Bond Evaluation, Selection, and Management

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Second Edition

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To Jan

To all my current and past students at Xavier University and to the students and alums of the Xavier Student Investment Fund

Contents

Preface

In 1985, Merrill Lynch introduced the liquid yield option note (LYON). The LYON is a zero-coupon bond, convertible into the issuer's stock, callable (with the call price increasing over time), and putable (with the put price increasing over time). The LYON is a good example of how innovative the investment community can be in structuring debt instruments with option clauses. Probably nowhere has this innovation been more pervasive than in the construction of mortgage-backed securities (MBSs). Faced with the problem of prepayment risk, various types of mortgage securities with different claims (sequential-pay tranches, planned amortization classes, interest-only MBSs, and principal-only MBSs) were created in the 1980s. The option features embedded in MBSs, LYONs, and many other bonds have made the evaluation of these securities more difficult. A callable 10-year bond issued when interest rates are relatively high may be more like a three-year bond, given that a likely interest rate decrease would lead the issuer to buy the bond back. Determining the value requires taking into account not only the value of the bond's cash flows, but also the value of the call option embedded in the bond.

In addition to the innovations in financial instruments, the investment and management of bonds and debt by financial and non-financial corporations has also experienced significant developments over the last two decades. Today, bond investors use strategies such as cash-flow matching, immunization, cell matching, contingent immunization, and bond selection based on forecasting yield curve shifts or the narrowing or widening of the quality yield spread. Similarly as striking as the growth in bond evaluation and management over the last twenty years is the growth in derivative products. Since the 1970s, the U.S. economy has experienced relatively sharp swings in interest rates. The resulting volatility in rates, in turn, has increased the exposure of many debt positions to market risk. Faced with this risk, many corporate borrowers, money managers, intermediaries, and bond portfolio mangers have increased their use of futures and options contracts on debt securities as a hedge against such risk. Today, futures and options contracts on debt securities, as well as such hybrid derivatives as swaps, interest rate options, caps, and floors, are used by banks and financial intermediaries to manage the maturity gaps between loans and deposits, by corporations and financial institutions to fix or cap the rates on future loans, and by fixed-income portfolio managers, money managers, investment bankers, and security dealers in locking in the future purchase or selling price on their fixed-income securities.

Many economists argue that the 2008–2009 financial crisis and recession was a correction to a major overshooting of the U.S. and world economies. From 2000–2006, expansionary U.S. monetary actions, China's large investment in U.S. Treasury securities, and liberal credit policies by financial institutions led to excessive overshooting, especially in the housing industry. This ultimately led to too many risky mortgage loans whose default eventually led to the subprime mortgage meltdown of 2007 and the near collapse of the financial system providing those loans and the overall economy. The overshooting may also have been the result of the myriad financial innovations that were introduced over the last three decades.

Today, understanding the dynamic and innovative fixed-income investment environment requires that finance students and professionals understand the markets for an increasing number of debt securities, the process of securitization and how securitized derivatives are formed and valued, and the fundamental, as well as advanced, bond investment strategies. The purpose of this book is to provide finance students and professionals with a bond and debt management exposition that will take them from the basic bond investment theories and fundamentals that can be found in many investment books to a more detailed understanding of the markets and strategies. It is my hope that this synthesis of fundamental and advanced topics will provide students of finance with a better foundation in understanding the complexities and subtleties involved in the evaluation and selection of bonds and debt positions with detailed structures. The book is written for MBA, MS, and advanced undergraduate finance students, as a training and instructional source for those involved in bond and debt management, and as a reference for CFA preparation for investment professionals. As a text, the book is designed for a one-semester course. Undergraduate students should have had an introductory course in investments, and MBA students should have had an introductory corporate finance course. Some basic statistics and math are used. At the back of the text are appendixes on exponents and logarithms, statistics, and the time value of money to help students who need a review.

OVERVIEW OF THE CONTENTS

All securities can be evaluated in terms of the characteristics common to all assets: value, return, risk, maturity, marketability, liquidity, and taxability. In Part 1, debt securities are analyzed in terms of these characteristics. Chapter 1 presents an overview of the investment environment, examining the nature of financial assets, the types of securities that exist, the nature and types of markets that securities give rise to, and the general characteristics of assets. With this background, the next four chapters examine bonds in terms of their characteristics: Chapter 2 looks at how debt instruments are valued and how their rates of return are measured; Chapters 3 and 4 examine the level and term structure of interest rates and show how such factors as market expectations, economic conditions, and risk-return preferences are important in determining the level and structure of rates; Chapter 5 describes three types of bond risk—default, call, and market risk—and introduces two measures of bond volatility: duration and convexity.

Part 2 delineates the different debt securities and their markets in terms of the rules, participants, and forces that govern them. Chapter 6 describes the debt claims of businesses; Chapters 7 and 8 look at the types and markets for government securities—Treasury, federal agencies, and municipals; Chapters 9 and 10 examine intermediary and foreign debt securities. In Chapters 11 and 12, mortgage-backed and asset-backed securities are examined. Chapter 11 looks at mortgage-backed securities and explains prepayment risk and how mortgage-backed derivatives are constructed to address the problems of prepayment risk. Chapter 12 looks at the market for other asset-backed securities: commercial mortgage-backed securities, receivable-backed securities, and collateralized debt obligations.

Part 3 examines bond strategies and the valuation of bonds with embedded options. In Chapter 13, bond analysis is extended from evaluation to investment and management by examining a number of active and passive bond management strategies, including cash matching, bond immunization, indexing, and contingent immunization. With many bonds having some option clause, the binomial interest rate approach to valuing bonds has become an important pricing model for bonds. Chapter 14 describes how the binomial interest rate tree is used to price bonds with call and put options, sinking fund agreements, convertible clauses, and prepayment options. Chapter 15, in turn, addresses the technical problem of how to construct the tree, examining two models that have been used to estimate the binomial interest rate trees—the arbitrage-free calibration model and the equilibrium model.

Part 4 consists of four chapters covering bond derivatives. Chapters 16 and 17 provide overviews of the markets, uses, and pricing of interest rate futures and options contracts. These chapters provide a foundation for the more detailed analysis of hedging and debt management strategies using futures and options contracts that are examined in Chapters 18 and 19. In those chapters, futures and options contracts on debt securities are examined in terms of how they are used by banks and financial intermediaries to manage the maturity gaps between loans and deposits, by corporations and financial institutions to fix the rates on floating-rate loans, and by fixed-income portfolio managers, money managers, investment bankers, and security dealers in locking in the future purchase or selling price on their fixed-income securities. Part 5 completes the analysis of derivatives with three chapters devoted to swaps: interest rate swaps, swap derivatives, and currency and credit default swaps.

The book is a revised second edition that draws from the first edition and two other texts of the author: *Options and Futures* (West Publishing, 1995) and *Introduction to Derivatives* (Oxford University Press, 2009). The text stresses concepts, model construction, and numerical examples. This is done to empower the reader to understand the tools with which answers can be found rather than just the answers. A number of review questions, problems, and Web exercises are provided at the end of each chapter to reinforce concepts; solutions for many of the problems are provided at the end of the text. A number of the problems can be done using Excel. Readers, in turn, can access Excel programs for working many of those problems, as well as find chapter PowerPoint files and additional resources by going to the book's complementary Web site.

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Bond Evaluation, Selection, and Management

Bond Evaluation, Selection, and Management, Second Edition
by R. Stafford Johnson

PART One

Bond Evaluation

CHAPTER 1

Overview of the Financial System

1.1 REAL AND FINANCIAL ASSETS

Most new businesses begin when an individual or a group of individuals come up with an idea: manufacturing a new type of computer, developing land for a future housing subdivision, or launching a new Internet company. To make the idea a commercial reality, though, requires funds that the individual or group generally lacks or personally does not want to commit. Consequently, the fledgling business sells ***financial claims or instruments*** to raise the funds necessary to buy the capital goods (equipment, land, etc.), as well as the human capital (architects, engineers, lawyers, etc.) needed to launch the project. Technically, such instruments are claims against the income of the business represented by a certificate, receipt, or other legal document. In this process of initiating and implementing the idea, both real and financial assets are therefore created. The ***real assets*** consist of both the tangible and intangible capital goods, as well as human capital, which are combined with labor to form the business. The business, in turn, transforms the idea into the production and sale of goods or services that will generate a future stream of earnings. The ***financial assets***, on the other hand, consist of the financial claims on the earnings. Those individuals or institutions that provided the initial funds and resources hold these assets. Furthermore, if the idea is successful, then the new business may find it advantageous to initiate other new projects that it again may finance through the sale of financial claims. Thus, over time, more real and financial assets are created.

The creation of financial claims, of course, is not limited to the business sector. The federal government's expenditures on national defense and the space program and state governments' expenditures on the construction of highways, for example, represent the development of real assets that these units of government often finance through the sale of financial claims on either the revenue generated from a particular public sector project or from future tax revenues. Similarly, the purchase of a house or a car by a household often is financed by a loan from a savings and loan or commercial bank. The loan represents a claim by the financial institution on a portion of the borrower's future income, as well as a claim on the ownership of the real asset (house or car) in the event the household defaults on its promise.

Modern economies expend enormous amounts of money on real assets to maintain their standards of living. Such expenditures usually require funds that are beyond the levels a business, household, or unit of government has or wants to commit at a given point in time. As a result, to raise the requisite amounts, economic entities sell financial claims. Those buying the financial claims therefore supply funds to the economic entity in return for promises that the entity will provide them with a future flow of income. As such, financial claims can be described as financial assets.

All financial assets provide a promise of a future return to the owners. Unlike real assets, though, financial assets do not depreciate (since they are in the form of certificates or information in a computer file), and they are ***fungible***, meaning they can be converted into cash or other assets. There are many different types of financial assets. All of them, though, can be divided into two general categories—equity and debt. Common stock is the most popular form of equity claim. It entitles the holder to dividends or shares in the business's residual profit and participation in the management of the firm, usually indirectly through voting rights. The stock market, where existing stock shares are traded, is the most widely followed market in the world and it receives considerable focus in many investments and securities analysis texts. The focus of this book, though, is on the other general type of financial asset—debt. Businesses finance more of their real assets and operations with debt than equity, whereas governments and households finance their entire real assets and operations with debt. This chapter provides a preliminary overview of the types of debt securities and markets, whereas Chapters 6–12 provide a more detailed analysis.

1.2 TYPES OF DEBT CLAIMS

Debt claims are loans whereby the borrower agrees to pay a fixed income per period, defined as a coupon or interest, and to repay the borrowed funds, defined as the principal (also called redemption value, maturity value, par value, and face value). Within this broad description, debt instruments can take on many different forms. For example, debt can take the form of a loan by a financial institution such as a commercial bank, insurance company, or savings and loan bank. In this case, the terms of the agreement and the contract instrument generally are prepared by the lender/creditor, and the instrument often is nonnegotiable, meaning it cannot be sold to another party. A debt instrument also can take the form of a bond or note, whereby the borrower obtains her loan by selling (also referred to as issuing) contracts or IOUs to pay interest and principal to investors/lenders. Many of these claims, in turn, are negotiable, often being sold to other investors before they mature.

Debt instruments also can differ in terms of the features of the contract: the number of future interest payments, when and how the principal is to be paid—at maturity (i.e., the end of the contract) or spread out over the life of the contract (amortized)—and the recourse the lender has should the borrower fail to meet her contractual commitments (i.e., collateral or security). For many debt instruments, standard features include the following:

1. **Term to Maturity:** Number of years over which the issuer promises to meet the obligations. (Maturity refers to the date that the debt will cease to exist.) Generally, bonds with maturities between 1 and 5 years are considered short term; those with maturities between 5 and 12 years are considered intermediate-term; and those with maturities greater than 12 years are considered long term.
2. **Principal:** The amount that the issuer/borrower agrees to repay the bond-holder/lender.

3. **Coupon Rate** (or Nominal Rate): The rate the issuer/borrower agrees to pay each period. The dollar amount is called the coupon. There are, though, zero coupon bonds in which the investor earns interest between the price paid and the principal, and floating-rate notes where the coupon rate is reset periodically based on a formula.
4. **Amortization:** The principal repayment of a bond can be repaid either at maturity or over the life of the bond. When principal is repaid over the life of the bond, there is a schedule of principal repayments. The schedule is called the amortization schedule. Securities with an amortization schedule are called amortizing securities, whereas securities without an amortized schedule (those paying total principal at maturity) are called nonamortizing securities.
5. **Embedded Options:** Bonds often have embedded option features in their contracts, such as a call feature giving the issuer the right to buy back the bond from the bondholder before maturity at a specific price—a ***callable bond.***

Finally, the type of borrower or issuer—business, government, household, or financial institution—can differentiate the debt instruments. Businesses sell three general types of debt instruments, ***corporate bonds, medium-term notes,*** and ***commercial paper***, and borrow from financial institutions, usually with long-term or intermediate-term loans from commercial banks or insurance companies and with short-term ***lines of credit*** from banks. The corporate bonds they sell usually pay the buyer/lender coupon interest semiannually and a principal at maturity. For example, a manufacturing company building a $10 million processing plant might finance the cost by selling 10,000 bonds at a price of $1,000 per bond, with each bond promising to pay $50 in interest every June 15th and January 15th for the next 10 years and a principal of $1,000 at maturity. In general, corporate bonds are long-term securities, sometimes secured by specific real assets that bondholders can claim in case the corporation fails to meet its contractual obligation (defaults). Corporate bonds also have a priority of claims over stockholders on the company's earnings and assets in the case of default. Medium-term notes (MTNs) issued by a corporation are debt instruments sold through agents on a continuing basis to investors who are allowed to choose from a group of bonds from the same corporation, but with different maturities and features. Such instruments allow corporations flexibility in the way in which they can finance different capital projects. Commercial paper is a short-term claim (less than one year) that usually is unsecured. Typically, commercial paper is sold as a zero-discount note in which the buyer receives interest equal to the difference between the principal and the purchase price. For example, a company might sell paper promising to pay $1,000 at the end of 270 days for $970, yielding a dollar return of $30. Term loans to businesses have intermediate- to long-term maturities, often with the principal amortized. Like all debt instruments, these loans have a priority of claims on income and assets over equity claims, and the financial institution providing the loan often requires collateral. Finally, lines of credit are short-term loans provided by banks and other financial institutions in which the business can borrow up to a maximum amount of funds from a checking account created for it by the institution.

The federal government sells a variety of financial instruments, ranging from short-term ***Treasury bills*** to intermediate- and long-term ***Treasury notes*** and ***Treasury bonds***. These instruments are sold by the Treasury to finance the federal deficit and to refinance current debt. In addition to Treasury securities, agencies of the federal government, such as the Tennessee Valley Authority, and government-sponsored corporations, such as the Federal National Mortgage Association and the Federal Farm Credit Banks, also issue securities, classified as ***Federal Agency Securities***, to finance a variety of government programs ranging from the construction of dams to the purchase of mortgages to provide liquidity to mortgage lenders. The agency sector includes securities issued by federal agencies and also federally related institutions, referred to as ***government-sponsored enterprises***. Similarly, state and local governments, agencies, and authorities also offer a wide variety of debt instruments, broadly classified as either ***general obligation bonds*** or ***revenue bonds***. The former are bonds financed through general tax revenue, whereas the latter are instruments financed from the revenue from specific state and local government projects and programs.

Finally, there are financial intermediaries such as commercial banks, savings and loans, credit unions, savings banks, insurance companies, and investment funds that provide debt claims. These intermediaries sell or provide financial claims to investors, and then use the proceeds to purchase debt and equity claims or to provide direct loans. In general, financial institutions, by acting as intermediaries, control a large amount of funds and thus have a significant impact on financial markets. For borrowers, intermediaries are an important source of funds; they buy many of the securities issued by corporations and governments and provide many of the direct loans. For investors, intermediaries create a number of securities for them to include in their short-term and long-term portfolios. These include negotiable certificates of deposit, bankers' acceptances, mortgage-backed instruments, asset-backed securities, collateralized debt obligation, investment fund shares, annuities, and guaranteed investment contracts.

1.3 FINANCIAL MARKET

Markets are conduits through which buyers and sellers exchange goods, services, and resources. In an economy there are three types of markets: a product market where goods and services are traded, a factor market where labor, capital, and land are exchanged, and a financial market where financial claims are traded. The financial market, in turn, channels the savings of households, businesses, and governments to those economic units needing to borrow.

The financial market can be described as a market for loanable funds. The supply of loanable funds comes from the savings of households, the retained earnings of businesses, and the surpluses of governments. The demand for loanable funds emanates from businesses who need to raise funds to finance their capital purchases of equipment, plants, and inventories; households who need to purchase houses, cars, and other consumer durables; and the Treasury, federal agencies, and municipal governments who need to finance the construction of public facilities, projects, and operations. The exchange of loanable funds from savers to borrowers is done either directly through the selling of financial claims (stock, bonds, commercial paper, etc.) or indirectly through financial institutions.

The financial market facilitates the transfer of funds from ***surplus economic units*** to ***deficit economic units***. A surplus economic unit is an entity whose income from its current production exceeds its current expenditures; it is a saver or net lender. A deficit unit, on the other hand, is an entity whose current expenditures exceed its income from its current production; it is a net borrower. Although businesses, households, and governments fluctuate from being deficit units one period to surplus units in another period, on average, households tend to be surplus units whereas businesses and government units tend to be deficit units. A young household usually starts as a deficit unit as it acquires homes and cars financed with mortgages and auto loans. In its midlife, the household's income usually is higher and its mortgage and other loans are often paid; at that time the household tends to become a surplus unit, purchasing financial claims. Finally, near the end of its life, the household lives off the income from its financial claims. In contrast, businesses tend to invest or acquire assets that cost more than the earnings they retain. As a result, businesses are almost always deficit units, borrowing or selling bonds and stocks; furthermore, they tend to remain that way throughout their entire life. Similarly, the federal government's expenditures on defense, education, and welfare have more often exceeded its revenues from taxes. Thus, the federal government, as well as state and local units, tend to be deficit units.

1.4 TYPES OF FINANCIAL MARKETS

Financial markets can be classified in terms of whether the market is for new or existing claims (primary or secondary market), for short-term or long-term instruments (money or capital market), for direct or indirect trading between deficit and surplus units (direct or intermediary market), for domestic or foreign securities, and for immediate, future, or optional delivery (cash, futures, or options markets).

Primary and Secondary Market

The ***primary market*** is that market where financial claims are created. It is the market in which new securities are sold for the first time. Thus, the sale of new government securities by the U.S. Treasury to finance a government deficit, or a $100 million bond issue by Procter and Gamble to finance the construction of a new soap manufacturing plant, are examples of securities transactions occurring in the primary market. The principal function of the primary market is to raise the funds needed to finance investments in new plants, equipment, inventories, homes, roads, and the like—it is where capital formation begins.

The ***secondary market*** is the market for the buying and selling of existing assets and financial claims. Its economic function is to provide marketability—ease or speed in trading a security. Given the accumulation of financial claims over time, the volume of trading on the secondary market far exceeds the volume in the primary market. The buying and selling of existing securities is done primarily through a network of brokers and dealers who operate through organized securities exchanges and the ***over-the-counter*** (***OTC***) market. Brokers and dealers serve the function of bringing buyers and sellers together by finding opposite positions or by taking positions in a security. By definition, ***brokers*** are agents who bring securities buyers and sellers together for a commission. ***Dealers***, in turn, provide markets for investors to buy and sell securities by taking a temporary position in a security; they buy from investors who want to sell and sell to those who want to buy. Dealers receive compensation in terms of the spread between the ***bid price*** at which they buy securities and the ***asked price*** at which they sell securities. Whereas brokers and dealers serve the function of bringing buyers and sellers together, exchanges serve the function of linking brokers and dealers together to buy and sell existing securities. In the United States, there is the ***New York Stock Exchange (NYSE)*** and several regional organized exchanges. Outside the United States, there are major exchanges in such cities as London, Tokyo, Hong Kong, Singapore, Sydney, and Paris. In addition to organized exchanges, a large number of existing securities and a large proportion of bonds are traded on the OTC market.

New York Stock Exchange The NYSE was formed in 1792 by a group of merchants who wanted to trade notes and bonds. Since then it has grown to an exchange in which stocks and a limited number of bonds, Exchange-Traded Funds (ETFs), and other securities are traded. The NYSE can be described as a corporate association consisting of member brokers. Most brokerage firms with membership (seats) on the NYSE function as commission brokers, executing buy and sell orders on behalf of their clients. The NYSE and a number of other organized exchanges provide a continuous market. A continuous market attempts to have constant trading in a security. To have such a feature, time discrepancies caused by different times when investors want to sell and when others want to buy must be eliminated or at least minimized. In a continuous market this is accomplished by having ***specialists*** or ***designated market makers (DMMs)***. Specialists and DMMs are dealers who are part of the exchange and who are required by the exchange to take opposite positions in a security if conditions dictate. Under a specialist system, the exchange board assigns a specific security to a specialist to deal. In this role, a specialist acts by buying the stock from sellers at low bid prices and selling to buyers at (they hope!) higher asked prices. Specialists and DMMs quote a bid price to investors when selling the security and an asked price to investors interested in buying. They hope to profit from the difference between the bid and asked prices; that is, the ***bid-asked spread***. In addition to dealing, the NYSE and other exchanges using a specialist system also require that the specialists maintain the ***limit order book*** (which appears on their computer screens) on the securities they are assigned and that they execute these orders. A ***limit order*** is an investor's request to his broker to buy or sell a security at a given price or better. On the NYSE, such orders are taken by commission brokers and left with the specialist in that security for execution.[1]

In April 2007, the NYSE became part of NYSE Euronext, a holding company created by combining the NYSE Group, Inc. and Euronext N.V. NYSE Euronext can be described as a transatlantic exchange group that brings together six equities exchanges and six derivatives exchanges, providing physical and electronic trading in stocks, bonds, and derivatives. In the United States, NYSE Euronext includes the NYSE physical exchange and ***NYSE Arca***. NYSE Arca is a fully electronic stock exchange, trading more than 8,000 exchange-listed equity securities. NYSE Arca's trading platform links traders to multiple U.S. market centers and provides customers with fast electronic execution and open, direct, and anonymous market access. NYSE Arca's functions are based on a price-time priority system.[2]

Over-the-Counter Market The OTC market is an informal exchange for the trading of stocks, corporate and municipal bonds, investment fund shares, asset-backed securities, shares in limited partnerships, and Treasury and federal agency securities. It can be described as a fragmented, noncentralized market of brokers and dealers linked to each other by a computer, telephone, and telex communication system. To trade, dealers must register with the ***Securities and Exchange Commission (SEC)***. As dealers, they can quote their own bid and asked prices on the securities they deal, and as brokers, they can execute a trade with a dealer providing a quote. The securities traded on the OTC market are those in which a dealer decides to take a position. Dealers on the OTC market range from regional brokerage houses making a market in a local corporation's stocks or bonds, to large financial companies (such as Merrill Lynch) making markets in Treasury securities, to investment bankers dealing in the securities they had previously underwritten, to dealers in federal agency securities and municipal bonds. Like the specialist on the organized exchanges, each dealer maintains an inventory in a security and quotes a bid and an asked price at which she is willing to buy and sell. Initially, the ***National Association of Securities Dealers (NASD)*** regulated OTC trading. In July 2007, the ***Financial Industry Regulatory Authority*** (***FINRA***), the largest independent regulator for all securities firms doing business in the United States, consolidated NASD and the member regulation, enforcement, and arbitration functions of the NYSE. Although no physical exchange exists, communication among brokers and dealers takes place through a computer system known as the ***National Association of Securities Dealers Automatic Quotation System, NASDAQ***. NASDAQ is an information system in which current bid-asked quotes of dealers are offered, and also a system that sends brokers' quotes to dealers, enabling them to close trades.[3]

Electronic Trading Market There are several other types of secondary market trading for stock. For example, the NYSE features both a physical auction convened by DMMs and a completely automated auction that includes algorithmic quotes from DMMs and other participants. As noted, NYSE Arca is an electronic stock exchange, trading more than 8,000 exchange-listed (NASDAQ included) equity securities. NYSE Euronext also has ArcaEdge, which is an all-electronic matching system trading OTC stocks. The ArcaEdge platform offers best-price executions based on liquidity, transparency, speed and anonymity. There are also ***crossing network*** and ***electronic communication network (ECN)*** systems. The crossing network system allows institutional investors to cross order, matching buy and sell orders directly via computers. The ECN is a privately-owned broker-dealer network that operates with NASDAQ.

Secondary Market for Bonds The secondary market for bonds in the United States and throughout the world is not centralized, but rather is part of the OTC market. As noted, the OTC market consists of a network of noncentralized or fragmented market makers who provide bid and offer quotes for each issue they participate in. There are some corporate bonds that are listed on physical exchanges. Such bonds are sometimes said to be trading in the "Bond Room." Although they may be listed, they are more likely to be traded via dealers on the OTC market than on the exchange. There is also a transition to electronic trading. For example, NYSE Euronext recently began offering an all-electronic platform for trading NYSE bonds based on a price-time priority system. There are developing multi-dealer systems that allow customers to execute bond trades from multiple quotes. The systems display the best bid or offer prices of those posted by all dealers. The participating dealers usually act as the principal in the transaction. There are also developing single-dealer systems that allow investors to execute transactions directly with the specific dealers desired.

WEB INFORMATION

For information on NYSE Euronext, go to www.nyse.com.

For information on the OTC market, go to www.finra.org/index.htm and www.nasdaq.com.

For financial information on securities, market trends, and analysis, see:

- www.Finance.Yahoo.com
- www.hoovers.com
- www.bloomberg.com
- www.businessweek.com
- www.ici.org
- http://seekingalpha.com
- http://bigcharts.marketwatch.com
- www.morningstar.com
- http://free.stocksmart.com
- http://online.wsj.com/public/us

Direct and Intermediate Financial Markets

In addition to being classified as primary or secondary, markets for financial instruments can also be classified in terms of being either part of the direct financial market or the intermediary financial market.

Direct Financial Market The *direct financial market* is where surplus units purchase claims issued by the ultimate deficit unit. This market includes the trading of stocks, corporate bonds, Treasury securities, federal agency securities, and municipal bonds. The claims traded in the direct financial market are referred to as ***primary securities***.[4]

As is the case with many securities markets, the direct financial market can be divided into primary and secondary markets. The secondary market for direct financial claims takes place in both the organized exchanges and the OTC market just discussed. In the primary market, new securities are sold either in a negotiated market or an open market. In a ***negotiated market***, the securities are issued to one or just a few economic entities under a private contract. Such sales are referred to as a ***private placement***. In an open market transaction, the securities are sold to the public at large. The key participant in ***open market trades*** is the ***investment banker***. The investment banker is a middleperson or matchmaker who, for a fee or share in

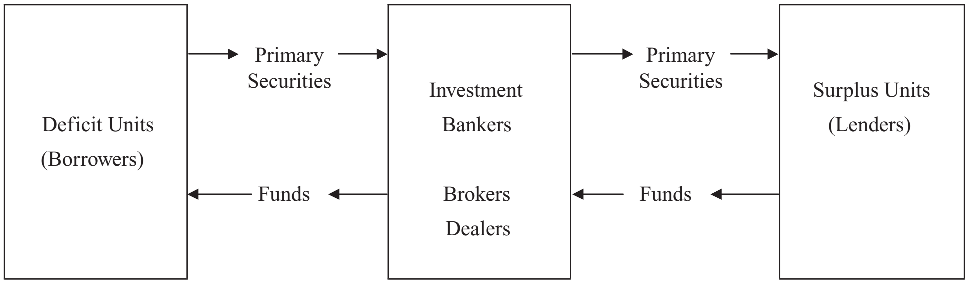

FIGURE 1.1 Direct Financial Market

the trading profit, finds surplus units who want to buy the security being offered by a deficit unit (see Figure 1.1). The major investment bankers include such firms as Merrill Lynch and Goldman Sachs. Investment bankers sell a security issue for the issuer for a commission (i.e., for a percentage of the total issue's value) using their ***best effort***, ***underwrite*** the securities (i.e., buy the securities from the issuer and then sell them at hopefully a higher price), or form an ***underwriting syndicate*** whereby a group of investment bankers buys and sells the issue. Whatever the arrangements, the primary function of the investment banker is to match the needs of the surplus and deficit units. By performing this function the investment banker reduces the search and information costs to both the investors and the issuer, facilitating the efficient operation of the primary market.

Intermediary Financial Market The intermediary financial market consists of financial institutions such as commercial banks, savings and loans, credit unions, insurance companies, pension funds, trust funds, and mutual funds. In this market, the financial institution, as shown in Figure 1.2, sells financial claims (checking accounts, savings accounts, certificates of deposit, mutual fund shares, payroll deduction plans, insurance plans, and the like) to surplus units, and uses the proceeds to purchase claims (stocks, bonds, etc.) issued by ultimate deficit units or to create financial claims in the form of term loans, lines of credit, and mortgages. Through their intermediary function, financial institutions in turn create intermediate securities, referred to as ***secondary securities***.

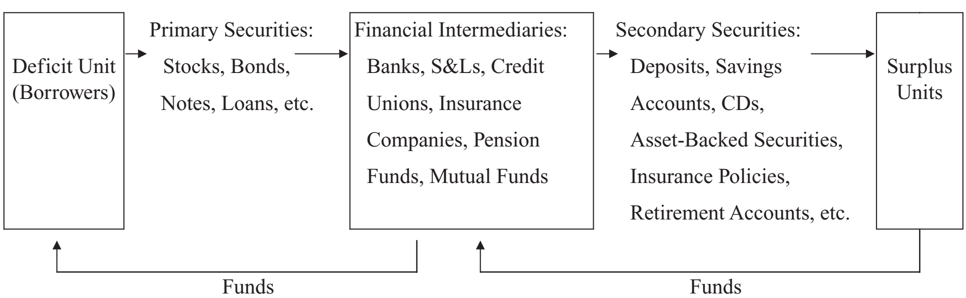

FIGURE 1.2 Intermediary Financial Market

Financial institutions can be divided into three categories: ***depository institutions, contractual institutions,*** and ***investment companies***. Depository institutions include commercial banks, credit unions, savings and loans, and savings banks. These institutions obtain large amounts of their funds from deposits, which they use primarily to fund commercial and residential loans and to purchase Treasury, federal agency, and municipal securities. Contractual institutions include life insurance companies, property and casualty insurance companies, and pension funds. They obtain their funds from legal contracts to protect businesses and households from risk (premature death, accidents, etc.) and from savings plans. Investment companies include mutual funds, money market funds, and real estate investment trusts. These institutions raise funds by selling equity or debt claims, and then use the proceeds to buy debt securities, stocks, real estate, and other assets. The claims they sell entitle the holder/buyer either to a fixed income each period or a pro rata share in the ownership and earnings generated from the asset fund. Also included with investment company securities are ***securitized assets***. Banks, insurance companies, and other financial intermediaries, as well as federal agencies, sell these financial assets. In creating a securitized asset, an intermediary will put together a package of loans of a certain type (mortgages, auto loans, credit cards, etc.). The institution then sells claims on the package to investors, with the claim being secured by the package of assets—securitized assets. The package of loans, in turn, generates interest and principal that is passed on to the investors who purchased the securitized asset.[5]

Some of the financial claims created in the intermediary financial market do not have a secondary market; that is, secondary markets where investors sell their bank saving accounts or insurance or pension plans to other investors are rare. However, there are secondary markets for many intermediary securities: negotiable certificates of deposit, mutual fund shares, and securitized assets.

WEB INFORMATION

Data on most financial intermediaries is prepared by the Federal Reserve and is published in the ***U.S. Flow of Funds*** report. The report can be accessed from www.federalreserve.gov/releases (click "Flow of Funds Account" tab).

For additional information on investment funds, see the Investment Company Institute's Web site: www.ici.org.

Money and Capital Markets

Financial markets can also be classified in terms of the maturity of the instrument traded. Specifically, the ***money market*** is defined as the market where short-term instruments (by convention defined as securities with original maturities of one year or less) are traded, and the ***capital market*** is defined as the market where long-term securities (original maturities over one year) are traded. The former includes such securities as certificates of deposit, commercial paper, Treasury bills, savings accounts, and shares in money market investment funds, whereas the latter includes corporate bonds, municipal bonds, securitized assets, Treasury bonds, and investment fund shares, as well as corporate stock. Investors with long-term liabilities or investment horizons buy securities in the capital markets. This includes many institutional investors, such as life insurance companies and pensions. The issuers of capital market securities include corporations and governments who use the market to finance their long-term capital formation projects. Investors use the money market to earn interest on excess funds that they expect to have only temporarily. They also hold funds in money market securities as a store of value when they are waiting to take advantage of investment opportunities or when they fear precarious economic conditions are possible. The sellers of money market securities use the market to raise funds to finance their short-term assets (inventory or accounts receivable), to take care of cash needs resulting from the lack of synchronization between cash inflows and outflows from operations, or in the case of the U.S. Treasury to finance the government's deficit or to refinance its maturing debt. It should be noted that the money market functions primarily as a ***wholesale market***, in which many of the transactions are done by large banks and investment firms who buy and sell in large denominations. This feature helps to promote the popularity of money market funds. These funds pool the investments of small investors and invest them in money market securities, providing small investors an opportunity to obtain higher returns than they could obtain from individual bank savings accounts.

Foreign Securities Markets

Over the last three decades there has been a substantial growth in the number of equity and fixed-income securities traded globally. This growth in the size of world equity and debt markets is reflected by the significant increase in global securities investments among nonresidents. The popularity of global investments is generally attributed to the growing number of corporations, governments, and financial intermediaries issuing securities in foreign countries; to the emergence of currency futures, options, and swaps markets that have made it possible for investors to better manage exchange-rate risk; and to the potential diversification benefits investors can obtain by adding foreign stocks and bonds to their portfolios.

In general, an investor looking to internationally diversify his bond portfolio has several options. First, he might buy a bond of a foreign government or foreign corporation that is issued in the foreign country or traded on that country's exchange. These bonds are referred to as ***domestic bonds***.[6] Secondly, the investor might be able to buy bonds issued in a number of countries through an international syndicate. Such bonds are known as ***Eurobonds***. Finally, an investor might be able to buy a bond of a foreign government or corporation being issued or traded in his own country. These bonds are called ***foreign bonds***. If the investor were instead looking for short-term foreign investments, his choices would similarly include buying short-term domestic securities such as commercial paper, CDs, and Treasuries issued in those countries, Eurocurrency CDs issued by Eurobanks, and foreign money market securities issued by foreign corporations and governments in the local country. Similarly, a domestic financial institution or nonfinancial multinational corporation looking to raise funds may choose to do so by selling debt securities or borrowing in the company's own financial markets, the foreign markets, or the Eurobond or Eurocurrency markets. The markets where domestic, foreign, and Euro securities are issued and traded can be grouped into two categories—the ***internal bond market*** and the ***external bond market***. The internal market, also called the ***national market***, consists of the trading of both domestic bonds and foreign bonds; the external market, also called the ***offshore market***, is where Eurobonds and Eurodeposits are bought and sold.

For foreign investors, one of the most important factors to consider is that their price, interest payments, and principal are denominated in a different currency. This currency component exposes them to ***exchange rate risk*** and affects their returns and overall risk. Most of the currency trading takes place in the ***Interbank Foreign Exchange Market***. This market consists primarily of major banks that act as currency dealers, maintaining inventories of foreign currencies to sell to or buy from their customers (corporations, governments, or regional banks). The price of foreign currency or the exchange rate is defined as the number of units of one currency that can be exchanged for one unit of another. It is determined by supply and demand conditions affecting the foreign currency market.

WEB INFORMATION

For an explanation of various investments, including foreign bonds, go to www.finpipe.com.

For information on historical exchange rates and trade, go to http://research.stlouisfed.org/fred (click "exchange rate" tab).

For information on current exchange rates and foreign interest rates, go to www.fxstreet.com.

Spot, Futures, Options, and Swap Markets

A ***spot market*** (also called a ***cash market***) is one in which securities are exchanged for cash immediately (usually within one or two business days). An investor buying a Treasury bill, for example, is a transaction that takes place in the spot market. Not all securities transactions, though, call for immediate delivery. A ***futures*** or ***forward contract*** calls for the delivery and purchase of an asset (either real or financial) at a future date, with the terms (price, amount, etc.) agreed upon in the present. For example, a contract calling for the delivery of a Treasury bill in 70 days at a price equal to 97% of the bill's principal would represent a futures contract on a Treasury bill. This agreement is distinct from buying a Treasury bill from a Treasury dealer in the spot market, where the transfer of cash for the security takes place almost immediately. Similar to a futures contract, an option is a security that gives the holder the right (but not the obligation) either to buy or to sell an asset at a specific price on or possibly before a specific date. Options include calls, puts, warrants, and rights. Both futures and options are traded on organized exchanges and through dealers on the OTC market. In the United States, the major futures exchange is the Chicago Mercantile Exchange and the major option exchange is the Chicago Board of Option Exchange. Options and futures are referred to as ***derivative securities***, since their values are derived from the values of their underlying securities. In contrast, securities sold in the spot market are sometimes referred to as ***primitive securities***.

Derivative debt securities have become important to both borrowers and investors in managing the risk associated with issuing and buying fixed income securities. Part 4 of this book focuses on the markets and uses of debt derivative securities.

In addition to derivative securities, bonds often have ***embedded option*** features in their contracts. As noted earlier, many bonds have a call feature giving the issuer the right to buy back the bond from the bondholder before maturity at a specific price. In addition to these so-called callable bonds, there are putable bonds in which the bondholder has the right to sell the bond back to the issuer at a specified price, sinking fund clauses in which the issuer is required to orderly retire the bond by either buying bonds in the market or by calling them at a specified price, and convertible bonds that give the bondholder the right to convert the bond into a specified number of shares of stock. The inclusion of option features in a bond contract makes the valuation of such bonds more difficult. The valuation of bonds with embedded options is the subject of Chapters 14 and 15.

Today there is a large swap market. A ***swap*** is an exchange of cash flows: It is a legal arrangement between two parties to exchange specific payments. There are four types of swap:

1. **Interest Rate Swaps**: Exchange of fixed-rate payments for floating-rate payments
2. **Currency Swaps**: Exchange of liabilities in different currencies
3. **Cross-Currency Swaps**: Combination of interest rate and currency swap
4. **Credit Default Swaps**: Exchange of premium payments for default protection

The swap market primarily consists of financial institutions and corporations who use swap contracts to hedge more efficiently their liabilities and assets. For example, many institutions create synthetic fixed or floating-rate assets or liabilities with better rates than the rates obtained on direct liabilities and assets. The markets and uses of swaps are examined in Part 5 of this book.

WEB INFORMATION

For information on derivatives, see:

- CME Group: www.cmegroup.com.
- Chicago Board Options Exchange: www.cboe.com.

1.5 REGULATIONS

Prior to the enactment of federal securities laws in 1933 and in 1934, the regulation of securities trading in the United States came under the auspices of state governments who had passed a number of laws to prevent fraud and speculative schemes. The state securities laws, known as ***blue-sky laws***, were often hard to enforce since many fraudulent promoters could operate outside a state's jurisdiction. With the passage of the ***Securities Act of 1933*** and the ***Securities Exchange Act of 1934***, though, securities regulations came more under the providence of the federal government. The 1933 act, known as the "***truth in securities" law***, requires registration of new issues, disclosure of pertinent information by issuers, and prohibits fraud and misrepresentation. The Securities Exchange Act of 1934 established the Securities and Exchange Commission, extended the disclosure requirements of the 1933 act to include traders and participants in the secondary market, and outlawed fraud and misrepresentation in the trading of existing securities. Today, five commissioners, appointed by the president and confirmed by the Senate for five-year terms, run the SEC. The SEC is responsible for the administration of both the 1933 and 1934 acts, as well as the administration of a number of other securities laws that have been enacted since then. The 1934 act gave the SEC authority over organized exchanges. Historically, the SEC has exercised its authority by setting only general guidelines for the bylaws and rules of an exchange, allowing the exchanges to regulate themselves. The SEC does have the power, though, to intervene and change bylaws, as well as close exchanges. Exhibit 1.1 summarizes the securities acts of 1933 and 1934, and Exhibit 1.2 describes some of the other important securities laws in the United States.

The 1933 and 1934 securities acts are aimed at ensuring that information is disseminated efficiently to all investors and that fraud and misrepresentation are outlawed. There are also laws, regulations, and regulatory agencies that work to ensure the financial system is sound. Of particular note is the Federal Reserve System. Created in 1913, the Federal Reserve (Fed) is the most important central bank in the world. The Fed is responsible for managing the economy's money supply and the general level of interest rates. As we will discuss in more detail in later chapters, the Fed does this by open market operations, changing the reserve requirements banks maintain, and changing the discount rate they charge commercial banks on loans.

WEB INFORMATION

For information on the laws, regulations, and litigations of the SEC, go to www.sec.gov.

For information on monetary policy, economic data, and research from the Federal Reserve, go to www.federalreserve.gov.

Identification

There are hundreds of thousands of bonds issues. Most securities, though, can be identified by a nine-character ***CUSIP*** number. CUSIP stands for the Committee on Uniform Securities Identification Procedures. CUSIP is owned by the American Bankers Association and operated by Standard & Poor's. It is used to identify trades and for clearing. There is also a 12-character foreign securities identification system known as ***CINS***.

EXHIBIT 1.1 Securities Acts of 1933 and 1934

The Securities Act of 1933, also known as the "***truth in securities" law***, required registration of new issues and disclosure of pertinent information by issuers, and prohibited fraud and misrepresentation. To comply with this act today, a company selling securities across state lines is required to submit a prospectus and audited financial statements on the company's condition to a federal agency or the Securities and Exchange Commission. Once approved, the prospectus is sent to potential investors. Furthermore, any fraud or misrepresentation is subject to legal actions.

The Securities Exchange Act of 1934 established the ***Securities and Exchange Commission***, extended the disclosure requirements of the 1933 act to include traders and participants in the secondary market, and outlawed fraud and misrepresentation in the trading of existing securities. Today, five commissioners, appointed by the president and confirmed by the Senate for five-year terms, run the SEC. The SEC is responsible for the administration of both the 1933 and 1934 acts, as well as the administration of a number of other securities laws that have been enacted since then. The 1934 act gave the SEC authority over organized exchanges.

Financial Disclosure Requirements: To comply with the disclosure provisions of the Securities Exchange Act (and its 1964 amendments), companies listed on the exchanges and those traded on the OTC market with assets over $13 million are required to file with the SEC ***10-K reports***, which are audited financial statement forms, ***10-Q reports***, which are quarterly unaudited financial statement forms, and ***8-K forms***, which report significant developments by the company.

Fraud and Misrepresentation Provisions: The Security Exchange Act of 1934 in particular outlaws price manipulation schemes such as wash sales, pools, churning, and corners. A wash sale is a sale and subsequent repurchase of a security or purchase of an identical security. It is done in order to establish a record to show, for example, a capital loss for tax purposes or to deceive investors into thinking there is large activity on the stock. A pool is an association of people formed to manipulate the price of a security. Churning occurs when a broker manipulates his client to make frequent purchases and sales of a security in order to profit from increased commissions. A corner occurs when someone buys up all of the security (or commodity) in order to have the monopolistic power to raise its price and to pressure short sellers to sell at higher prices. An investor or group of investors who try to corner the market could do so by forming pools to manipulate the security's price. In addition to outlawing wash sales and pools, the Security Exchange Acts also require that all officers, directors, and owners of more than 10% of the company file an insider report each month in which they trade their securities. The purpose of this requirement is to eliminate an insider from profiting from inside information.

1.6 EFFICIENT FINANCIAL MARKETS

As defined earlier, an asset is any commodity, tangible or intangible good, or financial claim that generates future benefits. The value of an asset is equal to the current value of all of the asset's future expected cash flows; that is, the present value of the expected cash flow. Thus, if an investor requires a rate of return (R) of 10% per year on investments in government securities that mature in one year, he would value (V_0) a government bond promising to pay $100 interest and $1,000 principal at the end of one year as worth $1,000 today.

EXHIBIT 1.2 U.S. Federal Laws Related to Securities Trading

Glass-Steagall Act (enacted 1933; major provisions repealed 1999): The Glass-Steagall Act, also known as the Banking Act of 1933, prohibited commercial banks from acting as investment bankers. Enacted after the 1929 stock market crash, the act also prohibited banks from paying interest on demand deposits (a prohibition that was later eliminated under Monetary Control Act of 1980), and created the Federal Deposit Insurance Company. As a result of the Glass-Steagall Act, for years most commercial banks in the United States were not allowed to underwrite securities, act as brokers or dealers, or offer investment company shares. The Glass-Steagall Act also served to differentiate U.S. banking activities from those of many countries in which banks were allowed to provide investment banking and securities services (merchant banking). Recognizing these differences, the U.S. Congress repealed many of the provisions of the Glass-Steagall Act.

Financial Services Modernization (Gramm-Leach-Bliley) Act (1999): The act permits finance companies and banks to form financial holding companies to offer banking, insurance, securities, and other financial services under one controlling corporation.

Federal Reserve Regulations T and U: Regulations T and U give the Board of Governors of the Federal Reserve the authority to set margin requirements for security loans made by banks, brokers, and dealers. Regulation T sets loan limits made by brokers and dealers, and Regulation U sets loan limits made by banks for securities transactions. Since 1934, these requirements have ranged from 40% to 100%. Note: Brokerage houses and securities exchanges set maintenance margins.

Maloney Act (1936): This act requires associations such as NASD to register with the SEC and allows them to regulate themselves within general guidelines specified by the SEC.

Trust Indenture Act (1939): This act gives the SEC the authority to ensure that there are no conflicts of interest between bondholders, trustees, and issuer. The act was in response to abuses in the 1930s that resulted from the issuer having control over the trustee. Among its provisions, the act requires that the bond indenture clearly delineate the rights of the bondholders, that periodic financial reports be given to the trustee, and that the trustee act judiciously in bringing legal actions against the issuer when conditions dictate.

Investment Company Act (1940), ICA: This act extends the provisions of the securities acts of 1933 and 1934 to investment companies. Like the securities acts, it requires a prospectus to be approved and issued to investors with full disclosure of financial statements, and it outlaws fraud and misrepresentations. In addition, the act requires investment companies to state their goals (growth, balance, income, etc.), to have a management firm approved by the investment company's board, and to manage funds for the benefit of the shareholders. The 1940 act was amended in 1970 (Investment Company Amendments Act of 1970) with provisions calling for certain restrictions on management fees and contracts.

Investment Advisors Act (1940), IAA: This act requires individuals and firms providing investment advice for a fee to register with the SEC. The act does not, however, require certification of an advisor's qualifications. The act also outlaws fraud and misrepresentation.

Employee Retirement Income Security Act (1974), ERISA: This act requires that managers of pension funds adhere to the ***prudent man rule*** (a common-law principle) in managing retirement funds. When applied to investment management, this rule requires average portfolio returns and risk levels to be consistent with that of a prudent man. The objective (which is subject to legal testing) is that pension managers be adequately diversified to minimize the risk of large losses.

Sarbanes-Oxley Act of 2002: The act mandated a number of reforms to enhance corporate responsibility, enhance financial disclosure, and combat corporate and accounting fraud, and created the Public Company Accounting Oversight Board, also known as the PCAOB, to oversee the activities of the auditing profession.

$$V_0 = \frac{\text{Interest} + \text{Principal}}{1 + R} = \frac{\$100 + \$1{,}000}{1.10} = \$1{,}000$$

Similarly, an investor who expected ABC stock to pay a dividend of \$10 and to sell at a price of \$105 one year later would value the stock at \$100 if she required a rate of return of 15% per year on such investments:

$$V_0 = \frac{\text{Dividend} + \text{Expected price}}{1 + R} = \frac{\$10 + \$105}{1.15} = \$100$$

(See Appendix C for a primer on the time value of money.)

In the financial market, if stock investors expecting ABC stock to pay a \$10 dividend and be worth \$105 one year later required a 15% rate of return, then the equilibrium price of the stock in the market would be \$100. Similarly, if government bond investors required a 10% rate of return, then the equilibrium price of the government bond would be \$1,000. The equilibrium price often is ensured by the activities of ***speculators***: those who hope to obtain higher rates of return (greater than 15% in this case of the stock or 10% in the case of the bond) by gambling that security prices will move in certain directions. For example, if ABC stock sold below the \$100 equilibrium value, then speculators would try to buy the underpriced stock. As they try to do so, though, they would push the underpriced ABC stock towards its equilibrium price of \$100. On the other hand, if ABC stock were above \$100, investors and speculators would be reluctant to buy the stock, lowering its demand and the price. These actions might also be reinforced with some speculators selling the stock short. In a ***short sale***, a speculator sells the stock first and buys it later, hoping to profit, as always, by buying at a low price and selling at a high one. For example, if ABC stock is selling at \$105, a speculator could borrow a share of ABC stock from one of its owners (i.e., borrow the stock certificate, not money), and then sell the share in the market for \$105. The short seller/speculator would now have \$105 cash and would owe one share of stock to the share lender. Since the speculator believes the stock is overpriced, she is hoping to profit by the stock decreasing in the near future. If she is right such that ABC stock decreases to its equilibrium value of \$100, then the speculator could go into the market and buy the stock for \$100 and return the borrowed share, leaving her with a profit of \$5. However, if the stock goes up and the share lender wants his stock back, then the short seller would lose when she buys back the stock at a price higher than \$100. In general, speculators help to move the market price of a security to its equilibrium value.

Theoretically, a market in which the price of the security is equal to its equilibrium value at all times is known as a ***perfect market***. For a market to be perfect requires, among other things, that all the information on which investors and speculators base their estimates of expected cash flows be reflected in the security's price. Such a market is known as an ***efficient market***. In a perfect market, speculators, on average, would not earn abnormal returns (above 15% in our stock example). However, if the information the market receives is ***asymmetrical*** in the sense that some speculators have information that others don't, or some receive information earlier than others, then the market price will not be equal to its equilibrium value at all times. In this inefficient market, there would be opportunities for speculators to earn abnormal returns.

Efficient markets would also preclude arbitrage returns. An ***arbitrage*** is a risk-free opportunity. Such opportunities come from price discrepancies among different markets. For example, if the same car sells for $1,000 in Boston but $2,000 in New York, an ***arbitrageur*** (one who exploits such opportunities) could earn a risk-free profit by buying the car in Boston and selling it in New York (assuming, of course, that the transportation cost are less than $1,000). In the financial markets, arbitrageurs tie markets together. For example, suppose there were two identical government bonds, each paying a guaranteed interest and principal of $1,100 at the end of one year, but with one selling for $1,000 and the other selling for $900. With such price discrepancies, an arbitrageur could sell short the higher priced bond at $1,000 (borrow the bond and sell it for $1,000) and buy the underpriced one for $900. This would generate an initial cash flow for the arbitrageur of $100 with no liabilities. That is, at maturity the arbitrageur would receive $1,100 from the underpriced bond that he could use to pay the lender of the overpriced bond. Arbitrageurs, by exploiting this arbitrage opportunity, though, would push the price of the underpriced bond up and the price of the overpriced one down until they were equally priced and the arbitrage was gone. Thus, arbitrageurs would tie the markets for the two identical bonds together.

WEB INFORMATION

For more on the efficient market hypothesis, go to www.investorhome.com/emh.htm.

1.7 CHARACTERISTICS OF ASSETS

The preceding discussion on the types of financial claims and their markets suggests that there are considerable differences among assets. All assets, though, can be described in terms of a limited number of common characteristics. These common properties make it possible to evaluate, select, and manage assets by defining and comparing them in terms of these properties. In fact, as an academic subject, the study of investments involves the evaluation and selection of assets. The evaluation of assets consists of describing assets in terms of their common characteristics, whereas selection involves selecting assets based on the trade-offs between those characteristics (e.g., higher return for higher risk). The characteristics common to all assets are value, rate of return, risk, maturity, divisibility, marketability, liquidity, and taxability.

Value: As defined earlier, the value of an asset is the present value of all of the asset's expected future benefits. Moreover, if markets were efficient, then, in equilibrium, the value of the asset would be equal to its market price.

Rate of Return: The rate of return on an asset is equal to the total dollar return received from the asset per period of time expressed as a proportion of the price paid for the asset. The total return on the security includes the income payments the security promises (interest on bonds, dividends on stock, etc.), the interest from reinvesting the coupon or dividend income during the life of the security, and any capital gains or losses realized when the investor sells the asset. Thus, if a corporate bond cost $P_0 = \$1{,}000$ and were expected to pay a coupon interest of $C = \$100$ and a principal of $F = \$1{,}000$ at the end of the year, then its annual rate of return would be 10% if all the expectations hold true:

$$R = \frac{C + (F - P_0)}{P_0} = \frac{\$100 + (\$1{,}000 - \$1{,}000)}{\$1{,}000} = .10$$

It should be noted that value (or price) and rate of return are necessarily related. If an investor knows the price she will pay for a security and the security's expected future benefits, then she can determine the security's rate of return. Alternatively, if she knows the rate of return she wants or requires and the security's expected future benefits, then she can determine the security's value or price.

Risk: The third property of an asset is its risk. Investment risk can be defined as the possibility that the rate of return an investor will obtain from holding an asset will be less than expected. Risk can result, for example, out of a concern that a bond issuer might fail to meet his contractual obligations (default risk) or it could result from an expectation that conditions in the market will change, resulting in a lower price of the security than expected when the holder plans to sell the asset (market risk). Bond investors are exposed to one or more of the following risks:

1. ***Interest-Rate Risk***: The risk that interest rates change, causing the bond price to change (part of market risk).
2. ***Reinvestment Risk***: The risk that the cash flows on the bond are reinvested at lower rates (part of market risk).
3. ***Call Risk***: The risk that the issuer will call the bond prior to maturity and the investor will have to reinvest in a market with lower rates.
4. ***Credit Risk*** or ***Default Risk***: The risk that the issuer/borrower will fail to meet contractual obligations. Such risk is evaluated in term of quality ratings by rating agencies (Moody's, Standard & Poor's, and Fitch). Ratings range from triple A (high quality, low credit risk) to C.
5. ***Credit Spread Risk***: The risk that the bond's credit risk will increase causing the bond's price to decrease relative to other bonds.
6. ***Liquidity Risk***: The risk that the bond will be hard to sell at a price near it value.
7. ***Risk Risk***: The risk of not being able to fully understand the risk of the security due to unexpected future events.

Risk, rate of return, and the value of an asset are necessarily related. In choosing between two securities with the same cash flows but with different risks, most investors will require a higher rate of return from the riskier of the two securities. For example, we would expect investors averse to risk to require a higher rate of return on a corporate bond issued by a fledgling company than on a U.S. government bond. If for some reason both securities traded at prices that yielded the same expected rates, then we would expect that investors would want the government bond, but not the corporate. If this were the case, the demand and price of the government bond would increase and its rate of return would decrease, while the demand and price of the corporate would fall and its rate of return would increase. Thus, if investors are risk averse, riskier securities must yield higher rates of return in the market or they will languish untraded.

Maturity: The fourth characteristic of an asset is its maturity. Maturity is the length of time from the present until the last contractual payment is made. Maturity can vary anywhere from one day to indefinitely, as in the case of stock or a consul (a bond issued with no maturity). Maturity can be used as a measure of the life of an asset. In defining a bond's life in terms of its maturity, though, one should always be aware of provisions such as a sinking fund or a call feature that modifies the maturity of a bond. For example, a 10-year callable bond issued when interest rates are relatively high may be more like a 5-year bond given that a likely interest rate decrease would lead the issuer to buy the bond back.

Divisibility: The fifth attribute, divisibility, refers to the smallest denomination in which an asset is traded. Thus a bank savings deposit account, in which an investor can deposit as little as a penny, is a perfectly divisible security; a jumbo certificate of deposit, with a minimum denomination of $10 million, is a highly indivisible security. Moreover, one of the economic benefits that investment funds provide investors is divisibility. That is, an investment company, by offering shares in a portfolio of high-denomination money market securities, makes it possible for small investors to obtain a higher rate of return than they could obtain by investing in a smaller-denomination money market security.

Marketability: The sixth characteristic is marketability. It can be defined as the speed in which an asset can be bought and sold. As a rule, for an asset to be highly marketable its price should be independent of the time spent searching for buyers or sellers. Many tangible assets, such as houses, as well as a number of financial assets, require a certain length of time before they can be bought or sold at their fair market values. This does not mean that they can't be sold in a short period of time; but if they must be, they typically fetch a price substantially lower than what the market would yield if adequate time were allowed. In general, highly marketable securities tend to be very standardized items with a wide distribution of ownership. Thus the stock of large corporations listed on the NYSE or Treasury issues are highly marketable securities that can be bought or sold on the exchanges or through a dealer in the OTC market in a matter of minutes. One way to measure the degree of marketability of a security is in terms of the size of the bid and asked spread offered by dealers in the OTC or a designated market maker. Dealers who make markets in less marketable securities necessarily set wider spreads than dealers who have securities that are bought and sold by many investors and therefore can be traded more quickly.

Liquidity: The seventh property, liquidity, is related to marketability. Liquidity can be defined as how cashlike a security is. For an instrument to be liquid it must be highly marketable and have little, if any, short-run risk. Thus, a Treasury security that can be sold easily and whose rate of return in the short run is known with a high degree of certainty is said to be liquid. On the other hand, a security such as a NYSE-listed stock is marketable but is not considered liquid given its day-to-day price fluctuations. Technically, the difference between marketability and liquidity is the latter's feature of low or zero risk that makes the security cashlike. It should be noted that although there is a difference between marketability and liquidity, the term liquidity is often used to describe a security's marketability.

Taxability: The eighth characteristic of an asset is taxability. Taxability refers to the claims that the federal, state, and local governments have on the cash flows of an asset. Taxability varies in terms of the type of asset. For example, the coupon interest on a municipal bond is tax exempt whereas the interest on a corporate bond is not. To the investor, the taxability of a security is important because it affects his after-tax rate of return.

1.8 CONCLUSION

In this chapter, we have given an overview of the financial system by examining the nature of financial assets, the types of markets that they give rise to, and their general characteristics. With this background, we now take up the study of the evaluation and selection of debt claims. In the next four chapters, debt securities are analyzed in terms of their characteristics: Chapter 2 looks at how debt instruments are valued and how their rates of return are measured; Chapters 3 and 4 examine respectively the level and structure of interest rates and explain how such factors as market expectations, economic conditions, taxability, and risk-return preferences are important in determining the level and structure of interest rates; Chapter 5 describes bond risk—default, call, and market risk—and introduces two measures of bond volatility—duration and convexity.

KEY TERMS

arbitrage
arbitrageur
asked price
asymmetrical
banker bourse
best effort
bid-asked spread
bid price
blue-sky laws
brokers
callable bond
capital market
cash market
certificates of deposit
CINS
commercial paper
contractual institutions
corporate bonds
crossing network
CUSIP
dealers
debt claims
deficit economic units
depository institutions
derivative securities
designated market makers (DMMs)
direct financial market
disintermediation
divisibility
domestic bonds
electronic communication network (ECN)
efficient market
embedded option
Eurobonds
exchange rate risk
external bond market
Federal Agency Securities
financial assets
financial claims
Financial Industry Regulatory Authority (FINRA)
foreign bonds
forward contract
fungible
futures
general obligation bonds
government-sponsored enterprises
Interbank Foreign Exchange Market
internal bond market
investment banker
investment companies
limit order
limit order book
lines of credit
liquidity

marketability
maturity
medium-term notes
money market
National Association of Securities Dealers (NASD)
National Association of Securities Dealers Automatic Quotation System (NASDAQ)
national market
negotiated market
New York Stock Exchange (NYSE)
NYSE Arca
offshore market
open-auction or crieé system
open market trades
over-the-counter (OTC)
perfect market
primary market
primary securities
primitive securities
private bourse
private placement
public bourse
rate of return
real assets
revenue bonds
risk
secondary market
secondary securities
Securities Act of 1933
Securities Exchange Act of 1934
securitized assets
Securities and Exchange Commission
short sale
specialists
speculators
spot market
surplus economic units
swaps
taxability
Treasury bills
Treasury bonds
Treasury notes
"truth in securities" law
U.S. Flow of Funds
underwrite
underwriting syndicate
value
wholesale market

PROBLEMS AND QUESTIONS

1. Explain how real and financial assets are created through the capital formation process in both the private and public sectors.
2. Comment on what is meant by the statement, "The financial markets are markets for loanable funds."
3. Describe the following markets and their features:
   a. Primary and secondary markets
   b. Direct and intermediary markets
   c. Money and capital markets
4. Define the following types of primary market sales and participants:
   a. Negotiated market and private placement
   b. Open market sales
   c. Investment banker
   d. Best effort
   e. Underwrite
   f. Underwriting syndicate
5. Explain the difference between a broker and dealer.
6. Describe the organizational structure of the New York Stock Exchange.
7. Define the following:
   a. NYSE Euronext
   b. NYSE Arco
   c. ArcaEdge

8. Define and explain the role of the specialist or market maker in ensuring a continuous market.
9. Describe the following aspects of the over-the-counter market:
   - **a.** How the market trades
   - **b.** Types of securities
   - **c.** Number of securities
   - **d.** National Association of Securities Dealers
   - **e.** FINRA
   - **f.** National Association of Securities Dealers Automatic Quotation System
10. Define the following financial institutions and explain their function in the intermediary financial market:
    - **a.** Depository institutions
    - **b.** Contractual institutions
    - **c.** Investment companies
11. What are securitized assets? How are they created?
12. Define the following international bonds and markets:
    - **a.** Eurobond market
    - **b.** Foreign bond
    - **c.** Internal market or national market
    - **d.** External market or offshore market
    - **e.** Interbank Foreign Exchange Market
13. Define a forward contract and an option contract. What is the main difference between the contracts?
14. Define a swap contract and list the major types.
15. List some of the major provisions in the securities acts of 1933 and 1934.
16. What is an efficient market? What is an inefficient market?
17. Define the characteristics of assets.
18. What is the difference between liquidity and marketability?

WEB EXERCISES

1. Learn about the NYSE Euronext by going to www.nyse.com and clicking on "About Us." At the site, check a stock by going to "Quick Quote" and entering a company's ticker symbol.
2. Find some current prices of OTC securities by going to www.pinksheets.com.
3. Brokerage firms provide a number of services. Identify some of those services by going to the Merrill Lynch site: www.ml.com.
4. Two important securities laws are the Securities Act of 1933 and the Securities Exchange Act of 1934. Learn more about these acts and others (e.g., Sarbanes-Oxley Act of 2002), as well as the activities of the Securities and Exchange

Commission, by going to www.sec.gov. At the site, go to "Site Map" and click on "Laws and Regulations" and then "Securities Act of 1933," "Securities Exchange Act of 1934," and "Sarbanes-Oxley Act of 2002." As part of the 1933 and 1934 securities acts, traded companies are required to submit quarterly and annual financial statements. These statements can be found at the SEC site by going to "Site Map" and "Filings and Forms." Select a company and then look up its reported financial statements.

5. There are a number of Web sites that provide information on current and historical stock prices of companies, as well as fundamental information.
   a. The NASDAQ site is a good source for stock information. Select a stock and examine its price trends and fundamentals by going to www.nasdaq.com. At the site, enter the company's symbol (you can enter as many as 25 companies).
   b. Another Web site for securities information is the *Wall Street Journal* site: http://online.wsj.com/public/us. For securities information and quotes, click "Markets" and "Market Data" tabs.
   c. Obtain information on a company, such as its profile, fundamentals, and price charts, by going to FINRA: http://finra.org. On the FINRA site, go to "Site Map" and then to "Company Information."

6. Explore some of the useful financial information from the following sites:
   a. www.Finance.Yahoo.com
   b. www.hoovers.com
   c. www.bloomberg.com
   d. www.businessweek.com
   e. www.ici.org
   f. http://seekingalpha.com
   g. http://bigcharts.marketwatch.com
   h. www.morningstar.com
   i. http://free.stocksmart.com

7. Start monitoring several stocks, interest rates, and other market information by downloading MarketBrowser: www.marketbrowser.com.

NOTES

1. Although U.S. exchanges use specialists to ensure continuous trading, the exchanges in some countries trade a security only once or just a few times during a day. These so-called "call" markets use an ***open-auction*** or ***crieé system*** in which interested traders gather in a designated trading area when the security is called. An exchange clerk then calls out prices until one is determined that clears all trades. In addition to continuous and call markets, there are also exchanges that have elements of both.
2. In October 2008, the NYSE Euronext also acquired the American Stock Exchange and formed the NYSE Amex that trades in small- and microcap listed companies.
3. For a security to qualify for the system it must have at least two market makers and its issuer must meet certain financial requirements. For a company to have its stock listed on the NASDAQ system it must satisfy requirements related to its net worth and shares outstanding.

4. Some scholars refer to direct financial claims as those in which only the ultimate borrowers and lenders trade with each other and a *semidirect market* as one in which brokers and dealers bring borrowers and lenders together. The definition of direct financial market here includes both of these markets.
5. An occasional trend in the financial markets is towards disintermediation. *Disintermediation* refers to the shifting from intermediary financing to direct financing. This occurs when a surplus unit withdraws funds from a financial institution and invests the funds by buying primary claims from an ultimate borrower.
6. Securities exchanges in different countries can be grouped into one of three categories: public bourse (exchange), private bourse, and banking bourse. A ***public bourse*** is a government securities exchange in which listed securities (usually both bonds and stocks) are bought and sold through brokers who are appointed by the government. A ***private bourse*** is a securities exchange owned by its member brokers and dealers. In countries where there are private exchanges a number of the exchanges will usually compete with each other; this is not the case in countries using a public bourse structure. A ***banker bourse*** is a formal or informal market in which securities are traded through bankers. This type of trading typically occurs in countries where historically commercial and investment banking have not separated.

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Bond Evaluation, Selection, and Management, Second Edition
by R. Stafford Johnson

CHAPTER 2

Bond Value and Return

2.1 INTRODUCTION

All securities can be evaluated in terms of the characteristics common to all assets: value, return, risk, maturity, marketability, liquidity, and taxability. In this and the next three chapters, we will analyze debt securities in terms of these characteristics. In this chapter, we look at how debt instruments (which we will usually refer to here as bonds) are valued and how their rates of return are measured. It should be noted that this chapter is very technical, entailing a number of definitions. Understanding how bonds are valued and their rates determined, though, is fundamental to being able to evaluate and select bonds.

2.2 BOND VALUATION

Pricing Bonds

An investor who has purchased a bond can expect to earn a possible return from the bond's periodic coupon payments; from capital gains (or losses) when the bond is sold, called, or matures; and from interest earned from reinvesting coupon payments. Given the market price of the bond, the bond's yield is the interest rate that makes the present value of the bond's cash flow equal to the bond price. This yield takes into account these three sources of return. In Section 2.3 we will discuss how to solve for the bond's yield given its price. Alternatively, if we know the rate we require to buy the bond, then we can determine its value.

Like the value of any asset, the value of a bond is equal to the sum of the present values of its future cash flows:

$$V_0^b = \sum_{t=1}^{M} \frac{CF_t}{(1+R)^t} = \frac{CF_1}{(1+R)^1} + \frac{CF_2}{(1+R)^2} + \cdots + \frac{CF_M}{(1+R)^M} \tag{2.1}$$

where V_0^b is the value or price of the bond, CF_t is the bond's expected cash flow in period t, including both coupon income and repayment of principal, R is the discount rate, and M is the term to maturity on the bond. The discount rate is the required rate; that is, the rate investors require to buy the bond. This rate is typically estimated by determining the rate on a security with comparable features: same risk, liquidity, taxability, and maturity.

Many bonds pay a fixed coupon interest each period, with the principal repaid at maturity. The coupon payment, C, is often quoted in terms of the bond's coupon rate, C^R. The coupon rate is the contractual rate the issuer agrees to pay on the bond. This rate is often expressed as a proportion of the bond's face value (or par) and is usually stated on an annual basis. Thus, a bond with a face value (F) of \$1,000 and a 10% coupon rate would pay an annual coupon of \$100 each year for the life of the bond: $C = C^R F = (.10)(\$1{,}000) = \$100$.

The value of a bond paying a fixed coupon interest each year (annual coupon payment) and the principal at maturity, in turn, would be

$$V_0^b = \sum_{t=1}^{M} \frac{C}{(1+R)^t} + \frac{F}{(1+R)^M}$$

$$V_0^b = \frac{C}{(1+R)^1} + \frac{C}{(1+R)^2} + \cdots + \frac{C}{(1+R)^M} + \frac{F}{(1+R)^M} \tag{2.2}$$

where M = Number of years to maturity

With the coupon payment fixed each period, the C term in Equation (2.2) can be factored out and the bond value can be expressed as

$$V_0^b = C \sum_{t=1}^{M} \frac{1}{(1+R)^t} + \frac{F}{(1+R)^M}$$

The term $\Sigma 1/(1+R)^t$ is the present value of \$1 received each period for N periods (M years in the above case). It is defined as the present value interest factor (PVIF). The PVIF for different terms and discount rates can be found using PVIF tables found in many finance text books. It also can be calculated using the following formula: $PVIF(R, N) = \frac{1-[1/(1+R)^N]}{R}$

Thus, if investors require a 10% annual rate of return on a 10-year, investment-grade corporate bond paying a coupon equal to 9% of par each year and a principal of \$1,000 at maturity ($M = 10$ years), then they would price the bond at \$938.55:

$$V_0^b = \sum_{t=1}^{M} \frac{C}{(1+R)^t} + \frac{F}{(1+R)^M}$$

$$V_0^b = \sum_{t=1}^{10} \frac{\$90}{(1.10)^t} + \frac{\$1{,}000}{(1.10)^{10}}$$

$$V_0^b = \$90 \sum_{t=1}^{M} \frac{1}{(1.10)^t} + \frac{\$1{,}000}{(1.10)^{10}}$$

$$V_0^b = \$90[\text{PVIF}(10\%, 10 \text{ yrs})] + \frac{\$1{,}000}{(1.10)^{10}}$$

$$V_0^b = \$90\left[\frac{1-[1/1.10]^{10}}{.10}\right] + \frac{\$1{,}000}{(1.10)^{10}}$$

$$V_0^b = \$938.55$$

Bond Price Relations

Relation between Coupon Rate, Required Rate, Value, and Par Value The value of the bond in the above example is not equal to its par value. This can be explained by the fact that the discount rate and coupon rate are different. Specifically, for investors in the above case to obtain the 10% rate per year from a bond promising to pay an annual rate of $C^R = 9\%$ of par, they would have to buy the bond at a value, or price, below par: The bond would have to be purchased at a discount from its par, $V_0^b < F$. In contrast, if the coupon rate is equal to the discount rate (i.e., $R = 9\%$), then the bond's value would be equal to its par value, $V_0^b = F$. In this case, investors would be willing to pay $1,000 for this bond, with each investor receiving $90 each year in coupons. Finally, if the required rate is lower than the coupon rate, then investors would be willing to pay a premium over par for the bond, $V_0^b > F$. This might occur if bonds with comparable features were trading at rates below 9%. In this case, investors would be willing to pay a price above $1,000 for a bond with a coupon rate of 9%. Thus, the first relationship to note is that a bond's value (or price) will be equal to, greater than, or less than its face value depending on whether the coupon rate is equal to, less than, or greater than the required rate.

BOND-PRICE RELATION 1

If $C^R = R \Rightarrow V^b = F$: Bond valued at par
If $C^R < R \Rightarrow V^b < F$: Bond valued at discount
If $C^R > R \Rightarrow V^b > F$: Bond valued at premium

In addition to the preceding relations, the relation between the coupon rate and required rate also explains how the bond's value changes over time. If the required rate is constant over time, and if the coupon rate is equal to it (i.e., the bond is priced at par), then the value of the bond will always be equal to its face value throughout the life of the bond. This is illustrated in Figure 2.1 by the horizontal line that shows

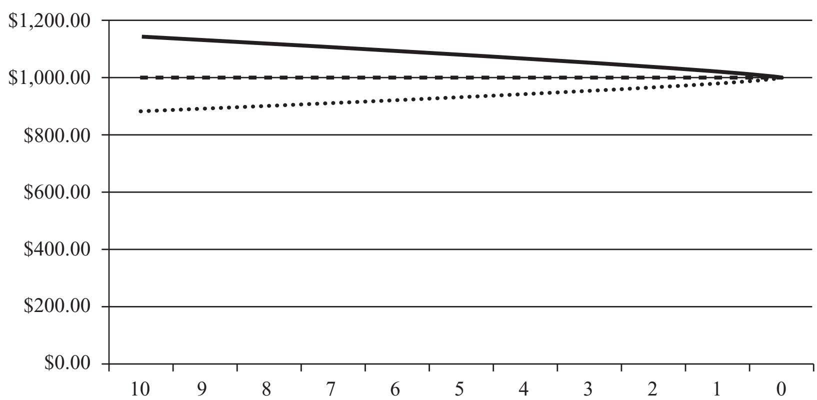


FIGURE 2.1 The Value over Time of an Original 10-Year, 9% Annual Coupon Bond Selling at Par, Discount, and Premium

the value of the 9% coupon bond is always equal to the par value. Here investors would pay $1,000 regardless of the terms to maturity. On the other hand, if the required rate is constant over time and the coupon rate is less (i.e., the bond is priced at a discount), then the value of the bond will increase as it approaches maturity; if the required rate is constant and the coupon rate is greater (i.e., the bond is priced at a premium), then the value of the bond will decrease as it approaches maturity. These relationships are also illustrated in Figure 2.1.

Relation between Value and Rate of Return Given known coupon and principal payments, the only way an investor can obtain a higher rate of return on a bond is for its price (value) to be lower. In contrast, the only way for a bond to yield a lower rate is for its price to be higher. Thus, an inverse relationship exists between the price of a bond and its rate of return. This, of course, is consistent with Equation (2.1) in which an increase in R increases the denominator and lowers V_0^b. Thus, the second bond relationship to note is that there is an inverse relationship between the price and rate of return on a bond.

BOND-PRICE RELATION 2

If $R \uparrow \Rightarrow V_0^b \downarrow$
If $R \downarrow \Rightarrow V_0^b \uparrow$

The inverse relation between a bond's price and rate of return is illustrated by the negatively sloped ***price-yield curve*** shown in Figure 2.2. The curve shows the different values of a 10-year, 9% annual coupon bond given different rates. As shown, the 10-year bond has a value of $938.55 when $R = 10\%$ and $1,000 when

Bond Relation 2: Price-yield curve depicts the inverse relation between V and R. The price-yield curve for the 10-year, 9% coupon bond:

Rate	Bond Value
7.00%	$1,140.47
7.50%	$1,102.96
8.00%	$1,067.10
8.50%	$1,032.81
9.00%	$1,000.00
9.50%	$968.61
10.00%	$938.55
10.50%	$909.78
11.00%	$882.22
11.50%	$855.81
12.00%	$830.49

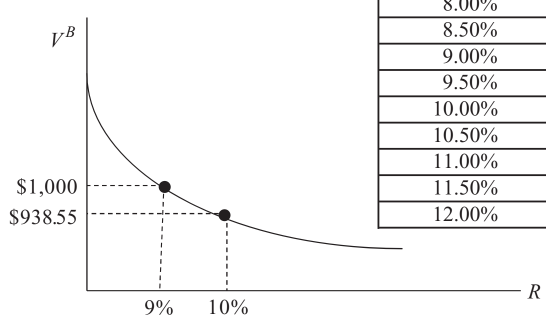


FIGURE 2.2 Price-Yield Relation, 10-Year, 9% Annual Coupon Bond

$R = 9\%$. In addition to showing a negative relation between price and yield, the price-yield curve is also convex from below (bow shaped). This convexity implies that for equal increases in yields, the value of the bond decreases at a decreasing rate (for equal decreases in yields, the bond's price increases at an increasing rate). The bow shape of a bond's price-yield curve is referred to as the bond's ***convexity***. It has important implications related to bond investment and management that we will examine in more detail later.

The Relation between a Bond's Price Sensitivity to Interest Rate Changes and Term to Maturity The third bond relationship to note is the relation between a bond's price sensitivity to interest rate changes and its maturity. Specifically:

BOND-PRICE RELATION 3

The greater the bond's term to maturity, the greater its price sensitivity to a given change in interest rates.

This relationship can be seen by comparing the price sensitivity to interest rate changes of the 10-year, 9% coupon bond in our above example with a one-year, 9% coupon bond. As shown in the exhibit box, if the required rate is 10%, then the 10-year bond would trade at $938.55, whereas the one-year bond would trade at $990.91 ($1,090/1.10). If the interest rate decreases to 9% for each bond (a 10%

Bond Relation 3: The greater the bond's maturity, the greater its price sensitivity to a change in interest rates
• **10-year, 9% annual coupon bond** $R = 10\% \Rightarrow V_0^b = \$938.55$ $R = 9\% \Rightarrow V_0^b = \$1{,}000$ $\%\Delta R = -10\% \Rightarrow \%\Delta V_0^b = 6.55\%$
• **1-year, 9% annual coupon bond** $R = 10\% \Rightarrow V_0^b = \$990.91$ $R = 9\% \Rightarrow V_0^b = \$1{,}000$ $\%\Delta R = -10\% \Rightarrow \%\Delta V_0^b = 0.9\%$

EXHIBIT 2.1 Bond Relation 3

change in rates), both bonds would increase in price to $1,000. For the 10-year bond, the percentage increase in price would be 6.55% [($1,000 − $938.55)/$938.55], whereas the percentage increase for the one-year bond would be only 0.9%. Thus, the 10-year bond's price is more sensitive to the interest rate change than the one-year bond. In addition, the greater price sensitivity of longer maturity bonds also implies that their price-yield curves are more convex than the price-yield curves for smaller maturity bonds.

The Relation between a Bond's Price Sensitivity to Interest Rate Changes and Coupon Payments Consider two 10-year bonds, each priced at a discount rate of 10% and each paying a principal of $1,000 at maturity, but with one bond having a coupon rate of 10% and priced at $1,000, and the other having a coupon rate of 2% and priced at $508.43:

$$V_0^b = \sum_{t=1}^{10} \frac{\$100}{(1.10)^t} + \frac{\$1,000}{(1.10)^{10}} = \$1,000$$

$$V_0^b = \sum_{t=1}^{10} \frac{\$20}{(1.10)^t} + \frac{\$1,000}{(1.10)^{10}} = \$508.43$$

Now suppose that the rate required on each bond decreases to a new level of 9%. The price on the 10% coupon bond, in turn, would increase by 6.4% to equal $1,064.18, whereas the price on the 2% coupon bond would increase by 8.3% to $550.76:

$$V_0^b = \sum_{t=1}^{10} \frac{\$100}{(1.09)^t} + \frac{\$1,000}{(1.09)^{10}} = \$1,064.18$$

$$\text{Proportional change} = \frac{\$1064.18 - \$1,000}{\$1,000} = .064$$

$$V_0^b = \sum_{t=1}^{10} \frac{\$20}{(1.090)^t} + \frac{\$1,000}{(1.09)^{10}} = \$550.76$$

$$\text{Proportional change} = \frac{\$550.76 - \$508.43}{\$508.43} = .083$$

In this case, the lower coupon bond's price is more responsive to a given interest rate change than the price of the higher coupon bond. Thus:

BOND-PRICE RELATION 4

The lower a bond's coupon rate, the greater its price sensitivity to changes in discount rates.

Pricing Bonds with Different Cash Flows and Compounding Frequencies

Equation (2.2) defines the value of a bond that pays coupons on an annual basis and a principal at maturity. Bonds, of course, differ in the frequency in which they pay coupons each year, and many bonds have maturities less than one year. Also, when investors buy bonds they often do so at noncoupon dates. Equation (2.2), therefore, needs to be adjusted to take these practical factors into account.

Semiannual Coupon Payments Many bonds pay coupon interest semiannually. When bonds make semiannual payments, three adjustments to Equation (2.2) are necessary: (1) the number of periods is doubled; (2) the annual coupon rate is halved; (3) the annual discount rate is halved. Thus, if our illustrative 10-year, 9% coupon bond trading at a quoted annual rate of 10% paid interest semiannually instead of annually, it would be worth $937.69:

$$V_0^b = \sum_{t=1}^{20} \frac{\$45}{(1.05)^t} + \frac{\$1{,}000}{(1.05)^{20}} = \$937.69$$

$$V_0^b = \$45 \left[\frac{1 - [1/(1.05)]^{20}}{.05}\right] + \frac{\$1{,}000}{(1.05)^{20}} = \$937.69$$

Note that the rule for valuing semiannual bonds is easily extended to valuing bonds paying interest even more frequently. For example, to determine the value of a bond paying interest four times a year, we would quadruple the periods and quarter the annual coupon payment and discount rate. In general, if we let n be equal to the number of payments per year (i.e., the compoundings per year), M be equal to the maturity in years, R^A be the discount rate quoted on an annual basis (simple annual rate), and R be equal to the periodic rate, then we can express the general formula for valuing a bond as follows:

$$V_0^b = \sum_{t=1}^{N} \frac{C^A/n}{(1 + (R^A/n))^t} + \frac{F}{(1 + (R^A/n))^{Mn}} \tag{2.3}$$

C^A = Annual coupon = $(C^R)(F)$
n = number of payments per year
Periodic coupon = Annual coupon/n
M = term to maturity in years
N = number of periods to maturity = $(n)(M)$
Required periodic rate = R = Annual rate/n = R^A/n

Thus, the value of a 20-year, 6% coupon bond, with semiannual payments, a par value of $1,000, and a required return of 8% would be $802.07:

$$V_0 = \sum_{t=1}^{N} \frac{C}{(1+R)^t} + \frac{F}{(1+R)^N}$$

$$V_0 = \sum_{t=1}^{40} \frac{\$30}{(1.04)^t} + \frac{\$1,000}{(1.04)^{40}}$$

$$V_0 = \$30\left[\frac{1 - 1/(1.04)^{40}}{.04}\right] + \frac{\$1,000}{(1.04)^{40}}$$

$$V_0 = \$593.78 + \$208.29 = \$802.07$$

N = number of periods = 40 [= (20 years)(2)]
F = \$1,000
C = Semiannual coupon = (.06/2)(\$1,000) = \$30
R = Required semiannual rate = .08/2 = .04

Compounding Frequency The 10% annual rate in the first example and the 8% rate in the second is a simple annual rate: It is the rate with one annualized compounding. With one annualized compounding and a 10% annual rate, we earn 10% every year and a \$100 investment would grow to equal \$110 after one year: \$100(1.10) = \$110. If the simple annual rate were expressed with semiannual compounding, then we would earn 5% every six months with the interest being reinvested; in this case, \$100 would grow to equal \$110.25 after one year: $\$100(1.05)^2 = \$110.25$. If the rate were expressed with monthly compounding, then we would earn 0.8333% (10%/12) every month with the interest being reinvested; in this case, \$100 would grow to equal \$110.47 after one year: $\$100[1 + (.10/12)]^{12} = \$110.47$. If we extend the compounding frequency to daily, then we would earn 0.0274% (10%/365) daily, and with the reinvestment of interest, a \$100 investment would grow to equal \$110.52 after one year: $\$100[1 + (.10/365)]^{365} = \$110.52$. Note that the rate of 10% is the simple annual rate, whereas the actual rate earned for the year is $[1+(R^A/n)]^n - 1$. This rate that includes the reinvestment of interest (or compounding) is known as the ***effective rate***.

When the compounding becomes large, such as daily compounding, then we are approaching continuous compounding with the n term in Equation (2.3) becoming very large. For cases in which there is continuous compounding, the future value (FV) for an investment of A dollars M years from now is equal to

$$FV = Ae^{RM}$$

where e is the natural exponent (equal to the irrational number 2.71828). Thus, if the 10% simple rate were expressed with continuous compounding, then \$100 (A) would grow to equal \$110.52 after one year: $\$100e^{(.10)(1)} = \$110.52$. (After allowing for some slight rounding differences, this is the value obtained with daily compoundings.) After two years, the \$100 investment would be worth \$122.14: $\$100e^{(.10)(2)} = \$122.14$.

Note that from the FV expression, the present value (A) of a future receipt (FV) is

$$A = PV = \frac{FV}{e^{RM}} = FVe^{-RM}$$

If $R = .10$, a security paying \$100 two years from now would be worth \$81.87, given continuous compounding: $PV = \$100e^{-(.10)(2)} = \$81.87$. Similarly, a security paying \$100 each year for two years would be currently worth \$172.36:

$$PV = \sum_{t=1}^{2} \$100e^{-(.10)(t)} = \$100e^{-(.10)(1)} + \$100e^{-(.10)(2)} = \$172.36$$

Thus, if we assume continuous compounding and a discount rate of 10%, then the value of our 10-year, 9% bond would be \$908.82:

$$V_0^b = \sum_{t=1}^{M} C^A e^{-Rt} + Fe^{-RM}$$

$$V_0^b = \sum_{t=1}^{10} \$90e^{-(.10)(t)} + \$1{,}000e^{-(.10)(10)} = \$908.82$$

It should be noted that most practitioners use interest rates with annual or semiannual compounding. Most of our examples in this book, in turn, will follow that convention. However, continuous compounding is often used in mathematical derivations, and we will make some use of it when it is helpful.

Valuing Bonds with Maturities Less than One Year Some bonds do not make any periodic coupon payments. Instead the investor realizes interest as the difference between the maturity value and the purchase price. These bonds are called ***zero-coupon bonds*** (also called ***zeros*** or ***pure discount bonds, PDBs***). The value of a zero-coupon bond is

$$V_0^b = \frac{F}{(1+R)^N}$$

For example, a zero-coupon bond maturing in 10 years and paying a maturing value of \$1,000 would be valued at \$385.54 if the required rate is 10% and annual compound is assumed:

$$V_0^b = \frac{1{,}000}{(1.10)^{10}} = \$385.54$$

If the convention is to double the number of years and half the annual discount rate, then the bond would be valued at \$376.89 to yield a semiannual rate of 5%, simple annual rate of 10%, and effective annual rate of 10.25% [= 1.05)^2 − 1]:

$$V_0^b = \frac{1{,}000}{(1.05)^{20}} = \$376.89$$

Many zero-coupon bonds have maturities less than a year. In valuing such bonds, the convention is to discount by using an annual rate and to express the bond's maturity as a proportion of a year. Thus, on March 1 a zero-coupon bond promising to pay \$100 on September 1 (184 days) and trading at an annual discount rate of 8% would be worth \$96.19:

$$V_0^b = \frac{\$100}{(1.08)^{184/365}} = \$96.19$$

The \$96.19 bond value reflects a maturity measure in terms of the actual number of days between March 1 and September 1 (184) and 365 days in the year. If we had instead assumed 30-day months and a 360-day year, then the maturity expressed as a proportion of year would be 0.5 and the value of the bond would be \$96.225 [= \$100/(1.08)$^{.5}$]. The choice of time measurement used in valuing bonds and computing accrued interest is known as the ***day count convention***. The day count convention is defined as the way in which the ratio of the number of days to maturity (or days between dates) to the number of days in the reference period (e.g., year) is calculated. The bond value of \$96.19 is based on a day count convention of actual days to maturity to actual days in the year (actual/actual), whereas the value of \$96.225 is based on a day count convention of 180 days to maturity (30 days times 6) to 360 days in the year (30/360). For short-term U.S. Treasury bills and other money market securities, the convention is to use actual number of days based on a 360-day year (actual/360).

Valuing Bonds at Noncoupon Dates Equations (2.2) and (2.3) can be used to value bonds at dates in which the coupons are to be paid in exactly one period. However, most bonds purchased are not bought on coupon dates, but rather at dates in between coupon dates. An investor who purchases a bond between coupon payments must compensate the seller for the coupon interest earned from the time of the last coupon payment to the settlement date of the bond.[1] This amount is known as ***accrued interest (AI)***. The formula for determining accrued interest is

$$\text{AI} = \text{Coupon}\left[\frac{\text{Number of days from last coupon to settlement date}}{\text{Number of days in the coupon period}}\right]$$

For U.S. Treasury coupon securities, the convention is to use the actual number of days since the last coupon date and the actual number of days between coupon payments: an actual/actual ratio. For example, consider a T-note whose last coupon payment was on March 1 and whose next coupon is six months later on September 1. Suppose the note is purchased with a settlement date of July 20. The actual number of days in the coupon period (sometime referred to as the basis) is 184 days and the actual number of days to the next coupon is 43:

- July 20 to July 31 = 11 days
- August = 31 days
- September 1 = 1 day
- Total = 43 days

For corporate, agency, and municipal bonds, the practice is to use 30-day months and a 360-day year: a 30/360 ratio. Each month is assumed to have 30 days and each year is assumed to have 360 days. If the preceding T-note were a corporate credit with a 30/360 day count convention, then the number of days in the coupon period would be 180 and the days to the next coupon would be 41:

- Remainder of July = 10 days
- August = 30 days
- September 1 = 1 day
- Total = 41 days

In trading bonds on a noncoupon date, the amount the buyer pays to the seller is the agreed-upon price plus the accrued interest. This amount is often called the ***full price*** or ***dirty price***. The price of a bond without accrued interest is called the ***clean price***:

$$\text{Full price} = \text{Clean price} + \text{Accrued interest}$$

The full price of the bond can be found by

1. Determining the number of days between the settlement date and the next coupon date
2. Determining the number of days in the coupon period
3. Computing the following:

$$v = \frac{\text{Number of days between settlement and next coupon date}}{\text{Number of days in the coupon period}}$$

4. Computing the value:
   For a bond with N semiannual coupon payments of C remaining, the full-price value would be

$$V_0 = \sum_{t=1}^{N} \frac{C}{(1+R)^{t-1+v}} + \frac{F}{(1+R)^{N-1+v}}$$

$$V_0 = \frac{C}{(1+R)^{v}} + \frac{C}{(1+R)^{1+v}} + \frac{C}{(1+R)^{2+v}} + \cdots + \frac{C}{(1+R)^{N-1+v}} + \frac{F}{(1+R)^{N-1+v}}$$

As an example, suppose a corporate bond with an annual coupon rate of 8%, semiannual payments, face value of $100, and maturing in 2016 is purchased with a settlement date of July 20, 2010. If the required yield is 10% (semiannual rate of 5%), the price of the bond will be $94.636:

Period t	$t-1$	Period	Cash flow per $100 par	PV of CF at 5%
1	0	0.227778	$ 4.00	$ 3.955793
2	1	1.227778	$ 4.00	$ 3.767422
3	2	2.227778	$ 4.00	$ 3.588021
4	3	3.227778	$ 4.00	$ 3.417163
5	4	4.227778	$ 4.00	$ 3.254441
6	5	5.227778	$ 4.00	$ 3.099467
7	6	6.227778	$ 4.00	$ 2.951873
8	7	7.227778	$ 4.00	$ 2.811308
9	8	8.227778	$ 4.00	$ 2.677436
10	9	9.227778	$ 4.00	$ 2.549939
11	10	10.227778	$ 4.00	$ 2.428514
12	11	11.227778	$104.00	$60.134623
	$v = 41/180 = 0.227778$			$94.636

Alternatively, the full price can also be found by

1. Moving to the next coupon date and determining the value of the bond at that date based on the future coupons and principal
2. Adding the coupon at the next coupon date to the value of bond
3. Discounting the bond value plus coupon back to the current date

Thus, the 8% corporate bond maturing in 2016 purchased with a settlement date of July 20, 2010 at a required yield of 10% would have value of 91.693586 per $100 face at the next coupon date. Discounting the sum of that value and the $4 coupon back to the settlement date ($v = .227778$ years) yields the full-bond price of $94.636:

- Value of the bond at next coupon date in 41 days or 41/180 = .227778 year:

$$V^B = \sum_{t=1}^{11} \frac{\$4}{(1.05)^t} + \frac{\$100}{(1.05)^{11}} = \$91.693586$$

- Value of the bond at next coupon date plus coupon paid at that date:

$$\$91.693586 + \$4 = \$95.693586$$

- Current value of the bond—full price:

$$V^B = \frac{\$95.693586}{(1.05)^{.227778}} = \$94.636$$

The full or dirty price of \$94.636 includes the portion of the coupon interest the buyer will receive but the seller has earned. Although the price the buyer pays the seller is the full price, in the United States the convention is to quote a bond's clean price. In this example, given there are 41 days to the next coupon and 180 days in the coupon period, the number of days from the last coupon is 139 (= 180 − 41). The accrued interest (AI) per \$100 par is \$3.0222 and the clean price or flat price is \$91.547 (full price minus the AI):

$$AI = \$4(139/180) = \$3.089$$

$$\text{Clean price} = 94.636 - \$3.089 = \$91.547$$

Price Quotes, Fractions, and Basis Points Whereas many corporate bonds pay principals of \$1,000, this is not the case for many noncorporate bonds and other fixed income securities. As a result, many traders quote bond prices as a percentage of their par value. For example, if a bond is selling at par, it would be quoted at 100 (100% of par); thus, a bond with a face value of \$10,000 and quoted at 80-1/8 would be selling at (.80125)(\$10,000) = \$8,012.50. When a bond's price is quoted as a percentage of its par, the quote is usually expressed in points and fractions of a point, with each point equal to \$1. Thus, a quote of 97 points means that the bond is selling for \$97 for each \$100 of par. The fractions of points differ among bonds. Fractions are either in 100ths, eighths, quarters, halves, or 64ths. On a \$100 basis, a 1/2 point is \$0.50 and a 1/32 point is \$0.03125. A price quote of 97–4/32 (97–4) is 97.125 for a bond with a 100 face value. It should also be noted that when the yield on a bond or other security changes over a short period, such as a day, the yield and subsequent price changes are usually quite small. As a result, fractions on yields are often quoted in terms of basis points (bp). A bp is equal to 1/100 of a percentage point, or 100 bp = 1%. Thus, 6.5% may be quoted as 6% plus 50 bp or 650 bp, and an increase in yield from 6.5% to 6.55% would represent an increase of 5 bp.

