</i> :	To find <i>A</i> , given <i>P</i> :	To find <i>F,</i> given <i>A</i> :	To find <i>P</i> , given <i>A</i> :	
		$\frac{1}{(1+i)^n}$	$\frac{i}{(1+i)^n-1}$	$\frac{i(1+i)^n}{(1+i)^n-1}$	$\frac{(1+i)^n-1}{i}$	$\frac{(1+i)^n-1}{i(1+i)^n}$	
и	$(f/p)_n^9$	$(p/f)_n^9$	$(alf)_n^9$	$(a/p)_{n}^{9}$	$(f/a)_n^9$	$(p/a)_n^9$	u
	1.090	0.9174	1.00000	1.09000	1.000	0.917	1
2	1.188	0.8417	0.47847	0.56847	2.090	1.759	7
ю	1.295	0.7722	0.30505	0.39505	3.278	2.531	ю
4	1.412	0.7084	0.21867	0.30867	4.573	3.240	4
ŋ	1.539	0.6499	0.16709	0.25709	5.985	3.890	5
9	1.677	0.5963	0.13292	0.22292	7.523	4.486	9
4	1.828	0.5470	0.10869	0.19869	9.200	5.033	4
8	1.993	0.5019	0.09067	0.18067	11.028	5.535	8
6	2.172	0.4604	0.07680	0.16680	13.021	5.995	6
10	2.367	0.4224	0.06582	0.15582	15.193	6.418	10
11	2.580	0.3875	0.05695	0.14695	17.560	6.805	11
12	2.813	0.3555	0.04965	0.13965	20.141	7.161	12
13	3.066	0.3262	0.04357	0.13357	22.953	7.487	13
14	3.342	0.2992	0.03843	0.12843	26.019	7.786	14
15	3.642	0.2745	0.03406	0.12406	29.361	8.061	15
16	3.970	0.2519	0.03030	0.12030	33.003	8.313	16
17	4.328	0.2311	0.02705	0.11705	36.974	8.544	17
18	4.717	0.2120	0.02421	0.11421	41.301	8.756	18
19	5.142	0.1945	0.02173	0.11173	46.018	8.950	19
20	5.604	0.1784	0.01955	0.10955	51.160	9.129	20
21	6.109	0.1637	0.01762	0.10762	56.765	9.292	21

22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	06	95	100
9.442	9.580	9.707	9.823	9.929	10.027	10.116	10.198	10.274	10.343	10.406	10.464	10.518	10.567	10.757	10.881	10.962	11.014	11.048	11.070	11.084	11.094	11.100	11.104	11.106	11.108	11.109
62.873	69.532	76.790	84.701	93.324	102.723	112.968	124.135	136.308	149.575	164.037	179.800	196.982	215.711	337.882	525.859	815.084	1260.092	1944.792	2998.288	4619.223	7113.232	10950.556	16854.444	25939.000	39917.378	61422.544
0.10590	0.10438	0.10302	0.10181	0.10072	0.09973	0.09885	0.09806	0.09734	0.09669	0.09610	0.09556	0.09508	0.09464	0.09296	0.09190	0.09123	0.09079	0.09051	0.09033	0.09022	0.09014	0.09009	0.09006	0.09004	0.09003	0.09002
0.01590	0.01438	0.01302	0.01180	0.01072	0.00973	0.00885	0.00806	0.00734	0.00669	0.00610	0.00556	0.00508	0.00464	0.00296	0.00190	0.00123	0.00079	0.00051	0.00033	0.00022	0.00014	0.0000	0.00006	0.00004	0.00003	0.00002
0.1502	0.1378	0.1264	0.1160	0.1064	0.0976	0.0895	0.0822	0.0754	0.0691	0.0634	0.0582	0.0534	0.0490	0.0318	0.0207	0.0134	0.0087	0.0057	0.0037	0.0024	0.0016	0.0010	0.0007	0.0004	0.0003	0.0002
6.659	7.258	7.911	8.623	9.399	10.245	11.167	12.172	13.268	14.462	15.763	17.182	18.728	20.414	31.409	48.327	74.358	114.408	176.031	270.864	416.730	641.191	986.552	1517.948	2335.501	3593.513	5529.089
22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	55	60	65	70	75	80	85	06	95	100

10%	To find E	To find D	To find A	To find 4	To find E	To find D	
	given P: $(1 + i)^n$	given F:	given F:	given P:	given A:	given A:	
		$\frac{1}{(1+i)^n}$	$i \over (1+i)^n - 1$	$\frac{i(1+i)^n}{(1+i)^n-1}$	$\frac{(1+i)^n-1}{i}$	$\frac{(1+i)^n-1}{i(1+i)^n}$	
u	$(f/p)_{n}^{10}$	$(p/f)_{n}^{10}$	$(a f)_n^{10}$	$(a/p)_{n}^{10}$	$(f/a)_{n}^{10}$	$(p/a)_{n}^{10}$	u
-	1.100	0.9091	1.00000	1.10000	1.000	606.0	1
2	1.210	0.8264	0.47619	0.57619	2.100	1.736	7
ю	1.331	0.7513	0.30211	0.40211	3.310	2.487	ю
4	1.464	0.6830	0.21547	0.31547	4.641	3.170	4
IJ	1.611	0.6209	0.16380	0.26380	6.105	3.791	IJ
9	1.772	0.5645	0.12961	0.22961	7.716	4.355	9
4	1.949	0.5132	0.10541	0.20541	9.487	4.868	4
8	2.144	0.4665	0.08744	0.18744	11.436	5.335	8
6	2.358	0.4241	0.07364	0.17364	13.579	5.759	6
10	2.594	0.3855	0.06275	0.16275	15.937	6.144	10
11	2.853	0.3505	0.05396	0.15396	18.531	6.495	11
12	3.138	0.3186	0.04676	0.14676	21.384	6.814	12
13	3.452	0.2897	0.04078	0.14078	24.523	7.103	13
14	3.797	0.2633	0.03575	0.13575	27.975	7.367	14
15	4.177	0.2394	0.03147	0.13147	31.772	7.606	15
16	4.595	0.2176	0.02782	0.12782	35.950	7.824	16
17	5.054	0.1978	0.02466	0.12466	40.545	8.022	17
18	5.560	0.1799	0.02193	0.12193	45.599	8.201	18
19	6.116	0.1635	0.01955	0.11955	51.159	8.363	19
20	6.727	0.1486	0.01746	0.11746	57.275	8.514	20
21	7.400	0.1351	0.01562	0.11562	64.002	8.649	21