Figure 2.3 shows the price quotes of several U.S. Treasury bonds, Treasury bills, and several corporate credits offered for sale or purchase by dealers. The T-bond and corporate bond quotes were downloaded from the Yahoo Finance site (http://finance.yahoo.com/bonds), and the T-bill quotes were downloaded from the *Wall Street Journal* site: www.wsj.com/free. The first box in the figure shows a partial listing of the Treasury bonds traded on Friday, August 28, 2009. T-bonds and T-notes both pay semiannual coupon interest and principal at maturity. They differ in their maturities: Treasury bonds have original maturities greater than 10 years, whereas notes have original maturities less than 10 years. In the figure, the first four columns provide information on the issue, the closing price, coupon rate, and maturity. The coupon rate is quoted as a percentage of par. Thus, the Treasury bond maturing in February 2024 is paying a coupon of 7.5% of its face value. Given a face value of \$1,000, the annual coupon interest would be \$75, paid in two semiannual installments. The price of the bond shown in Column 2 is the average of the dealer's bid and asked prices, expressed as a percentage of the face value, or equivalently, as the price of a bond with a \$100 par value. The bid price is the price the dealer is willing to pay for the bond and the asked is what she is willing to sell it for (bid and ask quotes on T-bonds and notes can be found by going to the *Wall Street Journal* site: www.wsj.com/free). Column 5 shows the T-bond's yield to maturity.

T-Bonds, August 31, 2009

1	2	3	4	5	6
Issue	Price	Coupon(%)	Maturity	YTM(%)	Current Yield(%)
T-BOND 7.500 15-Nov-2024	139.1	7.5	15-Nov-24	4.068	5.391
T-BOND 7.625 15-Feb-2025	141.12	7.625	15-Feb-25	4.06	5.403
T-BOND 6.875 15-Aug-2025	132.82	6.875	15-Aug-25	4.087	5.176
T-BOND 6.000 15-Feb-2026	121.96	6	15-Feb-26	4.163	4.919
T-BOND 6.750 15-Aug-2026	131.56	6.75	15-Aug-26	4.163	5.13
T-BOND 6.500 15-Nov-2026	128.42	6.5	15-Nov-26	4.189	5.061
T-BOND 6.625 15-Feb-2027	130.16	6.625	15-Feb-27	4.195	5.09
T-BOND 6.375 15-Aug-2027	127.02	6.375	15-Aug-27	4.232	5.019
T-BOND 6.125 15-Nov-2027	123.81	6.125	15-Nov-27	4.25	4.946
T-BOND 5.500 15-Aug-2028	116.02	5.5	15-Aug-28	4.269	4.74
T-BOND 5.250 15-Nov-2028	112.93	5.25	15-Nov-28	4.265	4.648
T-BOND 5.250 15-Feb-2029	112.9	5.25	15-Feb-29	4.274	4.65
T-BOND 6.125 15-Aug-2029	124.98	6.125	15-Aug-29	4.268	

T-Bills, August 31, 2009

Traded on Friday, August 28, 2009

Maturity	Bid	Asked	Chg	Asked Yield
2009 Oct 22	0.115	0.108	0.01	0.109
2009 Nov 05	0.11	0.1	–0.003	0.101
2009 Dec 10	0.14	0.135	–0.007	0.137
2010 Jan 14	0.168	0.153	0.003	0.155
2010 Jan 21	0.168	0.158	unch.	0.16
2010 Mar 04	0.25	0.243	unch.	0.246
2010 Jun 03	0.308	0.303	unch.	0.307
2010 Jul 29	0.38	0.375	–0.013	0.381
2010 Aug 26	0.425	0.415	–0.01	0.422

Corporate Bonds

Issue	Price	Coupon (%)	Maturity	YTM (%)	Current Yield(%)	Fitch Ratings	Callable
DUKE CAP CORP	104.00	8.000	1-Oct-19	7.45	7.692	BBB	No
KRAFT FOODS INC	105.80	6.000	11-Feb-13	4.439	5.671	BBB	No
DOW CHEM CO	107.45	6.000	1-Oct-12	3.847	5.584	A	No
CENTEX CORP	87.00	4.550	1-Nov-10	12.7	5.23	BB	No
APACHE CORP	105.62	5.250	15-Apr-13	3.816	4.971	A	No
GENERAL ELECTRIC CO	104.02	5.000	1-Feb-13	3.925	4.807	AAA	No
CATERPILLAR INC DEL	118.08	7.900	15-Dec-18	5.514	6.69	A	No
AOL TIME WARNER INC	99.65	6.875	1-May-12	6.991	6.899	BBB	No
VERIZON COMMUNICATIONS INC	102.27	5.550	15-Feb-16	5.164	5.427	A	No
BANK OF AMERICA CORPORATION	103.70	5.625	14-Oct-16	5.044	5.424	AA	No
CONOCOPHILLIPS	104.81	4.400	15-May-13	3.212	4.198	A	No
TARGET CORP	106.72	5.875	1-Mar-12	3.612	5.505	A	No

FIGURE 2.3 Financial Quotes: Treasury Bonds and Notes, Treasury Bills, and Corporate Bonds
Source: http://finance.yahoo.com/bonds; *Wall Street Journal* site (www.wsj.com/free).

This is the average annual rate earned on the bond based on the price (this concept of yield to maturity is discussed in Section 2.3). Finally, Column 6 shows the bond's ***current yield***. This is the annual coupon expressed as percentage of the current bond price.

The second box in Figure 2.3 shows a partial listing for Treasury bills traded on August 28, 2009 as reported by the *Wall Street Journal* (www.wsj.com/free). U.S. T-bills are zero-coupon bonds with maturities less than one year. The *Wall Street Journal's* listing includes the dealer quotes in terms of the bid yield and asked yield. The bid yield is the annualized return expressed as a percent of the par value that the dealer wants if she buys the bill. The asked yield is the rate that the dealer is offering to sell the bills. Both yields are calculated as a ***discount yield***. The discount yield is the annualized return specified as a proportion of the bill's par value (F):

$$\text{Annual discount yield} = R_D = \frac{F - P_0}{F}\,\frac{360}{\text{Days to maturity}}$$

Given the dealer's discount yield, the bid or ask price can be obtained by solving the yield equation for the bond's price, P_0. Doing this yields

$$P_0 = F\,[1 - R_D(\text{Days to maturity}/360)]$$

In looking at the bid and asked yields, the reader should note that the yields are expressed as a proportion of the par value and not the current price, and that the yields are annualized on a 360-day year, instead of 365 days. The "Chg" column in the T-bill figure box indicates how much the asked yield changed from the previous day. The .01 change in the asked discount yield is an increase of 1 basis point. Finally, the last column shows the yield on the Treasury based on the days to maturity and the asked quote. The yield is expressed as a percentage. Thus, the T-bills maturing on August 26, 2010 are priced to yield only .422%!

The last box in Figure 2.3 shows quotations on several corporate bonds traded on December 23, 2008, that were found with the Yahoo! Finance search tool. As shown, the information provided includes the bond's coupon rate, maturity, current yield, yield to maturity, and closing price. For example, the information on the Duke Energy bond indicates an annual coupon rate of 8% and maturity year of 2019. The Duke credit closed at 104% of par on December 23, 2008 ($1,040 given a par of $1,000), providing a yield (yield to maturity) of 7.45% and a current yield of 7.692% (annual coupon/closing price = 8/104).

2.3 THE YIELD TO MATURITY AND OTHER RATES OF RETURN MEASURES

The financial markets serve as conduits through which funds are distributed from borrowers to lenders. The allocation of funds is determined by the relative rates paid on bonds, loans, and other financial securities, with the differences in rates among claims being determined by risk, maturity, and other factors that serve to differentiate the claims. There are a number of different measures of the rates of return on bonds and loans. Some measures, for example, determine annual rates based on cash flows received over 365 days, whereas others use 360 days; some measures determine rates that include the compounding of cash flows, whereas some do not; and some measures include capital gains and losses, whereas others exclude price changes. In this section, we examine some of the measures of rates of return, including the most common measure—the yield to maturity—and in Sections 2.4, 2.5, 2.6, and 2.7 we look at other important rate measures: the spot rate, the total return, and the geometric mean.

Common Measures of Rates of Return

When the term *rate of return* is used it can mean a number of different rates, including the interest rate, coupon rate, current yield, or discount yield. The term ***interest rate*** sometimes refers to the price a borrower pays a lender for a loan. Unlike other prices, this price of credit is expressed as the ratio of the cost or fee for borrowing and the amount borrowed. This price is typically expressed as an annual percentage of the loan (even if the loan is for less than one year). Today, financial economists often refer to the yield to maturity on a bond as the interest rate. In this book, the term *interest rate* will mean yield to maturity.

Another measure of rate of return is a bond's ***coupon rate***. As noted in the last section, the coupon rate, C^R, is the contractual rate the issuer agrees to pay each period. It is usually expressed as a proportion of the annual coupon payment to the bond's face value:

$$C^R = \frac{\text{Annual coupon}}{F}$$

Unless the bond is purchased at par, the coupon rate is not a good measure of the bond's rate of return because it fails to take into account the price paid for the bond.

In examining corporate bond quotes, the current yield on a bond is often provided. As noted, this rate is computed as the ratio of the bond's annual coupon to its current price. This measure provides a quick estimate of a bond's rate of return, but in many cases not an accurate one because it does not capture price changes. The current yield is a good approximation of the bond's yield, if the bond is selling at or near its face value or if it has a long maturity. That is, we noted earlier that if a bond is selling at par, its coupon rate is equal to the discount rate. In this case, the current yield is equal to the bond's yield to maturity. Thus, the closer the bond's price is to its face value, the closer the current yield is to the bond's yield to maturity. As for maturity, note that a coupon bond with no maturity or repayment of principal, known as a ***perpetuity*** or ***consul***, pays a fixed amount of coupons forever. The value of such a bond is

$$V_0^b = \frac{C}{R}$$

If the bond is priced in the market to equal V_0^b, then the rate on the bond would be equal to the current yield: $R = C/V_0^b$. Thus, when a coupon bond has a long-term maturity (e.g., 20 years), then it is similar to a perpetuity, making its current yield a good approximation of its rate of return.

Finally, the discount yield is the bond's return expressed as a proportion of its face value. For example, a one-year zero-coupon bond costing \$900 and paying a par value of \$1,000 yields \$100 in interest and a discount yield of 10%:

$$\text{Discount yield} = \frac{F - P_0}{F} = \frac{\$100}{\$1000} = .10$$

The discount yield used to be the rate frequently quoted by financial institutions on their loans (because the discount rate is lower than a rate quoted on the borrowed amount). The difficulty with this rate measure is that it does not capture the conceptual notion of the rate of return being the rate at which the investment grows. In this example, the \$900 bond investment grew at a rate of over 11%, not 10%:

$$\frac{F - P_0}{P_0} = \frac{\$100}{\$900} = .111$$

Because of tradition, the rates on Treasury bills are quoted by dealers in terms of the bills' discount yield. Whereas Treasury bills have maturities less than one year, the discount yields are quoted on an annualized basis. As we noted in the last section, dealers quoting the annualized rates use a day count convention of actual days to maturity but with a 360-day year:

$$\text{Annual discount yield} = \frac{F - P_0}{F}\,\frac{360}{\text{Days to maturity}}$$

Yield to Maturity

The most widely used measure of a bond's rate of return is the ***yield to maturity (YTM)***. As noted earlier, the YTM, or simply the yield, is the rate that equates the purchase price of the bond, P_0^b, with the present value of its future cash flows. Mathematically, the YTM (y) is found by solving the following equation for y (YTM):

$$P_0^b = \sum_{t=1}^{M} \frac{CF_t}{(1+y)^t} \tag{2.4}$$

The YTM is analogous to the internal rate of return used in capital budgeting. It is a measure of the rate at which the investment grows. From our first example, if the 10-year, 9% annual coupon bond were actually trading in the market for \$938.55, then the YTM on the bond would be 10%. Unlike the current yield, the YTM incorporates all of the bond's cash flows (CFs). It also assumes the bond is held to maturity and that of all CFs from the bond are reinvested to maturity at the calculated YTM.

Estimating YTM: Average Rate to Maturity If the cash flows on the bond (coupons and principal) are not equal, then Equation (2.4) cannot be solved directly for the

YTM. Alternatively, one must use an iterative (trial and error) procedure, substituting different y values into Equation (2.4) until that y is found that equates the present value of the bond's cash flows to the market price. An estimate of the YTM, however, can be found using the bond's ***average rate to maturity, ARTM*** (also referred to as the ***yield approximation formula***). This measure determines the rate as the average return per year as a proportion of the average price of the bond per year. For a coupon bond with a principal paid at maturity, the average return per year on the bond is its annual coupon plus its average annual capital gain. For a bond with an M-year maturity, its average gain is calculated as the total capital gain realized at maturity divided by the number of years to maturity: $(F - P_0^b)/M$. The average price of the bond is computed as the average of two known prices, the current price and the price at maturity (F): $(F + P_0{}^b)/2$. Thus the ARTM is

$$\text{ARTM} = \frac{C + [(F - P_0^b)/M]}{(F + P_0^b)/2} \tag{2.5}$$

The ARTM for the 10-year, 9% annual coupon bond trading at \$938.55 is 0.0992:

$$\text{ARTM} = \frac{\$90 + [(\$1{,}000 - \$938.55)/10]}{(\$1{,}000 + \$938.55)/2} = .0992$$

Bond Equivalent Yields

The YTM calculated above represents the yield for the period (in the above example this was an annual rate, given annual coupons). If a bond's CFs were semiannual, then solving Equation (2.4) for y would yield a 6-month rate; if the CFs were monthly, then solving (2.4) for y would yield a monthly rate. To obtain a ***simple annualized rate*** (with no compounding), y^A, one needs to multiply the periodic rate, y, by the number of periods in the year. Thus, if a 10-year bond paying \$45 every six months and \$1,000 at maturity were selling for \$937.69, its 6-month yield would be .05 and its simple annualized rate, y^A, would be 10%:

$$\$937.69 = \sum_{t=1}^{20} \frac{\$45}{(1+y)^t} + \frac{\$1{,}000}{(1+y)^{20}} \Rightarrow y = .05$$

$$y^A = \text{Simple annualized rate} = (n)(y) = (2)(.05) = .10$$

In this example, the simple annualized rate is obtained by determining the periodic rate on a bond paying coupons semiannually and then multiplying by two. Because Treasury bonds and many corporate bonds pay coupons semiannually, the rate obtained by multiplying the semiannual periodic rate by two is called the ***bond-equivalent yield.*** Bonds with different payment frequencies often have their rates expressed in terms of their bond-equivalent yields so that their rates can be compared to each other on a common basis. This bond-equivalent yield, though, does not take into account the reinvestment of the bond's cash flows during the year. Therefore, it underestimates the actual rate of return earned. Thus, an investor earning 5% semiannually would have \$1.05 after six months from a \$1 investment that she can reinvest for the next six months. If she reinvests at 5%, then her annual rate would be 10.25% [= $(1.05)(1.05) - 1 = (1.05)^2 - 1$], not 10%. As noted earlier, the 10.25% annual rate, which takes into account compounding, is known as the effective rate.

Yield to Call, Yield to Put, and Yield to Worst

Yield to Call Many bonds have a call feature that allows the issuer to buy back the bond at a specific price known as the call price. (Call features and other option features will be discussed in some detail in Chapters 5 and 14.) Given a bond with a call option, the ***yield to call (YTC)*** is the rate obtained by assuming the bond is called on the call date. Like the YTM, the YTC is found by solving for the rate that equates the present value of the CFs to the market price:

$$P_0^b = \sum_{t=1}^{N_{CD}} \frac{CF_t}{(1+y)^t} + \frac{CP}{(1+y)^{N_{CD}}}$$

where $CP =$ Call price
$N_{CD} =$ Number of periods to the call date

Thus, a 10-year, 9% coupon bond callable in 5 years at a call price of \$1,100, paying interest semiannually and trading at \$937.69, would have a YTM of 10% and an annualized YTC of 12.2115%:

$$\$937.69 = \sum_{t=1}^{10} \frac{\$45}{(1+y)^t} + \frac{\$1100}{(1+y)^{10}} \Rightarrow \text{YTC} = .0610575$$

$$\text{Simple annualized YTC} = (2)(.0610575) = .122115$$

When the bond may be called and at what price are specified in the indenture. For some issues, the call price is the same. For other callable bonds, the call price depends on when the bond is called. There is a call schedule that specifies the call price for each call dates. The convention is to calculate the YTC for each date and call price.

Yield to Put An issue can be putable, allowing the bondholder the right to sell the bond back to the issuer at a specified price—put price, PP. As with callable bonds, putable bonds can have a constant put price or a put schedule. When a bond is putable, the convention is to calculate the yield to put, YTP. Like the YTM and YTC, the ***yield to put (YTP)*** is found by solving for the rate that equates the present value of the CFs to the market price:

$$P_0^B = \sum_{t=1}^{N_{PD}} \frac{CF_t}{(1+YTP)^t} + \frac{PP}{(1+YTP)^{N_{PD}}}$$

where $PP =$ Put price
$N_{PD} =$ Number of periods to the put date

A 10-year, 9% coupon bond, first putable in 5 years at a put price of \$950, paying interest semiannually and trading at \$937.69 would have an annualized YTP of 9.807741%:

$$\$937.69 = \sum_{t=1}^{10} \frac{\$45}{(1+\text{YTP})^t} + \frac{\$950}{(1+\text{YTP})^{10}} \Rightarrow \text{YTP} = .04903870$$

Simple annualized YTP = (2)(.04903870) = .09807741

Yield to Worst Many investors calculate the YTC for all possible call dates and the YTP for all possible put dates, as well as the YTM. They then select the lowest of the yields as their yield return measure. The lowest yield is sometimes referred to as the ***yield to worst***.

Cash Flow Yield Fixed-income securities whose cash flows (CF) include scheduled principal payments prior to maturity are ***amortized securities***. Their CFs include principal and interest payments. If the CFs are constant, then the yields can be found by solving for the yield that equates the present value of the CF to the current price:

$$P_0^B = \sum_{t=1}^{N} \frac{CF}{(1+YTM)^t}$$

For example, a 10-year, fully amortized bond that pays an interest of 8% on a semiannual basis and has a principal of \$100 would make semiannual payments of \$7.358175 per \$100 par:

$$P_0^B = \sum_{t=1}^{N} \frac{CF}{(1+YTM)^t}$$

$$P_0^B = CF \sum_{t=1}^{N} \frac{1}{(1+YTM)^t}$$

$$100 = CF\left[\frac{1-1/(1+R)^N}{R}\right]$$

$$CF = \frac{100}{\left[\frac{1-1/(1+R)^N}{R}\right]}$$

$$CF = \frac{100}{\left[\frac{1-1/(1+(.08/2))^{20}}{.08/2}\right]} = 7.358175$$

If the bond were priced at 95, its semiannual yield would be 4.586656% and it simple annualized yield would be 9.1733%:

$$\$95 = \sum_{t=1}^{20} \frac{\$7.358175}{(1+\text{YTM})^t} \Rightarrow \text{YTM} = .04586656$$

$$\text{Simple annualized YTM} = (2)(.04586656) = .091733$$

Examples of amortized securities include many securitized securities, such as mortgage-backed securities and asset-backed securities. As we will see in Chapter 11, many of these securities have prepayment options and, as a result, their CFs are not fixed over the life of the securities.

Bond Portfolio Yields

The yield for a portfolio of bonds is found by solving the rate that will make the present value of the portfolio's cash flow equal to the market value of the portfolio. For example, a portfolio consisting of a two-year, 5% annual coupon bond priced at par (100) and a three-year, 10% annual coupon bond priced at 107.87 to yield 7% (YTM) would generate a three-year cash flow of \$15, \$115, and \$110 and would have a portfolio market value of \$207.87. The rate that equates this portfolio's cash flow to its portfolio value is 6.2%:

$$\$207.87 = \frac{\$15}{(1+y)^1} + \frac{\$115}{(1+y)^2}\frac{\$110}{(1+y)^3} \Rightarrow y = .062$$

Note that this yield is not the weighted average of the YTM of the bonds comprising the portfolio. In this example, the weighted average (R_P) is 6.04%:

$$R_P = w_1(YTM_1) + w_2(YTM_2)$$

$$R_P = \left[\frac{\$100}{\$207.87}\right](.05) + \left[\frac{\$107.87}{\$207.87}\right](.07) = .0604$$

Thus, the yield for a portfolio of bonds is not the average of the YTMs of the bonds making up the portfolio.

2.4 RATES ON ZERO-COUPON BONDS

Formula for the Rate on a Zero-Coupon Bond

Whereas no algebraic solution for the YTM exists when a bond pays coupons and principal that are not equal, a solution does exist in the case of a zero-coupon bond or pure discount bonds, PDBs (we will use both expressions), in which there is only one cash flow (F). That is,

$$P_0^b = \frac{F}{(1 + YTM_M)^M}$$

$$(1 + YTM_M)^M = \frac{F}{P_0^b}$$

$$YTM_M = \left[\frac{F}{P_0^b}\right]^{1/M} - 1 \tag{2.6}$$

where M = maturity in years. Thus, a zero-coupon bond with a par value of \$1,000, a maturity of three years, and trading for \$800 would have an annualized YTM of 7.72%:

$$\text{YTM}_3 = \left[\frac{\$1{,}000}{\$800}\right]^{1/3} - 1 = .0772$$

If the convention is to assume semiannual compounding, then the semiannual YTM would be 3.798% and the simple annual rate or bond-equivalent yield would be 7.596%:

$$M = 3 \text{ years}$$

$$n = \text{Compound frequency} = 2$$

$$N = n\ M = (2)(3) = 6$$

$$\text{Semiannual YTM} = \left[\frac{\$1000}{\$800}\right]^{1/6} - 1 = .03789$$

$$\text{Bond} - \text{equivalent yield} = (2)(.03789) = .075782$$

Similarly, a pure discount bond paying \$100 at the end of 182 days and trading at \$96 would yield an annual rate using a 365-day year of 8.53%:

$$\text{YTM} = \left[\frac{\$100}{\$96}\right]^{365/182} - 1 = .0853$$

Rate on a Zero-Coupon Bond with Continuous Compounding

Using the properties of logarithms (see Appendix A at the end of the book for a primer on logarithms), the rate on a zero-coupon bond with continuous compounding is

$$P_0^b e^{Rt} = F$$

$$e^{Rt} = \frac{F}{P_0^b}$$

$$\ln(e^{Rt}) = \ln\left[\frac{F}{P_0^b}\right]$$

$$Rt = \ln\left[\frac{F}{P_0^b}\right]$$

$$R = \frac{\ln[F/P_0^b]}{t}$$

A zero-coupon bond selling for \$96 and paying \$100 at the end of 182 days would yield an annual rate of 8.1868% with continuous compounding:

$$R = \frac{\ln[\$100/\$96]}{182/365} = .081868$$

When the rate of return on a security is expressed as the natural log of the ratio of its end of the period value to its current value, the rate is referred to as the ***logarithmic return***. Thus, a bond currently priced at \$96 and expected to be worth \$100 at the end of the period would have an expected logarithmic return of 4.082%: $R = \ln(\$100/96) = .04082$

It should be noted that the rate on a zero-coupon bond is called the ***spot rate***. As discussed in Section 2.6, spot rates are important in determining a bond's equilibrium price.

2.5 TOTAL RETURN

Equation (2.6) provides the formula for finding the YTM for a zero-discount bond. A useful extension of Equation (2.6) is the ***total return (TR)*** also called the ***realized return*** and ***average realized return (ARR)***. The total return is the yield obtained by assuming the cash flows are reinvested to the investor's ***horizon*** at an assumed reinvestment rate and at the horizon the bond is sold at an assumed rate given the horizon is not maturity or pays its principal if the horizon is maturity. The TR is computed by first determining the investor's horizon, HD; next finding the HD value, defined as the total funds the investor would have at HD; and third solving for the TR using a formula for the zero-coupon bond (Equation 2.6).

To illustrate, suppose an investor buys a four-year, 10% coupon bond, paying coupons annually and selling at its par value of \$1,000. Assume the investor needs cash at the end of year 3 (HD = 3), is certain he can reinvest the coupons during the period in securities yielding 10%, and expects to sell the bond at his HD at a rate of 10%. To determine the investor's TR, we first need to find the HD value. This value is equal to the price the investor obtains from selling the bond at the HD and the value of the coupons at the HD. In this case, the investor, at his HD, will be able to sell a one-year bond paying a \$100 coupon and a \$1,000 par at maturity for \$1,000, given the assumed discount rate of 10%:

$$P_0^b = \frac{\$100 + \$1{,}000}{(1.10)^1} = \$1{,}000$$

Also at the HD, the \$100 coupon paid at the end of the first year will be worth \$121, given the assumption it can be reinvested at 10% for two years and there is annual compounding, $\$100(1.10)^2 = \$121$, and the \$100 received at the end of year 2 will, in turn, be worth \$110 in cash at the HD, \$100(1.10) = \$110. Finally, at the HD the investor would receive his third coupon of \$100. Combined, the investor would have \$1,331 in cash at the HD: HD value = \$1,331. The horizon value of \$1,330 consists of a bond valued at \$1,000, coupons of \$300, and interest earned from reinvesting coupons of \$31 (HD coupon value − total coupon received = \$331 − \$300 = \$31); see Figure 2.4A. Note that if the rates at which coupons can be reinvested (reinvestment rates) are the same (as assumed in this example), then the coupon value at the horizon would be equal to the period coupon times the future value of an annuity ($FVIF_a$):

$$\text{Coupon value at HD} = \sum_{t=0}^{HD-1} C(1+R)^t$$

$$\text{Coupon value at HD} = C \sum_{t=0}^{HD-1} (1+R)^t$$

$$\text{Coupon value at HD} = C\ FVIF_a$$

$$\text{Coupon value at HD} = C\left[\frac{(1+R)^{HD}-1}{R}\right]$$

$$\text{Coupon value at HD} = \$100\left[\frac{(1.10)^3-1}{.10}\right] = \$331$$

The reinvestment income or interest earned from reinvesting coupons (interest on interest), in turn, is equal to the coupon value at HD minus the total coupons received, (N)(C):

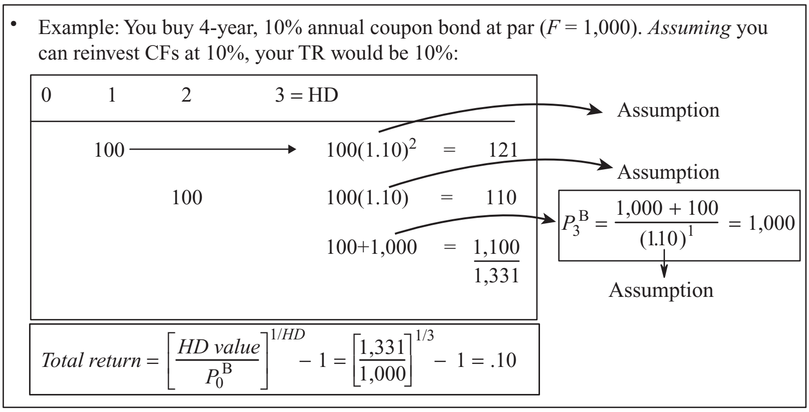
$$\text{Reinvestment income} = \text{Coupon value at HD} - \text{Total coupons}$$

$$\text{Reinvestment income} = \sum_{t=0}^{HD-1} C(1+R)^t - NC$$

$$\text{Reinvestment income} = \$100\left[\frac{(1.10)^3-1}{.10}\right] - (3)(\$100)$$

$$\text{Reinvestment income} = \$331 - \$300 = \$31$$

Given the HD value of \$1,331, the TR is found in the same way as the YTM for a zero-coupon bond. In this case, a \$1,000 investment in a bond returning \$1,331 at the end of year 3 yields a total return of 10%:



$$\textit{Coupon value} = \sum_{t=0}^{HD-1} C(1+R)^t = \$100\left[\frac{(1.10)^3-1}{.10}\right] = \$331$$

$$\textit{Interest on interest} = \$331 - \$300 = \$31$$

$$P_{HD}^B = \frac{\$1{,}000 + \$100}{1.10} = \$1{,}000$$

$$\textit{Horizon value} = \textit{Coupon value} + P_{HD}^B = \$331 + \$1{,}000 = \$1{,}331$$

$$\$1{,}000 = \frac{\$1{,}331}{(1+\textit{Total return})^3}$$

$$\textit{Total return} = \left[\frac{\$1{,}331}{\$1{,}000}\right]^{1/3} - 1 = .10$$

FIGURE 2.4A Total Realized Return

$$P_0^b = \frac{\text{HD value}}{(1+TR)^{HD}}$$

$$(1+TR)^{HD} = \frac{\text{HD value}}{P_0^b}$$

$$TR = \left[\frac{\text{HD value}}{P_0^b}\right]^{1/HD} - 1 \tag{2.7}$$

$$TR = \left[\frac{\$1{,}331}{\$1{,}000}\right]^{1/3} - 1 = .10$$

Note that the total return is the rate that makes the initial investment grow to the horizon value. That is, \$1,000 grows at an annual rate of 10% to equal a horizon value of \$1,331 at year 3:

$$\$1{,}000(1.10)^3 = \$1{,}331$$

The total return can be applied to any period. For example, if the four-year bond purchased by the investor made semiannual payments and the six-month yield were at 5%—a simple annual yield of 10% and an effective annual yield of 10.25% [= $(1.05)^2$ −1]—then the investor's coupon value, reinvestment income, price at HD, and HD value at his HD would respectively be $340.10, $40.10, $1,000, and $1,340.10 (see Figure 2.4B):

$$\text{Coupon value} = \sum_{t=0}^{6-1} \$50\,(1.05)^t = \$50\left[\frac{(1.05)^6 - 1}{.05}\right] = \$340.10$$

$$\text{Reinvestment income} = \$340.10 - (6)(\$50) = \$40.10$$

$$\text{HD price} = \sum_{t=1}^{6} \frac{\$50}{(1.05)^t} + \frac{\$1000}{(1.05)^6} = \$50\left[\frac{1 - (1/(1.05)^6)}{.05}\right] + \frac{\$1{,}000}{(1.05)^6} = \$1{,}000$$

$$\text{HD value} = \$340.10 + \$1{,}000 = \$1{,}340.10$$

The investor's semiannual total return would be 5%, the simple annual rate would be 10%, and the effective annual rate would be 10.25%:

$$\text{Semiannual total return} = \left[\frac{\$1{,}340.10}{\$1{,}000}\right]^{1/6} - 1 = .05$$

$$\text{Simple annual rate} = 2\left[\left[\frac{\$1{,}340.10}{\$1{,}000}\right]^{1/6} - 1\right] = .10$$

$$\text{Effective annual rate} = (1.05)^2 - 1 = .1025$$

Market Risk and Horizon Analysis

In the above example, the semiannual TR is 5%, which is the same rate at which the bond was purchased; that is, a 10% coupon bond, paying interest twice a year, and selling at par, yields a semiannual YTM of 5% and bond-equivalent yield of 10%. In this case, obtaining a total return equal to the initial YTM should not be surprising because the coupons are assumed to be reinvested at the same semiannual rate as the initial YTM (5%) and the bond is also assumed to be sold at that rate as well (recall, the YTM measure assumes that all coupons are reinvested at the calculated YTM). If the coupons were expected to be reinvested at different rates or the bond sold at a different YTM, then a total return equal to the initial YTM would not have been realized. For example, if the rates on all maturities were to increase from 10% to 12% (simple annual rate) just after the four-year, 10% bond was purchased at par, then the total semiannual total return would decrease to 4.8735% and the simple annual rate would be 9.7471%.

Maturity = 4 years, annual coupon rate = 10%, interest paid semiannually, par = $1,000, reinvestment rate = 5% semiannually, purchase price = $1,000, horizon = 3 years, bond expected to sell at the HD at a 5% semiannual rate.
Horizon value = $1,340, semiannual total return = 5%, simple total return = 10%, and effective total rate = 10.25%:

Year	0.0	0.5	1.0	1.5	2.0	2.5	3.0		Values
		$50.00						$50(1.05)^5^	$63.81
			$50.00					$50(1.05)^4^	$60.78
				$50.00				$50(1.05)^3^	$57.88
					$50.00			$50(1.05)^2^	$55.13
						$50.00		$50(1.05)^1^	$52.50
							$50.00	$50	$50.00
							$1,000.00	$50/1.05 + ($1,050/(1.05)^2^	$1,000.00
								Horizon value	$1,340.10

$$\textit{Coupon value} = \sum_{t=0}^{6-1} \$50\ (1.05)^t = \$50\left[\frac{(1.05)^6 - 1}{.05}\right] = \$340.10$$

$$\textit{Interest on interest} = \$340.10 - \$300 = \$40.10$$

$$\textit{Horizon price} = P_{HD}^B = \frac{\$50}{1.05} + \frac{\$50 + \$1{,}000}{(1.05)^2} = \$1{,}000$$

$$\textit{HD value} = \$340.10 + \$1{,}000 = \$1{,}340.10$$

$$\textit{Initial price} = \sum_{t=1}^{6} \frac{\$50}{(1.05)^t} + \frac{\$1{,}000}{(1.05)^6} = \$50\left[\frac{1 - (1/(1.05)^6)}{.05}\right] + \frac{\$1{,}000}{(1.05)^6} = \$1{,}000$$

$$\textit{Semiannual total return} = \left[\frac{\$1{,}340.10}{\$1{,}000}\right]^{1/6} - 1 = .05$$

$$\textit{Simple annualized rate} = 2\left[\left[\frac{\$1{,}340.10}{\$1{,}000}\right]^{1/6} - 1\right] = .10$$

$$\textit{Effective annual rate} = (1.05)^2 - 1 = .1025$$

FIGURE 2.4B Total Realized Return

In this case, the rate increase has increased the reinvestment income by $0.67 from to $48.10 to $48.77, but it has also lowered the horizon price by $18.33 from $1,000 to $981.67. As a result, the price decrease has reduced the total return more than the interest-on-interest increase has increased the realized return. Thus, the increase in rates from 10% to 12% lowers the total return from 10% to 9.7471%.

$$\text{Coupon value at HD} = \sum_{t=0}^{HD-1} \$50(1+R)^t = \$50\left[\frac{(1.06)^6 - 1}{.06}\right] = \$348.77$$

$$\text{Reinvestment income} = \$348.77 - (6)(\$50) = \$48.77$$

$$\text{HD price: } P_{HD}^b = \frac{\$50}{(1.06)} + \frac{\$1{,}000 + \$50}{(1.06)^2} = \$981.67$$

$$\text{Horizon value} = \text{Total dollar return} + P_{HD}^B = \$348.77 + \$981.67$$

$$= \$1{,}330.44$$

$$\text{Total return} = \left[\frac{\$1{,}330.44}{\$1{,}000}\right]^{1/6} - 1 = .048735$$

$$\text{Simple annualized total return} = (2)(.048735) = .097471$$

The possibility of the actual return on a bond deviating from the expected return because of a change in interest rates is known as ***market risk***. As illustrated in the above total return example, a change in interest rates has two effects on a bond's return. First, interest rate changes affect the price of a bond; this is referred to as ***price risk***. If the investor's horizon is different from the bond's maturity date, then the investor will be uncertain about the price he will receive from selling the bond (if HD < M), or the price he will have to pay for a new bond (if HD > M). Also, as we noted earlier in discussing the properties of bonds, the price of a bond is inversely related to interest rates and is more price responsive to a change in interest rates if it has a longer term to maturity and its coupon rates are less. Thus, if interest rates change, the price effect on the total return will be negative (i.e., lower rates increase the horizon price and therefore the total return), with the effect being greater for bonds with greater terms to maturity and lower coupon rates. Secondly, interest rate changes affect the return the investor expects from reinvesting the coupon—***reinvestment risk***. If an investor buys a coupon bond, he automatically is subject to market risk. Thus, if interest rates change, the interest-on-interest effect on the total return will be direct (i.e., greater rates increase the reinvestment return and therefore the total return), with the effect being greater for bonds with greater coupon rates.

One way to evaluate market risk for a bond is to estimate the bond's total returns given different interest rate scenarios. Such analysis is known as ***horizon analysis***. Moreover, by conducting horizon analysis on one or more bonds or bond portfolios, an investor or portfolio manager can project the performance of the bonds or portfolios and can compare different bonds or bond portfolios based on a planned investment horizon and expectations concerning the market. To illustrate, Figure 2.5 shows the total returns for a three-year horizon for four bonds with different coupons and maturities under three interest rate scenarios: yields stay at 7.5%, yields decrease to 5%, and yields increase to 10%. For each scenario, it is assumed the reinvestment

Total returns for a three-year horizon for four bonds under three interest rate scenarios: Yields stay at 7.5%, yields decrease to 5%, and yields increase to 10%. For each scenario it is assumed the reinvestment rate and the rate for determining the horizon price of the bond are equal to the scenario yield.

Bond	Annual Coupon Rate	Maturity	Price at 7.5%	Total Return 5.00%	Total Return 7.50%	Total Return 10.00%
A	10.00%	3 yrs	106.61	7.23%	7.50%	7.77%
B	7.50%	5 yrs	100.00	8.59%	7.50%	6.48%
C	10.00%	10 yrs	117.37	10.85%	7.50%	4.47%
D	5.00%	15 yrs	77.71	13.80%	7.50%	2.01%

Horizon = 3 years
Semiannual payments
Yield same on all maturities

FIGURE 2.5 Horizon Analysis

rate and the rate for determining the horizon price of the bond are equal to the scenario yield. In terms of market risk, Bond A has the smallest deviations in total returns, with the lowest rate being 7.23% and the highest being 7.77%. Bond A's total return also decreases when rates decrease and increases when rates increase, suggesting Bond A's interest-on-interest effect dominates its price effect. In contrast, longer term Bond D has the greatest market risk, with the range in total returns being 2.01% to 13.8%, and with its total return increasing when rates decrease and decreasing when rates increase, implying its price effect dominates its interest-on-interest effect. Ultimately, which bond an investor should select depends on her expectations about future interest rates and the degree of market risk she wants to assume. Horizon analysis, though, is a useful tool for analyzing market risk and facilitating bond investment decisions.

2.6 SPOT RATES AND EQUILIBRIUM PRICES

The rate on a zero-coupon bond is called the spot rate. We previously examined how bonds are valued by discounting their cash flows at a common discount rate. Given different spot rates on similar bonds with different maturities, the correct approach to valuing a bond, though, is to price it by discounting each of the bond's cash flows, CFs, by the appropriate spot rates for that period (S_t). Theoretically, if the market does not price a bond with spot rates, arbitrageurs would be able to realize a risk-free return by buying the bond and stripping it into a number of zero-coupon bonds (Chapter 7 discusses strip securities), or by buying strip bonds and bundling them into a coupon bond to sell. Thus, in the absence of arbitrage, the ***equilibrium price*** of a bond is determined by discounting each of its CFs by their appropriate spot rates.

To illustrate this relationship, suppose there are three risk-free zero-coupon bonds, each with principals of $100 and trading at annualized spot rates of $S_1 = 7\%$, $S_2 = 8\%$, and $S_3 = 9\%$, respectively. If we discount the CF of a three-year, 8% coupon bond, paying an $8 coupon annually and a principal of $100 at maturity at these spot rates, its equilibrium price, P_0^*, would be $97.73:

$$P_0^* = \frac{C_1}{(1+S_1)^1} + \frac{C_2}{(1+S_2)^2} + \frac{C_3 + F}{(1+S_3)^3}$$

$$P_0^* = \frac{\$8}{(1.07)^1} + \frac{\$8}{(1.08)^2} + \frac{\$108}{(1.09)^3} = \$97.73$$

Suppose this coupon bond were trading in the market at a price (P_0^M) of $95.03 to yield 10%:

$$P_0^M = \sum_{t=1}^{3} \frac{\$8}{(1.10)^t} + \frac{\$100}{(1.10)^3} = \$95.03$$

At the price of $95.03, an arbitrageur could buy the bond, then strip it into three risk-free zero-coupon bonds: a one-year zero paying $8 at maturity, a two-year zero paying $8 at maturity, and a three-year zero bond paying $108 at maturity. If the arbitrageur could sell the bonds at their appropriate spot rates, she would be able

Market price of 3-*year,* 8% *coupon bond* = \$95

Arbitrage:

Buy the bond for \$95

Sell three stripped zeroes:

$$1\text{-}year\ zero\ with\ F = 8{:}\ P_0 = \frac{8}{1.07} = 7.4766$$

$$2\text{-}year\ zero\ with\ F = 8{:}\ P_0 = \frac{8}{(1.08)^2} = 6.8587$$

$$3\text{-}year\ zero\ with\ F = 108 : P_0 = \frac{108}{(1.09)^3} = 83.3958$$

Sale of stripped bonds = \$97.73

CF_0 = \$97.73 − \$95 = \$2.73

FIGURE 2.6 Equilibrium Bond Price Arbitrage When Bond Is Underpriced

to realize an initial cash flow (CF_0) from the sale of \$97.73 and a risk-free profit of \$2.70 (see Figure 2.6). Given this risk-free opportunity, this arbitrageur, as well as others, would exploit this strategy of buying and stripping the bond until the price of the coupon bond was bid up to equal its equilibrium price of \$97.73.

On the other hand, if the 8% coupon bond were trading above its equilibrium price of \$97.73, then arbitrageurs could profit by reversing the above strategy. For example, if the coupon bond were trading at \$100, then arbitrageurs would be able to go into the market and buy proportions (assuming perfect divisibility) of the three pure discount bonds (8% of Bond 1, 8% of Bond 2, and 108% of Bond 3) at a cost of \$97.73 and bundle them into one three-year, 8% coupon bond to be sold at \$100. As shown in Figure 2.7, this strategy would result in a risk-free cash flow of \$2.27.

Market price of 3-*year*, 8% *coupon bond* = \$100

Arbitrage:

Buy 3 *zeroes*:

$$8\%\ of\ 1\text{-}year\ zero\ with\ F = 100{:}\quad Cost = (.08)\frac{100}{1.07} = 7.4766$$

$$8\%\ of\ 2\text{-}year\ zero\ with\ F = 100{:}\quad Cost = (.08)\frac{100}{(1.08)^2} = 6.8587$$

$$1.08\%\ of\ 3\text{-}year\ zero\ with\ F = 100{:}\quad Cost = (1.08)\frac{100}{(1.09)^3} = 83.3958$$

Cost = 7.4766 + 6.8587 + 83.3958 = \$97.73

Bundle the bonds and sell them as 3-*year*, 9% *coupon bond for* \$100

CF_0 = \$100 − \$ 97.73 = \$2.27

FIGURE 2.7 Equilibrium Bond Price Arbitrage When Bond Is Overpriced

Estimating Spot Rates: Bootstrapping

Many investment companies use spot rates to value bonds, and there are a number of securities that have been created by arbitrageurs purchasing bonds valued at rates different from the spot rates, stripping the securities, and then selling them. One problem in valuing bonds with spot rates or in creating stripped securities is that there are not enough longer term zero-coupon bonds available to determine the spot rates on higher maturities. As a result, long-term spot rates have to be estimated.

One estimating approach that can be used is a sequential process commonly referred to as ***bootstrapping***. This approach requires having at least one zero-coupon bond, such as a Treasury bill. Given this bond's rate, a coupon bond with the next highest maturity is used to obtain an implied spot rate; then another coupon bond with the next highest maturity is used to find the next spot rate, and so on. As an example, consider the three risk-free bonds in Figure 2.8. Bond 1 is a one-year zero bond selling at \$100 and paying \$107 at maturity. The one-year spot rate using this bond is 7%: $S_1 = (\$107/\$100) - 1$. Bond 2 is an 8% annual coupon bond selling at par to yield 8% (YTM = 8%). Using bootstrapping, the spot rate on a two-year risk-free bond is found by setting this bond's price equal to the equation for its equilibrium price, P_0^*, and then solving the resulting equation for the two-year spot rate (S_2). Doing this yields a two-year spot rate of 8.042% (see Figure 2.8). Similarly, given one-year and two-year spot rates, the three-year spot rate can be found by setting the price of the three-year annual coupon bond equal to its equilibrium price, and

Maturity	*Annual coupon*	*F*	P_0^b
1 *year*	7%	100	100
2 *years*	8%	100	100
3 *years*	9%	100	100

S_1

$$100 = \frac{107}{1+S_1} \Rightarrow S_1 = \left[\frac{107}{100}\right] - 1 = .07$$

S_2

$$P_0^b = \frac{CF_1}{(1+S_1)^1} + \frac{CF_2}{(1+S_2)^2}$$

$$100 = \frac{8}{1.07} + \frac{108}{(1+S_2)^2}$$

$$92.52 = \frac{108}{(1+S_2)^2} \Rightarrow S_2 = \left[\frac{108}{92.52}\right]^{1/2} - 1 = .08042$$

S_3

$$P_0^b = \frac{CF_1}{(1+S_1)^1} + \frac{CF_2}{(1+S_2)^2} + \frac{CF_3}{(1+S_3)^3}$$

$$100 = \frac{9}{1.07} + \frac{9}{(1.08042)^2} + \frac{109}{(1+S_3)^3}$$

$$83.88 = \frac{109}{(1+S_3)^3} \Rightarrow Ss_3 = \left[\frac{109}{83.88}\right]^{1/3} - 1 = .0912$$

FIGURE 2.8 Generating Spot Rates Using Bootstrapping then solving the resulting equation for S_3. Doing this yields a three-year spot rate of 9.12%.

It should be noted that the equilibrium prices of other one-, two-, or three-year bonds can be obtained using these spot rates. For example, the equilibrium price of a risk-free, three-year, 10% annual coupon would be \$102.57:

$$P_0^* = \frac{\$10}{(1.07)^1} + \frac{\$10}{(1.08042)^2} + \frac{\$110}{(1.0912)^3} = \$102.57$$

2.7 GEOMETRIC MEAN

Another useful measure of the return on a bond is its ***geometric mean***. Conceptually, the geometric mean can be viewed as an average of current and future rates. To see this, consider one of our previous examples in which we computed a YTM of 7.72% for a zero-discount bond selling for \$800 and paying \$1,000 at the end of year 3. The rate of 7.72% represents the annual rate at which \$800 must grow to be worth \$1,000 at the end of three years assuming annual compounding. If we do not restrict ourselves to the same rate in each year, then there are other ways \$800 could grow to equal \$1,000 at the end of three years. For example, suppose one-year bonds are currently trading at a 10% rate, a one-year bond purchased one year from the present is expected to yield 8% ($R_{Mt} = R_{11} = 8\%$), and a one-year bond to be purchased two years from the present is expected to yield 5.219% ($R_{Mt} = R_{12} = 5.219\%$). With these rates, \$800 would grow to \$1,000 at the end of year 3. Specifically, \$800 after the first year would be \$880 = \$800(1.10); after the second, \$950.40 = \$800(1.10)(1.08); and after the third, \$1,000 = \$800(1.10)(1.08)(1.05219). Thus, an investment of \$800 that yielded \$1,000 at the end of three years could be thought of as an investment that yielded 10% the first year, 8% the second, and 5.219% the third. Moreover, 7.72% can be viewed not only as the annual rate at which \$800 grows to equal \$1,000, but also as the average of three rates: one-year rates today ($R_{Mt} = R_{10}$), one-year rates available one year from the present ($R_{Mt} = R_{11}$), and one-year rates available two years from the present ($R_{Mt} = R_{12}$):

$$P_0^b(1 + \text{YTM}_\text{M})^\text{M} = F = P_0^b[(1 + \text{YTM}_1)(1 + R_{11})(1 + R_{12})(1 + R_{13}) \cdots (1 + R_{1,M-1})]$$

$$(1 + \text{YTM}_\text{M})^\text{M} = \frac{F}{P_0^b} = [(1 + \text{YTM}_1)(1 + R_{11})(1 + R_{12})(1 + R_{13}) \cdots (1 + R_{1,M-1})] \tag{2.8}$$

$$(1.0772)^3 = \frac{\$1{,}000}{\$800} = [(1.10)(1.08)(1.05219)]$$

Mathematically, the expression for the average rate on an M-year bond in terms of today's and future one-year rates (and assuming annual compounding) can be found by solving Equation (2.8) for YTM_M:

$$YTM_M = [(1 + YTM_1)(1 + R_{11})(1 + R_{12})(1 + R_{13}) \cdots (1 + R_{1,M-1})]^{1/M} - 1$$
$$YTM_3 = [(1.10)(1.08)(1.05219)]^{1/3} - 1 = .0772 \quad (2.9)$$

Equation (2.9) defines the rate of return on an M-year bond in terms of expected future rates. A more practical rate than an expected rate, though, is the implied forward rate.

Implied Forward Rate

An ***implied forward rate***, f_{Mt}, is a future rate of return implied by the present interest rate structure. This rate can be attained by going long and short in current bonds. To see this, suppose the rate on a one-year, zero-coupon bond is 10% (i.e., spot rate is $S_1 = 10\%$) and the rate on a similar two-year zero is $S_2 = 9\%$. Knowing these current rates, we could solve for f_{11} in the equation below to determine the implied forward rate. That is,

$$S_2 = [(1 + S_1)(1 + f_{11})]^{1/2} - 1$$
$$f_{11} = \frac{(1 + S_2)^2}{(1 + S_1)} - 1$$
$$f_{11} = \frac{(1.09)^2}{(1.10)} - 1 = .08$$

With one-year and two-year zeros presently trading at 9% and 10%, respectively, the rate implied on one-year bonds to be bought one year from the present is 8%. This 8% rate, though, is simply an algebraic result. This rate actually can be attained, however, by implementing the following locking-in strategy:

1. Sell the one-year zero-coupon bond short (or borrow an equivalent amount of funds at the one-year zero-coupon or spot rate).
2. Use the cash funds from the short sale (or loan) to buy a multiple of the two-year zero.
3. Cover the short sale (or pay the loan principal and interest) at the end of the first year.
4. Collect on the maturing two-year bond at the end of the second year.

In terms of the above example, obtain the 8% implied forward rate:

1. Execute a short sale by borrowing the one-year bond and selling it at its market price of \$909.09 = \$1,000/1.10 (or borrowing \$909.09 at 10%).
2. With two-year bonds trading at \$841.68 = \$1,000/$(1.09)^2$, buy \$909.09/\$841.68 = 1.08 issues of the two-year bond.

3. At the end of the first year, cover the short sale by paying the holder of the one-year bond his principal of \$1,000 (or repay loan).
4. At the end of the second year, receive the principal on the maturing two-year bond issues of (1.08)(\$1,000) = \$1,080.

With this locking-in strategy the investor does not make an investment until the end of the first year when he covers the short sale; in the present, the investor simply initiates the strategy. Thus, the investment of \$1,000 is made at the end of the first year. In turn, the return on the investment is the principal payment of \$1,080 on the 1.08 holdings of the two-year bonds that comes one year after the investment is made. Moreover, the rate of return on this one-year investment is 8% [(\$1,080 − \$1,000)/\$1,000]. Hence, by using a locking-in strategy, an 8% rate of return on a one-year investment to be made one year in the future is attained, with the rate being the same rate obtained by solving algebraically for f_{11}.[2]

Given the concept of implied forward rates, the geometric mean now can be formally defined as the geometric average of the current one-year spot rate and the implied forward rates. That is

$$YTM_M = [(1 + YTM_1)(1 + f_{11})(1 + f_{12})(1 + f_{13}) \cdots (1 + f_{1,M-1})]^{1/M} - 1 \tag{2.10}$$

Two points regarding the geometric mean should be noted. First, the geometric mean is not limited to one-year rates. That is, just as 7.72% can be thought of as an average of three one-year rates of 10%, 8% and 5.219%, the implied rate on a two-year bond purchased at the end of one year, $f_{Mt} = f_{21}$, can be thought of as the average of one-year implied rates purchased one and two years, respectively, from now. Accordingly, the geometric mean could incorporate an implied two-year bond by substituting $(1+f_{21})^2$ for $(1+f_{11})(1+f_{12})$ in Equation (2.10). Similarly, to incorporate a two-year bond purchased in the present period and yielding YTM_2, one would substitute $(1+\text{YTM}_2)^2$ for $(1+S_1)$ $(1+f_{11})$. Thus:

$$YTM_3 = [(1 + YTM_1)(1 + f_{11})(1 + f_{12})]^{1/3} - 1$$
$$YTM_3 = [(1 + YTM_1)(1 + f_{21})^2]^{1/3} - 1$$
$$YTM_3 = [(1 + YTM_2)^2(1 + f_{12})]^{1/3} - 1$$

Secondly, note that for bonds with maturities of less than one year, the same general formula for the geometric mean applies. For example, the annualized YTM on a zero-coupon maturing in 182 days (YTM_{182}) is equal to the geometric average of a current 91-day bond's annualized rate (YTM_{91}) and the annualized implied forward rate on a 91-day investment made 91 days from the present, $f_{91,91}$:

$$YTM_{182} = [(1 + YTM_{91})^{91/365}(1 + f_{91,91})^{91/365}]^{365/182} - 1$$

Thus, if a 182-day zero-coupon bond were trading at $P_0^b(182) = \$97$ per \$100 face value and a comparable 91-day bond were at $P_0^b(91) = 98.35$, then the implied forward rate on a 91-day bond purchased 91 days later would be 5.7%:

$$YTM_{182} = \left[\frac{100}{97}\right]^{365/182} - 1 = .063$$

$$YTM_{91} = \left[\frac{100}{98.35}\right]^{365/91} - 1 = .069$$

$$f_{91,91} = \left[\frac{(1 + YTM_{182})^{182/365}}{(1 + YTM_{91})^{91/365}}\right]^{365/91} - 1$$

$$f_{91,91} = \left[\frac{(1.063)^{182/365}}{(1.069)^{91/365}}\right]^{365/91} - 1 = .057$$

Usefulness of the Geometric Mean

One of the practical uses of the geometric mean is in comparing investments in bonds with different maturities. For example, if the present interest rate structure for zero-coupon bonds were such that two-year bonds were providing an average annual rate of 9% and one-year bonds were at 10%, then the implied forward rate on a one-year bond one year from now would be 8%. With these rates, an investor could equate an investment in the two-year bond at 9% as being equivalent to an investment in a one-year bond today at 10% and a one-year investment to be made one year later yielding 8% (possibly through a locking-in strategy). Accordingly, if the investor knew with certainty that one-year bonds at the end of one year would be trading at 9% (a rate higher than the implied forward rate), then he would prefer an investment in the series of one-year bonds over the two-year bond. That is, by investing in a one-year today and a one-year bond one year from now the investor would obtain 10% and 9%, respectively, for an average annual rate on the two-year investment of 9.5%; specifically,

$$\text{Series equivalent } YTM_2 = [(1 + YTM_1)(1 + \text{Expected spot rate})]^{1/2} - 1$$

$$\text{Series equivalent } YTM_2 = [(1.10)(1.09)]^{1/2} - 1 = .095$$

This, of course, exceeds the 9% average annual rate the investor would obtain if he bought the two-year bond; thus in this case, the series of one-year bonds represents the better investment. In contrast, if the investor expected with certainty that, at the end of one year, one-year bonds would be trading at 6% (a rate below the implied forward rate), then a series of one-year bonds at 10% and 6% would yield a two-year average annual rate of only 8% (equivalent $\text{YTM}_2 = [(1.10)\ (1.06)]^{1/2} - 1 = .08$), a rate below the 9% average annual rate on the two-year bond. Thus, in this case, the investor would prefer the two-year bond to the series of one-year bonds. Finally, if the investor expects the rate in the future to equal the implied rate, then we can argue that he would be indifferent to an investment in a two-year bond and a series of one-year bonds (see Figure 2.9).

In general, the investor's decision to invest in an M-year bond or a series of one-year bonds, or some combination with the equivalent maturity, depends on what the

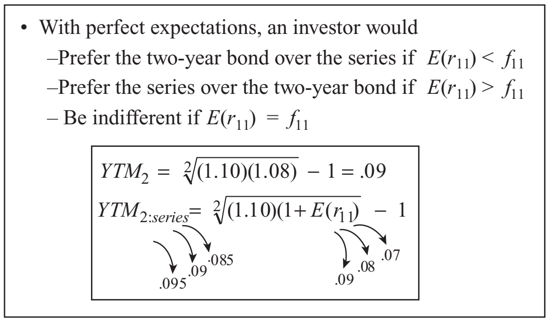


FIGURE 2.9 Geometric Mean

investor expects rates will be in the future relative to the forward rates implied by today's interest rate structure.

2.8 CONCLUSION

For most investors, the most important characteristic of an asset is its rate of return. In this chapter we have examined how the rate of return on a bond and its related characteristic, value, can be measured. We are now in a position to take up the question of what determines the rate of return on a bond. Our preceding analysis suggests that the characteristics of bonds—taxability, liquidity, maturity, and risk—ultimately determine the yield on a bond. The two most important of these characteristics are maturity and risk—the subjects for the next three chapters.

WEB INFORMATION

(See Appendix D for Web Site Information and Examples.)
Treasury Information:
Investinginbonds.com

1. Go to http://investinginbonds.com/.
2. Click "Government Market-at-a-Glance."
3. Treasury yields are based on price data for U.S. Treasury securities of different maturities. The yields are provided by GovPX, Inc. The issues listed are "on-the-run" issues for the date, meaning they are the most recently issued in that maturity.
4. The Treasury yield data can be exported to Excel (right click and click "Export to Excel").

FINRA

1. Go to www.finra.org/index.htm, Sitemap, Market Data, and Bonds.
2. Click "Treasury and Agency" tab and then click "Advanced Bond Search" to find Treasury and Agency bonds with certain features.
3. Enter Treasury or Agency Symbol (e.g., FHLMC) and then click "Search."

Wall Street Journal

1. Go to http://online.wsj.com/public/us, Market Data, Bonds, Rates & Credit Markets, and Treasury Quotes.
2. The Treasury yield data can be exported to Excel (on data quote table, right click and then click "Export to Excel")
3. Go to http://online.wsj.com/public/us, Market Data, Bonds, Rates & Credit Markets, and Treasury Strips.

Yahoo.com

1. Go to http://finance.yahoo.com/bonds, click "Advanced Bond Screener," and click "Treasury" or "Treasury Zero Coupon" (Treasury strips).

Corporate Bond Information:
Investinginbonds.com

1. Go to http://investinginbonds.com/, click "Corporate Market at-a-Glance," Search by Issuer.

FINRA

1. Go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds."
2. For a bond search, click "Corporate" and then click "Advanced Bond Search" to find corporate bonds with certain features or issues of specific issuer (click "Search").

Wall Street Journal

1. Go to http://online.wsj.com/public/us, Market Data, Bonds, Rates & Credit Markets, and "Corporate Bonds Most Active" and "Corporate Gainers and Losers."

Yahoo.com

1. Go to http://finance.yahoo.com/bonds, click "Advanced Bond Screener" and click Corporate Tab, then provide information for search.
2. Go to http://finance.yahoo.com/bonds, enter "Name of Issuer" and click "Search."

Finance Calculator—FICALC

The online FICALC calculator computes a bond's price given its yield or its yield given its price, as well as other information, such as cash flows and total returns. The FICALC calculator is designed so it can either be used as a stand-alone fixed income calculator, or integrated into a Web site that has information on fixed income securities.

www.ficalc.com/calc.tips

KEY TERMS

accrued interest
amortized securities
average rate to maturity (ARTM)
average realized return
bond-equivalent yield
bootstrapping
cash flow yield
clean price
convexity
coupon rate
current yield
day count convention
discount yield
effective rate
equilibrium price
full price or dirty price
geometric mean
horizon
horizon analysis
implied forward rate
interest rate
logarithmic return
market risk
perpetuity or consul
price risk
price-yield curve
pure discount bonds (PDBs)
realized return
reinvestment risk
simple annualized rate
spot rate
total return (TR)
yield to call
yield to put (YTP)
yield to worst
yield approximation formula
yield to maturity (YTM)
zero-coupon bonds

PROBLEMS AND QUESTIONS

Note: For problems requiring a number of calculations, readers may want to use their own Excel program, a financial calculator, or the Bond Excel Program available on the text's Web site.

1. Given a five-year, 8% coupon bond with a face value of $1,000 and coupon payments made annually, determine its values given it is trading at the following yields: 8%, 6%, and 10%. Comment on the price and yield relation you observe. What are the percentage changes in value when the yield goes from 8% to 6% and when it goes from 8% to 10%?
2. Given a 10-year, 8% coupon bond with a face value of $1,000 and coupon payments made annually, determine its value for the following yields: 8%, 6%, and 10%. What are the percentage changes in value when the yield goes from 8% to 6% and when it goes from 8% to 10%? Comment on the price and interest rate relation you observe for this 10-year bond and the price and interest rate relation you observe for the five-year bond in question 1.
3. Determine the value of a five-year, zero-coupon bond with a face value of $1,000 given it is trading at the following yields: 8%, 6%, and 10%. What are the percentage changes in value when the yield goes from 8% to 6% and when it goes from 8% to 10%? Comment on the price and interest rate relation you observe for this zero-coupon bond and the price and interest rate relation you observe for the five-year, 8% coupon bond you observe in question 1.
4. Given a five-year, 8% coupon bond with a face value of $1,000 and trading at a simple annual rate of 9%, determine the values and effective annualized rates given the bond has the following payment or compounding frequencies:

a. Semiannual
b. Monthly
c. Weekly
Comment on the relation you observe.

5. Given a two-year, zero-coupon bond with a face value of $100 and trading at a simple annual rate of 10%, determine the bond values given following compounding frequencies:

a. Monthly
b. Weekly
c. Daily
d. Continuously
Comment on the relation you observe.

6. Generate the price-yield curve for a zero-coupon bond with a face value of $100 and 260 actual days to maturity using the following annual yields: 4%, 4.25%, 4.5%, 4.75%, 5%, 5.25%, 5.5%, 5.75%, 6%, 6.25%, 6.5%, 6.75%, 7%, 7.25, 7.5%, 7.75%, and 8%. Use actual/actual day count convention.

7. Given a 10-year, 8% coupon bond with a face value of $100 and semiannual coupon payments:

a. Generate the bond's price-yield curve using annual yields ranging from 5% to 10% and differing by .5%.
b. What is the price change when the yield increases from 8% to 8.5%?
c. What is the price change when the yield decreases from 8% to 7.5%?
d. Comment on the capital gain and capital loss you observe in (b) and (c).
e. Comment on the features of the price-yield curve.

8. Suppose an investor bought a 10-year, 10% annual coupon bond at par (face value of $1,000 and paying coupons annually) and then sold it 3.5 years later at a yield of 8%. Determine the full price, clean price, and accrued interest the investor would receive when he sold the bond. Use a 30/360 day count convention.

9. What would an investor pay for a four-year, 9% annual coupon bond (face value of $1,000 and paying coupons annually) if the bond were trading to yield 10%? What would the investor receive (full price) if she sold the bond 3.5 years later and bonds with maturities of .5 years were trading at 8%? Use 30/360 day count convention.

10. Determine the actual prices a dealer would pay (bid) or sell (ask) on the following bonds:

a. A Treasury bond with $1,000 face value quoted by a dealer at a bid price of 95-4 per $100 face value and fractions in 32nds.
b. A Treasury bond with $1,000 face value quoted by a dealer at an asked price of 110-4+ per $100 face value with 4+ indicating fractions in 64ths.
c. A corporate bond with $1,000 face value quoted by a dealer at a bid price of 97 1/2 per $100 face value with fractions in 100ths.

d. A zero-coupon bond with maturity of one year quoted at an asked price to yield 550 basis points.
e. A T-bill maturing in 52 days and paying $10, 000 face value and quoted by a dealer at an asked annual discount yield of 4%.

11. Suppose you have an A-rated bond with an 8% annual coupon, face value of $1,000, and due to mature in five years. Presently, the YTM on such bonds is 10%. You expect the Federal Reserve will tighten credit and force yields up by 50 basis points in the near future. Determine today's price and the expected price.

12. Suppose the AIF Company sold a bond with a 10-year maturity, $1,000 principal and an annual 10% coupon paid semiannually. What would be the price of the bond if two years after the bond were issued the promised YTM were 12%? What is the effective YTM?

13. Define the following rates of return measures:
a. Discount yield
b. Interest rate
c. Coupon rate
d. Current yield
e. Rate on perpetuity
f. Yield to maturity
g. Average rate to maturity (yield approximation formula)
h. Bond equivalent yield
i. Yield to call
j. Yield to put
k. Yield to worst
l. Bond portfolio yield
m. Logarithmic return
n. Spot rate
o. Total return
p. Geometric mean

14. Using the average rate to maturity (yield approximation formula) approach, estimate the YTM on a 20-year, 7% annual coupon bond, with a face value of $1,000, annual coupon payments, and currently priced at $901.82. What is the value of the bond using the ARTM as the discount rate?

15. Suppose the 20-year, 7% annual coupon bond in question 14 had a call option giving the issuer the right to buy the bond back after five years at a call price of $1,000. Given the bond is priced at $901.82, estimate its yield to call using the yield approximation formula (average rate to call, ARTC) approach.

16. A zero-coupon Treasury bill maturing in 150 days is trading at $98 per $100 face value. Determine the following rates for the T-bill:
a. Dealer's annual discount yield
b. YTM (use an actual/365 day count convention)
c. Logarithmic return (use actual/365 day count convention)
Explain the differences in the rates.

17. ABC Trust has the following bond portfolio:

Bond	Maturity (Years)	Annual Coupon	Face Value	Price per $100 Face Value
A	1	0	100	92.59
B	2	8%	100	100.00
C	3	7%	100	97.42
D	4	10%	100	106.62
E	5	9%	100	104.00
				500.63

The coupon bonds in the portfolio all pay coupons annually and all the bond prices are quoted per $100 face value to yield 8%.

a. Explain how the bond portfolio's YTM is calculated.
b. Determine the bond portfolio's YTM using a financial calculator, Excel program, or by trial and error (hint: try YTM = 8%).
c. Does the portfolio's YTM equal the weighted average yield of the bonds? If so, is this always the case?

18. Bond A is a 10-year, 10% coupon bond with a face value of $1,000 and annual coupon payments. The bond is currently priced at $1,064.18 to yield 9%.

a. Define the bond-equivalent yield.
b. Explain how Bond A's bond equivalent yield is calculated.
c. Calculate Bond A's bond equivalent yield using a financial calculator, Excel program, or by trial and error.
d. What is the importance of the bond-equivalent yield?

19. Suppose A-rated bonds were trading in the market at YTM of 10% on all maturities, and you bought an A-rated, 10-year, 9% coupon bond with face value of $1,000 and annual coupon payments. Suppose that immediately after you bought the bond the yield on such bonds dropped to 8% on all maturities and remained there until you sold the bond at your horizon date at the end of four years.

a. What price did you pay for the 10-year, 9% coupon bond?
b. Show in a flow matrix (similar to Figure 2.4A) the coupons you received on the bond and their values at your horizon date from reinvesting.
c. What is the price of the original 10-year bond at your horizon date?
d. What is your horizon date value and total return?

20. Given a 10-year, 10% coupon bond with semiannual payments, $1,000 face value, and currently trading at par, calculate the total return for an investor with a five-year horizon date, given the following interest rate scenarios:

a. Yields on such bonds stay at 10% on all maturities until the investor sells the bond at her horizon date.
b. Immediately after the investor buys the bond, yields on such bonds drop to 8% on all maturities and remain there until the investor sells the bond at her horizon date.

c. Immediately after the investor buys the bond, yields on such bonds increase to 12% on all maturities and remain there until the investor sells the bond at her horizon date.

Comment on the relation between total return and interest rates.

21. Given the following spot rates on one-year to four-year zero-coupon bonds:

Year	Spot Rate
1	8.0%
2	8.5%
3	9.0%
4	9.5%

a. What is the equilibrium price of a four-year, 9% coupon bond paying a principal of $100 at maturity and coupons annually?
b. If the market prices the four-year bond such that it yields 10%, what is the bond's market price?
c. What would arbitrageurs do given the prices you determined in (a) and (b)? What impact would their actions have on the market price?
d. What would arbitrageurs do if the market price exceeded the equilibrium price? What impact would their actions have on the market price?

22. Given a one-year zero-coupon bond trading at $100 and promising to pay $106 at maturity and a two-year 6% coupon bond with face value of $100, annual payments, and trading at $96.54:

a. Determine the one-year and two-year spot rates.
b. What is the equilibrium price of a comparable two-year 8% annual coupon bond ($F = 100$)?

23. Using the geometric mean, show four expressions for the yield to maturity on a four-year bond, YTM_4.

24. Bond X is a one-year zero with face value of $1,000 trading at $945 and Bond Y is a two-year zero with a face value of $1,000 trading at $870:

a. Determine algebraically the implied forward rate f_{11}.
b. Explain how the forward rate can be attained by a locking-in strategy.

25. Explain how you would lock in the following implied forward rates: f_{11}, f_{21}, and f_{23}.

26. Given that current 182-day T-bills are trading at a YTM of 4% and 91-day bills are trading at YTM of 3.75%, what is the implied forward rate on a 91-day T-bill 91 days from now? Explain how you would lock in the implied forward rate.

WEB EXERCISES

1. Bond Finance Calculator: www.ficalc.com/calc.tips

Use the FICALC calculator to compute a bond's price given its yield or its yield given its price.

Exercise:

Go to www.ficalc.com/calc.tips and click the "Use Calculator" tab.

Select bond type (Example: fixed-income corporate).

Input information and set settings for information and calculations (e.g., cash flow, total return, etc.).

Click the "Calculation" tab.

Input terms:

Issue Date: This is the date upon which the security starts accruing interest.

First Coupon Date: This is the date upon which the first coupon payment is made.

Settlement Date: This is the date for which all calculations are performed.

2. Treasury Bond Search and Value and Yield Calculation

Exercise:

Use Yahoo.com to find Treasury bonds or notes: http://finance.yahoo.com/bonds.

Using Yahoo's Advanced Bond Screener, search for Treasury bonds with certain characteristics, such as maturity and coupon rate.

Select one or more of the bonds from the search and find their prices, cash flow, and total return using the finance calculator: www.ficalc.com/calc.tips.

Calculate the price yourself based on the YTM (remember you are at a non-coupon date).

Generate a price-yield curve for your selected T-bond by varying the yield using the finance calculator.

3. Corporate Bond Search and Value and Yield Calculation

Exercise:

Use Yahoo.com to find a corporate bond or note: http://finance.yahoo.com/bonds.

Use Yahoo's Advanced Bond Screener, search for corporate bonds with certain characteristics, such as maturity and coupon rate.

Select one or more of the bonds from the search and compute their prices, cash flow, and total return using the finance calculator: www.ficalc.com/calc.tips.

Calculate the price yourself based on the YTM (remember you are at a non-coupon date).

Generate a price-yield curve for your selected corporate by varying the yield using the finance calculator.

4. This chapter discusses how the equilibrium price of a bond is based on spot rates and how arbitrageurs can buy or sell strip securities to ensure this condition. Yahoo.com and the *Wall Street Journal* site (http://online.wsj.com/public/us) provide market quotes on Treasury strips.

   Exercise:

   Using Yahoo! (http://finance.yahoo.com/bonds) and its Bond Screener, search for Treasury zero-coupon bonds with certain maturities.

   Using the WSJ site's (http://online.wsj.com/public/us) "Markets" and "Market Data," examine the prices and yield on Treasury strips.

5. Rates on Treasury securities and other bonds can be found on www.bloomberg.com. Using Bloomberg, go to "Market Data" and "Rates & Bonds" and explain the relation you observe between coupon rates, yields, and bond prices that you find on Treasuries and municipals.
6. Bond Search using Financial Industry Regulatory Authority Site: www.finra.org/

   Exercise:

   Go to the FINRA site to find information on a corporate bond: Go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds."

   Use Quick Bond Search to find bonds with certain features.

   Click one of the bonds to get more information.

   At the bottom of the information page, do a search of the bond's trading activity.

   Select one or more of the bonds from the search and, using the finance FICALC calculator (www.ficalc.com/calc.tips), compute their YTM given price, yield to worse given the bond's call price and call schedule (if the bond is callable), price-yield curve, and cash flow.

   Go back to the home page, click "Company Information," and select the corporation whose bond you are analyzing to find more information about the company.

7. Bond Search using Investinginbonds.com site: www.Investinginbonds.com

   Exercise:

   Go to http://investinginbonds.com/.

   Click "Corporate."

   Click "Corporate Market at-a-Glance."

   On Search Box, enter the corporation's name and then select bond.

   On Trade Date Information Table, click "Graph Trade Data" (above box) to obtain the last five months of trading activity.

   On Trade Date Information Table, click "Run Calculations" to obtain more information on the bond and a price/yield calculator set at the price for the trade time you selected.

Using the Price Calculator, examine the price-yield relation by varying the yield.

Using the Price Calculator, examine the price-yield relation by varying the price.

Using the Price Calculator, examine the value of the bond at different trade dates.

8. Information Exercise

Create a folder in "Favorites" to bookmark studies, stories, and other information related to the financial markets and debt management.

Example: Financial Crisis Folder

Content: From the *Wall Street Journal* site: "End of Wall Street: An Oral History"

http://online.wsj.com/public/page/wall-street-in-crisis.html.

NOTES

1. An exception to this rule would be when a bond is in default. Such a bond is said to be quoted flat; that is, without accrued interest.
2. In Chapter 16 we will show that there is good reason to expect that the implied forward rate is equal to the rate implied on a futures contract to buy or sell a debt security at some future date.

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Radcliffe, Robert C. *Investment Concepts, Analysis, and Strategy*. Glenview, IL: Scott, Foresman and Company, 1982.

Rose, Peter S. *Money and Capital Markets*. New York: McGraw-Hill/Irwin, 2003.

Bond Evaluation, Selection, and Management, Second Edition
by R. Stafford Johnson

CHAPTER 3

The Level and Structure of Interest Rates

3.1 INTRODUCTION

Over the last three decades interest rates have often followed patterns of persistent increases or persistent decreases with fluctuations around those trends. This is illustrated in Figure 3.1 where 10-year Treasury rates and Baa corporate bond yields are shown from 1960 to 2009. As shown, in the late 1970s and early 1980s, interest rates increased dramatically, increasing from 7.38% in February 1977 to 15.75% in November 1981. This period was marked by high inflation and recession (stagflation) and by the implementation of a contractionary U.S. monetary policy in which the Federal Reserve raised discount rates, increased reserve requirements, and lowered monetary growth. This period of increasing rates was followed by a long period of declining rates from the early 1980s to 2009, with fluctuations around this declining trend. As shown in Figure 3.1, troughs in Treasury yields occurred in February 1983 at 10.42%, April 1987 at 7.68%, January 1998 at 4.77%, June 2003 at 3.38%, and December 2008 at 2.25%, with each trough lower than the previous one.

In addition to the observed trends in interest rate levels, there have also been observed differences or spreads between the interest rates on bonds of different categories over this same period. The spreads can be seen in Figure 3.2: the top panel shows five-year Treasury yields along with the yields on high quality AAA corporate bonds and riskier Baa quality corporate bonds from 1960 to 2009; the bottom panel shows the spreads between BBB corporate yields and the Treasury yields and between AAA corporate yields and the Treasury yields over the same time period. From 1960 to 2002, the relatively narrow spreads of BBBs over Treasuries range between 0.14% in February 1966 to 1.82% in 1984, and the relatively wider spreads range from 2.78% in December 1998 to 4.57% in November 2002. More recently, the spread of BBB corporate credits over Treasuries was at 1.62% in November 2006 and at 6.91% in December 2008. In general, spreads are explained by differences in each bond's characteristics: risk, liquidity, and taxability. For example, the spread between yields on BBBs and Treasuries are wider in the recessionary periods and tighter in periods of economic growth.

Finally, interest rate differences can be observed between similar bonds with different maturities. Figure 3.3 shows various plots of the YTM on bonds with different maturities. The graphs are known as ***yield curves*** and they illustrate what is referred to as the ***term structure of interest rates***. Figure 3.3 shows yield curves generated

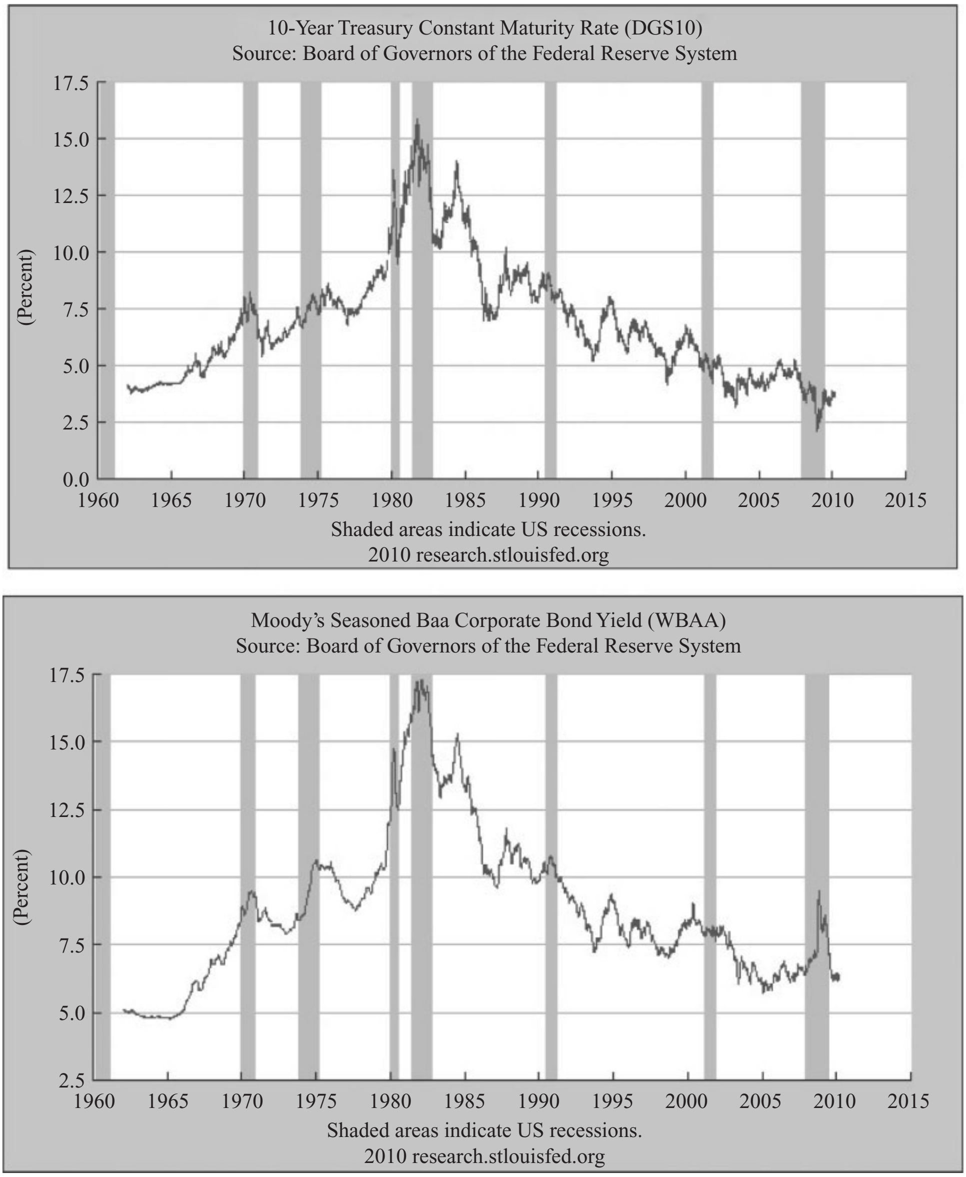
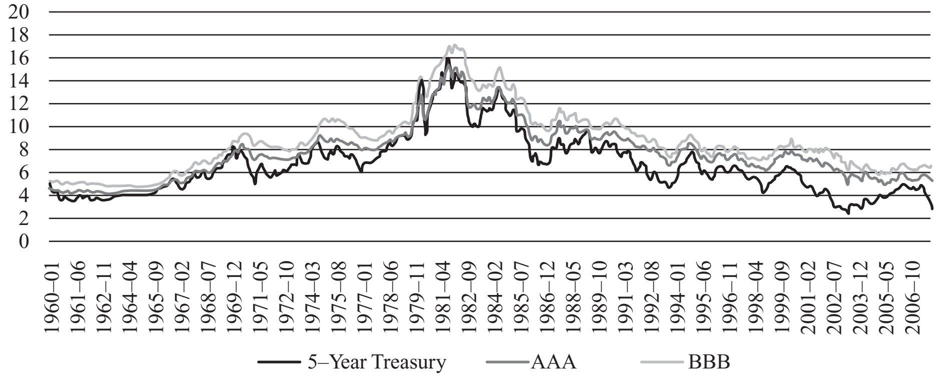


FIGURE 3.1 10-Year Treasury Rates and Baa Corporate Bond Yields: 1960–2009
Source: Federal Reserve: www.research.stlouisfed.org/fred2.

from U.S. securities and AAA-quality and BB-quality industrial bonds on October 17, 2008; Figure 3.4 shows Treasury yield curves for eight select times from 1973 to 2007. The October 2008 yield curves and a number of yield curves in Figure 3.4 are positively-sloped with rates on short-term securities lower than intermediate-term and long-term ones. There are some cases, though, in which the yield curve is relatively flat and two cases (May 1981 and August 1973) in which the curve is negatively sloped with short-term rates higher than intermediate-term and long-term ones.



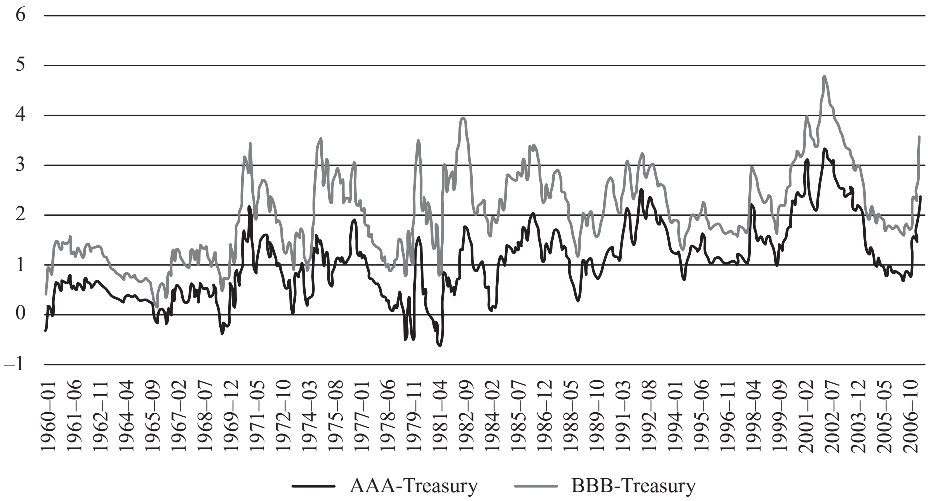


FIGURE 3.2 Treasury Bond, AAA Corporate, BBB Corporate, 1960–2009

Understanding what determines both the overall level and structure of interest rates is an important subject in financial economics. In this chapter, we examine the factors that are important in explaining the level and differences in interest rates. We begin by examining the behavior of overall interest rates using basic supply and demand analysis and treating debt and bonds as one general type of security. With this foundation, we then look at how risk, liquidity, and taxes explain the differences in the rates on bonds of different categories. Finally, in Chapter 4, we complete our examination of the structure of interest rates by looking at four well-known theories that explain the term structure on interest rates.

3.2 LEVEL OF INTEREST RATES

In general, the overall level of interest rates in an economy is determined by economic and financial factors that affect the demand and supply of bonds (or loanable funds). For example, if an economy is expanding and as a result corporations are selling more

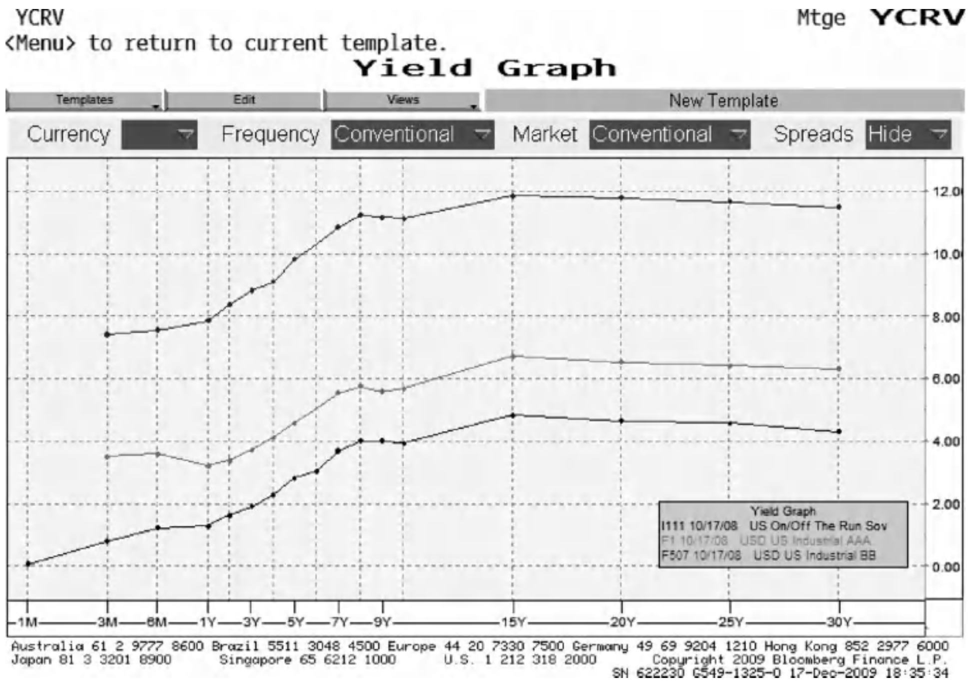


FIGURE 3.3 Treasury and Corporate Yield Curves: October 17, 2008
Source: Copyright © 2009 Bloomberg Finance L.P. All rights reserved. Used with permission.

bonds, then the bond market will initially experience an excess supply of bonds at the current level of rates. In the financial markets, the excess supply will cause bond prices to fall and bond yields to increase. The general level of interest rates therefore depends on identifying the important factors determining the aggregate demand and supply of existing bonds. Among the determining factors are (1) the overall state of the economy, (2) government policies, such as monetary and fiscal policy, (3) bond risk relative to the risk of investing in other assets such as equity or real estate, (4) international factors, such as foreign interest rates and exchange rates that influence the inflow and outflow of foreign capital, and (5) the current level of inflation and the expectation about future inflation. The impacts these economic factors have on the level of interest rates can be analyzed by using fundamental supply and demand analysis (see Figure 3.5).

Note: In determining the supply and demand for bonds, different bonds are treated as being alike. In this case, the generic bond is assumed to be a 1-period, zero-coupon bond paying a principal of F equal to 100 at maturity and priced at P_0 to yield a rate of y:

$$P_0 = \frac{F}{1+y} = \frac{100}{1+y}$$

$$y = \frac{F - P_0}{P_0} = \frac{F}{P_0} - 1 = \frac{100}{P_0} - 1$$

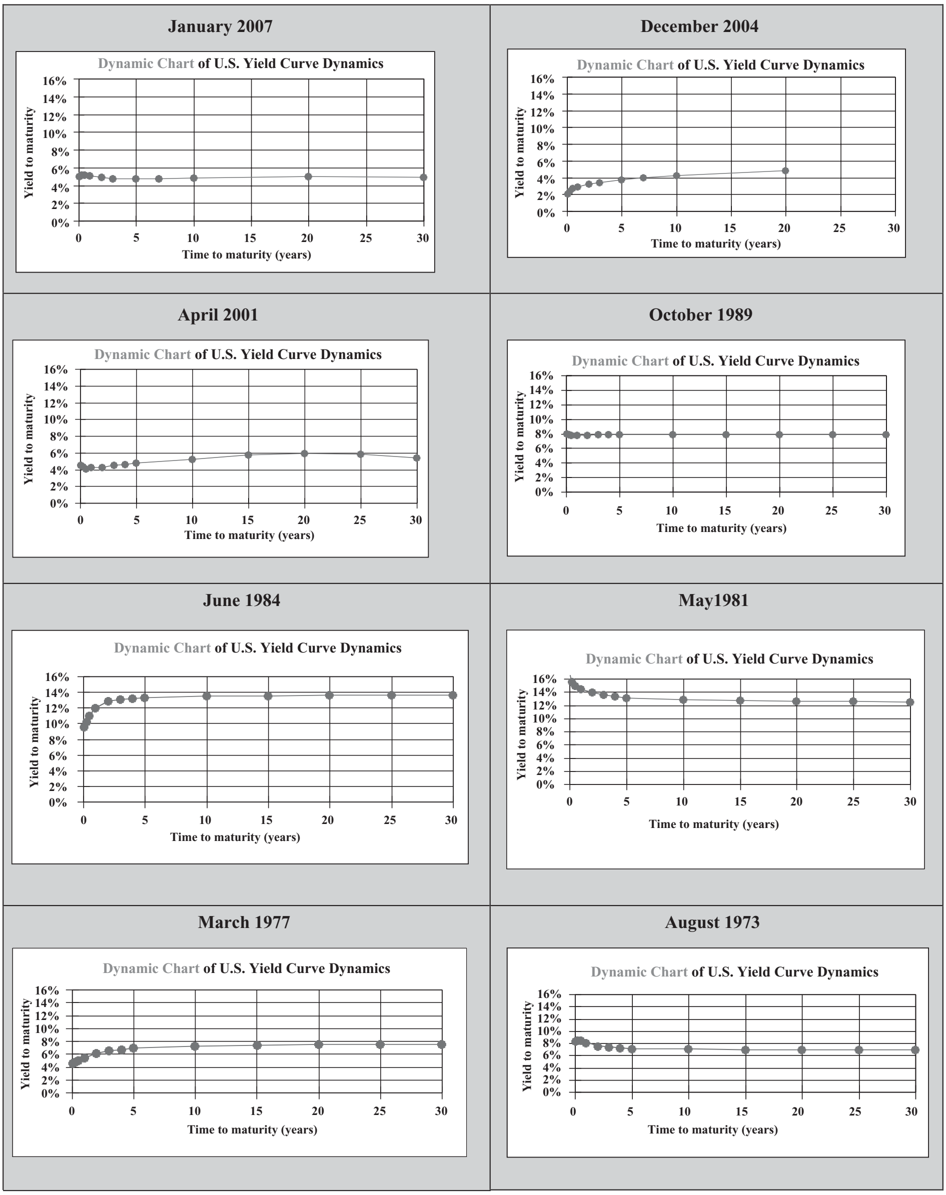


FIGURE 3.4 Yield Curve for U.S. Treasuries
Source: Craig Holden's Dynamic Chart of U.S. Yield Curve Dynamics. Professor Craig W. Holden, Kelley School of Business, Indiana University, Bloomington, IN 47405.