	$\infty \bigcirc \neg \land \land \land \land \neg \land \land$		2 1 2 3 4 7 5 3 4 7 5 3 4 7 5 3 4 7 5 3 4 7 5 3 4 7 5 5 5 4 7 5 5 5 5 5 5 5 5 5 5 5 5 5	82 24 25 77 25 38 43 26 27 27 28 28 29 20 00 20 20 20 20 20 20 20 20 20 20 20	0 0 7 2 3 8 3 7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0 0 2 3 5 4 2 8 3 6 3 0 8 0 0 2 3 5 4 2 3 8 6 3 7 2 3 8 6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	8 2 3 3 5 4 2 2 3 8 5 3 5 3 6 0 0 3 3 6 0 0 0 0 0 0 0 0 0 0 0 0 0	88 16 10 00 20 30 57 20 38 59 57 10 00 10 16 10 00 20 30 57 20 20 38 59 57 20 20 20 20 20 20 20 20 20 20 20 20 20	88 00 01 01 02 03 03 05 05 05 05 05 05 05 05 05 05 05 05 05	88 00 22 27 22 38 43 41 00 20 33 24 27 22 38 20 20 33 24 27 22 38	88 27 0 0 26 0 27 2 27 2 28 4 29 6 29 6 29 7 29 7 20 2 29 7 20 2 29 7 20 2 29 7 20 2 20 2 20 2 20 2 20 2 20 2 20 2 20	88 27 2 2 16 1 6 0 2 3 5 4 2 2 2 2 2 2 2 2 2 2 3 5 5 3 5 5 5 5 5 5	8 2 2 2 2 4 2 6 2 8 2 7 2 8 8 7 1 0 8 8 9 2 4 6 6 6 6 7 8 9 7 2 8 8 7 2 8 8 7 8 6 6 6 6 6 7 9 6 6 6 6 6 7 9 7 9 6 6 6 6	91.0 91.0
.0916 109.1 .0826 121.1 .0825 121.1 .0673 134.2 .06673 148.6 .0608 164.4 .0550 181.9	.0916 109.18 .0826 121.10 .0825 134.21 .0673 148.63 .0668 164.49 .0608 181.94 .0550 181.94 .0497 201.13	.0916 109.182 .0826 121.100 .0745 134.210 .0673 148.631 .0608 164.494 .0550 181.943 .0497 201.138 .0450 222.252	.0916 109.182 .0826 121.100 .0745 134.210 .06673 148.631 .0608 164.494 .0550 181.943 .0497 201.138 .0450 222.252 .0407 245.477 .0407 245.477 .0407 245.477 .0369 271.024	.0916 109.182 .0826 121.100 .0745 124.210 .06673 148.631 .06608 164.494 .0550 181.943 .0497 201.138 .0450 222.252 .0407 245.477 .0369 271.024 .0226 442.593	.0916 109.182 .0826 121.100 .0745 134.210 .0673 148.631 .0668 164.494 .0550 181.943 .0497 201.138 .0450 222.252 .0369 242.533 .0369 242.533 .0369 271.024 .0139 718.905 .0139 718.905	.0916 109.182 .0826 121.100 .0745 124.10 .0673 148.631 .0668 164.494 .06550 181.943 .0497 201.138 .0450 222.252 .04607 245.477 .0369 245.477 .0369 245.477 .0369 271.024 .0139 273.024 .0086 1163.909	.0916 109.182 .0826 121.100 .0673 134.210 .0673 148.631 .0663 164.494 .06550 181.943 .06550 181.943 .06497 201.138 .0497 201.138 .0497 201.138 .0407 222.252 .0407 245.477 .0369 245.477 .0369 271.024 .0136 1163.905 .00139 1163.905 .0053 1880.591	.0916 109.182 .0826 121.100 .0745 134.210 .0673 148.631 .0608 164.494 .0650 181.943 .0497 201.138 .0450 222.252 .0407 245.477 .0369 271.024 .0139 718.905 .0053 180.591 .0053 3034.816	.0916 109.182 .0826 121.100 .0745 134.210 .0673 148.631 .0608 164.494 .06550 181.943 .06497 201.138 .0450 222.252 .0369 271.024 .0139 718.905 .0133 7163.909 .0033 3034.816 .0020 4893.707	.0916 109.182 .0826 121.100 .0745 134.210 .0673 148.631 .0608 164.494 .0608 164.494 .06550 181.943 .0497 201.138 .0450 222.252 .0369 271.024 .0369 271.024 .0013 718.905 .0033 3034.816 .0013 7887.470 .0013 7887.470	(0916 109.182 0826 121.100 0745 134.210 0673 148.631 06673 148.631 0668 134.210 06673 148.631 0668 181.943 07450 201.138 0497 201.138 0407 245.477 0369 242.593 0139 718.905 0139 718.905 0133 3034.816 00053 1880.591 00013 7887.470 00013 7887.470 00013 708.954	(0916 109.182 0826 121.100 0745 134.210 0673 148.631 0608 181.943 06550 181.943 0497 201.138 0407 201.138 0407 201.138 0369 181.943 01350 245.477 0369 271.024 01369 271.024 0133 2442.593 01139 718.905 01033 303.4.816 00053 1880.591 00033 303.4.816 00013 7789.591 00013 7789.591 00005 20474.002	.0916 109.182 .0826 121.100 .0673 134.210 .0673 148.631 .0608 134.210 .0608 181.943 .06570 181.943 .06550 181.943 .0497 201.138 .0450 222.252 .0369 271.024 .0369 271.024 .0136 271.024 .0036 1163.909 .0013 718.905 .0020 4893.707 .0013 7887.470 .0013 7383.4.816 .0013 7383.4.816 .0013 7383.4.905 .0013 7387.470 .0013 7289.591 .0013 72708.954 .0003 32979.690 .0003 32979.690	.0916 109.182 .0826 121.100 .0673 134.210 .0673 148.631 .0608 134.210 .0608 1494 .0550 181.943 .0551 181.943 .0552 201.138 .0497 201.138 .0369 227.252 .0407 245.477 .0369 271.024 .0139 271.024 .0036 271.024 .0053 1163.909 .0139 718.905 .0013 718.905 .0033 3034.816 .0003 3034.816 .0003 7887.470 .0003 32979.690 .0003 32979.690 .0003 32979.690
.00826 0.10826 11 .00745 0.10745 11 .00673 0.10673 1- .00608 0.10608 1- .00550 0.10550 1-	.00826 0.10826 11 .00745 0.10745 11 .007673 0.10673 12 .00668 0.10668 14 .00550 0.10650 14 .00550 0.10650 12	.00826 0.10826 11 .00745 0.10745 11 .00673 0.10673 11 .00663 0.10673 11 .00663 0.10673 11 .00663 0.10673 11 .006608 0.10673 11 .00550 0.10650 11 .00550 0.10550 12 .00497 0.10497 22 .00450 0.10450 22	.00826 0.10826 11 .00745 0.10745 11 .00745 0.10745 11 .00673 0.10673 11 .00663 0.10663 11 .00560 0.10668 11 .00550 0.10650 11 .00497 0.10497 21 .00450 0.10447 22 .00407 0.10447 22 .00369 0.10369 2	.00826 0.10826 11 .00745 0.10745 11 .00673 0.10745 11 .00663 0.10673 14 .00608 0.10668 14 .00650 0.10650 11 .00647 0.10647 21 .00497 0.10457 22 .00467 0.10450 22 .00369 0.10369 22 .00369 0.10369 22 .00369 0.10369 23	.00826 0.10826 11 .00745 0.10745 11 .00673 0.10673 14 .006608 0.10663 14 .00550 0.10668 14 .00647 0.10668 14 .00550 0.10668 14 .00450 0.10450 22 .00369 0.10467 22 .00369 0.10369 27 .00369 0.10369 27 .00139 0.10139 7	0.00826 0.10826 1.00745 1.00745 1.00745 1.00673 0.10673 1.00673 1.00673 1.00068 <td>.00826 0.10826 11 .00745 0.10745 11 .00745 0.10673 11 .00663 0.10673 11 .00668 0.10668 11 .00550 0.10650 11 .00550 0.10650 11 .00550 0.10497 21 .00450 0.10450 22 .00407 0.10467 22 .00369 0.10369 22 .000369 0.10369 27 .000369 0.10139 7 .000369 0.10036 11 .000369 0.10369 23 .000369 0.10369 11</td> <td>0.00826$0.10826$$1.00745$$1.00745$$1.00745$$1.00673$$0.10673$$1.00673$$0.10673$$1.16008$$0.00608$$0.106608$$0.10650$$116008$$1160086$$0.00497$$0.10497$$2.001497$$2.000407$$0.00407$$0.10447$$2.010467$$2.000369$$0.00369$$0.104407$$2.000369$$0.100369$$0.000226$$0.100369$$0.10036$$2.000086$$0.00053$$0.10053$$0.10053$$1160068$$0.00033$$0.10033$$0.10033$$3.00003$</td> <td>0.00826$0.10826$$1.10745$$1.10745$$1.10745$$1.100673$$1.10673$$1.10608$$1.00608$$0.10673$$0.10673$$1.100086$$1.100086$$1.000450$$0.10497$$0.10497$$2.2$$0.00467$$0.10467$$2.2$$0.10369$$2.2$$0.00369$$0.10467$$2.2$$0.10369$$2.2$$0.00369$$0.10369$$0.10369$$2.7$$0.00366$$0.100369$$0.100369$$1.1$$0.00086$$0.10033$$0.10033$$1.8$$0.00033$$0.10033$$0.10033$$3.0$$0.00033$$0.10033$$0.10033$$3.0$</td> <td>0.00826$0.10826$$1.10745$$1.10745$$1.10745$$1.100603$$0.00673$$0.10673$$1.10603$$1.100033$$0.006497$$0.106608$$1.10497$$2.1000450$$0.00450$$0.104407$$0.104407$$2.2000369$$0.00369$$0.102407$$2.2000369$$0.10369$$0.00369$$0.10226$$4.400033$$2.100036$$0.00033$$0.100369$$0.100369$$1.100033$$0.00033$$0.10033$$0.10033$$3.000033$$0.00003$$0.100133$$0.10013$$7.70033$$0.00003$$0.100133$$0.100133$$7.80033$</td> <td>0.00826$0.10826$$1.00745$$0.10745$$1.00745$$0.00745$$0.10673$$1.1673$$1.160083$$0.00550$$0.10608$$0.10608$$1.000450$$0.00450$$0.104497$$2.2000369$$0.104607$$0.00369$$0.104607$$2.2000369$$0.100369$$0.00369$$0.100369$$0.10026$$4.000139$$0.00033$$0.10036$$0.10033$$300000033$$0.00013$$0.10013$$0.10013$$2.8000033$$0.00013$$0.10013$$0.10013$$2.8000033$$0.00013$$0.10013$$0.10013$$2.8000033$$0.00013$$0.100013$$0.100013$$2.8000033$</td> <td>0.00826$0.10826$$1.00745$$0.10745$$1.00745$$1.00745$$1.00673$$0.10673$$1.00608$$1.000608$$1.000608$$1.000608$$1.000608$$1.000450$$0.100497$$2.00000000000$$2.000000000000000000000000000000000000$</td> <td>0.00826$0.10826$$1.00745$$0.10745$$1.00745$$1.00745$$1.00673$$0.10673$$1.00673$$1.00608$$0.00550$$0.10608$$0.10650$$1.000450$$0.10497$$2.000450$$0.00450$$0.104407$$2.0010407$$2.000369$$0.10407$$2.000369$$0.00226$$0.10369$$0.10026$$4.2$$0.00226$$0.100369$$0.10139$$7.7$$0.00226$$0.10033$$0.10033$$300$$0.00033$$0.10033$$0.10033$$300$$0.00033$$0.10033$$0.10013$$78$$0.00033$$0.10013$$0.10013$$78$$0.00013$$0.10013$$0.10003$$204$$0.00003$$0.10003$$0.10003$$204$$0.00003$$0.10003$$0.10003$$329$$0.00003$$0.10003$$0.10003$$329$</td> <td>0.00826$0.10826$$1.00745$$0.10745$$1.00745$$1.00745$$1.00673$$0.10673$$1.00673$$1.00608$$1.00050$$0.10668$$1.000369$$1.000369$$0.10497$$2.2$$0.00450$$0.10467$$0.10497$$2.2$$2.000369$$0.10369$$2.2$$0.00369$$0.10369$$0.100369$$2.100226$$4.4$$0.00326$$0.100369$$0.100369$$1.100266$$4.4$$0.000226$$0.100369$$0.100369$$1.100033$$3.0000033$$0.00033$$0.10033$$0.10033$$3.000033$$0.100033$$2.0467$$0.000133$$0.100033$$0.100033$$0.100033$$2.0467$$0.000033$$0.100033$$0.100033$$0.100033$$2.046$$0.000033$$0.100033$$0.100033$$2.046$$0.000033$$0.100033$$0.100033$$2.046$$0.000033$$0.100033$$0.100033$$2.046$$0.000033$$0.100033$$0.100033$$2.046$$0.000033$$0.100033$$0.100033$$2.046$$0.000033$$0.100033$$0.100033$$2.046$$0.000033$$0.100033$$0.100033$$2.046$</td>	.00826 0.10826 11 .00745 0.10745 11 .00745 0.10673 11 .00663 0.10673 11 .00668 0.10668 11 .00550 0.10650 11 .00550 0.10650 11 .00550 0.10497 21 .00450 0.10450 22 .00407 0.10467 22 .00369 0.10369 22 .000369 0.10369 27 .000369 0.10139 7 .000369 0.10036 11 .000369 0.10369 23 .000369 0.10369 11	0.00826 0.10826 1.00745 1.00745 1.00745 1.00673 0.10673 1.00673 0.10673 1.16008 0.00608 0.106608 0.10650 116008 1160086 0.00497 0.10497 2.001497 2.000407 0.00407 0.10447 2.010467 2.000369 0.00369 0.104407 