Bond Demand and Supply Curve Analysis

Figure 3.5 shows a bond demand curve $B^D B^D$. The curve shows the relationship between the aggregate level of bond demand in the economy, B^D, and their average price, P, or interest rate, y, with the assumption that the other factors, such as the state of the economy and relative risk, are constant. To illustrate the relation

- Supply and Demand for Bonds
- Equilibrium is at 94.33 and $y^* = 6\%$

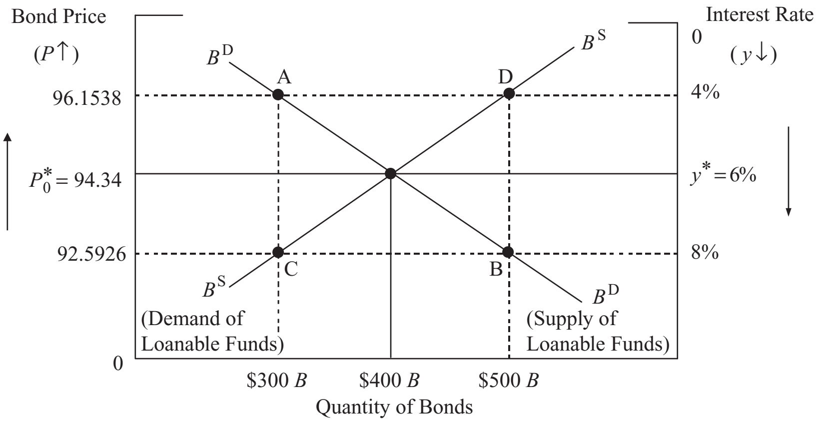


FIGURE 3.5 Bond Demand and Supply

between bond demand and both its price and interest rate, Figure 3.5 has two vertical axes. The left axis shows the bond's price, with the price increasing as we go from the bottom up. The right vertical axis shows the associated interest rate, with the interest rate increasing as we move from the top to the bottom. Combined, the two axes capture the inverse relation between a bond's price and interest rate. Thus, at point A on $B^D B^D$, the average price of a bond is at 96.1538 and its associated interest rate is 4%; at point B the price is 92.5926 and the associated rate is 8%. The bond demand curve in Figure 3.5, in turn, is negatively sloped. This reflects the fundamental assumption that investors will demand more bonds the lower the price or equivalently the greater the interest rate. Thus, at point B investors are shown to demand $500 billion worth of the bond (number of bonds times the face value) when P is 92.5926 and y is 8%, and at point A, bond demand is shown to be only $300 billion when P is 96.1538 and y is 4%. It should be noted that because buying or demanding a bond is equivalent to supplying a loan, the bond demand curve in Figure 3.5 can also be identified as a ***supply of loanable funds*** curve.

The bond supply curve, $B^S B^S$, in Figure 3.5, shows the relation between the quantity supplied of bonds by corporations, governments, and intermediaries, B^S, and the average bond price and interest rate, given other determining factors are constant. The $B^S B^S$ curve is positively sloped. This reflects the fundamental assumption that corporations, governments, and financial intermediaries will sell more bonds the greater the bond's price or equivalently the lower the interest rate. Thus, at point C issuers are shown to supply only $300 billion worth of the bond when P is 92.5926 and y is 8%, whereas at point D bond supply is shown to be $500 billion when P is 96.1538 and y is 4%. It should be noted that because selling or supplying a bond is equivalent to obtaining or demanding a loan, the bond supply curve in Figure 3.5 can also be identified as a ***demand for loanable funds*** curve.

Market Equilibrium

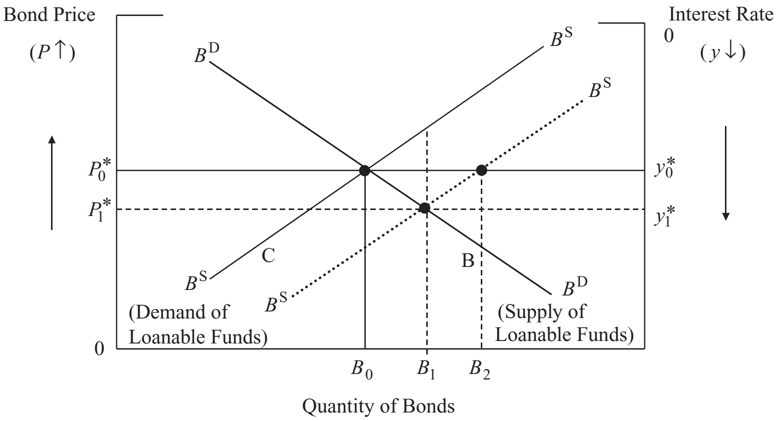
The bond price or interest rate that ultimately prevails in the market is the one at which the quantity demanded of bonds equals the quantity supplied: $B^D = B^S$. The equilibrium rate, y^* and price, P_0^*, are graphically defined by the intersection of the supply and demand curves. In Figure 3.5, this occurs at $P_0^* = 94.34$ and $y^* = 6\%$ where the quantity demanded and supplied are both \$400 billion. The equilibrium interest rate of 6% is the market-clearing interest rate and the equilibrium price of 94.34 is the market-clearing price. If the bond price were below this equilibrium price (or equivalently the interest rate were above the equilibrium rate), then investors would want more bonds than issuers were willing to sell. This excess demand would drive the price of the bonds up, decreasing the demand (movement down along the demand curve) and increasing the supply (movement along the supply curve) until the excess was eliminated. On the other hand, if the price on bonds were higher than the equilibrium price (or interest rates lower that the equilibrium rate), then bondholders would want fewer bonds, whereas issuers would want to sell more bonds. This excess supply in the market would lead to lower prices and higher interest rates, increasing bond demand (movement along the bond demand curve) and reducing bond supply (movement along the supply curve) until the excess supply was eliminated. Thus, only at P_0^* and y^*, where bond demand equals bond supply, is there an equilibrium where bondholders and suppliers do not want to change.

Comparative Equilibrium

The insights that one can gain from supply and demand analysis come from identifying the important factors that affect the positions of the demand and supply curves. Analytically, changes in these factors cause shifts in the demand or supply curves that, in turn, lead to a new equilibrium bond price and interest rate levels.

For example, on the bond supply side, if an economy is expanding and as a result corporations are selling more bonds, then the bond market will initially experience an excess supply of bonds at the current level of rates. The excess supply will cause bond prices to fall and bond yields to increase. This is shown graphically in Figure 3.6 where the B^SB^S curve is shown shifting to the right from the solid B^SB^S line to the dashed B^SB^S line that reflects higher economic growth. At the initial interest rate of y_0^*, there is an excess supply of bonds of $B_2 - B_0$. This excess supply in the market will lead to lower prices and higher interest rates, increasing bond demand (movement along the bond demand curve) and reducing bond supply (movement along the supply curve) until the excess supply is eliminated. On the bond demand side, an increase in demand will have the opposite impact on rates. For example, if stocks become riskier relative to bonds, then bond demand will increase, leading to an excess demand for bonds, increasing their price and lowering their rates. This is shown graphically in Figure 3.7 where the B^DB^D curve is shown shifting to the right from the solid B^DB^D line to the dashed B^DB^D line. At the initial interest rate of y_0^*, there is an excess demand of bonds of $B_2 - B_0$. This excess demand in the market will lead to higher prices and lower interest rates, decreasing bond demand (movement along the bond demand curve) and increasing bond supply (movement along the supply curve) until the excess demand is eliminated.

- Supply and Demand for Bonds
- Rightward Shift in $B^S B^S$ Curve

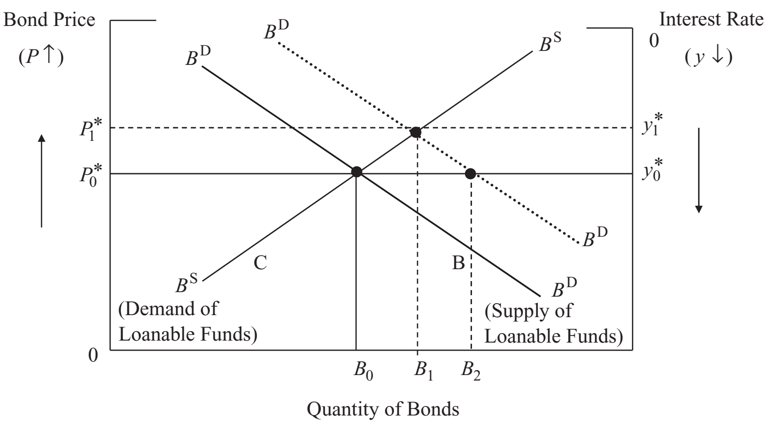


$B^S = f$[y or P_0, GDP, Treasury financing, OMO, Intl. capital flows, Inflation, E(Inflation)]

FIGURE 3.6 Increase in Bond Supply

As noted, some of the important economic factors influencing the supply and demand for bonds are the overall state of the economy, monetary and fiscal policy, international capital flows, relative risk, inflation, and expected inflation. Some of these factors affect both the supply and demand for bonds, such as the state of the economy (as measured by GDP or aggregate wealth) and expected inflation; some factors are unique to bond supply, such as Treasury financing, certain types

- Supply and Demand for Bonds
- Rightward Shift in $B^D B^D$ Curve



$B^D = f$[y or P_0, GDP, ΔR_D, ΔRR, Relative risk), E (Inflation)]

FIGURE 3.7 Increase in Bond Demand

of monetary policies such as open market operations (OMO), international capital flows, and actual inflation; some factors impact only bond demand, such as relative risk and certain types of monetary policies such as changing the discount rate (ΔR_D) and changing reserve requirements (ΔRR):

$B^S = f$[y or P_0, GDP, Treasury financing, OMO, International capital flows, Inflation, E(Inflation)]

$B^D = f$[y or P_0, GDP, ΔR_D, ΔRR, E(Inflation), Relative risk, International capital flows]

Economic Conditions

A priori, we would expect an increase in the both the demand and supply of bonds in periods when the economy is growing and aggregate wealth and GDP are increasing, and a decrease in bond demand and supply in periods when the economy is in a recession or aggregate wealth is decreasing. On the supply side, when an economy is expanding, business demand for both short-term assets, such as inventories and accounts receivable, and long-term assets, such as plants and equipment, increase. As a result, companies find themselves selling more bonds (demanding more loans) to finance the increases in their short-term and long-term capital formation. In addition to changes in corporate bond supply, aggregate economic growth is also likely to increase both the purchases of cars and homes by household and the number of public projects by municipal government (e.g., roads), augmenting the supply of bonds by financial intermediaries and state and local governments. Thus, we would expect the supply of bonds to increase in periods of economic growth (rightward shift in the $B^S B^S$ curve), causing an excess supply that would serve to lower bond prices and increase rates.

On the demand side, investments in bonds and other securities also tend to increase in periods of economic growth. An increase in the demand for bonds would have the opposite impact on bond prices and rates, causing bond demand to increase ($B^D B^D$ line to shift right), and leading to an excess demand for bonds. The excess demand in the market would lead to higher prices and lower interest rates.

It should be noted that the direct impact of economic growth on bond demand can be explained not only by the growth in GDP, but also a wealth effect that often accompanies economic conditions or in some cases precedes economic changes. That is, a country's economic state is measured not only in terms of the aggregate production of final goods and services (i.e., the flow of GDP), but also on the value of its assets or aggregate wealth: the value of its equity, real estate, business debt, and government debt. Typically, when economies grow, aggregate wealth also tends to increase, raising stock market values and increasing housing and real estate values. This increase in wealth serves to increase the demand for bonds. Conversely, when economies decline, aggregate wealth also tends to decrease. There are also times when changes in aggregate wealth precede changes in economic growth. For example, in 2008, the decline in real estate values and the subsequent decrease in equity values preceded the slowdown in the United States and global economies.

In recessionary periods, there is less capital formation and fewer bonds being sold by corporations, governments, and intermediaries. This decrease in bond supply (leftward shift in the $B^S B^S$ curve), would cause an excess demand or a shortage of bonds that would tend to increase bond prices and decrease rates. At the same time, in recessionary periods when GDP and aggregate wealth are declining, the demand for bonds would tend to decrease (leftward shift in the $B^D B^D$ curve), causing bond prices to fall and rate to increase.

In general, the net effect of economic growth or decline on interest rates depends on the relative impacts the economy has on the supply and demand for bonds (the slopes of the bond demand and supply curves and the shifts in the curves in response to changes economic growth). In growth periods, rates would increase (decrease) if the supply (demand) impact on interest rates dominates the demand (supply) impact. In receding economic periods, rates would decrease (increase), if again the supply (demand) impact on interest rate dominates the demand (supply) impact. Historically, in some recessionary periods, rates have had a tendency to decline (1987–1988 and 1999–2003); and in some periods of expansion, rates have tended to increase (1993–1999). These trends in rates tend to suggest that the supply impact of corporate, municipal, and intermediate borrowing dominates the demand impact on rates.

Government Monetary and Fiscal Policy Treasury Financing

Government monetary and fiscal policies can indirectly impact interest rates by affecting the overall economy. Such policies can also have a direct impact on rates. Consider the actions of the Treasury in its financing of the federal government's budget. If the federal government has a deficit (government expenditures exceeding tax revenues), then the Treasury will be raising funds in the financial market by selling more Treasury securities. At current interest rate levels, this would create an excess supply of bonds in the market, pushing bond prices down and rate up (rightward shift in the $B^S B^S$ curve). In contrast, if there were a government surplus, the bond supply would decrease if the Treasury decided to use the surplus to buy up existing Treasury securities in order to reduce the government's outstanding debt. In this case, the supply of bonds would decrease and the resulting excess demand would push bond prices up and rate down (leftward shift in the $B^S B^S$ curve).

In discussing the effects of Treasury financing on interest rates, it should be pointed out that the impact of the financing of a government deficit on the financial markets depends on the size of the deficit relative to the size of the overall economy. For example, in 1983 the financing of a \$208 billion deficit when the GDP of the U.S. economy was \$2.7 trillion had a larger impact on interest rates than the financing of a \$160 billion deficit in 2007 when the level of GDP was approximately \$13.6 trillion. Similar to the 1980 deficits, the impact of the projected U.S. deficits of approximately \$1.8 trillion in 2009 would be expected to have a more significant impact on interest rates than the financing of the 2007 deficit of \$160 billion.

Monetary Policy

In addition to Treasury financing, bond supply is also affected by central bank policies. One important monetary tool central banks employ is an open market operation.

In order to stimulate the economy, the central bank often uses an expansionary ***open market operation (OMO)*** to lower interest rates. In an expansionary OMO, the central bank buys existing securities, usually Treasuries. If we limit the definition of bond supply to those bonds held by the public and not the central bank, then an expansionary OMO (or contractionary bond policy) leads to a decrease in the supply of bonds (leftward shift in the $B^S B^S$ curve).

In contrast, when the central bank is fighting inflation, it may try to slow the economy by increasing interest rates through a contractionary OMO. Here the bank sells some of its security holdings (expansionary bond policy), increasing the supply of bonds (rightward shift in the $B^S B^S$ curve).

In addition to open market operations, two other traditional monetary tools of note are changing the discount rate, ΔR_D, that the central bank charges banks for borrowing, and changing the amount of reserves, ΔRR, banks are required to maintain in order to secure their deposits. These monetary actions, in turn, change the amount of loans banks are willing to offer. A change in the supply of loanable funds in our analysis is equivalent to change in bond demand. Thus, if the central bank were to increase the amount of reserves banks were required to maintain to back their deposits or if they were to increase the discount rate they charge banks for borrowing, we would expect the supply of loanable funds (bond demand) made by banks to decrease, shifting the bond demand curve to the left.

For developed economies, monetary policy can be an effective tool for stimulating economic growth or fighting inflation provided interest rates are not too low or too high. Often contractionary policies are implemented when an economy is experiencing inflationary pressures, whereas expansionary policies are used when the economy is beset by unemployment, excess capacity, and recessionary pressures. Two of the most dramatic monetary actions occurred in the late 1970s and early 1980s, when the U.S. Federal Reserve implemented a major contractionary monetary policy, and more recently in 2008, when the Federal Reserve and other European central banks implemented expansionary monetary actions to offset the global financial crisis.

Relative Risk

When bonds become riskier relative to other investments, we expect a decrease in bond demand, and when they become less risky relative to other investments, we expect an increase in bond demand. For example, if the chance of default on bonds increases or if other investment alternatives such as stocks become less risky, then bond demand will fall. On the other hand, if bonds become less risky relative to other investments, then bonds will become relatively more attractive, causing the demand for bonds to increase. Often relative risk is related to the overall state of economy. For example, in a recessionary time or in a period in which economic decline is expected, there is a usually a ***flight to safety*** where investors tend to move their investments from riskier stock and riskier bonds to less risky bonds (Treasury and high-quality bonds). The substitution of high-quality bonds for equity has the impact of pushing bond demand and prices up and rates down (rightward shift in the $B^D B^D$ curve). In such periods, there is also likely to be a move from lower quality (high risk) bonds to higher quality (low risk) bonds. Furthermore, such periods are often accompanied with an increase in credit risk that leads to a tightening of credit provided by financial institutions. A credit tightening causes a decrease in the supply of loanable funds, or in our model, a decrease in bond demand. An increase in credit risk combined with an increase in credit tightening would lead to an increase in the demand for Treasury and higher quality bonds and a decrease in the demand for lower quality bonds. These changes would, in turn, lead to a widening of the spread between investment grade and speculative grade bonds (this is discussed in Section 3.3). In contrast, in periods of economic expansion, investors tend to move their investments from bonds to stocks that are now viewed as relatively less risky. The substitution of equity for bonds would have the impact of pushing bond demand and prices down, and rates up (leftward shift in the $B^D B^D$ curve). In such periods, there is also likely to be a move from higher quality to lower quality bonds, as well as a credit loosening, which would lead to a tightening of the spread between investment grade and speculative grade bonds.

The impact of changes in relative risk on interest rates was recently observed during the early stages of the 2008 financial crisis. The extraordinary turmoil in the credit markets and the slowdown in the overall U.S. and global economies in 2008 led to a precipitous decline in equity values. After an impressive rise in the S&P 500 index from 1,000 in 2003 to just over 1,500 at the end of 2007, the stock market began to fall during the first nine months of 2008, decreasing 13% from 1,500 to just below 1,300, and then decreasing an astonishing 30% in October when it went from 1,300 to 900. Equally as striking as the declines in the stock market were the increases in volatility. For the period from end of 2003 to the end of 2007, the historical and implied volatilities of the S&P 500 had been relatively stable, fluctuating within a range between 10% and 20% variability for the implied volatility and 5% and 10% for the historical volatility.[1] In October 2008, the implied volatility increased from 20% to over 50% and the historical volatility went from 17% to 27% (see Figure 3.8). During this period, equity and low-quality debt became riskier relative to Treasury and high-quality bonds. As a result, a flight to safety ensued in which investors moved from stocks and lower-quality bonds to riskless Treasuries and higher-quality bonds. This portfolio substitution increased Treasury and high-quality bond prices and lowered their yields (see Figure 3.8).

International Capital Flows

The globalization of financial markets over the last 25 years has led to significant increases in investment flows in and out of countries. These international capital flows also influence the supply of securities. For example, over the last 10 years China has invested a significant amount of its international currency reserves (U.S. dollars) resulting from its balance of payments surpluses in intermediate-term U.S. Treasury securities. These investments have contributed significantly to keeping U.S. intermediate rates low—in some periods lower than short-term T-bill yields. The National Bureau of Economic Research (NBER) estimates that in 2007 Asian investments decreased the yields on U.S. intermediate securities by as much as 70 basis points.

The ability of China to invest heavily in liquid U.S. Treasuries gives them influence over U.S. rates similar to an exogenous central bank open market operation. That is, if we limit the definition of bond supply to those bonds held by the public and not the central bank or foreign central banks, then, like an expansionary OMO,

S&P 500 Price and Volatility Trends

Treasury Yield Curves: Quarterly 2007–2008

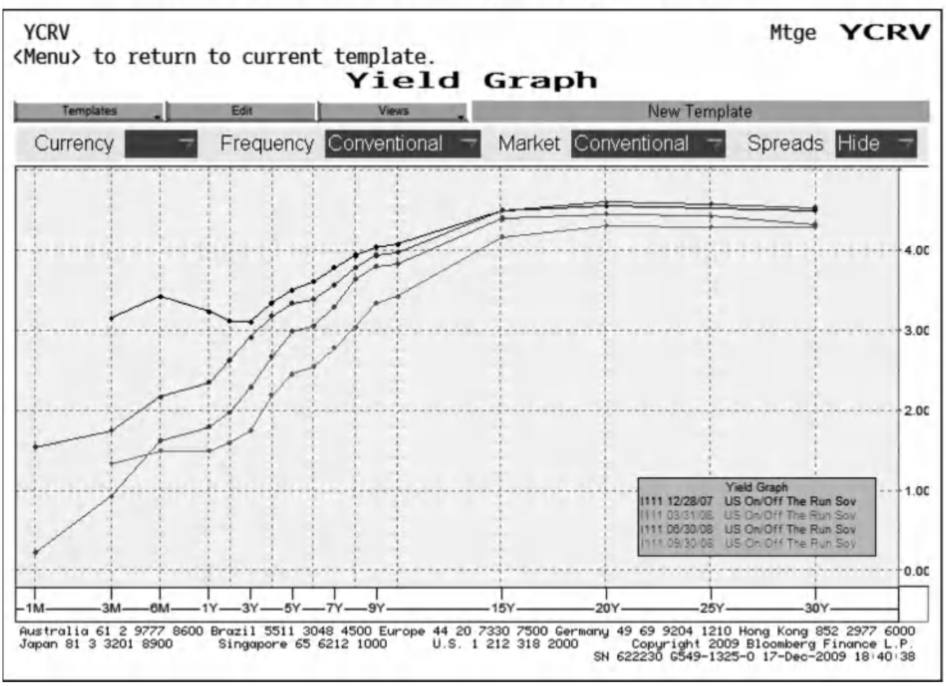


FIGURE 3.8 Price and Volatility Trends; Treasury Yield Curves: Quarterly 2006–2007
Source: Copyright © 2009 Bloomberg Finance L.P. All rights reserved. Used with permission.

the purchase of U.S. Treasuries by China would lead to a decrease in the supply of bonds (leftward shift in the $B^S B^S$ curve), creating an excess demand that would push U.S. bond prices up and rates down.

In general, when a country operates with a persistent balance of payments surplus, then there is often an accumulation of foreign currency by the country's central banks. In an effort to keep their currency values low (to maintain their export sales), such countries often invest their currency reserves in the country whose currency they are holding. Such actions describe the policies of China over the last 10 years, as well as a number of energy-producing countries in the Middle East. Furthermore, many of these surplus countries also have sovereign wealth funds that also invest in foreign assets. In China, for example, there is the state-owned China Development Bank that buys foreign assets as a ***sovereign wealth fund***.

In addition to foreign investments from central banks and sovereign wealth funds, a large amount of private institutional funds also invest globally. Global investment funds earn returns not only from the foreign securities in which they invest, but also from the currency positions that they assume. Figure 3.9 shows the dollar/British pound spot exchange rate and the dollar/euro rate from 2000 to 2009. For a number of subperiods between 2000 and 2009, the dollar prices of the BP and euro were increasing (dollar depreciation). For dollar investors, the dollar depreciations over these periods would have made investments in foreign securities very attractive, whereas British pound investors would have found investing in dollar-denominated assets less attractive. (For an example of the relationship between exchange rates and global investments, see Appendix D.)

In general, when a currency like the dollar is experiencing sustained depreciation, more investment dollars can flow out as global institutional funds try to take advantage of expected dollar depreciation, and less investment dollars can flow in. Just the opposite occurs when the currency is experiencing sustained appreciation. These flows, in turn, can affect the demand for domestic bonds and therefore the level of interest rates. Like investors, borrowers also may find it advantageous to borrow or issue securities outside the country where they are incorporated and in different currency denominations. Today, multinational corporations raise funds to finance their extensive global operations. As noted in Chapter 1, this has given rise to a global debt market consisting of Eurobonds, foreign bonds, and Eurocurrency loans. The impact these external sources of funding have on domestic interest rates depends on the extent to which domestic (U.S.) capital formation is financed domestically (bonds sold in United States) or globally (bonds sold outside the United States).

Actual and Expected Inflation

On the supply side, actual inflation increases borrowing and the supply of bonds as corporations and other economic entities are forced to borrow more funds to finance their inflated capital cost. In addition to actual inflation, the expectation of higher inflation can also lead to an increase in bond supply. That is, if inflation is expected to be higher in the future, then expected borrowing costs will be higher in the future and more funds (inflated funds) will be needed to finance capital formation. As a result, corporations will find it advantageous to borrow more funds now. Thus, both actual and expected inflation can increase current bond supply, lowering bond prices and increasing yields (rightward shift in the $B^S B^S$ curve).

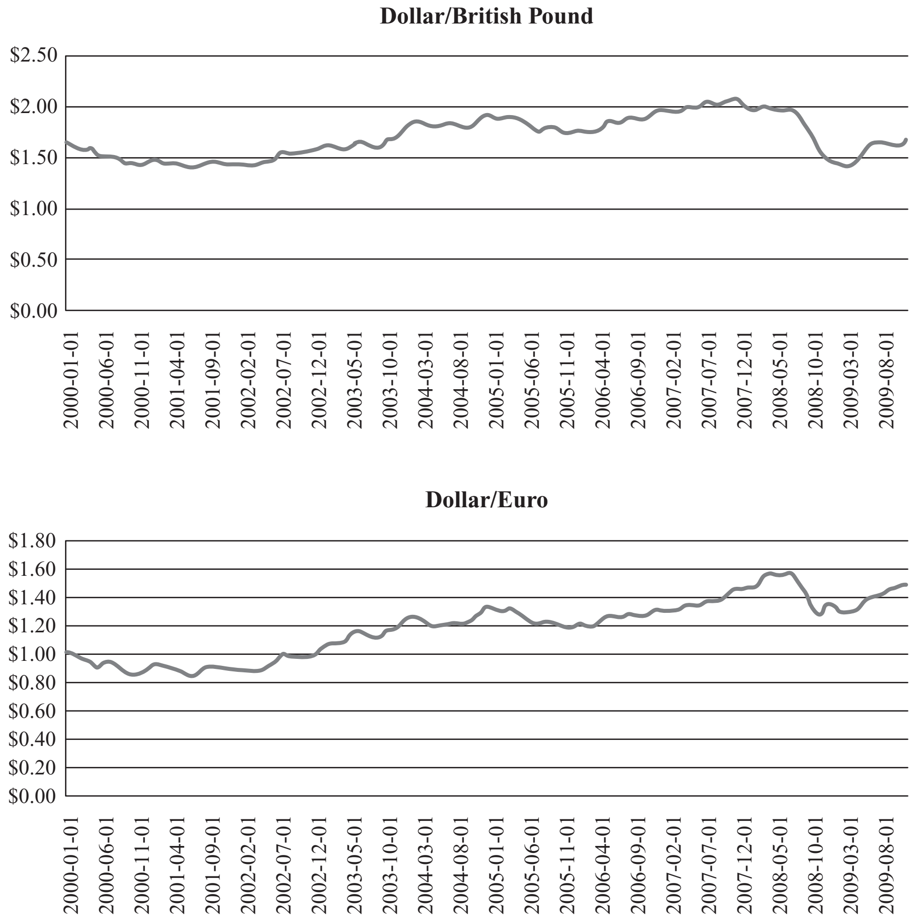


FIGURE 3.9 U.S. Dollar Exchange Rates
Source: Federal Reserve: www.federalreserve.gov/releases/h15/data.htm.

On the demand side, it is expected inflation that is the important factor influencing bond demand. If investors expect the prices on goods and services, as well as cars, houses, and other consumer durables, to be higher in the future, they will decrease their current purchases of bonds and other securities and buy more consumption goods and consumer durables. This decrease in demand will decrease bond prices and increase yields (leftward shift in the $B^D B^D$ curve). Thus, actual inflation, by increasing bond supply (rightward shift in the $B^S B^S$ curve), leads to a decrease in interest rates; and expected inflation, by increasing bond supply (rightward shift in the $B^S B^S$ curve) and decreasing bond demand (leftward shift in the $B^D B^D$ curve), also leads to a increase in interest rates.

It should be noted that in inflationary times, there is not only current inflation but often the expectation of higher inflation. Thus, in an inflationary climate characterized by actual and expected inflation, interest rates would increase (see Figure 3.10).

In contrast, actual and expected deflation lowers bond supply as corporations find they need less funds (deflated funds) to finance capital formation and also as they

Impact of Inflation

- An increase in actual and expected inflation increases bond supply, shifting the bond supply curve to the right; an increase in expected inflation decreases bond demand, shifting the bond demand curve to left.
- Impact: Interest Rates Increase

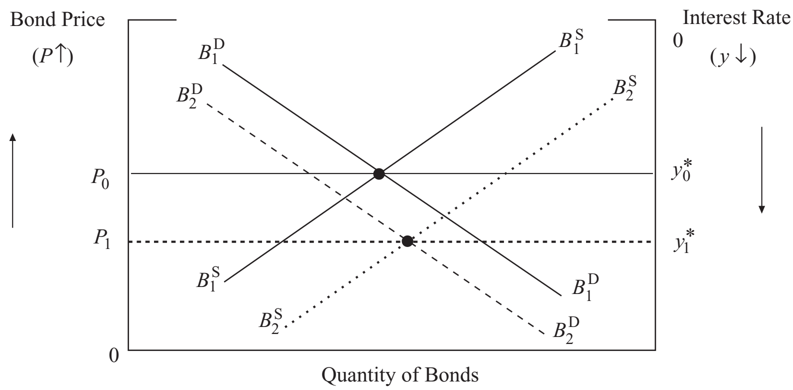


FIGURE 3.10 Inflation Impact on Interest Rates

find it advantageous to defer funding in expectation of lowering borrowing needs (deflated capital cost) in the future. On the demand side, expected deflation increases bond demand. That is, if investors expect the prices on goods and services, cars, houses, and other consumer durables to be lower in the future, they will increase their current purchases of bonds and other securities so that they can buy more consumption goods and consumer durables later after prices have fallen. Thus, actual deflation, by decreasing bond supply (leftward shift in the B^SB^S curve), leads to a decrease in interest rates; and expected deflation, by decreasing bond supply (leftward shift in the B^SB^S curve) and increasing bond demand (rightward shift in the B^DB^D curve), also leads to a decrease in interest rates. Thus, in a deflationary climate in which there is actual and expected deflation, interest rates would fall.

The direct relationship between inflation and interest rates is referred to as the ***Fisher effect***, after Irving Fisher, the economist who broached this relation. The Fisher effect is helpful in explaining not just the relatively high interest rate periods of the late 1970s when the U.S. inflation rate was relatively high, but also low interest rate periods when inflation expectations are low. For example, in the late 1990s, Japan experienced very low interest rates and deflation, with Treasury rates actually becoming slightly negative in 1998. Using our analysis, on the bond supply side, expected deflation (often combined with recession) decreases the bond supply, whereas on the demand side, the expected deflation increases bond demand. Together the demand and supply response to expected deflation would increase the price of bonds and lower their rate.

Interest Rate Expectations and Liquidity

Expectations about future interest rate levels and bond liquidity relative to the liquidity of other investments are two other factors affecting the demand and supply for bonds. However, interest rate expectations and liquidity tend to impact the relative demands and supplies of different types of bonds rather than the relative supplies and demands between the broader equity and debt security classes. As a result, interest rate expectations and liquidity are better explained in terms of the structure of rates than the level of rates. For example, a market expectation of higher interest rates in the future would likely lead to lower bond demand for intermediate and long-term bonds, and a greater demand for short-term bonds. Thus, interest rate expectations are more likely to lead to substitutions between bonds with different maturities, than between and aggregate bonds and equity groups.

Combined Effects The aggregate bond supply and demand model is a partial equilibrium analysis that can be used to explain the impacts of a change in a specific exogenous economics factor with other determining factors assumed constant. Extending this methodology to understand past or future interest rate levels requires identifying all of the important factors influencing rates. Consider two of the most dramatic interest rate periods: the late 1970s and early 1980s, when interest rates were at an unprecedented high level, and more recently in 2008 when rates were at a very low level. It can be argued that the high interest rates of the late 1970s and early 1980s were the result of the period's high inflation rate that averaged over 10% and the contractionary monetary policy undertaken by the Federal Reserve. In contrast, it can be argued that the low rates on securities in 2008 were caused by the slowdown in the world economy and the aggressive expansionary monetary actions of the Fed and other central banks (see Exhibit 3.1).

In summary, the study of what determines the level of interest rates depends on identifying the important factors and their influence on the supply and demand for bonds. Exhibit 3.1 summarizes several periods from 1970 to 2008 that are characterized by certain interest rate trends and offers some possible reasons for the trends based on some of factors we have examined here.

3.3 THE STRUCTURE OF INTEREST RATES

In addition to the level of interest rates, we also observe differences in rates among different types of bonds. The differences or spreads between rates, in turn, can be explained by differences in the fundamental features of bonds: risk, liquidity, taxability, and maturity.

Risk

Risk Premium Investment risk is the possibility that the actual rate of return realized from a security will differ from the expected rate. In the case of bonds, there are three general types of investment risk: (1) ***default risk*** (or ***credit risk***): the possibility that the issuer/borrower will fail to meet his contractual obligations to pay interest and principal, as well as other obligations specified in the indenture; (2) ***call risk***: the possibility that the issuer/borrower will buy back the bond, forcing the investor to reinvest in a market with lower interest rates; (3) ***market risk***: the uncertainty that interest rates will change, changing the price of the bond and the return earned from reinvesting coupons. In Chapter 5, we will examine in some detail each of these types

EXHIBIT 3.1 The Impact of Economic Events on the Level of Interest Rates: 1970–2009

Energy Cost and the Inflation of the 1970s

Nobel Laureate Paul Samuelson describes an economic state in which there is both inflation and recession as stagflation. In the 1970s, the United States and other industrial economies experienced severe stagflation resulting from increases in energy prices. Specifically, the price of OPEC oil increased from $3 per barrel in 1972 to approximately $35 per barrel in 1980. These energy price increases led to increases in the overall costs of production (which were passed on in the form of higher prices), and lower economic growth rates (in economics, the increase in resource cost is reflected by a leftward shift in the aggregate supply curve). The United States suffered recessions in 1973, 1975, and 1978, with each of the recessions accompanied by increases in the inflation rate. For the decade, the annual inflation rate averaged over 10%, whereas the growth rate in real gross domestic product for the decade was only 5% (in contrast, from 1982 to 1995, real aggregate output doubled). During this period, there was also a significant increase in interest rates in the United States and other industrial countries (see Figure 3.10). The increase can be explained primarily by the high actual and expected inflation. As shown in Figure 3.10, actual and expected inflation increases the real cost of borrowing and the need by corporations and others to borrow more funds, shifting the bond supply curve to the right; inflation also decreases the demand for bonds, shifting the bond demand curve to the left. Combined, the actual and expected inflation cause bond prices to fall (prices go from P_0 to P_1) and interest rates to increase (y_0^* to y_1^*).

Monetary Actions of the Late 1970s and Early 1980s

Beginning in October of 1979 and extending through October 1982, the Fed raised the discount rate, increased reserve requirements, and set lower monetary growth targets for its OMOs in an effort to combat high inflation and balance of payments problems in the United States. These actions, in turn, represented a directional change in the Fed's policies from the preceding three-year period in which they maintained lower discount rates and reserve requirements. Although higher energy prices had already contributed to inflation and high interest rates, these contractionary monetary actions served to push up rates even higher. By 1982, rates on Treasury bonds mortgages were 15%, the prime lending rate charged by banks was 21%, and the Dow Jones Industrial Average was at 700!

Fiscal Policy Stimulant and the Mid-1980s

Since the late 1960s, fiscal policy in the United States has been characterized in most years by deficit spending in which federal government expenditures exceeded tax revenues. The Reagan Administration in the 1980s, for example, increased government expenditures and substantially cut taxes. These expansionary fiscal policy actions were accompanied with an accommodating monetary policy by the Fed. The combined expansionary monetary and fiscal policy actions led to economic recovery in the United States during the second half of the 1980s and also explained the relatively lower interest rates.

Technological Changes and the 1990s

From 1984 to 1998, America's gross domestic product rose from $3 trillion to over $8 trillion, and the stock market, as measured by the Dow Jones Industrial Average, increased from 700 in 1984 to over 9,000 in 1998. Whereas some of this extraordinary growth can be explained by the expansionary monetary and fiscal policies of the mid-1980s, the decrease in energy prices, and world economic growth, much can be attributed to the advances in science and technology in such areas as computer technology, genetic engineering, and telecommunications. The advances in technology and science that were realized in the 1990s (though their development was much earlier) increased the productivity of labor and capital. As a result, for much of the 1990s the United States and other industrial economies enjoyed not only significant economic growth, but also stable prices. During this period, there was a gradual increase in interest rates in the United States. The increase can be explained primarily by the increase in borrowing (bond supply increase causing a rightward shift in the bond supply curve) needed to finance the capital formation during this period. The reason that interest rate increases were not significant during this extraordinary growth period can be explained in part by the relatively low inflation rate.

2008 to 2009: Overshooting Corrections and Financial Crisis

Although the industrial economies have experienced significant economic growth over the last century, such long-run trends have been characterized by business cycles in which economies have experienced peaks and troughs as they moved along their long-run growth trend. Economists have debated the cause of business cycles. In his classic 1939 work, the Austrian economist Joseph Schumpeter argued that fluctuations are the result of innovations, or the setting into use of new technological advancements. He argued that "what dominates the picture of capitalistic life and more than anything else is responsible for disequilibria is innovation, the intrusion into the system of new production functions." On a more contemporary level, Schumpeter's business cycle theory can be explained in terms of an overshooting phenomenon in which businesses have a tendency to overproduce when aggregate demand is high, leading to an excess supply and an equilibrium adjustment in which they have to cut output; on the other hand, businesses have a tendency to underproduce in response to declines in aggregate demand, leading to an excess demand and an eventual equilibrium adjustment in which they increase output. This overshooting phenomenon has also been explained by different financial lending and investing behaviors during different economic periods: When the economy is expanding, financial institutions tend to extend credit more liberally and investors tend to buy more securities, resulting in more loans, investments, and ultimately overproduction and overpriced asset values; when the economy is declining, financial institutions tend to tighten credit and investors tend to curb their investments, which results in underproduction and underpriced asset values. Thus, banks, financial institutions, and investors by their lending and investing behaviors tend to exacerbate the current economic trend, leading to an overshooting of the trend.

Many economists argue that the 2008 to 2009 financial crisis and recession was a correction to a major overshooting of the United States and world economies. From 2000 to 2006, expansionary U.S. monetary actions, China's large investment in U.S. Treasury securities, and liberal credit policies by financial institutions led to excessive overshooting, especially in the housing industry. This ultimately led to too many risky mortgage loans whose default eventually led to the subprime mortgage meltdown of 2007 and the near collapse of financial systems providing those loans and the overall economy. The Fed, Treasury, and other world banks initiated unprecedented monetary and fiscal policy actions to try to save the financial structure. These actions reduced Treasury rates to historic levels and widened the spreads between the yields on risky equity and bonds and less-risky Treasury and high-quality bonds.

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of bond risk. We can say, though, that in general a riskier bond will trade in the market at a price that yields a greater YTM than a less-risky bond. The difference in the YTM of a risky bond and the YTM of less-risky bond is referred to as a ***risk spread*** or ***risk premium***. The risk premium, RP, indicates how much additional return investors must earn to induce them to buy the riskier bond:

$$\text{RP} = \text{YTM on risky bond} - \text{YTM on less-risky bond}$$

Our supply and demand model can be used to explain why normally there is a positive risk premium and why the premium increases, the greater the risk. Figure 3.11 shows the demand and supply graphs for a risk-free Treasury bond and a corporate bond. If we initially assume that the corporate bond is also risk-free and has identical features to the Treasury, then we would expect the corporate bond to be priced at $P_0{}^C$ to yield a rate ($y_0{}^C$) that would be equal to the rate on the risk-free Treasury ($y_0{}^T$). In this case, the risk premium would be zero. Suppose, though, that economic conditions change such that there is now some chance of default on the corporate bond. The increased riskiness of the corporate bond would cause its demand to decrease, shifting its bond demand curve to the left. The corporate bond's riskiness would also make the Treasury security more attractive, augmenting its demand and shifting its demand curve to the right. At the new equilibriums, the corporate bond's price is lower ($P_1{}^C$) and its rate ($y_1{}^C$) greater than the Treasury's. As shown in Figure 3.11, the risk associated with the corporate bond leads to a market adjustment in which at the new equilibrium there is a positive risk premium: $\text{RP} = y_1{}^C - y_1{}^T$. In general, we can conclude that if a bond is risky, it will trade with a positive risk premium and that the premium will increase, the greater the bond's risk.

Types of Risk Spreads When the YTM on the less-risky bond is the minimum rate, the rate is referred to as the ***benchmark rate*** or base rate and the spread is referred to as a ***benchmark spread***. Typically, the benchmark rate is the YTM on a Treasury that is comparable in maturity and is recently issued and therefore liquid. For tax-exempt bonds, such as municipals, the benchmark for calculating spreads is often a generic AAA general obligation municipal bond with a specified maturity rather than a Treasury. The spread between the interest rates offered in two sectors of the bond market with the same maturity (e.g., Treasury and non-Treasury) is referred to as the ***intermarket sector spread***, whereas the spread between two issues within the same sector is called the ***intramarket sector spread***.[2]

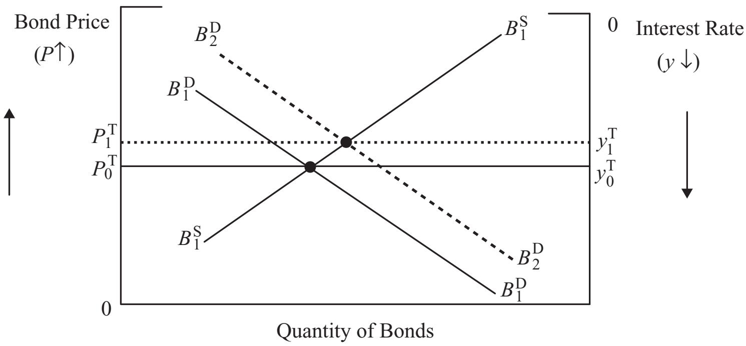
Two factors that affect the risk premium or spread are credit risk and embedded option provisions in a bond. The spread between a Treasury security and a non-Treasury security that is identical except for credit risk is referred to as the ***credit spread***. However, as discussed in Chapter 2, many bonds have embedded options that give either the issuer or the bondholder the right to take some actions. The most common is a call option giving the issuer the right to buy back the bonds at a specific price. There are also put options that give the holder the right to sell the bond back to the issuer at a specific price. In general the market will require a bigger spread with a call option that benefits the issuer and a smaller spread with a put option that benefits the holder. (Note: Given an embedded put option on a bond, it is possible for that bond to have a negative benchmark spread.)

In analyzing a bond, analysts try to separate the portion of the benchmark spread that can be attributed to the embedded options. The analytical measure used to estimate this portion of the spread is called the ***option-adjusted spread (OAS)***. Methods for estimating the OAS are examined in Chapter 15.

Risk Premiums and Investors' Return-Risk Preferences The size of the risk premium depends on investors' attitudes toward risk. To see this relation, suppose there are only two bonds available in the market: a risk-free bond and a risky bond. Suppose the risk-free bond is a zero-coupon bond promising to pay $1,000 at the end

Treasury Bond Market

- The riskiness of the corporate bond increases the demand for Treasury bonds, shifting the Treasury bond demand curve to the right.
- Impact: A Lower Interest Rate on Treasury Bonds



Corporate Bond Market

- The riskiness of the corporate bond decreases its demand, shifting the corporate bond demand curve to the left.
- Impact: A Higher Interest Rate on Corporate Bonds

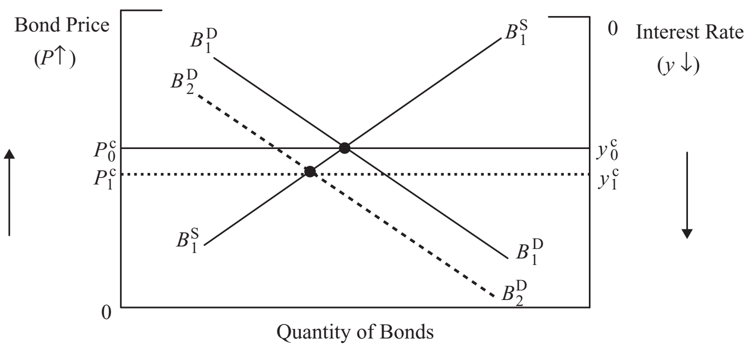


FIGURE 3.11 Risk Premium

of one year and that it currently is trading for \$909.09 to yield a one-year risk-free rate, R_f, of 10%:

$$P_0 = \frac{\$1{,}000}{1.10} = \$909.09$$

$$R_f = \frac{\$1{,}000}{\$909.09} - 1 = .10$$

Suppose the risky bond is also a one-year zero coupon bond with a principal of \$1,000, but there is a chance it could default and pay nothing. In particular, suppose there is a 0.8 probability the bond would pay its principal of \$1,000 and a 0.2 probability it would pay nothing. The expected dollar return from the risky bond is therefore \$800:

$$E(\text{return}) = .8(\$1{,}000) + .2(0) = \$800$$

Given the choice of two securities, suppose that the market were characterized by investors who were willing to pay \$727.27 for the risky bond, in turn yielding them an expected rate of return of 10%:

$$E(R) = \frac{E(return)}{P_0} - 1$$

$$E(R) = \frac{\$800}{\$727.27} - 1 = .10$$

By paying \$727.27, investors would have a 0.8 probability of attaining a rate of return of 37.5% [(\$1,000/\$727.27) − 1] and a 0.2 probability of losing their investment. In this case, investors would be willing to receive an expected return from the risky investment that is equal to the risk-free rate of 10%, and the risk premium, $E(R) - R_f$, would be equal to zero. In finance terminology, such a market is described as ***risk neutral***. Thus, in a risk neutral market, the required return is equal to the risk-free rate and the risk premium is equal to zero.

Instead of paying \$727.27, suppose investors like the chance of obtaining returns greater than 10% (even though there is a chance of losing their investment), and as a result are willing to pay \$750 for the risky bond. In this case, the expected return on the bond would be 6.67% and the risk premium would be negative:

$$E(R) = \frac{\$800}{\$750} - 1 = .0667$$

$$RP = E(R) - R_f = .0667 - .10 = -.033$$

By definition, markets in which the risk premium is negative are called ***risk loving***. Risk loving markets can be described as ones in which investors enjoy the excitement of the gamble and are willing to pay for it by accepting an expected return from the risky investment that is less than the risk-free rate. Even though there are some investors who are risk loving, a risk loving market is an aberration, with the exceptions being casinos, sports gambling markets, lotteries, and racetracks.

Whereas risk-loving and risk-neutral markets are rare, they do serve as a reference for defining the more normal behavior towards risk—***risk aversion***. In a risk-averse market, investors require compensation in the form of a positive risk premium to pay them for the risk they are assuming. Risk-averse investors view risk as a disutility, not a utility as risk-loving investors do. In terms of our example, suppose most of the investors making up our market were risk averse and as a result were unwilling to pay \$727.27 or more for the risky bond. In this case, if the price of the risky bond were \$727.27 and the price of the risk-free were \$909.09, there would be little demand for the risky bond and a high demand for the risk-free one. Holders of the risky bonds who wanted to sell would therefore have to lower their price, increasing the expected return. On the other hand, the high demand for the risk-free bond would tend to increase its price and lower its rate.[3] For example, suppose the markets cleared when the price of the risky bond dropped to \$701.75 to yield 14%, and the price of the risk-free bond increased to \$917.43 to yield 9%:

$$E(R) = \frac{\$800}{\$701.75} - 1 = .14$$

$$R_f = \frac{\$1{,}000}{\$917.43} - 1 = .09$$

In this case, the risk premium would be 5% and the market is defined as being risk averse.

In a risk-averse market, the positive risk premium required by investors to hold the riskier bond is partly the result of uncertainty and partly due to liquidity. For example, if investors knew that the probability of default was in fact 0.8, then they would know that by buying a portfolio of such bonds (e.g. 100 bonds like B), 80% of the portfolio would pay \$1,000 at the end of the year and 20% would pay nothing. Alternatively, if investors buy Bond Bs over time, then they would find that in eight out of 10 years, they would receive a \$1,000 principal and in two out of 10 years, they would receive nothing. Thus, if the price of Bond B were the risk-neutral price of \$727.27, then investors' average portfolio return or their average return over time would be 10%. It would appear that provided the 0.8 probability is known, investors would be indifferent between a portfolio of B bonds and Bond A, or a strategy of buying B bonds or A bonds over time. However, to obtain the certain portfolio return of 10% would require that investors buy a portfolio of B bonds or buy such bonds over an extended period to realize the 10% rate. This would require more funds or time than simply buying the risk-free Bond A. As a result, investors would demand less of B because of these marketability or liquidity requirements. The liquidity concern would, in turn, push the price of the B bond down and increase its yield above the risk-free rate. The market for our corporate bond depicted in Figure 3.11 could be categorized as a risk-averse market.

Historical Risk Premiums In practice, economic and financial conditions do change, affecting a bond's probability of default and changing investors' degree of risk aversion. As a result, the size of the risk premium does change as a result of changing economic conditions. In recessionary periods (or the expectation of recession), the risk premium tends to widen as investors move their investment holdings from lower quality, high-risk bonds to higher quality corporate credits and Treasuries. By contrast, in periods of economic growth (or the expectation of growth), the risk premium tends to narrow as investors move their investment holdings to lower quality, high-risk bonds from higher quality corporate credits or Treasuries.

An example of the responsiveness of risk premiums to economic conditions can be seen in comparing the risk premium in 2003, when the U.S. economy was growing and aggregate wealth as measured in terms of the overall stock market and real estate values were rising, to 2008, when the real estate and stock markets had collapsed and economic indicators were forecasting recession. Figure 3.12 shows the yield curves

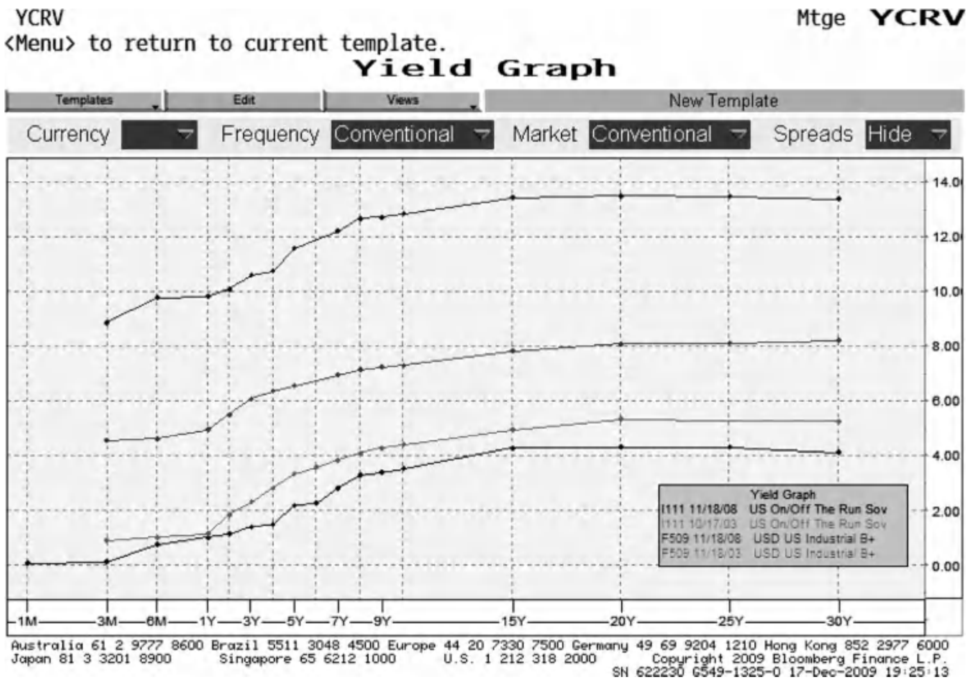


FIGURE 3.12 Comparative Yield Curves: Treasuries and B+ Industrials, November 2003 and November 2008
Source:

for Treasuries and B+ corporate industrial bonds on November 18, 2003 and on November 18, 2008. In comparing the yield curves at the different dates, observe the downward shift in the Treasury curve and the upward shift in the B+ industrial curve from November 2003 to November 2008. The opposite directions of the shifts, as well as their relative sizes, highlights the significant increase in the risk premiums that investors required to hold B-quality industrial corporate bonds, as well as the flight to safety to Treasuries. For example, in November 2003, 7-year B+ corporate credits were trading at approximately 7%, a 3% spread over 7-year Treasuries that were trading at a YTM of 4%. In November 2008, though, 7-year Treasuries were trading at a YTM of 3%, while 7-year B+ corporate credits were trading at approximately 12%—a 9% risk premium. Similar changes in risk premiums can also be observed for different maturities.

Liquidity

Liquidity Premium Liquid securities are those that can be easily traded and in the short run are absent of risk. Treasury securities, with their wide distribution of ownership, for example, are relatively easy to trade and are therefore more liquid than corporate bonds. In general, we can say that a less liquid bond will trade in the market at a price that yields a greater YTM than a more liquid one. The difference in the YTM of a less liquid bond and the YTM of a more liquid one is defined as the liquidity premium, LP:

$$LP = \text{YTM on less liquid bond} - \text{YTM on more liquid bond}$$

One way to measure the degree of marketability of a security is in terms of the size of the bid-asked spread that dealers offer in the market. Dealers who make markets in less marketable securities necessarily set wider spreads than dealers who have securities that are bought and sold by many investors and therefore can be traded more quickly.

Similar to bond risk, liquidity can be influenced by the overall state of the economy. In slow economic periods, there is often a tightening of credit combined with a flight to safety to Treasury securities that can often slow the sale of new debt and loans. This can lead to illiquid markets for many non-Treasury securities, even those with good credit ratings. In such periods, corporate and municipal bonds that have relatively low default risk take longer to sell, leading to not only greater bid-ask spreads, but also lower prices and higher yields and spreads. The spread can be explained partly by the increase in risk due to the economy and partly by the illiquid market conditions—***liquidity spread***.

Taxability Investors are more concerned with the after-tax yield on a bond than its pre-tax yield. An investor in a 40% income tax bracket who purchased a fully-taxable 10% corporate bond at par would earn an after-tax yield, ATY, of 6%: ATY = 10%(1 −.40). The ATY is found by solving for that yield, ATY, that equates the bond's price to the present value of its after-tax cash flows:

$$P_0 = \sum_{t=1}^{N} \frac{CF_t(1 - tax\ rate)}{(1 + ATY)^t}$$

Bonds that have different tax treatments but otherwise are identical will trade at a different pre-tax YTM. That is, the investor in the 40% tax bracket would be indifferent between the 10% fully-taxable corporate bond and a 6% tax-exempt municipal bond selling at par, if the two bonds were identical in all other respects. The two bonds would therefore trade at equivalent after-tax yields of 6%, but with a ***pre-tax yield spread*** of 4%: $y_0{}^C - y_0{}^M = 10\% - 6\% = 4\%$.

As we discussed in Chapter 1, taxability refers to the claims the government has on a security's cash flow. In general, bonds whose cash flows are subject to less taxes trade at a lower YTM than bonds that are subject to more taxes. Historically, taxability explains why some municipal bonds whose coupon interest is exempt from federal income taxes have traded at yields below default-free U.S. Treasury securities or AAA-quality corporate credits, even though many municipals are subject to credit risk (see Figure 3.13).

3.4 CONCLUSION

In this chapter, we have examined what determines the level and structure of interest rates. We started our analysis by first developing a supply and demand model

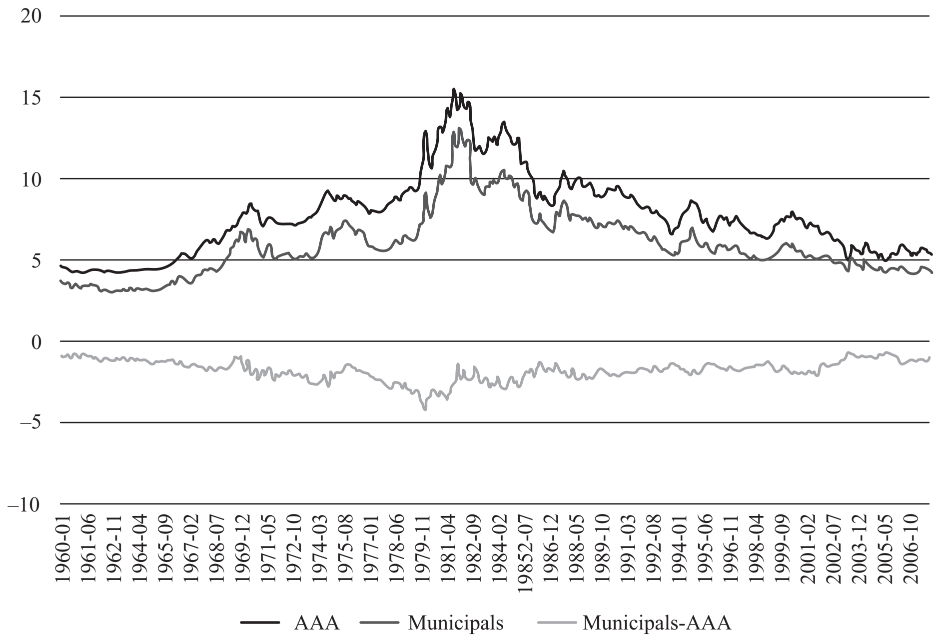


FIGURE 3.13 Historical Yields and Spreads on AAA Corporate Bonds and Municipals: 1960–2009
Source: Federal Reserve: www.research.stlouisfed.org/fred2.

for bonds. The model, in turn, helped us to explain how economic factors such as economic growth, monetary and fiscal policy, international capital flows, relative risk, and inflation affect the general level of interest rates. We next examined how differences in risk, liquidity, and tax features determine the spreads between interest rates on bonds of different categories. In the next chapter, we complete our analysis of the structure of interest rates by examining the term structure of interest rates—the relationship between yields and maturity.

KEY TERMS

benchmark rate
benchmark spread
call risk
credit spread
default risk
demand for loanable funds
Fisher effect
flight to safety
intermarket sector spread
intramarket sector spread
liquidity spread
market risk
open market operation (OMO)
option-adjusted spread
pre-tax yield spread
risk aversion
risk loving
risk neutral
risk premium
sovereign wealth funds
supply of loanable funds
term structure of interest rates
yield curve

WEB INFORMATION

1. Historical interest rate data on different bonds can be found at the following Federal Reserve sites:
   - www.federalreserve.gov/releases/h15/data.htm
   - FRED: www.research.stlouisfed.org/fred2
2. For information on Federal Reserve policies, go to www.federalreserve .gov/policy.htm.
3. For information on the European Central Bank, go to www.ecb.int.
4. For information on rates on bonds and the current yield curve, go to www.bloomberg.com.
5. Current yields on U.S. Treasuries, corporate bonds, and municipal bonds and different maturities and quality ratings can be found by going to http://bonds.yahoo.com and clicking on "Composite Bond Rates."
6. For information on the U.S. Treasury's debt, go to www.publicdebt .treas.gov.
7. For information on the distribution of U.S. debt, go to the Treasury Bulletin: www.fms.treas.gov/bulletin/.
8. For information on U.S. government expenditures, revenues, deficits and debt, go to: www.gpo.gov/fdsys/browse/collectionGPO.action?collection Code=BUDGET.
9. For downloadable tables on U.S. government expenditures, revenues, deficits and debt, go to: www.gpo.gov/fdsys/search/pagedetails.action ?granuleId=&packageId=BUDGET-2010-TAB.
10. For government information submitted by Congress, go to: www.gpo .gov/fdsys/search/home.action.
11. Information on the Federal Reserve System can be found by going to the Federal Reserve site: www.federalreserve.gov/pubs/Frseries/frseri.htm. The site has useful information on important monetary actions such as open market operations, changes in the discount rate, and reserve requirement changes.
12. For the Federal Reserve report on the state of the economy, go to the Federal Reserve Beige Book: www.minneapolisfed.org/bb/. The book provides analysis of current and future economic condition for the nation and regions.
13. For information on exchange rates and euro and yen bond yields, go to FXStreet.com: www.fxstreet.com.
14. Information on rates and other economic information can be found at www.economagic.com.
15. For information on security holding by foreigners, go to Treasury Tic Information: www.treas.gov/tic.
16. For information and reports from the U.S. Treasury, go to www.treas.gov/.
17. For information on economic indicators and economic performance from the Council of Economic Advisors at the Federal Reserve Archival System for Economic Research (FRASER), go to http://fraser.stlouisfed.org/ publications/ei/.

PROBLEMS AND QUESTIONS

1. Describe the bond demand and supply model presented in this chapter. Include in your description the definitions of the bond demand and supply curves, the important factors that shift the curves, equilibrium, and proof of equilibrium.
2. Using the bond demand and supply model presented in this chapter, explain the impacts of the following cases on the level of interest rates.
   a. Expansionary open market operation
   b. Economic recession
   c. Treasury financing of a government deficit
   d. Economic expansion
   e. China purchase of U.S. Treasury securities
3. Given two identical bonds that are priced at the same yields in their markets, explain using supply and demand analysis the adjustments that would take place if events were to occur that would make one of the bonds more risky.
4. Define risk-neutral, risk-averse, and risk-loving markets.
5. Given an economy with two bonds, (1) a one-year, risk-free zero-coupon bond paying a principal of $1,000 and priced at $952.38 to yield 5% and (2) a one-year risky zero-coupon bond with a .75 probability of paying $1,000 at the end of the year and a .25 probability of defaulting and paying only $100 from liquidation,
   a. What would be the price of the risky bond if the market were risk neutral?
   b. What would be the response of a risk-averse market if the risky bond were priced at its risk-neutral value?
   c. What would be the response of a risk-loving market if the risky bond were priced at its risk-neutral value?
6. Given two identical bonds that are priced with the same yields in their markets, explain using supply and demand analysis the adjustments that would take place if events were to occur that would make one of the bonds less liquid.
7. In Question 5 the risky bond paid a certain return of $1,000 and was priced to yield a return of 5% and the risky bond had a .75 probability of paying $1,000 and .25 probability of paying $100, for an expected return of $775. Suppose the probabilities of .75 and .25 and possible payoffs of $1,000 and $100 are known with absolute certainty (i.e., no one can question the statistics). Explain why the risky bond could still be priced to yield a rate exceeding 5%.
8. Given a fully taxable two-year, 8% annual coupon bond with a face value of $1,000 and with annual coupon payments and an identical bond except that it is tax free, what would the yield and price on the tax-free bond have to be for an investor in a 35% tax bracket to be indifferent between the two bonds?
9. Short-Answer Questions:
   a. What impact would an economic expansion have on default risk premiums?
   b. What would the discount rate on risky bonds be if the market were risk neutral?

c. Explain how the YTM on many municipal bonds that are subject to default has historically been less than the YTM on T-bonds that are default free.

WEB EXERCISES

1. In this chapter we discussed how risk spreads change as a result of economic conditions.

Go to Fred: www.research.stlouisfed.org/fred2.

In Category, click "Interest Rates."

Select series (e.g., Moody's Corporate AAA and BBB Yields).

For BBB and AAA graph, click DBAA and DAAA and "Download Data" to send to Excel.

Examine the level and difference in the yields during the recessionary periods.

2. In this chapter, we examined the impact of taxes on yield spreads.

Go to www.federalreserve.gov/releases/h15/data.htm and download to Excel the historical yields on 10-year Treasury bonds and the yields on state and local bonds.

Comment on the yield difference you observe.

3. Information on the Federal Reserve System can be found by going to the Federal Reserve site: www.federalreserve.gov/pubs/Frseries/frseri.htm. The site has useful information on important monetary actions such as open market operations, changes in the discount rate, and reserve requirement changes. Click on "Monetary Policy," "Reports," and "Beige Book." Comment on what you find about the Federal Reserve's outlook for the economy and your Federal Reserve district. Note that the Beige Book can also be found by going to www.minneapolisfed.org/bb/.

4. Find information on how the European Central Bank is structured by going to their site: www.ecb.int.

5. Examine the historical growth in the U.S. government's receipts, outlays, deficits and surpluses, and debt.

For tables on U.S. government's expenditures, revenues, deficits and debt, go to:

www.gpo.gov/fdsys/search/pagedetails.action?granuleId=&packageId=BUDGET-2010-TAB.

On Table 1.3, Summary of Receipts, Outlays, and Surpluses or Deficits (–) In Current Dollars, click "XLS" to send to Excel.

On Table 7.1, Federal Debt at the End of Year: 1940–2014, click "XLS" to send to Excel.

6. Information on economic indicators and economic performance from the Council of Economic Advisors can be found at the Federal Reserve Archival System for Economic Research (FRASER):

   http://fraser.stlouisfed.org/publications/ei/

   Select by year or subject.

   Examine some of the reports for different economic categories (e.g., consumer prices, national income, productivity, and balance of trade).

   Comment on the economic trends you observe.

NOTES

1. Implied volatility is measured as the volatility that equates the market price of an index option to the option pricing model's value.
2. Some analysts and market participants measure spreads on a relative basis: (Yield on A − Yield on B)/Yield on B or as a yield ratio: Yield A/ Yield B.
3. The risk-free bond is risk free because its principal payment is known with certainty; if the bond is sold before maturity, it is subject to market risk.

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Bond Evaluation, Selection, and Management, Second Edition
by R. Stafford Johnson

CHAPTER 4

The Term Structure of Interest Rates

4.1 INTRODUCTION

In Chapter 3, we examined how different economic factors affect the level of interest rates and how risk, liquidity, and taxes explain the differences in the rates on bonds of different categories. In this chapter, we continue our analysis of the structure of interest rates by examining how interest rate differences can be observed between similar bonds with different terms to maturity.

In the financial literature, the relationship between the yields on financial assets and their terms to maturity is referred to as the ***term structure of interest rates***. As noted in Chapter 3, term structure is often depicted graphically by a ***yield curve***: a plot of the YTM against the terms to maturity for bonds that are otherwise alike. A yield curve can be constructed from current observations. For example, one could take all outstanding corporate bonds from a group in which the bonds are almost identical except for their maturities, and then generate the current yield curve by plotting each bond's YTM against its maturity. For investors who are more interested in average yields instead of current ones, the yield curve could be generated by taking the average yields over a sample period (e.g., five-year averages) and plotting these averages against their maturities. Figure 4.1 shows yield curves for two types of corporate credits (A+ and B-quality industrials and AA and BBB-quality bank credits) and the Treasury yield curves for the European Union (euro-denominated) and Great Britain (sterling-denominated).

Figure 3.3 shows the yield curves for U.S. Treasuries securities. A yield curve constructed using Treasuries often serves as the benchmark yield curve for other sector yield curves. However, one of the problems in constructing a Treasury yield curve from observed yields is that many Treasuries with the same maturity carry different yields because they have different coupon rates. To redress this problem, a widely used approach for generating a Treasury yield curve is to use spot rates, which we examined in Chapter 2. The construction of benchmark Treasury yield curves is discussed in Section 4.7.

Whether yield curves are derived from current rates, averages, or spot rates, they tend to take on one of the three shapes shown in Figure 4.2. They can be positively-sloped with long-term rates being greater than short-term ones. Such yield curves are called ***normal*** or ***upward-sloping curves***. They are usually convex from below, with the YTM flattening out at higher maturities. Yield curves can also be negatively-sloped, with short-term rates greater than long-term ones. These curves are known as ***inverted*** or ***downward-sloping yield curves***. Like normal curves, these

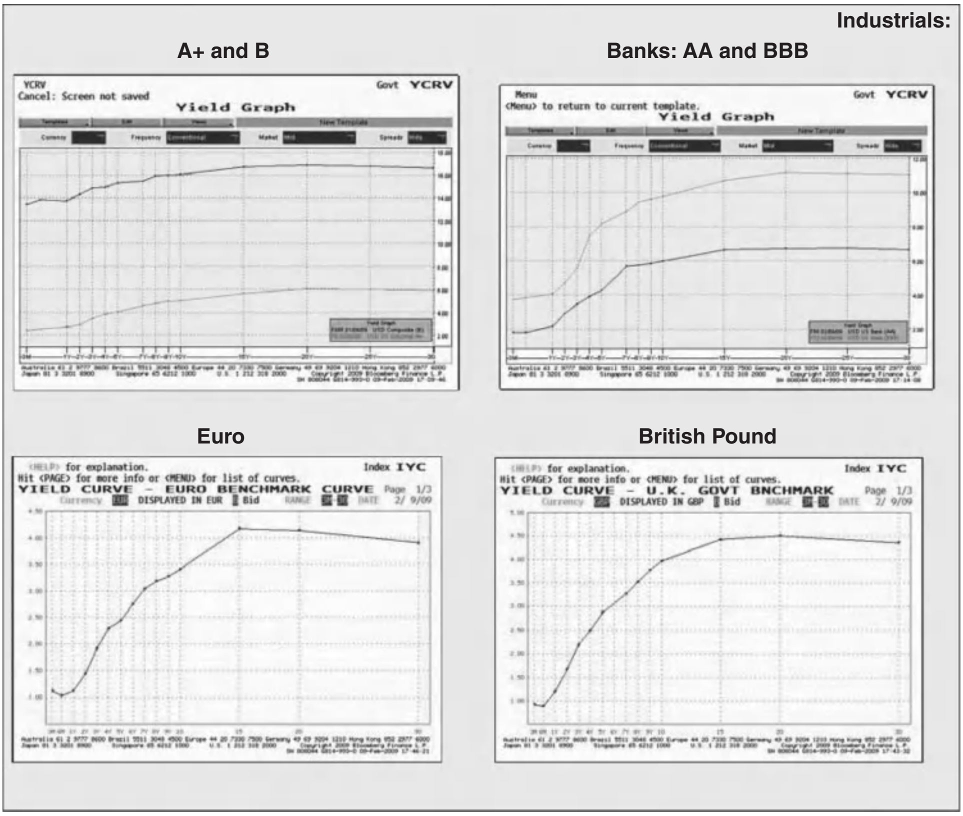


FIGURE 4.1 Yield Curves for Different Sectors and Currencies: February 10, 2009
Source: Copyright © 2009 Bloomberg Finance L.P. All rights reserved. Used with permission.

curves also tend to be convex, with the yields flattening out at the higher maturities. Finally, yield curves can be relatively flat, with the YTM being invariant to maturity. Occasionally a yield curve can take on a more complicated shape in which it can have both positively-sloped and negatively-sloped portions; these are often referred to as ***humped yield curves.***[1]

The actual shape of the yield curve depends on the types of bonds under consideration (e.g., AAA bond versus B bond), economic conditions (e.g., economic growth or recession, tight monetary conditions, etc.), the maturity preferences of investors and borrowers, and the market's expectations about future rates, inflation, and the state of economy. Two theories encompassing many of these considerations have evolved over the years to try to explain the shapes of yield curves: market segmentation theory (MST) and pure expectations theory (PET). There are also two extensions of these theories that are frequently used to explain the term structure: preferred habitat theory (PHT) and the liquidity premium theory (LPT). As we will see, each of these theories by itself is usually not sufficient to explain the shape of a yield curve; rather, the full explanation underlying the structure of interest rates depends on elements of all four theories.

• Shapes

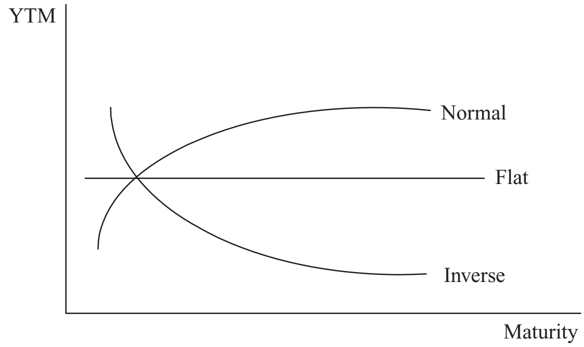


FIGURE 4.2 Yield Curve Shapes

4.2 MARKET SEGMENTATION THEORY

Market segmentation theory (MST) posits that investors and borrowers have strong maturity preferences that they try to attain when they invest in or issue fixed income securities. As a result of these preferences, the financial markets, according to MST, are segmented into a number of smaller markets, with supply and demand forces unique to each segment determining the equilibrium yields for each segment. Thus, according to MST, the major factors that determine the interest rate for a maturity segment are supply and demand conditions unique to the maturity segment. For example, the yield curve for high quality corporate bonds can be segmented into three markets: short-term, intermediate-term, and long-term. The supply of short-term corporate bonds, such as commercial paper, would depend on business demand for short-term assets such as inventories, accounts receivables, and the like, whereas the demand for short-term corporate bonds would emanate from investors looking to invest their excess cash for short periods. The demand for short-term bonds by investors and the supply of such bonds by corporations would ultimately determine the rate on short-term corporate bonds. Similarly, the supplies of intermediate and long-term bonds would come from corporations trying to finance their intermediate and long-term assets (plant expansion, equipment purchases, acquisitions, etc.), whereas the demand for such bonds would come from investors, either directly or indirectly through institutions (e.g., pension funds, mutual funds, insurance companies, etc.), who have long-term liabilities. The supply and demand for intermediate funds would, in turn, determine the equilibrium rates on such bonds, whereas the supply and demand for long-term bonds would determine the equilibrium rates on long-term debt securities.

Important to MST is the idea of unique or segmented markets. According to MST, the short-term bond market is unaffected by rates determined in the intermediate or long-term markets, and vice versa. This independence assumption is based on the premise that investors and borrowers have a strong need to match the maturities of their assets and liabilities. For example, an oil company building a refinery with an estimated life of 20 years would prefer to finance that asset by selling a 20-year bond.

If the company were to finance with a 10-year note, for example, it would be exposed to market risk in which it would have to raise new funds at an uncertain rate at the end of 10 years. Similarly, a life insurance company with an anticipated liability in 15 years would prefer to invest its premiums in 15-year bonds; a money market manager with excess funds for 90 days would prefer to hedge by investing in a money market security; a corporation financing its accounts receivable would prefer to finance the receivables by selling short-term securities. Moreover, according to MST, the desire by investors and borrowers to avoid market risk leads to hedging practices that tend to segment the markets for bonds of different maturities. It should be noted that MST does recognize the interdependence between markets in different sectors. For example, MST does assume that short-term investors will substitute between short-term Treasuries and corporate commercial paper depending on their relative rates or that long-term investors will substitute between long-term Treasuries and long-term corporate bonds depending on their relative rates.

In general, the positions and the shapes of the yield curves depend on the factors that determine the supply and demand for short-term, intermediate, and long-term bonds: economic state (GDP and wealth), expected inflation, credit risk, relative liquidity, and the sales and purchases by the Treasury, central bank, and foreign central banks.[2] Changes in these factors will cause a change in the structure of interest rates that will be reflected by different shifts and twists in the yield curves. For example, if the yield curve is initially positively sloped, then an economic or financial change that increases short-term Treasury rates will cause the yield curve to become flatter. If the yield curve is initially negatively sloped, then the rate change will cause the curve to become even more negatively sloped. In general, we can describe such an impact as having a tendency to cause the yield curve to become negatively sloped. A change in an economic or financial factor can have not only a direct impact on one sector (e.g., open market operations affecting Treasury rates), but also an indirect impact on another sector (change in the Treasury rate resulting from an OMO affecting the demand for corporate bonds). Several cases of yield curve shifts and twists are discussed below assuming a two-sector (Treasury and corporate) and two-segment [short-term (ST) and long-term (LT)] world.

Case 1: Economic Recession

Suppose the economy moved from a period of economic growth into a recession. As discussed in Chapter 3, when an economy moves into a recession, business demand for short-term and long-term assets tends to decrease. As a result, many companies find themselves selling fewer short-term bonds given that they plan to maintain smaller inventories and expect to have fewer accounts receivable. They also find themselves selling fewer long-term bonds, given that they tend to cut planned investments in plants, equipment, and other long-term assets. In the bond markets, these actions cause the short-term and the long-term supplies of bonds to decrease as the economy moves from growth to recession. At the initial interest rates, the decrease in bonds outstanding creates an excess demand, with bondholders now competing to buy fewer available bonds. This drives bond prices up and the YTM down, decreasing demand until a new equilibrium rate is attained.

As the rates on short-term and long-term corporate bonds decrease, short-term and long-term Treasury securities become relatively more attractive. As a result, the demands for short-term and long-term Treasuries increase, creating an excess demand in both the short-term and long-term Treasury markets at their initial rates. Like the corporate bond markets, the excess demand in the Treasury security markets will cause their prices to increase and their rates to fall until a new equilibrium is attained. Thus, a recession has a tendency to decrease both short-term and long-term rates for corporate bonds, and by a substitution effect, decrease short-term and long-term Treasury rates. Thus, a recession causes the yield curves for both sectors to shift down as shown in Exhibit 4.1.

In analyzing the impact of a recession on yield curves using the MST, several points should be noted. First, higher interest rates would occur if the economy moved from recession to economic expansion (see Exhibit 4.2). Second, note that with this level of analysis we cannot explain whether or not the slope of the yield curve also changes. To address that question we need information about the responsiveness of each sector's bond supply and demand to economic and interest rate changes. Finally, note that in this analysis, we have assumed that economic growth affects only the supply of bonds and not demand. As discussed in Chapter 3, economic growth can also affect the general demand for bonds, decreasing demand in recessions and pushing rates up, and increasing demand in economic expansion and pushing rates down. If we assume economic conditions impact the demand for bonds, then we would conclude that a recession would cause interest rates to decrease provided that the supply impact on interest rate dominates the demand impact. If the demand impact dominates, then a recession could push up rates.

Case 2: Recession, Credit Risk, and Credit Tightening

In a recessionary period (or in anticipation of an economic recession), there is often a flight to safety from stocks to Treasury and high quality bonds. Furthermore, the flight to safety is often accompanied by a tightening of credit by financial institutions. The credit tightening, in turn, causes a decrease in the supply of loanable funds and corporate bond demand. In our two-sector, two-segment analysis, an increase in credit risk combined with an increase in credit tightening would lead to an increase in the demand for short-term and long-term Treasury bonds and a decrease in the demand for corporate bonds. Thus, a recession, when accompanied by a flight to safety and a tightening of credit, would widen the spread between corporates and Treasuries. In contrast, if the economy moved from recession to economic expansion and the expansion was accompanied by a decrease in credit risk and credit tightening, the spread between corporates and Treasuries would narrow.

Case 3: Treasury Financing

Interest rates on government securities depend, in part, on the size and growth of the federal government debt. If federal deficits are increasing over time, then the Treasury will constantly be trying to raise funds in the financial market. The Treasury sells a number of short-term, intermediate, and long-term securities. Which securities the Treasury uses to finance a federal deficit affects the yield curve for Treasury securities and, through a substitution effect, the yield curve for corporate bonds. For example, if the Treasury were to finance a deficit by selling short-term Treasury securities, then there would be an increase in the supply of the short-term Treasuries.

EXHIBIT 4.1 MST Model: Economic Recession Case

Outline:

Corporate Market:

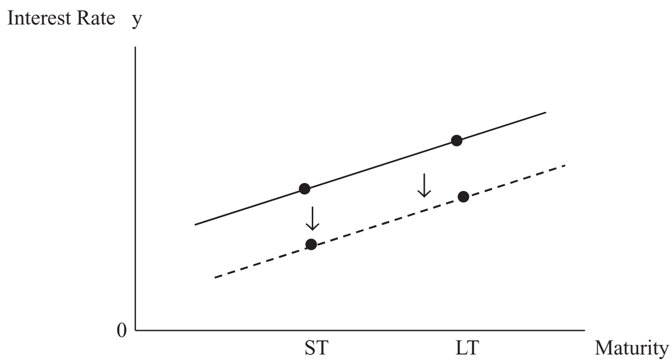
Decrease in capital formation (S-T and L-T) ⇒ Fewer corporate bonds sold (S-T and L-T) ⇒ Excess demand for corporate bonds (S-T and L-T) ⇒ Corporate bond prices increase and yields decrease ⇒ Downward shift in corporate YC

Treasury Market:

Substitution Effect: As corporate yields (S-T and L-T) decrease ⇒ Demand for Treasury securities (S-T and L-T) increases ⇒ Treasury bond prices increase and yields decrease ⇒ Downward shift in Treasury YC

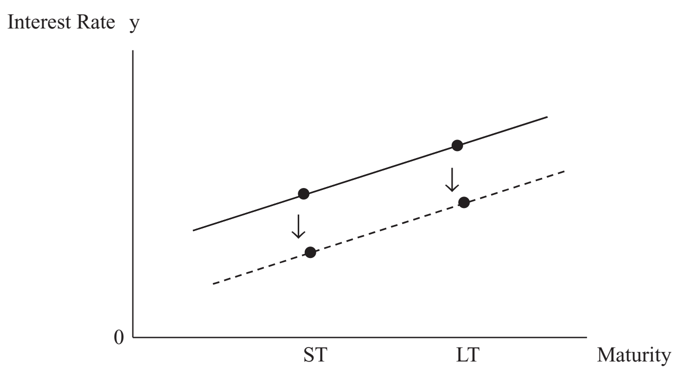
Corporate Bond

- Yield Curve



Treasury Bond

- Yield Curve



The increase in supply would push the price of the short-term government securities down, increasing their yield. In the corporate bond market, the higher rates on short-term government securities would lead to a "crowding-out" effect, decreasing the demand for short-term corporate securities. The decrease in demand would lead to an excess supply in that market as short-term corporate bondholders try to sell their corporate bonds to buy the higher-yielding Treasury securities. As bondholders try

EXHIBIT 4.2 MST Model: Economic Expansion Case

Outline:

Corporate Market:

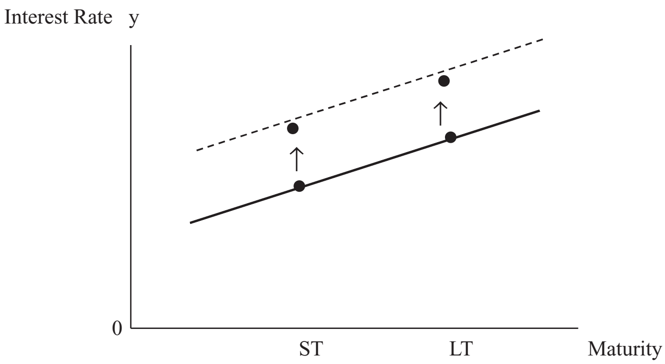
Increase in capital formation (S-T and L-T) ⇒ Greater corporate bonds sold (S-T and L-T) ⇒ Excess supply for corporate bonds (S-T and L-T) ⇒ Corporate bond prices decrease and yields increase ⇒ Upward shift in corporate YC

Treasury Market:

Substitution Effect: As corporate yields (S-T and L-T) increase, demand for Treasury securities (S-T and L-T) decreases ⇒ Treasury bond prices decrease and yields increase ⇒ Downward shift in Treasury YC

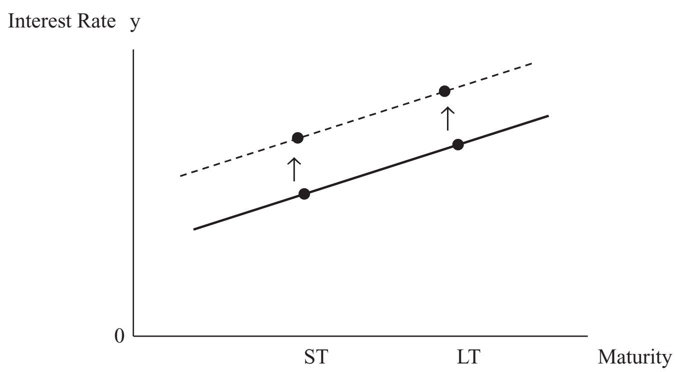
Corporate Bond

- Yield Curve



Treasury Bond

- Yield Curve



to sell their short-term corporate bonds, the prices on such bonds would decrease, causing the rates on short-term corporate bonds to rise until a new equilibrium is reached. Thus, the sale of the short-term Treasury securities increases both short-term government and short-term corporate rates. Because the long-term market is assumed to be independent of short-term rates, the total adjustment to the Treasury's sale of short-term securities would occur through the increase in short-term corporate and Treasury rates. Moreover, if corporate and Treasury yield curves are initially flat, as shown in Exhibit 4.3, the Treasury's sale of short-term securities would cause the yield curves to become negatively sloped. By contrast, if the Treasury had financed the deficit with long-term securities, the impact would have been felt in the long-term bond market. In this case, the Treasury and corporate bond yield curves would become positively sloped (see Exhibit 4.4).

In the opposite case of a budget surplus (such as the brief one that occurred in the United States in the late 1990s and early 2000), the yield curve could become negatively sloped if the Treasury used some of the surplus to buy up long-term Treasury securities as a policy to reduce the government's debt. That is, the Treasury's purchase of long-term securities would create an excess demand for long-term Treasury bonds, and by the substitution effect, an excess demand for long-term corporate bonds, leading to higher price and lower rates on long-term securities.

Case 4: Open Market Operations

The yield curve can also be affected by the direction of monetary policy and how it is implemented. For example, if the central bank were engaged in an expansionary open market operation (OMO) in which it were buying Treasury securities, there would be a tendency for the yield curve to become positively sloped if the central bank were buying short-term securities, and a tendency for the yield curve to be become negatively sloped if it were purchasing long-term securities.[3] On the other hand, in a contractionary OMO, there would be a tendency for the Treasury yield curve to become negatively sloped if the central bank were to sell some of its holdings of short-term bills and positively sloped if it were to sell some of its long-term security holdings.

In addition to affecting the Treasury yield curve, open market operations also change the yield curve for corporate securities through a substitution effect. For example, an expansionary OMO in which the Fed purchases short-term Treasury securities would tend to cause the yield curve for corporate securities to become positively sloped. That is, as the rate on short-term Treasury securities decreases as a result of the OMO, the demand for short-term corporate bonds would increase, causing higher prices and lower yields on the short-term corporate securities. Again, because the long-term market is assumed to be independent of short-term rates, the total adjustment to the central bank's purchases of short-term securities would only occur in the short-term corporate and Treasury market and not in the long-term markets. If both the Treasury and corporate yield curves were initially flat, as shown in Exhibit 4.5, then the expansionary OMO would result in new, positively-sloped yield curves.

Case 5: East Asian Treasury-Note Purchase

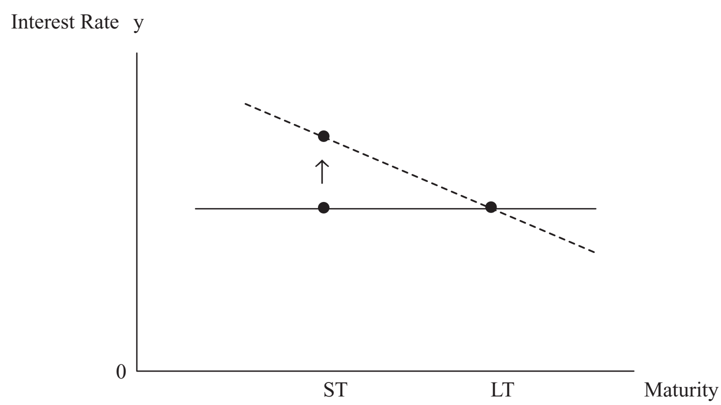
A number of economists have argued that the globalization of the financial markets has weakened the control many central banks have over their interest rates. For example, in early 2003, the Federal Reserve followed a monetary policy of increasing interest rates. During this period, short-term U.S. rates did increase, but intermediate rates actually fell. Undermining the U.S. Federal Reserve policy was China's large investment in liquid U.S. Treasury securities. As noted in Chapter 3, China's large investment in intermediate Treasury securities since 2003 has contributed significantly to keeping U.S. intermediate rates low and in some periods lower than T-bill yields.

EXHIBIT 4.3 MST Model: Treasury Issue of Short-Term Securities

Outline:
Treasury Market:
Treasury sells S-T Treasuries (T-bills) ⇒ T-bill prices decrease and yields increase.
∴ Tendency for Treasury YC to become negatively sloped.
Corporate Market:
Substitution effect: As S-T Treasury yields increase, the demand for S-T corporate securities decreases ⇒ S-T corporate bond prices decrease and their rates tend to increase.
∴ Tendency for Corporate YC to become negatively sloped.

Corporate Bond

- Yield Curve



Treasury Bond

- Yield Curve

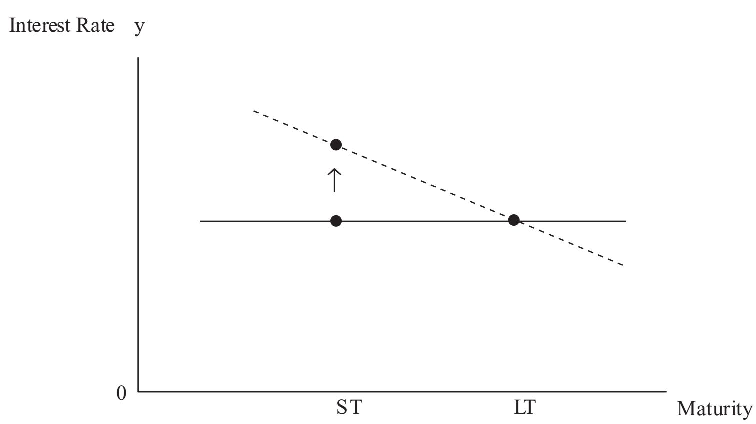
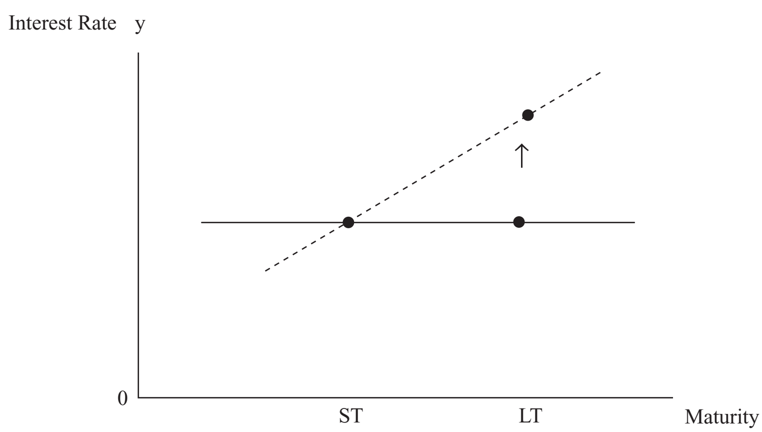


EXHIBIT 4.4 MST Model, Treasury Issue of Long-Term Securities

Outline:
Treasury Market:
Treasury sells L-T Treasuries (T-bonds) ⇒ T-bond prices decrease and yields increase.
∴ Tendency for Treasury YC to become positively sloped.
Corporate Market:
Substitution effect: As L-T Treasury yields increase, the demand for L-T corporate securities decreases ⇒ L-T corporate bond prices decrease and their yields increase.
∴ Tendency for corporate YC to become positively sloped.