2.000369 0.100369 0.000226 0.100369 0.10036 2.000086 0.00053 0.10053 0.10053 1160068 0.00033 0.10033 0.10033 3.00003	0.00826 0.10826 1.10745 1.10745 1.10745 1.100673 1.10673 1.10608 1.00608 0.10673 0.10673 1.100086 1.100086 1.000450 0.10497 0.10497 2.2 0.00467 0.10467 2.2 0.10369 2.2 0.00369 0.10467 2.2 0.10369 2.2 0.00369 0.10369 0.10369 2.7 0.00366 0.100369 0.100369 1.1 0.00086 0.10033 0.10033 1.8 0.00033 0.10033 0.10033 3.0 0.00033 0.10033 0.10033 3.0	0.00826 0.10826 1.10745 1.10745 1.10745 1.100603 0.00673 0.10673 1.10603 1.100033 0.006497 0.106608 1.10497 2.1000450 0.00450 0.104407 0.104407 2.2000369 0.00369 0.102407 2.2000369 0.10369 0.00369 0.10226 4.400033 2.100036 0.00033 0.100369 0.100369 1.100033 0.00033 0.10033 0.10033 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0.10013 0.10003 204 0.00003 0.10003 0.10003 204 0.00003 0.10003 0.10003 329 0.00003 0.10003 0.10003 329	0.00826 0.10826 1.00745 0.10745 1.00745 1.00745 1.00673 0.10673 1.00673 1.00608 1.00050 0.10668 1.000369 1.000369 0.10497 2.2 0.00450 0.10467 0.10497 2.2 2.000369 0.10369 2.2 0.00369 0.10369 0.100369 2.100226 4.4 0.00326 0.100369 0.100369 1.100266 4.4 0.000226 0.100369 0.100369 1.100033 3.0000033 0.00033 0.10033 0.10033 3.000033 0.100033 2.0467 0.000133 0.100033 0.100033 0.100033 2.0467 0.000033 0.100033 0.100033 0.100033 2.046 0.000033 0.100033 0.100033 2.046 0.000033 0.100033 0.100033 2.046 0.000033 0.100033 0.100033 2.046 0.000033 0.100033 0.100033 2.046 0.000033 0.100033 0.100033 2.046 0.000033 0.100033 0.100033 2.046 0.000033 0.100033 0.100033 2.046
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0.00745 0.1074 0.00673 0.1067 0.00608 0.1060 0.00550 0.1055	0.00745 0.10745 0.00673 0.10677 0.00608 0.10607 0.00550 0.10550 0.00497 0.1049	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
0.00673 0.10673 0.00608 0.10608 0.00550 0.10550	0.00673 0.10673 0.00608 0.10608 0.00550 0.10550 0.00497 0.10497	0.00673 0.10673 0.00608 0.10608 0.00550 0.10550 0.00497 0.10497 0.00450 0.10450	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{llllllllllllllllllllllllllllllllllll$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{llllllllllllllllllllllllllllllllllll$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.00673 0.10673 0.00608 0.10673 0.00550 0.10608 0.00497 0.10497 0.00450 0.10450 0.00467 0.10457 0.00407 0.10450 0.00407 0.10450 0.00369 0.10369 0.00369 0.10369 0.00369 0.10369 0.00326 0.10369 0.00139 0.10036 0.000133 0.10033 0.00020 0.10033 0.00013 0.10033 0.00013 0.10033 0.00008 0.10003 0.00008 0.100033 0.00008 0.100033	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{llllllllllllllllllllllllllllllllllll$
0.00550 0.10550 1	0.00550 0.10550 1 0.00497 0.10497 2	0.00550 0.10550 1 0.00497 0.10497 2 0.00450 0.10450 2	0.00550 0.10550 1 0.00497 0.10497 2 0.00450 0.10450 2 0.004407 0.104607 2 0.00369 0.10369 2	0.00550 0.10550 1 0.00497 0.10497 2 0.00450 0.10450 2 0.00450 0.10460 2 0.00369 0.10407 2 0.00369 0.10369 2 0.00226 0.10226 4	0.00550 0.10550 1 0.00497 0.10497 2 0.00450 0.10450 2 0.00407 0.10450 2 0.00369 0.10369 2 0.00226 0.10226 4 0.00139 0.10139 7	0.00550 0.10550 1 0.00457 0.10497 2 0.00450 0.10450 2 0.00407 0.10467 2 0.00369 0.10369 2 0.00226 0.101369 2 0.00139 0.10139 7 0.00086 0.10086 11	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
	0.00497 0.10497 201.	0.00497 0.10497 201. 0.00450 0.10450 222.	0.00497 0.10497 201. 0.00450 0.10450 222. 0.00407 0.10407 245. 0.00369 0.10369 271.	0.00497 0.10497 201. 0.00450 0.10450 222. 0.00407 0.10407 245. 0.00369 0.10369 271. 0.00226 0.10226 442.	0.00497 0.10497 201. 0.00450 0.10450 222. 0.00407 0.10407 245. 0.00369 0.10369 271. 0.00226 0.10226 442. 0.00139 0.10139 718.	0.00497 0.10497 201. 0.00450 0.10450 222. 0.00407 0.10407 245. 0.00369 0.10369 271. 0.00226 0.10226 442. 0.00139 0.10139 718. 0.00086 0.10086 1163.	0.00497 0.10497 201. 0.00450 0.10450 222. 0.00407 0.10407 245. 0.00369 0.10369 271. 0.00226 0.10136 271. 0.00139 0.10139 718. 0.00086 0.10086 1163. 0.00053 0.10053 1880.	0.00497 0.10497 201. 0.00450 0.10450 222. 0.00467 0.10467 245. 0.00369 0.10369 271. 0.00326 0.10369 271. 0.00139 0.10226 442. 0.00139 0.10139 718. 0.00053 0.10053 1880. 0.00033 0.10033 3034.	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.00497 0.10497 201. 0.00450 0.10450 222. 0.00407 0.10407 245. 0.00369 0.10407 245. 0.00369 0.10369 271. 0.00226 0.10226 442. 0.00139 0.10139 718. 0.00053 0.10053 1163. 0.00053 0.10053 3034. 0.00020 0.10013 7887. 0.00013 0.10013 7887. 0.00013 0.10013 7887. 0.00008 0.10013 7887.	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