Corporate Bond

- Yield Curve



Treasury Bond

- Yield Curve

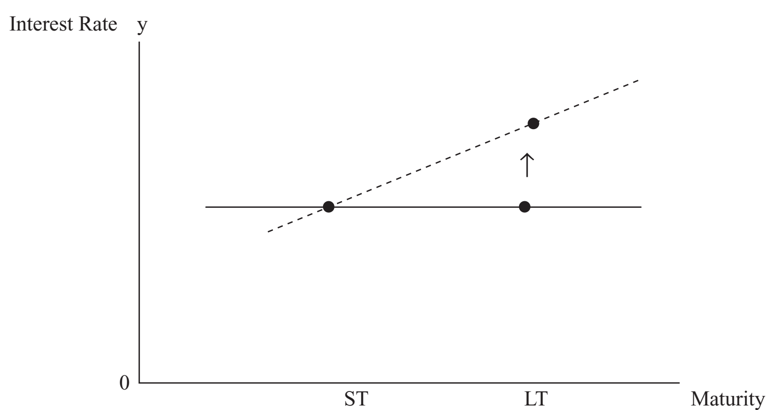


EXHIBIT 4.5 MST Model: Central Bank Open Market Purchase of Short-Term Treasury Securities

Outline:

Treasury Market:

Central Bank buys S-T Treasuries (T-bills) ⇒ T-bill prices increase and yields decrease.

∴ Tendency for Treasury YC to become positively sloped.

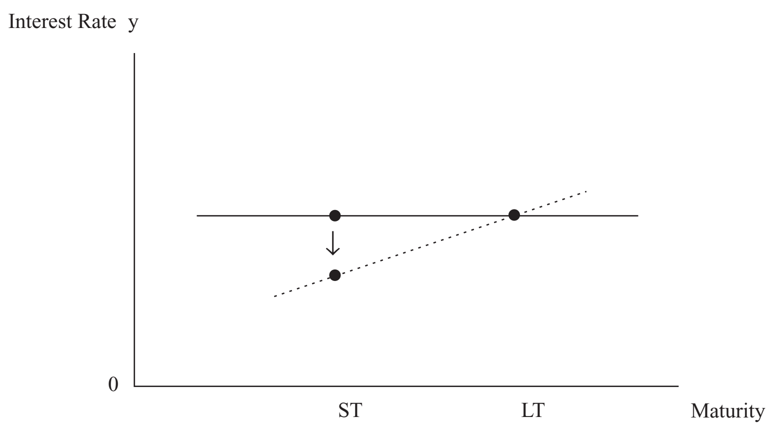
Corporate Market:

Substitution effect: As yields on S-T Treasury securities decrease, the demand for S-T corporate securities increases ⇒ Prices of S-T corporate bonds increase and their yields decrease.

∴ Tendency for corporate YC to become positively sloped.

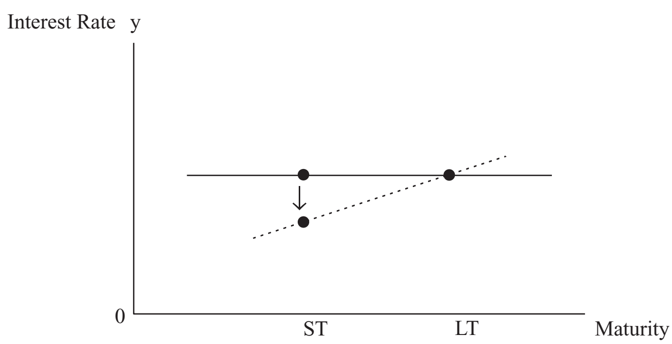
Corporate Bond

- Yield Curve



Treasury Bond

- Yield Curve



The ability of other countries to invest heavily in liquid U.S. Treasuries gives them influence over U.S. rates similar to an exogenous central bank open market purchase. For example, if the China Central Bank were to buy long-term U.S. Treasury securities, there would be a decrease in the supply of long-term Treasuries. The decrease in supply would push up the price of long-term government securities, resulting in a lower long-term Treasury yield. In the corporate bond market, the lower rates on long-term government securities would lead to an increase in the demand for long-term corporate securities, which, in turn, would lead to an excess demand in that market. As bondholders try to buy long-term corporate bonds, the prices on such bonds would increase, causing the yields on long-term corporate bonds to fall until a new equilibrium is reached. Thus, the purchase of the long-term Treasury securities decreases both long-term government and long-term corporate rates. Because the short-term market is assumed to be independent of long-term rates, the market adjustment to the Chinese purchase of long-term securities would occur through the decrease in long-term corporate and Treasury rates. If the corporate and Treasury yield curves were initially flat, then China's purchase of U.S. long-term Treasuries would cause the yield curves to become negatively sloped (see Exhibit 4.6).

Case 6: Expected Inflation

If investors expected inflation to be greater in the future, then as we examined in Chapter 3, the demands for all bonds (short-term and long-term corporate and Treasury) would decrease. This decrease would lead to an overall decrease in bond prices, increasing rates on all securities and leading to upward shifts in the yield curves for both corporate and Treasury. In addition, if corporations sold more bonds in inflationary times to avoid the expected higher future borrowing costs, then corporate bond supply would also increase. This would lower corporate bond prices further and increase rates more, leading to an ever greater upward shift in the corporate yield curve. Thus, expected inflation (deflation) affects the overall level of interest rates, shifting up (down) the yield curves for corporate and Treasury bonds.

Note that it may also be the case that inflation or deflation expectations impact the maturity segments differently. For example, an expectation of higher inflation may lead to a greater decrease in the demand for long-term bonds than short-term bonds. This, in turn, would lead to an increase in the demand for short-term bonds as more investors seek liquidity. If this were the case, then there would be a tendency for the yield curve to become steeper in inflationary times.

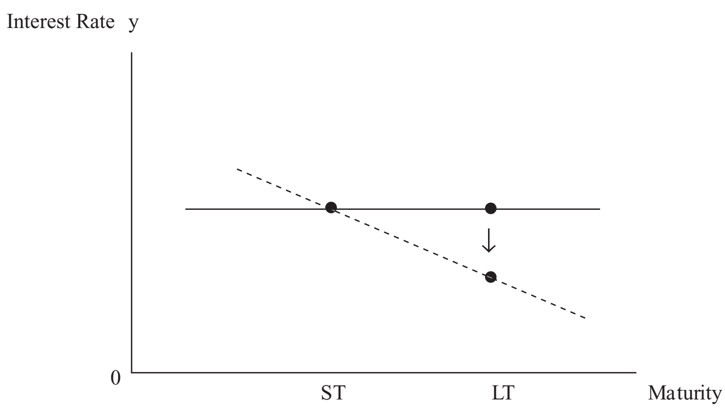
Summary of MST

The MST provides an economic foundation for explaining the shapes of yield curves in terms of fundamental supply and demand forces. As such, the model can be used to analyze the impacts of a number of economic activities on the term structure of interest rates. MST, though, has two shortcomings. First, by assuming independent markets, MST does not take into account cases in which bond yields in a particular maturity segment could increase to a level sufficient to induce investors to move out of their preferred segment. Secondly, MST does not take into account the role

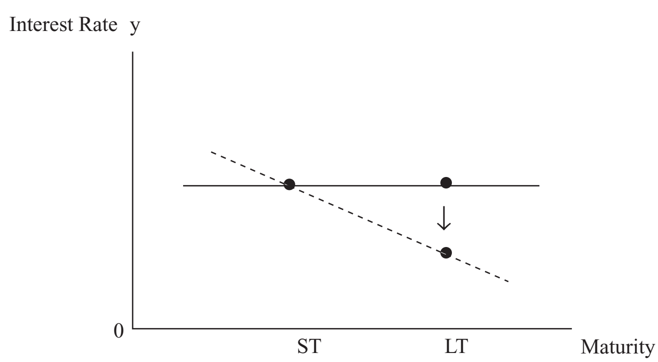
EXHIBIT 4.6 MST Model: China's Purchase of Long-Term U.S. Treasuries

Outline:
Treasury Market:
China buys L-T Treasuries (T-bonds) ⇒ T-bond prices increase and yields decrease.
∴ Tendency for Treasury YC to become negatively sloped.
Corporate Market:
Substitution effect: As L-T Treasury yields decrease, the demand for L-T corporate securities increases ⇒ L-T corporate bond prices increase and their yields to decrease.
∴ Tendency for corporate YC to become negatively sloped.

Corporate Bond
- Yield Curve



Treasury Bond
- Yield Curve



of expectations in determining the structure of interest rates. An investor with a two-year horizon, for example, might prefer a series of one-year bonds to a two-year bond if she expects relatively high yields on one-year bonds next year. If there are enough investors with such expectations, they could have an impact on the current demands for one- and two-year bonds. These limitations of MST are addressed in the other theories of term structures: preferred habitat theory, pure expectations theory, and liquidity preference theory.

4.3 PREFERRED HABITAT THEORY

MST assumes that investors and borrowers have preferred maturity segments or habitats, determined by the maturities of their securities, that they want to maintain. The ***preferred habitat theory (PHT)*** posits that investors and borrowers may stray away from their desired maturity segments if there are relatively better rates to compensate them. Furthermore, PHT asserts that investors and borrowers will be induced to forego their perfect hedges and shift out of their preferred maturity segments when supply and demand conditions in different maturity markets do not match.

To illustrate PHT, consider an economic world in which, on the demand side, investors in corporate securities, on average, prefer short-term to long-term instruments, whereas on the supply side, corporations have a greater need to finance long-term assets than short-term, and therefore prefer to issue more long-term bonds than short-term. Combined, these relative preferences would cause an excess demand for short-term bonds and an excess supply for long-term claims and an equilibrium adjustment would have to occur. The equilibrium adjustment is illustrated in Figure 4.3. The figure shows a short-term and long-term market for corporate bonds, where the bond demand in each segment is assumed to depend directly on its own segment's rate and the rate in the other segment, and where bond supply in each segment is assumed to depend inversely on its own rate and on the rate in the other segment. The positions of the bond demand and supply graphs in Figure 4.3 show an excess demand for short-term bonds and an excess supply of long-term bonds at $y^{ST}{}_0$ and $y^{LT}{}_0$. The excess supply in the long-term market would force issuers to lower their bond prices, increasing long-term bond yields, and the excess demand in the short-term market would cause short-term bond prices to increase and their yields to fall. As long-term bond yields increase and short-term yields decrease, some investors would change their short-term investment demands or preference, increasing their demand for higher yielding long-term bonds (movement down the long-term bond demand curve) and decreasing their demand for lower yielding short-term bonds (movement up the short-term demand curve). On the supply side, the decrease in short-term rates and the increase in long-term rates would induce some corporations to finance their long-term assets by selling short-term claims. This would lead to a substitution in which corporations would increase their sale of the lower yielding short-term bonds (movement up the short-term bond supply curve), and decrease their sale on the higher yielding long-term bonds (movement down the long-term supply curve). Ultimately, equilibriums in both markets would be reached with long-term rates higher than short-term rates, a premium necessary to compensate investors and borrowers/issuers for the risk they've assumed.

As an explanation of term structure, the PHT would suggest that yield curves are positively sloped if investors, on the average, prefer short-term to long-term investments and borrowers/issuers prefer to finance their assets with long-term debt instead of short-term debt. A priori, such preferences may be the case. That is,

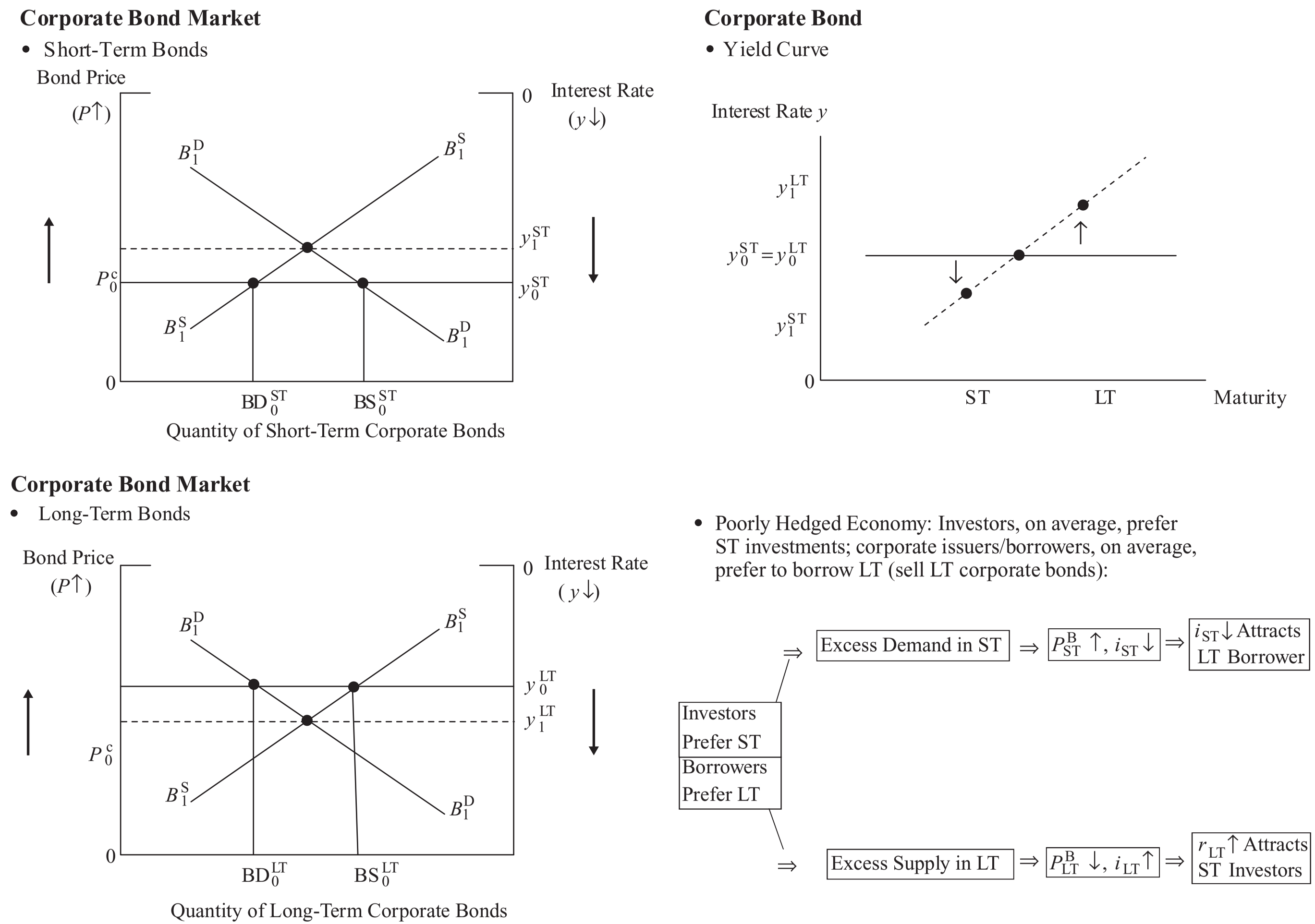


FIGURE 4.3 Preferred Habitat: Equilibrium Adjustment to Excess Demand in the Corporate Short-Term Market and Excess Supply in the Long-Term Market

investors may prefer short-term investments given that longer maturity bonds tend to be more sensitive to interest rate changes or because there are more investors in the upper middle-age class (with shorter investment horizons) than in the young adult or middle-age class (with longer horizons). Borrowers also may have greater long-term than short-term financing needs and thus prefer to borrow long-term. Hence, one could argue that the yield curve is positively sloped because investors' and borrowers' preferences make the economy poorly hedged. Of course, the opposite case, in which investors want to invest more in long-term securities than short-term and issuers desire more short-term to long-term debt, is possible. Under these conditions the yield curve would tend to be negatively sloped.

4.4 PURE EXPECTATIONS THEORY

Expectations theories try to explain the impact of investors' and borrowers' expectations on the term structure of interest rates. A popular model is the ***pure expectations theory (PET)***, also called the ***unbiased expectations theory (UET)***. Developed by Fredrick Lutz, PET is based on the premise that the interest rates on bonds of different maturities can be determined in equilibrium where implied forward rates are equal to expected spot rates.

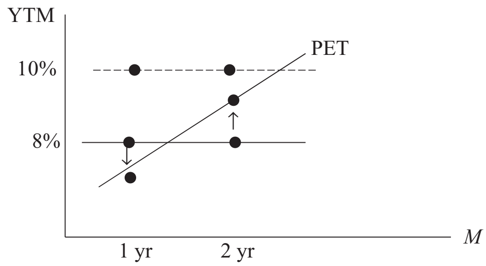
To illustrate PET, consider a market consisting of only two bonds: a risk-free one-year zero-coupon bond and a risk-free two-year zero-coupon bond, both with principals of $1,000. Suppose that supply and demand conditions are such that both the one-year and two-year bonds are trading at an 8% YTM. Second, suppose that the market expects the yield curve to shift up to 10% next year, but, as yet, has not factored that expectation into its current investment decisions (see Figure 4.4). Third, assume initially that one-year and two-year bond investors are willing to give up their preferred maturity habitats, assuming interest rate risk, but that issuers/borrowers have a strong maturity segmentation preference and are not willing to assume interest rate risk. This assumption suggests that those issuers financing one-year assets with one-year bonds and those issuers financing two-year assets with two-year bonds do not consider alternative segments (e.g. two-year borrowers financing with a series of one-year bonds). As a result, issuers' current financing decisions are not influenced by the expected rates on one-year and two-year bonds. In contrast, investors with one-year and two-year horizons are willing to consider alternative segments. Finally, assume the market is risk-neutral, such that investors do not require a risk premium for investing in risky securities (i.e., they will accept an expected rate on a risky investment that is equal to the risk-free rate).

To see the impacts of market expectations on the current structure of rates, consider first the case of investors with horizons of two years. These investors can buy the two-year bond with an annual rate of 8%, or they can buy the one-year bond yielding 8%, then reinvest the principal and interest one year later in another one-year bond expected to yield 10%. Given these alternatives, such investors would prefer the latter investment because it yields a higher expected average annual rate for the two years of 9%:

$$E(R) = [(1.08)(1.10)]^{1/2} - 1 = .09$$

- Expectations of rates increasing from 8% to 10%.
- Investors with HD of two years and those with HD of one year would prefer one-year bonds over two-year bonds.
- Market Response:

$B_2^D \downarrow \Rightarrow P_2^B \downarrow \Rightarrow \text{YTM}_2 \uparrow$ | $B_1^D \uparrow \Rightarrow P_1^B \uparrow \Rightarrow \text{YTM}_1 \downarrow$ | YC becomes *positively* sloped



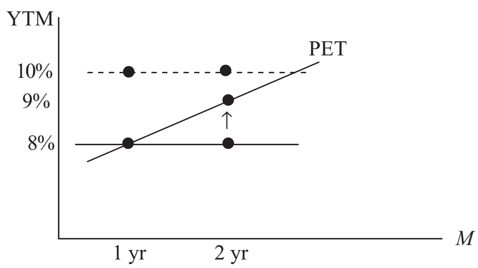
(a)

- If the market response to the expectation is only in terms of a change in the two-year bond, then the equilibrium yield on the two-year will be 9%.

$B_2^D \downarrow \Rightarrow P_2^B \downarrow \Rightarrow \text{YTM}_2 \uparrow$

$\text{YTM}_2 \uparrow$ until $\text{YTM}_2 = \text{YTM}_{2:\text{series}} = 9\%$

When $\text{YTM}_2 = 9\%$, $\text{YTM}_1 = 8\%$, then $f_{11} = E(R_{11}) = 10\%$.



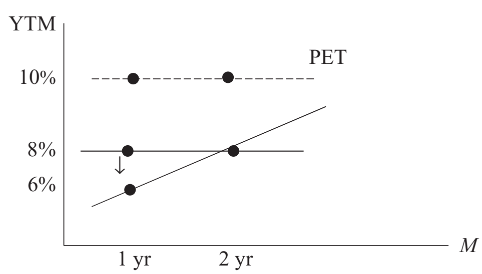
(b)

- If the market response to the expectation is only in terms of a change in the one-year bond, then the equilibrium yield on the one-year will be 6%.

$B_1^D \uparrow \Rightarrow P_1^B \uparrow \Rightarrow \text{YTM}_1 \downarrow$

$\text{YTM}_1 \downarrow$ until $\text{YTM}_1 = \text{YTM}_1$ (holding 2 yr, 1 yr) $= 6\%$

When $\text{YTM}_1 = 6\%$, $\text{YTM}_2 = 8\%$, then $f_{11} = E(R_{11}) = 10\%$.



(c)

FIGURE 4.4 Pure Expectations Theory: Market Expectation of Higher Interest Rates: Investors-Only Response

Similarly, investors with one-year horizons would also find it more advantageous to buy a one-year bond yielding 8% than a two-year bond (priced at $857.34 = $1,000/1.08^2$) that they would sell one year later to earn an expected rate of only 6%:

$$P_{Mt} = P_{2,0} = \frac{\$1,000}{(1.08)^2} = \$857.34$$

$$E(P_{11}) = \frac{\$1,000}{(1.10)^1} = \$909.09$$

$$E(R) = \frac{\$909.09 - \$857.34}{\$857.34} = .06$$

Thus, in a risk-neutral market with an expectation of higher rates next year, investors with both one-year horizons and two-year horizons would prefer to purchase one-year bonds instead of two-year ones. If enough investors do this, an increase in the demand for one-year bonds and a decrease in the demand for two-year bonds would occur until the average annual rate on the two-year bond is equal to the equivalent annual rate from the series of one-year investments or where the one-year bond's rate is equal to the rate expected on the two-year bond held one year (see Figure 4.4).

In the example, there are a number of possible one-year and two-year yields in which there is equilibrium. For example, suppose the equilibrium adjustment occurs only in the two-year market. In this case, if the price on the two-year bond fell such that it traded at a YTM of 9%, and the rate on a one-year bond stayed at 8%, then investors with two-year horizon dates would be indifferent between a two-year bond yielding a certain 9% and a series of one-year bonds yielding 10% and 8%, for an expected rate of 9%. Investors with one-year horizons would likewise be indifferent between a one-year bond yielding 8% and a two-year bond purchased at 9% and sold one year later at 10%, for an expected one-year rate of 8%. Thus, in this case, the impact of the market's expectation of higher rates would be to push the longer term rates up to 9% (see Figure 4.4). Another scenario would be for the adjustment to take place solely in the one-year market. In this case, if the price on the one-year bond increased such that it traded at a YTM of 6%, and the rate on the two-year bond stayed at 8%, then investors with one-year horizons would be indifferent between a one-year bond yielding a certain 6% and a two-year bond (priced at $857.34 = $1,000/1.08^2$) that they would sell one year later at an expected one-year yield of 10% to earn an expected rate of 6%. At one-year and two-year yields of 6% and 8%, respectively, investors with two-year horizons would be indifferent between a two-year bond yielding a certain 8% and a series of one-year bonds yielding 6% and 10%, for an expected rate of 8%. Thus in this adjustment scenario, the impact of the market's expectation of higher rates would be to push the shorter term one-year rates down to 6% (see Figure 4.4). In equilibrium, the one-year and two-year yields will be somewhere between 6% and 8% and 8% and 9%.

Recall that in the Chapter 2 we defined the implied forward rate as a future rate implied by today's rates. In this example, if the equilibrium YTM on the two-year bond is 9% and the equilibrium YTM on the one-year bond is 8%, then the implied forward rate is 10%, the same as the expected rate on a one-year bond, one year from now. Similarly, if the equilibrium YTM on the two-year bond is 8% and the equilibrium YTM on the one-year bond is 6%, then the implied forward rate is also

10%, the same as the expected rate on a one-year bond, one year from now. Thus, in equilibrium the yield curve will be governed by the condition that the implied forward rate is equal to the expected spot rate of 10%:

$$YTM_2 = [(1 + YTM_1)(1 + f_{11})]^{1/2} - 1$$

$$f_{11} = \frac{(1 + YTM_2)^2}{(1 + YTM_1)} - 1$$

$$f_{11} = \frac{(1.09)^2}{(1.08)} - 1 = .10$$

or

$$f_{11} = \frac{(1.08)^2}{(1.06)} - 1 = .10$$

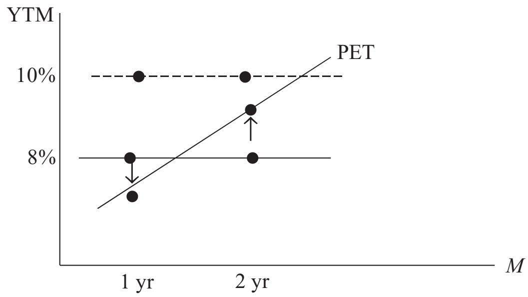
Bond Issuers' Response to Yield Curve Expectations

In the preceding example, we assumed that bond issuers or borrowers had strong maturity segmentation preferences such that they were not influenced by expected rates. Suppose we relax that assumption, positing that bond issuers and borrowers also consider expectation, and also like investors, they are risk neutral. In terms of the preceding example, the assumption that issuers/borrowers consider expectations in their current financing decisions suggests that a bond issuer financing a two-year asset—a two-year borrower—would consider issuing either a two-year bond at 8% or a series of one-year bonds: a one-year bond today at 8% and a one-year bond one year later at 10% for an average rate of 9%. Given this choice, the two-year borrower would therefore prefer to issue two-year bonds at 8%. On the other hand, a bond issuer financing a one-year asset—a one-year borrower—could issue either one-year bonds at 8% or issue a two-year bond at 8% [for example, borrowing 857.3388 = $1,000/(1.08)^2$] and then buy the bond back in the market (or prepay) one year later when rates are at 10% and the price on bond is 909.09 (= $1,000/1.10), paying a one-year borrowing rate of 6% [= (909.09/857.3388) – 1]. Thus, given this choice, a one-year borrower would also prefer to issue two-year bonds instead of one-year bonds. In the two-year market, the expectation of higher rates would cause the supply of two-year bonds to increase (rightward shift in the bond supply curve), lowering their price and increasing the two-year yield. This would, in turn, reinforce the demand impact where the demand and price for two-year bonds are decreasing, causing two-year yields to increase. In the one-year market, the expectation of higher rates would cause the supply of one-year bonds to decrease (leftward shift in the one-year bond supply curve), increasing their price and lowering the one-year yield. This would, in turn, reinforce the demand impact where the demand and price for one-year bonds are increasing, causing one-year yields to decrease.

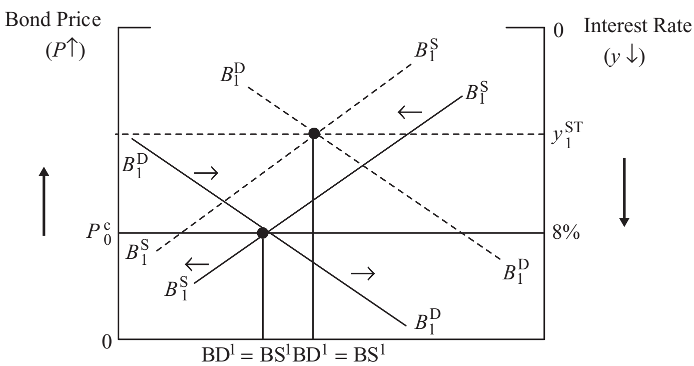
In equilibrium, the increase in the supply of two-year bonds (rightward shift in the two-year bond supply curve shown in Figure 4.5) and the decrease in the supply for one-year bonds (leftward shift in the one-year bond supply curve shown in Figure 4.5), combined with the demand adjustments of one-year bond demand increasing (one-year bond demand curve shifting right) and two-year bond demand decreasing (two-year bond demand curve shifting left) will continue until the average

- Expectations of rates increasing from 8% to 10%.
- Investors with HD of two years and those with HD of one year would prefer one-year bonds over two-year bonds.
- Two-year and one-year borrowers would prefer to finance with two-year bonds.
- Market Response:

$B_2^D\downarrow\Rightarrow P_2^B\downarrow\Rightarrow YTM_2\uparrow$	$B_1^D\uparrow\Rightarrow P_1^B\uparrow\Rightarrow YTM_1\downarrow$	YC becomes *positively* sloped
$B_2^S\uparrow\Rightarrow P_2^B\downarrow\Rightarrow YTM_2\uparrow$	$B_1^S\downarrow\Rightarrow P_1^B\uparrow\Rightarrow YTM_1\downarrow$	



One-Year Bond Market



Two-Year Bond Market

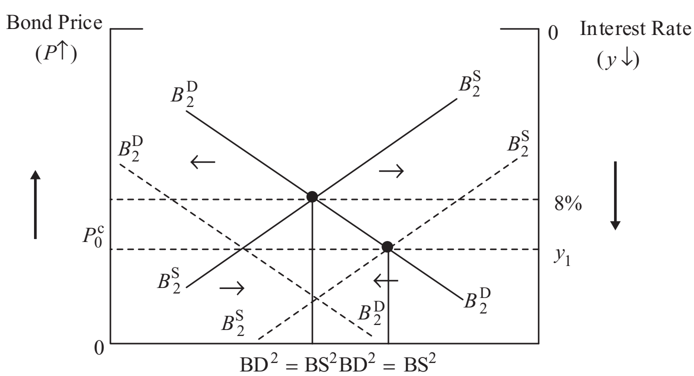


FIGURE 4.5 Pure Expectations Theory: Market Expectation of Higher Interest Rates: Investors' and Issuers' Response

annual rate on the two-year bond is equal to the equivalent annual rate from the series of one-year loans (or the one-year bond's rate is equal to the rate expected on the two-year bond held one year). This is the same equilibrium condition governing bond investors. Thus, similar to investors, the response of issuers/borrowers to the expectation of higher rates contributes to the steepening of the yield curves (see Figure 4.5).

Yield Curves that Incorporate Expectations

In the above example, the equilibrium yield curve is positively sloped, reflecting the market expectation of higher rates. By contrast, if the yield curve was currently flat at 10% and there was a market expectation that it would shift down to 8% next year, then the expectation of lower rates would cause the yield curve to become negatively sloped (see Figure 4.6). In this case, an investor with a two-year horizon date would prefer the two-year bond at 10% to a series of one-year bonds yielding an expected rate of only 9% [$E(R) = [(1.10)(1.08)]^{1/2} - 1 = .09$]. Similarly, an investor with a one-year horizon would also prefer buying a two-year bond that has an expected rate of return of 12% [$P_2 = 100/(1.10)^2 = 82.6446$, $E(P_{11}) = 100/1.08 = 92.5926$, $E(R) = [92.5926 - 82.6446]/82.6446 = .12$] to the one-year bond that yields only 10%. In the bond markets, the expectations of lower rates would cause the demand and price of the two-year bond to increase, lowering its rate, and the demand and price for the one-year bond to decrease, increasing its rate. These adjustments would continue until the rate on the two-year bond equaled the average rate from the series of one-year investments, or until the rate on the one-year bond equaled the expected rate from holding a two-year bond one year (or when the implied forward rate is equal to expected spot rates). In this case, if one-year rates stayed at 10%, then the demand for the two-year bond would increase until it was priced to yield 9%—the expected rate from the series: $[(1.10)(1.08)]^{1/2} - 1 = .09$ (see Figure 4.6). On the other hand, if two-year rates stayed at 10%, then the demand for the one-year bond would decrease until it was priced to yield 12%—the expected rate from buying a two-year bond at 826.4462 [$= (1{,}000/(1.10)^2$] and selling it one year later at 8% [$925.9259 = 1{,}000/1.08$]-: $R = (925.9259/826.4462) - 1 = .12$ (see Figure 4.6). Thus, in equilibrium, the one-year and two-year yields will be somewhere between 10% and 9% and 12% and 10%, with the resulting yield curve satisfying the condition that the implied forward rate is equal to the expected spot rate of 8%:

$$YTM_2 = [(1 + YTM_1)(1 + f_{11})]^{1/2} - 1$$

$$f_{11} = \frac{(1 + YTM_2)^2}{(1 + YTM_1)} - 1$$

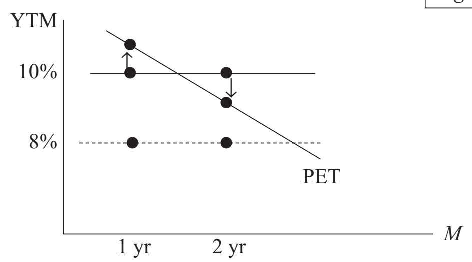
$$f_{11} = \frac{(1.09)^2}{(1.10)} - 1 = .08$$

or

$$f_{11} = \frac{(1.10)^2}{(1.12)} - 1 = .08$$

- Expectations of rates decreasing from 10% to 8%.
- Investors with HD of two years and those with HD of one year would prefer two-year bonds over one-year bonds.
- Market Response:

$B_1^D\downarrow \Rightarrow P_1^B\downarrow \Rightarrow \text{YTM}_1\uparrow$ | $B_2^D\uparrow \Rightarrow P_2^B\uparrow \Rightarrow \text{YTM}_2\downarrow$ | YC becomes *negatively* sloped

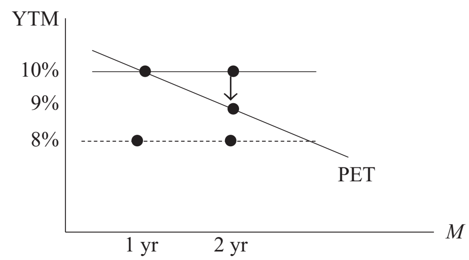


(a)

- If the market response to the expectation is only in terms of a change in the two-year bond, then the equilibrium yield on the two-year will be 9%.

$B_2^D\uparrow \Rightarrow P_2^B\uparrow \Rightarrow \text{YTM}_2\downarrow$
$\text{YTM}_2\downarrow$ until $\text{YTM}_2 = \text{YTM}_{2:\text{series}} = 9\%$

When $\text{YTM}_2 = 9\%$, $\text{YTM}_1 = 10\%$, then $f_{11} = E(R_{11}) = 8\%$.

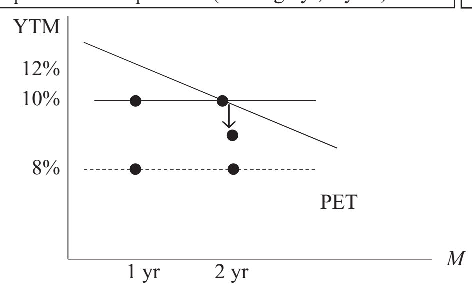


(b)

- If the market response to the expectation is only in terms of a change in the one-year bond, then the equilibrium yield on the one-year will be 12%.

$B_1^D\downarrow \Rightarrow P_1^B\downarrow \Rightarrow \text{YTM}_1\uparrow$
$\text{YTM}_1\uparrow$ until $\text{YTM}_1 = \text{YTM(holding 2yr, 1 year)} = 12\%$

When $\text{YTM}_2 = 10\%$, $\text{YTM}_1 = 12\%$, then $f_{11} = E(R_{11}) = 8\%$



(c)

FIGURE 4.6 Pure Expectations Theory: Market Expectation of Lower Interest Rates

If we assume that issuers/borrowers also consider expectations in their current financing decision, then a two-year borrower would favor financing with a series of one-year bonds at 10% and 8% for an average of 9% ($[(1.10)(1.08)]^{1/2} - 1 = .09$) to a two-year bond at 10%. Similarly, a one-year borrower would prefer current financing with one-year bonds at 10% to issuing a two-year bond at 10% [for example, borrowing $826.4463 = 1{,}000/(1.10)^2$] and then buying the bond back in the market one year later when rates are at 8% and the price on bond is 925.9259 (= $1,000/1.08), paying a one-year borrowing rate of 12% [= (925.9259/826.4436) – 1]. Both one-year and two-year borrowers would therefore prefer to issue one-year bonds instead of two-year bonds. In the market, this would cause the supply of two-year bonds to decrease (leftward shift in the bond supply curve), increasing their price and decreasing the two-year yield, and the supply of one-year bonds to increase (rightward shift in the one-year bond supply curve), decreasing their price and increasing the one-year yield. These adjustments, along with the demand adjustments of one-year bond demand decreasing and two-year bond demand increasing, would continue until the rate on the two-year bond equaled the average rate from the series of one-year investments, or until the rate on the one-year bond equaled the expected rate from holding a two-year bond one year (see Figure 4.7).

Finally, suppose the yield curve is currently positively sloped, with the yield on the one-year bond at 8% and the yield on the two-year bond at 9%. This time, though, suppose that the market expects no change in yields and that this expectation has not yet been reflected in the yield curve. In this scenario, investors with horizons of two years would prefer the two-year bond yielding 9% to a series of one-year bonds yielding 8% now and 8% expected at the end of one year (for a two-year equivalent of 8%). Similarly, investors with one-year horizons would also prefer the two-year bond held for one year in which the expected rate is 10%, compared to only 8% from the one-year bond. Given the investors' preference for two-year bonds, a market response would occur in which the demand for the two-year bond would rise, in turn increasing its price and lowering its yield, and the demand and price for the one-year bond would decrease, increasing its yield. Thus, investors would flatten the positively-sloped yield curve, given the expectation of no change in the yield curve.[4]

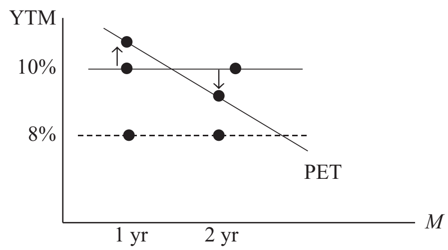
Features of Pure Expectations Theory

One of the features of the pure expectations theory is that in equilibrium the yield curve reflects current expectations about future rates. From our preceding examples, when the equilibrium yield curve was positively sloped, the market expected higher rates in the future; when the curve was negatively sloped, the market expected lower rates; when it was flat, the market expected no change in rates. Moreover, if PET strictly holds (i.e., we can accept all of the model's assumptions), then the expected future rates would be equal to the implied forward rates. As a result, one could forecast futures rates and future yield curves by simply calculating implied forward rates from current rates.

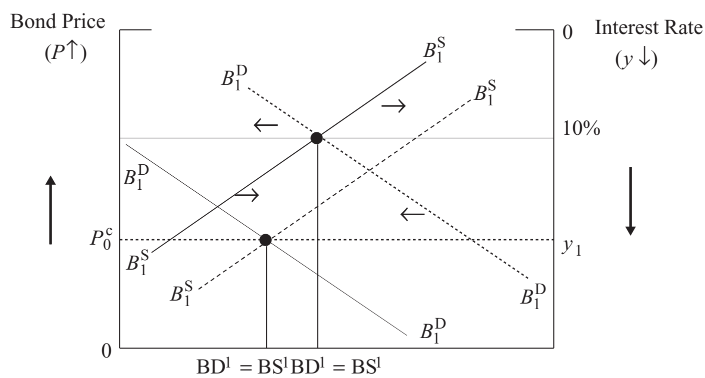
Forecasting Future Spot Yield Curves Figure 4.8 shows spot rates on bonds with terms to maturity ranging from one year to five years (Column 2) and with assumed

- Expectations of rates decreasing from 10% to 8%.
- Investors with HD of two years and those with HD of one year would prefer two-year bonds over one-year bonds.
- Two-year and one-year borrowers would prefer to finance with one-year bonds.
- Market Response:

$B_1^D \downarrow \Rightarrow P_1^B \downarrow \Rightarrow YTM_1 \uparrow$	$B_2^D \uparrow \Rightarrow P_2^B \uparrow \Rightarrow YTM_2 \downarrow$	YC becomes *negatively* sloped
$B_1^S \uparrow \Rightarrow P_1^B \downarrow \Rightarrow YTM_1 \uparrow$	$B_2^S \downarrow \Rightarrow P_2^B \uparrow \Rightarrow YTM_2 \downarrow$	



One-Year Bond Market



Two-Year Bond Market

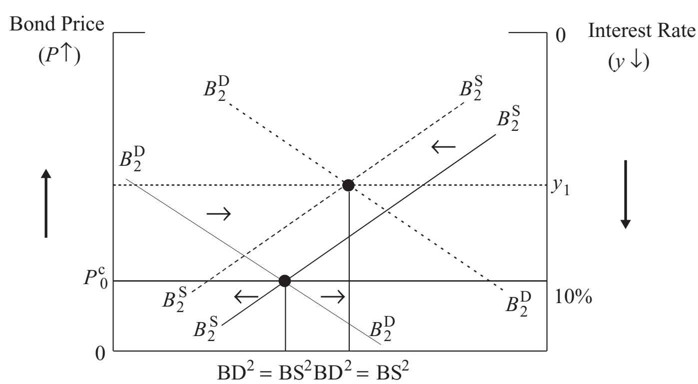


FIGURE 4.7 Pure Expectations Theory: Market Expectation of Lower Interest Rates: Investors' and Issuers' Response

(1)	(2)	(3)	(4)
Maturity	Spot Rates	Expected Spot Rates One Year from Present	Expected Spot Rates Two Years from Present
1	10.0%	f_{11} = 11%	f_{12} = 12%
2	10.5	f_{21} = 11.5%	f_{22} = 12.5%
3	11.0	f_{31} = 12.0%	f_{32} = 13%
4	11.5	f_{41} = 12.5%	
5	12.0		

$$f_{Mt} = \left[\frac{(1+S_{M+t})^{M+t}}{(1+S_t)^t}\right]^{1/M} - 1$$

f_{12}	f_{32}
$S_3 = [(1+S_1)(1+f_{11})(1+f_{12})]^{1/3} - 1$	$S_5 = [(1+S_1)(1+f_{11})(1+f_{12})(1+f_{13})(1+f_{14})]^{1/5} - 1$
$S_3 = [(1+S_2)^2(1+f_{12})]^{1/3} - 1$	$S_5 = [(1+S_2)^2(1+f_{32})^3]^{1/5} - 1$
$f_{12} = \frac{(1+S_3)^3}{(1+S_2)^2} - 1$	$f_{32} = \left[\frac{(1+S_5)^5}{(1+S_2)^2}\right]^{1/3} - 1$
$f_{12} = \frac{(1.11)^3}{(1.105)^2} - 1 = .12$	$f_{32} = \left[\frac{(1.12)^5}{(1.105)^2}\right]^{1/3} - 1 = .13$

FIGURE 4.8 Forecasting Yield Curves Using Implied Forward Rates

annual compounding. Using implied forward rates as estimates, expected spot rates (S_t) are generated for bonds one year from the present (Column 3) and two years from the present (Column 4) from the current spot rates. For example, the expected rate on a one-year bond one year from now $[E(S_{Mt}) = E(S_{11})]$ is equal to the implied forward rate of $f_{Mt} = f_{11} = 11\%$. This rate is obtained by using the geometric mean with the current two-year and one-year spot rates:

$$S_2 = [(1 + S_1)(1 + f_{11})]^{1/2} - 1$$

$$f_{11} = \frac{(1 + S_2)^2}{(1 + S_1)} - 1$$

$$f_{11} = \frac{(1.105)^2}{(1.10)} - 1 = .11$$

Similarly, the expected two-year spot rate one year from now is equal to the implied forward rate on a two-year bond purchased one year from now of $f_{Mt} = f_{21} = 11.5\%$. This rate is obtained using three-year and one-year spot rates:

$$S_3 = [(1 + S_1)(1 + f_{11})(1 + f_{12})]^{1/3} - 1$$

$$S_3 = [(1 + S_1)(1 + f_{21})^2]^{1/3} - 1$$

$$(1 + S_3)^3 = [(1 + S_1)(1 + f_{21})^2]$$

$$f_{21} = \left[\frac{(1 + S_3)^3}{(1 + S_1)}\right]^{1/2} - 1$$

$$f_{21} = \left[\frac{(1.11)^3}{(1.10)}\right]^{1/2} - 1 = .115$$

The other expected spot rates for next year are found repeating this process.

A similar approach also can be used to forecast the yield curve two years from the present. The expected one-year spot rate two years from now, $E(S_{12})$, is equal to the implied forward rate f_{12}, which is equal to 12% (see Figure 4.8). This rate is obtained from two-year and three-year spot rates using the geometric mean. The expected two-year spot rate, two years from now, is found by solving for f_{22}. Using four-year and two-year spot rates, f_{22} is equal to 12.5%. Finally, f_{32} of 13% is found using five-year and two-year bonds. All of these implied forward rates can be found using the following formula:

$$f_{Mt} = \left[\frac{(1 + S_{M+t})^{M+t}}{(1 + S_t)^t}\right]^{1/M} - 1$$

Using Forward Rates to Determine Expected Rates of Return According to PET, if the market is risk neutral, then the implied forward rate is equal to the expected spot rate, and in equilibrium, the expected rate of return for holding any bond for one year would be equal to the current spot rate on a one-year bond. Similarly, the rate expected to be earned for two years from investing in any bond or combination of bonds (e.g., such as a series of one-year bonds) would be equal to the rate on the two-year bond. This condition can be illustrated using the spot yield curve and expected yield curves (or forward rates) shown in Figure 4.8. For example, the expected rate of return from purchasing a two-year zero-coupon bond at the spot rate of 10.5% and selling it one year later at an expected one-year spot rate equal to the implied forward rate of f_{11} of 11% is 10%:

$$E(R) = \frac{90.09 - 81.8984}{81.8984} = .10$$

$$E(P_{11}) = \frac{100}{1.11} = 90.09$$

$$P_{20} = \frac{100}{(1.105)^2} = 81.8984$$

This is the same rate obtained from investing in a one-year bond. Similarly, the expected rate of return from holding a three-year bond for one year, then selling it at the implied forward rate of f_{21} is also 10%:

$$E(R) = \frac{80.43596 - 73.1191}{73.1191} = .10$$

$$E(P_{21}) = \frac{100}{(1.115)^2} = 80.43596$$

$$P_{30} = \frac{100}{(1.11)^3} = 73.1191$$

Any of the bonds with spot rates shown in Figure 4.8 would have expected rates for one year of 10% if the implied forward rate were used as the estimated expected rate. Similar results hold for a two-year investment period. That is, any bond held for two years and sold at its forward rate would earn the two-year spot rate of 10.5%. For example, a four-year bond purchased at the spot rate of 11.5% and expected to be sold two years later at $f_{22} = 12.5\%$ would have an expected rate of return of 10.5%—the same as the current two-year spot. Similarly, an investment in a series of one-year bonds at spot rates of $S_1 = 10.0\%$ and $E(S_{11}) = f_{11} = 11\%$ yields a two-year rate of 10.5% ($= [(1.10)(1.11)]^{1/2} - 1$).

Finally, the same conditions also hold for coupon bonds that are valued at spot rates and are expected to be sold at expected spot rates equal to the implied forward rates. For example, a four-year, 10% coupon bond with a face value of 100 would be worth 95.762 if it is discounted by the spot rates shown in Figure 4.5:

$$V_0 = \frac{10}{1.10} + \frac{10}{(1.105)^2} + \frac{10}{(1.11)^3} + \frac{110}{(1.115)^4} = 95.762$$

If that bond were sold one year later at the forward rates, its expected price would be 95.3484, yielding a one-year expected rate of 10%—the same as the one-year spot rate:

$$E(P) = \frac{10}{1.11} + \frac{10}{(1.115)^2} + \frac{110}{(1.12)^3} = 95.3484$$

$$E(R) = \frac{(95.3484 - 95.762) + 10}{95.762} = .10$$

Hedgeable Rates The implication that expected rates from holding any bond (or combination) for M years are equal to the rate on an M-year, zero-coupon bond is a direct result of the risk-neutrality assumption of PET. In reality, bond markets are not risk-neutral. As a result, forward rates may not necessarily be good predictors of future interest rates. Analysts often refer to forward rates as ***hedgeable rates***, and most do not consider forward rates as the market's consensus on expected future rates. The most practical use of forward rates is that they provide ***cut-off rates***, useful in evaluating investment decisions. For example, an investor with a one-year horizon date should only consider investing in the two-year bond in our above example if she expected one-year rates one year later to be less than $f_{11} = 11\%$; that is, assuming she is not risk-loving and wants an expected rate greater than 10%. Thus, forward rates serve as a good cut-off rate for evaluating investments.

4.5 LIQUIDITY PREMIUM THEORY

The fourth term structure theory is the ***liquidity premium theory (LPT)***, also referred to as the ***risk premium theory (RPT)***. LPT posits that there is a liquidity premium for long-term bonds over short-term bonds. Recall that in Chapter 2 we examined how long-term bonds were more price sensitive to interest rate changes than short-term bonds. As a result, the prices of long-term securities tend to be more volatile and therefore more risky than short-term securities. According to LPT, if investors were risk averse, then they would require some additional return (liquidity premium) in order to hold long-term bonds instead of short-term ones. Thus, if the yield curve were initially flat, but had no risk premium factored in to compensate investors for the additional volatility they assumed from buying long-term bonds, then the demand for long-term bonds would decrease and their rates would increase until risk-averse investors were compensated. In this case, the yield curve would become positively sloped

LPT can be thought of as an extension of the pure expectations theory. That is, given that long-term bonds are more responsive to interest rate changes, there is more risk holding a longer term bond for one period, with the risk increasing the greater the bond's maturity. As a result, LPT asserts that investors will hold longer maturity bonds if they offer higher yields than the expected future rate, with the risk premium increasing with the terms to maturity.[5] This would suggest that the yield curve is governed by the condition that the expected spot rate is equal to the forward rate plus a liquidity premium (LP) equal to a maturity spread ($R_{LT} - R_{ST}$), with the premium increasing with maturity.[6]

4.6 SUMMARY OF TERM STRUCTURE THEORIES

As we noted earlier, the structure of interest rates cannot be explained in terms of any one theory; rather, it is best explained by a combination of theories. Of the four theories, the two major ones are MST and PET. MST is important because it establishes how the fundamental market forces governing the supply and demand for assets determine interest rates. PET, in turn, extends MST to show how expectations impact the structure of interest rates. PHT, by explaining how markets will adjust if the economy is poorly hedged, and LPT, by including a liquidity premium for longer-term bonds, both represent necessary extensions of MST and PET. Together, the four theories help us to understand how supply and demand, economic conditions, government deficits and surpluses, monetary policy, hedging, maturity preferences, and expectations all affect the bond market in general and the structure of rates in particular.

4.7 CONSTRUCTING THE BENCHMARK YIELD CURVE

Theoretical Spot Rate Curve

Treasury yield curves are the standard benchmark for yields on other sectors. One of the problems in constructing a benchmark yield curve from observed Treasury yields is that many Treasuries with the same maturity carry different yields because they have different coupon rates. To rectify this, the convention is to generate a ***spot rate curve*** showing the relation between the spot rates and maturity. A spot Treasury rate is the rate on a zero-coupon Treasury bond. Because such bonds lack coupons and have no default and option risk, they would seem ideal. Unfortunately, with no zero-coupon Treasury debt issues with maturities greater than one year, it is not possible to construct such a curve from observed Treasury security yields.

Recall that in Chapter 2 we examined how spot rates are estimated using the bootstrapping technique. A spot yield curve in which the spot rates are estimated using bootstrapping, in turn, is referred to as a ***theoretical spot rate curve***. Moreover, as also examined in Chapter 2, the equilibrium price of a bond is that price obtained by discounting its cash flows by spot rates. If the market prices a bond above its equilibrium value, then dealers/arbitrageurs can earn a risk-free profit by buying the bond and stripping it; if the market prices a bond below its equilibrium value, then dealers/arbitrageurs can realize a risk-free profit by buying stripped securities and then forming an identical coupon bond to sell. In practice, the process of stripping and rebundling causes the true yield curve for Treasury securities to approach the theoretical spot rate curve. As a result, the theoretical spot rate curve is often used by practitioners to price financial instruments and by dealers to identify arbitrage opportunities; as such, it represents a good estimate of the benchmark yield curve.

Estimating the Spot Rate Curve In deriving a spot rate curve, possible Treasuries to include are either liquid on-the-run Treasury issues or on-the-run Treasury issues and select off-the-run issues. On-the-run Treasuries are the most recently auctioned issues of a given maturity. They include three-month, six-month, two-year, five-year, 10-year, and 30-year issues. For on-the-run issues, the yield used is the one that makes the issue trade at par—the coupon rate; not the yield when the issue is not trading at par. The resulting yield curve is referred to as a ***par coupon yield curve***.

In constructing the theoretical spot rate curve, 60 semiannual spot rates from six-month rates to 30-year rates need to be estimated. For on-the-run issues there are only six issues. The other maturity points for the par yield curve are estimated using linear extrapolation. Linear extrapolation requires taking the yields at two maturity points and calculating Δy:

$$\Delta y = \frac{\text{Yield at higher maturity} - \text{Yield at lower maturity}}{[\text{Number of semiannual periods between the two maturities}] + 1}$$

Starting with the lower maturity yield, Δy is then sequentially added to each yield. For example, if the two-year yield were 4% and the five-year yield were 4.6%, then there would be five semiannual periods between the maturities, and Δy would be equal to 0.10%:

$$\Delta y = \frac{\text{Yield at higher maturity} - \text{Yield at lower maturity}}{[\text{Number of semiannual periods between the two maturities}] + 1}$$

$$\Delta y = \frac{4.6\% - 4.0\%}{[5] + 1} = .10\%$$

Starting at the two-year yield of 4% and sequentially adding 0.10% to each yield, we obtain the following extrapolated yields:

- 2.5 years: 4.00% + .10% = 4.10%
- 3.0 years: 4.10% + .10% = 4.20%
- 3.5 years: 4.20% + .10% = 4.30%
- 4.0 years: 4.30% + .10% = 4.40%
- 4.5 years: 4.40% + .10% = 4.50%
- 5.0 years: 4.50% + .10% = 4.60%

One of the obvious problems with generating the theoretical spot rate curve from on-the-run issues is the significant proportion of yields that are generated from extrapolation. To minimize this problem, some analysts use select off-the-run Treasury issues. Combining on-the-run and select off-the run issues, a par coupon curve with fewer extrapolations can be generated.

Given the resulting par coupon curve, bootstrapping is then used to estimate the theoretical spot rate curve. As described in Chapter 2, the bootstrapping technique requires taking at least one zero-coupon bond and then sequentially generating other spot rates from the coupon bonds. To illustrate, consider the par yield curve shown in Table 4.1. There are two T-bills with maturities of six months (.5 years) and one year, trading at yields of 5% and 5.25%. Since T-bills are zero-coupon bonds, these rates can be used as spot rates (S_t) for maturities of .5 years ($S_{.5}$) and one year (S_1).

TABLE 4.1 Estimating a Spot Rate Curve Using Bootstrapping

Security	Type	Maturity	Semiannual Coupon	Annual YTM	Face Value	Current Price	Spot Rate
1	T-Bill	.5 years	-	5%	100	97.561	5.00%
2	T-Bill	1.0 years	-	5.25%	100	94.9497	5.25%
3	T-Note	1.5 years	2.75	5.5%	100	100	5.551%
4	T-Note	2.0 years	2.875	5.75%	100	100	5.577%
5	T-Note	2.5 years	3	6.00%	100	100	6.03%
6	T-Note	3.0 years	3.125	6.25%	100	100	6.30%

The other bonds shown in the table are coupon bonds assumed to be trading at par and therefore with yields equal to their coupon rates. Using bootstrapping, the spot rate for 1.5 years is found by

- Taking the T-note with a maturity of 1.5 years and annual coupon rate of 5.5% (semiannual coupons of 2.75)
- Setting the par value of the 1.5-year bond equal to the present value of its cash flows discounted at known spot rates of $S_{.5}$ and S_1 and an unknown spot rate for 1.5 years, $S_{1.5}$
- Solving for the spot rate for 1.5 years.

Doing this yields an annualized spot rate of $S_{1.5} = 5.51\%$:

$$P_{1.5} = \frac{CF_{.5}}{(1+(S_{.5}/2))^1} + \frac{CF_{1.0}}{(1+(S_1/2))^2} + \frac{CF_{1.5}}{(1+(S_{1.5}/2))^3}$$

$$100 = \frac{2.75}{(1+(.05/2))^1} + \frac{2.75}{(1+(.0525/2))^2} + \frac{102.75}{(1+(S_{1.5}/2))^3}$$

$$94.705956 = \frac{102.75}{(1+(S_{1.5}/2)^3}$$

$$S_{1.5} = 2\left[\left[\frac{102.75}{94.705956}\right]^{1/3} - 1\right] = .0551$$

To obtain the spot rate for a two-year bond (S_2), we repeat the process using the two-year bond paying semiannual coupons of 2.875 and selling at par. This yields a spot rate of $S_2 = 5.577\%$. Continuing the process with the other securities in the table, we obtain spot rates for bonds with maturities of 2.5 years and 3 years: $S_{2.5} = 6.03\%$ and $S_3 = 6.30\%$ (see the last column of Table 4.1).

Note that if an arbitrageur prices T-notes at par, then she could earn a risk-free profit by buying the note and stripping it. For example, consider the arbitrage from buying and stripping the three-year, 6.25% coupon bond. A dealer could buy the issue at par and strip the issue with the expectation of selling the strips at the yields corresponding to their maturities. As shown in Table 4.2, the proceeds from selling the strips would be 100.1217, yielding the dealer a profit of \$0.1217 per \$100 face value. In contrast, if the dealer sells at the spot rates defining the theoretical spot rate curve, the profit is zero and the arbitrage disappears. Thus, the actions of arbitrageurs to exploit this opportunity would be to drive the prices and yields on Treasury securities to the theoretical spot rates where the arbitrage disappears.

Forward Yield Curves Once a theoretical spot rate curve is derived, it can then be used as the base yield curve, with other yield curves generated by adding estimated risk and liquidity premiums. As we showed in Section 4.4, the spot rate curve can also be used to estimate implied forward rates and to generate ***forward yield curves***.

Table 4.3 shows forward rates implied from the theoretical spot rate curve we just derived. As shown, using the semiannual spot rates, the implied forward rate on the six-month bill, six months from now is $f_{Mt} = f_{.5,.5} = 2.7502\%$ (annualized

TABLE 4.2 Cash Flows from Selling Strips at YTMs and at Spot Rates

Maturity Years	Semiannual Cash Flow	YTM	PV	Spot Rates	PV
0.5	3.125	0.0500	3.0488	0.0500	3.0488
1.0	3.125	0.0525	2.9672	0.0525	2.9672
1.5	3.125	0.0550	2.8807	0.0551	2.8804
2.0	3.125	0.0575	2.7900	0.0558	2.7994
2.5	3.125	0.0600	2.6957	0.0603	2.6936
3.0	103.125	0.0625	85.7394	0.0630	85.6210
			100.1217		100.01

rate of 5.5003%). This rate is obtained by defining the one-year spot rate as the geometric average of the current six-month spot rate and the implied forward rate on a six-month rate, six months forward, and then solving the equation for the implied forward rate:

$$S_1 = [(1 + S_{.5})(1 + f_{.5,.5})]^{1/2} - 1$$

$$f_{.5,.5} = \frac{(1 + S_1)^2}{(1 + S_{.5})} - 1$$

$$f_{.5,.5} = \frac{(1.02625)^2}{(1.025)} - 1 = 0.027502$$

$$\text{Annualized } f_{.5.5} = (2)(0.027502) = .055003$$

Similarly, the implied forward rate on a one-year bond, purchased 6 months from now ($f_{Mt} = f_{1,.5} = 2.8827\%$), is obtained by solving the 1.5-year geometric mean for $f_{1,.5}$. Similar approaches also can be used to generate forward yield curves one year, 1.5 years, and other years from the present. In general, implied semiannual forward rates on M-year bonds purchased t years from the present can be found using the following formula:

$$f_{Mt} = \left[\frac{(1 + S_{M+t})^N}{(1 + S_t)^i}\right]^{1/(N-i)} - 1$$

where S = Semiannual spot rate
M = Time to maturity in years
t = Time period from the present to the forward date in years
N = Number of semiannual periods to $M + t$
i = Number of semiannual periods to t

Thus, the implied forward rate on a .5-year bill, purchased one year from now, is 3.0155%:

- $M = .5$ years
- $t = 1$ year
- $N = 3$ (= number of semiannual periods to $M + t = 1.5$ years)
- $i = 2$ (= number of semiannual periods to $t = 1$ year)

TABLE 4.3 Forward Yield Curve

Security	Type	Maturity	Spot Rate	Semiannual	Forward rate t = .5	Annual	$E(R_{M,.5})$ Sold at Forward Rate	Forward rate t = 1	Annual	$E(R_{M,1})$ Sold at Forward Rate
1	T-Bill	.5 years	0.050000	0.025000	0.027502	0.055003	0.025000	0.030155	0.060310	
2	T-Bill	1.0 year	0.052500	0.026250	0.028827	0.057655	0.025000	0.029523	0.059045	0.026250
3	T-Note	1.5 years	0.055100	0.027550	0.028848	0.057697	0.025000	0.032758	0.065516	0.026250
4	T-Note	2.0 years	0.055770	0.027885	0.031442	0.062883	0.025000	0.034135	0.068270	0.026250
5	T-Note	2.5 years	0.060300	0.030150	0.032805	0.065610				
6	T-Note	3.0 years	0.063000	0.031500						

Implied forward rate on .5-year bond, .5 years forward:

$M = .5$ *years*

$t = .5$ *years*

$N = 2$ (*number of semiannual periods to* $M + t$)

$i = 1$ (*number of semiannual periods to* t)

$$f_{Mt} = \left[\frac{(1+S_{M+t})^N}{(1+S_t)^i}\right]^{1/(N-i)} - 1$$

$$f_{.5,.5} = \left[\frac{(1+S_1)^2}{(1+S_{.5})^1}\right]^{1/(2-1)} - 1$$

$$f_{.5,.5} = \frac{(1.02625)^2}{(1.025)^1} - 1 = 0.027502$$

Annualized $f_{.5.5} = (2)(0.027502) = .055003$

Implied forward rate on .5-year bond, one year forward:

$M = .5$ *years*

$t = 1$ *year*

$N = 3$ (*number of semiannual periods to* $M + t$)

$i = 2$ (*number of semiannual periods to* t)

$$f_{Mt} = \left[\frac{(1+S_{M+t})^N}{(1+S_t)^i}\right]^{1/(N-i)} - 1$$

$$f_{.5,1} = \left[\frac{(1+S_{1.5})^3}{(1+S_{.5})^2}\right]^{1/(3-2)} - 1$$

$$f_{.5,1} = \left[\frac{(1.027550)^3}{(1.02625)^2}\right]^1 - 1$$

$f_{.5,1} = 0.030155$

Annualized $f_{.5,1} = (2)(0.030155) = .060310$

Implied forward rate on one-year bond, .5 years forward:

$M = 1$ *year*

$t = .5$ *years*

$N = 3$ (*number of semiannual periods to* $M + t$)

$i = 1$ (*number of semiannual periods to* t)

$$f_{Mt} = \left[\frac{(1+S_{M+t})^N}{(1+S_t)^i}\right]^{1/(N-i)} - 1$$

$$f_{1,.5} = \left[\frac{(1+S_{1.5})^3}{(1+S_{.5})^1}\right]^{1/(3-1)} - 1$$

$$f_{1,.5} = \left[\frac{(1.027550)^3}{(1.025)^1}\right]^{1/2} - 1$$

$f_{1,.5} = 0.028827\%$

Annualized $f_{1,.5} = (2)(0.028827) = .057655$

Implied forward rate on one-year bond, one year forward:

$M = 1$ *year*

$t = 1$ *year*

$N = 4$ (*number of semiannual periods to* $M + t$)

$i = 2$ (*number of semiannual periods to* t)

$$f_{Mt} = \left[\frac{(1+S_{M+t})^N}{(1+S_t)^i}\right]^{1/(N-i)} - 1$$

$$f_{1,1} = \left[\frac{(1+S_2)^4}{(1+S_1)^2}\right]^{1/(4-2)} - 1$$

$$f_{1,1} = \left[\frac{(1.027885)^4}{(1.02625)^2}\right]^{1/2} - 1$$

$f_{1,1} = .0.029523$

Annualized $f_{1,1} = (2)(0.029523) = .059045$

$$f_{Mt} = \left[\frac{(1+S_{M+t})^N}{(1+S_t)^i}\right]^{1/(N-i)} - 1$$

$$f_{.5,1} = \left[\frac{(1+S_{1.5})^3}{(1+S_{.5})^2}\right] - 1$$

$$f_{.5,1} = \left[\frac{(1.027550)^3}{(1.02625)^2}\right] - 1$$

$$f_{.5,1} = 0.030155$$

$$\text{Annualized } f_{.5,1} = (2)(0.030155) = .060310$$

Note that in our illustration, if the implied forward rates six months from now are realized, then an investor with a six-month horizon would earn the same semiannual rate of 2.5% obtained on the six-month bill by buying any maturity bond and selling it six months later at its implied forward rate (see Table 4.3). Similarly, if the implied forward rates one year from now are realized, then an investor with a one-year horizon would earn the same semiannual rate of 2.6255% obtained on the one-year by buying any maturity bond and selling it one year later at its implied forward rate.

Other Benchmark Yield Curves

An alternative approach to estimating the benchmark yield curve is to simply use stripped securities generated from Treasury coupon securities; in fact, this would seem to be the most promising approach. However, there are two problems using a ***Treasury strip yield curve*** as the benchmark yield curve. First, the liquidity of the stripped securities is less than that of Treasury securities.[7] As a result, the yields on strips reflect a liquidity premium. Second, the tax treatment on strips differs from that of Treasury coupons in that the accrued interest on a strip is taxed even though no cash is received until maturity.

Another approach to estimating the benchmark yield curve that is gaining in popularity is to use the rates on interest rate swaps. In a generic interest rate swap contract, fixed interest payments are swaped for floating rates. The fixed rate on a swap contract is called the swap rate and is equal to a T-note rate plus basis points. Dealers in the market, in turn, quote swap rates for different maturities. The relation between the swap rate and maturity of a swap is called the ***swap rate yield curve***. Interest rate swaps are examined in Chapter 20.

4.8 CONCLUSION

In this chapter, we have examined four theories of the term structure of interest rates. The theories explain term structure in terms of such factors as market expectation, economic conditions, financial conditions, and risk-return preferences. In our discussion of the level and structure of interest rates over the last two chapters, we have looked at the general relationship between risk and return. In the next chapter, we turn our attention to the specific types of risk associated with bonds.

WEB INFORMATION

1. Yield curves from Investinginbonds.com:
   Go to http://investinginbonds.com/.
   Click "Government Market-at-a-Glance."
   Treasury yields shown are based on price data for U.S. Treasury securities of different maturities. The yields are provided by GovPX, Inc. The Treasury yield data can be exported to Excel (right click and then click "Export to Excel"). More detailed information is obtained by clicking "see data" at the bottom.

2. Yield curves from Bloomberg:
Go to www.bloomberg.com; click "Market Data" and "Rates and Bonds."
3. Treasury yields from the *Wall Street Journal* site:
The yields over the last five years on Treasuries with different maturities (one-month, three-month, two-year, five-year, 10-year, and 30-year) can be found by going to the *Wall Street Journal* site:
Go to http://online.wsj.com/public/us.
Click "Market Data" and "Bonds, Rates & Credit Markets" tabs.
Click "Main: Overview, Intraday Quotes, Links to All" tab to find interactive chart to study yields over different time periods.
4. Yield curves from FINRA:
Current yield curves and yield curves one year earlier for Treasuries and different quality corporate and municipal bonds are found on the FINRA site.
Go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds."
5. Foreign yields from FXstreet:
Go to FXstreet to find historical yields on U.S. dollar-denominated bonds, euro-denominated bonds, and yen-denominated bonds with five-, 10-, and 30-year maturities.
Go to www.fxstreet.com.
Click "Rates and Charts" and "Bond Yields."
6. Foreign Security Holders of U.S. Treasuries:
Go to Treasury Tic Information: www.treas.gov/tic.
Click "Foreign Holders of Treasuries" and "Major Foreign Holders of Treasury Securities."
Click "Additional Historical Data" to study historical trends.
For information on net foreign purchases of U.S. long-term securities by major foreign sectors, click:
U.S. Treasury Bonds & Notes
U.S. Gov't Corp. & Federally-Sponsored Agency Bonds
U.S. Corporate & Other Bonds
U.S. Stocks
7. U.S. Treasury: Studies, actions, and other information
Go to www.treas.gov/.
8. Information on Federal Reserve policies
Go to www.federalreserve.gov/policy.htm and click tabs under "Features":
Credit and Liquidity Programs and the Balance Sheets
Federal Open Market Committee Statement
Information Regarding Recent Federal Reserve Actions
9. For information on U.S. government expenditures, revenues, deficits, and debt, go to: www.gpo.gov/fdsys/browse/collectionGPO.action?collectionCode=BUDGET.
10. For downloadable tables on U.S. government expenditures, revenues, deficits, and debt, go to: www.gpo.gov/fdsys/search/pagedetails.action?granuleId= &packageId=BUDGET-2010-TAB.

(*continued*)

(*Continued*)

11. For government information submitted by Congress, go to: www.gpo.gov/fdsys/search/home.action
12. For information on the U.S. economy, *Survey of Current Business*, go to www.bea.gov.
13. Federal Reserve Archival System for Economic Research (FRASER): Information on economic indicators and performance from the Council of Economic Advisors:
    Go to http://fraser.stlouisfed.org/publications/ei/.
    Go to Economic Indicators and Inventory Listings.
14. Information from the European Central Bank: Go to www.ecb.int.

KEY TERMS

cut-off rates
forward yield curves
hedgeable rates
humped yield curve
inverted yield curves
liquidity premium theory (LPT)
market segmentation theory (MST)
normal yield curve
par coupon yield curve
preferred habitat theory (PHT)
pure expectations theory (PET)
risk premium theory (RPT)
spot rate curve
swap rate yield curve
term structure of interest rates
theoretical spot rate curve
yield curve

PROBLEMS AND QUESTIONS

1. Using the market segmentation theory, explain the impacts on the term structure of interest rates in the following cases:
    a. Economic recession
    b. Economic expansion
    c. Expansionary open market operation in which the central bank buys S-T Treasuries
    d. Treasury sale of long-term Treasury bonds
    e. Treasury purchase of long-term Treasury bonds
2. Explain the equilibrium adjustments that would occur in the short-term and long-term bond markets for the following cases:
    a. Investors in corporate securities, on average, prefer short-term to long-term instruments, whereas corporations have greater financial requirements to finance long-term assets than short-term, and therefore prefer to issue more long-term bonds than short-term.
    b. Investors in corporate securities, on average, prefer long-term to short-term instruments, whereas corporations have a greater need to finance

short-term assets than long-term, and therefore prefer to issue more short-term bonds than long-term.

3. Explain pure expectations theory intuitively and with an example. In your example assume a flat yield curve with one- and two-year bonds at 6% and an expectation of next year's yield curve being flat with one- and two-year bonds at 8%. Explain the theory only in terms of the response to the expectation by investors with one-year and two-year horizon dates.
4. Explain how borrowers/issuers with one-year and two-year assets to finance would respond to the market expectations case in Question 3. Do the impacts of their actions on the yield curve complement the actions of investors?
5. Outline the impacts the following market expectations have on the yield curve:
   a. A flat yield curve at 8% with a market expectation of a flat yield curve at 12% one year later
   b. A flat yield curve at 10% with a market expectation of a flat yield curve at 8% one year later
   c. A yield curve with one-year bonds at 6% and two-year bonds at 7%, with the expectation of a flat yield curve at 8% one year later
6. Assume the following yield curve for zero-coupon bonds with a face value of $100 and annualized compounding:

Maturity	YTM
1 Year	7%
2 Years	8%
3 Years	8%
4 Years	7%
5 Years	6%

   a. Using implied forward rates, estimate the yield curve one year from the present (rates on one-year, two-year, three-year, and four-year bonds).
   b. Using implied forward rates, estimate the yield curve two years from the present (rates on one-year, two-year, and three-year bonds).
   c. If you bought the three-year bond and held it one year, what would your expected rate of return be if your expectations were based on implied forward rates?
   d. Without calculating, if you bought a bond of any maturity and held it one year, what would your expected rate of return be if your expectations were based on implied forward rates?
   e. If you bought the four-year bond and held it two years, what would your expected rate of return be if your expectations were based on implied forward rates?
   f. Without calculating, if you bought a bond of any maturity and held it two years, what would your expected rate of return be if your expectations were based on implied forward rates?

7. Given the following spot yield curve:

Maturity	Spot Rate
1 Year	6.0%
2 Years	6.5%
3 Years	7.0%
4 Years	7.5%

a. What is the equilibrium price of a four-year, 7% coupon bond making annual coupon payments and paying a principal of $100 at maturity?
b. Using implied forward rates, estimate the yield curve one year from the present (one-year, two-year, and three-year spot rates).
c. What is the expected equilibrium price one year from now of a three-year, 7% annual coupon bond paying a principal of $100 at maturity?
d. What is the one-year expected rate of return from investing in the four-year, 7% coupon bond if your expectations are based on implied forward rates?
e. Using implied forward rates, estimate the yields for one-year and two-year spot rates two years from now.
f. Show that the expected rate from holding the four-year, 7% coupon bond for two years is equal to the two-year spot rate of 6.5%.

8. Using the theories of term structure of interest rates, identify several scenarios that would tend to cause the yield curve to become negatively sloped.

9. Explain the liquidity premium theory. How could the yield curves in Questions 6 and 7 be adjusted to reflect a liquidity premium?

10. Short-Answer Questions:
   1. If the yield curve includes investor expectations, then a positively sloped yield curve would reflect what type of expectations about future interest rates?
   2. If the yield curve includes investor expectations, then a negatively sloped yield curve would reflect what type of expectations about future interest rates?
   3. How is an implied forward rate used as a cutoff rate?
   4. Define the preferred habitat theory.

WEB EXERCISES

1. Comment on what factors you think have contributed to the current term structure. Current yield curves can be found by going to www.bloomberg.com and clicking on "Market Data" and "Rates & Bonds." See "Web Information" for other sites providing yield curves.
2. Explore the yields on U.S. Treasuries over the last five years with different maturities (one-month, three-month, two-year, five-year, 10-year, and 30-year) by going to the *Wall Street Journal* site. Provide some economic and policy arguments that might explain the differences in yields you observe in different periods. Use some economic and government sites to find information to support your arguments.
   - Go to http://online.wsj.com/public/us.
   - Click "Market Data, Bonds, Rates & Credit Markets" tab.

- Click "Main: Overview, Intraday Quotes, Links to All" tab.
- Use the interactive chart to study yields over different time periods.
- Information can be exported to Excel.

Possible economic sites:

- http://fraser.stlouisfed.org/publications/ei/ Go to Economic Indicators and Inventory Listings.
- www.treas.gov/ Click Emergency Economic Stabilization Act.
- www.bea.gov *Survey of Current Business.*
- www.federalreserve.gov/policy.htm Reports and Beige Book.
- www.treas.gov/tic Foreign Security Holders.
- www.gpo.gov/fdsys/browse/collectionGPO.action?collectionCode= BUDGET Information on U.S. government's expenditures, revenues, deficits and debt.
- www.gpo.gov/fdsys/search/pagedetails.action?granuleId= &packageId=BUDGET-2010-TAB Downloadable tables on U.S. government's expenditures, revenues, deficits, and debt.
- www.gpo.gov/fdsys/search/home.action Government information submitted by Congress.

3. Compare current and previous year yield curves on Treasuries and different quality bonds by going to the FINRA site. Provide some economic and policy arguments that might explain the differences in yields.
   - Go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds."
   - On U.S. Treasury yield curve, click "View All."
4. Go to FXstreet to find historical yields on U.S. dollar-denominated bonds, euro-denominated bonds, and yen-denominated bonds with five-, 10-, and 30-year maturities. Comment on the correlation between the yields on the different denominated bonds.
   - Go to Exchange Rate Information: www.fxstreet.com
   - Click "Rates and Charts" and "Bond Yields"
5. Study the current public debt by going to
   - www.gpo.gov/fdsys/browse/collectionGPO.action?collectionCode= BUDGET Information on U.S. government's expenditures, revenues, deficits, and debt.
   - www.gpo.gov/fdsys/search/pagedetails.action?granuleId=&packageId= BUDGET-2010-TAB Downloadable tables on U.S. government's expenditures, revenues, deficits, and debt.
   - www.gpo.gov/fdsys/search/home.action Government information submitted by Congress.

NOTES

1. The convention is to refer to a yield curve as positively sloped if the spread between a six-month and 30-year bond is 300 bp, and as a steep yield curve when the spread is more than 300 bp. When the yield curve's maturity spread increases (widens), the yield curve is said to be steeper; when the yield curve's maturity spread decreases (narrows), the yield curve is said to flatten.
2. One way to examine how market forces determine the shape of yield curves is to examine MST using the supply and demand analysis that we introduced in Chapter 3. For such an

analysis, see Johnson, Zuber, and Gandar, "Re-Examination of the Market Segmentation Theory," *Journal of Financial Education*, forthcoming.

3. The open market operations of the U.S. Federal Reserve have historically been implemented through the purchase and sale of short-term Treasuries. In an effort to lower intermediate-term and long-term rates, the Fed in 2009 began purchasing intermediate and long-term Treasuries.
4. On the borrower side, bond issuers' response to the expectation of no change would also contribute to a yield curve flattening. That is, one-year borrowers would prefer issuing one-year bonds at 8% to issuing two-year bonds at 9% and then buying them back a year later at 8% rates for a one-year financing rate of 10%. Similarly, two-year borrowers would prefer financing with a series of one-year bonds at 8% now and a year later at an expected rate of 9% (average of 8.5%) to issuing two-year bonds at 9%. Given the borrowers' preference for one-year bonds, a market response would occur in which supply for the two-year bonds would decrease, in turn increasing its price and lowering its yield, and the supply for one-year bonds would increase, lowering its price and increasing its yield. Thus, bond suppliers would also contribute to a flattening of the positively-sloped yield curve given the expectation of no change in the yield curve.
5. See Cox, Ingersoll, and Ross, "A Re-examination of Traditional Hypotheses about the Term Structure of Interest Rates," pp. 774–775.
6. PET assumes a risk-neutral market in which investors and borrowers do not require compensation for assuming risk and where the implied forward rate is an unbiased estimate of the expected spot rate. In contrast, the liquidity preference theory is referred to as a *biased expectations theory*. That is, according to LPT, the implied forward rate will not be an unbiased estimate of the market's expectation of future interest rates. Another biased expectation theory is the *local expectations theory*. This theory posits that the returns on bonds of different maturities will be the same for short-run horizons, but not longer horizons.
7. The Treasury strip security market is discussed in more detail in Chapter 7.

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CHAPTER 5

Bond Risk

5.1 INTRODUCTION

Investment risk is the chance that the actual rate of return realized from an investment will differ from the expected rate. Bond investors face three types of risk: (1) ***default risk,*** the uncertainty that the issuer/borrower will fail to meet the contractual obligations specified in the indenture; (2) ***call risk,*** the uncertainty that the issuer/borrower will buy back the bond, forcing the investor to reinvest in a market with lower interest rates; and (3) ***market risk,*** the uncertainty that interest rates will change, changing the price of the bond and the return earned from reinvesting coupons. In this chapter, we examine these three types of risk and introduce two measures of bond volatility: duration and convexity.

5.2 DEFAULT RISK

Default risk (or ***credit risk***) is the risk that the borrower/issuer will not meet all promises at the agreed-upon times. A failure to meet any of the interest payments, the principal obligation, or other terms specified in the indenture (e.g., sinking fund arrangements, collateral requirements, or other protective covenants) places the borrower/issuer in default. When issuers default they can file for bankruptcy, their bondholders/creditors can sue for bankruptcy, or both parties can work out an agreement. Many large institutional investors have their own credit analysis departments to evaluate bond issues in order to determine the abilities of companies to meet their contractual obligations. These institutions, as well as individual investors, also rely on bond rating companies for evaluating the credit worthiness of bonds. Currently, the major rating companies in the United States are Moody's Investment Services, Standard & Poor's, and Fitch Ratings Service.

Moody's and Standard & Poor's have been rating bonds for almost 100 years. Today, they rate over 2,000 companies in additional to municipals and other debt obligations. Moody's, Standard & Poor's, and Fitch evaluate bonds by giving them a quality rating in the form of a letter grade (see Exhibit 5.1). The grades start at A with three groups: Triple A bonds (Aaa for Moody's and AAA for Standard & Poor's) for the highest grade bonds, double A (Aa or AA) for bonds that are considered prime, single A for those considered high quality. Grade A bonds are followed by medium-grade B-rated bonds, classified as either triple B (Baa or BBB), double B (Ba or BB), and single B. Finally, there are C-grade and lower-grade bonds. Moody's also breaks

EXHIBIT 5.1 Bond Ratings

	Very High Quality	High Quality	Speculative	Very Poor
Standard & Poor's	AAA AA	A BBB	BB B	CCC D
Moody's	Aaa Aa	A Baa	Ba B	Caa C

Moody's	S&P	DESCRIPTION
Aaa	AAA	Bonds have the highest rating. Ability to pay interest and principal is very strong.
Aa	AA	Bonds have a very strong capacity to pay interest and repay principal. Together with the highest ratings, this group comprises the high-grade bond class.
A	A	Bonds have a strong capacity to pay interest and repay principal, although they are somewhat susceptible to the adverse effects of changes in economic conditions.
Baa	BBB	Bonds are regarded as having an adequate capacity to pay interest and repay principal. Adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity to pay interest and repay principal for debt in this category than in higher-rated categories. These bonds are medium-grade obligations.
Ba B Caa Ca	BB B CCC CC	Bonds are regarded as predominately speculative with respect to capacity to pay interest and repay principal in accordance with the terms of the obligation. BB and Ba indicate the lowest degree of speculation, and CC and Ca the highest degree of speculation.
C	C	This rating is reserved for income bonds on which no interest is being paid.
D	D	Bonds rated D are in default, and payment of interest and/or repayment of principal is in arrears.

At times both Moody's and Standard & Poor's have used adjustments to these ratings. S&P uses plus and minus signs: A+ is the strongest A rating and A− the weakest. Moody's uses a 1, 2, or 3 designation, with 1 indicating the strongest.

Source: FINRA, www.finra.org/index.htm.

down bonds by using a 1, 2, and 3 designation, whereas Standard & Poor's does the same with a plus or minus designation. In interpreting these ratings, triple A bonds are considered to have virtually no default risk, whereas low B-rated or C-rated bonds are considered speculative with some chance of default. In general, bonds with a relatively low chance of default are referred to as ***investment-grade bonds***, with quality ratings of Baa (or BBB) or higher; bonds with a relatively greater chance of default are referred to as ***non-investment-grade, speculative-grade*** or ***junk bonds*** and have quality ratings below Baa. Table 5.1 shows the number of defaulted issues each year from 1985 to 2006 for all corporate bonds broken down by original ratings for investment-grade and non-investment grade. As expected, a higher percentage of default is shown for the non-investment grade.

TABLE 5.1 Annual Defaults by Original Ratings

The table shows the number of defaulted issues each year from 1985 to 2006 for all corporate bonds broken down by original ratings for investment-grade and non-investment grade. Higher percentage of default is shown for the non-investment grade.

Year	Number of Defaulted Issues	Originally Rated Investment Grade (%)	Originally Rated Non-investment Grade (%)
2006	52	13	87
2005	184	49	51
2004	79	19	81
2003	203	33	67
2002	322	39	61
2001	258	14	86
2000	142	16	84
1999	87	13	87
1998	39	31	69
1997	20	0	100
1996	24	13	87
1995	29	10	90
1994	16	0	100
1993	24	0	100
1992	59	25	75
1991	163	27	73
1990	117	16	84
1989	66	18	82
1988	64	42	58
1987	31	39	61
1986	55	15	85
1985	26	4	96

Source: http://pages.stern.nyu.edu/~ealtmanAboutCorporateDefaultRates.pdf.

Historical Default Rates

The term *junk bond* became part of the financial vernacular in the 1980s to refer to low-graded bonds sold by companies with financial problems, often referred to as Fallen Angels. During that decade, there was a rapid growth in low-rated debt issues. Led by Michael Milken of the investment banking firm of Drexel Burnham Lambert, much of the growth in junk bond financing was due to their use in many hostile takeovers in which many companies sold high-yielding, low-quality bonds to finance their acquisitions of other companies. As a result of this financing, many newly structured companies with high debt-to-equity ratios emerged. Because debt is tax deductible, the potential return to investors was augmented by this increase in leverage, but so also was their exposure to default risk.

Trading at yields 4% to 5% over comparable U.S. government securities, junk bonds were attractive investments to many institutional investors, including many savings and loans (at least until 1989 when Congress passed the ***Financial Institutions Reform, Recovery, and Enforcement Act*** outlawing the purchase of low quality bonds by federally-sponsored deposit institutions). In the early 1980s, the default experience for such bonds was relatively low. However, slower economic

TABLE 5.2 High-Yield Corporate Bond Default Rates in the United States and Canada: 1985–2006

Year	Default Rates (%)
2006	0.761
2005	3.375
2004	1.249
2003	4.661
2002	12.795
2001	9.801
2000	5.073
1999	4.147
1998	1.603
1997	1.252
1996	1.231
1995	1.896
1994	1.454
1993	1.105
1992	3.402
1991	10.273
1990	10.140
1989	4.285
1988	2.662
1987	5.778
1986	3.497
1985	1.798
Average	4.193
Standard Deviation	3.505
Default rate = Par value defaults/Par value outstanding	

Source: http://pages.stern.nyu.edu/~ealtman/About CorporateDefaultRates.pdf.

growth in 1990 and the recession in 1991 resulted in a high incidence of defaults by many companies that had issued junk bonds in the 1980s. In 1990, the default rate for high-yield non-investment-grade bonds in the United States and Canada was 10.14%, and in 1991, it was 10.273% (see Table 5.2 for a summary of U.S. and Canadian default rates for high-yield corporate bonds from 1985 to 2006).

The strong economic growth in the United States and the world in the 1990s saw default rates decline. By 1992, speculative-grade default rates had declined from a 10% level in 1991 to an average of 5%, and for the period from 1992 to 1999, the default rates on speculative-grade bonds ranged between 2.5% and 5% (similar trends also were found with global speculative-grade bonds). The economic slowdown in 2000 and 2001, though, saw the default rate on speculative bonds increase back to the 1991 level of 9% (global rates to 8%). From 2002 to 2006, the return of economic growth globally and in the United States resulted in a decline in default rates. In 2002, the U.S. speculative-grade default rate was 7.5%, followed by 5% in 2003, 3.5% in 2004, and approximately 2.5% in 2005 and 2006.[1] However, beginning in 2007, economic concerns over subprime mortgage loans and housing led to the beginning of another economic slowdown. In 2008, bankruptcy filings among publicly traded companies surged 74 percent to 136 bankruptcy filings by publicly traded companies in 2008, compared with 78 in 2007 (see: www.BankruptcyData.com). The value of the firms seeking protection also grew much faster: The 136 companies seeking bankruptcy protection in 2008 had approximately $1.16 trillion in assets, compared with just $70.5 billion in assets for firms filing for bankruptcy protection in 2007.

By September 2008, the subprime mortgage meltdown had developed into a global credit crisis, with the trillion dollar mortgage giants Fannie Mae and Freddie Mac being placed into conservatorship by the Treasury; Lehman Brothers filing for the largest bankruptcy in American history ($600 billion); American International Group (AIG) receiving an emergency $85 billion lifeline from the Federal Reserve as it teetered on the brink of insolvency; and with Washington Mutual being seized by the FDIC and its deposits sold to JPMorgan Chase. By the beginning of 2009, the financial crisis that had started on Wall Street was being felt on Main Street, with industrial giants such as General Motors filing for bankruptcy and with the default rate for speculative-grade bonds growing to 8% for the year.

Probability Intensities from Historical Default Rates

Historical default rates can be used as an estimate of a bond's probability of default. Table 5.3 shows three different probabilities for corporate bonds with quality ratings of AAA, AA, A, BBB, BB, B, and CCC: ***cumulative default rates***, ***unconditional probability rates***, and ***conditional probability rates***. The cumulative probabilities show the default chance through time. For example, the BBB corporate bonds have 0.18% chance of defaulting after one year, 0.51% chance after two years, 1.94% chance after five years, and 4.64% chance after 10 years. The cumulative probabilities shown in the table are the average historical cumulative default rates from 1977 to 2006 as compiled by Moody's. The unconditional probabilities are the probabilities of default in a given year as viewed from time zero. The unconditional probability of a bond defaulting during year t is equal to the difference in the ***cumulative probability*** in year t minus the cumulative probability of default in year $t - 1$. As shown in the table, the probability of a CCC bond default during year 4 is equal to 7.18% (= 46.90% − 39.72%). Finally, the conditional probability is the probability of default in a given year conditional on no prior defaults. Conditional probabilities of default are known as ***probability intensities***. This probability is equal to the unconditional probability of default in time t as a proportion of the bond's probability of survival at the beginning of the period. The probability of survival is equal to 100 minus the cumulative probability. For example, the probability that a CCC bond will survive until the end of year 3 is 60.28% (100 minus its cumulative probability 39.72%), and the probability that the CCC bond will default during year 4 conditional on no prior defaults is 11.91% (= 7.18%/60.28%).

The historical default rates in Table 5.3 show the 10-year cumulative default rate for AAA bonds is only .52%, and the average conditional probability of default is .0579%. The default rates, in turn, increase, the lower the quality rating:

- For BBB-rated bonds, their historical 10-year cumulative probability is 4.64% and their average probability intensity is .50656%

TABLE 5.3 Historical Default Rates: 1977–2006, Moody's

Year	1	2	3	4	5	6	7	8	9	10
Aaa										
Cumulative Probability (%)	0.00000	0.00000	0.00000	0.03000	0.10000	0.17000	0.25000	0.34000	0.42000	0.52000
Unconditional Probability (%)		0.00000	0.00000	0.03000	0.07000	0.07000	0.08000	0.09000	0.08000	0.10000
Conditional Probability *p* (%)		0.00000	0.00000	0.03000	0.07002	0.07007	0.08014	0.09023	0.08027	0.10042
Aa										
Cumulative Probability (%)	0.01000	0.02000	0.04000	0.11000	0.18000	0.26000	0.34000	0.42000	0.46000	0.52000
Unconditional Probability (%)		0.01000	0.02000	0.07000	0.07000	0.08000	0.08000	0.08000	0.04000	0.06000
Conditional Probability *p* (%)		0.01000	0.02000	0.07003	0.07008	0.08014	0.08021	0.08027	0.04017	0.06028
A										
Cumulative Probability (%)	0.02000	0.10000	0.22000	0.34000	0.47000	0.61000	0.76000	0.93000	1.11000	1.29000
Unconditional Probability (%)		0.08000	0.12000	0.12000	0.13000	0.14000	0.15000	0.17000	0.18000	0.18000
Conditional Probability *p* (%)		0.08002	0.12012	0.12026	0.13044	0.14066	0.15092	0.17130	0.18169	0.18202
Baa										
Cumulative Probability (%)	0.18000	0.51000	0.93000	1.43000	1.94000	2.45000	2.96000	3.45000	4.02000	4.64000
Unconditional Probability (%)		0.33000	0.42000	0.50000	0.51000	0.51000	0.51000	0.49000	0.57000	0.62000
Conditional Probability *p* (%)		0.33060	0.42215	0.50469	0.51740	0.52009	0.52281	0.50495	0.59037	0.64597
Ba										
Cumulative Probability (%)	1.21000	3.22000	5.57000	7.96000	10.22000	12.24000	14.01000	15.71000	17.39000	19.12000
Unconditional Probability (%)		2.01000	2.35000	2.39000	2.26000	2.02000	1.77000	1.70000	1.68000	1.73000
Conditional Probability *p* (%)		2.03462	2.42819	2.53098	2.45545	2.24994	2.01686	1.97697	1.99312	2.09418
B										
Cumulative Probability (%)	5.24000	11.30000	17.04000	22.05000	26.79000	30.98000	34.77000	37.98000	40.92000	43.34000
Unconditional Probability (%)		6.06000	5.74000	5.01000	4.74000	4.19000	3.79000	3.21000	2.94000	2.42000
Conditional Probability *p* (%)		6.39510	6.47125	6.03905	6.08082	5.72326	5.49116	4.92105	4.74041	4.09614
Caa										
Cumulative Probability (%)	19.48000	30.49000	39.72000	46.90000	52.62000	56.81000	59.94000	63.27000	66.28000	69.18000
Unconditional Probability (%)		11.01000	9.23000	7.18000	5.72000	4.19000	3.13000	3.33000	3.01000	2.90000
Conditional Probability *p* (%)		13.67362	13.27866	11.91108	10.77213	8.84339	7.24705	8.31253	8.19494	8.60024

- For B-rated bonds, their 10-year cumulative probability is 43.34% and their average probability intensity is 5.551%
- For CCC-rated bonds, their 10-year cumulative default rate is 69.18% and their average conditional probability of default is 10.093%.

A comparison of these default rates shows there is a high degree of default risk associated with low-quality bonds. Moreover, our discussion of the historical trends in default rates shows these rate to be high in slow or declining economic periods.[2]

Default and Recovery Rates

Holders of defaulted bonds usually recover a percentage of their investment—the recovery rate. As a result, the default loss rate from an investment is lower than the default rate:

$$\text{Default Loss Rate} = \text{Default Rate } (1 - \text{Recovery Rate})$$

For example, if the recovery rate from a defaulted bond is estimated to be 30% and the bond's default rate is 6%, then the default loss rate would be 4.2%:

$$\text{Default Loss Rate} = 6\% \ (1 - .30) = 4.2\%$$

Focusing on just the default rate highlights the worst possible scenario. Moody's, Fitch, and Standard & Poor's have developed ***recovery rating scale systems*** for secured corporate bonds. Introduced in December 2003, these rating scale systems estimate historical recovery rates based on collateral, subordination, debt in the capital structure, and the expected value of the issue in distress. The recovery rating scale systems of Standard & Poor's and Fitch are shown in Tables 5.4 and 5.5.

Downgrades and Ratings Transition Matrix

Credit risk involves not only the uncertainty related to actual default, but also the concern that the bond's default risk may increase, leading to a downgrade in its quality rating and a decrease in the bond's price. Prior to a downgrade, the market may have already anticipated a company's potential economic and financial

TABLE 5.4 Standard & Poor's Recovery Ratings

1	Highest Expectation of Full Recovery	100% of Principal
2	Substantial Recovery of Principal	80%–100%
3	Meaningful	50%–80%
4	Marginal	25%–50%
5	Negligible	0–25%

TABLE 5.5 Fitch Recovery Ratings

Rank	Prospect	Recovery
R1	Outstanding	91%–100%
R2	Superior	71%–90%
R3	Good	51%–70%
R4	Average	31%–50%
R5	Below Average	11%–30%
R6	Poor Recovery	0–10%

problems by trading the company's bonds at a lower price and wider spread. The general consensus among bond participants is that the market usually anticipates a ratings change. The rating agencies do publish notices of companies whose credit ratings are under scrutiny for a change that could be either an upgrade or downgrade. Usually, when an issuer is placed on a ***credit watch***, it is for a downgrade not an upgrade. Rating agencies also accumulate statistics on how ratings change over time. The information is presented in a ***ratings transition matrix***. The matrix is defined in terms of quality ratings at the beginning of the year (column) and ratings at the end of the year (row). Table 5.6 shows an illustrative ratings transition matrix. As shown in the table, 93% of the AA bonds at the beginning of the year remained that way at the end of the year, 2% of AA bonds at the beginning of the year were upgraded to AAA by the end of the year, and 5% of AA bonds at the beginning of the year were downgraded to A by the end of the year.

Default Risk and Credit Spreads

Because of the default risk on corporate, municipal, and other non-U.S. Treasury bonds, they trade with a ***default risk premium*** (also called a ***quality*** or ***credit spread***). As discussed in Chapter 3, for corporate bonds this premium is often measured as the spread between the rates on the credit and a U.S. Treasury security that is the same in all respects except for its default risk—the benchmark spread. For tax-exempt bonds, the benchmark for calculating spreads is not Treasuries, but instead a generic AAA general obligation bond with a comparable maturity.

TABLE 5.6 Ratings Transition Matrix

	Ratings at the End of the Year							
Ratings at the Start of the Year	AAA	AA	A	BBB	BB	B	CCC-D	Total
AAA	92%	8%	0.5%	0.5%	0	0	0	100%
AA	2%	93%	5%	0.5%	0.5%	0	0	100%
A	0.5%	2%	90%	5%	2%	.5%	0	100%
BBB	0	0.5%	3%	88%	6%	2%	0.5%	100%
BB	0	0	1%	4%	87%	6%	2%	100%

A credit spread on a bond can be thought of as bond investors' expected loss from the principal from default. To see this, consider a portfolio of five-year BBB bonds trading at a 2% credit spread. The 2% premium that investors receive from the bond portfolio represents their compensation for an implied expected loss of 2% per year of the principal from the defaulted bonds. If the spread were 2% and bond investors believed that the expected loss from default on such bonds would be only 1% per year of the principal, then the bond investors would want more BBB bonds, driving the price up and the yield down until the premium reflected a 1% spread. Similarly, if the spread were 2% and bond investors believed the default loss on a portfolio of BBB bonds would be 3% per year, then the demand and price for such bonds would decrease, increasing the yield to reflect a credit spread of 3%. Thus, in an efficient market, the credit spread on bonds represents the market's implied expectation of the expected loss per year from the principal resulting from default.

Over a period of time, probabilities of default and associated spreads will change, increasing or decreasing depending on the quality of the credit. Usually adverse economic conditions result in greater default probabilities and credit spreads. For corporate issues, such developments could be aggregate economic factors such as a recession, industry factors like declining sales due to competition, or firm factors related to a company's investment or financing decisions.[3] For municipal issues, adverse economic developments include declining property values, municipal government deficits, increasing regional unemployment, or increased use of debt reserves. Over the last two decades, the spread between high-yield, non-investment-grade bonds and Treasuries has ranged from 150 basis points (bp) to over 1,000 bp, reflecting economic conditions and default probabilities (see Table 5.7).

More recently, during the economic growth periods from 2002 to 2006, credit spreads were narrow, with many high-quality (AAA or AA) bonds trading only 20 to 30 basis points off the Treasury yields and with many BBB bonds trading only 150 to 200 basis points off. However, during the 2008 financial crisis, spreads widened significantly. Many of the corporate credits experiencing significant widening were financials. Moreover, with the uncertainty regarding the financial bailout policies of the Treasury and the Federal Reserve, many financial credits experienced significant fluctuations in their spreads. For example, in September 2008, just after the collapse of Washington Mutual and before the announcement by the Treasury of its takeover by Wells Fargo, Wachovia Bank, which was deep in subprime mortgage holdings, had its bonds trading with 15% to 25% spreads. Once acquired, the bonds were trading with a 6% to 8% spread as a Wells Fargo Bond.

Studies on Default Risk Premiums

A number of empirical studies have examined the factors that determine default risk premiums. For example, early studies by Salomon Brothers and Hutzler looked at the relationship between default risk premiums and the state of the economy, and a study by R. E. Johnston examined yield curves for different quality bonds.

In both the Salomon Brothers and the Hutzler studies, a moderate widening in the yield spread between moderate (BB) and high-grade bonds was observed during recessions, whereas the spread narrowed during economic growth. These studies suggest that during recessions investors are more concerned with safety than during

TABLE 5.7 High-Yield Spreads

Year	High-Yield YTM (%)	Treasury YTM (%)	Spread
2005	7.98	3.94	4.04
2004	7.35	4.21	3.14
2003	8.00	4.26	3.74
2002	12.38	3.82	8.56
2001	12.31	5.04	7.27
2000	14.56	5.12	9.44
1999	11.41	6.44	4.97
1998	10.04	4.65	5.39
1997	9.20	5.75	3.45
1996	9.58	6.42	3.16
1995	9.76	5.58	4.18
1994	11.50	7.83	3.67
1993	9.08	5.80	3.28
1992	10.44	6.69	3.75
1991	12.56	6.70	5.86
1990	18.57	8.07	10.50
1989	15.17	7.93	7.24
1988	13.70	9.15	4.55
1987	13.89	8.83	5.06
1986	12.67	7.21	5.46
1985	13.50	8.99	4.51
Average	11.60	6.31	5.30
Standard Deviation	2.78	1.70	2.14

Source: Edward Altman, et al., "High Yield Bond and Distressed Debt Default and Returns," NYU, Stern School of Business.

expansionary times. As a result, a relatively low demand for lower grade bonds occurs, leading to lower prices for the lower grade bonds and thus a higher interest premium. On the other hand, during periods of economic expansion there seems to be less concern about default. This tends to increase the demand for lower grade bonds relative to higher grade, causing a smaller premium. It should be noted that these studies suggest that speculators could profit from a strategy of investing in low-grade bonds at the trough of a cycle, when demand and prices are low, and selling at the peak of the cycle, when demand and prices are high. In contrast, speculators would find it more profitable to buy high-grade bonds at the peak of the economic cycle, when their demands are relatively low, and then sell at the trough of the cycle. This, of course, assumes that a speculator can reasonably forecast the troughs and peaks of economic cycles.

In looking at variability in spreads on high-yield bonds and their historical default frequencies, a relevant question is whether or not the returns on high-yield bonds justify their risk. More recent studies by Edward Altman examining the total return (not spreads) over different time periods have found that high-yield bonds on average tend to outperform investment-grade bonds but underperform stock (see Table 5.8).[4]

TABLE 5.8 High-Yield Total Returns

Year	High Yield Total Return (%)	Treasury Total Return (%)	Spread
2004	10.79	4.87	5.92
2003	30.62	1.25	29.37
2002	−1.53	14.66	−16.19
2001	5.44	4.01	1.43
2000	−5.68	14.45	−20.13
1999	1.73	−8.41	10.14
1998	4.04	12.77	−8.73
1997	14.27	11.16	3.11
1996	11.24	0.04	11.20
1995	22.40	23.58	−1.18
1994	−2.55	−8.29	5.74
1993	18.33	12.08	6.25
1992	18.29	6.50	11.79
1991	42.23	17.18	25.05
1990	−8.46	6.88	−15.34
1989	1.98	16.72	−14.74
1988	15.25	6.34	8.91
1987	4.57	−2.67	7.24
1986	18.50	24.04	−5.54
1985	26.08	31.54	−5.46
Average	11.38	9.44	1.94
Standard Deviation	12.96	10.51	13.16

Source: Edward Altman, et al., "High Yield Bond and Distressed Debt Default and Returns," NYU, Stern School of Business.

In the Johnston study, the spread between moderate-grade and high-grade bonds was found to increase as maturity increased, whereas the spread between low-grade and moderate- or high-grade bonds was found decreasing as maturity increased. Johnston's results are illustrated in Figure 5.1, where a hypothetical flat yield curve for an AAA-grade bond is shown along with a positively-sloped yield curve for the A-grade bond (moderate) and a negatively-sloped yield curve for the CCC-grade bond (low). The negatively-sloped yield curve for low-grade bonds suggests that investors have more concern over the repayment of principal (or the issuer's ability to refinance at favorable rates) than they do about the issuer meeting interest payments. This concern would explain the low demand and higher yields for short-term bonds, in which principal payment is due relatively soon, compared to long-term bonds.

Bond Diversification and Quality Ratings

In an interesting study, McEnally and Boardman examined the relationship between portfolio risk and size for bonds grouped in terms of their quality ratings. Using the same methodology employed by Evan and Archer in their portfolio risk and size study on stocks, McEnally and Boardman collected monthly rates of return for over 500 corporate and municipal bonds with quality ratings of Baa or greater. For each

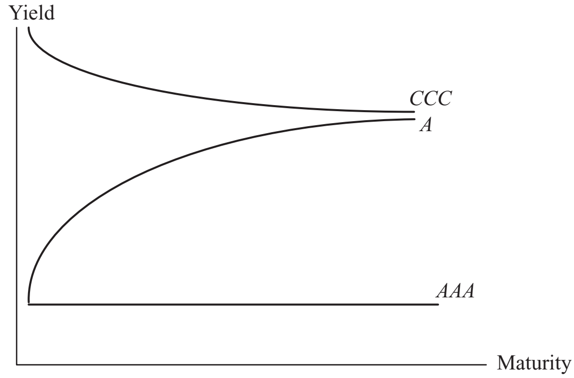

FIGURE 5.1 Yield Curves for Different Quality Bonds

quality group (Aaa, Aa, A, and Baa), they randomly selected portfolios with *n*-bonds (n = 2, 3, 4, ···· 40) and calculated the *n*-bond portfolio's average standard deviation. McEnally and Boardman's findings are displayed in Figure 5.2.

As shown in the figure, McEnally and Boardman found the same portfolio risk and size relationship for bonds as Evans and Archer had found for stocks. Specifically, as the size of the bond portfolio increased, the portfolio risk decreased asymptotically, with the maximum risk reduction being realized with a portfolio size of 20. More interestingly, though, McEnally and Boardman also found that the portfolios consisting of the lowest quality bonds had the lowest portfolio risk when sufficiently diversified.

This seemingly counterintuitive result can be explained in terms of the correlation between bonds in the same quality groups. Specifically, a lower quality bond, although having a greater variance than a higher quality bond, has lower correlations with other lower quality bonds than does a higher quality bond. The relatively lower correlations, in turn, cause the lower quality bond's portfolio variance to be smaller than the higher quality bond's portfolio variance. Intuitively, higher quality bonds

McEnally-Boardman Study:

- The study found that lower-quality bond portfolios had less portfolio risk than higher-quality bonds because of their lower correlation.

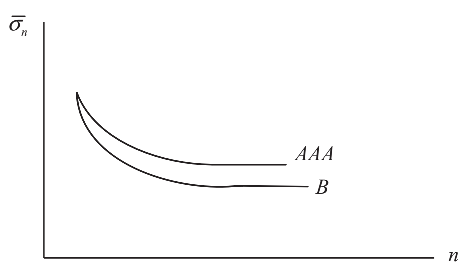

FIGURE 5.2 Bond Diversification

have less default risk. Consequently, such bonds are affected primarily by general interest rate changes and, therefore, tend to fluctuate together. With this high correlation, such bonds do not benefit from diversification. By contrast, lower quality bonds are affected by default risk as well as interest rate changes. They are also more subject to ***event risk*** in which their credit risk changes unexpectedly because of events: natural or industrial accidents, regulatory changes, takeovers, or corporate restructuring. The rates on these bonds are therefore affected by factors unique to the individual bond's company and industry. As a result, lower quality bonds are not as highly correlated with each other, and therefore their portfolio risk can be reduced with sufficient diversification.

A Note on Liquidity Risk and Credit Risk

Liquidity risk is the chance that market trading conditions will change, making the bond harder to trade and, in turn, causing the bond's price to decrease and its yield to increase. Occasionally in recessionary periods when credit is tight, some higher quality bonds can become more difficult to trade if the overall demand for bonds decreases or if investors begin holding such bonds longer. This decrease in liquidity will cause a widening in the bond's spread over Treasury yields. It is important for bond investors to be able to differentiate between bonds whose spreads increase due to credit conditions and those that increase due to liquidity conditions; that is, to differentiate between liquidity risk and credit risk.

5.3 CALL RISK

Call risk relates to the possibility that the issuer will call the bond. A call feature on a bond gives the issuer the right to buy back the bond before maturity at a stated price, known as the call price. The call price is often set a certain percentage above the bond's par value, say 110 ($1,100, given a par value of $1,000); for some bonds the call price may decrease over time (e.g., a 20-year bond's call price decreasing each year by 5%). Some callable bonds can be called at any time, whereas for others the call is deferred for a certain period, giving the investor protection during the deferment period. Also, some bonds, as part of their sinking fund arrangements, are retired over the life of the bond, usually with the issuer having the choice of purchasing the bonds directly at market prices or calling the bonds at a specified call price.

A call provision is advantageous to the issuer. If interest rates in the market decline, an issuer can lower his interest costs by selling a new issue at a lower interest rate, then use the loan proceeds to call the outstanding issue. What is to the advantage of the issuer, though, is to the disadvantage of the investor. When a bond is called, the investor's realized rate of return is affected in two ways. First, since the call price is typically above the bond's face value, the actual rate of return the investor earns for the period from the purchase of the bond to its call generally is greater than the yield on the bond at the time it was purchased. However, if an investor originally bought the bond because its maturity matched her horizon date, then she will be faced with the disadvantage of reinvesting the call and investment proceeds at lower market rates. Moreover, this second effect, known as ***reinvestment risk***, often dominates the

1	2	3 = Call Date
100 →		$100(1.10)^2 = 121$
	100 →	$100(1.10) = 110$
		$100 + 1{,}100 = 1{,}200$
		1,431

$$TR_C = \left[\frac{\text{Call date value}}{P_0^B}\right]^{1/CD} - 1$$

$$TR_C = \left[\frac{\$1{,}431}{\$1{,}000}\right]^{1/3} - 1 = .1269$$

$$TR_{HD} = \left[\frac{HD\ value}{P_0^B}\right]^{1/HD} - 1$$

$$TR_{10} = \left[\frac{\$1{,}431(1.08)^7}{\$1{,}000}\right]^{1/10} - 1 = .0938$$

$$TR_{10} = \sqrt[10]{(1.1269)^3(1.08)^7} - 1 = .0938$$

$TR_{HD} <$ YTM

$9.38\% < 10\%$

FIGURE 5.3 Cash Flow and Total Return for Callable Bond

first effect, resulting in a rate of return over the investor's horizon that is lower than the promised YTM when the bond was bought.

Example of Call Risk

To illustrate the nature of call risk, consider the case of an investor with a 10-year horizon who purchases a 10-year, 10% coupon bond at its par value of $1,000, with coupon payments paid annually and with the bond callable at a call price (CP) of $1,100. In addition, suppose that the yield curve for such bonds is flat at 10% and that it remains that way for the first three years the investor holds the bond. At the end of year 3, however, assume the yield curve shifts down to 8% and the issuer calls the bond. The investor's total return (TR) for the three-year period would be 12.69%. Specifically, at the end of year 3 the investor's cash value would be $1,431. This would include the $1,100 call price, $300 in coupons, and $31 in interest earned from investing the coupons (see Figure 5.3), yielding a TR of 12.69%:

$$TR_3 = \left[\frac{\$1{,}431}{\$1{,}000}\right]^{1/3} - 1 = .1269$$

Thus, for the call period, the investor earns a rate of return greater than the initial YTM. However, with a 10-year horizon, the investor must reinvest the $1,431 cash for seven more years at the lower market rate. If we assume she reinvests all coupons to the horizon date at 8%, then the $1,431 will grow at an 8% annual rate to equal $2,452.48 [= $\$1{,}431(1.08)^7$] at the end of the tenth year, yielding a total return for the 10-year period of 9.386%:

$$TR_{10} = \left[\frac{\$2{,}452.48}{\$1{,}000}\right]^{1/10} - 1 = .09386$$

Note that the 9.386% rate is also equal to the geometric average of the 12.69% annual rate earned for the three years and the 8% annual rate earned for seven years:

$$TR_{10} = \left[(1.1269)^3(1.08)^7\right]^{1/10} - 1 = .09386$$

With the total return of 9.386% less than the initial YTM of 10%, the second effect of reinvesting in a market with lower rates dominates the first effect of a call price greater than the bond's face value. This example shows that bonds that are callable are subject to the uncertainty that the actual rate will be less than the investor's expected YTM. Because of this call risk, there is usually a lower market demand and price for callable bonds than noncallable bonds, resulting in a higher rate of return or interest premium on callable over noncallable bonds. The size of this interest premium, in turn, depends on investors' and borrowers' expectations concerning interest rates. When interest rates are high and expected to fall, bonds are more likely to be called; thus, in a period of high interest rates, a relatively low demand and higher rate on callable over noncallable bonds would occur. In contrast, when interest rates are low and expected to rise, we expect the effect of call provisions on interest rates to be negligible. Moreover, several empirical studies tend to support these observations. For example, Jen and Wert examined interest premiums on immediately callable corporate bonds and deferred callable bonds and found the interest premium increased during high interest rate periods and decreased during low interest rate periods.

Note that the difference in yields between a bond with option features, such as a call option, sinking fund call, or put option, and an otherwise identical option-free bond is defined as the ***option adjusted spread (OAS)***. Estimating a bond's OAS is examined in Chapter 15.

Price Compression

In addition to reinvestment risk, callable bonds are also subject to ***price compression***: limitations on a bond's price. As we discussed in Chapter 2, there is an inverse relationship between interest rates and bond prices. For callable bonds, though, the percentage increases in their prices may be limited when interest rates decrease, given that the market expects the bonds to be redeemed at the call price. This limitation is illustrated in Figure 5.4. In the figure, the price-yield curve *PP* is shown for a noncallable bond. This curve is negatively sloped and convex from below. The curve *PC* represents the price-yield curve for a comparable callable bond. As shown, this

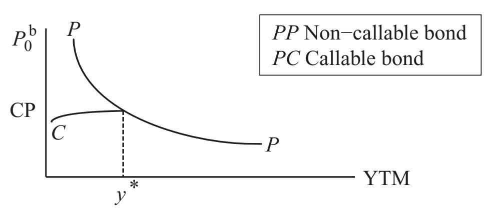

FIGURE 5.4 Price-Yield Curve for Callable Bond

curve flattens out and becomes concave (negative convexity) at rate y^*, where y^* represents a threshold rate that corresponds to a bond price equal, or approximately equal, to the call price. Since the callable bond would likely be called if rates are at y^* or less, we would not expect investors to pay a price for this bond that is greater than the call price. Thus, the price-yield curve for the callable bond would tend to flatten out at y^*, as shown in Figure 5.4.

Valuation of Callable Bond

When valuing a callable bond, one needs to take into account the possibility that interest rates could decrease, leading to the bond being called. If called, the bond's cash flow patterns would be different than if rates increased and the bond was not called. Given the uncertainty of the bond's cash flows, valuing callable bonds and other bonds with embedded option features is more difficult than valuing option-free bonds. One approach to valuing callable bonds is to incorporate interest rate volatility by using a ***binomial interest rate tree***. Another is to determine the value of the call feature. Conceptually, when an investor buys a callable bond, she implicitly sells a call option to the bond issuer, giving the issuer the right to buy the bond from the bondholder at a specified price before maturity. Theoretically, the price of a callable bond should therefore be equal to the price of an identical, but noncallable, bond minus the value of the call feature or call premium. The value of the call feature can be estimated using the option pricing model developed by Black and Scholes. The application of this model and the binomial interest rate model for valuing callable bonds and other option features embedded in bonds are examined in Chapters 14 and 15.

5.4 MARKET RISK

Market risk is the risk that interest rates in the market will change, causing the actual rate of return earned on the bond to differ from the expected return. As noted in our discussion of the total return in Chapter 2, a change in interest rates has two effects on a bond's return. First, interest rate changes affect the price of a bond; this is referred to as ***price risk***. If the investor's horizon date, HD, is different from the bond's maturity date, then the investor will be uncertain about the price he will receive from selling the bond (if HD $< M$), or the price he will have to pay for a new bond (if HD $> M$). Secondly, interest rate changes affect the return the investor expects from reinvesting the cash flows—***reinvestment risk***. Thus, if an investor buys a coupon bond, he automatically is subject to market risk. One obvious way an investor can eliminate market risk is to purchase a zero-coupon bond with a maturity that is equal to the investor's horizon date. However, such a bond may not exist (or if it does, it may not yield an adequate rate). As a result, most bond investors are subject to some degree of market risk.

Market Risk Example 1

To illustrate market risk, consider the case of an investor with a horizon of 3.5 years who buys a 10-year, 10% annual coupon bond at its par value of $1,000 to yield 10%. If the yield curve were initially flat at 10% and if there were no changes in the yield curve in the ensuing years, then the investor would realize a rate of return (as measured by her TR) of 10% (see Figure 5.5). That is, with no change in the flat yield curve, the investor would be able to reinvest each of her coupons at a rate of 10%, yielding a coupon value of $347.16 at year 3.5. The $347.16 coupon value consists of $300 in coupons and $47.16 in interest earned from reinvesting the coupon; that is, ***interest on interest*** of $47.16. In addition, with no change in the flat yield curve, the investor would be able to sell the original 10-year bond (now with a maturity of 6.5 years) for $1,048.81 at the end of 3.5 years.[5] Note, since this bond is being sold at a non-coupon date, its price is determined by discounting the value of the bond at the next coupon date (year 4) when the bond has six years left to maturity (P_4) plus the $100 coupon received on that date back .5 years to the HD. That is:

$$P_4 = \sum_{t=1}^{6} \frac{\$100}{(1.10)^t} + \frac{\$1,000}{(1.10)^{10}} = \$1,000$$

$$P_{3.5} = \frac{\$1,000 + \$100}{(1.10)^{.5}} = \$1048.81$$

Combined, the selling price of $1,048.81 and the coupon value of $347.16 yield an HD value of $1,395.97, which equates to a total return of 10% for the 3.5 years. This is the same rate as the initial YTM:

$$\$1,000 = \frac{\$1,395.97}{(1 + TR)^{3.5}}$$

$$TR = \left[\frac{\$1,395.97}{\$1,000}\right]^{1/3.5} - 1 = .10$$

As we first discussed in Chapter 2, the TR will equal the initial YTM if the yield curve is flat and remains that way to the horizon. Suppose that shortly after the investor purchased the bond, though, the flat 10% yield curve shifted up to 12% and remained there for the 3.5 years. As shown in Figure 5.5, at her HD the investor would be able to sell the bond for only $961.70, resulting in a capital loss of $38.30. This loss would be partly offset, though, by the gains realized from reinvesting the coupons at 12%. Combined, the investor's HD value would be $1,318.81— $77.16 less than the HD value of $1,395.97 realized if rates had remained constant at 10%. As shown in Figure 5.5, the TR would be only 8.23%. In contrast, if the yield curve had shifted down from 10% to 8% and remained there, then the investor would have gained on the sale of the bond (selling it at a price of $1,147.44) but would have earned less interest from reinvesting the coupons. In this case, the HD value increases to $1,484.82 to yield a TR of 11.96% (see Figure 5.5).

In these examples, note that interest rate changes have two opposite effects on the total return. First, there is a direct interest-on-interest effect in which an interest rate increase (decrease) causes the interest earned from reinvesting coupons to be greater (less), augmenting (decreasing) the TR. Second, there is a negative price effect, in which an interest rate increase (decrease) lowers (augments) the price of the bond, causing the TR to decrease (increase). Whether the TR varies directly or

a. If there is no change in the yield curve, then the TR for 3.5 years is 10%

$$\text{HD value} = 100(1.10)^{2.5} + 100(1.10)^{1.5} + 100(1.10)^{.5} + P^B_{3.5} = 1{,}395.97$$

$$\text{where: } P^B_{3.5} = \frac{1{,}000 + 100}{(1.10)^{.5}} = 1{,}048.81$$

$$P^B_4 = \sum_{t=1}^{6} \frac{100}{(1.10)^t} + \frac{1{,}000}{(1.10)^6} = 1{,}000$$

$$TR_{3.5} = \left[\frac{1{,}395.97}{1{,}000}\right]^{1/3.5} - 1 = .10$$

b. If the yield curve shifts to 12%, then the TR for 3.5 years is 8.23%:

$$\text{HD value} = 100(1.12)^{2.5} + 100(1.12)^{1.5} + 100(1.12)^{.5} + P^B_{3.5} = 1{,}318.81$$

$$\text{where: } P^B_{3.5} = \frac{917.77 + 100}{(1.12)^{.5}} = 961.70$$

$$P^B_4 = \sum_{t=1}^{6} \frac{100}{(1.12)^t} + \frac{1{,}000}{(1.12)^6} = 917.77$$

$$TR_{3.5} = \left[\frac{1{,}318.81}{1{,}000}\right]^{1/3.5} - 1 = .0823$$

c. If the yield curve shifts to 8%, then the TR for 3.5 years is 11.96%:

$$\text{HD value} = 100(1.08)^{2.5} + 100(1.08)^{1.5} + 100(1.08)^{.5} + P^B_{3.5} = 1{,}484.82$$

$$\text{where: } P^B_{3.5} = \frac{1{,}092.46 + 100}{(1.08)^{.5}} = 1{,}147.44$$

$$P^B_4 = \sum_{t=1}^{6} \frac{100}{(1.08)^t} + \frac{1{,}000}{(1.08)^6} = 1{,}092.46$$

$$TR_{3.5} = \left[\frac{1{,}484.82}{1{,}000}\right]^{1/3.5} - 1 = .1196$$

FIGURE 5.5 Total Return for 10-Year, 10% Coupon Bond, HD 3.5 Years, Evaluated at Rates of 10%, 12%, and 8%

indirectly with interest rate changes depends on which effect dominates. If the price effect dominates, as in the case described above, then the TR will vary inversely with interest rates. If the interest-on-interest effect dominates, though, the TR will vary directly with the interest rate changes. For example, suppose our investor had purchased a four-year, 20% annual coupon bond when the yield curve was flat at 10% (price of $1,317). As shown in Figure 5.6, if the yield curve shifted up to 12% shortly after the purchase and remained there, then the investor would have realized a TR of 10.16%. In this case, the additional interest earned from reinvesting coupons more than offsets the capital loss. But if the yield curve had shifted down to 8%, the investor would have realized a lower TR of 9.845%. With an HD of 3.5 years, the four-year, 20% bond has an interest-on-interest effect that dominates the price effect, resulting in the direct relationship between the TR and interest rate changes.

a. If the yield curve shifts to 12%, the TR for 3.5 years is 10.16%:

$$\text{HD value} = 200(1.12)^{2.5} + 200(1.12)^{1.5} + 200(1.12)^{.5} + P^{B}_{3.5} = 1{,}848.12$$

$$\text{where: } P^{b}_{3.5} = \frac{1{,}000+200}{(1.12)^{.5}} = 1{,}133.89$$

$$TR_{3.5} = \left[\frac{1{,}848.12}{1{,}317}\right]^{1/3.5} - 1 = .1016$$

$$\text{where: } P^{b}_{0} = \sum_{t=1}^{4}\frac{200}{(1.10)^{t}} + \frac{1{,}000}{(1.10)^{4}} = 1{,}317$$

b. If the yield curve shifts to 8%, the TR for 3.5 years is 9.845%:

$$\text{HD Value} = 200(1.08)^{2.5} + 200(1.08)^{1.5} + 200(1.08)^{.5} + P^{B}_{3.5} = 1{,}829.45$$

$$\text{where: } P^{b}_{3.5} = \frac{1{,}000+200}{(1.08)^{.5}} = 1{,}154.70$$

$$TR_{3.5} = \left[\frac{1{,}829.45}{1{,}317}\right]^{1/3.5} - 1 = .09845$$

$$\text{where: } P^{b}_{0} = \sum_{t=1}^{4}\frac{200}{(1.10)^{t}} + \frac{1{,}000}{(1.10)^{4}} = 1{,}317$$

FIGURE 5.6 Total Return for Four-Year, 20% Coupon Bond, HD 3.5 Years, Evaluated at Rates of 12% and 8%

Finally, it is possible to select a bond in which the two effects exactly offset each other. When this occurs, the TR will not change as rates change, and the investor will not be subject to market risk. For example, suppose our investor had purchased a four-year, 9% annual coupon for \$968.30 to yield 10%. As shown in Figure 5.7, if the flat yield curve shifted to 12%, 8%, or any other rate, the TR would remain at 10%. To reiterate, what is occurring in this case is that we have a bond with price and interest-on-interest effects that are of the same magnitude in absolute value; thus, when rates change the two effects cancel each other out.

Duration and Bond Immunization

The last example illustrates how an investor with an HD = 3.5 years can apparently eliminate market risk by buying a four-year, 9% annual coupon bond. Note, the investor can do this by buying a bond that pays a coupon and has a maturity different than her HD (i.e., it is not a zero coupon bond with a maturity equal to her HD). What is distinctive about this four-year, 9% coupon bond is that it has a duration equal to 3.5 years—the same as the HD. A bond's ***duration (D)*** can be defined as the weighted average of the bond's time periods, with the weights being each time period's relative present value of its cash flow:

$$D = \sum_{t=1}^{N} t\frac{PV(CF_t)}{P^{b}_{0}} \tag{5.1}$$

a. If the yield curve shifts to 12%, the TR for 3.5 years is 10%:

$$\text{HD value} = 90(1.12)^{2.5} + 90(1.12)^{1.5} + 90(1.12)^{.5} + P^{B}_{3.5} = 1{,}351.35$$

$$\text{where: } P^{b}_{3.5} = \frac{1{,}000+90}{(1.12)^{.5}} = 1{,}029.95$$

$$TR_{3.5} = \left[\frac{1{,}351.35}{968.30}\right]^{1/3.5} - 1 = .10$$

$$\text{where: } P^{b}_{0} = \sum_{t=1}^{4} \frac{90}{(1.10)^{t}} + \frac{1{,}000}{(1.10)^{4}} = 968.30$$

b. If the yield curve shifts to 8%, the TR for 3.5 years is 10%:

$$\text{HD value} = 90(1.08)^{2.5} + 90(1.08)^{1.5} + 90(1.08)^{.5} + P^{B}_{3.5} = 1{,}352.49$$

$$\text{where: } P^{b}_{3.5} = \frac{1{,}000+90}{(1.08)^{.5}} = 1{,}048.85$$

$$TR_{3.5} = \left[\frac{1{,}352.49}{968.30}\right]^{1/3.5} - 1 = .10$$

$$\text{where: } P^{b}_{0} = \sum_{t=1}^{4} \frac{90}{(1.10)^{t}} + \frac{1{,}000}{(1.10)^{4}} = 968.30$$

FIGURE 5.7 Total Return for Four-Year, 9% Coupon Bond, HD 3.5 Years, Evaluated at Rates of 12% and 8%

In our example, the duration of a four-year, 9% annual coupon bond is 3.5 years, given a flat yield curve at 10% (see Table 5.9). It should be noted that duration also extends to a portfolio of bonds. The duration of a bond portfolio, D_p, is simply the weighted average of each of the bond's durations (D_i), with the weights being the proportion of investment funds allocated to each bond (w_i):

$$D_p = \sum_{i=1} w_i D_i \tag{5.2}$$

Thus, instead of selecting a specific bond with a desired duration, an investor could determine the allocations (w_i) for each bond in his portfolio that would yield the desired portfolio duration.

In Chapter 13, we will examine bond management strategies; one of them is called ***bond immunization***. The objective of bond immunization is to minimize

TABLE 5.9 Duration of Four-Year, 9% Coupon Bond with Annual Coupon Payments and YTM of 10%

t	CF_t	$CF_t/(1.10)^t$	$PV(CF_t)/P^B$	$t[PV(CF_t)/P^B]$
1	90	81.818	.084496	.084496
2	90	74.380	.076815	.153630
3	90	67.618	.069832	.209496
4	1090	744.485	.768857	3.075428
		$P^B = 968.30$		D = 3.52

market risk. As our discussion here indicates, one way to achieve this goal is to select a bond or portfolio of bonds with a duration matching the investor's horizon date. Duration, and a related characteristic known as convexity, are also important parameters in describing a bond or bond portfolio's volatility in terms of its price sensitivity to interest rate changes.

Market Risk Example 2

Suppose an investor with a horizon of five years bought a five-year, 9% coupon bond paying interest semiannually when the yield curve was flat at 9% (bond-equivalent yield). If the investor could invest the semiannual coupons at a reinvestment rate of 4.5% for the five-year horizon, her horizon value would be \$1552.97, her total dollar return would be \$552.97, and her annualized TR would be 9%:

$$\text{Investment}: P_5 = \sum_{t=1}^{10} \frac{\$45}{(1.045)^t} + \frac{\$1{,}000}{(1.045)^{10}} = \$1{,}000$$

$$\text{Coupon value} = \sum_{t=0}^{10-1} \$45(1.045)^t = \$45\left[\frac{(1.045)^{10}-1}{.045}\right] = \$552.97$$

$$\text{Interest on Interest} = \$552.97 - (\$45)(10) = \$102.97$$

$$\text{Horizon price} = F = \$1{,}000$$

$$\text{HD value} = \$1{,}000 + \$552.97 = \$1{,}552.97$$

$$\text{Total dollar return} = \text{HD value} - \text{Investment} = \$1{,}552.97 - \$1{,}000 = \$552.97$$

$$\text{Semiannual total return} = \left[\frac{\$1{,}552.97}{\$1{,}000}\right]^{1/10} - 1 = .045$$

$$\text{Simple annualized rate} = 2(.045) = .09$$

$$\text{Effective annual rate} = (1.045)^2 - 1 = .092025$$

With the maturity of the bond matching the investor's horizon, the investor would have no price risk, but she would be subject to reinvestment risk. Of the total dollar return of \$552.97, 18.62% comes from the interest-on-interest (\$102.97/\$552.91), 81.38% from coupons (\$450/\$552.97), and zero from capital gains. If the flat yield curve were to shift down to 8% and remain there for the next five years, then the investor's HD value would decrease to \$1,540.27, with interest-on-interest decreasing \$12.70 from \$102.97 to \$90.27, reducing the annualized total return by 1.94% to 8.8285%.

In this case, the bond's maturity matches the investor's horizon, and in the absence of a default or a call, the investor knows she will receive the face value of \$1,000 at her horizon. As a result, the investor's total return depends only on interest-on-interest; that is, market risk consists only of reinvestment risk. Note that the five-year, 9% coupon bond has a duration lower than 5 (D = 3.9546). If the investor wanted to minimize market risk, she would need to find a bond with a duration closer to her horizon of five years. Such a bond would have a maturity

TABLE 5.10 Duration of 6.5-Year, 9% Coupon Bond with Semiannual Payment and YTM of 9%

t	*CF*	*CF/*(1.045)	*PV(CF)/P*	*t[PV(CF)/P]*
1	$45.00	$43.06	0.0431	0.0431
2	$45.00	$41.21	0.0412	0.0824
3	$45.00	$39.43	0.0394	0.1183
4	$45.00	$37.74	0.0377	0.1509
5	$45.00	$36.11	0.0361	0.1806
6	$45.00	$34.56	0.0346	0.2073
7	$45.00	$33.07	0.0331	0.2315
8	$45.00	$31.64	0.0316	0.2531
9	$45.00	$30.28	0.0303	0.2725
10	$45.00	$28.98	0.0290	0.2898
11	$45.00	$27.73	0.0277	0.3050
12	$45.00	$26.53	0.0265	0.3184
13	$1,045.00	$589.66	0.5897	7.6656
		$1,000.00		10.1186

Annualized Duration = 10.1186/2 = 5.0593

greater than five years given it pays coupons. Thus, at the horizon, if interest rates were lower, the resulting lower interest-on-interest would be offset by a higher selling price on the bond. As shown in Table 5.10, a 9% bond with a maturity of 6.5 years (13 semiannual periods) has an annualized duration of 5.0593 (a duration equal to 10.1186 in semiannual periods). At the investor's horizon, this bond would have a maturity of 1.5 years. If the yield curve had shifted down to 8%, the $12.70 decrease in interest-on-interest from $102.97 to $90.27 would have been offset by a $13.88 increase in price to $1,013.88:

$$P_4 = \sum_{t=1}^{3} \frac{\$45}{(1.04)^t} + \frac{\$1{,}000}{(1.04)^3} = \$1{,}013.88$$

The total return from the investment would therefore stay at approximately 9%:

$$\text{Investment}: P_{6.5} = \sum_{t=1}^{13} \frac{\$45}{(1.045)^t} + \frac{\$1{,}000}{(1.045)^{13}} = \$1{,}000$$

$$\text{Coupon value} = \sum_{t=0}^{10-1} \$45(1.04)^t = \$45\left[\frac{(1.04)^{10}-1}{.04}\right] = \$540.27$$

$$\text{Interest on interest} = \$540.27 - (\$45)(10) = \$90.27$$

$$\text{Horizon price} = P_{1.5} = \sum_{t=1}^{3} \frac{\$45}{(1.04)^t} + \frac{\$1{,}000}{(1.04)^3} = \$1{,}013.88$$

$$\text{HD value} = \$1{,}013.88 + \$540.27 = \$1{,}554.15$$

$$\text{Investment}: \sum_{t=1}^{50} \frac{\$45}{(1.045)^t} + \frac{\$1{,}000}{(1.045)^{50}} = \$1{,}000$$

Horizon value :

$$\text{Coupon value} = \sum_{t=0}^{20-1} \$45\ (1.04)^t$$

$$\text{Coupon value} = \$45\left[\frac{(1.04)^{20} - 1}{.04}\right] = \$1{,}340$$

$$\text{Interest on Interest} = \$1{,}340 - (\$45)(20) = \$440$$

$$\text{Horizon price} = \sum_{t=1}^{30} \frac{\$45}{(1.04)^t} + \frac{\$1{,}000}{(1.04)^{30}} = \$1{,}086.46$$

$$\text{HD value} = \$1{,}086.46 + \$1{,}340 = \$2{,}426.46$$

$$\text{Semiannual total return} = \left[\frac{\$2{,}426.46}{\$1{,}000}\right]^{1/20} - 1 = 0453$$

$$\text{Simple annualized rate} = 2(.0453) = .0906$$

$$\text{Effective annual rate} = (1.0453)^2 - 1 = .09265$$

$$\text{Investment}: \sum_{t=1}^{50} \frac{\$45}{(1.045)^t} + \frac{\$1{,}000}{(1.045)^{50}} = \$1{,}000$$

Horizon value :

$$\text{Coupon value} = \sum_{t=0}^{20-1} \$45\ (1.05)^t$$

$$\text{Coupon value} = \$45\left[\frac{(1.05)^{20} - 1}{.05}\right] = \$1{,}487.97$$

$$\text{Interest on Interest} = \$1{,}487.97 - (\$45)(20) = \$587.97$$

$$\text{Horizon price} = \sum_{t=1}^{30} \frac{\$45}{(1.05)^t} + \frac{\$1{,}000}{(1.05)^{30}} = \$923.14$$

$$\text{HD value} = \$1{,}487.97 + \$923.14 = \$2{,}411.11$$

$$\text{Semiannual total return} = \left[\frac{\$2{,}411.11}{\$1{,}000}\right]^{1/20} - 1 = .045$$

$$\text{Simple annualized rate} = 2(.045) = .09$$

$$\text{Effective annual rate} = (1.045)^2 - 1 = .092025$$

FIGURE 5.8 Total Return for 25-Year, 9% Coupon Bond with a Duration of 10, HD 10 Years, Evaluated at Rate of 8% and 10%

$$\text{Semiannual total return} = \left[\frac{\$1{,}554.15}{\$1{,}000}\right]^{1/10} - 1 = .045$$

$$\text{Simple annualized rate} = 2(.045) = .09$$

$$\text{Effective annual rate} = (1.045)^2 - 1 = .092025$$

It should be noted that the greater the maturity and coupon on a bond, the more dependent its total return is on interest-on-interest to realize its YTM. Longer term, higher coupon bonds are therefore subject to more reinvestment risk. For example, if the investor's horizon were 10 years and she had bought a 10-year, 9% coupon bond at par, her total dollar return would be $1,411.71, with 36.25% coming from the interest-on-interest ($511.71/$1,411.71). The 10-year, 9% bond has a duration of 6.7966 when yields are at 9%. To offset the large interest-on-interest effect would require buying a bond with a duration of 10 years. For a 9% coupon, the investor would have to buy a bond with a maturity of 25 years to realize a duration of 10 years. As shown in Figure 5.8, if the yield curve were to shift up or down and remain there for the next 10 years, then the investor would have a bond with an interest-on-interest effect and price effect that offset each other, leaving the investor with a total return of 9%.

5.5 DURATION AND CONVEXITY

Duration Measures

Immunizing a bond against market risk by buying a bond whose duration equals the investor's HD is a relatively new technique in finance. The concept of duration and its applications, though, are not new. Duration was introduced in international economics in the 1800s as a way of reducing exchange-rate risk. Its introduction to finance came later when, in 1938, Frederick Macaulay suggested using the weighted average of a bond's time periods as a better measure of the life of a bond than maturity. J.R. Hicks in 1939, Paul Samuelson in 1945, and F.M. Redington in 1952 also came up with duration measures, each somewhat different, to explain the relationship between price and the life of a bond. In 1971, duration attracted widespread attention when Fisher and Weil published their work on the use of duration as a way of minimizing market risk.

Though duration is defined as the weighted average of a bond's time periods, it is also an important measure of volatility. As a measure of volatility, duration is defined as the percentage change in a bond's price ($\%\Delta P = \Delta P/P_0$) given a small change in yield, dy. Mathematically, duration is obtained by taking the derivative of the equation for the price of a bond with respect to the yield, then dividing by the bond's price and expressing the resulting equation in absolute value (this derivation is presented in Exhibit 5.2).

Doing this yields the following duration measure:

$$\text{Duration} = \frac{dP/P}{dy} = \frac{1}{(1+y)}\left(\sum_{t=1}^{N} t\frac{PV(CF_t)}{P_0^B}\right) \tag{5.3}$$

where dP/P_0 = Percentage change in the bond's price
dy = Small change in yield
N = Number of periods to maturity (M)

The bracketed expression in Equation (5.3) is the weighted average of the time periods, defined in the last section as duration. Formally, the weighted average of the time periods is called ***Macaulay's duration***, and Equation (5.3), which defines the percentage change in the bond's price for a small change in yield in absolute value, is called the ***modified duration***.[6] Thus, the modified duration is equal to Macaulay's duration divided by $1 + y$:

$$\text{Modified duration} = \frac{1}{(1+y)}\left[\text{Macaulay's duration}\right]$$

$$\text{Macaulay's duration} = \left(\sum_{t=1}^{N} t\frac{PV(CF_t)}{P_0^B}\right)$$

The four-year, 9% annual coupon bond (used in the first illustrative example in the last section) has a Macaulay duration of 3.5 years, and given the initial yield of 10%, a modified duration of 3.18:[7]

$$\text{Modified duration} = \frac{1}{(1+y)}\left[\text{Macaulay's duration}\right]$$

$$\text{Modified duration} = \frac{1}{(1.10)}[3.5] = 3.18$$

EXHIBIT 5.2 Derivation of Duration and Convexity

Duration:

$$P_0^B = \sum_{t=1}^{M} \frac{CF_t}{(1+y)^t} = \sum_{t=1}^{M} CF_t(1+y)^{-t}$$

$$P_0^B = CF_1(1+y)^{-1} + CF_2(1+y)^{-2} + \cdots + CF_M(1+y)^{-M}$$

Take the derivative with respect to y

$$\frac{dP^B}{dy} = (-1)CF_1(1+y)^{-2} + (-2)CF_2(1+y)^{-3} + \cdots + (-M)CF_M(1+y)^{-(M+1)}$$

Factor out $-(1+y)^{-1} = -\frac{1}{(1+y)^1}$

$$\frac{dP^B}{dy} = -\frac{1}{(1+y)}\left((1)CF_1(1+y)^{-1} + (2)CF_2(1+y)^{-2} + \cdots + (M)CF_M(1+y)^{-M}\right)$$

$$\frac{dP^B}{dy} = -\frac{1}{(1+y)}\left((1)\frac{CF_1}{(1+y)^1} + (2)\frac{CF_2}{(1+y)^2} + \cdots + (M)\frac{CF_M}{(1+y)^M}\right)$$

$$\frac{dP^B}{dy} = -\frac{1}{(1+y)}\left((1)PV(CF_1) + (2)PV(CF_2) + \cdots + (M)PV(CF_M)\right)$$

Divide through by P:

$$\frac{dP/P}{dy} = -\frac{1}{(1+y)}\left(\sum_{t=1}^{M} t\frac{PV(CF_t)}{P_0^B}\right)$$

Modified duration is *dP/P/dy* expressed in absolute value:

Convexity:

Take the derivative of

$$\frac{dP^B}{dy} = (-1)CF_1(1+y)^{-2} + (-2)CF_2(1+y)^{-3} + \cdots + (-M)CF_M(1+y)^{-(M+1)}$$

$$\frac{d^2P^B}{dy^2} = 2CF_1(1+y)^{-3} + 6CF_2(1+y)^{-4} + \cdots + M(M+1)CF_M(1+y)^{-(M+2)}$$

Divide through by P_0^B :

$$\frac{d^2P^B}{dy^2}\frac{1}{P_0^B} = \frac{d\Delta/P_0^B}{dy} = \text{Convexity} = \frac{1}{P_0^B}\left(\sum_{t=1}^{M} \frac{t(t+1)(CF_t)}{(1+y)^t}\right)$$

Mathematically, *dP/dy* is the slope of the price-yield curve. Thus, the modified duration is also the slope of price-yield curve divided by *dy* expressed in absolute value (see Figure 5.9). Also note that the price of a bond that pays coupons each period and its principal at maturity is

$$P_0^B = C\left[\frac{1-(1/(1+y))^N}{y}\right] + \frac{F}{(1+y)^N}$$

- Modified duration is the slope of price-yield curve (dP/dy) divided by P: $(dP/dy)(1/P) = dP/P/dy = \%\Delta P/dy$

$$\text{Duration} = \left|\frac{dP/P}{dy}\right| = \left|\frac{\%\Delta P}{dy}\right|$$

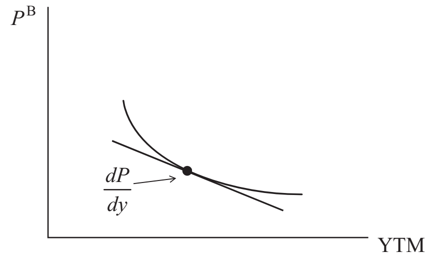

FIGURE 5.9 Duration—Slope of the Price-Yield Curve

Taking the first derivative of this equation, dividing through by P, and expressing the resulting equation in absolute value provides a measure of duration for a bond paying principal at maturity:[8]

$$\text{Modified duration} = \frac{\dfrac{C}{y^2}\left[1 - \dfrac{1}{(1+y)^N}\right] + \dfrac{N[F - (C/y)]}{(1+y)^{N+1}}}{P_0^b} \tag{5.4}$$

The above measures of duration are defined in terms of the length of the period between payments. Thus, if the cash flow is distributed annually, as in the above example, duration reflects years; if cash flow is semiannual, then duration reflects half years. The convention is to express duration as an annual measure. Annualized duration is obtained by dividing duration by the number of payments per year (n):

$$\text{Annualized duration} = \frac{\text{Duration for bond with } n - \text{payments per year}}{n}$$

Thus, the modified duration measured in half-years for a 10-year, 9% coupon bond selling at par ($F = 100$) with coupon payments made semiannually is 13 and its annualized duration is 6.5:

$$\text{Duration in half years} = \frac{\dfrac{4.5}{.045^2}\left[1 - \dfrac{1}{(1.045)^{20}}\right] + \dfrac{20[100 - (4.5/.045)]}{(1.045)^{21}}}{100} = 13$$

$$\text{Annualized duration} = \frac{13}{2} = 6.5$$

Properties and Uses of Duration

Recall in Chapter 2 we described the relationship between a bond's price sensitivity to interest rate changes and its maturity and coupon rate. Since duration is the

TABLE 5.11 Bonds with Different Durations and Convexities

Bond	Annual Coupon Rate	Maturity	Price at 7.5%	Modified Duration	Macaulay's Duration	Convexity
A	0	3 yrs	80.18	2.8916	3.0000	9.7548
B	10.00%	3 yrs	106.61	2.5800	2.6768	8.3619
C	10.00%	5 yrs	110.27	3.9571	4.1055	19.5273
D	5.00%	10 yrs	82.63	7.4675	7.7479	69.0235
E	10.00%	10 yrs	117.37	6.5833	6.8302	57.3956

percentage change in a bond's price to a small change in yield, these relationships also can be defined in terms of duration. Specifically, the greater a bond's maturity, the greater its duration, and therefore the greater its price sensitivity to interest rate changes; the smaller a bond's coupon rate, the greater its duration, and therefore the greater its price sensitivity to interest rate changes. Thus, in addition to identifying bonds for immunization strategies, duration is also an important descriptive parameter, defining a bond's volatility as measured by its price sensitivity to interest rate changes. To summarize, the following properties apply to duration:

1. The lower the coupon rate, the greater the duration.
2. The longer the term to maturity, the greater the duration.
3. For zero-coupon bonds, Macaulay's duration is equal to the bond's term to maturity (N) and modified duration is equal $N/(1+y)$.
4. The higher the yield to maturity, the lower the duration. (That is, the slope of the price-yield curve is flatter at higher yields.)

The durations for five bonds shown in Table 5.11 illustrate these properties.

Knowing a bond or bond portfolio's duration is important in formulating bond strategies. For example, a bond speculator who is anticipating a decrease in interest rates across all maturities (downward parallel shift in the yield curve) could realize a potentially greater expected return, but also a greater risk, by purchasing a bond with a relatively large duration. In contrast, a bond portfolio manager expecting a parallel upward shift in the yield curve could take defensive action against possible capital losses by reallocating his portfolio such that it would have a lower portfolio duration. Such strategies are discussed further in Chapter 13.

A second application of duration is its use as an estimate of the percentage change in a bond's price for a small change in rates. Consider again the 10-year, 9% coupon bond selling at 100 to yield 9%. If the yield were to increase by 10 annual basis points (from 9% to 9.10%), then using Equation (5.3) the bond would decrease by approximately 0.65%:

$$\%\Delta P^b = \frac{dP^b}{P_0^b} = \left[\text{Modified duration}\right] dy$$

$$\%\Delta P^b = \frac{dP^b}{P_0^b} = [6.5]\,[.0910 - .09] = .0065$$

This is very close to the actual percentage change of $-.6476\%$.[9]

A 100 bp decrease in rates, in turn, would lead to an estimated price increase of 6.5%:

$$\%\Delta P^b = [6.5]\,(.10 - .09) = .065$$

Observe that in this case the modified duration equals the estimated $\%\Delta P$ of 6.5%; thus, the modified duration can be interpreted as an approximate percentage change in the price of a bond for a 100 bp change in yield.

It should be noted that duration as an estimator is only good for measuring small changes in yields. For example, if the yield had increased by 200 basis points to 11%, instead of only 10 basis points, the approximate percentage change using the duration measure would be −13%:

$$\%\Delta P^b = [6.5]\,[.11 - .09] = .13$$

This contrasts with the actual percentage change of −11.95%.[10] Thus, the greater the change in yields, the less accurate duration is in estimating the approximate percentage change in price. This is because duration does not take into account the convexity of the price-yield curve. In addition, using duration to estimate discrete changes does not capture the asymmetrical gain and loss relation that characterizes the price-yield curve. For yield increases, duration overestimates the price change; for yield decreases, duration underestimates the price change.

Portfolio Duration

As noted, the duration of a portfolio is the weight average of the durations of the bonds comprising the portfolio:

$$D_p = \sum w_i D_i$$

A bond portfolio formed with the five bonds in Table 5.11 is shown in Table 5.12. The portfolio has a modified duration of 5.1251. If the yields on each of the bonds change by 100 bp, the portfolio value would change by approximately

TABLE 5.12 Portfolio Duration

Bond	Annual Coupon Rate	Maturity	Price	Market Value ($)	Portfolio Weights	Modified Duration	Weighted Duration
A	0	3 yrs	80.18	10,000,000	0.0909	2.6916	0.2629
B	10.00%	3 yrs	106.61	20,000,000	0.1818	2.5800	0.4691
C	10.00%	5 yrs	110.27	30,000,000	0.2727	3.9571	1.0792
D	5.00%	10 yrs	82.63	40,000,000	0.3636	7.4675	2.7156
E	10.00%	10 yrs	117.37	10,000,000	0.0909	6.5833	0.5984
				110,000,000	1.0000		5.1251

5.1251%. Each bond's weighted duration, $w_i D_i$, measures that bond's contribution to the overall portfolio's duration.

Table 5.13 shows the portfolio duration of the Xavier Student Investment Fund (XSIF) as of September 1, 2009.[11] The fund consists of Treasuries, Federal Agencies, and investment-grade corporate credits and is managed against the Barclay's Government Credit Index (an index consisting of approximately 5,000 Treasuries, agencies, and investment-grade corporate bonds). The fund has a duration of 4.97 compared to the index's duration of 5.26. With a portfolio duration less than the index's duration, the fund is positioned to outperform the index if yields across all maturities and sectors increase and underperform if yields increase. The last column in Table 5.13 shows each security's contribution to the overall duration. For example, the Treasury sector contributes 40.58% to fund's duration of 4.97, agencies contribute 8.22%, and the corporate sector contributes 52.27%.

Knowing how a portfolio's duration breaks down in terms of sectors and maturity segments is important to the overall management of a bond portfolio. For example, the size of the Treasury sector is important because Treasury securities are more liquid than agencies and corporate credits and, as a result, they are often used to manage a portfolio's overall duration in response to expected interest rate changes. For non-Treasury sectors, their prices are subject to changes in not only interest rate levels, but also spreads. As a result, managers often divide their portfolios into two durations: (1) a duration for the portfolio, sectors, and issues due to a change in Treasury yields, and (2) a duration for the portfolio, sectors, and issues due to changes in the spread. The latter is referred to as the sector's (or bond's) ***spread duration***. Knowing the spread duration is important in estimating the impact that a widening or a narrowing of spreads in different sectors or for different bonds would have on the overall portfolio's value. Finally, yield curve changes are often characterized by a steepening or flattening instead of a parallel shift. This makes it important to know not only the portfolio's duration, but also the portfolio duration's maturity distribution. For example, Figure 5.10 shows the duration distribution of the XSIF fund as of September 1, 2009. Relative to the index, the XSIF fund has a lower duration allocation to long-term bonds and a greater duration allocation to bonds with two- to three-year maturities.

Convexity

Duration is a measure of the slope of the price-yield curve at a given point (dP/dy). As we noted in Chapter 2, the price-yield curve is not linear, but convex from below (bow-shaped). Convexity means that the slope of the price-yield curve ($\Delta = dP/dy$) gets smaller as you move down the curve or as the YTM increases (see Figure 5.11). This, in turn, implies that for a given absolute change in yields, the percentage increase in price will be greater in absolute value for the yield increase than the percentage decrease in price in absolute value for the yield decrease. For an investor who is long in a bond, its convexity suggests that the capital gain resulting from a decrease in rates will be greater than the capital loss resulting from an increase in rates of the same absolute magnitude. That is, bonds or a bond portfolio with greater convexity have a greater asymmetrical gain-loss relation. Thus, all other things equal, the greater a bond convexity the more valuable the bond.

TABLE 5.13 Bond Investment Fund XSIF: September 2009

1	2	3	4	5	6	7	8	9	10	11
ISSUER	Ratings Moody's	COUPON (%)	MATURITY	PRICE	MKT VAL [USD 000]	% OF PORT	YTM/C	MOD DUR	Duration Contribution	Percentage Contribution to Port. Duration
Cash					CASH				CASH	
CASH & EQUIVALENTS	AAA	0.19	9/17/2009	100	11	0.89	0.19	0.08	0.000712	0.0143%
CASH & EQUIVALENTS	AAA	0.19	9/17/2009	100	12	0.97	0.19	0.08	0.000776	0.0156%
TREASURY					TREASURY				TREASURY	
UNITED STATES TREAS NTS	TSY	4.625	12/31/2011	107.891	22	1.76	1.235	2.25	0.039600	0.7972%
UNITED STATES TREAS NTS	TSY	4.75	1/31/2012	108.438	27	2.20	1.247	2.33	0.051260	1.0319%
UNITED STATES TREAS NTS	TSY	4.75	5/15/2014	110.844	39	3.18	2.322	4.24	0.134832	2.7144%
UNITED STATES TREAS NTS	TSY	2.625	6/30/2014	101.008	71	5.75	2.404	4.54	0.261050	5.2553%
UNITED STATES TREAS NTS	TSY	4.5	11/15/2015	109.383	11	0.90	2.848	5.42	0.048780	0.9820%
UNITED STATES TREAS NTS	TSY	4.5	2/15/2016	109.18	38	3.10	2.937	5.66	0.175460	3.5323%
UNITED STATES TREAS NTS	TSY	4.625	11/15/2016	109.68	39	3.14	3.121	6.13	0.192482	3.8750%
UNITED STATES TREAS BDS	TSY	9.125	5/15/2018	143.063	29	2.36	3.392	6.45	0.152220	3.0644%
UNITED STATES TREAS NTS	TSY	4.75	8/15/2017	110.164	28	2.23	3.294	6.7	0.149410	3.0079%
UNITED STATES TREAS NTS	TSY	4.25	11/15/2017	106.406	43	3.48	3.354	6.9	0.240120	4.8340%
UNITED STATES TREAS BDS	TSY	8.75	8/15/2020	143.609	29	2.33	3.848	7.79	0.181507	3.6540%
UNITED STATES TREAS BDS	TSY	7.25	8/15/2022	132.281	53	4.29	4.034	9.07	0.389103	7.8333%
AGENCY					AGENCY				AGENCY	
FEDERAL NATL MTG ASSN	AGY	7.125	6/15/2010	105.483	27	2.16	0.479	0.81	0.017496	0.3522%
FEDERAL NATL MTG ASSN	AGY	6.625	11/15/2010	107.326	22	1.77	0.701	1.19	0.021063	0.4240%
FEDERAL NATL MTG ASSN	AA2	5.25	8/1/2012	106.816	21	1.73	2.829	2.74	0.047402	0.9543%
FEDERAL HOME LOAN BANKS	AGY	4	9/6/2013	105.814	22	1.74	2.483	3.68	0.064032	1.2891%
FEDERAL NATL MTG ASSN	AGY	5	4/15/2015	110.164	17	1.36	3.032	4.89	0.066504	1.3388%
FEDERAL HOME LN MTG CORP	AGY	6.25	7/15/2032	120.026	18	1.47	4.799	13.02	0.191394	3.8531%

CREDITS					CREDITS				CREDITS	
WACHOVIA CORP GLOBAL MTN	A1	4.375	6/1/2010	102.441	26	2.09	1.254	0.77	0.016093	0.3240%
NORFOLK SOUTHERN CORP	BAA1	6.75	2/15/2011	106.117	21	1.72	2.552	1.43	0.024596	0.4952%
TARGET CORP	A2	5.875	3/1/2012	109.123	22	1.81	2.162	2.32	0.041992	0.8454%
PROLOGIS	BAA2	5.5	4/1/2012	98.336	20	1.63	6.194	2.35	0.038305	0.7711%
CATERPILLAR FINL SVCS MTNS B	A2	4.7	3/15/2012	104.105	53	4.30	3.032	2.38	0.102340	2.0603%
APACHE CORP	A3	6.25	4/15/2012	109.543	45	3.62	2.518	2.42	0.087604	1.7636%
AOL TIME WARNER INC	BAA2	6.875	5/1/2012	109.345	22	1.80	3.237	2.44	0.043920	0.8842%
SBC COMMUNICATIONS INC	A2	5.875	8/15/2012	109.269	44	3.54	2.635	2.76	0.097704	1.9669%
DUKE ENERGY CORP	A3	5.625	11/30/2012	109.287	22	1.79	2.654	2.99	0.053521	1.0775%
UNITED STATES TREAS NTS	TSY	4	11/15/2012	107.531	27	2.20	1.608	3.03	0.066660	1.3420%
GENERAL ELECTRIC CO	AA2	5	2/1/2013	105.648	16	1.29	3.258	3.17	0.040893	0.8232%
DU PONT E I DE NEMOURS & CO	A2	5	7/15/2013	108.109	27	2.20	2.796	3.55	0.078100	1.5723%
NEW YORK LIFE GLOBAL FD 144A	AAA	5.375	9/15/2013	103.982	27	2.15	4.299	3.57	0.076755	1.5452%
US BK NATL ASSN MINN SUB MTN	AA2	6.3	2/4/2014	110.836	22	1.80	3.647	3.91	0.070380	1.4169%
SOUTHERN PWR CO	BAA1	4.875	7/15/2015	102.866	26	2.09	4.32	5.09	0.106381	2.1416%
BANK OF AMERICA CORPORATION	A2	4.75	8/1/2015	98.225	20	1.60	5.099	5.11	0.081760	1.6460%
VERIZON GLOBAL FDG CORP	A3	4.9	9/15/2015	105.915	22	1.75	3.8	5.16	0.090300	1.8179%
CVS CORP	BAA2	5.75	6/1/2017	106.111	16	1.30	4.799	6.22	0.080860	1.6278%
KRAFT FOODS INC	BAA2	6.125	2/1/2018	108.1	33	2.63	4.941	6.62	0.174106	3.5050%
GOLDMAN SACHS GRP INC MTN BE	A1	7.5	2/15/2019	116.522	35	2.83	5.266	6.98	0.197534	3.9767%
JPMORGAN CHASE	AA3	6.3	4/23/2019	108.852	33	2.69	5.128	7.19	0.193411	3.8937%
WAL MART STORES INC	AA2	7.55	2/15/2030	126.44	32	2.56	5.401	11.49	0.294144	5.9216%
CONOCOPHILLIPS	A1	5.9	10/15/2032	103.329	26	2.13	5.64	12.48	0.265824	5.3515%
DOMINION RES INC VA NEW	BAA2	5.95	6/15/2035	102.026	21	1.67	5.797	13.11	0.218937	4.4075%
XSBIF PORTFOLIO			Av. Life		PORTFOLIO				PORTFOLIO	
# of Bonds 44	AA2	5.36%	6.65 yrs.	110.3	1,234	**100%**	**3.24%**	**4.97**	**4.967323**	**100.0000%**
BARCLAY'S					BARCLAY'S				BARCLAY'S	
	AA1	4.50%	7.54 yrs.	NA	NA	**NA**	**3.26%**	**5.26**	**5.26**	

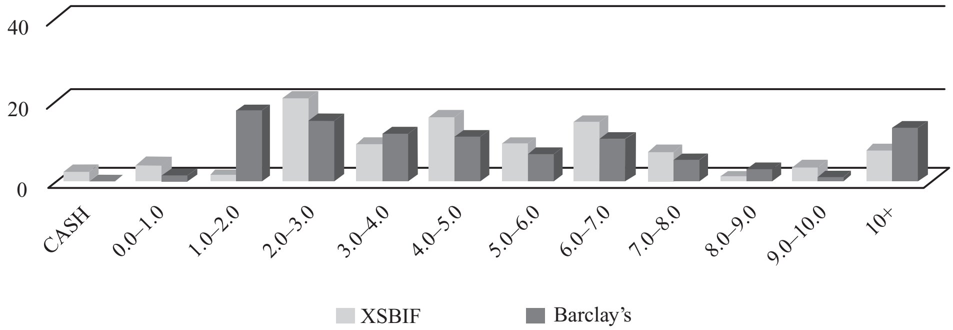

FIGURE 5.10 Duration Distribution by Maturity

Convexity Measures

Mathematically, ***convexity*** is the change in the slope of the price-yield curve for a small change in yield; it is the second-order derivative. It is derived by taking the derivative of Equation (5.3) with respect to a change in yield and dividing the resulting equation by the current price. (This derivation is presented in Exhibit 5.2.)

Doing this yields

$$\text{Convexity} = \frac{1}{P_0^b}\left[\sum_{t=1}^{N}\frac{t(t+1)(CF_t)}{(1+y)^{t+2}}\right] \tag{5.5}$$

The convexity of a bond that pays a fixed coupon each period and the principal at maturity is obtained by taking the derivative of Equation (5.4):

$$\text{Convexity} = \frac{\dfrac{2C}{y^3}\left[1-\dfrac{1}{(1+y)^N}\right]-\dfrac{2CN}{y^2(1+y)^{N+1}}+\dfrac{N(N+1)[F-(C/y)]}{(1+y)^{N+2}}}{P_0^b} \tag{5.6}$$

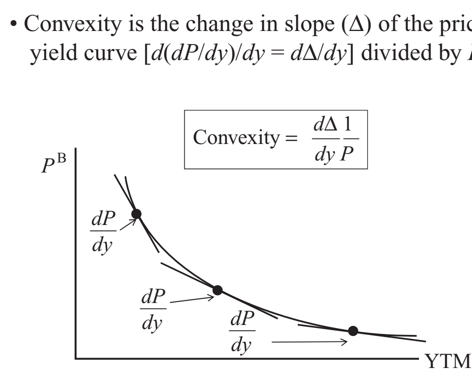

FIGURE 5.11 Convexity

Like duration, convexity reflects the length of periods between cash flows. The annualized convexity is found by dividing convexity, measured in terms of n-payments per year, by n^2:

$$\text{Annualized convexity} = \frac{\text{Convexity for bond with } n - \text{payments per year}}{n^2}$$

Thus, the convexity in half-years for the 10-year, 9% coupon bond with semi-annual payments is 225.43, and its annual convexity is 56.36:

$$\text{Convexity}(1/2\,\text{yrs}) = \frac{\left[\begin{array}{l} \dfrac{2(4.5)}{.045^3}\left[1 - \dfrac{1}{(1.045)^{20}}\right] - \dfrac{2(4.5)(20)}{(.045)^2(1.045)^{21}} \\ + \dfrac{(20)(21)[100 - (4.5/.045)]}{(1.045)^{22}} \end{array}\right]}{100} = 225.43$$

$$\text{Annualized convexity} = \frac{225.43}{2^2} = 56.36$$

Properties and Uses of Convexity

As we discussed earlier, given two bonds that are similar except for their convexity, the one with greater convexity is more valuable since it provides greater capital gains and smaller capital losses for the same absolute changes in yields. This is illustrated in Figure 5.12 where, for the same changes in rates, Bond B with the greater convexity than Bond A has a greater capital gain and smaller capital loss than Bond B.

In addition to describing the asymmetrical gain-loss feature of a bond, convexity also can be used with duration to estimate the percentage change in a bond's price given a change in yield. Unlike duration, which can only provide a good estimate when the yield changes are small, incorporating convexity allows for better estimate of large yield changes. The formula for estimating the percentage change in price for a large change in yields is derived using Taylor expansion. This expansion yields

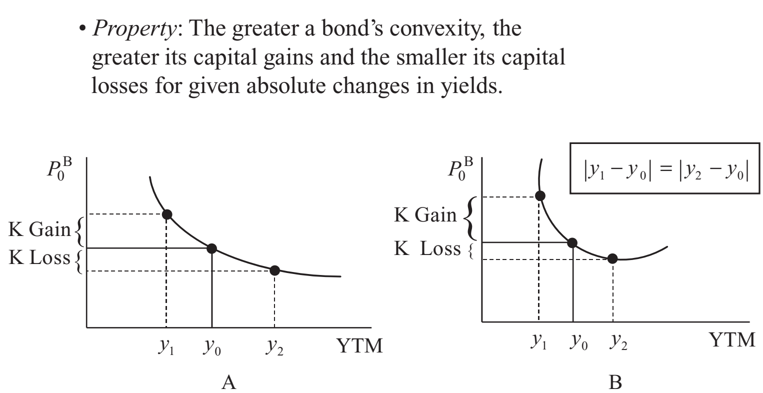

FIGURE 5.12 Convexity—Asymmetrical Gain-Loss Relation

$$\%\Delta P^b = [\text{Modified duration}]\Delta y + \frac{1}{2}[\text{Convexity}](\Delta y)^2$$

The estimated percentage change in the price of our 10-year, 9% coupon bond given a 200 basis point increase in yields is 11.87%:

$$\%\Delta P^b = [-6.5](.02) + \frac{1}{2}[56.36](.02)^2 = -.1187$$

The 11.87% decrease is closer to the actual decrease of 11.95% than the estimated 13% decrease obtained using the duration measure. The above formula also results in non-symmetrical percentage increases and decreases. For example, if rates had decreased by 200 basis points, the percentage increase would be 14.13%, not 13% that the duration measure yields:

$$\%\Delta P^b = [-6.5](-.02) + \frac{1}{2}[56.36](.02)^2 = .1413$$

In general, the following properties apply to convexity:

1. As the yield increases (decreases), the convexity of the bond decreases (increases). This is referred to as positive convexity.
2. For a given yield and maturity, the lower the coupon, the greater the convexity.
3. For a given yield and modified duration, the lower the coupon, the smaller the convexity.

The convexity for the five bonds shown in Table 5.11 illustrates these properties.

The Value of Convexity Consider two similar bonds—X and Y—that are trading at the same yield (y_0) and have the same duration, but with Bond X having a greater convexity than Bond Y (Figure 5.13). In an efficient market, investors would take the greater convexity of X into account, pricing it higher than Y and accepting a

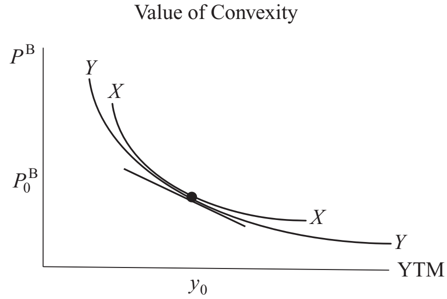

FIGURE 5.13 Convexity in Bond X and Y

lower yield. The question is, how much would the market pay for the convexity? The answer depends on how much the market expects rates to change. If investors expect yields to change very little—market with low interest rate volatility—then the advantage of X over Y is small. Thus, in this market, investors would pay little for convexity. On the other hand, if investors expect yields to change significantly—a market with high interest rate volatility—then the advantage of X over Y is more important. Thus, in this market, investors would pay more for convexity.

Given that longer term bonds have greater duration and therefore volatility, a priori, one would expect the yields on longer term bonds to be affected more by convexity than shorter term bonds. The influence of convexity on the shape of the yield curve and the relation between returns and duration is referred to as the ***convexity bias***. In studies by Ilmanwen, the relationship between total returns and durations were examined for Treasury securities.[12] Ilmanenm, in turn, found that the greater the duration, the greater the average return and the greater the liquidity premium. However, his studies also found that the bond risk premium increases up to a duration of three and then increases as a slower rate. Given that longer maturity bonds have greater convexity and therefore a greater asymmetrical gain/loss relation, Ilmanenm's findings suggest that convexity could contribute to the liquidity premium increasing at a slower rate as maturity and duration increase.

Alternative Formulas for Duration and Convexity

Duration and convexity can also be estimated by determining the price of the bond when the yield increases by a small number of basis points (e.g., 2 to 10 basis points), P_+, and when the yield decreases by the same number of basis points, P_-. These measures are referred to as ***approximate duration*** and ***approximate convexity*** and can be estimated using the following formulas:

$$\text{Approximate duration} = \frac{P_- - P_+}{2(P_0)(\Delta y)} \tag{5.7}$$

$$\text{Approximate convexity} = \frac{P_+ + P_- - 2P_0}{P_0(\Delta y)^2} \tag{5.8}$$

Caveats

Duration and convexity are important characteristics that can be used to determine a bond's price sensitivity to interest rates and the asymmetry of its capital gains to capital losses for given absolute changes in yields. However, there are two problems with the measures we have defined here for determining a bond's duration and convexity. First, in deriving duration and convexity, we assumed the cash flows were discounted at the same rate. This implies that we are assuming the yield curve is flat and all shifts in the curve are parallel. These assumptions create problems when we use the modified portfolio duration to estimate the impact of a change in interest rates on the portfolio value. That is, if the portfolio has bonds with different maturities, then the duration measure will not give good estimates if there is a nonparallel shift in the yield curve in which there are unequal changes in the yields on different maturities. There are several techniques that can be used to estimate a bond or bond portfolio's value to changes in rates resulting from a nonparallel shift in the yield curve. The most popular technique is called ***key rate determination***.[13]

A second and more serious problem with our measures is that they apply only to option-free bonds. Call, put, and sinking fund arrangements alter a bond's cash flow patterns and can dramatically change a bond's duration and convexity. Moreover, since many bonds have option features, adjusting their cash flow patterns to account for such features is important in measuring a bond's duration and convexity. In Chapters 14 and 15, we will show how bonds with embedded option features can be valued using a binomial interest rate tree. With this valuation model, one can estimate the prices of bonds with call and put option features, then substitute these prices into Equations (5.7) and (5.8) to estimate the duration and convexity of bonds with embedded options. The duration and convexity measures using this approach are referred to as ***effective duration*** and ***effective convexity***.

5.6 CONCLUSION

In a world of certainty, bonds with similar features would, in equilibrium, trade at the same rates. If this were not the case, then investors would try to buy bonds with higher rates and sell or short bonds with lower rates, causing their prices and rates to change until they were all equal. We, of course, live in a world of uncertainty. Issuers can default on their obligations, borrowers can redeem their bonds early, and the markets can change. This is why we have differences in the relative demands, prices, and yields on bonds. Thus, an important factor explaining different rates among debt instruments is uncertainty. In this chapter, we have examined the nature and impact of uncertainty by examining default, call, and market risk, and by introducing duration and convexity as volatility measures.

KEY TERMS

approximate convexity
approximate duration
binomial interest rate tree
bond immunization
call risk
conditional probability rates
convexity
convexity bias
credit spreads
credit watch
cumulative default rates
cumulative probability
default risk or credit risk
default risk premium
duration
effective convexity
effective duration
event risk
Financial Institutions Reform, Recovery, and Enforcement Act
interest on interest
investment-grade bonds
key rate determination
liquidity risk
Macaulay's duration
market risk
modified duration
non-investment-grade bond
option-adjusted spread (OAS)
price compression

price risk
probability intensity
ratings transitions matrix
recovery rating scale systems
reinvestment risk
speculative-grade or junk bonds
spread duration
unconditional probability rates

WEB INFORMATION

1. Rating Agencies
   www.moodys.com
   www.standardandpoors.com
2. Moody's
   Go to www.moodys.com (registration required).
   To download Moody Study of Historical Default Rates:
   Click "Ratings, Methodologies, and Performance"
   Historical Performance
   Corporate Default and Recovery Rates (Archive)
   Corporate Bond Ratings Transition Matrix
   Municipal Defaults
   Sovereign Defaults
   To evaluate a credit's credit rating enter name on "Quick Search" and click "go."
   To find credits on Moody's Watch List, click "WATCHLIST."
3. FINRA
   Go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds."
   For a bond search click "Corporate" tab and then click "Advanced Bond Search" to find corporate bonds with certain features.
   For bonds of a specific issuer, enter issuer's symbol and then click "Search."
4. Wall Street Journal
   Go to http://online.wsj.com/public/us.
5. Yahoo.com
   Go to http://finance.yahoo.com/bonds, click "Advanced Bond Screener" and click "Corporate" tab, and then provide information for search.
   Go to http://finance.yahoo.com/bonds, enter name of issuer and click "Search."
6. Investinginbonds.com
   Go to http://investinginbonds.com/.
7. Online Finance Calculator
   www.ficalc.com/calc.tips
   Use to calculate duration and convexity.
8. Federal Reserve
   Historical interest rate data on different bonds can be found at the Federal Reserve site: www.federalreserve.gov/releases/h15/data.htm and www.research.stlouisfed.org/fred2.

PROBLEMS AND QUESTIONS

Note: Many of these problems can be done in Excel by either writing a program or using one of the Excel programs; many also can be done using a financial calculator. For problems requiring a number of calculations, the reader may want to use Excel or a financial calculator.

1. Explain why default risk premiums widen during recessionary periods and narrow during growth periods.
2. Explain why the yield curve for lower quality bonds could be negatively sloped when the yield curves for other bonds are not.
3. Explain the McEnally-Boardman study. What were the findings of the study? How do you explain their findings?
4. The table below shows the historical cumulative probabilities for corporate bonds with quality ratings of AA and B:

Cumulative Probabilities (%)

Year	1	2	3	4	5
AA	0.10	0.40	1.10	1.50	1.75
B	6.00	13.00	20.00	28.00	36.00

   a. Determine the unconditional and conditional default probabilities from the cumulative probabilities shown in the table.
   b. What is the relation between conditional probabilities and credit spreads?

5. What impact does the exercising of a call option by an issuer generally have on the investor's rate of return earned for the period from the purchase of the bond to its call and the rate for the investor's horizon period?
6. Explain the relationship between call risk premiums and the level of interest rates in the economy.
7. Explain how interest rate changes affect a bond's return.
8. The AIF Company issued a 10-year bond at par ($F = \$1,000$) that pays a coupon of 11% on an annual basis and is callable at $1,100. Suppose you bought the bond when it was issued, and at the time of your purchase suppose the yield curve was flat and continued to remain at that level during years 1 through 4. However, at the start of year 5 (or end of year 4), suppose the yield curve dropped to 8% and AIF called the bond. Assume you reinvest your investment funds in a new six-year bond at par and the yield curve remains flat at 8%. What is your total return for the call period? What is your total return for the 10-year period? How do the total returns compare to the YTM when you purchased the bond?

9. Suppose you have a horizon of 10 years and bought a 10-year, 8% coupon bond at par (F = \$1,000) that pays coupons semiannually and is callable at a call price of \$1,100. Assume the yield curve is flat at an 8.16% effective yield (or 4% semiannual yield) and remains constant for three years. At the end of year 3, suppose the yield curve drops to an effective yield of 5.0625% (2.5% semiannual) and the issuer calls the bond. Assume you reinvest your investment funds in a new seven-year bond paying coupons semiannually and the yield curve remains flat at 5.0625%. What is your semiannual rate of return and effective annual rate of return for the call period? What is your semiannual and effective annual rate for the 10-year period? How do your rates compare to the YTM when you purchased the bond?

10. The yield curve for AA-rated bonds is presently flat at a promised YTM of 9%. You buy a 10-year, 8% coupon bond with face value of \$1,000 and annual coupon payments. Suppose your horizon is at the end of four years. What would your total return be given the following cases:

a. Immediately after you buy the bond the yield curve drops to 8% and remains there until you sell the bond at your horizon date.
b. Immediately after you buy the bond the yield curve increases to 10% and remains there until you sell the bond at your horizon date.
c. What type of risk is your investment subject to? How could the risk be minimized?

11. Suppose you have a horizon at the end of six years and buy an eight-year, 8.5% coupon bond with face value of \$1,000 and annual coupon payments when the applicable yield curve is flat at 10%. What would your total return be given the following cases:

a. Immediately after you buy the bond the yield curve drops to 8% and remains there until you sell the bond at your horizon date.
b. Immediately after you buy the bond the yield curve increases to 12% and remains there until you sell the bond at your horizon date.
c. Is there any market risk? If not, why?

12. Calculate both Macaulay and modified durations of the eight-year, 8.5% coupon bond given a flat yield curve at 10% in Question 11. Given your answers in Question 11, comment on duration, horizon date, and bond immunization.

13. Assume the following yield curve for zero-coupon bonds:

Maturity	YTM
1 year	5%
2 years	6%
3 years	7%
4 years	8%
5 years	9%

a. What is the Macaulay duration of each of the bonds?
b. Assume your HD is three years and you want to buy bonds with one-year and four-year maturities. What percentage investment should be made in each to assure a fully immunized portfolio?

14. Calculate Macaulay's duration, the modified duration, and the convexity of the following bonds (annualize the parameters). Assume all of the bonds pay principal at their maturity.

a. Four-year, 9% coupon bond with a principal of $1,000 and annual coupon payments trading at par.
b. Four-year zero-coupon bond with a principal of $1,000 and priced at $708.42 to yield 9%.
c. Five-year, 9% coupon bond with a principal of $1,000 and annual coupon payments trading at par.
d. 10-year, 7% coupon bond with a principal of $1,000 and semiannual coupon payments (3.5%) and priced at par.
e. Three-year, 7% coupon bond with a principal of $1,000 and semiannual coupon payments (3.5%) and priced at par.
f. Three-year zero-coupon bond with a principal of $1,000 and priced at $816.30 to yield 7%.

15. Given your duration and convexity calculations in Question 14, answer the following:

a. Which bond has the greatest price sensitivity to interest rate changes?
b. For an annualized 1% decrease in rates, what would be the approximate percentage change in the prices of Bond d and Bond e?
c. Which bond has the greatest asymmetrical capital gain and capital loss feature?
d. If you were a speculator and expected yields to decrease in the near future by the same amount across all maturities (a parallel downward shift in the yield curve), which bond would you select?
e. If you were a bond portfolio manager and expected yields to increase in the near future by the same amount across all maturities (a parallel upward shift in the yield curve), which bond would you select?
f. Comment on the relation between maturity and a bond's price sensitivity to interest rate changes.
g. Comment on the relation between coupon rate and a bond's price sensitivity to interest rate changes.

16. Short-Answer Questions:

1. Define market risk.
2. Define call risk.
3. Define default risk.
4. Define liquidity risk.
5. Explain the difference between the default rate, recovery rate, and default loss rate.
6. Define a rating transitions matrix.
7. How does price compression apply to callable bonds?

8. When would there be a direct relation between the total return and interest rate changes?
9. When would there be an inverse relation between the total return and interest rate changes?
10. When would the total return be invariant to interest rate change?

WEB EXERCISES

1. Use Yahoo.com to compare the yields on bonds with different quality ratings.
   - Go to http://finance.yahoo.com/bonds.
   - Use Yahoo's Advanced Bond Screener to search for corporate bonds with different quality ratings.
2. Use the FINRA site to compare the yields on bonds with different quality ratings.
   - Go to www.finra.org/index.htm, "Sitemap," "Market Data," "Bonds."
   - Use Quick Bond Search to find bonds with different quality ratings.
   - Select several corporate or Treasury bonds with different maturities using Yahoo.com or FINRA: http://finance.yahoo.com/bonds or www.finra.org/index.htm.
   - Using the online finance calculator (www.ficalc.com/calc.tips), calculate the duration and convexity for each bond and generate its price-yield curve. Comment on the relation between duration and the price-yield curve and convexity and the price-yield curve in terms of the bonds you selected.
3. Select a bond from http://finance.yahoo.com/bonds or www.finra.org/index.htm and then go to Moody's to study its credit history and profile:
   - Go to www.moodys.com (registration required).
   - Enter name on "Quick Search" and hit "Go."
   - Click "WATCHLIST," enter name on "Quick Search" and hit "Go."
   - Study some of the recent companies, municipalities, and sovereigns added to Moody's "Watch List."

   Moody's also provides information on default rates, ratings changes, and other credit information. Examine some of their information.
   - Go to www.moodys.com.
   - Click "Ratings, Methodologies, and Performance."
   - "Historical Performance."
   - Examine some of the following:
     - Corporate Default and Recovery Rates (Archive)
     - Corporate Bond Ratings Transition Matrix
     - Municipal Defaults
     - Sovereign Defaults
4. Go to the FINRA site to find information on callable corporate bonds (go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds"). Select one or more of the bonds from the search and compute using the finance FICALC calculator (www.ficalc.com/calc.tips) their YTM given price and their yield-to-worse given their call price and call schedule.

5. Examine the yield spreads of bonds with different quality ratings.
   - Go to http://investinginbonds.com/.
   - Click "Corporate Price Date."
   - Click "Corporate Market Data-at-a-Glance."
   - Investigate the graphs showing yield spreads for corporate, investment-grade and speculative-grade bonds.
6. In this chapter we discussed how credit spreads changed as a result of economic conditions.
   - Go to Fred: www.research.stlouisfed.org/fred2.
   - In "Category," click "Interest Rates."
   - Select series (e.g., Moody's Corporate AAA and BBB Yields).
   - Examine the level and difference in the yields during the recessionary periods.
7. Many bonds were issued in the 1980s to finance corporate takeovers. Examine that period by going to www.encyclopedia.com and searching for "Junk Bonds" and "Michael Milken."

NOTES

1. See Moody's study of historical default rates. To download, go to http://v2.moodys.com/cust/content/Content.ashx?source=StaticContent/Free Pages/Products and Services/Downloadable Files/August Default Report.pdf
2. For studies on cumulative defaults see Asquith, Mullins, and Wolff, *Journal of Finance* (September 1989); Altman, *Journal of Applied Corporate Finance* (Summer 1990); and Douglass and Lucas, Moody's Investor's Service (July 1989).
3. In a widely cited study by Lawrence Fisher, yield premiums for corporate securities were found to be directly related to a company's volatility in earnings and inversely related to the company's equity-to-debt ratio, number of outstanding bonds, and how long the company had been solvent.
4. See Marshall Blume, D. B. Keim, and S. A. Patel, *Journal of Finance*, March 1991; Bradford Cornell and K. Green, *Journal of Finance*, March 1991; and Rayner Cheung, J. C. Bencivenga, and F. Fabozzi, *Journal of Fixed Income*, September 1992.
5. Note: For this example, we are assuming annual coupon payments.
6. The proportional change in the bond price to a change in yield ($dP/P/dy$) is negative (inverse relationship between price and yield). The convention, though, is to express modified duration in absolute value.
7. The modified duration is the most commonly used measure of duration. Some applications of duration use the *dollar duration*. The dollar duration is the change in the bond price given a small change in yield (dP/dy). The dollar duration is obtained by multiplying both sides of Equation (5.3) by P_0: Dollar duration = (Modified duration) P_0.
8. For a zero-coupon bond, Macaulay's duration would be equal to the bond's maturity, whereas the modified duration would be less than the maturity.
9. The actual percentage change is

$$\%\Delta P^b = \frac{\$99.3524 - \$100}{\$100} = -.006476$$

$$\text{where}: P = \sum_{t=1}^{20} \frac{\$4.50}{(1.0455)^t} + \frac{\$100}{(1.0455)^{20}} = \$99.3524$$

10. The actual percentage change is

$$\%\Delta P^b = \frac{\$88.0496 - \$100}{\$100} = -.1195$$

$$\text{where}: P = \sum_{t=1}^{20} \frac{\$4.50}{(1.055)^t} + \frac{\$100}{(1.055)^{20}} = \$88.0496$$

11. The Xavier Student Investment Fund is a student-managed fund consisting of an equity fund and a bond fund. The bond fund manages approximately $1m of Xavier University's endowment as a fixed-income fund. The fund is managed against the Barclay's Government Credit Index.
12. Antti Ilmanen, "Convexity Bias in the Yield Curve," in *Advanced Fixed-Income Valuation Tools*, Chapter 3 (New York: Wiley, 2000); Antti Ilmanen, "Does Duration Extension Enhance Long-Term Expected Return?" *Journal of Fixed Income,* September 1996, pp. 23–36.
13. See Donald Chamber and Willard Carleston, "A Generalized Approach to Duration," *Research in Finance* 7 (1988); Robert Reitana, "Non-Parallel Yield Curve Shifts and Duration Leverages," *Journal of Portfolio Management*, Summer 1990, pp. 62–67; and Thomas S. Y. Ho, "Key Rate Determination: Measure of Interest Rate Risk," *Journal of Fixed Income*, September 1992, pp. 29–44.