To find P , given A : $(1+i)^n - 1$ $i(1+i)^n$	$(p/a)_n^{12}$ <i>n</i>	0.893 1	1.690 2	2.402 3	3.037 4	3.605 5	4.111 6	4.564 7	4.968 8	5.328 9	5.650 10	5.938 11	6.194 12		6.424 13	6.628 13 6.628 14	6.628 14 13	6.424 13 6.628 14 6.811 15	6.424 13 6.628 14 6.811 15 6.974 16	6.424 13 6.628 14 6.811 15 6.974 16 7.120 17	6.424 13 6.628 14 6.811 15 6.974 16 7.120 17 7.250 18	6.424 13 6.628 14 6.811 15 6.974 16 7.120 17 7.250 18 7.366 19	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
To find F , given A : $(1+i)^n - 1$ i	$(f/a)_{n}^{12}$	1.000	2.120	3.374	4.779	6.353	8.115	10.089	12.300	14.776	17.549	20.655	24.133		28.029	28.029 32.393	28.029 32.393	28.029 32.393 37.280	28.029 32.393 37.280 42.753	28.029 32.393 37.280 42.753 48.884	25.029 32.393 37.280 42.753 48.884 55.750	28.029 32.393 37.280 42.753 48.84 55.750 63.440	28.029 32.393 37.280 42.753 48.884 55.750 63.440 63.440
To find A_r given P : $\frac{i(1+i)^n}{(1+i)^n-1}$	$(a/p)_{n}^{12}$	1.12000	0.59170	0.41635	0.32923	0.27741	0.24323	0.21912	0.20130	0.18768	0.17698	0.16842	0.16144	0.15568		0.15087	0.15087	0.15087 0.14682	0.15087 0.14682 0.14339	0.15087 0.14682 0.14339 0.14046	0.15087 0.14682 0.14339 0.14046 0.13794	0.15087 0.14682 0.14682 0.14046 0.13794 0.13776	0.15087 0.14682 0.14682 0.14046 0.13794 0.13576 0.13388
To find A , given F : i $(1+i)^n - 1$	$(a/f)_{n}^{12}$	1.00000	0.47170	0.29635	0.20923	0.15741	0.12323	0.09912	0.08130	0.06768	0.05698	0.04842	0.04144	0.03568		0.03087	0.03087	0.03087 0.02682	0.03087 0.02682 0.02339	0.03087 0.02682 0.02339 0.02046	0.03087 0.02682 0.02339 0.02339 0.02046 0.01794	0.03087 0.02682 0.02339 0.02046 0.01794 0.01576	0.03087 0.02682 0.02339 0.02046 0.01794 0.01576 0.01388
To find P , given F : $\frac{1}{(1+i)^n}$	$(p/f)_{n}^{12}$	0.8929	0.7972	0.7118	0.6355	0.5674	0.5066	0.4523	0.4039	0.3606	0.3220	0.2875	0.2567	0.2292		0.2046	0.2046	0.2046 0.1827	0.2046 0.1827 0.1631	0.2046 0.1827 0.1631 0.1456	0.2046 0.1827 0.1631 0.1456 0.1300	0.2046 0.1827 0.1631 0.1456 0.1456 0.1300 0.1161	0.2046 0.1827 0.1631 0.1456 0.1300 0.1161 0.1161 0.1037
To find F , given P : $(1 + i)^n$	$(f/p)_{n}^{12}$	1.120	1.254	1.405	1.574	1.762	1.974	2.211	2.476	2.773	3.106	3.479	3.896	4.363	100 4	4.00/	4.00/	4.887 5.474	4.887 5.474 6.130	4.887 5.474 6.130 6.866	4.887 5.474 6.130 6.866 7.690	4.887 5.474 6.130 6.866 7.690 8.613	4.887 5.474 6.130 6.866 7.690 9.646
0/ 71	и	1	2	С	4	ß	9	4	8	6	10	11	12	13	14	-	H	15	15 16	15 16 17	15 16 17 18	15 16 17 18 19	15 16 17 18 19 20

22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	
7.645	7.718	7.784	7.843	7.896	7.943	7.984	8.022	8.055	8.085	8.112	8.135	8.157	8.176	8.244	8.283	8.305	
92.503	104.603	118.155	133.334	150.334	169.374	190.699	214.582	241.333	271.292	304.847	342.429	384.520	431.663	767.091	1358.230	2400.018	
0.13081	0.12956	0.12846	0.12750	0.12665	0.12590	0.12524	0.12466	0.12414	0.12369	0.12328	0.12292	0.12260	0.12232	0.12130	0.12074	0.12042	
0.01081	0.00956	0.00846	0.00750	0.00665	0.00590	0.00524	0.00466	0.00414	0.00369	0.00328	0.00292	0.00260	0.00232	0.00130	0.00074	0.00042	
0.0826	0.0738	0.0659	0.0588	0.0525	0.0469	0.0419	0.0374	0.0334	0.0298	0.0266	0.0238	0.0212	0.0189	0.0107	0.0061	0.0035	
12.100	13.552	15.179	17.000	19.040	21.325	23.884	26.750	29.960	33.555	37.582	42.091	47.142	52.800	93.051	163.988	289.002	
22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	

Appendix B

15%	To find <i>F</i> ,	To find <i>P</i> ,	To find <i>A</i> ,	To find <i>A</i> ,	To find <i>F</i> ,	To find <i>P</i> ,	
	given <i>P</i> : $(1 + i)^n$	given F:	given F:	given <i>P</i> :	given A:	given A:	
		$\frac{1}{(1+i)^n}$	$\frac{i}{(1+i)^n-1}$	$\frac{i(1+i)^n}{(1+i)^n-1}$	$\frac{(1+i)^n-1}{i}$	$\frac{(1+i)^n-1}{i(1+i)^n}$	
и	$(f/p)_{n}^{15}$	$(p/f)_{n}^{15}$	$(a f)_{n}^{15}$	$(a/p)_{n}^{15}$	$(f/a)_{n}^{15}$	$(p/a)_{n}^{15}$	u
1	1.150	0.8696	1.00000	1.15000	1.000	0.870	1
7	1.322	0.7561	0.46512	0.61512	2.150	1.626	2
ю	1.521	0.6575	0.28798	0.43798	3.472	2.283	ю
4	1.749	0.5718	0.20027	0.35027	4.993	2.855	4
ß	2.011	0.4972	0.14832	0.29832	6.742	3.352	5
9	2.313	0.4323	0.11424	0.26424	8.754	3.784	9
~	2.660	0.3759	0.09036	0.24036	11.067	4.160	7
8	3.059	0.3269	0.07285	0.22285	13.727	4.487	8
6	3.518	0.2843	0.05957	0.20957	16.786	4.772	6
10	4.046	0.2472	0.04925	0.19925	20.304	5.019	10
11	4.652	0.2149	0.04107	0.19107	24.349	5.234	11
12	5.350	0.1869	0.03448	0.18448	29.002	5.421	12
13	6.153	0.1625	0.02911	0.17911	34.352	5.583	13
14	7.076	0.1413	0.02469	0.17469	40.505	5.724	14
15	8.137	0.1229	0.02102	0.17102	47.580	5.847	15
16	9.358	0.1069	0.01795	0.16795	55.717	5.954	16
17	10.761	0.0929	0.01537	0.16537	65.075	6.047	17
18	12.375	0.0808	0.01319	0.16319	75.836	6.128	18
19	14.232	0.0703	0.01134	0.16134	88.212	6.198	19
20	16.367	0.0611	0.00976	0.15976	102.444	6.259	20
21	18.821	0.0531	0.00842	0.15842	118.810	6.312	21

22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	
6.359	6.399	6.434	6.464	6.491	6.514	6.534	6.551	6.566	6.579	6.591	6.600	6.609	6.617	6.642	6.654	6.661	
137.631	159.276	184.168	212.793	245.711	283.569	327.104	377.170	434.745	500.956	577.099	664.664	765.364	881.170	1779.090	3585.128	7217.716	
0.15727	0.15628	0.15543	0.15470	0.15407	0.15353	0.15306	0.15265	0.15230	0.15200	0.15173	0.15150	0.15131	0.15113	0.15056	0.15028	0.15014	
0.00727	0.00628	0.00543	0.00470	0.00407	0.00353	0.00306	0.00265	0.00230	0.00200	0.00173	0.00150	0.00131	0.00113	0.00056	0.00028	0.00014	
0.0462	0.0402	0.0349	0.0304	0.0264	0.0230	0.0200	0.0174	0.0151	0.0131	0.0114	0.0099	0.0086	0.0075	0.0037	0.0019	0.0009	
21.645	24.891	28.625	32.919	37.857	43.535	50.066	57.575	66.212	76.143	87.565	100.700	115.805	133.176	267.863	538.769	1083.657	
22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	