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PART Two

Debt Markets

Bond Evaluation, Selection, and Management, Second Edition
by R. Stafford Johnson

CHAPTER 6

Corporate Debt Securities

6.1 INTRODUCTION

Today's corporations can be viewed as perpetual investment machines: constantly developing new products and technologies, regularly expanding their markets, and from time to time acquiring other companies. To finance these investments, corporations obtain funds both internally and externally. With ***internal financing***, companies retain part of their earnings that otherwise would go to existing shareholders in the form of dividends, whereas with ***external financing***, companies generate funds from outside by selling new shares of stock, selling debt instruments, or borrowing from financial institutions. From the corporation's perspective, decisions on internal versus external financing depend on the dividend policy it wants to maintain and the cost of raising funds from the outside. The company's choice of financing with debt or equity, in turn, depends on the return-risk opportunities management wants to provide its shareholders. Since debt instruments have provisions that give creditors legal protection in the case of default, the rate corporations are required to pay creditors for their investments is typically smaller than the rate their shareholders require. As a result, a firm that tends to finance its projects with relatively more debt than equity (i.e., a ***leveraged firm***) benefits its shareholders with the relatively lower rates it pays to creditors. In addition, debt financing also has a major tax advantage to corporations: the interest payments on debt are treated as an expense by the Internal Revenue Service (IRS) and are therefore tax deductible, whereas the dividends a corporation pays its shareholders are not tax deductible. The relatively lower rates required by creditors and the tax advantage of debt make debt financing cheaper than equity financing for a corporation, all other things being equal. The lower rates on debt, though, are not without costs. Unlike equity financing in which funds are paid to shareholders only if they are earned, the obligations of debt instruments are required to be made. Thus, if a company has a period with poor sales or unexpected high costs, it still has to make payments to the bondholders, leaving fewer earnings available for shareholders. Moreover, very low sales or very high costs could lead to the company being unable to meet its interest and/or principal payments. In this case, the creditors can sue the company, forcing it to sell company assets to meet its obligations, or the company can petition the courts to reorganize.

When a corporation decides to finance its investments with debt, it will do so either by selling corporate bonds or notes or by securing a loan from a financial institution.[1] The larger corporations, whose credit standings are often strong, prefer to finance their long-term and intermediate-term assets by selling corporate bonds and notes or issuing, though an agent, medium-term notes, and a number of corporations finance their short-term assets by selling commercial paper. These securities, in turn, offer different investment features to investors. In this chapter, we examine these features and the markets for corporate bonds, medium-term notes, and commercial paper.

6.2 CORPORATE BONDS

A corporate bond is a debt obligation with an original maturity of over five years, whereas a corporate note is an obligation with an original maturity of less than five years. Since bonds and notes are similar, we will follow the custom of referring to both as corporate bonds.

Corporate bondholders have a legal claim over common and preferred shareholders as to the income and assets of the corporation. Their contractual claim is specified in the bond's ***indenture***. An indenture is the contract between the borrower and the lender (all the bondholders). The document is very extensive, detailing all the characteristics of the bond issue, including the time, amounts, manners in which interest and principal are to be paid, the type of collateral, and all restrictive covenants or clauses aimed at protecting the bondholders. In addition to the indenture, a corporation issuing a bond must also file a ***prospectus*** with the Securities and Exchange Commission. This smaller document is a summary of the main provisions included in the indenture.

By federal law, all corporations offering bonds in excess of $5 million and sold interstate must have a ***trustee***. A trustee is a third party, often a commercial or investment bank or the trust department of a bank, who is selected to represent the bondholders. The trustee has three major responsibilities: (1) bond certification, which entails ensuring that the bond issue has been drawn up in accordance with all legal requirements; (2) overseeing the issue, which requires ensuring the issuer is meeting all of the prescribed functions specified in the indenture; and (3) taking legal action against the corporation if it fails to meet its interest and principal payments or satisfy other terms specified in the indenture.

General Features of Corporate Bonds

The characteristics of many security issues often are determined by the underlying real assets they are financing. For example, to finance the construction of a $1 billion nuclear generating plant with an estimated economic life of 40 years, a utility company might sell one million corporate bonds priced at $1,000 par, with each bond promising to pay $100 each year for 40 years plus a principal of $1,000 at maturity. Given the wide variety of assets and bonds financing them, the differences in corporate bonds can best be explained by examining their general characteristics. These include how they pay interest (fixed, zero, or floating coupon rates) and principal, and their maturities, call features, protective covenants, and collateral.

Coupon Bonds In the United States, many corporate bonds are coupon bonds paying interest semiannually. For U.S. bonds, the coupon interest is typically based on a 360-day year and 30-day month (30/360 day-count convention). Thus, a $1,000 par value bond with a 9% coupon would pay $90 per year and its interest would accrue at a rate of $7.50 per month ($90/12) and $0.25 per day ($90/360).[2] Technically, a coupon bond is one that has a series of coupons attached to the bond certificate, which the holder cuts out at specified times and sends to a designated party (e.g., trustee) for collection. At one time, most bonds were sold with attached coupons. Such bonds were called ***bearer bonds*** since their coupon payment was made to whoever had physical possession of the bond. Bearer bonds have been replaced by ***registered bonds***. The interest on registered bonds is paid by the issuer or a third party (usually the trustee) to all bondholders who are registered with the issuer or the trustee. If the bond is sold, the issuer or trustee must cancel the name of the old holder and register the new one. In addition, issuers of U.S. bonds are required by law to report to the IRS all bondholders receiving interest. A variation of a registered bond is a bond sold in ***book-entry form***. Such bonds have one master certificate with all bondholder names. A depository holds the certificate and issues ownership receipts to each bondholder.

Zero-Coupon Bonds As noted in previous chapters, bonds that pay no coupon interest are referred to as zero-discount bonds or as pure discount bonds. In discussing zero-discount bonds or zeros, the difference between the bond's face value and the offering price when the bond is issued is called the ***original-issue discount (OID)***. Zero-coupon bonds were first issued in the U.S. corporate market during the high interest-rate period of the early 1980s. In 1982, for example, Beatrice Foods sold a 10-year, $250 million zero-coupon bond priced at $255 per $1,000 face value. In addition to zeros, many firms also issue ***deep-discount bonds*** that pay low coupon interest and sell at a price below par. With little or no reinvestment risk, some fund managers find zero-coupon and deep-discount bonds attractive investments for matching their future liabilities. In addition, many of these bonds also have call prices equal to their par values, making it unlikely they would be called. Thus, the bonds are characterized by having low market and call risk.

Floating Rate Notes During the high and often volatile interest-rate period of the late 1970s and early 1980s, a number of companies began selling floating rate notes (FRNs). Similar to variable rate loans offered by financial institutions, FRNs pay a coupon rate that can vary in relation to another bond, benchmark rate, or formula. Floating-rate securities originated in Europe and were introduced in the United States in 1974 when Citigroup issued a $650 million floating-rate note. Citigroup's note was reset semiannually to be one percent above the rate on a three-month Treasury-bill rate. Subsequently, Standard Oil, Georgia Pacific, and other corporations issued FRNs. By 1990, there were approximately 500 floating-rate offerings, with two-thirds being offered by banks and financial service companies. Today, floating-rate securities are issued primarily by financial institutions.

The term floating-rate note or ***floater*** is often used to define any bond with an interest rate that is adjusted periodically. Technically, though, an FRN is defined as a debt instrument with the coupon based on a short-term index (e.g., Treasury-bill rate) and reset more than once a year, whereas an ***adjustable-rate note*** or ***variable-rate note*** is defined as a debt security with its coupon based on a longer-term rate.

The rate on a floater is reset based on a reference rate plus a quoted margin or spread:

$$\text{Reset rate} = \text{Reference rate} + \text{Margin}$$

The reference rate is based on different benchmarks—Treasury rates, commercial paper rate, ***prime rate***, and ***London interbank offer rate***, ***LIBOR*** (this is the average bank rate paid by London Eurocurrency bank; discussed in Chapter 10). The quoted margin is the additional rate the issuer agrees to pay above the reference rate (typically related to credit risk). Floaters might also have restrictions placed on the coupon rate. For example, a floater could have a cap (maximum coupon rate) or a floor (minimum rate).

FRN issues are sometimes sold with sweeteners, such as convertibility to stock, a put option giving the holder the right to sell the bond back, or a drop-lock rate (i.e., a rate that will be fixed if it is hit). A number of FRNs are also inversely related to the reference rate. An ***inverse FRN*** or ***inverse floater*** is typically constructed from a fixed-income security (the collateral) and sold along with a floater. The two notes are created such that the interests paid on both are equal to or less than the interest paid on the collateral. The coupon rate on the floater is usually set equal to a reference rate such as LIBOR, whereas the rate on the inverse floater is determined by a formula that is inversely related to the reference rate. See Exhibit 6.1 for an example of how the rates on floaters and inverse floaters are related to a fixed rate on the collateral.

EXHIBIT 6.1 Rates on Floaters, Inverse Floaters, and Collateral

Example:
Collateral is $30 million, 7.5% coupon bond rate.
$22.5 million floater is issued.
$7.5 million inverse floater is issued.
The rate on the floater, R_{FR}, is set to the LIBOR plus 50 basis points, with the maximum rate permitted being 10%.
The rate on the inverse floater, R_{IFR}, is determined by the following formula:

$$R_{IFR} = 28.5 - 3\text{ LIBOR}$$

This formula ensures that the weighted average coupon rate (WAC) of the floater and inverse floater will be equal to the coupon rate on the collateral of 7.5%, provided the LIBOR is less than 9.5%.
For example, if the LIBOR is 8%, then the rate on the floater is 8.5%, the inverse floater's rate is 4.5%, and the WAC of the floater and inverse floater is 7.5%:

$$LIBOR = 8\%$$
$$R_{FR} = LIBOR + 50bp = 8.5\%$$
$$R_{IFR} = 28.5 - 3LIBOR = 4.5\%$$
$$WAC = .75R_{FR} + .25R_{IFR} = 7.5\%$$

Note: If the LIBOR is greater than 9.5%, then the rate on the inverse floater will be negative. To prevent this from occurring, the inverse floater would have to have a floor equal to zero. Moreover, a floor of zero on the inverse floater would require that a cap of 10% be set on the floater.

Maturity The maturities of corporate bonds vary from intermediate-term bonds with original maturities of five years or less to long-term bonds with maturities of over five years. Today, the rapid change in technology has led to more corporate bonds being issued with original maturities averaging 15 years.[3] This contrasts with the 1950s and 1960s when the original maturities on corporate bonds ranged from 20 years to 30 years.

Call and Redemption Features A call provision or option redemption provision in an indenture gives the issuer the right to redeem some or all of the issue for a specific amount (call price) before maturity. Bonds issued without a call or other embedded options are referred to as ***bullet bonds***. Callable bonds are often issued when interest rates are relatively high. The call provision often requires that the company redeem the bonds at a price greater than the par value. The additional amount is defined as the ***call premium***. For some issues, the call price (CP) is the same; for other callable bonds, the call price depends on the time the bond is called. Such issues will have a call schedule indicating the call price for each call date. For example, some companies set the premium equal to one year's interest for the first year if the bond is called, with the premium declining thereafter. For instance, a 10-year, 10%, $1,000 par value bond might be called the first year at a call price of $1,100 [$100 premium = (.10)($1,000)], the second year for $1,090 [$90 premium = (9/10)(.10)($1,000)], and so on, with the premium rate declining by 1/10 each year. Finally, instead of a specified call price or call schedule, some callable bonds have a ***make-whole premium provision*** (also called a ***yield maintenance provision***). This provision specifies that the amount of the premium be such that when it is added to the principal and reinvested at the redemption date in U.S. Treasury securities with the same remaining life as the bond, the yield would equal the bond's original yield. A make-whole premium provision provides some protection to investors.

As we discussed in Chapter 5, a call option is to the advantage of the issuer. For example, during a period of high interest rates, a corporation might sell a 20-year callable bond with a 12% coupon rate at its par value of $1,000. Suppose two years later, though, interest rates on all bonds dropped and bonds similar to this corporation's were selling at a 10% rate. Accordingly, the company might find it advantageous to sell a new 18-year, 10% callable bond with the funds of the new issue used to redeem the 12% issue. If a company decides to call its bond issue, it would send a ***notice of redemption*** to each holder and then at a specified time a check equaling the call price.

To the investor, a bond being called provides a benefit to the extent that the call price exceeds the par value. However, if a bond is called, the investor is forced to reinvest her proceeds in a market in which rates are generally lower. Consequently, on balance, call provisions tend to work against the investor. As a result, the issuer, in addition to the call premium, might provide the investor with some call protection. For example, although it is often standard for the entire issue to be called, provisions could be included in which only a certain proportion of the bonds issued could be called for a specified period, possibly with those selected to be determined by the trustee by lot. Investor protection might also be provided with a ***deferred call*** feature that prohibits the issuer from calling the bond before a certain period of time has expired. The investor would therefore have call protection for the period. However, a more common practice is to prohibit the issuer from buying back the bonds during a specified period (e.g., 5 years) from proceeds from a debt issue that ranks senior or par with the bond. Under this type of provision, the issuer has the right to redeem the bonds from excess cash or from the proceeds from the sale of equity, property, or higher interest rate debt. This type of redemption is called ***refunding***: the replacement of an old issue with a new one at a lower cost.

As an example of a callable bond with a redemption provision, consider a 20-year, 10% coupon bond with a par value of $1,000 issued by the ABC Company with the following call provisions:

- The first-year call price is equal to the public offering price plus the coupon; thereafter, the call price decreases by equal amounts for 15 years to equal par and thereafter the call price is equal to par.
- During the first five years, the issuer can only exercise the call from proceeds coming from cash, equity, or debt inferior to the issue.

Suppose the bonds were issued at 105 ($1,050 per $1,000 par). The first year call price would then be 115 (price + coupon = 105 + 10). The call price would then decrease by $1.00 each year [= (115 − 100)/15] to equal 100 in year 15; and in years 16 to 20 the call price would be 100. Finally, for the first five years the bond would not be refundable out of any debt issue that ranks senior or on par with the bond.

Year	Call Price
1	115
2	114
3	113
4	112
5	111
•	•
•	•
15	101
16–20	100

In this example, the refunding provision does not give the bondholders call protection during the first five years—they only have protection against certain types of redemptions. To reiterate, refunding occurs when the issuer sells bonds with the proceeds used to redeem an earlier issue, whereas redemption is simply a call of the bond. Moreover, there have been court cases in which bondholders have challenged bonds that were called under refunding clauses. For example, in 1983 Archer Daniels Midland issued a 16% bond due in 2011 with a refunding provision similar to the one just described. The bonds were called in 1991, with the source of the refunding being a stock offering. Bondholders subsequently brought legal actions to stop the redemption. The court, though, ruled in favor of Archer Daniels Midland, allowing the redemption. Interestingly, the company later issued new bonds.[4] Note that if the issue does not have any protection against early call, it is said to be a ***currently callable issue.***

Sinking Fund Most corporations sell their bonds with a principal of $1,000 that is usually paid at maturity. This contrasts with real estate mortgages and consumer loans made by financial institutions. These loans are usually ***fully amortized*** with the borrower making payments for both interest and principal during the life of the loan such that the loan is gradually repaid by installments before maturity arrives. Given many corporate bonds are not amortized, some corporations do sell their bonds as a ***serial bond*** issue. This type of bond issue consists of a series of bonds with different maturities. Such bonds serve to reduce a bondholder's concern over the payment of principal. A more common feature to allay principal risk is the inclusion of a ***sinking fund*** provision in the indenture. A sinking fund used to be simply a provision requiring that the issuer make scheduled payments into a fund often maintained by the trustee, or in some cases to certify to the trustee that the issuer had added value to investments. Today, though, many sinking fund agreements have provisions requiring an orderly retirement of the issue. In recent years, this has been commonly handled by the issuer being required to buy up a certain portion of bonds each year either at a stipulated call price or in the secondary market at its market price. Usually the sinking fund's call is the par value unless the issue is originally sold above par; then the call price is typically the issuance price.

The sinking fund call provision benefits the issuer and is a disadvantage to the bondholder. If interest rates are relatively high, then the issuer will be able to buy back the requisite amount at a relatively low market price, and if rates are low and bond prices are high, the issuer will be able to buy back the bonds at the sinking fund call price. A sinking fund with a call option is therefore valuable to the issuer and should trade at a lower price in the market than an otherwise identical non-sinking-fund bond (the valuation of sinking-fund bonds is discussed in Chapter 14).

Sinking funds are usually applied to a particular bond issue. There are, though, nonspecific sinking funds (sometimes referred to as ***tunnel, funnel,*** or ***blanket sinking funds***) that are applied to a company's total outstanding bonds. For most bonds, the periodic sinking fund payments are the same each period. Some indentures do allow the sinking fund to increase over time or to be determined by the level of earnings, and some bonds have accelerated sinking fund provisions giving the issuer the option to redeem more than the stipulated amount.

It should be noted that since many sinking fund provisions require the repayment of the debt in installments, they effectively reduce the life of the bond. As such, a better measure of a sinking fund bond's life than its maturity is its ***average life***. The average life is the average amount of time the debt will be outstanding. It is equal to the weighted average of the time periods, with the weights being relative principal payments:

$$Average\ life = \frac{\sum_{t=1}^{M} t(A_t)}{F}$$

where A_t is the sinking fund due at time t. Thus a bond that matures in 10 years and requires equal sinking fund payments each year would have an average life of 5.5 years [average life $= .1(1) + .1(2) + \cdots + .1(10)$].

Protective Covenants The board of directors hires the managers and officers of a corporation. Since the board represents the stockholders, this arrangement can create a moral hazard in which the managers may engage in activities that could be detrimental to the bondholders. For example, the managers might use the funds provided by creditors to finance projects different and riskier than bondholders were expecting. Since bondholders cannot necessarily seek redress from managers after they have made decisions that could harm them, they need to include rules and restrictions on the company in the bond indenture. Such provisions are known as ***protective*** or ***restrictive covenants***.

The covenants often specify the financial criterion that must be met before borrowers can incur additional debt (debt limitation) or pay dividends (dividend limitations). For example, a debt limitation covenant would be one that prohibits a company from incurring any new long-term debt if it causes the company's interest-coverage ratio (earnings before interest and taxes/interest) to fall below a specified level. In addition to limits on debt and dividends, other possible covenants include limitations on liens, borrowing from subsidiaries, asset sales, mergers and acquisitions, and leasing.

PROTECTIVE COVENANTS

1. Limitations on debt
2. Limitations on dividends
3. Limitations on share repurchases
4. Limitations that prohibit issuing long-term debt if certain ratios (e.g., interest coverage ratio) are lowered
5. Limitations on liens
6. Limitations on borrowing from subsidiaries
7. Limitations on mergers, acquisitions, and asset sales
8. Limitations on selling assets and leasing them

Over the last two decades, there has been an increase in the number of mergers, corporate restructurings, and stock and bond repurchases. Often these events benefit the stockholders at the expense of the bondholders, resulting in a downgrade in a bond's quality ratings and a lowering of its price. Bond risk resulting from such actions is known as ***event risk***.[5] Certain protective covenants such as poison puts and net worth maintenance clauses have been used to minimize event risk. A ***poison put*** clause in the indenture gives the bondholders the right to sell the bonds back to the issuer at a specified price under certain conditions arising from a specific event such as a takeover, change in control, or an investment ratings downgrade. A ***net worth maintenance clause***, in turn, requires that the issuer redeem all or part of the debt or give bondholders the right to sell (***offer-to-redeem clause***) their bonds back to the issuer if the company's net worth falls below a stipulated level.

Secured, Unsecured, and Guaranteed Bonds

In most loans, the lender, whether it is a financial institution or a bondholder, has three questions to pose to a borrower: What does the borrower need the money for?

How much does the borrower need? What does the borrower plan to do if the idea doesn't work? The latter question usually translates into the type of collateral the borrower intends to pledge in order to pay the lender if he is unable to meet the interest and principal obligations. In the case of corporate bonds, the bonds can be either ***secured bonds***, backed by a specific asset, or ***unsecured bonds***, backed by a general creditor's claim but not by a specified asset. The latter are called ***debentures***. A secured bond is defined as one that has a lien giving the bondholder, via the trustee, the right to sell the pledged asset in order to pay the bondholders if the company defaults. Secured bonds can be differentiated in terms of the types of collateral pledged and the priority of the lien.

Types of Collateral Assets that can be used as security are real, financial, or personal. A mortgage bond and an equipment-trust bond are bonds secured by real assets. A ***mortgage bond*** has a lien on real property or buildings whereas an ***equipment trust bond*** has a lien on specific equipment, such as airplanes, trucks, or computers. A ***collateral trust bond***, in turn, is secured by a lien on equity shares of a company's subsidiary, holdings of other company's stocks and bonds, government securities, and other financial claims. Finally, a company might secure its debt with personal property, such as the corporation's cash or liquid assets, accounts receivable, or inventory. Since these assets are short term in nature, they are usually used as collateral for short-term debt obligations.

Mortgage Bonds: In the case of a mortgage bond, if the issuer defaults and the assets are liquidated, then the mortgage bondholders can claim the underlying asset and sell it to pay off their obligation, or if the issuer defaults and the company is reorganized, then the bondholders' mortgage lien will give them a stronger bargaining position relative to other creditors on any new securities created. Often in a mortgage bond, there are provisions in the indenture that allow the mortgaged asset to be sold provided it is replaced with a suitable substitute; some mortgage bonds also have a ***release and substitution provision*** that allows for the asset to be sold with the proceeds used to retire the bonds. Mortgage bonds are sometimes sold in a series, similar to a serial bond issue, with the bonds of each series secured by the same mortgage. Generally, it is more efficient for a company to issue a series of bonds under one mortgage and one indenture than it is to arrange collateral and draw up a new indenture for each new bond issue.

Equipment Trust Bonds: Usually equipment trust bonds are secured by one piece of property (e.g., a truck) and often are named after the security (e.g., truck bond). Equipment trust bonds are sometime formed through a lease and buyback agreement with a third party or trustee. Under this type of agreement, a trustee (e.g., bank, leasing company, or the manufacturer) might purchase the equipment (plane, machine, etc.) and lease it to a company who would agree to take title to the equipment at the termination date of the lease. Alternatively, the company could buy the equipment and sell it the trustee who would then lease it to them. The trustee would finance the equipment purchase from the company or the manufacturer by selling equipment trust bonds (sometime called equipment trust certificates). Such bonds often have a maturity that reflects the life of the equipment and the terms of the lease, and often the principal is amortized. Each period, the trustee would then collect rent from the company and pay the interest and principal on the certificates. At maturity, the certificates would be paid off, the trustee would transfer the title of the equipment to the company, and the lease would be terminated. This arrangement (sometimes referred to as the ***Philadelphia Plan*** and ***rolling stock***) and other variations work well when the underlying equipment is relatively standard (e.g., plane, railroad car, or computer) and therefore can be easily sold in the event the company defaults on the lease. Airlines and railroad companies are big users of this type of financing.

Collateral Trust Bonds: A collateral trust bond is secured by a lien on the company's holdings of other company's stocks and bonds, other securities and financial claims, or the issuer's subsidiaries. The legal arrangements governing collateral trust bonds generally require the issuer to deliver to the trustee the pledged securities (if the securities are stock or the stock of a subsidiary, the company still retains its voting rights). The company is usually required to maintain the value of the securities, positing addition collateral (e.g., cash or more securities) if the collateral decreases in value. There are also provisions in the indenture allowing for the withdrawal of the collateral provided there is an acceptable substitute. Finally, some collateral trust bonds are sold as a series like some mortgage bonds, with the same indenture and financial collateral defining each series.

Priority of Claims In designating an asset as collateral, it is possible for a company to have more than one bond issue or debt obligation secured by that asset. When this occurs, debt obligations must be differentiated in terms of the priorities of their claims. A ***senior lien***, or first lien, has priority over a ***junior lien*** (second or third lien). Thus, if a company defaults and the real property pledged is sold, then the senior bondholders would be paid first, with the second or third lien holders being paid only after the senior holders have been paid in full.[6] It should be noted that such bonds are typically not defined in their title as second or third lien bonds (e.g., second mortgage bond) because of the insinuation of weakness.

Closely associated with priorities in claims are clauses in the indenture that specify the issuer's right to incur additional debt secured by the assets already encumbered. At one extreme, there are ***closed-end bonds*** (usually mortgage bonds) that prohibit the company from incurring any additional debt secured by a first lien on the assets already being used as security. For example, a company with a processing plant and land valued at $20 million might use those assets as security for a $14 million bond issue. If the issue were closed-ended, then no other debt obligation with first liens could be obtained. In contrast, an issue silent on this point is an ***open-end bond***; it allows for more debt to be secured by the same collateral. Thus in the case of the company with a $14 million secured bond, if the company were to later sell a new $6 million bond issue, it could secure the new debt with the $20 million plant and land assets, provided the earlier issue was open ended. In turn, if the company defaulted and the assets were sold for only $14 million, then the first bondholders would receive only 70 cents on each dollar of their loan, compared to a dollar on a dollar if their issue had been closed-end.

Because of the adverse effects to investors, most open-end bonds include certain covenants that limit the amount of additional indebtedness the company can incur. A typical case is an open-end bond accompanied by an ***after-acquired property clause***. This clause dictates that all property or assets acquired after the issue be added to the property already pledged. Finally, within the extremes of open- and closed-end bonds are bonds with limited open-end clauses that allow the company to incur additional debt secured by assets up to a certain percentage of the pledged asset's value.

Debenture The majority of all corporate bonds are unsecured. As noted, such bonds are defined as ***debentures***. Even though such instruments lack asset-specific collateral, they still make the holder a general creditor. As such, debenture holders are protected by assets that are not already pledged, and they also have a claim on pledged assets to the extent that those assets have values in excess of the secured debt. For investors, it is important to distinguish between strong companies that sell debentures and have no bonds secured with pledged assets and companies that sell debentures and have bond secured with pledged assets—the latter needs closer scrutiny.

Debentures can be issued with a number of protective covenants. For example, the indenture might include a restriction on additional debt that can be incurred or specifications that new debt can only be incurred if earnings grow at a certain level or if certain financial ratios are met. Debentures can also be classified as either ***subordinate*** or ***unsubordinate***. In the case of liquidation, subordinated debt (junior security) has a claim only after an unsubordinated claim (senior claim) has been met. Accordingly, a debenture can be made subordinate to other claims such as bank loans or accounts payable. Subordination may be the result of the terms agreed to by the firm in its other debt obligations. For example, a bank might require that all future debts of a company be made subordinate to its loans.[7] Since subordinated debenture bondholders are last in line among creditors if the issuer defaults, they are sometimes sold with a sweetener or inducement such as an option to convert to shares of the company's stock or a put option giving the holders the right to sell the bond back to the issuer at a specific price.

Guaranteed Bonds Bonds issued by one company and guaranteed by another economic entity are defined as ***guaranteed bonds***. The guarantee ensures that the bondholders will be paid interest and principal in the event the issuer defaults. With the guarantee, the default risk of the bond shifts from the borrower to the financial capacity of the insurer.

The guarantor could be the parent company or another company securing the issue in return for an option on an equity interest in the project the bond is financing. There may also be multiple guarantors. In a joint venture, for example, a limited partnership may be formed with several companies who jointly agree to guarantee the bond issue of the venture. For some corporate issues, a financial institution may provide the guarantee. For example, banks for a fee provide corporations with ***credit enhancements*** in the form of ***letters of credit*** that guarantee the interest and principal payment on the corporation's debt obligation. Similarly, insurance companies have expanded their insurance coverage of municipal bonds that they began providing in the 1970s to the coverage of corporate bonds. Often the insurance is provided in the form of a ***surety bond***; this is an insurance policy written by an insurance company to protect a party against loss or the violation of a contract. Finally, municipal governments and governmental agencies sometimes offer guarantees. For example, to promote the gasohol program in the late 1970s, the Farmers' Home Administration provided loan guarantees to companies that developed alcohol-fuel plants.

Corporate Bonds with Special Features

The discussion to this point has focused on the general characteristics of corporate bonds. Many corporate bonds also have special features included in their covenants that make them more identifiable with stocks, options, or other securities. Bonds with special features may be created out of Chapter 7 corporate reorganization or they may be high-yield bonds. High-yield bonds used to have a conventional structure. In the 1980s, though, many companies that were active in leveraged buyout acquisitions or in debt-for-equity recapitalization faced cash flow constraints. To minimize their cash flow problems or to try to maintain quality ratings, they often issued bonds with special features.

Income Bonds *Income bonds* are instruments that pay interest only if the earnings of the firm are sufficient to meet the interest obligations; principal payments, however, are required. Thus, a failure by the issuer to pay interest does not constitute a default. Because income bonds are rare, companies who have been reorganized because of financial distress sometimes issue them. In general, because the interest payments are not required unless earnings hit a certain level, income bonds are similar to preferred stock. In fact, some income bonds have the cumulative dividend feature of preferred stock: If interest is not met, it accumulates. Similarly, some income bonds permit voting or limited voting rights (usually if interest is not paid). Unlike preferred stock, though, income bonds do provide corporations with the tax advantage of interest deductibility. Finally, income bonds often include such features as sinking fund arrangements and convertibility to the company's stock.

Participating Bonds *Participating bonds* provide a guaranteed minimum rate, as well as additional interest up to a certain point if the company achieves a certain earnings level. Like income bonds, participating bonds are similar to preferred stock, except for the interest deductibility benefit. However, for obvious reasons participating bonds are not very popular to shareholders. As a result such bonds are very rare.

Deferred Coupon Bonds Some corporations sell bonds with a deferred coupon structure that allows the issuer to defer coupon interest for a specified period. Included in the group of deferred coupon bonds are ***deferred-interest bonds (DIBs)***, ***reset bonds***, ***extendable reset bonds***, and ***payment-in-kind (PIK) bonds***. Many of these debt securities with special features were created during the merger period of the 1980s. For example, in 1989, the RJR leveraged buyout created convertible and exchangeable debentures that had both payment-in-kind and reset features.

A deferred-interest bond (DIB) has its coupon interest deferred for a specified period. They are often structured so that they do not pay coupons for a specified number of years (e.g., five years). At the end of the deferred-interest period, they begin to pay interest, usually semiannually, until they mature or are called. Such bonds sell originally like deep discount bonds. A reset bond or step coupon bond is similar in structure to a DIB except that it starts with a low coupon interest, which is later increased. A reset bond may have a call option that is likely to be exercised as the coupon level increases. An extendable reset bond has a rate that is reset to reflect the current level of interest and credit spread (usually determined by an independent investment firm or firms). Extendable reset bonds are like FRNs. Finally, a payment-in-kind bond (PIK) gives the issuer the option on the interest-payment date to pay the coupon interest either in cash or in kind, usually by issuing the bondholder a new bond. In essence, a PIK allows coupons to be paid in units of the security (baby bonds). If the issuer pays in kind, then at maturity the investor would own a number of bonds and the cash flow from her PIK would be similar to that of a zero-coupon bond.

Tax-Exempt Corporate Bonds To promote investments in projects that are in the public interest, Congress grants tax-exempt status for bonds used for specified purposes. When a project qualifies for tax-exemption, the holders of the bond do not have to pay federal income tax on the interest they receive. As a result, investors in tax-exempt bonds will accept a lower interest rate, lowering the interest cost to the issuing corporation. Prior to 1986, a number of activities qualified for tax-exempt status. The Tax Reform Act of 1986, though, significantly reduced the number of eligible activities. Examples of eligible tax-exempt activities would be the construction of solid and hazardous waste disposal facilities.

Bonds with Warrants A *warrant* is a security or a provision in a security that gives the holder the right to buy a specified number of shares of stock or another designated security at a specified price. It is a call option issued by the corporation. As a sweetener, some corporate bonds, such as a subordinated debenture, are sold with warrants. A warrant that is attached to the bond can only be exercised by the bondholder. Often, the warrant can be detached from the bond as of a particular date and sold separately.

Convertible Bonds A ***convertible bond*** is one that has a conversion provision that grants the bondholder the right to exchange the bond for a specified number of shares of the issuer's stock. A convertible bond is similar to a bond with a non-detachable warrant. Like a regular bond, it pays interest and principal, and like a warrant, it can be exchanged for a specified number of shares of stock. Convertible bonds are often sold as a subordinate debenture (convertible debentures). The conversion feature of the bond, in turn, serves as a sweetener to the bond issue. Note: Some convertibles can be converted into other securities. For example, a company owning a significant proportion of another company could issue a convertible bond giving the convertible bondholders the right to convert the bond into shares the issuer owns of the other company. Similarly, a gold mining company could issue a bond convertible into gold claims.

To issuers, convertibles tend to lower the interest costs on their debt. The conversion feature may also make it possible for issuers to reduce the number of protective covenants they normally would include in their debt obligations. In general, convertibles give issuers the opportunity to sell stock at a better price via the convertible than the stock price they currently would receive if they sold directly. This advantage could be negated later if the stock were sold on the convertible at a price below the market. However, most convertibles have a call option that the issuer can use to force the conversion to a price that in some cases would be higher than the price realized in the market. To the investor, convertible bonds provide a floor against a stock price decrease. That is, if the stock price decreases, the value of the convertible will only drop to its value as a straight bond. On the other hand, if the stock price increases, then the convertible bond's price will also increase, providing upside potential. The disadvantage of a convertible to investors is that the yield on the bond is less than the yield on a comparable nonconvertible, and the issuer can call the convertible, forcing the conversion. The features and valuation of convertible bonds are discussed in more detail in Chapter 14.

Putable Bonds A ***putable bond*** or ***put bond*** gives the holder the right to sell the bond back to the issuer at a specified price. In contrast to callable bonds, putable bonds benefit the holder: If interest rates increase and as a result the price of the bond decreases below the specified price, then the bondholder can sell the bond back to the issuer and reinvest in a market with higher rates. As we noted earlier, a bond with a put option may also be used to protect the bondholder against a decrease in the price of the bond due to a downgrade in its quality rating.

Extendable Bonds *Extendable bonds* have an option to extend the maturity of the bond. Typically, the bond issuer holds the option. Some extendable bonds give the holder the right to extend and some give both the issuer and the investor the extension option.

Credit-Sensitive Bonds Credit-sensitive bonds are bonds with coupons that are tied to the issuer's credit ratings. For example, the coupon rate may be 10% if the bond has a quality rating of A or better, 10.25% if the rating is BBB, 10.5% if the rating is BB, and so on. Such bonds provide bondholders some protection against management pursuing risky investments or diluting the quality of current bonds by management's increase use of debt financing. However, such clauses also increase the company's interest costs at a time when it may not need higher rates.

Commodity-Linked Bonds A ***commodity-linked bond*** is one that has its coupons and possibly principal tied to the price of a particular commodity. The bonds are designed to provide a company a hedge against adverse changes in the price of a commodity. For example, an oil-producing company might sell an oil-index bond in which the interest is tied to the price of crude oil.

Voting Bonds As the name indicates, ***voting bonds*** give voting privileges to the holders. The vote is usually limited to specific corporate decisions under certain conditions.

Assumed Bonds An *assumed bond* is one whose obligations are taken over or assumed by another company or economic entity. In many cases, such bonds are the result of a merger. That is, when one firm takes over or buys a second firm, the second firm usually loses its identity (legally and in name). As a result, the first company takes over the liabilities of the second. Accordingly, the bonds of the second are assumed by the first firm's promise to pay, often with additional security pledged by the first company in order to allay any fears of the creditors.

6.3 BANKRUPTCY

A number of factors can lead to the financial distress and deterioration of a company: poor investments, competition, excessive debt, litigation, and poor management. One of the main risks that an investor assumes when she buys a bond is the chance the company will become financially distressed and the issuer will default. If a corporation defaults, the amount the investor receives depends, in part, on the security pledged and the priority of the claim; however, equally important is how the bankruptcy is handled.

A company is considered bankrupt if the value of its liabilities exceeds the value of its assets; it is considered in default if it cannot meet its obligations. Technically, default and bankruptcy are dependent. On the one hand, a company with liabilities exceeding assets (bankrupt) will inevitably be in default when the future income from its assets is insufficient to cover future obligations on its liabilities. On the other hand, a company that is presently unable to meet its current obligations will, if conditions persist, have its assets' prices decline. It should be noted that bankruptcy is not limited to size. There have been many large corporations that have declared bankruptcy: GM, Chrysler, Enron, Texaco, Federated Department Stores, Continental Airlines, Penn Central, Eastern Airlines, Southland Corporation, and Pan Am. Also, there are occasions when a company is currently solvent but files for bankruptcy in order to obtain protection against future claimants. This was the motivation for the bankruptcy petition filed by the Manville Corporation in 1982: The company was solvent but had legal claims against it due to asbestos-related diseases.

In the United States, when a company defaults on its obligations to bondholders and other creditors, the company can voluntarily file for bankruptcy with the courts; the bondholders (via their trustee) and other creditors can sue for bankruptcy; or both parties can try to work out an agreement. In the first two cases, the court will decide whether the assets should be liquidated or whether the company should be reorganized. In the third case, the parties can settle by extending or changing the composition of the debt with minimum court involvement. In the United States, the Bankruptcy Reform Act of 1994 governs bankruptcies. The act is composed of 15 chapters. ***Chapter 7*** deals with liquidation of a corporation and ***Chapter 11*** deals with reorganization. Technically, liquidation means that all of the assets will be distributed to the holders of claims and the corporate entity will not survive. In contrast, when there is reorganization a new entity emerges, with claim holders getting new securities in the new corporation or cash and new securities. The Bankruptcy Act, in turn, provides the framework under which liquidation and reorganization are considered (see Exhibit 6.2 for a summary of the bankruptcy process). In addition, the law also provides stay protection for the distressed company from its creditors.

If the court decides on asset liquidation, creditors with security pledged will receive, to the extent possible, the par value of their debt from the sale of the secured

EXHIBIT 6.2 U.S. Bankruptcy Process for Reorganization

Filing:

A bankruptcy filing by creditors or the debtor (distressed company) is done in the appropriate circuit and district court. Appropriate can mean the court with jurisdiction over the company's headquarters or its principal place of business.

- **a.** The filing requires the best estimate of the value of the company's assets and liabilities and a listing of its 20 largest creditors.
- **b.** The company files a petition for protection, creditors are contacted, and a meeting is set up.

Debtor-in-Possession:

When a company files for protection, it becomes a debtor-in-possession. As a debtor-in-possession, the company continues to operate, but under the supervision of the court.

Court supervision includes the court's approval on major transactions, the appointment of a trustee to oversee, and the possible appointment of an examiner. In certain cases, the court may appoint a trustee to take over control of the business.

- **a.** The bankruptcy judge issues an automatic stay.
- **b.** All debt is frozen: Creditors are precluded from trying to enforce collection.
- **c.** Lawsuits are suspended.

Formulation of a Plan:

A committee consisting of officers and representatives for creditors and possibly shareholders is formed to formulate a plan of reorganization.

The debtor must file the plan in 120 days, although the length can be extended. No other plans can be filed during this period. Thereafter any interested party can submit a plan.

Plans usually consider reorganization, the creation of new financial securities, elimination or changing of expensive contracts (e.g., leases or union contracts), and substantial consolidation.

Under substantial consolidation, all assets and liabilities of all of the company's subsidiaries are pooled and used; this can have important ramifications for security holders.

Disclosure Statement:

Once the committee approves the plan for reorganization, the debtor produces and files for approval a disclosure statement.

The disclosure statement summarizes the plan. It also includes pro formas and a liquidation analysis supporting the claim that the creditors will receive more under the reorganization plan than liquidation.

- **a.** If the court approves the disclosure statement, then it is sent to all impaired parties for approval.
- **b.** Parties are given 30 days to vote.
- **c.** To be accepted, at least two-thirds of the impaired parties and half of the claimants must accept the plan.
- **d.** If approved, the court sets a date for the reorganization.
- **e.** If the required number of creditors do not approve, the plan may be approved under a cram-down provision. Approval under this provision requires meeting several specified criteria.

assets. Next, the sale of unsecured assets and any excesses from the secured assets' sale will be used to satisfy priority creditors. Finally, what is left will be used to pay unsecured creditors, followed by shareholders. Thus, when a company is liquidated, senior creditors are paid in full before junior are paid, and secured creditors and unsecured creditors have senior claim over equity holders. This distribution of assets to creditors is referred to as the ***absolute priority rule (APR)***. In the case of liquidation, the bankruptcy courts have generally upheld this rule in their decisions.

Alternatively, if a court decides that the value of the company's operation is worth more if it continues as a business than if it is liquidated, then the court may order reorganization. For reorganization to be feasible (or preferable to liquidation), the causes of the firm's insolvency must be rectified and the prospects of a profitable future must be defended. Moreover, to achieve profitability, reorganization often requires a restructuring of the debt. When this occurs, creditors are usually given new claims on the reorganized firm that are at least equal in value to an amount estimated to be received if liquidation had occurred. This could take the form of debenture holders receiving long-term income bonds, stock, or convertible bonds, and short-term creditors receiving long-term claims. In contrast to liquidations, there have been numerous cases in which the absolute priority rule (APR) has been violated in Chapter 11 reorganizations; that is, where the actual distribution of assets was different from what the terms called for in the debt agreement.[8] Such violations can occur as a result of efforts to reach an agreement during the bankruptcy process amongst all impaired parties, including equity holders. In such cases, unsecured creditors may end up bearing a disproportionate cost of the reorganization, with possibly equity holders benefiting.

In summary, the amount of funds the bondholder will ultimately receive if the issuer defaults depends on whether the bankruptcy is handled through liquidation, reorganization, or voluntary settlement. Current U.S. law generally favors reorganization. Since bankruptcy proceedings can take some time, some speculators specialize in buying defaulted issues. They, in turn, can profit from such investments if the present value of the cash received at liquidation or the value of the new instrument (replacing the defaulted bond) from reorganization exceeds the price they paid for the defaulted bonds.

6.4 THE MARKETS FOR CORPORATE BONDS

Primary Market

Billions of dollars of corporate bonds are sold each year in the primary market. The new corporate bonds are sold either in the open market or privately placed to a limited number of investors.

Open-Market Sales Bonds sold in the open market (***open market sales***) are handled through investment bankers. Investment bankers may underwrite the issue themselves or with other investment bankers as a syndicate, or they may use their best effort: selling the bonds on commission at the best prevailing price.

The way a company chooses to offer an issue to the public depends, in part, on the size of the issue and the risk of a price decrease during the time the issue is being sold. For relatively strong companies, the investment banker often underwrites the issue, buying the issue at an agreed-upon price and then selling it in the market at hopefully a higher price. Such an agreement is referred to as a ***firm commitment***. The issuer may choose the investment banker or syndicate, either individually or by a bid process, selecting the underwriting group with the highest price. With an underwriting arrangement, the selected investment banker will try to profit from the spread between the selling price (retail) and the price paid to the issuer. The spread represents the ***floatation cost*** to the issuer; it is usually slightly less than 1% of the total value of the issue.

When a new issue is underwritten, the investment banker underwriting the issue bears the risk that the price of the issue could decrease during the time the bonds are being sold. A classic example illustrating such risk was the $1 billion bond issue of IBM in 1979. This issue was underwritten by a syndicate just before the announcement by the Federal Reserve System of a major change in the direction of monetary policy. The Federal Reserve announcement, in turn, led to a substantial increase in interest rates and a decrease in bond prices, causing substantial losses for the underwriters. To avoid such underwriting risk, the investment banker may choose to hedge the issue by taking a position in the futures market. Alternatively, the investment banker may elect to sell the issue on a best-effort basis or use a combination of underwriting and best effort by using a ***standby underwriting agreement***. In this latter agreement, the investment banker sells the issue on a commission, but agrees to buy all unsold securities at a specified price.

Before the issue is sold to the public, the issuer must file registration statements with the Securities and Exchange Commission (SEC). These statements include the relevant business and financial information of the firm. Once the company has registered, it must then wait while the SEC verifies all the information. Typically the investment banker uses this period to advertise the offering and to distribute to potential buyers a preliminary prospectus called a ***red herring*** that details all the pertinent information the official prospectus will have, except the price. Finally, after the SEC confirms the registration statements, the indenture and prospectus become official and the investment banker offers the issue for sale.

In selling the bond issue, the investment banker often forms a selling group. This group consists of the investment banker who, as an underwriter, acts as a wholesaler (or initial distributor if best-effort is being used) by selling the issue to a number of dealers who, in turn, sell to their clients. The arrangements between the investment banker and the selling group are specified in a ***selling group agreement*** (described in the prospectus). The agreement defines the period of time the members of the group have to sell their portion of the issue, commissions that they can charge, and restrictions such as prohibiting members from selling below a certain price.

In summary, the floating of a bond issue can be quite complex, involving the preparation of an indenture, the selection of a trustee, and the formation of a selling group. Since 1983 some corporations have been able to shorten this process, as well as reduce the floatation costs of issuing bonds, by taking advantage of SEC ***Rule 415***. Rule 415, known as the ***shelf registration rule,*** allows a firm to register an inventory of securities of a particular type for up to two years. The firm can then sell the securities whenever it wishes during that time—the securities remain on the shelf. To minimize costs, a company planning to finance a number of projects over a period of time could register a large issue, and then sell parts of the issue at different times.

Private Placement An alternative to selling securities to the public is to sell them directly to institutional investors through a private placement. One of the attractions of privately placed bonds is that they are exempt from SEC registration because they do not involve a public offering. During the 1980s, an increasing proportion of new corporate bonds were sold through ***private placement.*** Because they are sold through direct negotiation with the buyer, privately placed bonds usually have fewer restrictive covenants than publicly issued ones, and they are more tailor-made to both the buyer's and seller's particular needs.[9] Historically, one of the disadvantages of privately placed bonds was their lack of marketability due to the absence of an active secondary market. Under the Securities Act of 1933, firms could only offer securities privately (which did not require SEC registration) to investors deemed sophisticated—insurance companies, pension funds, banks, and endowments. In 1991, the SEC adopted Rule 144A under the Securities Act of 1933. Under this rule, issuers could sell unregistered securities to one or more investment bankers who could resell the securities to ***qualified investment buyers (QIBs)***. QIBs could then sell freely to each other in securities that have not been registered. The adoption of ***SEC Rule 144A*** eliminated some of the restrictions on the secondary trading of privately placed bonds by institutional investors. As such, it opened up the secondary market for privately placed bonds.

Another reason for the growth in privately placed bonds during the 1980s was their use in financing many of the corporate mergers and takeovers. During this period, many corporations and investment groups sold bonds and borrowed from financial institutions to finance their corporate acquisitions. Because privately placed bonds had less restrictive covenants, they were frequently used to finance these leveraged buyout acquisitions. Moreover, many of these bonds were non-investment grade bonds. By the late 1980s, these bonds accounted for approximately a third of the new corporate bonds offered, with two-thirds of those bonds being used to finance mergers or corporate restructurings aimed at stopping a corporate takeover. Exhibit 6.3 provides a brief history of the growth in this market and the rise and fall of one of its major participants—Drexel Burnham Lambert.

Today, the market for privately placed issues consists of medium-sized, less well-known companies, who see private placement as an alternative to the corporate bank loan market. For these companies, not all their private placements are Rule 144A placements. The market does have some large issuers whose placements make use of Rule 144A. Many of these issues are underwritten similar to publically issued bonds.

Secondary Market

The maturity and fixed income features of corporate bonds make them a good investment for large institutional investors. Life insurance companies, followed by corporate and private pension funds, dominate the ownership of existing corporate bonds.

Since most corporate bonds are transferable, a secondary market for corporate bonds exists. Some existing corporate bonds are listed on organized exchanges. Much of the trading of existing corporate bonds, though, takes place on the

EXHIBIT 6.3 The Rise and Fall of Drexel Burnham Lambert

Since the U.S. federal tax code allows interest, but not dividends, to be tax deductible, leveraged companies structured with greater debt-to-equity ratios have more of their earnings going to investors (creditors and shareholders) and less to the government, all other factors constant. As the company's debt-to-equity ratio increases, though, its expected bankruptcy costs also increase, augmenting the rate required on the debt and lowering its quality ratings.

In the 1970s, most lower-quality bonds were those of fallen angels: companies with investment-grade debt that had been downgraded. In the early 1980s, this changed with the emergence of many leveraged buyout companies (LBOs) formed, in part, to take advantage of the tax law. These LBOs would issue bonds to finance their corporate acquisitions. After the acquisition, the newly structured company would be more highly leveraged, with a greater proportion of the firm's investors now being creditors. With the interest tax deductible, though, the new company would be able to pay less corporate taxes, enabling it to pay a higher interest to its creditors.

With stock prices relatively low and yielding poor returns during the 1970s, Michael Milken of the investment banking firm Drexel Burnham Lambert was one who saw the potential of selling to institutional investors, as a substitute for stock, high-yielding corporate bonds created from financing mergers. During the 1980s, some 1800 corporations issued low-quality, high-yielding junk bonds to finance their acquisitions and to change their capital structure. In underwriting a number of these issues, Drexel Burnham Lambert earned fees as high as 2% to 3%. In addition, to facilitate the marketability of these bonds, Milken and Drexel Burnham Lambert also improved the creditworthiness of the bonds by standing ready to renegotiate the debt or to loan funds if the company were in jeopardy of default. The investment company also acted as market makers, providing a secondary market for junk bonds.

Unfortunately, the economic recession of the late 1980s and early 1990s depressed the earnings of many leveraged companies to levels that were not sufficient to pay their high-interest obligations. Over 250 companies defaulted between 1989 and 1991, including Drexel Burnham Lambert, who filed for bankruptcy in 1990 due to losses on its holdings of junk bonds. The junk bond market did eventually recover from its near collapse in the early 1990s. Today, it is a market used by medium-sized companies to raise funds.

As for Michael Milken, he was convicted of insider trading resulting from feeding information on target companies to Ivan Boesky, a Wall Street hedge fund player, and others. He was sentenced to three years in prison; his net worth, though, was reported by *Fortune* to be over $400 million in 1993.

over-the-counter (OTC) market, where brokers and dealers specializing in certain types of issues handle trading. In the OTC market, a core of large dealers dominates the corporate bond market. These dealers buy and sell existing corporate bonds to and from life insurance companies, pension funds, and other institutional investors. They also provide an important wholesale market in which they trade with other dealers and brokers who are executing buy and sell orders from the customers.

Although the amount of corporate bonds outstanding is large, the secondary market activity is somewhat limited compared to the activity in the secondary markets for stocks. The relatively thin secondary market for corporate bonds is due to the passive investment practices of large institutions that tend to buy and hold their corporate bonds to maturity. It is important to remember that the degree of trading activity determines a bond's degree of marketability and the spread between a dealer's bid and asked prices. In the corporate bond markets, the spreads range from a low of 1/4 to 1/2 of point (good marketability) to as high as 2% (poor marketability). For an investor who plans to buy a bond at its initial offering and hold it to its maturity, a thin market is not a concern; it is a major concern, though, to a bond speculator or a fund manager who needs marketability or whose profit margins could be negated by a large spread.

In 2002, the National Association of Security Dealers, NASD, established mandatory reporting requirement of OTC market transactions to make the secondary market for bonds more transparent. The reporting system that was established was the Trading Reporting and Compliance Engine—TRACE. By 2005 TRACE included all corporate bonds publically traded (29,000). TRACE provides information on price and quantity traded at all execution dates. Table 6.1 shows the price and quantity information from 01/07/09 to 01/08/09 accessed from TRACE using the FINRA site (www.finra.org/index.htm) for a Kraft bond with a 6.13% coupon and maturity of 2018.

TABLE 6.1 Price and Quantity Quotes from TRACE for a Kraft 6.13%, 2018 Bond from 01/07/09 to 01/08/09

Execution Date	Time	Quantity	Price	Yield
1/8/2009	10:28:37	10,000	102.399	5.78
1/8/2009	10:28:29	10,000	102.399	5.78
1/8/2009	10:27:32	42,000	102.45	5.773
1/8/2009	10:27:19	42,000	102.45	5.773
1/8/2009	9:53:30	25,000	102.399	5.78
1/8/2009	9:53:18	25,000	102.399	5.78
1/8/2009	9:18:44	4,000	102.471	5.77
1/8/2009	9:18:09	4,000	102.471	5.77
1/8/2009	8:57:18	5,000	103.5	5.625
1/8/2009	8:57:09	5,000	103.5	5.625
1/8/2009	8:54:00	40,000	102.783	5.726
1/8/2009	8:43:30	40,000	102.621	5.748
1/7/2009	17:07:44	1,000,000	101.648	5.886
1/7/2009	16:50:37	3,800,000	99.954	6.131
1/7/2009	16:01:51	70,000	99.182	6.244
1/7/2009	15:56:34	70,000	99.486	6.199
1/7/2009	15:45:56	3,000	104.721	5.456
1/7/2009	15:45:46	3,000	104.721	5.456
1/7/2009	15:33:07	750,000	102.04	5.831
1/7/2009	15:25:56	10,000	104.625	5.469
1/7/2009	15:25:00	10,000	103.083	5.684
1/7/2009	15:08:02	50,000	104.122	5.539
1/7/2009	15:06:09	50,000	102.685	5.739
1/7/2009	15:03:00	11,000	103.747	5.591
1/7/2009	14:49:02	1,000,000	100.574	6.041

6.5 MEDIUM-TERM NOTES

A ***medium-term note (MTN)*** is a debt instrument sold on a continuing basis to investors who are allowed to choose from a group of bonds from the same corporation, but with different maturities. MTNs were first introduced in the 1970s when General Motors Acceptance Corporation (GMAC) sold such instruments to finance its automobile loans. However, the market for MTNs did not take off until the early 1980s when Merrill Lynch began acting as an agent in issuing MTNs and also as a dealer by making a secondary market for the notes. Since then, the MTN market has grown significantly. The market's growth can be attributed to the flexibility MTN issues provide corporations in both the types of securities they can offer, and with SEC Rule 415, the times when they can offer them. As noted earlier, Rule 415 allows issuers to sell several issues over different periods without having to go through costly registration procedures each time.

Issuing Process

A corporation planning to issue an MTN first files a shelf registration form with the SEC. The filing includes a prospectus of the MTN program (different notes, their maturities, par values, and the like). By filing a shelf registration form, the corporation is able to enter the market constantly or intermittently, giving it the flexibility to finance a number of different short-, intermediate-, and long-term projects over a two-year period. Typically, the MTNs are sold through investment banking firms who act as agents. The agents will often post the maturity range for the possible notes in the program and their offering rates. The rates are often quoted in terms of a spread over a Treasury security with a comparable maturity (see Table 6.2). The minimum denominations on MTNs range from $1 million to $25 million. An investor interested in one of the note offerings will notify the agent who, in turn, contacts the issuing corporation for a confirmation. Once an MTN issue is sold, then the company can file a new registration to sell a new MTN issue—an action known as reloading.

In addition to providing the issuing corporations with flexibility in their capital budget, an MTN program also gives institutional investors the opportunity to choose notes whose maturities best fit their liabilities, thereby minimizing their market risk. In many instances, the market for MTNs starts with institutional investors indicating to agents the type of maturity they want; this is known as ***reverse inquiry***. On a

TABLE 6.2 MTN Program

MTN	Yield
12 months to 18 months	6.25
2 years to 3 years	6.45
3 years to 4 years	6.65
4 years to 5 years	6.75
5 years to 6 years	6.85
6 years to 7 years	7.00

reverse inquiry, the agent will inform the corporation of the investor's request; the corporation could then agree to sell the notes with that maturity from its MTN program, even if they are not posted.

Special Features

Today, MTNs are issued not only by corporations, but also by bank holding companies, government agencies, supranational institutions, and sovereign countries. MTNs vary in terms of their features. Some, for example, are offered with fixed rates where others pay a floating rate; some are unsecured whereas others are secured (e.g., equipment trust MTNs). Floaters and inverse floaters are also created from MTNs and some MTNs have currency clauses. It is also common for MTNs to be offered with derivatives (swaps, caps, floors, futures, and forward contracts). MTNs that are combined with other instruments are referred to as a ***structured MTNs***. The most common derivative used is a swap. A corporation, for example, might issue MTNs with floating rates and then take a position in an interest rate swap contract to form a synthetic fixed-rate debt. Interest rate swaps and other interest rate derivatives are examined in Parts 4 and 5.

6.6 COMMERCIAL PAPER

Commercial paper (CP) is a short-term debt obligation usually issued by large, well-known corporations. As a source of corporate funds, CP is a substitute for a bank's line of credit and other short-term loans provided by a financial institution. Some companies use the proceeds from CP sales to finance their cash flow needs between the time when they pay workers, resource suppliers, and the like, and the time when they sell their products. Other companies use CP to provide their customers with financing for the purchase of their products, and some companies use CP as bridge financing for their long-term investments, including corporate takeovers. For example, a company might sell CP to finance the construction of a plant or office building, with the CP paid off with long-term permanent financing from a loan from a financial institution or from a bond sale.

CP investors include money market funds, pensions, insurance companies, bank trust departments, other corporations, and governments. Many of the institutional investors purchase CP as part of their liquidity investments. Other corporations and state and local governments usually buy CP when they have temporary excess cash balances that they want to invest for a short period before they are needed to pay workers, accounts payable, accrued expenses, and other short-term liabilities. Finally, money market funds include CP in their portfolios with other money market securities. Money market funds are one of the largest investors in CP.

Direct Paper

CP issuers can be divided into finance companies and nonfinance companies, with the paper being either direct paper or dealer paper. As the name suggests, ***direct paper*** is sold by the issuing company directly to investors, instead of through dealers. The issuing companies include the subsidiaries of large companies, referred to as ***captive***

finance companies, bank holding companies, independent finance companies, and non-financial corporations. Frequently, these companies employ sales forces to place their CP with large institutional investors. The major captive finance companies selling direct paper are General Motors Acceptance Corporation (GMAC), Ford Credit Corporation, and GE Capital. These companies use the proceeds from their CP sales to finance installment loans and other credit loans extended to customers buying the products of their parent companies. GE Capital, for example, has had between $50 billion and $70 billion CP outstanding over the last several years and has been issuing CP for 50 years. Bank holding companies use CP sales to finance equipment purchases they lease to businesses, working capital loans, and installment loans.

Dealer Paper

Dealer paper, also called ***industrial paper***, is the CP of corporations sold through CP dealers. Historically, the dealer's market for CP has been dominated by the major investment banking firms. In 1987, though, the Federal Reserve gave the subsidiaries of bank holding companies permission to underwrite CP. This action served to increase the competition among CP dealers. Some of the major CP dealers include Bank of America, Goldman Sachs, First Boston Credit Suisse, Citicorp, and Banker's Trust. These dealers usually buy the CP from the issuer, mark it up (usually about one percent), and then resell it.

Features of Commercial Paper

Zero-Coupon Most CP issues are sold on a pure discount basis, although there are some that are sold with coupon interest. CP is quoted on a discount basis like T-bills, with a year being 360 days. The yields on CP are higher than the yields on T-bills, reflecting the credit and liquidity risk associated with CP.

Maturity The original maturities of CP range from three days (weekend paper) to 270 days, with the average original maturity being 60 days. The Securities Act of 1933 exempts companies issuing CP from registering with the SEC if the issue is less than 270 days. As a result of this provision, many CP issues have original maturities of less than 270 days; this reflects the desire by issuers to avoid the time-consuming SEC registration. CP can also be used as collateral for a bank that wants to borrow from the Fed discount window, provided the CP's maturity does not exceed 90 days. As a result, many CP issues have maturities of less than 90 days.

Denominations CP issues are usually sold in denominations of $100,000, although some are sold in $25,000 denominations. CP investors tend to hold their paper to maturity. As a result, the secondary market of CP is small.

Security and Line of Credit CP is often described as unsecured. The unsecured feature of CP means that there is no specific asset being pledged to secure the issue. Many CP issuers back up their paper with an unused line of credit from a bank. The line of credit is a safeguard in the event the CP issuer cannot pay off the principal or sell new CP to finance the principal payment on the maturing issue. CP issuers often roll CP, selling new issues to pay off maturing ones. For this commitment, the bank

TABLE 6.3 CP Ratings

Category	S&P	Moody's	Fitch
Investment Grade	A-1+	P-1	F-1+
	A-1	P-2	F-1
	A-2	P-3	F-2
	A-3		F-3
Noninvestment	B	Not Primed	F-S
Grade	D		
In Default	D		D

charges a fee of between 0.5% and 1% of the issue. In return, the CP issuer is able to reduce default risk and lower the rate he has to pay by an amount at least equal to the fee.

Credit Enhancements CP is assigned quality ratings by the major rating companies (see Table 6.3). Issuers of CP tend to have high credit ratings. Some smaller and less well-known companies also issue CP. These companies often issue CP with credit supports—***credit-supported CP***. Credit-supported CP includes issues backed by letters of credit. Paper sold with this type of credit enhancement is called ***LOC paper*** or ***documented paper***. Credit enhancements can also take the form of a surety bond from an insurance company. Finally, instead of a credit enhancement, some companies collateralize their issue with other assets—***asset-backed CP***. Included in this group of asset-based paper is securitized CP, often issued by a bank holding company. In these cases, a bank holding company sells CP to finance a pool of credit card receivables, leases, or other short-term assets, with the assets being used to secure the CP issue.

In recent years, there has been a decline in the market for medium- and low-quality CP. This decline is partly due to the reluctance of banks to provide backup line of credit facilities and partly due to SEC Rule 2a-7 of the Investment Company Act of 1970 that governs the quality standards of CP held by money market funds. This rule constrains money market funds to investments in ***eligible paper*** as defined by Tier-1 (eligible paper that is rated "1" by at least two of the rating agencies) and Tier-2 (eligible paper that is not Tier-1). SEC Rule 2a-7 specifies that money market funds may hold no more than 5% of their assets in Tier-1 paper in any industrial issuer and no more than 1% of their assets in Tier-2 paper of any industrial issuer, and that Tier-2 paper may not represent more than 5% of the fund's assets.

6.7 CONCLUSION

In the early 1980s, Chrysler Corporation issued a variable-rate subordinated debenture with a maturity of 10 years and exchangeable at Chrysler's option into a 10-year, fixed-rate note with the rate to be set at 124% above the 10-year rate on Treasury notes. The variable rate paid on the subordinated security made the note relatively attractive to investors given this period of high interest rates, and the option to exchange to a fixed-rate note was potentially beneficial to Chrysler given interest rates were declining at the time of the issue. This security is only one example of the different ways in which corporations structure debt instruments. Given the types of assets being financed and the conditions and risk-return preferences of the financial markets, there are many different types of corporate debt securities extant in the market. The differences that we observe among bonds, in turn, are reflected in different interest rates payments (fixed, floating, or discount), original maturities (CP, medium-term notes, corporate notes, and corporate bonds), option features (callable bonds, redemption features, and putable bonds), sinking fund arrangements, security (collateral, credit enhancements, and guarantees), and protective covenants. In this chapter, we have delineated many of these features that serve to differentiate the many types of corporate debt securities offered in the financial markets. In the next two chapters, we continue the same analysis for securities issued by the various government bodies: Treasury, federal agencies, and municipalities.

KEY TERMS

absolute priority rule (APR)
adjustable-rate note or variable-rate note
after-acquired property clause
asset-backed CP
assumed bond
average life
bearer bonds
book-entry form
bullet bonds
call premium
captive finance companies
Chapter 11
Chapter 7
closed-end bonds
collateral trust bond
commodity-linked bond
convertible bond
credit enhancements
credit-supported CP
currently callable issue
dealer paper or industrial paper
debentures
deep-discount bonds
deferred call
deferred-interest bonds
direct paper
documented paper
eligible paper
equipment trust bond
event risk
extendable bonds
extendable reset bonds
external financing
firm commitment
floatation cost
floater
fully amortized
guaranteed bonds
income bonds
indenture
internal financing
inverse floater
junior lien
letters of credit
leveraged firm
LOC paper
London interbank offer rate, LIBOR
make-whole premium provision
medium-term notes (MTNs)
mortgage bond
net worth maintenance clause
notice of redemption
offer-to-redeem clause
open market sales
open-end bond
original-issue discount (OID)
participating bonds
payment-in-kind (PIK) bonds
Philadelphia Plan and rolling stock
poison put
prime rate
private placement

prospectus
protective or restrictive covenants
putable bond or put bond
qualified investment buyers (QIBs)
red herring
refunding
registered bonds
release and substitution provision
reset bonds
reverse inquiry
rule 415, shelf registration rule
SEC Rule 144A
secured bonds
selling group agreement
senior lien
serial bond
sinking fund
standby underwriting agreement
structured MTNs
subordinate debentures
surety bond
trustee
tunnel, funnel or blanket sinking funds
unsecured bonds
unsubordinate debentures
voting bonds
warrant
yield maintenance provision

WEB INFORMATION

Corporate Bond Information:

1. Corporate Bond Price Information:
   FINRA
   Go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds." For a bond search click "Corporate Bond" tab and then click "Advanced Bond Search" to find corporate bonds with certain features.
   Wall Street Journal
   Go to http://online.wsj.com/public/us, "Market Data" and "Bonds, Rates, and Credit Markets."
   Yahoo.com
   Go to http://finance.yahoo.com/bonds, click "Advanced Bond Screener" and click "Corporate Bond" tab, and then provide information for search.
   Investinginbonds.com
   Go to http://investinginbonds.com/; click "Corporate Market At-a-Glance."
2. For recent information and reports on the corporate bond market trends go to investinginbonds.com: www.investinginbonds.com/.
   Click "Recent Commentary."
3. Rating Agencies:
   - www.moodys.com
   - www.standardandpoors.com
   - http://reports.fitchratings.com
4. For the Moody Study of historical default rates for corporate bonds, go to "Ratings, Methodologies, and Performance," "Historical Performance," and look for corporate default information:
   - www.moodys.com

(*continued*)

(*Continued*)

5. For information on security laws, go to www.sec.gov and click on "Market Regulations" under "SEC Divisions."
6. For more information on corporate bankruptcy, see "What Every Investor should Know about Corporate Bankruptcy," at www.sec.gov/investor/pubs/bankrupt.htm.
7. For information on MTNs, go to www.federalreserve.gov/releases/medterm/about.htm.
8. For information on the size of the market for MTNs, go to www.federalreserve.gov/releases/medterm.

PROBLEMS AND QUESTIONS

1. What are the major benefits and costs to a corporation of financing its operations with debt instead of equity?
2. Define and briefly explain the following terms:
   a. Amortization
   b. Deep discount bond
   c. Floater
   d. Protective covenants
   e. Serial bond
   f. Option redemption provision
   g. Deferred call feature
   h. Nonrefundable clause
   i. Debenture
   j. Sinking fund requirement
   k. Average life
   l. Registered bond
   m. Bearer bond
3. Explain some of the common features included in a sinking fund requirement.
4. Comment on the following statement: "By reducing the investor's principal risk, a sinking fund provision benefits the investor."
5. What is the average life of a debt issue with a $100 million par value and 10-year maturity that has a sinking fund that makes equal payments in years 7 through 10?
6. What feature of a zero-coupon or deep discount bond does an institutional investor such as a pension fund find attractive?
7. In the early 1980s, Beatrice Foods issued a 10-year, $250 million zero-coupon issue priced at $255 per $1,000 face value. What was the bond's initial YTM?
8. What is the difference between call protection and refunding protection?

9. ABC is issuing a bond with a maturity of 25 years and 10% coupon. The bond is callable with the first-year call price equal to the offering price plus the coupon; thereafter the call price decreases by equal amounts to equal par at year 20; thereafter the call price is equal to par. If the bond were sold at 95 ($950 for F =$1,000), what would be the call prices for each year?
10. Define the major types of secured bonds.
11. What is a release and substitution provision?
12. Explain how an equipment trust bond is created as part of a lease-and-buy-back arrangement.
13. What are some of the provisions that are included in a collateral trust bond?
14. Define the following:
    a. Priority of claim
    b. Closed-end bond
    c. Open-end bond
    d. After-acquired property clause
    e. Subordinate debenture
    f. Guaranteed bond
    g. Credit enhancement
15. What are some of the provisions in a debenture that enhance its creditworthiness?
16. Why are guaranteed bonds not considered risk free?
17. G&P is planning to construct a $250 million manufacturing and processing plant for the national production of its patented calorie-free chips. G&P's Marketing Research Division has estimates that G&P will gain a significant market share of the U.S. snack-food market and should maintain that share for a period of at least 10 years—a period extending beyond the expiration of its patent. Suppose you work for a large investment-banking firm that advises G&P on its debt issues and who would like to eventually bid on underwriting the issue. To help you in providing advice on the debt issue, identify the pertinent features of the debt issue that need to be considered, the alternatives, and factors that need to be considered in structuring the bond issue.
18. Define each of the following bonds and their features:
    a. Income bond
    b. Participating bond
    c. Deferred coupon bond
    d. Payment-in-kind bond
    e. Tax-exempt bond
    f. Bonds with warrants
    g. Convertible bond
    h. Voting bonds
    i. Assumed bond
    j. Bonds with put options
    k. Credit-sensitive notes

l. Extendable notes
m. Commodity-linked bonds

19. Discuss the nature of protective covenants.
20. Discuss the types of protective covenants that can be found in a bond contract.
21. Define event risk and the protective covenants that can be used to protect bondholders against such risk.
22. Explain the distinction between
    a. Insolvency and default
    b. Insolvency and illiquidity
23. Explain the steps in a bankruptcy process that are taken to determine reorganization.
24. List the steps involved in an open market sale of a new bond issue.
25. What is underwriting risk? Provide an example.
26. Define SEC Rule 415. What is the significance of the rule?
27. What is a privately placed bond issue and how does it differ from an open market issue?
28. Define SEC Rule 144A. What is the significance of the rule?
29. List some of the features that characterize the secondary market for corporate bonds.
30. List some of important features of commercial paper.
31. Explain the typical process a corporation would go though in selling medium-term notes.
32. What is meant by reverse inquiry?
33. What is the main feature contributing to the growth of the MTN market?

WEB EXERCISES

1. Many corporate bond issues have different features. Identify some of these issues by going to the FINRA, Investinginbonds.com, or Yahoo! sites:
   - FINRA, www.finra.org/index.htm; "Sitemap," "Market Data," and "Bonds," then click "Corporate" tab and "Advanced Bond Search."
   - Yahoo!, http://finance.yahoo.com/bonds; click "Advanced Bond Screener"
   - Investinginbonds.com site: www.Investinginbonds.com; click "Corporate Price Data" and "Corporate-Market Data at a Glance."
2. Study the different types of credit issued by a particular company by going to FINRA:
   - FINRA: www.finra.org/index.htm
   - Click "Sitemap."

- Click "Company Information."
- Enter company.
- Under Bonds Issued, click one of the company's issues to obtain information on the credit.

3. Select a corporate bond from FINRA (or the one from Exercise 2) and then go to Moody's to study its credit history and profile:
   - Go to www.moodys.com.
   - Enter CUSIP on "Quick Search" and click "Go."
   - Click "Credit History."
4. Investinginbonds.com provides a review of conditions and trends in the corporate bond market. Go to their site and examine recent conditions: www.investinginbonds.com, "Corporate Market At-a-Glance." Look at "Market Headline News" and "Commentary and Analysis."
5. Study some of the recent corporate credits added to Moody's "Watch List." Go to www.moodys.com and click on "WATCHLIST."
6. Moody's provides information on default rates, ratings changes, and other credit information. Examine some of their information. To download the Moody study of historical default rates:
   - Go to www.moodys.com (registration required).
   - Click "Ratings, Methodologies, and Performance."
   - "Historical Performance."
   - Review the "Corporate Default" section.

NOTES

1. Small companies, as well as some large ones, obtain their funds from loans from financial institutions. As noted in Chapter 1, many companies obtain the financing of their short-term assets by obtaining lines of credit from commercial banks. The rates banks provide on the short-term loans are often quoted in terms of the prime rate (also called reference rate). This is the rate banks provide to their most creditworthy clients. Banks and other financial institutions also provide businesses with intermediate and long-term loans. Bank loans and the market for bank loans are examined in Chapter 9.
2. For a corporate bond that pays 12% per $1,000 par, the accrued interest is $10 per month or $0.3333 per day. Thus, for 3 months the accrued interest is $30, and for 3 months and 25 days the accrued interest is $38.33 [= 30 + (25)($0.3333)].
3. An exception to this trend was the $150 million bond issue of Coca Cola in 1993 that had a maturity of 100 years.
4. For more discussion of this and some other cases, see Fabozzi, Wilson, and Todd, "Corporate Bonds," in *The Handbook of Fixed Income Securities,* ed. F. Fabozzi, 6th ed. (New York: McGraw-Hill, 2001), 265–271.
5. Event risk is default risk resulting from dramatic and unexpected changes. As such, it includes not only takeovers and corporate restructuring, but also natural or industrial accidents or government regulatory change. In the case of the latter, event risk can have external or spillover effects on other firms in the industry.
6. It should be noted that in terms of priorities it could be the case that a default could lead to the company being acquired by another company. The new company, in turn, may

be able to get the bondholders to agree to subordinate their debt to a new issue; this is quite possible if the bondholders determined that the sale of the pledged assets would be inadequate.

7. Companies often view subordinate debentures as an alternative to preferred stock financing. Preferred stock can be thought of as a limited ownership share. It provides its owners with only limited income potential in the form of a stipulated dividend (preferred dividend) that is usually expressed as a percentage of a stipulated par value. Preferred stock also gives its holders fewer voting privileges and less control over the business than common stock does. To make preferred stock more attractive, companies frequently sell preferred with special rights. Among the most common of these special rights is the priority over common stockholders over earnings and assets upon dissolution and the right to cumulative dividends: If preferred dividends are not paid, then all past dividends must be paid before any common dividends are paid. Subordinated debentures, in turn, are often sold at rates comparable to preferred stock; as a debt instrument, though, subordinated debt's interest is tax deductible whereas preferred dividends are not.
8. See Julian R. Franks and W.N. Torous, *Journal of Finance*, 1989; Lawrence A. Weiss, *Journal of Financial Economics*, 1990; Frank Fabozzi et al., *Journal of Fixed Income*, 1993; D. G. Braid and T. H. Jackson, *University of Chicago Law Review*, 1988; Karen Wruck, *Journal of Financial Economics*, 1990; and Michael Jensen, *Harvard Business Review*, 1989.
9. Investment banking firms often assist firms in privately placing securities, often using best effort.

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CHAPTER 7

Treasury and Agency Securities

7.1 INTRODUCTION

The U.S. Treasury security market began over 230 years ago when Alexander Hamilton, the first Treasury secretary, sold government securities to finance the new country's debt. The U.S. debt in 1790 consisted of $54 million in national debt and $25 million in assumed state debt, with most of the debt incurred as a result of the Revolutionary War. Just as it is today, the U.S. debt in 1790 was quite large compared to the debt of other sovereign nations. Today, the U.S. Treasury is the largest debt issuer in the world. Its size, as well as its wide distribution of ownership and default risk-free feature, makes the rates on Treasury securities the benchmark for all other securities. In this chapter, we extend our analysis of debt securities by examining the markets for U.S. Treasury securities and debt instruments issued by U.S. federal agencies.

7.2 TREASURY INSTRUMENTS

The U.S. Treasury is responsible for implementing the fiscal policy of the federal government and managing the federal government's enormous debt. As shown in Table 7.1, in the year 2008 the federal government raised over $2.524 trillion in revenue from income (45%), social insurance (36%), and corporate taxes (12%), and it spent over $2.983 trillion on welfare and individual security programs [social security, health care, and income security programs (64%)], national defense (21%), interest on the federal government debt (8%), physical resources [energy, commerce and housing, transportation, and regional development (5%)] and other expenditures (2%). The government's excess of tax expenditures over revenues in 2008 equated to a deficit of $459 billion.[1]

Since 1930 the U.S. federal government has operated with a deficit in almost every year, with the deficits growing dramatically over the last two decades and projected to reach over $1.8 trillion in 2009 (see Table 7.2). One noticeable exception to annual deficits was the three-year period from 1999 to 2001 when the U.S. government operated with surpluses of $126 billion in 1999, $236 billion in 2000, and $128 billion in 2001. The accumulation of deficits over the years has, in turn, contributed to a total government indebtedness of over $12.687 trillion as of 2009.

To finance the government's deficit each year and to manage its debt (refinancing maturing issues), the Treasury sells a number of securities. All of the securities sold

TABLE 7.1 Federal Government Revenues and Expenditures by Source, 2005–2009

Fiscal Year	2005	2006	2007	2008	2009[Est]
Revenue by Source (in millions of $)					
Individual income taxes	927,222	1,043,908	1,163,472	1,145,747	953,006
Corporation income taxes	278,282	353,915	370,243	304,346	146,758
Social insurance and retirement receipts	794,125	837,821	869,607	900,155	899,217
Excise taxes	73,094	73,961	65,069	67,334	66,280
Other	81,136	97,649	99,848	106,744	91,393
Total receipts	2,153,859	2,407,254	2,568,239	2,524,326	2,156,654
Expenditures by Source (in millions of $)					
National defense	495,326	521,840	551,286	616,097	690,308
Human resources	1,586,122	1,672,076	1,758,493	1,895,740	2,160,041
Education, training, and social services	97,567	118,560	91,676	91,306	79,342
Health	250,614	252,780	266,435	280,646	353,445
Medicare	298,638	329,868	375,407	390,758	430,779
Income security	345,847	352,477	365,975	431,313	519,289
Social security	523,305	548,549	586,153	617,027	680,509
Veterans benefits and services	70,151	69,842	72,847	84,690	96,677
Physical resources	130,177	164,800	133,859	161,950	931,424
Energy	429	782	−860	628	8,773
Natural resources and environment	28,023	33,055	31,759	31,883	42,187
Commerce and housing credit	7,567	6,188	488	27,871	758,152
Transportation	67,894	70,244	72,905	77,616	94,310
Community and regional development	26,264	54,531	29,567	23,952	28,002
Net interest	183,986	226,603	237,109	252,757	142,738
Other functions	141,818	138,366	130,431	142,579	165,012
International affairs	34,595	29,549	28,510	28,917	34,722
General science, space, and technology	23,628	23,616	25,566	27,793	31,150
Agriculture	26,566	25,970	17,663	18,388	20,398
Administration of justice	40,019	41,016	41,244	47,138	53,313
General government	17,010	18,215	17,448	20,343	21,854
Allowances	....	....	....	....	3,575
Undistributed offsetting receipts	−65,224	−68,250	−82,238	−86,242	−91,681
Total, federal outlays	2,472,205	2,655,435	2,728,940	2,982,881	3,997,842

Source: U.S. Government Printing Office: www.gpo.gov/fdsys/search/pagedetails.action?granuleId=&packageId=BUDGET-2010-TAB.

TABLE 7.2 Federal Government Receipts, Outlays, and Surplus and Deficits (–): 1980–2009

	In Current Dollars (billions)			As Percentages of GDP		
Fiscal Year	Receipts	Outlays	Surplus or Deficit(−)	Receipts	Outlays	Surplus or Deficit(−)
1980	$517.1	$590.9	−$73.8	19.0	21.7	−2.7
1981	$599.3	$678.2	−$79.0	19.6	22.2	−2.6
1982	$617.8	$745.7	−$128.0	19.2	23.1	−4.0
1983	$600.6	$808.4	−$207.8	17.4	23.5	−6.0
1984	$666.5	$851.9	−$185.4	17.3	22.1	−4.8
1985	$734.1	$946.4	−$212.3	17.7	22.8	−5.1
1986	$769.2	$990.4	−$221.2	17.5	22.5	−5.0
1987	$854.4	$1,004.1	−$149.7	18.4	21.6	−3.2
1988	$909.3	$1,064.5	−$155.2	18.1	21.2	−3.1
1989	$991.2	$1,143.8	−$152.6	18.3	21.2	−2.8
1990	$1,032.1	$1,253.1	−$221.0	18.0	21.8	−3.9
1991	$1,055.1	$1,324.3	−$269.2	17.8	22.3	−4.5
1992	$1,091.3	$1,381.6	−$290.3	17.5	22.1	−4.7
1993	$1,154.5	$1,409.5	−$255.1	17.5	21.4	−3.9
1994	$1,258.7	$1,461.9	−$203.2	18.1	21.0	−2.9
1995	$1,351.9	$1,515.9	−$164.0	18.5	20.7	−2.2
1996	$1,453.2	$1,560.6	−$107.4	18.9	20.3	−1.4
1997	$1,579.4	$1,601.3	−$21.9	19.3	19.6	−0.3
1998	$1,722.0	$1,652.7	$69.3	20.0	19.2	0.8
1999	$1,827.6	$1,702.0	$125.6	20.0	18.6	1.4
2000	$2,025.5	$1,789.2	$236.2	20.9	18.4	2.4
2001	$1,991.4	$1,863.2	$128.2	19.8	18.5	1.3
2002	$1,853.4	$2,011.2	−$157.8	17.9	19.4	−1.5
2003	$1,782.5	$2,160.1	−$377.6	16.5	20.0	−3.5
2004	$1,880.3	$2,293.0	−$412.7	16.3	19.9	−3.6
2005	$2,153.9	$2,472.2	−$318.3	17.6	20.2	−2.6
2006	$2,407.3	$2,655.4	−$248.2	18.5	20.4	−1.9
2007	$2,568.2	$2,728.9	−$160.7	18.8	20.0	−1.2
2008	$2,524.3	$2,982.9	−$458.6	17.7	21.0	−3.2
2009 estimate	$2,156.7	$3,997.8	−$1,841.2	15.1	28.1	−12.9
2010 estimate	$2,332.6	$3,591.1	−$1,258.4	15.8	24.4	−8.5
2011 estimate	$2,685.4	$3,614.8	−$929.4	17.3	23.3	−6.0
2012 estimate	$3,075.3	$3,632.7	−$557.4	18.7	22.1	−3.4
2013 estimate	$3,305.1	$3,817.5	−$512.3	18.9	21.8	−2.9
2014 estimate	$3,480.1	$4,016.0	−$535.9	18.9	21.8	−2.9

Source: U.S. Government Printing Office: www.gpo.gov/fdsys/search/pagedetails.action?granuleId=&packageId=BUDGET-2010-TAB.

by the Treasury are backed by the full faith and credit of the U.S. government. As such, they are considered default free.[2] The Treasury's securities can be broken into marketable and nonmarketable securities. Of the total government debt of $12.687 trillion in 2009, investors held approximately $8.531 trillion in marketable securities (see Table 7.3). Marketable securities include Treasury bills (T-bills), T-notes,

TABLE 7.3 Federal Debt, 1990–2009

	In Millions of Dollars					As Percentages of GDP				
			Equals: Held by the Public					Equals: Held by the Public		
End of Fiscal Year	Gross Federal Debt	Less: Held by Federal Government	Total	Federal Reserve System	Other	Gross Federal Debt	Less: Held by Federal Government Accounts	Total	Federal Reserve System	Other
1990	3,206,290	794,733	2,411,558	234,410	2,177,147	55.9	13.9	42.0	4.1	37.9
1991	3,598,178	909,179	2,688,999	258,591	2,430,408	60.6	15.3	45.3	4.4	41.0
1992	4,001,787	1,002,050	2,999,737	296,397	2,703,341	64.1	16.1	48.1	4.7	43.3
1993	4,351,044	1,102,647	3,248,396	325,653	2,922,744	66.1	16.8	49.4	5.0	44.4
1994	4,643,307	1,210,242	3,433,065	355,150	3,077,915	66.7	17.4	49.3	5.1	44.2
1995	4,920,586	1,316,208	3,604,378	374,114	3,230,264	67.2	18.0	49.2	5.1	44.1
1996	5,181,465	1,447,392	3,734,073	390,924	3,343,149	67.3	18.8	48.5	5.1	43.4
1997	5,369,206	1,596,862	3,772,344	424,518	3,347,826	65.6	19.5	46.1	5.2	40.9
1998	5,478,189	1,757,090	3,721,099	458,182	3,262,917	63.5	20.4	43.1	5.3	37.8
1999	5,605,523	1,973,160	3,632,363	496,644	3,135,719	61.4	21.6	39.8	5.4	34.4
2000	5,628,700	2,218,896	3,409,804	511,413	2,898,391	58.0	22.9	35.1	5.3	29.9
2001	5,769,881	2,450,266	3,319,615	534,135	2,785,480	57.4	24.4	33.0	5.3	27.7
2002	6,198,401	2,657,974	3,540,427	604,191	2,936,235	59.7	25.6	34.1	5.8	28.3
2003	6,760,014	2,846,570	3,913,443	656,116	3,257,327	62.6	26.3	36.2	6.1	30.2
2004	7,354,657	3,059,113	4,295,544	700,341	3,595,203	63.9	26.6	37.3	6.1	31.3
2005	7,905,300	3,313,088	4,592,212	736,360	3,855,852	64.6	27.1	37.5	6.0	31.5
2006	8,451,350	3,622,378	4,828,972	768,924	4,060,048	65.0	27.8	37.1	5.9	31.2
2007	8,950,744	3,915,615	5,035,129	779,632	4,255,497	65.6	28.7	36.9	5.7	31.2
2008	9,985,757	4,183,032	5,802,725	491,127	5,311,598	70.2	29.4	40.8	3.5	37.3
2009[Est]	12,867,455	4,336,088	8,531,367	N/A	N/A	90.4	30.4	59.9	N/A	N/A

Source: U.S. Government Printing Office: www.gpo.gov/fdsys/search/pagedetails.action?granuleId=&packageId=BUDGET-2010-TAB.

T-bonds, and Treasury inflation-indexed bonds and notes.[3] The Treasury sells these securities using an auction method and there is an active secondary market trading the existing marketable Treasury securities. Nonmarketable Treasury debt, in turn, includes the state and local government series (SLGS), ***government account series***, U.S. savings bonds, and nonmarketable securities sold to foreign governments. Original investors hold these securities until they mature or are redeemed. The government account series is one of the largest portions of the nonmarketable securities sold. This series includes Treasury securities sold to government agencies such as Social Security and the Tennessee Valley Authority. These agencies use their excess funds to purchase Treasury securities. Currently, the series accounts for over 30% of the Treasury's total debt holdings.

Treasury Bills

Treasury bills are short-term instruments sold on a pure discount basis in multiples of $1,000 (par) from the minimum, with the minimum denomination being $1,000. The interest on a T-bill is the difference between the face value and the price paid. This interest, in turn, is subject to federal income taxes, but not state and local taxes. T-bills with original maturities of 13 weeks (91 days) and 26 weeks (182 days) are sold weekly on a regular basis. The Treasury also sells special types of T-bills on an irregular basis. Included with this irregular series are ***strip bills***. This is a package of T-bills with different maturities in which the buyer agrees to buy bills at their bid price for several weeks. The Treasury also issues additional amounts of an existing security (T-bills, T-bonds, and T-notes). Such offerings are known as ***reopenings***. For example, the Treasury may offer 13-week T-bills as a reopening of a previously issued 26-week T-bill. All T-bills are issued and registered in a book-entry form, with the computerized record of ownership maintained by the Federal Reserve at their offices in Washington. At maturity, the Treasury sends a check to the investor of record, unless the holder has requested payment in terms of new T-bills.[4] In an effort to make the securities more attractive, the Fed has set up a direct purchase option, allowing investors to purchase T-bills over the Internet.

As noted in Chapter 2, the *Wall Street Journal* and other sites (*Wall Street Journal* site, http://online.wsj.com/public/us) provide information on existing T-bill issues (see Figure 2.3 and Table 7.6 for quotes of Treasury securities).

Recall, T-bill yields are calculated as an annualized discount yield (R_D) (also called the ***banker's discount yield***); this is the annualized return [principal (F) minus price (P_0)] specified as a proportion of the bill's principal:[5]

$$R_D = \frac{F - P_0}{F}\frac{365}{Days\ to\ maturity}$$

For example, a 182-day bill sold at auction at 97.606194 would be quoted at a discount rate of 4.735%:

$$R_D = \frac{100 - 97.606194}{100}\frac{360}{182} = .04735$$

Solving the equation for the discount yield for P_0 gives us the formula for the bid or ask price given the dealer's discount yield:

$$P_0 = F[1 - (R_D)(Days\ to\ maturity/360]$$
$$P_0 = 100[1 - (.04735)\ (182/360)] = 97.606194$$

In order to compare the yield on T-bills with other securities, one can use the annual YTM with a 365-day year [YTM = $(F/P_0)^{365/\text{Days to maturity}} - 1$; YTM = $(100/97.606194)^{365/182} - 1 = .04979$] or calculate the bond-equivalent yield. As noted in Chapter 2, the bond-equivalent yield is obtained by doubling the semiannual rate that equates the price of the bond to the present value of its cash flows. For T-bills with days to maturity (N) less than 182, the formula for the bond-equivalent (BE) yield given the discount yield is

$$BE\ yield = \frac{365\,R_D}{360 - N(R_D)}$$

Thus:

$$BE\ yield = \frac{365(.04735)}{360 - 182(.04735)} = .049185$$

For maturities greater than 182 days, the formula for the bond-equivalent yield given the price of the T-bill per \$1 face value (p) is

$$BE\ yield = \frac{-(2N/365) + 2\sqrt{(N/365)^2 - [(2N/365) - 1][1 - (1/p)]}}{(2N/365) - 1}$$

Treasury Bonds and Notes

Treasury bonds and notes are the Treasury's coupon issues. Both are identical except for maturity: T-notes have original maturities up to 10 years, whereas T-bonds have maturities ranging between 10 and 30 years. Both are sold in denominations of \$1,000 or more, and both pay semiannual coupon interest. Like all Treasury securities, interest income from T-bonds and T-notes is subject to federal taxes, but not state and local. Since 1984, the Treasury has not issued callable bonds.

All Treasury coupon securities are issued by the Treasury at prices approximately equal to par.[6] Like T-bills, notes and bonds are issued in a book-entry form with the investor's name and amount maintained in a computerized account. The Treasury currently issues notes with two-, five-, and 10-year maturities. They also resumed issuing the 30-year T-bond in February 2006. New two-year T-notes are sold every month, while five-year and 10-year notes are sold quarterly and T-bonds are sold semiannually (see Table 7.4). Sometimes when bonds or notes are sold in order to refund an earlier issue, they are offered only to the holders of the bond being replaced. This special type of issue takes the form of an advance refunding in which the Treasury sets the maturity of the new issue equal to the remaining life of the bond

TABLE 7.4 Treasury Auction Schedule

Issue	Frequency
13-week T-bill	Weekly
26-week T-bill	Weekly
Two-year T-note	Monthly
Five-year T-note	Quarterly
10-year T-note	Quarterly
30-year T-bond	Semiannual

Electronic Treasury—Public Debt Marketable Securities, September 30, 2009

Marketable Issues (electronic)	Total Securities Issued Dollar Value (billions)	Total Securities Outstanding Dollar Value (billions)
Bills	\$386	\$1,993
Notes	\$172	\$3,774
Bonds	\$ 12	\$ 680
TIPS	(\$1)	\$ 552
Totals	\$569	\$6,999

Source: U.S.Treasury: www.treasurydirect.gov/govt/reports/pd/pd_electreas.htm.

being replaced to ensure the holder his original maturity. In addition, the Treasury established in 2000 a debt buyback program in which it buys back outstanding issues by purchasing them in the secondary market.