20%							
	To find F , given P : $(1 + i)^n$	To find <i>P</i> , given <i>F</i> :	To find <i>A</i> , given <i>F</i> :	To find <i>A</i> , given <i>P</i> :	To find <i>F,</i> given <i>A</i> :	To find <i>P</i> , given A:	
		1	i	$i(1+i)^n$	$(1+i)^n - 1$	$(1 + i)^n - 1$	
		$(1+i)^n$	$(1+i)^n-1$	$(1+i)^n-1$	1:	$i(1+i)^n$	
и	$(f/p)_{n}^{20}$	$(p/f)_n^{20}$	$(a/f)_{n}^{20}$	$(a/p)_{n}^{20}$	$(f/a)_n^{20}$	$(p/a)_n^{20}$	и
1	1.200	0.8333	1.00000	1.20000	1.000	0.833	1
0	1.440	0.6944	0.45455	0.65455	2.200	1.528	2
ю	1.728	0.5787	0.27473	0.47473	3.640	2.106	ю
4	2.074	0.4823	0.18629	0.38629	5.368	2.598	4
IJ	2.488	0.4019	0.13438	0.33438	7.442	2.991	IJ
9	2.986	0.3349	0.10071	0.30071	9.930	3.326	9
~	3.583	0.2791	0.07742	0.27742	12.916	3.605	7
8	4.300	0.2326	0.06061	0.26061	16.499	3.837	8
6	5.100	0.1938	0.04808	0.24808	20.799	4.031	6
10	6.192	0.1615	0.03852	0.23852	25.959	4.192	10
11	7.430	0.1346	0.03110	0.23110	32.150	4.327	11
12	8.916	0.1122	0.02526	0.22526	39.581	4.439	12
13	10.699	0.0935	0.02062	0.22062	48.497	4.533	13
14	12.839	0.0779	0.01689	0.21689	59.196	4.611	14
15	15.407	0.0649	0.01388	0.21388	72.035	4.675	15
16	18.488	0.0541	0.01144	0.21144	87.442	4.730	16
17	22.186	0.0451	0.00944	0.20944	105.931	4.775	17
18	26.623	0.0376	0.00781	0.20781	128.117	4.812	18
19	31.948	0.0313	0.00646	0.20646	154.740	4.843	19
20	38.338	0.0261	0.00536	0.20536	186.688	4.870	20

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	
4.891	4.909	4.925	4.937	4.948	4.956	4.964	4.970	4.975	4.979	4.982	4.985	4.988	4.990	4.992	4.997	4.999	4.999	
225.025	271.031	326.237	392.484	471.981	567.377	681.853	819.223	984.068	1181.881	1419.257	1704.108	2045.930	2456.116	2948.339	7343.858	18281.331	45497.191	
0.20444	0.20369	0.20307	0.20255	0.20212	0.20176	0.20147	0.20122	0.20102	0.20085	0.20070	0.20059	0.20049	0.20041	0.20034	0.20014	0.20005	0.20002	
0.00444	0.00369	0.00307	0.00255	0.00212	0.00176	0.00147	0.00122	0.00102	0.00085	0.00070	0.00059	0.00049	0.00041	0.00034	0.00014	0.00005	0.00002	
0.0217	0.0181	0.0151	0.0126	0.0105	0.0087	0.0073	0.0061	0.0051	0.0042	0.0035	0.0029	0.0024	0.0020	0.0017	0.0007	0.0003	0.0001	
46.005	55.206	66.247	79.497	95.396	114.475	137.371	164.845	197.813	237.376	284.851	341.822	410.186	492.223	590.668	1469.772	3657.258	9100.427	
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	40	45	50	



OPEC proven crude oil reserves, end 2010 (billions barrels)

Venezuela	296.50	24.8%	Iraq	143.10	12.0%	Libya	47.10	3.9%	Algeria	12.20	1.0%
Saudi Arabia	264.52	22.2%	Kuwait	101.50	8.5%	Nigeria	37.20	3.1%	Angola	9.50	0.8%
Iron, I.R.	151.17	12.7%	United Arab Emirates	97.80	8.2%	Qatar	25.38	2.1%	Ecuador	7.21	0.5%

COLOR FIGURE 1.2

OPEC share of world oil reserves end 2010. (From *OPEC Annual Statistical Bulletin*, Vienna, 2011. With permission.)



COLOR FIGURE 1.3

Production of refined products (mb/d), 1960–2010. (From OPEC Annual Statistical Bulletin, Vienna, 2010. With permission.)



COLOR FIGURE 1.6

Proven gas reserves (trillion cm), 1960–2010. (From *OPEC Annual Statistical Bulletin*, Vienna, 2011. With permission.)



COLOR FIGURE 3.1 Semi-integrated chemical-fuel refinery.



COLOR FIGURE 4.2

Values of the deposit for simple and compound interest after 4 years.



COLOR FIGURE 4.3

Interest at the end of one year for compounded monthly and semiannual interest.



COLOR FIGURE 4.4 Change in future values of \$100 principal with time, using different types of interest.



COLOR FIGURE 5.4 Annual depreciation per year.



COLOR FIGURE 5.7 Comparison for the first year.



COLOR FIGURE 5.8 Comparison for the second year.

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COLOR FIGURE 9.7

Solution of Example 9.7 by Microsoft Excel Solver.

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COLOR FIGURE 9.8

Solution of Example 9.7 by Microsoft Excel Solver (final).



COLOR FIGURE 15.1 Water-in-oil emulsion.



* Includes refinery fuel oil.

** Includes bitumen, lubricants, waxes, still gas, coke, sulphur, direct use of crude oil, etc.

COLOR FIGURE 18.3

Worldwide demand for refined products: 2010 and 2035 in million b/d. (From OPEC, Oil Demand by Product, *World Oil Outlook*, Vienna, 2011. With permission.)



COLOR FIGURE 18.4

Regional refining margins (in U.S. \$/b) for refineries in the U.S. Gulf Coast, North West Europe, and Singapore. (From Leighton, P., Potential of Integrated Facilities—Finding Value Addition, *World Refining Association: Petchem Arabia*, 4th Annual Meeting, Abu Dhabi, October 2009. With permission.)



COLOR FIGURE 20.1 Oil tanker fleet, million DWT, 1976-2012

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