Information on T-notes and T-bonds can be found at various Web sites (e.g., *Wall Street Journal* site, http://online.wsj.com/public/us, FINRA, www.finra.org/index.htm, and Investinginbonds, http://investinginbonds.com/). The information at the Wall Street Journal site includes the dealer's bid and asked prices expressed as a percentage of the face value, or equivalently, as the price of a bond with a \$100 par value. Recall from Chapter 2 that the numbers to the right of the decimals on the bid and ask prices are in 32nds and not the usual 100s (e.g., 98–14 or $98^{14/32}$ or 98.4375). A plus sign next to the decimal means that $^1/_2$ of a 32nd is added to the price (97–14+ or $97^{14.5/32} = 98.453125$).

Treasury Inflation-Indexed Bonds: TIPS

Although Treasury securities are considered default free, they are subject to market risk that we examined in Chapter 5 and also to ***purchasing-power risk***. Purchasing-power risk is the risk that the rate of return earned from an investment is less than the inflation rate. Equivalently, it is the risk of a negative real interest rate, where the ***real interest rate*** is the actual or nominal rate minus the inflation rate: Real interest = Nominal interest – Inflation.

To address purchasing-power risk, in 1997 the Treasury began offering Treasury inflation-indexed bonds called ***Treasury Inflation Protection Securities*** or simply ***TIPS***. Inflation-adjusted securities, though, are not new or unique. Many countries have offered such securities for a number of years, and a number of corporations, agencies, and municipalities offer or have offered inflation-adjusted bonds. The U.S.

TABLE 7.5 Treasury Strip Cash Flow

Year	Annual Inflation	Semiannual Inflation	Inflation-Adjusted Principal	TIPS Cash Flow
0.5	3.00%	1.50%	$1,015.00	$20.30
1.0	3.00%	1.50%	$1,030.23	$20.60
1.5	3.00%	1.50%	$1,045.68	$20.91
2.0	3.00%	1.50%	$1,061.36	$21.23
2.5	3.00%	1.50%	$1,077.28	$21.55
3.0	3.00%	1.50%	$1,093.44	$21.87
3.5	3.00%	1.50%	$1,109.84	$22.20
4.0	3.00%	1.50%	$1,126.49	$22.53
4.5	3.00%	1.50%	$1,143.39	$22.87
5.0	3.00%	1.50%	$1,160.54	$23.21
5.5	3.00%	1.50%	$1,177.95	$23.56
6.0	3.00%	1.50%	$1,195.62	$23.91 + $1,195.62

Source: Wall Street Journal site, http://online.wsj.com/public/us.

Treasury's TIPS are patterned after the successful inflation-adjusted bonds introduced in Great Britain. They are structured so that each period's coupon payment is equal to a specified fixed rate times an inflation-adjusted principal, and at maturity, the bond pays the larger of the inflation-adjusted principal or the original par value. For example, suppose the Treasury issues three-year TIPS with a nominal principal of $1,000 and semiannual coupon rate of 2%. If there is no inflation in the ensuing three years, then the TIPS will pay bondholders $20 semiannually for three years and $1,000 at maturity. If there is inflation, as measured by the consumer price index, CPI-u, then the Treasury will adjust the nominal principal.[7] For example, suppose the United States experiences an annual inflation rate of 3% or a 1.5% semiannual rate for the first semiannual period of the bond. In this case, the inflation-adjusted principal would be $1,015 and a bondholder would receive a semiannual coupon of $20.30 [= ($1,015)(.02)]. If the 1.5% semiannual inflation continues for each period of the bond, then the bondholder would receive coupon interests in each of the next five semiannual periods shown in Table 7.5 and a principal at maturity of $1,195.62.

The inflation-adjusted returns from TIPS offset the loss in purchasing power resulting from inflation. Thus, even though the principal and interest fluctuate with the CPI, the purchasing power of each payment and the real rate of return are fixed. Because the real rate of return is fixed, Treasury inflation-indexed bonds are attractive to retirees who have their retirement funds invested in fixed-income securities. Inflation-indexed bonds also have relatively low correlations with other bonds, and as such provide some diversification benefits when included in a fixed-income portfolio. Table 7.6 shows a partial listing of bid and ask prices of TIPS offered by dealers in the secondary market on October 6, 2009.

Treasury Strips

In the 1980s, one of the more innovative instruments was introduced—the Treasury stripped security or ***Treasury strips***. A Treasury strip is formed by a dealer who

TABLE 7.6 Select T-Bonds, T-bills, TIPS, and Treasury Strips, October 6, 2009

T-bonds, August 31, 2009

1	2	3	4	5	6
Issue	Price	Coupon(%)	Maturity	YTM(%)	Current Yield(%)
T-BOND 7.500 15-Nov-2024	139.1	7.5	15-Nov-24	4.068	5.391
T-BOND 7.625 15-Feb-2025	141.12	7.625	15-Feb-25	4.06	5.403
T-BOND 6.875 15-Aug-2025	132.82	6.875	15-Aug-25	4.087	5.176
T-BOND 6.000 15-Feb-2026	121.96	6	15-Feb-26	4.163	4.919
T-BOND 6.750 15-Aug-2026	131.56	6.75	15-Aug-26	4.163	5.13
T-BOND 6.500 15-Nov-2026	128.42	6.5	15-Nov-26	4.189	5.061
T-BOND 6.625 15-Feb-2027	130.16	6.625	15-Feb-27	4.195	5.09
T-BOND 6.375 15-Aug-2027	127.02	6.375	15-Aug-27	4.232	5.019
T-BOND 6.125 15-Nov-2027	123.81	6.125	15-Nov-27	4.25	4.946
T-BOND 5.500 15-Aug-2028	116.02	5.5	15-Aug-28	4.269	4.74
T-BOND 5.250 15-Nov-2028	112.93	5.25	15-Nov-28	4.265	4.648
T-BOND 5.250 15-Feb-2029	112.9	5.25	15-Feb-29	4.274	4.65
T-BOND 6.125 15-Aug-2029	124.98	6.125	15-Aug-29	4.268	

T-bills, August 31, 2009

Traded on Friday, August 28, 2009

Maturity	Bid	Asked	Chg	Asked Yield
2009 Oct 22	0.115	0.108	0.01	0.109
2009 Nov 05	0.11	0.1	–0.003	0.101
2009 Dec 10	0.14	0.135	–0.007	0.137
2010 Jan 14	0.168	0.153	0.003	0.155
2010 Jan 21	0.168	0.158	unch.	0.16
2010 Mar 04	0.25	0.243	unch.	0.246
2010 Jun 03	0.308	0.303	unch.	0.307
2010 Jul 29	0.38	0.375	–0.013	0.381
2010 Aug 26	0.425	0.415	–0.01	0.422

TIPS

Maturity	Coupon	Bid	Asked	Chg	Yield*	Accrued Principal
2010 Jan 15	4.25	100.3	100.3	1	0.757	1280
2013 Jul 15	1.875	103.18	103.19	5	0.904	1173
2017 Jul 15	2.625	108.31	109	3	1.399	1039
2019 Jul 15	1.875	103.15	103.16	5	1.49	1009
2028 Jan 15	1.75	95.23	95.24	−7	2.03	1028
2032 Apr 15	3.375	123.08	123.09	−11	2.076	1213

*-Yld. to maturity on accrued principal

(continued)

TABLE 7.6 (*Continued*)

STRIPS Maturity	Bid	Asked	Chg	Asked Yield
Treasury Bond, Stripped Principal				
2009 Nov 15	99.989	99.999	0.001	0.01
2024 Nov 15	53.629	53.639	0.043	4.16
2025 Feb 15	52.88	52.89	0.024	4.19
2030 May 15	42.36	42.37	−0.104	4.21
2031 Feb 15	41.078	41.088	−0.105	4.21
2038 May 15	31.029	31.039	−0.138	4.13
Treasury Note, Stripped Principal				
2009 Nov 15	99.992	100	unch.	0
2015 Nov 15	84.994	85.004	−0.062	2.68
2018 Nov 15	74.027	74.036	−0.001	3.33
Stripped Coupon Interest				
2009 Nov 15	99.999	100	unch.	0
2010 Feb 15	99.993	100	unch.	0
2010 May 15	99.988	99.998	unch.	0
2010 Aug 15	99.795	99.805	0.014	0.23
2017 Aug 15	77.205	77.215	−0.031	3.32
2017 Nov 15	76.207	76.217	−0.031	3.38
2020 Nov 15	65.562	65.572	0.051	3.84
2021 Feb 15	64.852	64.862	0.015	3.85
2026 Nov 15	48.441	48.451	0.084	4.28
2030 Aug 15	41.452	41.462	−0.104	4.27
2030 Nov 15	41.017	41.027	−0.104	4.26
2031 Feb 15	40.586	40.596	−0.104	4.27
2031 May 15	40.139	40.149	−0.104	4.27
2031 Aug 15	39.717	39.727	−0.105	4.27
2038 May 15	30.179	30.189	−0.147	4.23

Source: Wall Street Journal site, http://online.wsj.com/public/us.

purchases a T-bond or T-note and then creates two general types of zero-coupon securities to sell to investors: a ***principal-only security*** (***PO***, also called the ***corpus*** and denoted *np* when quoted) and an ***interest-only security*** (***IO***, denoted *i* when quoted). As the name suggests, the PO security is a zero-discount bond that pays the T-bond's principal at its maturity; the IO securities are zero-discount instruments, with each paying a principal equal to the T-bond's coupon and with a maturity coinciding with the bond's coupon date. To create Treasury strips, a dealer could take a six-year U.S. Treasury note and strip it into 13 discount bonds, one maturing in 6 years and paying the T-bond's principal, the others paying principals equal to the bond's coupon interest and maturing on the coupon dates. For example, suppose a dealer purchased $500 million of five-year, 6% notes. The cash flow from the securities would be 10 semiannual payments of $15 million and a repayment of principal of $500 million at maturity. From this, the dealer could create 10 IO strips with each one representing a single-payment claim on each interest payment and with a maturity date matching the coupon payment dates and a PO strip paying the principal and maturing at the end of five years.

Merrill Lynch and Salomon Brothers were the first to create and market stripped securities. Both investment banking firms introduced their securities in 1982. Merrill Lynch called its stripped securities ***Treasury Income Growth Receipts*** (***TIGRs***), and Salomon Brothers referred to theirs as ***Certificates of Accrual on Treasury Securities*** (***CATS***). To create these strips, the companies would purchase a Treasury-coupon security and deposit it in a bank custodial account. They would then sell to investors separate IO receipts representing ownership of a coupon and a PO receipt representing ownership of the principal. Following the lead of Merrill Lynch and Salomon Brothers, other investment firms such as Lehman Brothers, E. F. Hutton, and Dean Witter Reynolds introduced their own stripped securities, with colorful names such as LIONS, GATORS, COUGARS, and DOGS. Collectively, these receipts were called ***trademarks***. One of the problems with trademarks was that dealers only made markets in their own strip securities. In an effort to expand the market, a group of dealers introduced Treasury receipts. Different from trademarks, which represented ownership in a custodial account, Treasury receipts represented ownership in the Treasury security. The U.S. Treasury facilitated the market for these generic stripped securities when in 1985 it initiated the ***Separate Trading of Registered Interest and Principal of Securities*** (***STRIPS***) program to aid dealers in stripping Treasury securities.[8] The securities created under the STRIPS program were, in turn, deemed direct obligations of the government. Furthermore, for clearing and payment purposes, the names of the buyers of these securities were included in the book entries of the Treasury and cleared through the Federal Reserve's book-entry system, thus eliminating the need to set up custodial accounts and therefore trademarks.

Stripped securities are attractive investments for institutional investors who buy the strips with maturities that match the maturities of their liabilities, thereby eliminating reinvestment risk. In addition to Treasuries, dealers also strip mortgage-backed, agency and municipal securities. There is also a secondary market for existing stripped securities. Table 7.6 shows a partial listing of bid and ask prices of strips offered by dealers in the secondary market on October 6, 2009.

For dealers, the creation of strip securities represents an arbitrage. As we examined in Chapter 2, the equilibrium price of a bond is that price obtained by discounting its cash flows by spot rates. If the market prices a bond above its equilibrium value, then dealers can earn a risk-free profit by buying the bond and stripping it into IO and PO bonds; if the market prices a bond below its equilibrium value, then dealers can realize a risk-free profit by buying stripped securities and then forming an identical coupon bond to sell. This process is known as ***rebundling*** or ***reconstruction***. Recall, in Chapter 4, we examined how spot rates and the ***theoretical spot rate curve*** are estimated using the bootstrapping technique. As noted then, the process of stripping and rebundling causes the actual yield curve for Treasury securities to approach the theoretical spot rate curve. As a result, the theoretical spot rate curve is often used by practitioners to price financial instruments and by dealers to identify arbitrage opportunities.

Like all Treasuries, strips are subject to federal taxes. However, with strips the accrued interest is taxed each year even though the interest is not paid (i.e., taxes are paid on interest earned, not on interest received). Thus, for taxable entities, strips generate a negative cash flow until maturity. It should be noted that the tax codes in some countries treat the return from PO strips as capital gains instead of ordinary income. With the capital gain tax rate lower, investors from these countries find PO strips relatively attractive.

The Markets for Treasury Securities

New issues of Treasury securities are sold through a sealed-bid auction process in which the Treasury announces an issue and dealers and investors submit either a competitive or a noncompetitive bid. With a competitive bid, an investor specifies the yield (annualized discount yield for T-bills and annualized YTM for Treasury coupon issues) and the quantity he wants, whereas with a noncompetitive bid an investor specifies only the amount he wants and accepts the weighted average price. Bidders must file tender forms with one of the Federal Reserve banks or branches or with the Treasury Bureau of Public Debt. The distribution is determined by first subtracting the noncompetitive bids and nonpublic purchases (such as those from the Federal Reserve) from the total bids; the remainder represents the amount that is awarded to competitive bidders. The distribution to competitive bidders is then determined by arraying the bids from lowest yield (highest price) to highest yield (lowest price). The lowest price at which at least some bills are awarded is called the ***stop price*** or ***stop-out price***. Those bidding above the stop price are awarded the quantity they requested, whereas those with bids below the stop price do not receive any bills. The bids at the stop price are awarded a proportion of the remaining bids. At the completion of the auction, the Treasury will adjust the coupon rate on the issue to reflect the stop-out yield, selecting that rate that will bring the price closest to par without exceeding it. An example of a T-bill auction is presented in Exhibit 7.1.

This auction process used by the Treasury is known as an ***English auction*** or ***first-price sealed-bid auction***. For bidders, an English auction may lead them to either overbid and pay too much for the securities, or underbid and be shut out of the auction. Those shut out, though, can buy them as a secondary market transaction from one of the successful bidders. This English system also may encourage collusion and a cornering of the market. In 1991 Salomon Brothers, for example, was charged with trying to corner the Treasury note market (see Exhibit 7.2). An alternative to the English auction is the ***Dutch auction system*** in which securities are ranked, but all are sold at just one price. Finally, there is a ***when-issued market*** or ***wi market*** for Treasuries. When-issued securities are those that have been announced for auction but have not yet been issued. In this market, Treasuries are traded to the time they are issued and settle on the issue date.

The secondary market for Treasury securities is very large. This market is part of the over-the-counter (OTC) market and is handled by Treasury security dealers. It is a 24-hour market with major dealers in New York, London, and Tokyo. The OTC market consists of both investors who prefer to buy from dealers instead of through the auctions, investors buying and selling outstanding securities, and the Federal Reserve, which buys T-bills and other Treasury securities as part of their open markets operations. In the secondary market, the recently issued ***on-the-run issues*** Treasury securities are the most liquid securities with a very narrow bid-ask spread; approximately 70% of the total secondary market trading involves on-the-run issues. In contrast, ***off-the-run*** Treasury securities issued earlier are not quite as liquid and can have slightly wider spreads.

EXHIBIT 7.1 T-Bill Auction Example

Three-month T-bills (13-week or 91-day bills) and six-month T-bills (26-week or 182-day bills) are auctioned every Monday. Announcements of the auction are made on the Thursday preceding the Monday offering. In the announcement, the Treasury indicates the size of auction, the proportions of the offering that will be used to replace maturing debt and for new funding, and their estimate of cash needs for the remainder of the quarter. Bidders must file tender forms by Monday. Competitive tenders submit a quantity bid and a yield bid based on a discount yield basis; noncompetitive tenders submit only a quantity bid up to $1million face value. Recall, the discount yield is

$$R_D = \frac{F - P_0}{F} \frac{360}{\textit{Days to maturity}}$$

The price given the discount yield and face value, F, of 100 is

$$P_0 = 100\,[1\text{- } R_D \text{ (Days to maturity/360)}]$$

The distribution is determined by first subtracting the noncompetitive bids from the total bids; the remainder represents the amount that is awarded to competitive bidders. For example, if the volume of bills requested is $12.5 billion and the amount accepted by the Treasury is $11 billion with $1.5 billion being noncompetitive, then the issue would be oversubscribed, with $1.5 billion going to noncompetitive bidders and $9.5 billion going to competitive bidders.

Volume of bills requested	$12.5 billion
Volume of bills accepted by the Treasury	$11 billion
Volume of noncompetitive offers accepted	$1.5 billion
Volume of competitive offers	$9.5 billion

The distribution to competitive bidders is determined by arraying the bids from lowest yield (highest price) to highest yield (lowest price). The discount yield is carried out to three decimal points. In this example, suppose we have the following competitive bids:

Bid Discount Yield %	Bid Price	Quantity Bid in Billions of $	Cumulative Bids in Billions of $
3.750	99.052	.20	.20
3.755	99.051	.57	.77
3.760	99.050	.78	1.55
3.765	99.048	1.25	2.80
3.770	99.047	1.30	4.10
3.775	99.046	1.50	5.60
3.800	99.039	1.70	7.30
3.825	99.033	1.85	9.15
3.830	99.032	.75	9.90
3.835	99.031	.75	10.65
3.840	99.029	.35	11.00
		11	

The Treasury allocates to competitive bidders until $9.5 billion is distributed. The lowest price at which at least some bills are awarded is the stop price. In this case, the stop price is 3.83% or 99.032. Those bidding above the stop price of 3.83% are awarded the quantity they requested, while those with bids below the stop price do not receive any bills—they are shut out. The bids at the stop price are awarded a proportion of the

(continued)

EXHIBIT 7.1 *(Continued)*

remaining bids. At 3.83%, there are $0.35 billion left in bids ($9.5 billion − $9.15 billion) and $0.75 billion requested at the stop price. Each of the bidders at 3.83% would therefore receive 46.667% (= .35/.75) of his bid. The weighted average bid that the noncompetitive bidders receive is 3.786%:

Average Bid = ($.2/$9.5)(3.75%) + ($.57/9.5)(3.755%) + ⋯ + (.35/9.5)(3.83%) = 3.786%

The difference between the stop bid yield and average yield is called the tail; in this case the tail is .044% (= 3.83% − 3.786%).

Government Security Dealers

Because of the difficulty in determining the best price to bid, individual investors, regional banks, fund managers, and corporations often prefer to buy new Treasury securities from dealers who specialize in the Treasury auction market rather than buy them directly at the auction. Although any firm can deal in government securities, the Treasury auction is principally carried out with ***primary dealers***. Primary dealers are those firms that trade with the Federal Reserve Bank of New York as part of their open market operations. For a firm to be on the primary dealer's list, it must have adequate capital and be willing to trade securities at any time. As of 2009, there were 18 primary dealers participating in the market (see Table 7.7 for a list of these dealers). The dealers include many of the major investment banking firms, such as Bank of America, Barclays Capital, and Goldman Sachs, as well as a number of foreign dealers. For new issues, primary dealers distribute new Treasury securities to nonprimary dealers and institutional investors. They also maintain large dealer positions in the secondary market.

In addition to the primary and secondary markets for Treasury securities, there is also an ***interdealer market*** in which primary and nonprimary dealers trade billions of dollars each day amongst themselves. This interdealer market functions through government security brokers who for a commission match dealers and other investors who want to sell with those wanting to buy.[9] The government brokers include

EXHIBIT 7.2 Cornering the Treasury Market

In 1991, Salomon Smith-Barney disclosed that they had violated the rules to corner the market. In cornering the Treasury market, Salomon Smith-Barney would purchase the permitted 35% maximum of an issue in its own name by submitting a high bid. They would then buy additional securities in the name of their customers without their knowledge and subsequently would buy them from the customers. From these acquisitions Salomon Smith-Barney was able to successfully corner the market, enabling them to sell their holdings at a monopoly premium. In one of the February 1991 auctions, they were awarded 57% of the issue, and in a May 1991 auction, they were able to attain 94% of the market. Following this scandal, the Treasury enacted new rules to ensure competitiveness in the Treasury auction market.

For more information, see the *Joint Report on the Government Securities Market*, Department of Treasury and the Board of Governors of the Federal Reserve System, January 1992.

TABLE 7.7 Primary Government Security Dealers, 2009

BNP Paribas Securities Corp.	HSBC Securities (USA) Inc.
Bank of America Securities LLC	Jefferies & Company, Inc.
Barclays Capital Inc.	J. P. Morgan Securities Inc.
Cantor Fitzgerald & Co.	Mizuho Securities USA Inc.
Citigroup Global Markets Inc.	Morgan Stanley & Co. Incorporated
Credit Suisse Securities (USA) LLC	Nomura Securities International, Inc.
Daiwa Securities America Inc.	RBC Capital Markets Corporation
Deutsche Bank Securities Inc.	RBS Securities Inc.
Goldman, Sachs & Co.	UBS Securities LLC

Source: Federal Reserve Bank of New York: www.newyorkfed.org/markets/pridealers_current.html.

such firms as Brokertec, Cantor Fitzgerald, Garban-Intercapital, Hilliard Farber and Company, and Tullett Liberty.

By taking temporary positions, dealers hope to profit from two sources: carry income and position profit. ***Carry income*** is the difference between the interest dealers earn from holding the securities and the interest they pay on the funds they borrow to purchase the securities. When dealers acquire securities, they often finance the purchase by borrowing from banks, other dealers, and other institutions. One major source of funds for them is demand loans from banks. Demand loans are short-term loans to dealers (one or two days), secured by the dealer's securities. These loans are usually renewable and often can be called at any time by the bank. Another important source of dealer funding is repurchase agreements; these are discussed in the next section. When dealers sell their securities, the invoice price is equal to the agreed-upon price plus the accrued interest. Generally, dealers profit with a positive carry income by earning higher accrued interest than the interest they pay on their loans.

The ***position profit*** of dealers comes from long positions, as well as short positions. In a long position, a dealer purchases the securities and then holds them until a customer comes along. The dealer will realize a position profit if rates decrease and prices increase during the time she holds the securities. In contrast, in a short position, the dealer borrows securities and sells them hoping that rates will subsequently increase and prices will fall by the time he purchases the securities to close the short position. To minimize their exposure to position risk, dealers do make use of futures and other derivative contracts to hedge against interest rate changes (interest rate futures and other derivatives are discusses in Part 4).

WEB INFORMATION

1. U.S. Treasury's debt: www.publicdebt.treas.gov.
2. Reports from the U.S. Treasury: www.treas.gov/.
3. Treasury auctions:
   - www.treasurydirect.gov/indiv/products/prod_auctions_glance.htm.

(*continued*)

(*Continued*)

- Recent bill auctions: www.treasurydirect.gov/RI/OFBills.
- Recent bonds, notes, and TIPS auctions: www.treasurydirect.gov/RI/OFNtebnd.

4. For the distribution of U.S. debt, go to the Treasury Bulletin: www.fms.treas.gov/bulletin/.
5. U.S. government's expenditures, revenues, deficits, and debt: www.gpo.gov/fdsys/browse/collectionGPO.action?collectionCode=BUDGET.
   For downloadable tables on U.S. government's expenditures, revenues, deficits, and debt, go to: www.gpo.gov/fdsys/search/pagedetails.action?granuleId=&packageId=BUDGET- 2010-TAB.
6. Government fiscal information submitted by Congress: www.gpo.gov/fdsys/search/home.action.
7. Marketable Treasury securities issued and outstanding: www.treasurydirect.gov/govt/reports/pd/pd_electreas.htm.
8. Primary securities dealers: www.newyorkfed.org/markets/pridealers_current.html.
9. Treasury debt market, securities, prices, and other U.S. Treasury market information:
   http://investinginbonds.com/ and click "Government Market-at-a-Glance."
10. FINRA
    - Go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds."
    - Click "Treasury and Agency" tab and then click "Advanced Bond Search" to find Treasury and agency bonds with certain features.
11. *Wall Street Journal*
    - Go to http://online.wsj.com/public/us, Market Data, Bonds, Rates & Credit Markets, and Treasury Quotes.
12. Yahoo.com
    - Go to http://finance.yahoo.com/bonds, click "Advanced Bond Screener," and click Treasury or Treasury Zero Coupon (Treasury strips).
13. U.S. security-holding by foreigners:
    - www.treas.gov/tic.
14. Historical interest rate data on Treasuries from the Federal Reserve:
    - www.federalreserve.gov/releases/h15/data.htm.
    - FRED: www.research.stlouisfed.org/fred2.

7.3 REPURCHASE AGREEMENTS

As noted above, many dealers finance their temporary holdings of securities with ***repurchase agreements***, also called ***repos*** (***RPs***). Under a repurchase agreement, the dealer sells securities to a lender, such as a commercial bank, with an agreement that he will buy the securities back at a later date and price. To the dealer, the RP represents a collateralized loan, with the Treasury securities serving as the collateral. Government security dealers often use overnight repos to finance their positions, agreeing to buy back the securities in the next day or two. To the lender, the RP position, defined as a ***reverse repo***, represents a secured short-term investment. Banks, investment banking firms, dealers, corporations, state and local governments, and other institutions find reverse RPs attractive securities for investing their excess cash. The terms repo and reverse repo can be confusing. Some practitioners refer to the party with the repo position (sale/repurchase) as the ***collateral seller*** and the party with the reverse repo (purchase/resale) as the ***collateral buyer***.

As an example, suppose a Treasury dealer plans to buy $20 million of T-notes on the Treasury auction day and anticipates holding them for one day before selling the securities to her customers for $20 million plus a premium. To finance the purchase, the dealer could buy the notes and simultaneously enter a repurchase agreement with an investor/lender. Per the agreement, the dealer would agree to deliver the notes for $20 million minus interest to be paid to the lender. In this market, dealers and lenders state the interest in terms of an annualized ***repo rate*** based on a 360-day year. If this rate were 6%, then the interest would be $3,333 and the price the dealer would sell the notes for on the repo agreement would be $19,996,667:

$$\text{Interest} = (\text{Principal})\ (\text{Repo rate})\ (\text{Length of loan}/360)$$

$$\text{Interest} = (\$20{,}000{,}000)\ (0.06)\ (1/360)$$

$$\text{Interest} = \$3{,}333$$

$$\text{Price sold} = \$20{,}000{,}000 - \$3{,}333 = \$19{,}996{,}667$$

$$\text{Repurchase price} = \$20{,}000{,}000$$

One day later, the dealer would buy back the T-notes on the repurchase agreement for $20 million and then sell them to her customers for hopefully $20 million or more plus the one-day accrued interest.

Types of Repurchase Agreements

The RP market is not limited to overnight repos or those arranged just with Treasury securities. Repos can have maturities that range from one day (overnight) to one year. An RP that is not overnight is called a ***term repo***.[10] There is also an ***open repo*** that has no maturity; this is typically an overnight repo that is automatically rolled over into another overnight repo until one of the parties closes (after giving the other party proper notification). For example, the Treasury dealer in the preceding example might estimate that it could take one to three days, if not longer, to sell her $20 million of T-notes and would therefore find an open repo as the best way to finance her long position. In addition to Treasury securities, repos are also arranged with federal agency securities, municipals, mortgage-back securities, and money-market securities. There are also ***dollar repos*** that permit the borrower to repurchase with securities similar, but not identical, to the securities initially sold. Finally, there are repos that allow the borrower to replace the collateral. A dealer who foresees the possibility of having to deliver the collateral to a customer during the term of the repo might want such a provision in the repo agreement. With a replacement provision, the dealer could then request a substitution whereby he would agree to deliver another security of equivalent value to the lender who would return the original collateral.

The Market for Repurchase Agreements

The repo market consists of investors with excess cash who find reverse repos an attractive investment alternative to other money market securities and dealers and holders of securities who want to borrow short-term funds and find repurchase agreements an efficient and less-expensive financing alternative. Many financial and non-financial corporations take both repo and reverse repo positions. A bank, for example, might loan funds to a dealer with an open reverse repo (collateral buyer), while financing part of its short-term loan portfolio with term repos (collateral seller). Similarly, security dealers use repos to set up long positions and reverse repos to form short positions. In the case of a long position, a Treasury security dealer might finance her security acquisition with a repurchase agreement as described above. If interest rates decrease and prices increase, the dealer would then profit from the long position. For short positions, a dealer might use the repo market to cover a short position. For example, suppose a dealer shorted $20 million of notes (i.e., borrowed the notes and sold them for $20 million) and one week later had to cover the short position (deliver the securities). To cover, the dealer could enter a reverse repo agreement (i.e. buy the securities and then agree to sell them back). If rates have increased over the past week, then the dealer will be able to buy the securities on the reverse repo at a lower price. The dealer, though, will eventually have to buy the securities to sell them back per his reverse repo agreement. Note that to avoid price risk, a dealer might hedge a long position financed with a repurchase agreement with a futures contract, earning just the carry income equal to the difference between the accrued interest earned on the securities held and the rate paid on the repurchase agreement—the repo rate. Finally, dealers use the repo market to run a match book. A match book involves taking repo and reverse repo positions with the same maturity. For example, a dealer might enter a five-day repo with a money market fund and a five-day reverse repo with a bank. The dealer in effect is borrowing from the money market fund on the repo agreement and lending to the bank on the reverse repo. If the repo rate is 6.5% and the reverse repo rate is 6.55%, then the dealer would gain .05% from the match.

Another important use of repurchase agreements is by depository institutions as a way to finance their federal funds position. Federal funds are deposits of banks and deposit institutions with the Federal Reserve that are used to maintain the bank's reserve position required to support their deposits. Banks maintain federal funds desks where they manage their federal funds positions: borrowing funds when they are deficient and lending funds when they have an excess amount of reserves. Two common ways depository institutions finance a deficient reserve position are through the federal funds market and the repo market. The ***federal funds market*** is a market in which depository institutions with excess reserves lend to institutions that are deficient. Federal funds market loans are typically overnight to one week, unsecured, and traded directly between the lending bank (usually a small regional bank) and the borrowing bank (often a money center bank). This contrasts with the repo loan that is secured and often offered by dealers making a market. Because of the security backing the repurchase agreement, their rates tend to be less than the federal fund's rate.

On balance, banks and dealers are net collateral sellers (net repo position), whereas money market funds, bank trust departments, municipalities, and non-financial corporations are net collateral buyers (net reverse repo position). The repo market also includes the Federal Reserve as one of its participants. In addition to its open market operations, the Federal Reserve uses the repo market to influence interest rates. To lower rates, they will buy collateral in the repo market (a reverse repo agreement) to inject money into the financial markets; this action is called a ***system repo***. To increase rates, the Federal Reserve will buy collateral (enter repo agreements) via the repo market to contract money from the financial markets; this action is called a ***customer repo.***

Risk

Because RPs are secured by Treasury or other high-quality securities, they are considered to be high-quality. The lender (collateral buyer), though, is subject to some risk. For example, if the borrower (collateral seller) cannot buy back the underlying securities, the lender is left with securities whose price could or may already have decreased. To minimize such risk, a repo agreement may require that the borrower set up an initial margin in the form of cash, pledge additional collateral, or sell securities in excess of the amount of the principal of the trade. In some cases, the agreement may include a maintenance margin provision requiring the borrower to deliver a requisite amount of cash or money market securities if the collateralized securities lose value by a certain amount.

Another risk facing the lender (collateral buyer) is that the borrower may use the collateral fraudulently as security for other repurchase agreements. This could occur when the repurchase agreement allows for the borrower/collateral seller to hold the securities in a separate customer account maintained by the seller (known as a ***hold-in-custody repo*** or ***letter repo***), instead of actually delivering the securities to the lender/collateral buyer or delivering them to a custodial account set up with a third party. The extent of this type of risk was brought to light in 1985 when it was learned that several government security dealers who had declared bankruptcy (ESM Government Securities and Bevil, Bresler, and Schulman) had used the same securities as collateral on different repurchase agreements. Investors who had purchased these repos included a number of municipalities and state-insured thrifts; their losses were over $500 million. Since that scandal, lenders have tried to avoid such risk by requiring more detailed accounting and notification of any ownership transfer of the securities. In addition, the scandal led to the passage of the Government Security Act of 1986 that required greater disclosure of repurchase agreements and gave the Treasury the authority to oversee dealers in the repo market.

It should be noted that instead of using a repo agreement, some dealers borrow the securities from financial institutions.[11] The borrowing or short sale of securities is often secured by other securities and either the lender or borrower can usually terminate the security loan.[12]

7.4 FEDERAL AGENCY SECURITIES

The U.S. Treasury is responsible for financing and managing the government's debt. In addition to the Treasury, there are also ***federal agencies*** and quasi-government corporations that issue securities. Many of these agencies were established to ensure that sufficient credit or liquidity was provided to certain segments of the economy having difficulties raising funds. Among those entities are farmers, students, homeowners, small businesses, and international businesses. These federal agencies raise funds by issuing short-, intermediate-, and long-term debt securities. They, in turn, use the proceeds to directly provide loans to farmers, students, and businesses or to provide loan guarantees and liquidity to private lenders who make loans to those entities. The securities they sell are part of the ***federal agency security market***. In addition, some of the entities, such as the Federal National Mortgage Association (FNMA or Fannie Mae) and the Government National Mortgage Association (GNMA or Ginnie Mae), are major players in the mortgage-backed securities market. In that market, they buy mortgages to securitize and market as mortgage-backed securities or they insure the mortgage-backed securities (mortgage-backed securities are examined in Chapter 11).

The Federal credit agencies can be divided into two groups: ***Government-sponsored enterprises*** (***GSE***) and ***non-GSE federal agencies***. GSEs can, in turn, be divided into publicly owned corporations and federally chartered bank lending institutions. The former includes the Federal National Mortgage Association (Fannie Mae), Federal Agriculture Mortgage Corporation (FAMC or Farmer Mac), the Federal Home Loan Mortgage Corporation (FHLMC or Freddie Mac), and (at the time of this writing) the Student Loan Marketing Association (SLMA or Sallie Mae). The federally chartered bank lending institutions include the Federal Home Loan Banks and the Federal Farm Credit Banks. The GSEs sell securities and use the proceeds to provide loans and liquidity to support the housing industry, banking system, agriculture sector, and college loan programs.

Non-GSE federal agencies are true federal agencies created by the U.S. government. Included in this group are the Export-Import Bank, the Private Export Funding Corporation, Tennessee Valley Authority (TVA), Federal Housing Administration (FHA), and Small Business Administration (SBA). Some of the non-GSEs, like the TVA, issue their own securities. Those agencies that do not issue their own debt use the Federal Financing Bank to raise their funds. Finally, there are some outstanding issues of one-time GSE issuers. These include the Resolution Trust Corporation, the Farm Credit Assistance Program, and the Financing Corporation (see Exhibit 7.3 for a listing of GSEs and non-GSE federal agencies and their Web sites.)

Collectively, the debt claims sold by GSEs and federal agencies are referred to as agency securities. For investors, these claims have been considered virtually default free because of the agency's or company's affiliation with the federal government (some federal agency issues are backed by Treasury bonds and many agencies have lines of credit with the Treasury). Following the bailout of savings and loans institutions and banks in the 1980s and early 1990, the General Accounting Office and the Treasury began requiring that all ***federally sponsored agencies*** maintain a triple-A credit rating or lose their government support. Following the 2008 financial crisis, the securities of Fannie Mae and Freddie Mac received backing by the U.S. Treasury as part of the Treasury's bailout program. The yields on agency securities are highly correlated with the yields on Treasuries, with the yield spread of federal over Treasury being positive.

Federal agency issues vary from short- to long-term in maturity, ranging from overnight issues to bonds with original 30-year terms to maturity. Agency money-market securities are sold as zero-discount bonds, whereas intermediate and long-term notes and bonds are sold as coupon bonds; agencies also sell floating-rate bonds.

EXHIBIT 7.3 Government-Sponsored Enterprises, Federal Agencies, and Related Entities

GOVERNMENT-SPONSORED ENTERPRISES (GSEs)

Federal National Mortgage Association (FNMA, Fannie Mae)
- Home page: www.fanniemae.com/
- Investments: www.fanniemae.com/investors/index.html

Federal Home Loan Mortgage Corporation (FHLMC, Freddie Mac)
- Home page: www.freddiemac.com/
- Debt securities: www.freddiemac.com/debt/

Federal Agriculture Mortgage Corporation (FAMC, Farmer Mac)
- Home page: www.farmermac.com/
- Debt securities: www.farmermac.com/investors/debtsecurities/index.aspx

Federal Home Loan Bank (FHLB)
- Home page: www.fhlbanks.com/
- Financial information: www.fhlb-of.com/specialinterest/financialframe2.html

Federal Farm Credit Bank System
- Home page: www.farmcredit-ffcb.com/farmcredit/fcsystem/overview.jsp

Federal Farm Credit Funding Corporation
- www.farmcredit-ffcb.com/farmcredit/index.jsp

NON-GSE FEDERAL AGENCIES

Export-Import Bank
- www.exim.gov/

Private Export Funding Corporation
- www.nndb.com/company/895/000127514/

Small Business Administration
- www.sba.gov/

Federal Housing Administration (FHA)
- www.hud.gov/offices/hsg/fhahistory.cfm

Tennessee Valley Authority (TVA)
- www.tva.gov/

RELATED ENTITIES

Government National Mortgage Association (GNMA, Ginnie Mae)
- Home page: www.ginniemae.gov/
- Investors: www.ginniemae.gov/investors/investors.asp?Section=Investors

Federal Financing Bank
- www.treas.gov/ffb/

Student Loan Marketing Association (SLMA, Sallie Mae)
- Home page: www.salliemae.com/
- Investors: www.salliemae.com/about/investors/
- Debt securities: www.salliemae.com/about/investors/debtasset/default.htm

International Bank for Reconstruction and Development—World Bank
- Home page: www.worldbank.org/
- Debt securities: http://treasury.worldbank.org/cmd/htm/index.html

The denominations on the bonds vary from $1,000 to $50,000 and up. Fannie Mae, Freddie Mac, the Federal Home Loan Bank system, and several other agencies also offer ***agency benchmark programs*** similar to corporate medium-term note issues. These programs provide for the regular issuance of coupon securities covering a range of maturities. In general, there are a variety of different types of agencies with different features: callable and non-callable, fixed and floating, coupon and zero coupon, and agencies denominated in different currencies. Unlike Treasury securities, which are exempt from state and local taxes, and municipal bonds, which are exempt from federal taxes, some agency securities are fully taxable. Because of their tax status, as well as their maturities and relatively low risk, agency claims are attractive investments for pension and trust funds, state and local governments, banks, and corporations. The Federal Reserve System also trades in some federal agency securities.

The primary market for many federal agency securities is handled through a network of federal security fiscal agents, brokers, and dealers. Depending on the type of security, a new issue can be sold through dealers, by auction, or by direct sales. Short-term securities are sold on a continuous basis, whereas intermediate issues are sold on a monthly basis, and long-term bonds are offered several times a year. Similar to the primary market for Treasuries, there is also an interdealer market that helps to improve the efficiency of the market. Intermediate- and long-term issues are usually sold through a ***solicitation method***. Under this method, a fiscal agent puts the selling group together. The selling group then provides potential investors with information on the issues. Given that information, the potential investor indicates the amount of the issue they plan to buy and the price they believe the issue should be. The fiscal agent then sets the price based on the inputs of the potential investors. The secondary market for agency securities is handled through dealers on the over-the-counter market. Information on the trading of existing agency securities can be found at Web sites of the *Wall Street Journal,* FINRA, Yahoo!, and Investinginbonds.com. Table 7.8 shows dealer price quotes and corresponding yields for a select number of GSEs and agencies as of October 6, 2009. Like the quotes of Treasury bonds and notes, the dealer's ask and bid quotes for the GSEs and agencies are in 32nds, and the yields are based on the dealer's ask price.

Although different agency securities have many similar characteristics, the purpose for which each agency and government-sponsored company uses its funds varies considerably. As noted, the major areas of financing for federal agencies are housing, agriculture, savings and loans and bank funding and reorganizing, student loans, and international business. Table 7.9 summarizes the functions of these various federal agencies.

Housing and Real Estate Financing

The Federal National Mortgage Association, the Government National Mortgage Association, the Federal Home Loan Mortgage Corporation (FHLMC or Freddie Mac), and the Federal Agriculture Mortgage Corporation are all GSEs or agencies involved with providing funding and liquidity for the mortgage industry. Until the introduction of mortgage-backed securities, the primary function of Fannie Mae, Ginnie Mae, and Freddie Mac was to provide a secondary market for mortgages. They did this by issuing their own securities and then using the proceeds to buy

TABLE 7.8 Select GSE and Agency Quotes, October 6, 2009

Fannie Mae Issues					Freddie Mac				
Maturity	**Rate**	**Bid**	**Asked**	**Yield**	**Maturity**	**Rate**	**Bid**	**Asked**	**Yield**
10-Jan	7.25	101:30	101:31	...	10-Feb	4.88	101:19	101:20	0.03
11-Nov	5.38	108:29	108:30	1.07	11-Sep	5.5	108:22	108:23	0.94
12-Mar	6.13	111:18	111:19	1.28	12-Jul	5.13	109:28	109:29	1.46
13-May	4.63	105:16	105:17	2.98	13-Jan	4.5	108:24	108:25	1.73
14-Jan	5.13	106:24	106:25	3.39	14-Jan	4.5	109:03	109:04	2.24
29-May	6.25	122:19	122:21	4.5	31-Mar	6.75	130:17	130:19	4.51
30-Jan	7.13	134:12	134:14	4.51	Jul-32	6.25	124:18	124:20	4.51

Federal Home Loan Bank					Tennessee Valley Authority				
Maturity	**Rate**	**Bid**	**Asked**	**Yield**	**Maturity**	**Rate**	**Bid**	**Asked**	**Yield**
11-May	6	108:08	108:09	0.77	11-Jan	5.63	106:13	106:14	0.56
12-May	5.75	111:01	111:02	1.41	13-Aug	4.75	109:11	109:12	2.17
13-Sep	4.5	108:31	109:00	2.11	17-Dec	6.25	118:03	118:04	3.66
14-Jun	5.25	112:08	112:09	2.46	25-Nov	6.75	123:13	123:15	4.66
					30-May	7.13	127:28	127:30	4.95

Source: Wall Street Journal, http://online.wsj.com/public/us.

mortgages from savings and loans, mortgage banks, and commercial banks. (See Exhibit 7.4 for a brief history of the secondary mortgage market.) In the 1980s, Fannie Mae, Ginnie Mae, and Freddie Mac began offering ***pass-through securities*** or ***participation certificates (PCs)***. These instruments are securitized assets formed by pooling a group of mortgages and then selling a security representing interest in the pool and entitling the holder to the income generated from the pool of mortgages. In 2000, Fannie Mae and Freddie Mac activities in the sub-prime mortgage market led to their near collapse and precipitated the 2008 financial crisis. In September 2008, Fannie Mae and Freddie Mac were taken over by the U.S. government.

In addition to mortgage-backed securities, Fannie Mae issues a variety of short-, intermediate-, and long-term debt securities: benchmark bills, notes, and bonds, callable benchmark notes, subordinated benchmark notes, callable securities, and bullet and callable medium-term notes. Similarly, Freddie Mac issues its own bullet and callable benchmarks, called ***reference*** bills, notes, and bonds; they also issue euro-denominated notes, medium-term notes, and global bonds. Many of Freddie Mac's debt securities are purchased by agents and dealers and then stripped. In 2001, both Fannie Mae and Freddie Mac issued subordinate securities—Fannie Mae Subordinated Benchmark Notes and the Freddie Mac Subs.

The Federal Agriculture Mortgage Corporation (Farmer Mac) was established in 1988 to provide a secondary market for agriculture real estate loans and rural home-owner and business loans, and to improve agriculture and rural credit and liquidity by the securitization of agriculture mortgage loans. Similar to Fannie Mae, Ginnie Mae, and Freddie Mac, Farmer Mac buys loans, pools them, and sells claims on the pool. These securitized assets are called ***agriculture mortgage-backed securities***

TABLE 7.9 Federal Agencies: Functions and Securities Issued

Agency	Function	Major Securities Issued
Federal National Mortgage Association (FNMA or Fannie Mae)	Buys mortgages and issues mortgage-backed securities.	Sells pass-through securities or participation certificates, which entitle the holder to a cash flow from a pool of mortgage securities.
Government National Mortgage Association (GNMA or Ginnie Mae)	Buys mortgages and issues mortgage-backed securities.	Sells pass-through securities or participation certificates, which entitle the holder to a cash flow from a pool of mortgage securities.
Federal Home Loan Mortgage Corporation (FHLMC or Freddie Mac)	Buys mortgages and issues mortgage-backed securities.	Sells pass-through securities or participation certificates, which entitle the holder to a cash flow from a pool of mortgage securities.
Federal Home Loan Bank System (FHLBS)	FHLBS consists of 12 district Federal Home Loan Banks. Provides loans to qualified S&Ls.	Sell bonds separately and jointly for Federal Home Loan Banks.
Student Loan Marketing Association (SLMA or Sallie Mae)	Provides funds for lenders participating in the Federally Guaranteed Student Loan Program and PLUS loan programs (loans to parents of undergraduate students).	Issues discount notes, long-term bonds, and pure discount bonds.
Federal Farm Credit Bank System (FFCBS)	Provides credit to agriculture sector distributed through Farm Credit Banks.	Issues bills, notes, and bonds.

(AMBS). Farmer Mac also finances its purchase of loans by selling discount notes and medium-term notes.

Agriculture Credit Financing

The Federal Farm Credit Bank System (FFCBS) originally consisted of 12 Federal Land Banks (FLB), 12 Federal Intermediate Credit Banks (FICB), and 12 Banks for Cooperatives. The Federal Land Banks (created by an act of Congress in 1916) made mortgage loans and provided financial funds to farmers and ranchers for purchasing or improving their farms and ranches; the Federal Intermediate Credit Banks (created in 1923) provided short-term loans for farmers; the Banks for Cooperatives (created in 1933) provided seasonal loans to farm cooperatives. Prior to 1979, these banks sold their own securities. In 1979, though, they consolidated their financing and sold separate and joint obligations of the Federal Farm Credit Bank System (FFCBS). In 1987, the FFCBS was reorganized under the Agriculture Credit Act.

EXHIBIT 7.4 Brief History of the Secondary Mortgage Market

In the early 1900s, the lack of standardized mortgage contracts and regional differences in real estate inhibited the development of an efficient secondary market for existing mortgage loans. Moreover, the lack of such a market translated into liquidity problems for lenders who were unable to sell their mortgage notes. In 1938, the Federal National Mortgage Association, Fannie Mae (FNMA), was formed with the initial purpose of acquiring mortgage notes. Combined with Federal Housing Administration (FHA) and Veterans Administration (VA) default insurance programs, Fannie Mae helped develop the present secondary mortgage market. Specifically, the FHA's program of providing insurance for mortgage lenders against defaults and the VA's program of guaranteeing veterans' mortgages had two major effects on the real estate lending market. First, as intended, it reduced the default risk characteristics of the mortgage, and secondly, FHA and VA by providing such benefits were able to insist on certain provisions governing the maturity, interest, down payments and amortization schedule. This led to a more standardized mortgage contract across the country. Moreover, the combination of uniformity and protection in mortgage notes—qualities that enhanced marketability—facilitated the mortgage purchases made by Fannie Mae and, in turn, accelerated the development of the secondary mortgage market.

In 1968 Fannie Mae was transformed from a federal agency into a privately owned company. Fannie Mae was classified as a government-sponsored company. More precisely, Fannie Mae was defined as private corporation with a public purpose. As a result, Fannie Mae's charter then stipulated that five of its 15 directors must be appointed by the President, that its debt limit and debt-to-equity ratio be regulated, and that the Treasury hold a certain level of its debts. These actions ensured some limited government control over this once public institution. Operationally, Fannie Mae raised funds through the sale of short-term and intermediate-term securities. From the proceeds they acquired mortgages.

Since Fannie Mae had historically tended to buy from mortgage companies, Congress created the Federal Home Loan Mortgage Company, Freddie Mac (FHLMC), in 1970 to specialize in the purchase of mortgages from savings and loans. Operating under the Federal Home Loan Bank Board, Freddie Mac served to not only help savings and loans, but it also extended the secondary market from FHA- and VA-secured mortgages to conventional mortgages (not federally insured). The final participant of note in the secondary market was the Government National Mortgage Association, Ginnie Mae (GNMA). Created after Fannie Mae was converted to a private company, Ginnie Mae, under its liquidity management program, initially bought mortgages from savings and loans and mortgage bankers and then sold them under a tandem program to Fannie Mae and Freddie Mac or packaged the mortgages and sold them to large institutional investors.

Beginning in the 1980s, Fannie Mae, Freddie Mac, and Ginnie Mae became the principals in the agency mortgage-backed securities market. In 2000, Fannie Mae and Freddie Mac activities in the sub-prime mortgage market led to their near collapse. In 2008, Fannie Mae and Freddie Mac were taken over by the U.S. government.

This reorganization led to the merger of the regional FLBs, FICBs, and Banks for Cooperatives into four regional Farm Credit Banks (FCBs), the Agriculture Credit Bank, and approximately 100 Production Credit Associations, Federal Land Credit Associations, and Agriculture Credit Associations.

The Federal Farm Credit Bank System (FFCBS) issues a variety of debt securities, ranging from short-term money market securities with maturities ranging from five to 270 days, to short-term bonds with maturities from three to nine months, to intermediate bonds with maturities ranging from one to 10 years. All of the FFCBS obligations are handled by the Federal Farm Credit Bank Funding Corporation, which sells new issues through a selling group of brokers and dealers. In addition to FFCBS, Congress in 1987 created the Farm Credit Finance Assistance Corporation (FACO). This government-sponsored corporation provided capital to the FFCBS when it was facing financing difficulties from loan defaults by farmers during the early 1980s.

Savings and Loans and Bank Financing

The savings and loan and banking crises of the 1980s led to the passage of the Federal Institutional Reform, Recovery, and Enforcement Act (FIRREA) of 1990. This act restructured some existing agencies and created several new programs involved in regulating and insuring commercial banks, savings and loans, and savings banks. Prior to the act, the Federal Home Loan Bank Board (FHLBB) was responsible for regulating federally chartered savings and loans and federally insured state-chartered savings and loans. The passage of FIRREA significantly curtailed these responsibilities, shifting the FHLBB regulatory authority to the Office of Thrift Supervision and dismantling the board but keeping the system of 12 federally chartered, privately owned Federal Home Loan Banks (FHLBank System). These 12 privately owned banks provide loans to over 8,000 member bank institutions for the financing of their residential mortgage, small business, rural, and agriculture loans. FHLBank issues a number of securities, including benchmark securities, notes, and medium-term notes.

The Federal Institutional Reform, Recovery, and Enforcement Act (FIRREA) also took away the FHLB's supervision of the insurance programs for savings and loans. Today, the Federal Deposit Insurance Corporation (FDIC) provides deposit insurance for banks (Bank Insurance Fund: BIF) and savings and loans (Savings and Loan Insurance Fund: SAIF). In this capacity, the FDIC also acts as a receiver for failed banks, acquires the assets of insolvent banks, and sets up bridge banks (new banks set up to manage a failed bank until it is sold). To facilitate the funding of such activities during the savings and loan and bank crises, Congress in 1987 created the Financing Corporation (FICO) to issue debt. FICO, in turn, issued approximately $11 billion of bonds secured by the Treasury.[13] The funding for FICO, though, was not adequate to redress the insolvent savings and loan problems. As a result, a provision in FIRREA established the Resolution Trust Corporation (RTC) with the responsibility of bailing out bankrupt savings and loans. The Resolution Funding Corporation (REFCORP) also was established to issue bonds to finance the RTC, with an initial authorization to issue up to $40 billion in long-term bonds secured by Treasury bonds. The RTC was responsible for selling over $450 billion of real estate owned by failed savings and loans. From 1989 to 1995 (when it stopped operations), the RTC seized approximately 750 insolvent savings and loans, selling over 95% of them.

College Student Loans

The Student Loan Marketing Association (Sallie Mae) was founded in 1972 to provide funds and guarantees for lenders who provide college students loans through the Federal Guaranteed Student Loan Program and college loans to parents of undergraduates through the PLUS loan program (Parent Loans for Undergraduate Students program). Sallie Mae issues a variety of different debt securities, ranging from zero-coupon bonds to long-term fixed-rate securities. Sallie Mae was reorganized in 1997 and was phased out as a GSE in 2005.

Tennessee Valley Authority

The Tennessee Valley Authority (TVA) was created by an act of Congress in 1933 to create and promote electrification and the development of the Tennessee Valley. Like the U.S. Postal Service, TVA is defined as a ***government-owned corporation*** or a wholly owned corporate agency. It is the largest public power system in the United States. Similar to stockholder-owned power companies, TVA issues a number of debt obligations to finance its power programs and development projects. The debt obligations it issues are not guaranteed by the government, but are rated triple A. The TVA offers a variety of debt securities, many with interesting features and structures. For example, they issue or have issued putable bonds, reset bonds, global bonds, and estate bonds (bonds that can be redeemed at par upon the passing of the bondholder).

Global Financing

In addition to federal agencies, international organizations such as the International Bank for Reconstruction and Development (World Bank) and various development banks also raise funds through the sale of bonds to finance development and guarantee programs. The World Bank provides loans to private and government sectors in various countries (generally when such loans are not available from private sources) to finance public and quasi-public projects such as educational establishments, power plants, transportation systems, dams, harbors, and other infrastructures. The loans made by the World Bank usually have maturities between 10 and 30 years and often the principal is amortized. Loans are typically made to governments or to firms with guarantees from the government, with an emphasis on building a country's infrastructure. To finance these projects, the World Bank borrows directly from the United States and other developed countries as well as issuing short-term and intermediate-term World Bank bonds.

Similar in structure to the World Bank are various regional development banks such as the Inter-American Development Bank (IDB), Asian Development Bank (ADB), and the African Development Bank. These institutions provide financing to support private and infrastructure developments in their specific regions.

GSE AND AGENCY WEB SITE INFORMATION

1. See Exhibit 7.3 for GSE and agency Web sites.
2. For recent information and reports on the government securities market (Treasury and agency) go to investinginbonds.com:
   - www.investinginbonds.com/ Recent commentary

(*continued*)

(*Continued*)

- SIFMA Quarterly Government Securities Issuance and Rate Forecast www.investinginbonds.com/news.asp?id=3100&catid=36 Full report

3. For information on specific GSEs and agencies, go to
   - FINRA: www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds." Click "Treasury and Agency" tab and then click "Advanced Bond Search" to find GSE agency bonds with certain features.
   - *Wall Street Journal*: http://online.wsj.com/public/us, Market Data, Bonds, Rates & Credit Markets, and Government Agency.

7.5 CONCLUSION

In this chapter we have examined the markets for Treasury and agency securities. These securities are used to finance not only the U.S. government's debt, but also a number of programs and operations of the federal government. For investors, the securities offered by the U.S. government vary from short-term to long-term, from zero-coupon to fixed-income securities, and from generic fixed-income to inflation-adjusted coupon issues. There is also an extensive market for government securities with special features such as stripped Treasury securities. In the next chapter, we continue the analysis of government securities by examining the many securities offered by municipal governments.

KEY TERMS

agency benchmark programs
agriculture mortgage-backed securities (AMBS)
banker's discount yield
carry income
certificates of accrual on Treasury securities (CATS)
collateral buyer
collateral seller
corpus
customer repo
dollar repos
Dutch auction system
English auction or first-price sealed-bid auction
federal agencies
federal agency security market
federal funds market
federally sponsored agencies
government account series
government-sponsored enterprises
hold-in-custody repo or letter repo
interest-only (IO) security
interdealer market
non-GSE federal agencies
off-the-run issues
on-the-run issues
open repo
participation certificates (PC)
pass-through securities
primary dealers
principal-only (PO) security
position profit
purchasing power risk

real interest rate
rebundling or reconstruction
reference
reopenings
repos (RPs)
repo rate
repurchase agreements
reverse repo
Separate Trading of Registered Interest and Principal of Securities (STRIPS)
solicitation method
stop price or stop-out price
strip bills
system repo
tax-anticipation bills
term repo
theoretical spot rate curve
trademarks
treasury income growth receipts (TIGRs)
treasury inflation protection securities (TIPS)
treasury strips
when-issued (wi) market

PROBLEMS AND QUESTIONS

1. Define and list the features of the following:
   a. T-bills
   b. T-bonds and T-notes
   c. Nonmarket series
   d. Treasury strip bills
   e. TIPS
   f. Treasury STRIPs
2. Determine the annual cash flows from an investment in a four-year, 3% TIP bond with an original principal of $1,000, given a 2% inflation rate each year for the next four years.
3. Discuss the history of the stripped security market.
4. Given the following Treasury securities and their current prices:

1	2	3	4	5	6	7
Security	Type	Maturity (Years)	Semiannual Coupon	Annualized YTM	Par	Current Price
1	T-bill	0.50	0.000	0.0500	100	97.5610
2	T-bill	1.00	0.000	0.0525	100	94.9497
3	T-note	1.50	3.000	0.0600	100	100.0000
4	T-note	2.00	3.250	0.0650	100	100.0000
5	T-note	2.50	3.500	0.0700	100	100.0000
6	T-note	3.00	3.750	0.0750	100	100.0000

   a. Using the bootstrapping approach discussed in Chapters 2 and 4, generate a theoretical spot rate curve for maturities from .5 years to 3 years.
   b. Suppose the yields on the actual spot yield curve were equal to the YTMs on the Treasuries securities shown in Column 5. Determine the market prices of Treasury strips with maturities from .5 years to 3 years created from the

three-year T-note with semiannual coupons of 3.75. Determine the values of the strips in terms of semiannual YTM equal to (Annualized YTM)/2.

c. Explain the arbitrage that exists from buying the three-year note, stripping it, and selling the strip securities.

d. Suppose the actual spot yield curve converges to the theoretical spot yield curve. Determine the market prices of Treasury strips with maturities from .5 years to 3 years created from the three-year T-note with semiannual coupons of 3.75. Does an arbitrage exist from buying the three-year note, stripping it, and selling the strip securities?

e. Comment on the actual yield curve being the theoretical spot rate curve.

5. Outline how the Treasury auction process works.

6. Given the following information on a Treasury auction for a 91-day T-bill issue:

- Volume of T-bills requested by Treasury = $15B
- Total volume of T-bills bids submitted = $16.5B
- Volume of noncompetitive T-bill bids submitted = $2B
- Volume of competitive T-bills bids submitted = $14B, broken down as follows:

Bid Yield	Bid Price	Quantity of Bid (in Billions)
2.750	99.3049	0.4
2.755	99.3036	0.5
2.760	99.3023	0.8
2.765	99.3011	1.3
2.770	99.2998	1.4
2.775	99.2985	1.5
2.800	99.2922	1.6
2.825	99.2859	1.7
2.830	99.2846	1.4
2.835	99.2834	1.3
2.840	99.2821	1.6
2.845	99.2808	0.5
2.850	99.2796	0.5
	Total	14.5

Questions:

a. What is the auction's stop price?

b. How many bids (in dollars) are accepted above the stop price?

c. What proportion of their request do bids at the stop price receive?

d. What is the weighted average bid that the noncompetitive bidders receive?

e. What is the tail?

7. What is the difference between the English auction used by the Treasury and a Dutch auction?

8. What are the sources of income for security dealers?

9. What are on-the-run and off-the-run issues?

10. What is the interdealer market?
11. Explain how a repurchase agreement and reverse repurchase agreement are created.
12. A security dealer plans to purchase $100 million of T-notes at the next auction. She anticipates holding the securities one day and plans to finance the purchase with an overnight repurchase agreement. Currently overnight repo rates are at 3%.
    a. Explain how the repurchase agreement would work in this case.
    b. Determine the dollar interest, selling price, and repurchase price on the repurchase agreement.
    c. What type of repurchase agreement would the dealer need if she thought there was a possibility that it could take several days to sell her securities?
13. Explain how the following institutions use repurchase agreements:
    a. Commercial banks
    b. Security dealers
    c. Federal Reserve
14. Explain the default risk associated with a repurchase agreement and how the risk can be reduced.
15. What was the 1985 repo scandal? What factors contributed to the scandal? What were some of the reforms that resulted from the scandal?
16. Explain the role of federal agencies in the capital formation process.
17. Briefly explain the history of the secondary mortgage market.
18. Explain the purposes of the following:
    a. Fannie Mae, Ginnie Mae, and Freddie Mac
    b. Federal Agriculture Mortgage Corporation
    c. Federal Farm Credit Banks System
    d. Federal Home Loan Bank System
    e. Student Loan Marketing Association, Sallie Mae
19. Describe the solicitation method used by fiscal agents, brokers, and dealers to sell federal agency securities in the primary market.

WEB EXERCISES

1. Find the yields on T-bills and T-bonds at recent Treasury auctions by going to
    - www.treasurydirect.gov/indiv/products/prod_auctions_glance.htm
    - Recent Bills Auctions: www.treasurydirect.gov/RI/OFBills
    - Recent Bonds, Notes, and TIPs Auctions: www.treasurydirect.gov/RI/OFNtebnd
2. Check to see if there have been any changes in the Primary Security Dealers list shown in Table 7.7 by going to www.newyorkfed.org/markets/pridealers_current.html.

3. Treasury-Inflation Index securities and STRIPS are popular Treasuries. Find the current yields on some select TIPs and STRIPS by going to the *Wall Street Journal* site, http://online.wsj.com/public/us.

4. Find the current yields on some select GSEs by going to the *Wall Street Journal* site, http://online.wsj.com/public/us.

5. Explore the programs and functions of the some of the following GSEs and agencies:
   - Federal National Mortgage Association (FNMA, Fannie Mae)
     - Home page: www.fanniemae.com/
     - Investments: www.fanniemae.com/investors/index.html
   - Federal Home Loan Mortgage Corporation (FHLMC, Freddie Mac)
     - Home page: www.freddiemac.com/
     - Debt securities: www.freddiemac.com/debt/
   - Federal Agriculture Mortgage Corporation (FAMC, Farmer Mac)
     - Home page: www.farmermac.com/
     - Debt securities: www.farmermac.com/investors/debtsecurities/index.aspx
   - Federal Home Loan Bank (FHLB)
     - Home page: www.fhlbanks.com/
     - Financial information: www.fhlb-of.com/specialinterest/financialframe2.html
   - Federal Farm Credit Bank System
     - Home page: www.farmcredit-ffcb.com/farmcredit/fcsystem/overview.jsp
     - Federal Farm Credit Funding Corporation: www.farmcredit-ffcb.com/farmcredit/index.jsp
   - Tennessee Valley Authority (TVA): www.tva.gov/
   - Government National Mortgage Association (GNMA, Ginnie Mae)
     - Home page: www.ginniemae.gov/
     - Investors: www.ginniemae.gov/investors/investors.asp?Section=Investors
   - Federal Financing Bank: www.treas.gov/ffb/
   - Student Loan Marketing Association (SLMA, Sallie Mae)
     - Home page: www.salliemae.com/
     - Investors: www.salliemae.com/about/investors/
     - Debt securities: www.salliemae.com/about/investors/debtasset/default.htm
   - International Bank for Reconstruction and Development—World Bank
     - Home page: www.worldbank.org/
     - Debt securities: http://treasury.worldbank.org/cmd/htm/index.html

6. The 2008 financial crisis led to the American Recovery and Reinvestment Act and the Emergency Economic Stabilization Act and Troubled Asset Relief Program (TARP). Review these and other acts by going to the U.S. Treasury's site. Reports from the U.S. Treasury: www.treas.gov/.

7. Changes in government debt depend on federal government expenditures and revenues. Tables on U.S. government's expenditures, revenues, deficits, and debt to view or download can be found at www.gpo.gov/fdsys/search/pagedetails.action?granuleId=&packageId=BUDGET-2010-TAB. Table 1.3 provides a summary of receipts, outlays, and surpluses or deficits (–) in current dollars and Table 7.1 shows federal debt at the end of year: 1940–2014. Comment on the number of budget deficits that have occurred over the last 20 years. Comment on the deficit increases after 2008.
8. Determine the growth in the government's debt over the last 20 years by going to www.gpo.gov/fdsys/search/pagedetails.action?granuleId=&packageId=BUDGET-2010-TAB and Table 7.1.
9. Determine the current distribution of U.S. debt by going to the Treasury Bulletin: www.fms.treas.gov/bulletin/. See "Ownership of Federal Securities" and "Federal Debt."
10. Fannie Mae and Freddie Mac provide forecasts of housing starts and the economy. Review their latest forecast by going to
    - FNMA site: www.fanniemae.com/investors/index.html, "Monthly Economic Outlook"
    - FHLMC site: www.freddiemac.com/debt/, "Economic and Housing Research"
11. Recent information and reports on the Treasury and agency security markets can be found at investinginbonds.com. The reports often include information on recent issuance and trends and a forecast of yields. Summarize their most recent findings:
    - www.investinginbonds.com/ Recent commentary
    - SIFMA Quarterly Government Securities Issuance and Rate Forecast: www.investinginbonds.com/news.asp?id=3100&catid=36 Full report
12. Price and yield quotes can be found at FINRA and Yahoo! Use the advance search at those sites to find Treasuries, STRIPS, GSEs, and Agencies with certain features.
    - FINRA: Go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds."
    - Click "Treasury and Agency" tab and then click "Advanced Bond Search" to find Treasury and Agency bonds with certain features.
    - Yahoo.com: Go to http://finance.yahoo.com/bonds, click "Advanced Bond Screener," and click Treasury or Treasury Zero Coupon (Treasury strips).

NOTES

1. During 2009 the projected revenue for the year was $2.157 trillion: 44% from income, 42% from social insurance, and 7% from corporate. The projected expenditures for 2009 were $3.998 trillion: 54% for individual security programs, 17% for national defense, 4% for interest on the federal government debt, and 23% for physical resources (the latter

expenditure reflecting the fiscal policy stimulant). The government's projected excess of tax expenditures over revenues in 2009 equates to a deficit of $1.841 trillion.

2. There was a threat of default in 1996 due to a political debate over the budget between President Clinton and the U.S. Congress. The debate led to a temporary impasse in which Congress refused to approve a spending program.
3. Marketable securities are sometimes classified as fixed-principal securities (T-bills, T-notes, and T-bonds) and inflation-indexed securities (TIPS).
4. The Treasury also sells ***tax-anticipation bills*** to corporations four times a year. The bills mature one week after a corporate tax payment is due. They are used to pay corporate taxes.
5. In the secondary market, T-bills are typically quoted two decimal places and three decimal places for more active issues.
6. When auctioned, Treasury bids are quotes on a yield basis. The coupon rate, though, is not set until auction is completed.
7. The inflation index used is the non-seasonally adjusted all-items consumer price index for all urban consumers—CPI-u.
8. At one point, the U.S. Treasury was not a supporter of strips because of their lower tax liability.
9. Dealers give government security brokers their bid or offer price. The brokers then display the highest bid and lowest offer through a computer network tied to trading desks of dealers.
10. Note that in the case of term repos, the collateral seller still receives any interest paid on the securities.
11. Some financial institutions prefer lending securities to repurchase agreements because the loan appears as a footnote instead of as an asset or liability on their balance sheet.
12. Pensions are often prohibited from investing in reverse repos; thus, lending securities gives them the equivalent of a reverse repo.
13. FICO was created as a provision in the Competitive Equality and Banking Act of 1987.

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CHAPTER 8

Municipal Securities

8.1 INTRODUCTION

In the United States, it is estimated that there are as many as 80,000 state, county, and municipal governments and government authorities (agencies created by the government with the authority to float bonds).[1] Over the last 30 years, there has been a significant increase in borrowing by these governments and authorities, with the debt being financed primarily through the sale of municipal securities. In 2007, total debt outstanding of all state and local governments was approximately $2.414 trillion. In this chapter, we examine the types of securities municipal governments issue and the markets in which they trade.

8.2 MUNICIPAL FINANCING

Over the last two decades, population growth, shifts from central cities to subdivisions, and other demographic changes have led to substantial increases in the expenditures of municipal governments. In 2007, the total expenditure for all state and local governments was approximately $2.657 trillion. The major expenditures were allocated to education (29%) and social services (14%), with the remainder being spread among a variety of state and local government expenditures: transportation, correctional facilities, police and fire protection, public safety, environmental cleanup, and interest on debt. The total revenue generated in 2007 by all state and local governments to support these expenditures was approximately $3.063 trillion. The major sources for this revenue came from taxes on sales, property, individual income, corporations, and other sources such as user fees, lotteries, and federal government transfers (see U.S. Census Bureau: www2.census.gov/govs/estimate/0700ussl_1.txt).

Like corporations, state and local governments undertake many long-term capital projects such as the construction of school buildings, highways, water treatment facilities, airports, hospitals, inner-city housing, and infrastructures that facilitate economic growth and job creation. To finance these long-term capital investments, they sell two types of notes and bonds: general obligation bonds and revenue bonds. In addition to their long-term investments, state and local governments also have short-term cash flow needs created from differences in their expenditure and revenue patterns and time gaps between when projects begin and when the permanent financing supporting the projects is received. To finance their short-term cash needs, they sell short-term anticipation notes. Security traders collectively refer to short-term anticipation notes, general obligation bonds, and revenue bonds as municipals or munis.

8.3 TAX-EXEMPT STATUS

Although there are a number of different types of municipal debt securities, one feature most of them have in common is their tax-exempt status. Specifically, the interest on municipals (but not the capital gain) is exempt from federal income taxes (both personal and corporate). In addition, most states also exempt the coupon interest earned on in-state issues from state income taxes and personal property taxes where it is applicable. This tax-exempt feature makes municipals very attractive to individuals and corporations in the higher income tax brackets and investment funds whose clients are in the higher brackets. An investor in the 35% tax bracket would be indifferent, with all other factors equal, to a fully taxed bond yielding 10% and a tax-exempt bond yielding 6.5%. Alternatively stated, if the yield on a tax-exempt bond is 6.5%, the ***investor's equivalent taxable yield*** on a fully taxable bond is 10%:

$$\text{Equivalent taxable yield} = \frac{\text{Tax-exempt yield}}{(1 - \text{Tax rate})} = \frac{.065}{1 - .35} = .10$$

Note that if a municipal issue is selling at a discount from par, then the after-tax yield to maturity is calculated by solving for the yield that makes the present value of the municipal's after-tax cash flow equal to its current price.

Changes in tax codes affect the demand for municipal securities. If the federal tax rate increases (decreases), then the tax-exempt feature of municipal securities becomes more (less) valuable. Thus, a tax increase (decrease), or its expectation, would cause the demand and price of municipals to increase (decrease) and their yields to fall (decline). The demand for municipals decreased notably after the Tax Reform Act of 1986 lowered rates, reducing the tax-exempt benefits of municipals; the demand, though, increased in 1993 when marginal tax rates were increased. The possibility of changes in the tax rates, in turn, exposes municipal investors to additional risk—***tax risk***. Changes in the tax codes can also change the relative preferences among different groups of municipal bond investors. Until the Tax Reform Act of 1986, commercial banks were one of the largest purchasers of municipals. The lowering of the top corporate tax rate from 46% to 35%, though, led to a sizable reduction in their investment in municipal bonds.

It should be noted that the tax code governing tax-exempt securities is quite complex. For example, consider a municipal bond purchased at a discount from its par at the initial offering, referred to as an ***original-issue discount bond (OID)***. An investor who buys an OID and holds it to maturity can treat the difference between the issue price and the par value as tax-exempt interest. If the bondholder subsequently sells the OIB before maturity, any increase in its price up to its par value is generally considered interest income and is tax exempt, whereas any increase above the par value is considered a capital gain and is taxable. For an investor who buys an OIB in the secondary market, though, the tax treatment depends on the purchase price relative to what the IRS defines as the market discount cutoff price (the price defining the allowable discount) and the revised issue price (price reflecting the price change over time that must be accreting). Another factor investors need to consider in determining tax liabilities is whether they leveraged the investment. In general, the interest expense on funds borrowed to purchase securities is tax deductible, except when the funds are used to purchase or carry tax-exempt securities. At one point, banks were exempt from this rule and were allowed to deduct all interest expense; later they were allowed to deduct 85% of the interest expense; finally, in 1986, their interest deductibility for financing tax-exempt bonds was eliminated unless the issue was a bank-qualified issue (generally this is tax-exempt bonds by small issuers and purchased for an investment portfolio). In addition to defining tax-exempt interest and deductibility of interest expenses, there are other tax considerations individual and institutional investors need to consider, such as the adjustments for specified tax preferences that some taxpayers are allowed. Finally, there are different state and local tax treatments that can also be complex.[2]

Although the tax-exempt feature of municipals is one of their most attractive features to investors, there are taxable municipal bonds, in which the interest is subject to federal income taxes. Like corporate credits, ***taxable municipals*** sell at a lower price and higher yield than comparable tax-exempt municipals. The types of taxable municipals include those used to finance projects (e.g., Build America Bonds to finance local capital projects) in which there are federal restrictions (many specified in the Tax Reform Act of 1986) that limit or prohibit financing with tax-exempt municipal bonds. Taxable municipals are attractive investments to tax-exempt investors (e.g., private trusts) or foreign investors who cannot benefit from the tax benefits of tax-exempt municipals.

8.4 TYPES OF MUNICIPALS

Municipal securities can be short-, intermediate- or long-term obligations, and they can be funded either from general tax revenues, the revenues from specific projects, or both. As previously noted, there are three types of municipals: anticipation notes, general obligation bonds, and revenue bonds.

Anticipation Notes

To finance short- to intermediate-term cash needs municipal governments issue several types of securities. These securities are sold to obtain funds in lieu of anticipated revenues. They include ***tax-anticipation notes*** (***TANs***), ***revenue-anticipation notes*** (***RANs***), ***grant-anticipation notes*** (***GANs***), ***bond-anticipation notes*** (***BANs***), and ***municipal tax-exempt commercial paper***. These obligations are usually secured by the issuer's taxing power and sell at yield spreads over Treasuries that reflect their tax-exempt status and credit ratings. Most are sold on a pure discount basis, with face values ranging from $5,000 to $1 million and maturities ranging from one month to three years.

TANs and RANs (also referred to as TRANs) are used to cover regular recurring government expenses before taxes and other anticipated revenues are received. BANs are used as temporary financing or construction financing for long-term projects (e.g., roads, correctional facilities, university buildings, and libraries), with the principal paid from the proceeds from the sale of a long-term municipal bond. For example, the construction funds needed to build a new College of Business building at the state university could be raised by the state selling a BAN, with the BAN being paid off from the proceeds from a long-term revenue bond. Finally, municipal tax-exempt commercial paper is sold primarily to finance recurring expenses. Like CP sold by corporations, tax-exempt CP has a maturity ranging from 30 days to 270 days and is often secured with a bank letter of credit, line of credit, or a purchase agreement in which the bank agrees to buy the bond if the issuer fails. Some municipal governments also finance their short-term cash needs with a tax-exempt ***variable-rate demand obligation*** (also called a ***floating-rate obligation***). These obligations have a long-term maturity but with a coupon reset frequently (e.g., every day or week) and often with embedded put and call options.

General Obligation Bonds

General obligation bonds (***GOs***) are intermediate- and long-term debt obligations that are secured by the issuing government's general taxing power and that can pay interest and principal from any revenue source. The GOs issued by states and large municipal governments that have a number of tax revenue sources and unlimited tax power are referred to as ***unlimited tax GO***s. They are considered backed by the ***full faith and credit*** of the issuer. The GOs issued by smaller municipalities or authorities whose revenues are limited to only one or two sources (e.g., property tax) or who have statutory limits on the tax rate that the issuer may levy to finance the debt are known as ***limited-tax GO bonds***. Like corporate bonds, the contract between the issuer and the investor is specified in the GO's trust indenture. With municipals, this document is usually accompanied by an official statement and a legal opinion. The ***official statement*** is a document, similar to the prospectus for a stock or corporate bond, which details the return, risk, and other characteristics of the issue and provides information on the issuer (see Table 8.1). The ***legal opinion*** is a document that interprets legal issues related to the bond's collateral, priority of claims, and the like. Municipal bond attorneys with Wall Street-based law firms, as well as local firms, prepare legal opinions. In addition, many bank and investment banking firms have their own counsel to review and prepare such documents.

TABLE 8.1 Information in Official Statement

Amount of the issue
Credit rating
Information on issuer
Names of the underwriters
Selling group
Sources of payments
Sources and uses of fund statement
Financial statements
Debt service required
Notice of any pending legislation
Bond insurance (if any)

The municipal defaults that have occurred over the last 20 years and the subsequent problems related to legally defining the security, revenue sources, and priorities have made the legal opinion an important information source for assessing a municipal bond's credit risk. In evaluating the creditworthiness of a GO bond, investors need to review the legal opinion to determine the state or local government's unlimited taxing authority. The legal opinion should identify if there are any statutory or constitutional limitations on the jurisdiction's taxing power, as well as any priority of claims on general funds. Finally, the legal opinion should also specify what the bondholders' redress is in the case of a default and whether there are any statutory or constitutional questions involved. Municipal defaults are usually handled through a restructuring, which makes the security and priorities as defined in the indenture and explained in the legal opinion important in establishing the types of new debt the municipal bondholders might receive.

Revenue Bonds

Revenue bonds are municipal securities paid by the revenues generated from specific public or quasi-public projects, by the proceeds from a specific tax, or by a special assessment on an existing tax. Occasionally, revenue bonds are issued with some general obligation backing and thus have characteristics of both GOs and revenue bonds. For example, some revenue bonds are secured by user charges as well as a GO pledge. Such bonds are referred to as being ***double-barreled.*** Similarly, some revenue bonds are secured by and paid from more than one revenue source. For example, some school districts issue bonds to finance certain capital projects that are paid for by property taxes earmarked for the project and are also secured by special funds of the state such that, in the event of a default, the investors can go to the state. Finally, there are ***dedicated tax-backed revenue bonds*** that are paid from dedicated revenues such as a tobacco settlement, lottery, or special fee. Some of the more common revenue bonds and their characteristics are summarized in Exhibit 8.1.

To the issuer, revenue bonds are an important source of funding for a number of public projects. They are used to finance major capital projects such as roads, bridges, tunnels, airports, hospitals, power-generating facilities, water treatment plants, and municipal and university buildings; they also support educational programs, inner-city housing development, and student loan programs. The revenues used to pay the interest and principal payments on these bonds are usually project specific and include tolls, rents, user charges, earmarked revenues from fees, and specific taxes. It should be noted that many revenue bonds are used to support not just public projects, but also those projects that benefit both public and private interests. In the 1980s, many private sector companies began to use tax-exempt revenue bonds to finance industrial parks, electricity-generating plants, and other capital projects. One popular revenue bond used to support private-public sector projects is the industrial development bond (IDB), also called an industrial revenue bond (IRB). Many state and local governments or authorities sell IDBs to finance the expansion of an area's industrial base or to attract new industries. Typically, the government or authority floats a bond issue and then uses the proceeds to build a plant or an industrial facility; it then leases the facility to a company or provides a low interest loan for the company to acquire the asset. Because of the tax-exempt status of municipal bonds,

EXHIBIT 8.1 Types of Revenue Bonds

Highway Revenue Bonds: These revenue bonds are used to finance highway systems and their related infrastructures (bridges, tunnels, etc.). There are two general types of highway bonds, classified in terms of how the transportation facility or highway is financed. If the highway, bridge, or tunnel is financed by a toll, then the toll revenue pays the bondholder's interest and principal payments. The quality of these bonds depends on the ability of the project to be self-supporting. Financial problems that have arisen from these bonds are often based on poor traffic projections. Alternatively, earmarked revenues from gasoline taxes, driver license fees, or auto registration fees may finance the transportation system. Analysts who evaluate the creditworthiness of bonds often look at such coverage ratios as earmarked revenue to debt service.

Water and Sewer Revenue Bonds: These bonds are issued to finance the building of water treatment plants, pumping stations, and sewers. Local governments or special bond authorities usually issue them. The bonds, in turn, are usually paid for by user charges. Covenants in the indentures may also specify that user charges be a specified proportion of debt services and reserves.

Lease Rental Bonds: These bonds are used to finance the construction of public office buildings, stadiums, university facilities, and the like, and for the purchase of computers and other types of capital equipment. The bonds are paid for by the rents generated from the users: rents, tuitions, earmarked revenues, annual appropriation of a general fund, or stadium receipts. These bonds are sometimes referred to by the facility they are financing: for example, Convention Center Revenue Bond or Sport Stadium Bond.

Hospital Revenue Bonds: These bonds are used to build or expand hospitals, to purchase medical equipment, and so on. The revenues used to finance the bonds are usually established by formulas involving a number of different levels of government and the medical facility's major source of revenue (e.g., Medicare and/or Medicaid).

Airport Revenue Bonds: Airport revenue bonds are used to finance the construction, expansion, or improvements of municipal airports. The bonds are usually secured by leases with the major airlines for the use of the terminals or by the revenues obtained from landing fees or fueling fees paid by the airlines and by the concession fees paid by terminal store users.

Industrial Development Bonds: State and local governments or authorities sell industrial development or revenue bonds (IDBs) to finance the expansion of an area's industrial base or to attract new industries. Typically, the government or authority floats a bond issue and then uses the proceeds to build a plant or an industrial facility; it then leases the facility to a company or provides a low interest loan for the company to acquire the asset. IDBs have been used to finance industrial parks, electricity-generating plants, and other projects.

Lottery Bonds: These are secured by expected future lottery revenue. They are often used to finance the construction of new school facilities.

Pollution Control Bonds: These bonds are used to help corporations purchase pollution control equipment. Often a municipal government will buy the equipment through the sale of the bonds and then lease the equipment to the corporation.

Resource Recovery Revenue Bonds: These bonds finance resource recovery operations that convert solid waste into commercially recoverable products and landfill residue. Revenue from these operations comes from fees for removing garbage or sales from the products generated.

Public Power Revenue Bonds: These bonds are used to finance construction of electricity-generating power plants and distribution systems. The bonds may be issued to finance the construction of one or several power plants with two or more utility companies. In this case, the issue is referred to as joint-power financing.

EXHIBIT 8.1 (*Continued*)

Sports Complex and Convention Center Bonds: These bonds are issued as permanent financing of sports stadiums and arenas and convention centers. Bond may be lease-rental bonds paid for by rental income from the facility or they may be paid from revenue generated from outside revenue sources such as local hotel taxes or city or county taxes.

Life-Care Revenue Bonds: These bonds are issued by state and local development agencies to finance the construction of long-term residential care facilities for the elderly managed by nonprofit agencies or religious groups. Revenues supporting the bonds are generated from lease rentals or lump-sum payment made by the residents.

Multi-Family Mortgage Revenue Bonds: These bonds are used to finance multi-family structures for low-income families and senior citizens. Some of the facilities are federally secured or provide interest cost subsidies or property tax reductions.

Single-Family Mortgage Revenue Bonds: These bonds are used to secure mortgages on single-family homes insured by FHA, VA, or private mortgage insurance.

Section 8 Bonds: These are municipal bonds issued to finance low- and middle-income rental housing. The bonds are issued under terms specified by the Federal Housing Act. Under Section 8 of the act, the U.S. Department of Housing and Urban Development (HUD) maintains a cash reserve for each project to protect against the failure of residents to pay rent. In some cases, eligible low-income tenants pay 15% to 30%, with the government subsidizing the remainder. The risk to bondholders comes from the apartment building not maintaining a sufficiently high occupancy rate.

College and University Revenue Bonds: These bonds are used as permanent financing of college buildings (libraries, classroom buildings, dormitories, and the like) and other university capital projects (computers and networks). Bondholders' interest and principal is paid from tuition, dormitory rental fees, and special fees. They also fall under the category of lease-rental bonds.

Student-Loan Revenue Bonds: These are bonds issued by state government agencies with the proceeds used to support loans to college students. The proceeds from these bonds are often used for purchasing federally guaranteed student loans made by local banks.

Tax Allocation Bonds: These bonds are issued to finance office and property development in blighted or low-income areas. Bondholders are paid from property taxes that are expected to increase from improved real estate values.

this type of financial arrangement benefits all parties: Investors receive a higher after-tax yield, corporations receive lower interest rates on loans or lower rental rates, and the area benefits from a new or expanding industry.[3]

As with GOs, revenue bonds have an indenture, legal opinion, and an official statement. Some of the important provisions delineated in the indenture and legal opinion include (1) whether the issuer can increase the tax or user's fee underlying the revenue source; (2) whether the issuer can incur additional debt secured by the revenue of the project (referred to as minimum revenue clauses) or under what conditions new debt can be incurred; (3) how the revenues of the project are to be directed—if they are to be paid to bondholders after operating expenses but before other expenses (this is called a net revenue-structured revenue bond) or to bondholders first (this is called a gross revenue-structured bond); and (4) whether there is any additional collateral or guarantees.

8.5 SPECIAL FEATURES OF MUNICIPALS

Two features that are more common among municipal bonds than corporate debt securities are serialization and default insurance. As noted in Chapter 6, serialization refers to the breaking up of a bond issue into different maturities. For example, to finance a $20 million convention center, a county might sell a serial issue with four types of securities, each with a face value of $5million, but with one maturing in year 5, one in year 10, one in year 15, and one in year 20. Instead of a serial issue, some municipals are sold with a serial maturity structure that requires a portion of the debt to be repaid each year. Like many corporate bonds, a number of municipals are sold with the principal paid at maturity. Many of these term bonds often include sinking fund arrangements. There are also several GOs and revenue bonds sold as zero-discount bonds in which the interest is equal to the difference between the purchase price and the face value. Municipal governments and authorities also sell a variation of a zero-coupon bond known as a ***municipal multiplier***, accretion, or compound interest bond. This bond pays coupon interest, which is not distributed, but rather is reinvested to the bond's maturity, making the bond similar to a zero-coupon bond.

Insured municipal bonds are ones secured by an insurance company. Insurance is provided by single-line or monoline insurance companies, whose primary business is providing municipal bond insurance, and by large diversified insurance companies. There are also multiline property and casualty insurance companies who provide municipal bond insurance along with their other types of insurance products. Major monoline insurers include AMBAC Indemnity Corporation (AMBAC), Municipal Bond Investors Assurance Company (MBIA), and Financial Guaranty Insurance Company (FGIC). The insurers write insurance policies in which they agree to pay interest and principal to bondholders in the event the issuer fails to do so. The municipal issuer and not the bond investor usually pays the insurance premium. Once the insurance is issued, the insurance company has a contractual commitment to pay the bondholders if they do not receive interest or principal payments from the issuer. Municipal insurance can also be obtained from the bondholder or issuer after it is issued. This is referred to as secondary market insurance.

Prior to 2007, about 50% of all new municipal bond issues were insured, with many trading with AAA quality ratings. After the downgrades of several monoline insurers from subprime mortgage losses in 2007 and 2008, many insured municipal bonds were downgraded.

In addition to insured bonds, two other types of municipal bonds carrying external protection are letter-of-credit (LOC) bonds and refunded bonds. Letter-of-credit-backed (LOC-backed) municipal bonds are secured by a letter of credit from a commercial bank. In some cases, the government unit must maintain a certain investment quality rating to maintain the guarantee. Refunded bonds are municipal bonds secured by an escrow fund consisting of high-quality securities such as Treasuries and federal agencies. There are also refunded municipals backed by an escrow fund consisting of a mix of Treasuries and non-Treasuries such as municipals. Because of their backing, insured municipal bonds, LOC-backed municipals, and refunded bonds all sell at yields lower than they would without such protection.

In contrast to bonds insured or backed by a bank or collateral, Mello-Roos and moral obligation bonds are bonds that are not fully backed. Mello-Roos bonds are municipal securities issued by local governments in California that are not backed by the full faith and credit of the government. These bonds were the result of

Proposition 13. This law, approved in 1978, set maximum property tax rates, prohibited statewide property taxes, and required a two-thirds vote of the legislature for approval of any increase in state taxes. With such constraints on revenue, some local governments in California were forced to issue municipal bonds without full backing. Moral obligation bonds, in turn, are bonds issued without the legislature approving appropriation. The bonds are therefore considered backed by the permissive authority of the legislature to raise funds, but not the mandatory authority.

A moral obligation bond is typically classified as an appropriation-backed bond. It represents a nonbinding pledge to approve appropriations and is considered a credit enhancement. In contrast, municipalities do issue credit-enhanced bonds that are legally binding. Such bonds often take the form of an obligation to withhold and/or provide government aid to pay any defaulted debt issue. A state might provide this type of public credit enhancement to debt securities issued by one of the state's school systems, with the state standing ready to cover shortfalls in the case of default. Often with this type of guarantee there is a provision that the state would withhold some state aid earmarked to the school system's municipality.

In addition to serial municipal bonds and insured bonds, there are also GOs and revenue bonds introduced over the last two decades with elaborate security structures and features. For example, there are municipal put bonds (bonds that can be cashed in at a specific value before maturity), municipals with warrants that allow the holder to buy additional bonds at set prices, municipal floaters (municipal bonds with floating rate tied to a reference rates such as T-bill rates, London Interbank Offer Rate, or municipal bond index), and minibonds—low denomination issues ($100, $500, and $1,000 par) that are sold directly to the public without an investment banker. Like Treasury stripped securities, the municipal bond market has developed interest-only and principal-only stripped municipals and floating-rate and inverse floating-rate stripped securities. Many of these municipal derivatives are created by investment banking firms, who buy municipals, place them in a trust, and then create derivative securities.

8.6 CREDIT RISK, QUALITY RATINGS, AND CREDIT SPREADS

In 1975, the Urban Development Corporation of the state of New York defaulted on a $100 million New York City obligation. Three years after that default, Cleveland became the first major U.S. city since the depression to default on its debt obligations. Although both were able to work out arrangements with local banks to pay their creditors, these events nevertheless raised immediate concern among investors over the quality of municipal bonds—bonds that up until then had been considered second in security to Treasury and federal agency securities. Unfortunately, these concerns did not abate. For example, consider the following:

- In the 1980s, cities such as Washington DC, Detroit, and Chicago experienced financial crises; approximately 70 municipalities filed for bankruptcy in 1980 (although none of them defaulted).
- In 1983 the Washington Public Power Supply System (WPPSS) defaulted on a $2 billion municipal government bond issue, with the courts ruling

that bondholders did not have claims to certain revenues identified in the indenture.

- In 1991, 260 municipal governments defaulted.
- In 1994, Orange County, California defaulted after some ill-conceived investment strategies by the county treasurer.
- In 2009, states such as California and a number of cities experienced budget deficit problems resulting from declines in tax revenues and increases in expenditure on welfare and education, crime prevention, and the like. States as a whole faced a $150 billion budget shortfall.

Exhibit 8.2 summarizes some of the defaults of Moody's rated municipals from 1970 to 2006 and Exhibit 8.3 quotes Moody's summary reports of the Orange County and the Washington Public Power Supply System defaults.

Although the aggregate economic growth and prosperity experienced for most of the decade of the 1990s served to increase the revenues of many state and local governments, the financial problems faced by many governments in the 1980s and early 1990s, pointed to the credit risk associated with municipals and the importance of careful and prudent credit analysis by investors in evaluating municipal bonds. More recently, the financial crisis and recession of 2008 and 2009 has again put many municipalities in precarious financial states, leading to defaults, downgrades, federal government assistance, and the issuance of IOUs.

Quality Ratings

To assist investors in determining the creditworthiness of municipals, Moody's, Standard & Poor's, and Fitch provide quality ratings on municipal securities similar to the ones they use for corporate bonds. Moody's has nine different categories from Aaa to C, with investment grades being Aaa to Ba. Moody's also includes a numerical modifier to indicate the degree of quality in each category (e.g., Aa1, Aa2, Aa3, A1, A2, and so on). They also use a prefix 'con' to indicate when a revenue bond is dependent on the completion of a project or when there is some current limiting condition. Standard & Poor's has 10 categories from AAA to D, with AAA to BBB being the investment grades. They use + and − signs to indicate relative strength and a 'p' to indicate a bond with provisional funds. Moody's and Standard & Poor's also rate notes and tax-exempt commercial paper. Finally, Fitch provides 10 rating categories similar to Standard & Poor's.

In determining ratings, Moody's, Standard & Poor's, and Fitch consider such factors as the amount of outstanding debt, the economic conditions of the area, the revenue sources backing the issue, the provisions specified in the indenture, and the legal opinion. If the bond is insured, Moody's and Standard & Poor's also look at the credit quality of the insurer. It should be noted that each company can rate an issue differently. When a bond is rated differently, it can reflect a different emphasis that each company places on certain parameters or differences in methodologies. Approximately 50% of municipal bonds have an A rating, with 10% having a triple-A rating. Included in this group are many of the insured bonds and refunded issues. Table 8.2 shows Moody's rating distribution by select sectors and Table 8.3

EXHIBIT 8.2 Moody's Defaults Summary: 1970–2006

General Obligation and Water/Sewer Enterprise:
1970–2000: 1 default
2001–2006: 0 defaults

Non-GO Obligations of State and Local Governments:
4 defaults
1970–2000: 3 defaults
2001–2006: 1 default
2001–2006:
Cicero Local Development Corporation—Annual Lease Appropriation Bonds Series 2001A (NY)

Electric Power:
2 defaults
1970–2000: 2 defaults
2001–2006: 0 defaults

Private Universities and Other Not-for-Profits:
1 default
1970–2000: 1 default
2001–2006: 0 defaults

Not-for Profit Healthcare:
17 defaults
1970–2000: 10 defaults
2000–2006: 7 defaults
2001–2006:
1. Citizens General Hospital (PA)
2. Genesee Hospital (NY)
3. Metro Health Center (PA)
4. St. Francis Medical Center (PA)
5. Mercy Hospital and Medical Center (IL)
6. National Benevolent Association (MO)
7. Fort Worth Osteopathic Hospital (TX)

State and Local Housing(Unaffiliated):
16 defaults
1970–2000: 2 defaults
2001–2006: 14 defaults
2001–2006:
1. Nebraska Investment Finance Authority, Section 8 Moderate Rehabilitation Program, Yorkshire Development Project (NE)
2. Indianapolis Economic Development Authority, Phoenix, IN (Meadows Project, Section 8) (IN)
3. Travis County Housing Finance Corporation, Lakeview Apartments (TX)
4. Tarrant County Housing Finance Corporation, Fair Oaks Apartments (TX)
5. Magnolia Park Housing Foundation, LLC, Magnolia Park Apartments(GA)
6. Tarrant County Housing Finance Corporation, Westridge Apartments (TX)
7. Maricopa County IDA, Bay Club at Mesa Cove Project (AZ)
8. Capital Trust Agency, River Bend Apartments (FL)
9. Tarrant County Housing Finance Corporation, Crossroads Apartments (TX)
10. South Carolina Jobs-Economic Development Authority, Legacy at Anderson Project (SC)
11. Texas State Affordable Housing Corporation, Ashton Place and Woodstock Apartments Project (TX)
12. American Opportunities Foundation, River Falls Project (CO)
13. Lee County Industrial Development Authority, Legacy at Lehigh Project (FL)
14. Greenville Housing Finance LLC, Cameron Crossing I and II Projects (CA)

1970–2000: 19 defaults
2001–2006: 22 defaults
TOTAL: 41 defaults

Source: Moody's: www.moodys.com, Ratings Methodology & Performance, Historical Performance, "Mapping to the Global Rating Scale and Assigning Global Scale Ratings to Municipal Obligations March 2007."

EXHIBIT 8.3 Moody's Case Summary of Municipal Defaults

Orange County, CA

- CUSIP: 68428LAN4
- Default date: December 6, 1994
- Obligor: Orange County, CA
- Issuer: Orange County, CA
- Defaulted bonds: Pension Obligation Series B; $110 million of debt affected
- Cause of default: Orange County Investment Pool's investment losses
- Recovery: Although the county was unable to fulfill its pledge to purchase any tendered bonds, all principal and interest payments were made. (*Source:* Moody's reports.)

On December 6, 1994, Orange County, California filed bankruptcy petitions for both itself and the Orange County Investment Pool (OCIP). The County had pledged that the OCIP would purchase any tendered Pension Obligation Series B bonds, but as a result of the bankruptcy filing, the OCIP was unable to fulfill this obligation and the bonds defaulted on December 8, 1994. The County did not default on the scheduled principal and interest payments of the Series B bonds or any of its other long-term obligations.

The Orange County bankruptcy was the largest municipal bankruptcy in U.S. history. Orange County's bankruptcy filing was a direct result of the investment losses incurred by the Orange County Investment Pool (OCIP), which amounted to approximately $1.5 billion of the $7.5 billion pool. The investment strategy of the county treasurer involved investing in high-risk, interest-rate sensitive securities and leveraging the pool to further increase returns. During the period when interest rates were on the decline and remained low, the OCIP succeeded in earning high returns. However, when interest rates began to rise in 1994, the OCIP experienced big losses. Adding to the financial distress of OCIP, when OCIP was unable to repay a $1.2 billion loan to a Wall Street creditor, the creditor refused to extend the loan and started liquidating the securities that OCIP had pledged as collateral for the loan. To protect itself from other creditors, Orange County filed for bankruptcy for itself and OCIP.

Rating History

RATING DATE	RATING	RATING ACTION
8 SEP 1994	A1	ASSIGNED
8 DEC 1994	Caa	DOWNGRADED
30 MAY 1996	Ba	UPGRADED
1 AUG 1996—WITHDRAWN		

Source: Moody's: www.moodys.com, Ratings Methodology & Performance, Historical Performance, "Mapping to the Global Rating Scale and Assigning Global Scale Ratings to Municipal Obligations March 2007." Reprinted with permission from Moody's.

Washington Public Power Supply System, WA (now Energy Northwest)

- CUSIP: 939821LN2
- Default date: August 1983
- Obligor: Washington Public Power Supply System (WPPSS)
- Issuer: Washington Public Power Supply System (WPPSS)
- Defaulted bonds: Nuclear Projects 4 & 5; approximately $2.25 billion of debt affected
- Cause of default: Declining demand for energy, rising construction costs
- Recovery: Approximately 40% after the settlement of a class action suit in December 1998 (*Source:* Moody's files.)

EXHIBIT 8.3 (*Continued*)

In August 1983, Washington Public Power Supply System (WPPSS) defaulted on $2.25 billion of revenue bonds for Nuclear Projects 4 and 5. Washington Public Power Supply System was organized in 1957 as a municipal corporation that allowed publicly owned utilities in the Pacific Northwest to jointly build power generation facilities. As part of the Ten-Year Hydro Thermal Power Plan, WPPSS and other Northwest utilities assumed that demand for electricity in the northwest region would double every 10 years beyond the capacity of current power sources. In the early 1970s WPPSS planned to construct five nuclear generation facilities to meet this forecasted demand. Bonds were sold to finance the cost of the power plants and were to be repaid through participation agreements with numerous municipal and cooperatively owned electric utilities. Construction delays and cost overruns on the sizable project and increased costs to meet newly required safety standards drove the cost of completion of the projects to three to four times the original estimates. At the same time, demand for energy was declining due to rising energy costs, conservation, and an economic slowdown in the area. In January 1982, WPPSS abandoned construction on Projects 4 and 5. In January 1983, the public utilities participating in WPPSS were obligated to begin repaying the debt incurred by the abandoned projects. In order to repay the debt, the utilities would have had to dramatically increase electricity rates on their customers to pay for the failed projects. The uproar due to the increasing rates resulted in challenges to the enforceability of the contracts with participants for repayment of the construction and operation costs of Projects 4 and 5 (including repayment of debt service). In 1983, the Washington Supreme Court ruled that the Washington state public agency participants in Projects 4 and 5 did not have the authority to enter into the Project 4 and 5 participation agreements, rendering void the agreements and the source of revenues to pay debt service. WPPSS became unable to service the debt on the $2.25 billion in bonds issued to finance construction of Projects 4 and 5, thereby precipitating the largest municipal bond payment default in history.

Rating History

RATING DATE	RATING	RATING ACTION
14 FEB 1977	A1	ASSIGNED
10 JUN 1981	Baa1	DOWNGRADED
1 JUN 1983	Caa	DOWNGRADED
16 JUN 1983—WITHDRAWN		

Source: Moody's: www.moodys.com, Ratings Methodology & Performance, Historical Performance, "Mapping to the Global Rating Scale and Assigning Global Scale Ratings to Municipal Obligations March 2007." Reprinted with permission from Moody's.

TABLE 8.2 Moody's Ratings Distribution by Sector

Sector	Aaa	Aa	A	Baa	Below Baa
State General Obligation	15%	80%	4%	0%	0%
Local General Obligation	3%	23%	55%	19%	0%
Airports	0%	13%	74%	14%	0%
Public Higher Education	2%	27%	67%	5%	0%
Hospitals	0%	17%	43%	31%	10%

Source: Moody's: www.moodys.com.

TABLE 8.3 Cumulative Municipal Default Rates: 1970–2005

Years	1	2	3	4	5	6	7	8	9	10
Aaa	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Aa	0.03%	0.03%	0.04%	0.04%	0.04%	0.05%	0.05%	0.05%	0.06%	0.06%
A	0.01%	0.01%	0.02%	0.02%	0.02%	0.03%	0.03%	0.03%	0.03%	0.03%
Baa	0.06%	0.07%	0.08%	0.09%	0.10%	0.10%	0.11%	0.12%	0.13%	0.13%
Ba	1.42%	1.56%	1.62%	1.70%	1.78%	1.98%	2.21%	2.41%	2.56%	2.65%
B	5.86%	7.71%	9.32%	10.64%	11.60%	11.86%	11.86%	11.86%	11.86%	11.86%
Caa-C	15.84%	16.58%	16.58%	16.58%	16.58%	16.58%	16.58%	16.58%	16.58%	16.59%
Investment-grade	0.03%	0.03%	0.04%	0.04%	0.05%	0.05%	0.05%	0.06%	0.06%	0.07%
Speculative-grade	2.42%	2.80%	3.08%	3.33%	3.54%	3.74%	3.93%	4.09%	4.22%	4.29%
All Municipals	0.05%	0.06%	0.06%	0.07%	0.08%	0.08%	0.09%	0.10%	0.10%	0.10%

Source: Moody's: www.moodys.com.

shows Moody's cumulative default rates based on historical default rates from 1970 to 2005.

Yield Spreads

The yields on different municipals reflect, in part, the differences in quality ratings. Like corporate bonds, the spreads between municipals with different quality ratings tend to narrow during economic upturns and widen during economic downturns, reflecting a flight to quality. The quality spread also tends to narrow during periods of low interest rates, reflecting a flight to higher yields. In addition to quality spreads, there are also geographical spreads, reflecting differences in state income taxes and perhaps some parochialism. The yields on in-state issues from states with high income taxes tend to be lower than the yields from those states with low or no income state income taxes (see Table 8.4 for price quotes and yields on select municipals obtained

TABLE 8.4 Municipal Bond Quotes, October 7, 2009

Issue	Coupon	Maturity	Price	Change	Bid Yield
AZ Phoenix Civic Improv Cp	5	7/1/1939	107.657	−0.42	4.05
Bay Area Tll Ath CA SnFrn	5.125	4/1/1939	107.012	−0.407	4.23
Bay Area Tll Ath CA SnFrn	5	4/1/1934	106.688	−0.407	4.15
Birmingham Wtrwrks & Swr	6	6/1/1939	112.013	−0.337	4.47
Broward Co FL airport sys Ser 20	5.375	10/1/2029	105.492	−0.165	4.69
CA state var purp gen obl	6	4/1/1938	110.176	−0.813	4.67
CA state var purp gen obl	6.5	4/1/1933	115.217	−0.841	4.53
California Hlth Facs Fing Auth	5.75	7/1/1939	105.637	−0.159	5.02
Charlotte−Mecklenburg Hosp NC hlthcr	5.25	1/15/1934	105.705	−0.157	4.5
Indianapls Pub Imp Bnd Bk	5.75	1/1/1938	108.564	−0.238	4.61
King Cnty WA swr rev bds 09	5	1/1/1939	106.933	−0.56	4.1
Metro Wshngtn Arpt Auth DC	5	10/1/1939	104.681	−0.249	4.42
Metro Wshngtn Arpt Auth DC	5.25	10/1/1944	106.549	−0.251	4.44
NC Muni Pwr Agy #1 Catawba	5	1/1/1930	105.36	−0.157	4.3
NC Tpke Auth Sr Lien rev	5.75	1/1/1939	107.706	−0.236	4.72
NC Tpke Auth Sr Lien rev	5.5	1/1/2029	108.424	−0.16	4.39
NJ Econ Dev Auth school facs constrct	5	9/1/1934	105.179	−0.249	4.36
NJ Tpke Auth rev Ser 09	5.25	1/1/1940	107.644	−0.24	4.25
OH State hosp rev bds Ser	5.5	1/1/1939	106.624	−0.156	4.62
PA Allghn Gen Obl Ser C-62	5	11/1/2029	106.172	−0.339	4.25
PA Econ Dev Fin Auth wtr	5	10/1/1939	105.301	−0.416	4.35
Penn Turnpke Comm Rv	5.25	6/1/1939	108.337	−0.499	4.2
Puerto Rico Pub Tax Fin	5.75	8/1/1937	107.929	−0.247	4.74
San Fran City & Cty Pub Util	5.125	11/1/1939	108.21	−0.432	4.13
San Fran Pub Utils Comm CA	5	11/1/1939	106.004	−0.422	4.27
Sarasota Co Pub Hosp Dist FL hosp	5.625	7/1/1939	104.762	−0.158	5.01
W Virginia Hosp Fin Auth	5.625	9/1/1932	102.783	−0.156	5.26

Source: Wall Street Journal, http://online.wsj.com/public/us.

from the *Wall Street Journal* site on October 7, 2009). Finally, the yield curves for municipals tend to be positively sloped and steeper than the Treasury yield curve. In addition, the municipal yield curves tend to be positively sloped even in periods when the Treasury yield curve is flat or negatively sloped. Exhibit 8.4 shows the Standard & Poor's yield curves for municipal composites and historical total returns and yields for select states.

8.7 MUNICIPAL BOND MARKETS

In the primary market, the sale of GOs and some revenue bonds is handled by investment bankers or a syndicate of commercial banks and dealers who underwrite the issue and then resell them in the open market. Traditionally, the selection of an underwriter or syndicate was done on a competitive bid basis, with many states requiring GOs to be marketed with competing bids. Because of the complexities with municipal bonds, more underwriters are being selected through negotiation. Many revenue bonds and some GOs are also sold through private placements with commercial banks, investment funds, insurance companies, and the like, and since 2000, some brokers and dealers auction some municipals, as well as deal in existing issues, over the Internet.[4] Information on upcoming municipal bond sales can be found in the *Bond Buyer*. The *Bond Buyer* is the trade publication of the municipal bond industry. The book or online subscription to its Web site provides information on future bond sales and the results from recent sales. Trade information and recent issues can also be found from *Muni Net Guide:* www.muninetguide.com/categories/municipal-bond-documents-nrmsirs.php. In 2007, the Securities and Exchange Commission designated four firms as Nationally Recognized Municipal Securities Information Repositories (NRMSIRs) to improve the disclosure of municipal bond information. Through NRMSIRs, these firms provide official statements and other information from municipal issuers (see Electronic Municipal Market Access: http://emma.msrb.org/).

In the secondary market, municipal bonds are primarily traded in the OTC market through municipal bond dealers specializing in particular issues. Local banks and regional brokerage firms often handle the issues of smaller municipalities (referred to as ***local credits***), while larger investment companies and the municipal bond departments of larger banks handle the issues of larger governments (referred to as ***general names***). Many dealers in the secondary market make bid and ask quotes in terms of the yield to maturity or yield to call. With the wide variety of municipal bonds, the spreads on municipals can range from $1/4$ to 1 point. Information on trading and prices can be obtained from the Securities Industry and Financial Markets Association (SIFMA) via their Web site: www.investinginbonds.com.

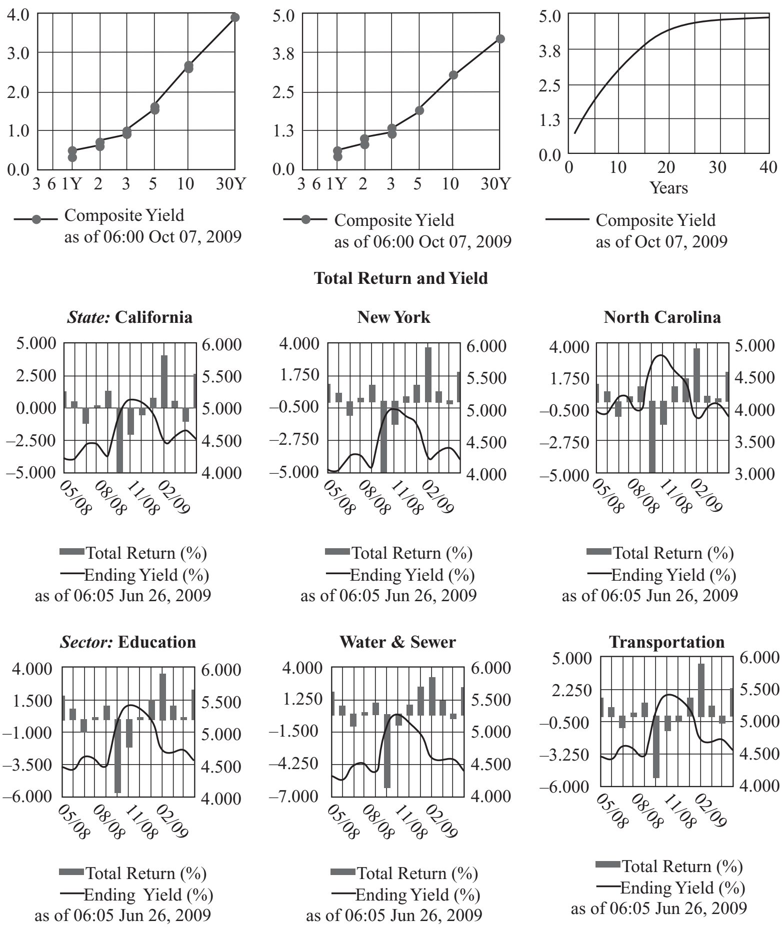
Regulations

From 1930 to 1970, the municipal bond market was relatively free of federal regulations. Municipal securities were exempted from the disclosure and reporting requirements defined under the security acts of 1933 and 1934. However, following some of the municipal defaults and state and local government budgetary problems of the

EXHIBIT 8.4 The S&P Composite Yield Table and Standard & Poor's/Investortools State and Sector Municipal Indexes

The Standard & Poor's (S&P) Composite Yield Table shows the yield composites of municipal bonds of various maturities from three months to 30 years.

The Standard & Poor's/Investortools state and sector Municipal Index is a composite that measures the overall performance of municipal bonds issued by state and local governments in a state. The composite includes any municipal bond from a state issuer held by a mutual fund that Standard & Poor's Securities Evaluations prices daily and that has an outstanding amount of at least $2,000,000. Changes in the Index over time show you whether overall returns in this state's issues are moving up or down. It also allows you to evaluate the performance of your specific municipal investments in this or other sectors against the performance of this market sector. The California sector is important because issuers in the state account for a significant portion of outstanding municipal bonds.



Source: Copyright 2005 Standard & Poor's Securities Evaluations Services ("S&P"). Downloaded from: www.Investinginbonds.com.

1970s, Congress passed the Security Act Amendment of 1975 that expanded federal regulations to the municipal bond market. Although the act did not require compliance to registration requirements under the 1933 act, it did put municipal bond dealers, brokers, and bankers under the SEC regulatory system. The amendment also mandated that the SEC establish the ***Municipal Securities Rule Board*** (MSRB), a self-regulatory board responsible for establishing rules for brokers, dealers, and banks operating in the municipal bond market (see www.msrb.org/msrb1/). As a result of this board, the Securities and Exchange Commission amended SEC Rule 15c2-12 to prohibit dealers from marketing new municipal issues if issuers did not agree to provide annual financial reports and disclose relevant events such as credit rating changes, property sales, and the like. The SEC also approved a rule limiting the campaign contributions that municipal security dealers, brokers, and bankers could make to government officials that they did business with who were running for office.

8.8 CONCLUSION

In this chapter we have examined the market for municipal securities. These securities are used to finance a myriad of capital projects, programs, and operations of state and local governments. The list of issuers of government securities is extensive: states, state agencies, cities, counties, and municipal authorities. For investors, municipals offer many types of instruments, from short-term municipal anticipation notes to long-term municipal GOs and revenue bonds. In the next chapter we will continue our analysis of securities by examining intermediary securities. As we will see, the number and different types of these securities, like corporate and government securities, is also extensive.

KEY TERMS

bond-anticipation notes (BANs)
dedicated tax-backed revenue bonds
double-barreled
floating-rate obligation
full faith and credit
general names
general obligation bonds (GOs)
grant-anticipation notes (GANs)
investor's equivalent taxable yield
legal opinion
letter-of-credit-backed (LOC-backed)
municipal bonds
limited-tax GO bonds
local credits
Mello-Roos bonds
moral obligation bonds
municipal multiplier
Municipal Securities Rule Board (MSRB)
municipal tax-exempt commercial paper
official statement
original-issue discount bond (OID)
refunded bonds
revenue-anticipation notes (RANs)
revenue bonds
taxable municipals
tax-anticipation notes (TANs)
tax risk
unlimited tax GOs
variable-rate demand obligation

WEB INFORMATION

1. Rating agencies
   - www.moodys.com
   - www.standardandpoors.com
   - http://reports.fitchratings.com
2. Moody's:
   - www.moodys.com (registration required)
   - For Moody's study of historical default rates for municipals, go to "Rating Methodologies and Performance," "Historical Performance," and look for municipal default information.
3. Price information:
   FINRA
   - Go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds."
   - For a bond search, click "Municipal" tab and then click "Advanced Bond Search" to find municipal bonds with certain features.

   Wall Street Journal
   - Go to http://online.wsj.com/public/us, "Market Data," "Bonds, Rates, and Credit Markets," and "Tax-Exempt Bonds."

   Yahoo.com
   - Go to http://finance.yahoo.com/bonds, click "Advanced Bond Screener" and click "Municipal" tab, and then provide information for search.

   Investinginbonds.com
   - Go to http://investinginbonds.com/; click "Municipal Bonds At-A-Glance."
4. For recent information and reports on municipal bond market trends, go to investinginbonds.com:
   - www.investinginbonds.com/ Recent commentary
   - SIFMA Municipal Bond Credit Report
   - www.investinginbonds.com/news.asp?id=3099&catid=36 Full report
   - The Standard & Poor's (S&P) Composite Yield Table and Index.
   - Indexes by state: www.investinginbonds.com/MarketAtAglance.asp?catid=32&id=349
   - Indexes by term: www.investinginbonds.com/MarketAtAglance.asp?catid=32&id=73
5. For information on sources of municipals, go to www.sec.gov/answers/nrmsir.htm.
6. Municipal Securities Rule Board (MSRB): www.msrb.org/msrb1/.
7. Electronic Municipal Market Access: http://emma.msrb.org/.
8. For news and information links on municipals, go to Muni Net Guide: www.muninetguide.com/categories/municipal-bond-documents-nrmsirs.php
9. For information on state and local government fiscal conditions, go to Bureau of Economic Analysis: www.bea.gov.

(*continued*)

(*Continued*)

10. For information on Ipreo, go to www.ipreo.com/
11. State and local government finances
    - www.census.gov/govs/estimate/index.html
    - www.census.gov/govs/www/financegen.html
12. Monoline insurers
    - MBIA: www.mbia.com/
    - AMBAC: www.ambac.com/
    - FGIC: www.fgic.com/aboutfgic/

PROBLEMS AND QUESTIONS

1. Define the following types of municipal bonds:
   a. General obligations
   b. Full faith and credit obligations
   c. Limited-tax general obligations
   d. Revenue bonds
   e. Double-barreled revenue bonds
2. What are some of the important provisions specified in the legal opinion of a municipal general obligation bond that a bond investor should consider in evaluating a municipal bond?
3. Describe some of the different types of municipal anticipation notes that are issued by state and local governments.
4. Explain how changes in federal tax rates affect the market for municipal securities.
5. How did the Tax Reform Act of 1986 affect commercial bank investments in municipal securities?
6. List some of the capital projects and programs municipal governments finance with revenue bonds.
7. Explain how municipal governments use industrial revenue bonds. Comment on why they are a popular source of financing for regional economic development.
8. What are some of the important provisions specified in the official statement and the legal opinion of a municipal revenue bond that a bond investor should consider in evaluating a municipal bond?
9. Explain the following special features and types of municipal bonds:
   a. Serial issue
   b. Insured bonds
   c. Letter-of-credit-backed municipal bonds
   d. Mello-Roos bonds

e. Refunded bond
f. Moral obligation bonds

10. How are many large municipal bond issues sold in the primary market?

11. Describe the secondary market for municipals.

12. What is the Municipal Securities Rule Board?

WEB EXERCISES

1. Find the current yields on municipal bonds given different quality ratings by going to http://bonds.yahoo.com and clicking on "Composite Bond Rates." What is the spread between the AAA 10-year municipal and 10-year T-bond? Comment on why the spread is small or even negative.
2. Find municipals for a particular state by going to www.investinginbonds.com and clicking "Municipal Market At-A-Glance."
3. Official statements are important sources of information on a municipal issue. Review the official statement on one of the bonds you found in Question 2. To access the statement, go to http://emma.msrb.org/ and enter the municipal's CUSIP in the Muni Search Box.
4. Use the "Advance Search" at http://bonds.yahoo.com to identify several municipal bonds that have certain features you select.
5. Many municipals have different features (floating rates, strips, convertible, etc.). Identify some of these issues by going to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds" and then click the "Municipal" tab and "Advanced Bond Search."
6. Taxable municipals are attractive investments for tax-exempt investors. Identify some of these issues by going to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds" and then click the "Municipal" tab and "Advanced Bond Search." At the search site, select "Taxable" for "Taxable Status."
7. SIFMA provides a review of conditions and trends in the municipal bond market, Standard & Poor's composite yield curves, and historical yields and total return for each state. Go to their site and examine recent conditions, yields curves, and returns of states of interest: www.investinginbonds.com, "Municipal Market At-A-Glance."
   - SIFMA Municipal Bond Credit Report www.investinginbonds.com/news.asp?id=3099&catid=36 Full report
   - The Standard & Poor's (S&P) Composite Yield Table and index: www.investinginbonds.com/MarketAtAglance.asp?catid=32&id=427
   - Indexes by state: www.investinginbonds.com/MarketAtAglance.asp?catid=32&id=349
   - Indexes by term: www.investinginbonds.com/MarketAtAglance.asp?catid=32&id=73
8. Trends and market conditions in the municipal market can be found at *Muni Net*. Visit the site to learn about some of the recent trends: www.muninetguide.com/categories/municipal-bond-documents-nrmsirs.php.

9. Select a municipal bond from www.investinginbonds.com and then go to Moody's to study its credit history and profile:
   - Go to www.moodys.com.
   - Enter CUSIP on "Quick Search" and click "Go."
10. Study some of the recent municipalities added to Moody's watch list:
    - Go to www.moodys.com, "U.S. Public Finance" and "Watchlist."
11. Moody's provides information on default rates, ratings changes, and other credit information. Examine some of their information. To download Moody's study of historical default rates:
    - Go to www.moodys.com.
    - Search for "Rating Methodologies and Performance."
    - Historical Performance.
    - Review the "Municipal Default" section.
12. Select a state and local government on Moody's watch list or with a recent rating change and do an analysis of it with information obtained from the Bureau of Economic Analysis Web site: www.bea.gov. At the site, go to "Regional" data. You may want to look at employment data by going to www.economagic.com. At the site, click "Browse by Region" or click "Bureau of Labor Statistics and States." You may also find information on a state's financing by going to the Census site: www.census.gov/govs/estimate/index.html and www.census.gov/govs/www/financegen.html.

NOTES

1. The actual number varies depending on how funding authorities are counted and how long they exist. Bloomberg, for example, estimates the total number to be closer to 60,000. Whether 60,000 or 80,000, the number is still impressive.
2. For a discussion of the tax provisions affecting municipals, see *The Handbook of Fixed-Income Securities*, ed. Frank Fabozzi, 2005, pp. 272–275.
3. The Deficit Reduction Act of 1984 places a limit of $40 million on small IDB issues, prohibits certain capital projects from IDB funding, and restricts the total amount of IDBs that can be issued by a state based on its population.
4. I-Deal and Ipreo are companies offering platforms for negotiating bond sales and providing a muni auction platform.

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CHAPTER 9

Intermediary Debt Securities

9.1 INTRODUCTION

The intermediary financial market consists of commercial banks, savings and loans, insurance companies, investment funds, and other financial intermediaries. These intermediaries sell financial claims to investors and then use the proceeds to purchase debt and equity claims or to provide direct loans. Commercial banks, for example, obtain funds from investors by providing demand deposits and money market accounts, selling certificates of deposit, and borrowing from other banks. They, in turn, use their funds to satisfy legal reserve requirements, to make loans, and to purchase financial securities. Savings and loans and savings banks function very similarly to commercial banks, except that their use of funds is directed more toward the creation of mortgage loans. Finally, life insurance companies, pensions, trust funds, and investment funds offer financial instruments in the form of insurance policies, retirement plans, and shares in stock or bond portfolios. The proceeds from their premiums, savings plans, and fund shares are used by these institutions to buy stocks, corporate bonds, Treasury securities and other debt instruments, as well as provide corporate, residential, and commercial loans.

In general, financial institutions, by acting as intermediaries, control a large amount of funds and thus have a significant impact on financial markets. For borrowers, intermediaries are an important source of funds; they buy many of the securities issued by corporations and governments and provide many of the direct loans. For investors, intermediaries create a number of securities for them to include in their short-term and long-term portfolios. These include negotiable certificates of deposit, banker's acceptances, mortgage- and asset-backed instruments, investment fund shares, annuities, and guaranteed investment contracts. In this chapter, we examine intermediaries and the markets for intermediary securities.

9.2 BANK SECURITIES

Negotiable Certificates of Deposit

A certificate of deposit (CD) is a certificate issued by a financial institution certifying that a specified sum of funds has been deposited at the issuing depository's institution. Banks and thrift institutions offer CDs to finance their loans and investments. The CD has a specified maturity date and interest rate and can be either nonnegotiable or negotiable. Nonnegotiable CDs are deposits that cannot be transferred and typically

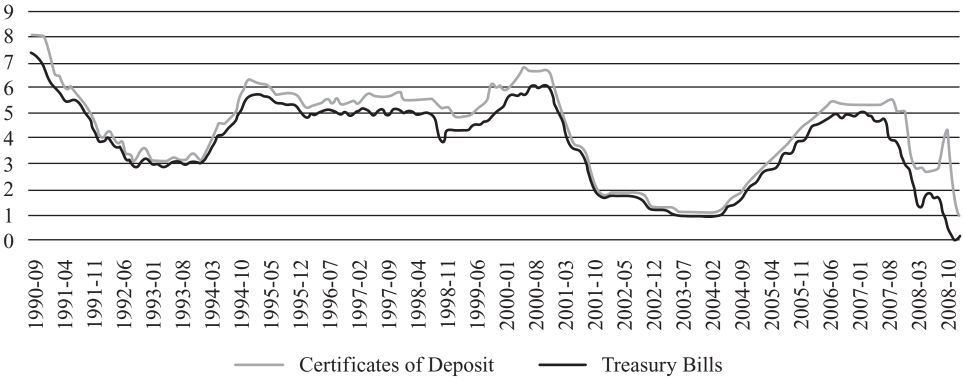

FIGURE 9.1 Rates on CDs and T-Bills, 1990–2009
Source: Federal Reserve, www.federalreserve.gov/releases/h15/data.htm.

must be held to maturity, with often a penalty applied for early withdrawal. They can be issued in any denomination. Negotiable CDs are higher-denomination certificates typically issued to institutions. Unlike nonnegotiable CDs, negotiable CDs can be sold to other investors in the secondary CD market.

Negotiable CDs are one of the more popular money market instruments. The maturities on negotiable CDs generally range from three to 18 months, although most have maturities of six months or less. CDs issued with maturities greater than one year are called ***term CDs***. CDs are interest-bearing notes, usually sold at their face value, with the principal and interest paid at maturity if the CD is less than one year and semiannually if it is a term CD. The minimum denomination on negotiable CDs is $100,000, with the average denomination being $1 million; there are also ***jumbo CDs*** with face values of $10 million or more. Like other bank and savings and loan deposits, the Federal Deposit Insurance Corporation (FDIC) insures CDs up to $100,000 against default. Most negotiable CDs, though, have denominations exceeding $100,000 and are therefore subject to default risk.[1] The yields on CDs generally reflect the risk of the issuing financial institution and the maturity of the CD. CD yields tend to exceed the rates on Treasury and short-term federal agency instruments (see Figure 9.1). Also, the yields for the CDs of larger (supposedly more secured) banks (called ***prime CDs***) tend to be lower than those of smaller banks (called ***nonprime CDs***). Finally, the rates that banks pay on CDs are quoted on a 360-day basis, instead of 365 days. Thus, an investor buying a $1 million CD, maturing in 180 days and paying a 6% interest, would receive $1,030,000 from the bank at maturity ($\$1{,}000{,}000[1 + .06(180/360)] = \$1{,}030{,}000$). Quotes of CDs trading in the secondary market, in turn, are made in terms of the bank's discount yield. Thus, if the CD yielding 6% were trading in the secondary market, its rate would be quoted at 5.8% [$= (\$1{,}030{,}000 - \$1{,}000{,}000)/\$1{,}030{,}000)\,(360/180) = .058$].

Markets

Today, dealers and brokers form the core of the primary and secondary markets for CDs, selling new CDs and trading and maintaining inventories in existing ones.

EXHIBIT 9.1 History of the Secondary CD Market

Prior to 1961, commercial banks lacked an effective instrument to compete for the temporary excess cash funds of corporations and state and local governments. At that time, there was no interest paid on demand deposits, and corporations were reluctant to tie their funds up in nonnegotiable CDs. Also with the rates paid on time deposits fixed by the Federal Reserve's Regulation Q, sometimes at a level below T-bill rates, banks had no security to offer corporations. Consequently, corporations tended to opt for T-bills instead of bank deposits when they invested their excess funds.

The solution to the problem for banks came in 1961 when First Bank of New York (now Citigroup) issued a negotiable CD, accompanied by an announcement by First Boston Corporation and Salomon Brothers that they would stand ready to buy and sell the CDs. Thus, the first secondary market for CDs was born. Moreover, what the secondary market provided was a way for banks to circumvent Regulation Q and offer investors rates competitive with T-bills and CP. To do this, the yield curve needed to be positively sloped and remain that way for the foreseeable future. With Regulation Q setting higher maximum rates on longer term CDs and the Fed rarely changing the maximum rate, these conditions were met. The existence of a secondary market meant that an investor could earn a rate higher than both the shorter or longer term CD, by buying the longer term CD and selling it later in the secondary market at a higher price associated with the short-term maturity. For example, if six-month CDs yielded 5% ($P = 100/(1.05)^{.5} = 97.59$) and a one-year CD yielded 6% ($P = 100/1.06 = 94.3396$), then an investor could buy the one-year CD for 94.3396, hold it for six months, and sell it for 97.59 (given the yield curve did not change) to realized an annualized yield of 7% [$= (97.59/94.3396)^{1/.5} - 1$]. Thus, to recapitulate, the significance of the secondary market for CDs was that it provided a way for banks to increase their CD yields to customers without violating Regulation Q.

Following First Bank of New York, Salomon Brothers, and First Boston's lead, other banks, brokers, and dealers quickly entered into the market for negotiable CDs.

Money market funds, banks, bank trust departments, state and local governments, foreign governments, central banks, and corporations are the major investors in CDs.[2] Since many of these investors hold their CDs until maturity, the secondary market for these instruments is not as active as the secondary T-bill market. However, it was the creation of the secondary market for CDs in 1961 that help make CDs popular. See Exhibit 9.1 for a brief history of the secondary market for CDs and its significance.

There are three types of CDs: domestic, foreign, and Eurodollar. From a U.S. investor's perspective, domestic CDs are those issued by U.S. banks, whereas foreign CDs are dollar-denominated CDs issued by foreign banks through their U.S. branches; they are often referred to as ***Yankee CDs***. Eurodollar CDs, in turn, are dollar-denominated CDs issued by foreign branches of banks from the United States, Europe, and Japan that are incorporated in countries with favorable banking laws. The yields on dollar-denominated Eurodollar CDs are higher than the yields on domestic CDs. This is because foreign subsidiary banks issuing the CDs are incorporated in countries with favorable banking laws such as low or zero reserve requirements.[3]

The growth of the CD market over the last two decades has been accompanied by innovations. In the 1980s, a ***floating-rate CD*** (***FRCD***) was introduced. The maturity on a FRCD ranges from 18 months to five years, with the coupon rates reset periodically to equal the rate on a comparable CD rate or the London Interbank

Offer Rate (this rate is discussed in Chapter 10). Other CDs with unique features that have been introduced over the years are ones with rates tied to the stock market (***bear and bull CDs***), longer term CDs with gradually increasing rates (***rising-rates CDs***), and contracts to buy CDs now and in the future (***forward CDs*** and ***rollover*** or ***roly-poly CDs***).

Bank Notes

In addition to CDs, commercial banks also issue ***bank notes***. Bank notes are similar to medium-term notes. They are sold as a program consisting of a number of notes with different maturities, typically ranging from one to five years, and offered either continuously or intermittently. Bank notes are usually sold to institutions in high denominations ranging from $5 million to $25 million, with the total offering ranging from $50 million to $1 billion. Different from corporate MTNs discussed in Chapter 6, bank notes are not registered with the SEC, unless it is the bank's holding company (and not the individual bank) issuing the MTN. Banks also sell bank notes and MTNs through international syndicates as part of the Eurocapital market (discussed in Chapter 10).

Leveraged Loans

Commercial bank loans can be classified as investment-grade loans and leveraged loans. ***Investment-grade loans*** are loans to borrowers with investment-grade ratings. They usually take the form of a revolving line of credit, with no maturity and with the bank setting a maximum amount that the company can borrow. Investment-grade loans set up as a line of credit are generally held by the originating bank and not sold. ***Leveraged loans*** are loans to corporations with below investment-grade ratings. These loans usually have a maturity and a floating rate, and in contrast to investment-grade bonds, they are often sold to institutional investors. As we will discuss in Chapter 12, some of these loans have been packaged and securitized as a collateralized debt obligations (CDOs).

Today leveraged bank loans and some investment-grade loans that have maturities are traded in secondary markets or securitized to create collateralized loan obligations. To institutional investors, leveraged loans represent an alternative to high-yield bonds. In contrast to high-yield bonds, leveraged loans are characterized by a floating- rate structure, shorter original maturities (five years on average compared to 10-years on high-yield bonds), no call provisions, a senior claim within the corporation's capital structure, more loan covenants, and higher recovery rates.

Syndicated Loans

Banks often form a syndicate to provide loans to borrowers who seek large amounts of funds. The resulting syndicated loan serves to spread the credit risk amongst the participating banks. The loan created usually has a senior claim with a priority position over subordinated lenders and bondholders, the terms to maturity are usually fixed, the loan often pays a floating rate, and depending on the use of the funds, the syndicated loan can be either an amortized or bullet loan. Like leveraged loans, there is also a secondary market for syndicated loans. This market is characterized by loan sales by the banks in the syndicate. Often banks in the group have the right to sell their parts of the loan to other banks by either assignment or participation. With assignment, the seller transfers all rights to the new holder, whereas with participation, the new holder does not become a party to the loan agreement but instead has a relation with the seller.[4]

9.3 BANKER'S ACCEPTANCES

Banker's acceptances (***BAs***) are time drafts (postdated checks) guaranteed by a bank—guaranteed postdated checks. The guarantee of the bank improves the credit quality of the draft, making it marketable. BAs are used to finance the purchase of goods that have to be transferred from a seller to the buyer. They are often created in international business transactions where finished goods or commodities have to be shipped.

To see how BAs originate, consider the case of a U.S. oil refinery that wants to import 100,000 barrels of crude oil at $50 per barrel ($5 million) from an oil producer in South America. Suppose the South American oil exporter wants to be paid before shipping, while the U.S. importer wants the crude oil before payment. To facilitate the transaction, suppose they agree to finance the sale with a BA in which the U.S. importer's banks will guarantee a $5 million payment 60 days from the shipment date. With this understanding, the U.S. oil importer would obtain a letter of credit (LOC) from his bank. The LOC would say that the bank would pay the exporter $5 million if the U.S. importer failed to do so. The LOC would then be sent by the U.S. bank to the South American bank of the exporter. Upon receipt of the LOC, the South American bank would notify the oil exporter, who would then ship the 100,000 barrels of crude oil. The oil exporter would then present the shipping documents to the South American bank and receive the present value of $5 million in local currency from the bank. The South American bank would then present a time draft to the U.S. bank who would stamp "accepted" on it, thus creating the BA. The U.S. importer would sign the note and receive the shipping documents. At this point, the South American bank is the holder of the BA. The bank can hold the BA as an investment or sell it to the American bank at a price equal to the present value of $5 million. If the South American bank opts for the latter, then the U.S. bank holds the BA and can either retain it or sell it to an investor such as a money market fund or to a banker's acceptance dealer. If all goes well, at maturity the oil importer will present the shipping documents to the shipping company to obtain his 100,000 barrels of crude oil, as well as deposit the $5 million funds in his bank; whoever is holding the BA on the due date will present it to the U.S. importer's bank to be paid.

The use of BAs to finance transactions is known as ***acceptance financing*** and banks that create BAs are referred to as ***accepting banks***. In the United States, the major accepting banks are the money center banks such as Citicorp and Bank of America, as well as some large regional banks. Many of the large Japanese banks have also been active in creating BAs. In the secondary market, BAs are traded as zero-coupon bonds, with the face value equal to the payment order and with the maturity between 30 and 270 days. With the bank guarantee, they are considered prime-quality instruments with relatively low yields. The secondary market trading of BAs takes place principally among banks and dealers. There are approximately 20 dealers who facilitate trading in the secondary market. The major dealers include the major investment banking firms and money center banks. Money market funds, banks, and institutional investors are the primary purchasers of BAs. The Federal Reserve also buys and sells BAs as part of their open market operations, and commercial banks use BAs as collateral for Federal Reserve loans.

The market for BAs has existed for over 70 years in the United States, although its origin dates back to the 12th century. In the United States, this market grew steadily in the 60s and 70s. From 1970 to 1985 the market accelerated from $7.6 billion in 1970 to almost $80 billion in 1985, reflecting the growth in world trade. Due to alternative financing, though, the BA market has declined marginally since 1985.

9.4 MORTGAGE-BACKED AND ASSET-BACKED SECURITIES

Up until the mid-1970s, most home mortgages originated when savings and loans, commercial banks, and other thrift institutions borrowed funds or used their deposits to provide loans to home purchasers, possibly later selling the resulting instruments in the secondary market to Fannie Mae, Ginnie Mae, or Freddie Mac. To a large degree, individual deposits financed real estate, with little financing coming from corporations or institutions. In an effort to attract institutional funds away from corporate bonds and other capital market securities, as well as to minimize their poor hedge (short-term deposit liabilities and long-term mortgage assets), financial institutions began to sell mortgage-backed securities in the 1970s. These securities provided them with an instrument that could compete more closely with corporate bonds for inclusion in the portfolios of institutional investors, and it provided the mortgage industry with more liquidity.

By definition, ***mortgage-backed securities (MBSs)*** are instruments that are backed by a pool of mortgage loans. Typically, a financial institution, agency, or mortgage banker buys a pool of mortgages of a certain type from mortgage originators (e.g., Federal Housing Administration-insured mortgages or mortgages with a certain minimum loan-to-value ratio or a specified payment-to-income ratio). This mortgage portfolio is financed through the sale of the MBS, which has a claim on the portfolio. The mortgage originators usually agree to continue to service the loans, passing the payments on to the mortgage-backed security holders. An MBS investor has a claim on the cash flows from the mortgage portfolio. This includes interest on the mortgages, scheduled payment of principal, and any prepaid principal. Since many mortgages are prepaid early as homeowners sell their homes or refinance their current mortgages, the cash flows from a portfolio of mortgages, and therefore the return on the MBS, can be quite uncertain. To address this type of risk, a number of derivative MBSs were created in the 1980s. For example, in the late 1980s Freddie Mac introduced ***collateralized mortgage obligations (CMOs)***. These securities had different maturity claims and different levels of prepayment risk.

An MBS is an asset-backed security created through a method known as ***securitization***. Securitization is the process of transforming illiquid financial assets into marketable capital market instruments. Today, it is applied not only to mortgages but also home equity loans, automobile loans, lines of credit, credit card receivables, and leases. Securitization is one of the most important financial innovations introduced in the last two decades; it is examined in detail in Chapters 11 and 12.

9.5 INVESTMENT FUNDS

Major investment firms and banks offer a wide variety of investment funds. For many investors, shares in these funds are an alternative to directly buying stocks and bonds. Fund investment provides several advantages over directly purchasing securities. First, investment funds provide divisibility. An investment company offering shares in a portfolio of negotiable, high-denomination CDs, for example, makes it possible for small investors to obtain a higher rate than they could obtain by investing in a lower yielding, small-denomination CD. Second, an investment in a fund consisting of a portfolio of securities often provides an investor more liquidity than forming his own portfolio; that is, it is easier for an individual investor to buy and sell a share in an investment fund than it is to try to buy and sell a number of securities. Third, the investment companies managing funds provide professional management. They have a team of security analysts and managers who know the markets and the securities available. They buy and sell securities for the fund, reinvest dividends and interest, and maintain records. Finally, since investment companies often buy large blocks of securities, they can obtain lower brokerage fees and commission costs for their investors. In summary, funds provide investors the benefits of divisibility, diversification, and lower transaction costs.

The Market for Funds

From the end of World War II to the late 1960s, investments in funds grew substantially, boasting as many as 40 million investors in the 1960s. Most of the investment funds consisted of stocks, with their popularity attributed primarily to the general rise in stock prices during that period. In the 1970s, investments in funds declined as stock prices fell due to rising energy prices, inflation, and recession. During this period, a number of funds specializing in debt securities were introduced. In the mid-1980s and in the 1990s, though, the popularity of fund investments rebounded. This more recent growth can be attributed to not only the bull market of the 1990s, but also to financial innovations. In addition to the traditional stock funds, investment companies today offer shares in bond funds (municipal bonds, corporate bonds, high-yield bonds, and foreign bonds), ***money market funds*** (consisting of CDs, CP, Treasury securities, etc.), ***index funds*** (funds whose values are highly correlated with a stock or bond index), funds with options and futures, ***global funds*** (funds with stocks and bonds from different countries), and even ***vulture funds*** (funds consisting of debt securities of companies that are in financial trouble or in Chapter 11 bankruptcy).

A number of investment companies, such as Fidelity and Vanguard, manage a family of mutual funds. From this family (sometimes referred to as a complex), these investment companies are able to offer investors different funds based on the investor's risk-return preferences. Currently there are over 8,500 funds in the United States, a number that exceeds the number of stocks listed on the NYSE. Contributing to this large number is the increased percentage of fund investment

TABLE 9.1 Investment Company Assets by Type: Billions of Dollars, Year-End, 1995–2008

	Mutual Funds	Closed-End Funds	ETFs	UITs	Total
1995	$2,811	$143	$ 1	$73	$3,028
1996	3,526	147	2	72	3,747
1997	4,468	152	7	85	4,712
1998	5,525	156	16	94	5,791
1999	6,846	147	34	92	7,119
2000	6,965	143	66	74	7,248
2001	6,975	141	83	49	7,248
2002	6,390	159	102	36	6,687
2003	7,414	214	151	36	7,815
2004	8,107	254	228	37	8,626
2005	8,905	277	301	41	9,524
2006	10,397	298	423	50	11,167
2007	12,000	313	608	53	12,974
2008	9,601	188	531	29	10,349

Source: 2009 Investment Company Fact Book, www.icifactbook.org/fb_sec1.html#investment.

coming from retirement investments such as individual retirement accounts (IRAs) and 401(k) accounts. As of year 2007, mutual funds accounted for approximately 25% ($4 trillion) of the estimated $16 trillion dollar retirement investment market.

Structure of Funds

There are three types of investment fund structures: open-end funds (also called mutual funds), closed-end funds, and unit investment trusts (UITs). The first two can be defined as managed funds, whereas the third is an unmanaged one. Table 9.1 shows the distribution of assets among investment companies by type—including exchange-traded funds (ETFs), discussed later—from 1995 to 2008.

Open-End Funds

Open-end funds (mutual funds) stand ready to buy back shares of the fund at any time the fund's shareholders want to sell, and they stand ready to sell new shares any time an investor wants to buy into the fund. Technically, a mutual fund is an open-end fund. The term mutual fund, though, is often used to refer to both open- and closed-end funds. With an open-end fund, the number of shares can change frequently. The price an investor pays for a share of an open-end fund is equal to the fund's ***net asset values*** (***NAV***). At a given point in time, the NAV of the fund is equal to the difference between the value of the fund's assets ($V^A{}_t$) and its liabilities ($V^L{}_t$) divided by the number of shares outstanding (N_t): $\text{NAV}_t = (V^A{}_t - V^L{}_t)/\, N_t$. For example, suppose a balanced stock and bond fund consists of a stock portfolio with a current market value of $100 million, a corporate bond portfolio with current market value of $100 million, liquid securities of $8 million, and liabilities of $8 million. The current net worth of this fund would be $200 million. If the fund, in turn, has 4 million shares outstanding, its current NAV would be $50 per share: NAV = ($208m – $8m)/4m = $50. This value, though, can change if the number of shares, the asset values, or the liability values change.

Open-end funds can be classified as ***load funds*** or ***no-load funds***. Load funds are sold through brokers or other intermediates; as such, the shares in load funds sell at their NAV plus a commission. The fees are usually charged up-front when investors buy new shares. Some funds charge a redemption fee (also called an exit fee or back-end load) when investors sell their shares back to the fund at their NAV. No-load funds, on the other hand, are sold directly by the fund and therefore sell at just their NAV. The fund does charge fees for management and for transferring individual investments from one fund to another.

Closed-End Funds

A ***closed-end fund*** has a fixed number of non-redeemable shares sold at its initial offering. Unlike an open-end fund, the closed-end fund does not stand ready to buy existing shares or sell new shares. The number of shares of a closed-end fund is therefore fixed. An investor who wants to buy shares in an existing closed-end fund can do so only by buying them in the secondary market from an existing holder. Shares in existing funds are traded on the over-the-counter market. Interestingly, the prices of many closed-end funds often sell at a discount from their NAVs.[5]

Unit Investment Trusts

Although the composition of open- and closed-end fund investments can change as managers buy and sell securities, the funds themselves usually have unlimited lives. In contrast, a ***unit investment trust*** has a specified number of fixed-income securities that are rarely changed, and the fund usually has a fixed life. A unit investment trust is formed by a sponsor (investment company) who buys a specified number of securities, deposits them with a trustee, and then sells claims on the security, known as ***redeemable trust certificates***, at their NAV plus a commission fee. These trust certificates entitle the holder to a proportional share in the income from the deposited securities. For example, an investment company might purchase $20 million worth of Treasury bonds, place them in a trust, and then issue 20,000 redeemable trust certificates at $1,025 per share: NAV + Commission = ($20 million/20,000) + $25 = $1,025. If the investment company can sell all of the shares, it will be able to finance the $20 million bond purchase and earn a 2.5% commission of $500,000.

Most unit investment trusts are formed with fixed-income securities: government securities, corporate bonds, municipal bonds, and preferred stock. The trustee pays all the interest and principal generated from the bonds to the certificate holders. Unlike open- and closed-end funds, when the securities in the pool mature, the investment trust ceases. Depending on the types of bonds, the maturity on a unit investment trust can vary from six months to 20 years. The holders of the securities, though, usually can sell their shares back to the trustee prior to maturity at their

NAV plus a load. To finance the purchase of the certificate, the trustee often sells a requisite amount of securities making up the trust.

Types of Investment Funds

A board of directors elected by the fund's shareholders determines the general investment policies of open- and closed-end investment funds. Typically, a management or investment advisory firm, often consisting of those who originally set up the fund, does the actual implementation and management of the policies. Some funds are actively managed, with fund managers aggressively buying stocks and bonds, whereas others follow a more passive buy-and-hold investment strategy.

One way of grouping the many types of funds is according to the classifications defined by Weisenberger's *Annual Investment Companies Manual* for growth funds, income funds, and balanced funds. ***Growth funds*** are those whose primary goal is in long-term capital gains. Such funds tend to consist primarily of those common stocks offering growth potential. Many of these are diversified stock funds, although there are some that specialize in certain sectors. ***Income funds*** are those whose primary goal is providing income. These funds are made up mainly of stocks paying relatively high dividends or bonds with high coupon yields. Finally, ***balanced funds*** are those with goals somewhere between those of growth and income funds. Balanced funds are constructed with bonds, common stocks, and preferred stocks that are expected to generate moderate income with the potential for some capital gains.

A second way of classifying funds is in terms of their specialization. There are four general classifications: equity funds, bond funds, hybrid funds (stocks and bonds), and money market funds. As shown in Table 9.2, each of these fund types can be broken down further by their specified investment objectives. Bond funds, for example, can be classified as corporate, municipal, government, high-yield, global, mortgage-backed securities, and tax-free. Each category reflects a different investment objective: Municipal bond funds, for example, specialize in providing investors with tax-exempt municipal securities; corporate bond funds are constructed to replicate the overall performance of a certain type of corporate bond, with a number of them formed to be highly correlated with a specific index; money market funds are constructed with money market securities in order to provide investors with liquid investments. See Figure 9.2 for a distribution of the size of mutual funds by types, Figure 9.3 for a breakdown of various bond funds by asset size, Figure 9.4 for the investment allocations of money market funds by types of securities, and Exhibit 9.2 for examples of the policy statements of several types of fixed-income funds.[6]

Accumulation Plans

Typically, most fund investors buy shares and receive cash from the fund when it is distributed. For investors looking for different cash flow patterns, investment funds also provide voluntary and contractual accumulation plans with different types of contributions and withdrawal plans. Included here are automatic reinvestment plans in which the net income and capital gains of the fund are reinvested, with the shareholders accumulating additional shares, and fixed contribution plans in which investors contribute (either contractually or voluntarily) a fixed amount on a regular basis for a set period.

TABLE 9.2 Categories of Investment Funds and Asset Distribution, 2007 and 2008

Equity funds
Value funds
Growth funds
Sector funds
World equity funds
Emerging market funds
Regional equity funds
Taxable bond funds (short-, intermediate-, and long-term)
Corporate bond funds
High-yield funds
Global bond funds
Government bond funds
Mortgage-backed securities
Tax-free bond funds (short-, intermediate-, and long-term)
State municipal bond funds
National municipal bond funds
Hybrid funds
Asset allocation funds
Balanced funds
Income-mixed funds
Money market funds
Taxable money market funds
Tax-exempt money market funds

Source: 2009 Investment Company Fact Book: www.icifactbook.org/index.html.

Taxes and Regulations

Most mutual funds make two types of payments to their shareholders: a net income payment from dividends and interest and a realized capital gain payment. If an

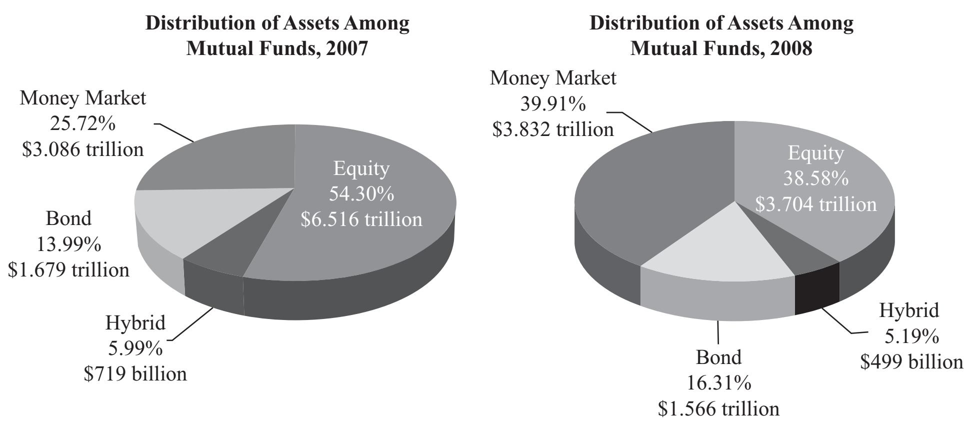

FIGURE 9.2 Mutual Fund Distributions, 2007 and 2008
Source: 2009 Investment Company Fact Book: www.icifactbook.org/index.html.

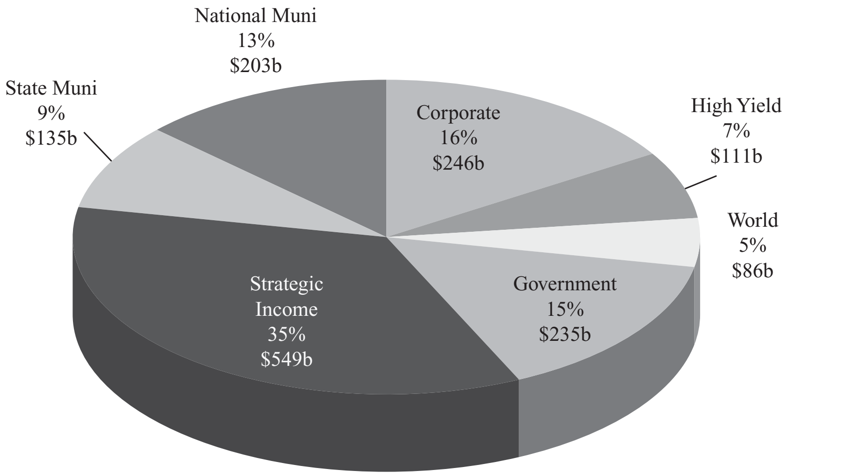

FIGURE 9.3 Net Assets Invested in Different Types of Bond Funds
Source: 2009 Investment Company Fact Book: www.icifactbook.org/index.html.

investment fund complies with certain rules, it does not have to pay corporate income taxes. To qualify for this favorable tax treatment, the company must have a diversified portfolio and it must pay out at least 90% of the fund's net income to shareholders. As a result, most investment companies distribute all of the net income from the fund to their shareholders. Investment companies can either distribute or retain their realized capital gains. Most investment companies distribute capital

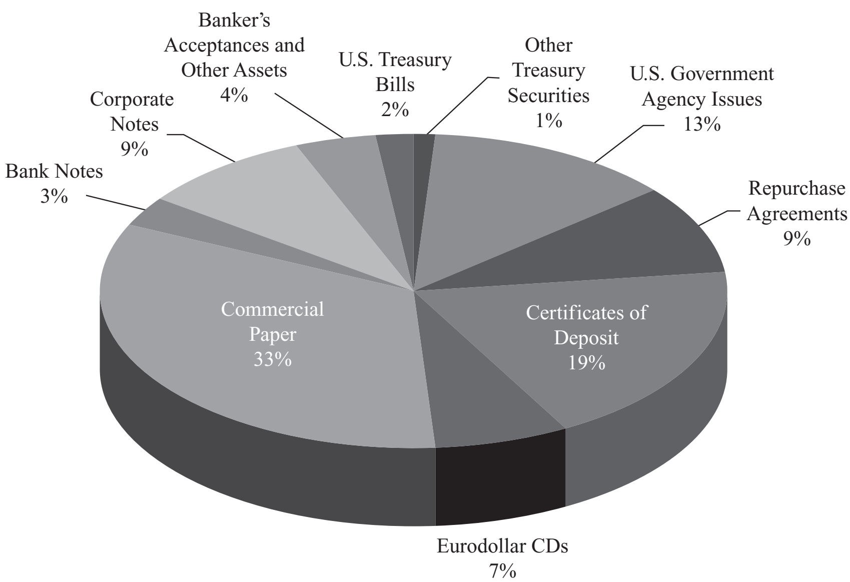

FIGURE 9.4 Asset Composition of Taxable Non-Government Money Market Funds as a Percent of Total Net Assets
Source: 2009 Investment Company Fact Book: www.icifactbook.org/index.html.

EXHIBIT 9.2 Examples of Fixed-Income Investment Funds, October 29, 2009

GuideStone Funds Global Bond GS4

The fund invests at least 80% of net assets in a diversified portfolio of fixed-income instruments of varying maturities and quality across different industries and sectors. It invests in high-yield and investment-grade corporate securities traded in the United States and non-United States markets; and in mortgage-backed and asset-backed securities. The fund may invest a substantial portion of total assets in securities denominated in foreign currencies. The average dollar-weighted duration normally varies between three and 10 years.

PIMCO Long-Term U.S. Government Admin

The investment seeks maximum total return consistent with preservation of capital and prudent investment management. The fund invests normally at least 80% of assets in a diversified portfolio of fixed-income securities that are issued or guaranteed by the U.S. government, its agencies or government-sponsored enterprises. The Fund may invest all of assets in derivative instruments, such as options, futures contracts, or swap agreements, or in mortgage-backed securities.

AIM High Yield A

The fund normally invests at least 80% of assets in non-investment grade debt securities. It may also invest up to 25% of assets in foreign securities, and up to 15% of assets in securities of companies located in developing markets.

American Century High-Yield C

The fund invests at least 80% of assets in high-yield corporate bonds and other debt instruments. It invests the remaining assets in common stocks or other equity-related securities. The fund may invest up to 40% of the assets in fixed-income obligations of foreign issuers and may invest up to 20% of assets, and for temporary defensive purposes up to 100% of its assets, in short-term money market instruments and U.S. government securities.

BlackRock High Yield Bond Instl

The fund invests primarily in non-investment grade bonds with maturities of 10 years or less. It invests at least 80% of assets in high-yield bonds, including convertible and preferred securities. The fund may invest up to 10% of assets in non-dollar denominated bonds of issuers located outside of the United States.

Franklin High Income Adv

The fund invests substantially in high-yield, lower-rated debt securities and preferred stocks. It may invest up to 100% of total assets in debt securities that are rated below investment grade. The fund may buy both rated and unrated debt securities including securities rated below B by Moody's or S&P.

Western Asset Global High Yield Bond I

The investment seeks total return. The fund invests primarily in high-yield fixed-income securities, including bonds, debentures, notes, equipment trust certificates, commercial paper, preferred stock and other obligations of United States and foreign issuers. The fund invests at least 80% of assets in high-yield bonds and related investments. It may also invest up to 20% of assets in equity and equity-related securities. The fund expects to invest in securities of issuers located in the United States and in approximately 7 to 15 foreign countries.

PIMCO Extended Duration P

The fund normally invests at least 65% of total assets in a diversified portfolio of fixed-income instruments of varying maturities that may be represented by forwards or derivatives such as options, futures contracts, or swap agreements. It invests primarily in investment-grade debt securities, but may invest up to 10% of total assets in high-yield securities.

(*continued*)

EXHIBIT 9.2 (*Continued*)

Rydex Govt Long Bond 1.2x Strategy C
The investment seeks returns that correspond to 120% of the price movement of the long Treasury bond. The fund employs as its investment strategy a program of investing in U.S. government securities and derivative instruments, which primarily consist of futures contracts, interest rate swaps, and options on securities and futures contracts. It invests at least 80% of net assets in fixed-income securities issued by the U.S. government. The fund also may invest in zero-coupons and U.S. Treasury bonds.

Columbia Strategic Income A
The fund invests primarily in debt securities issued by the U.S. government and its agencies, including mortgage- and other asset-backed securities. It also invests in debt securities issued by foreign governments, companies, or other entities, including in emerging market countries and non-dollar denominated securities, and securities below investment-grade corporate debt securities. The fund may invest in derivatives, including futures, forwards, options, swap contracts, and other derivative instruments.

Fidelity Advisor Strategic Income C
The fund invests primarily in debt securities, including lower-quality debt securities. It uses a neutral mix of approximately 40% high-yield securities, 30% U.S. government and investment-grade securities, 15% emerging markets securities, and 15% foreign developed markets securities.

AMF Intermediate Mortgage
The fund invests, under normal market conditions, primarily in mortgage-related investments paying fixed or variable rates of interest. It has no restriction as to the minimum or maximum maturity of any particular investment held. The fund also seeks to maintain a minimum duration of a two-year U.S. Treasury note, and a maximum duration equal to that of a four-year U.S. Treasury note.

BlackRock Low Duration Bd Inv B2
The fund invests primarily in investment-grade bonds and maintains an average portfolio duration that is within 20% of the duration of the benchmark. It may also invest up to 5% of assets in non-investment grade bonds or convertible securities with a minimum rating of B and up to 10% of assets in non-dollar denominated bonds of issuers located outside of the United States.

Fidelity Advisor Convertible Secs A
The fund normally invests at least 80% of assets in convertible securities. It may invest the balance of assets in corporate or U.S. government debt securities, common stocks, preferred stocks, and money market instruments. The fund may invest in securities rated below investment grade. It may also invest a substantial portion of assets in unrated securities. It may purchase foreign securities.

**Source:* Yahoo Fund Screener: http://screen.yahoo.com/funds.html.

gains. If they retain the gain, they are required to pay a tax equal to the maximum personal income tax rate; the shareholders, in turn, receive a credit for the taxes paid.

Bond Market Indexes

As noted, bond funds can be classified as corporate, municipal, government, high-yield, global, mortgage-backed securities, and tax-free. Each category reflects a different investment objective. The managers of these various bond funds, as well as the managers of pension, insurance, and other fixed-income funds, often evaluate the performance of their funds by comparing their fund's return with those of an appropriate bond index. In addition, many funds are constructed so that their returns replicate those of a specified index.

A number of bond indexes have been developed in recent years on which bond funds can be constructed or benchmarked. A number of investment companies also publish a variety of indexes; these include Barclays and Merrill Lynch. The indexes can be grouped into three categories: U.S. investment-grade bond indexes (including Treasuries), U.S. high-yield bond indexes, and global government bond indexes. Within each category, subindexes are constructed based on sector, quality ratings, or country. Table 9.3 summarizes some of the indexes.

TABLE 9.3 Bond Market Indexes

Index	Number of Issues	Maturity	Size	Subindexes
U.S. Investment-Grade Bond				
Barclays Aggregate	5,000	Over 1 year	Over $100 million	Government, corporate, government/corporate mortgage-backed, asset-backed
Merrill Lynch Composite	5,000	Over 1 year	Over $50 million	Government, corporate, government/corporate mortgage-backed
U.S. High-Yield Bond				
First Boston	423	All maturities	Over $75 million	Composite and by ratings
Barclays	624	Over 1 year	Over $100 million	Composite and by ratings
Merrill Lynch	735	Over 1 year	Over $25 million	Composite and by ratings
Global Government Bond				
Barclays	800	Over 1 year	Over $200 million	Composite and 13 countries in local currency and U.S.$
Merrill Lynch	9,735	Over 1 year	Over $100 million	Composite and 9 countries in local currency and U.S.$
J.P. Morgan	445	Over 1 year	Over $200 million	Composite and 11 countries in local currency and U.S.$

Source: Frank Fabozzi, ed. *The Handbook of Fixed-Income Securities*, 6th ed. (New York: McGraw-Hill), 158.

Exchange-Traded Funds

In 1993, the American Stock Exchange, AMEX, created an S&P 500 index fund called an ***exchange-traded fund (ETF)*** that could be traded continuously like a stock. This first ETF received exemptions from the Securities and Exchange Commission (SEC) from various provisions of the Investment Company Act of 1940. The exemptions made it possible for the ETF to be structured so that it could be listed and traded continuously. Until 2008, most of the ETFs introduced replicated designated indexes. These ETFs, commonly referred to as index-based ETFs, are designed to track the performance of their specified indexes or, in some cases, a multiple of or an inverse of their indexes. By 2008, there were over 400 separate ETFs; many with esoteric names such as Spiders (ETF that replicates the S&P 500), Qubes (an ETF indexed to the NASDAQ), Diamonds (ETF that replicates the Dow Jones Industrial Average), and Vipers (name for Vanguard ETS). Today, ETFs include most sectors, commodities, and investment styles. In early 2008, the SEC granted exemptive relief to several fund sponsors to offer actively managed ETFs that meet certain requirements. These actively managed ETFs, in turn, have led to new continuously traded exchange-traded products (ETPs) defined by a particular investment objective and policy. By 2010, the total number of index-based and actively managed ETFs had grown to over 728, with total net assets of over $530 billion (see Table 9.4).

Most ETFs originate with a sponsor, who defines the investment objective of the ETF and the method for tracking the performance. The sponsor of an index-based ETF, for example, would define the index (e.g., large U.S. bank sector) and the method of tracking it (e.g. a total-replication index method that holds every security in the target index or a sample index-based method that holds a representative sample of securities in the index). Given the fund objective and tracking method, a ***creation basket*** is identified that specifies the names and quantities of securities and other assets designed to track the performance of the index portfolio. ETF shares are created after an ***authorized participant*** (typically an institutional investor) deposits the creation basket and/or cash into the fund—the ETF. In return for the creation basket and/or cash, the authorized participant receives the block of ETF shares, referred to as a ***creation unit***. The authorized participant can then either keep the ETF shares that make up the creation unit or sell all or part of them on a stock exchange (see Figure 9.5).

TABLE 9.4 Exchange-Traded Funds: Total Net Assets by Type of Fund, 2003–2008 (in Millions of Dollars)

Year	Total	U.S. Equity Broad-Based	U.S. Equity Sector	Global/ International	Commodities	Hybrid	Bond
2003	150,983	120,430	11,901	13,984	–	–	4,667
2004	227,540	163,730	20,315	33,644	1,335	–	8,516
2005	300,820	186,832	28,975	65,210	4,798	–	15,004
2006	422,550	232,487	43,655	111,194	14,699	–	20,514
2007	608,422	300,930	64,117	179,702	28,906	119	34,648
2008	531,287	266,161	58,374	113,684	35,728	132	57,209

Source: 2009 Investment Company Fact Book: www.icifactbook.org/index.html.

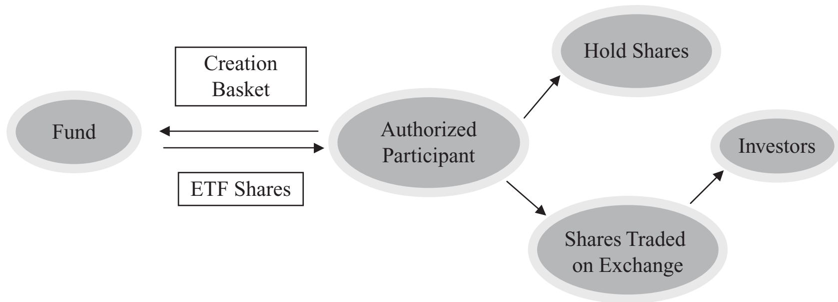

FIGURE 9.5 Creation Process of Exchange-Traded Funds
Source: 2009 Investment Company Fact Book: www.icifactbook.org/index.html.

ETFs are like mutual funds in that their value is derived from the underlying portfolio of securities. Different from mutual funds, ETFs trade like stocks: Investors can buy and sell them on a continuous basis and can execute trades with market or limit orders; they can also buy ETFs on margin and sell short. Also different from mutual funds, the price of an ETF is based on market supply and demand conditions. However, because of the disclosure requirements that require that the composition of the ETF's basket be made public, arbitrageurs are in a position to ensure that the price of an ETF trades close to the underlying net asset value of the securities held in the index basket.[7]

Not surprisingly, the demand for ETFs has accelerated in recent years with institutional investors increasingly using them to take positions on broad movements in the financial markets. Retail investors and households have also started to add ETFs to their portfolio holdings. According to the Investment Company Institute, an estimated 2.3 million households owned ETFs in 2008. Although many ETFs are equity based, there are an increasing number of fixed-income ETFs and exchange-traded products being offered both in and outside the United States. These fixed-income exchange-traded products vary from ETFs that are tied to bond indexes, to those linked to Treasury yields, to ETFs that are tied to a multiple of the Treasury yield. Table 9.5 shows a partial listing of fixed-income ETFs and their prices on October 29, 2009, along with a brief description of several of the funds' investment objectives.

Collateralized Debt Obligations

Collateralized debt obligations (CDOs) are securities backed by a diversified pool of one or more of fixed-income assets or derivatives. Like mortgage-backed securities, they are securitized assets. The portfolio of debt obligations underlying the CDO are referred to as the collateral, with the funds to purchase the collateral assets being obtained by the issuance of the debt obligations. Assets from which CDOs are formed include investment-grade corporate bonds, high-yield corporate bonds, asset-backed securities, leveraged bank loans, distressed debt, real estate mortgage-backed securities, commercial loans, commercial mortgage-backed securities, ***real estate investment trusts (REITs)***, municipal bonds, and emerging market bonds.

TABLE 9.5 U.S. Fixed-Income ETFs, October 29, 2009

Fund Name	Symbol	Last
WisdomTree:US ST Gov Inc	USY	25.05
Vanguard Tot Bd;ETF	BND	79.45
Vanguard Sh-Tm Bd;ETF	BSV	80.06
Vanguard Lg-Tm Bd;ETF	BLV	78.75
Vanguard Int-Tm Bd;ETF	BIV	80.40
Vanguard Ext Dur T;ETF	EDV	99.91
TDX Indep:In-Target	TDX	25.03
SPDR DB Intl Gvt IP Bd	WIP	56.67
SPDR Barclays TIPS	IPE	51.39
SPDR Barclays ST Intl Tr	BWZ	37.20
SPDR Barclays S-T Mun Bd	SHM	23.80
SPDR Barclays Lng Tm Trs	TLO	55.98
SPDR Barclays Hi Yld Bd	JNK	37.99
SPDR Barclays CA Mun Bd	CXA	22.36
SPDR Barclays Aggr Bd	LAG	56.18
SPDR Barclays 1-3Mo Tbll	BIL	45.88
PowerShares VRDO Wkly	PVI	25.00
PowerShares Preferred	PGX	13.07
PIMCO ETF:1-5 Yr US TIPS	STPZ	51.06
PIMCO ETF:1-3 Yr US Tres	TUZ	50.27
Mrkt Vctrs:Sh Muni Idx	SMB	17.19
iShares:S&P/Citi1-3 ITrs	ISHG	107.05
iShares:Barclays Agcy Bd	AGZ	108.75
iShares:Barc MBS Bond	MBB	107.27
iShares:Barc Aggreg Bond	AGG	104.80
iShares:Barc 7-10 Trs Bd	IEF	91.99
iShares:Barc 3-7 Trs Bd	IEI	112.44

Vanguard Lg-Tm Bd;ETF (BLV)
The ETF tries to match the investment performance of the Barclays Capital Mutual Fund Long Government/Corporate Index. Government/AAA = 52.6%, A-Rate = 19.81%, BBB Rated = 8.54%, and Equities and others = 1.86%.

SPDR Barclays Muni Bd (TFI)
The ETF tries to provide investment results that, before fees and expenses, correspond generally to the price and yield performance of an index that tracks the U.S. municipal bond market and provides income that is exempt from federal income taxes.

SPDR Barclays Mtge Bd (MBG)
The ETF seeks to provide investment results that correspond generally to the price and yield performance of an index that tracks the U.S. agency mortgage pass-through sector of the U.S. investment grade bond market.

PowerShares HY Corp Bd (PHB)
The ETF tries to provide investment results that correspond generally to the price and yield (before fees and expenses) of an index called Wachovia High Yield Bond Index. The Fund seeks to invest at least 80% of its total assets in high yield corporate bonds payable in U.S. dollars. BB/B-Rated = 72.06%, CCC, CC, and C Rated = 5.5%, Foreign Securities = 3.61%, and BBB-Rated = 1.9% (as of October 29, 2009).

TABLE 9.5 (*Continued*)

PowerShares EM Sov Dbt (PCY)
The ETF seeks investment results that correspond generally to the price and yield of an index called the DB Emerging Market USD Liquid Balanced Index. The Fund will normally invest at least 90% of its total assets in the securities that make up this Index. Latin America = 38.26%, Asia = 31.77%, Europe = 23.74%, and other = 6.23% (as of October 29, 2009).

iShares:iBoxx $IG Corp (LQD)
The ETF tries to provide investment results that correspond generally to the price and yield performance of a segment of the U.S. investment grade corporate bond market as defined by the GS $ InvesTop Index.

iShares:Barc Gvt/Crd Bd (GBF)
The ETF tries to provide investment results that correspond to the investment grade U.S. government and U.S. corporate securities of the U.S. bond market as defined by the Barclays Capital U.S. Government/Credit Index.

Direxion:30Y Trs Bull 3X (TMF)
The Fund seeks daily investment results of 300% of the price performance of the NYSE Current 30 Year U.S. Treasury Index. The Fund, under normal circumstances, creates long positions by investing at least 80% of its net assets in U.S. government securities that comprise the 30-Year Treasury Index.

iShares:S&P US P St Id (PFF)
The Fund tries to provide investment results that correspond generally to the price and yield performance, before fees and expenses, of the S&P U.S. Preferred Stock Index. The Index measures the performance of a selected group of preferred stocks listed on the NYSE, AMEX and the Nasdaq Stock Market, Inc.

Source: Wall Street Journal, http://online.wsj.com/public/us.

CDOs have a collateral manager who is responsible for managing the portfolio of debt obligations. Restrictions are, in turn, imposed on what the collateral manager can do. A CDO deal can be terminated early if an ***event of default*** occurs. Such an event relates to conditions that could significantly impact the performance of the collateral. This could include a failure to comply with certain coverage ratios, the bankruptcy of an issuing entity, or the departure of the collateral management team. Many of the CDOs that were based on subprime mortgage loans resulted in the CDOs issuing an ***Event of Default Notice***. The issuance of CDOs grew from the 1990s to 2007, but stopped in 2008 in the aftermath of the 2008 financial crisis. There are still, though, a number of issues outstanding. CDOs are discussed further in Chapter 12.

Other Investment Funds

In addition to open-end and closed-end investment funds, unit investment trusts, ETFs, and CDOs, two other investment funds of note are real estate investment trusts and ***hedge funds***.

Real Estate Investment Trust (REIT) A REIT is a fund that specializes in investing in real estate or real estate mortgages. The trust acts as an intermediary, selling stocks and warrants and issuing debt instruments (bonds, commercial paper, or loans from banks), and then using the funds to invest in commercial and residential mortgage loans and other real estate securities. REITs can take the form of an equity trust that invests directly in real estate, a mortgage trust that invests in mortgage loans or mortgage-backed securities, or a hybrid trust that invests in both. Many REITs are highly leveraged, making them more subject to default risks. REITs are tax-exempt corporations. To qualify for tax exemptions, the company must receive approximately 75% of its income from real estate, rents, mortgage interest, and property sales, and distribute 95% of its income to its shareholders. The stocks of many existing shares in REITs are listed on the organized exchanges and the OTC market.

Hedge Funds Hedge funds can be defined as special types of mutual funds. There are estimated to be as many as 9,000 such funds. They are structured so that they can be largely unregulated. To achieve this, they are often set up as limited partnerships. By federal law, as limited partnerships, hedge funds are limited to no more than 99 limited partners each with annual incomes of at least $200,000 or a net worth of at least $1 million (excluding home), or to no more than 499 limited partners each with a net worth of at least $5 million. Many funds or partners are also domiciled offshore to circumvent regulations. Hedge funds acquire funds from many different individual and institutional sources; the minimum investments range from $100,000 to $20 million, with the average investment being $1 million. Many of the funds also highly leverage their investments, borrowing funds to finance the investments with debt-to-equity ratio in some cases as high as 20 to 1. Hedge funds use their funds to invest or set up investment strategies reflecting pricing aberrations. Many of these strategies involve bond positions. One of the most famous is that of Long-Term Capital who in 1998 set up a quality swap position in T-bonds and long-term corporate bonds to profit from an expected narrowing of the default spread that instead widened. Other notable hedge fund collapses include Amaranth Advisors, Advanced Investment Management, Bayou Management, and Lipper Convertibles.

WEB INFORMATION

- *Wall Street Journal* site
  - http://online.wsj.com/public/us
  - Click "Market Data" tab.

  Fixed-income ETFs
  - Click "ETF" tab.
  - Click "ETF Screener."
  - Fixed-Income

  Mutual funds
  - Click "Mutual Fund" tab.
  - Use Screener.

  Money rates
  - Click "Bond Rates and Credit Market."
  - Click "Money Rate."

- FINRA
  - www.finra.org/index.htm
  - "Sitemap," "Mutual Funds" and "Fixed Income."
  - Click "Taxable" or "Tax-Exempt" searches.
- Yahoo!
  - Fund screener: http://screen.yahoo.com/funds.html
    Real estate investment trusts:
  - Price and other information on REITs can be found by going to Yahoo!, http://screen.yahoo.com/funds.html; Use Stock Screener to find REITs.
- Real estate investment trusts: www.nareit.com.
- Investment Company Institute: www.ici.org
  - Investment Company Institute Facts Book: www.icifactbook.org/index .html
- Investment funds and ratings:
  - http://investing.quicken.com/investing/
  - www.morningstar.com
  - www.lipper.com
- Money market funds: www.imoneynet.com
- Hedge funds:
  - www.thehfa.org
  - www.hedgefund.net
  - Hedge fund rankings: www.cta-online.com/hedgeland.asp
- Federal Reserve Flow of Fund Accounts:
  - www.federalreserve.gov/releases/z1/Current/
  - Go to PDF for Flow of Fund tables by sector and security type.
  - See Table L109-L113 for banks.
  - See Table L122 for mutual funds.
  - See Table L123 for ETFs.
  - See Table L128 for REITs.
  - Go to "Data Download Program" to download series to Excel.

9.6 INSURANCE COMPANIES AND PENSION FUNDS

Insurance companies and pension funds are important financial intermediaries. On the one hand, they use the premiums paid on various insurance policies and the investment funds from retirement and savings plans to invest in bonds, stocks, mortgages, and other assets. On the other hand, many individuals use insurance policies and pension plans as their primary investment conduit.

Insurance Companies

Insurance Products Insurance companies can be classified as either property and casualty companies or life insurance companies. Property and casualty companies provide property insurance to businesses and households against losses to their properties resulting from fire, accidents, natural disasters, and other calamities, and casualty (or liability) insurance to businesses and households against losses the policy owner may cause to others as a result of accidents, product failures, and negligence. Property and casualty insurance policies are short term, often renewed on an annual basis. Since the events being insured by the policies are difficult to predict, property and casualty insurance companies tend to invest the premiums from their policies into more liquid assets.[8]

Life insurance companies provide basic life insurance: protection in the form of income to beneficiaries in the event of the death of the policy owner. They also provide disability insurance, health insurance, annuities, and guaranteed investment contracts.

Life Insurance Policies: There are three general types of life insurance policies: term life, whole life, and universal life. A ***term life policy*** pays a lump sum benefit to the beneficiary if the policyholder dies when the policy is in effect. Whole life and universal life policies provide both life insurance benefits and a retirement plan. The premiums paid by the insured on whole life and universal life policies cover the cost of the life insurance and include a savings program. The interest earned from the savings part of the policy is also tax exempt until it is withdrawn (provided the investment value exceeds the death benefit). In the case of a ***whole life policy***, the insured pays a premium that in the earlier years of the policy exceeds the normal premium on term life. This excess accumulates into a cash value available to policyholders or their survivors: If the insured dies, the survivors receive both the death benefit and the cash value; if the insured lives past the policy's maturity, then the insured receives the cash value. In addition, many whole life policies often allow the policyholder to borrow against the cash value. Whole life policies began to decline in popularity in the early 1980s because of the low returns they were generating for their policyholders: In the early 1980s, whole life policyholders were significantly better off buying term life and investing the savings themselves. Life insurance companies responded to this decline by offering ***universal life policies***. Universal life is similar to whole life, except that the savings portion of the account accumulates at a greater rate.

Annuities: A life insurance company annuity pays the holder a periodic fixed income for as long as the policyholder lives in return for an initial lump-sum investment (coming, for example, from a retirement benefit or insurance cash value). Annuities provide policyholders protection against the risk of outliving their retirement income. Thus, in contrast to life insurance policies that provide insurance against dying too soon, annuities provide insurance against living too long. There are three general types of annuities: a ***life annuity***, which pays a fixed amount regularly until the investor's death; a ***last survivor's annuity***, which pays regular fixed amounts until both the investor and spouse die; a ***fixed-period annuity***, which makes regular fixed payments for a specified period (e.g., five, 10, or 20 years), with payments made to a beneficiary if the investor dies. These annuities are referred to as fixed annuities. They are constructed based on the rates of return insurance companies can obtain from investing an individual's payment for a period equal to the individual's life expectancy (fixed-life annuity), or for a prespecified period (fixed-period annuity). In addition to fixed annuities, insurance companies also offer a ***variable annuity*** in which regular payments are not fixed, but rather depend on the returns from the investments made by the insurance company (the insurance company sometimes invests in a mutual fund that they also manage). Finally, insurance companies offer ***deferred annuities***

(variable or fixed) that allow an investor to make a series of payments instead of a single payment.

Guaranteed Investment Contract: A ***guaranteed investment contract (GIC)*** is an obligation of an insurance company to pay a guaranteed principal and rate on an invested premium. For a lump-sum payment, the insurance company guarantees a specified dollar amount will be paid to the policyholder at a specified future date. For example, a life insurance company, for a premium of $1 million, guarantees the holder a five-year GIC paying 8% interest compounded annually. The GIC, in turn, obligates the insurance company to pay the GIC holder $1,469,328 [= $\$1,000,000(1.08)^5$] in five years.

Guaranteed investment contracts, also called guaranteed interest contracts and guaranteed insurance contracts, are similar to zero-coupon debentures issued by corporations. A debenture holder, though, is a general creditor, whereas a GIC holder is a policyholder who has a senior claim over general creditors of the insurance company. Pension funds are one of the primary investors in GICs. The GICs provide them not only an investment with a known payment but also an investment that always has a positive value to report; this contrasts with bond investments whose values may decrease if interest rates increase. The growth in GICs started in the 1980s with the increased investment in 401(k) plans. In addition to insurance companies, banks have also become active participants in this market, offering ***bank investment contracts*** (***BIC***). BICs are deposit obligations with a guaranteed rate and fixed maturity. BICs and GICs are sometimes referred to as ***stable value investments***.

The generic GIC, also called a ***bullet contract***, is characterized by a lump-sum premium, a specified rate and compounding frequency, and a lump-sum payment at maturity. Maturities can range from one year to 20 years, with the short-term GICs often set up like money market securities. There are several variations of the bullet contract. A window GIC, for example, allows for premium deposits to be made over a specified period, such as a year; they are designed to attract the annual cash flow from a pension or 401(k) plan. A GIC may consist of a single type of security, such as a mortgage-backed or other asset-backed security, or a portfolio of securities that are managed and immunized to the specified maturity date of the contract; some GICs may also be secured by letters of credit or other credit enhancements. Instead of a specified maturity date, the contract may specify that it will maintain a portfolio with a constant duration. There are also floating-rate GIC contracts in which the rate is tied to a benchmark rate, and there are GIC contracts in which the interest is paid periodically to the policyholder. Finally, a GIC may have a wrapped contract with clauses that give the holders the right to sell the contract and receive the book value or to change the rate under certain conditions.

In managing the funds from GICs, insurance companies may either pool the contracts into a general account (no separate identification of assets for a particular policy) or as a separate account (separate account for the GIC holder or group). The latter GIC is known as a ***separate account contract*** (SAC). When securities are separated from other liabilities of the insurer and managed separately in a SAC, they are considered legally protected against the liabilities arising from other businesses of the insurance company.

Health Insurance: Many life insurance companies offer health insurance. Most of their policies are offered through company-sponsored programs in which the company pays all or part of the premium.[9] With the increasing costs of health care over the last decade, the risk of many company-sponsored plans is borne by the company, with the insurance company responsible for managing the plan and providing for catastrophic expenses. To keep costs lower, insurance companies try to reduce health care cost by negotiating contracts with physician groups to provide services at a lower cost and through managed care in which services require prior approval. In such cases, the insurance company pays the health maintenance organization (HMO) a fixed payment per person who is covered in exchange for the service.[10]

Insurance Companies' Role in the Financial Market Insurance companies are a major player in the financial markets, investing billions of dollars of inflows received each year from the premiums from their insurance policies, their savings and investment products, and the pension and endowment funds they manage. In 2007, life insurance companies held approximately $5.1 trillion in assets. Since their liabilities tend to be more predictable and long term, life insurance companies tend to invest in long-term assets.[11] In 2008, about 40% of their assets were in corporate bonds, followed by equity (22%), government securities (11%), mortgages (7%), and various other assets. The allocations in 2007 differ significantly from their allocations in earlier periods. As shown in Figure 9.6, insurance companies invested almost 40% of their assets in mortgages and real estate in the early 1970s, compared to 7% in 2008.

In contrast to life insurance companies, property and casualty insurance companies insure against many different types of events, with the amount of potential losses on many of the events they insure difficult to predict. As a result, property and casualty companies, as previously noted, tend to invest in more liquid assets than life insurance companies.

Pension Funds

Pension funds are financial intermediaries that invest the savings of employees in financial assets over their working years, providing them with a large pool of funds at their retirements. Pension funds are one of the fastest growing intermediaries in the United States. The total assets of pension funds (private and state and local

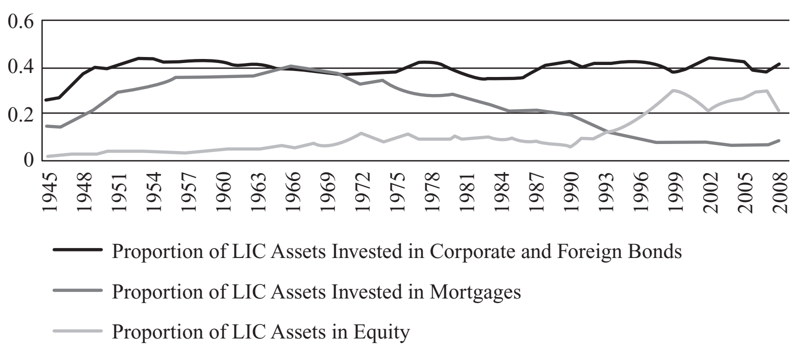

FIGURE 9.6 Proportion of LIC Assets Invested in Bonds, Mortgages, and Equity, 1945–2008
Source: Federal Reserve Flow of Fund Accounts, Table L117; www.federalreserve.gov/releases/z1/Current/.

government) have grown from $700 billion in 1980 to approximately $9 trillion in 2008. Part of this growth reflects a work force of baby boomers making contributions to their pensions. As this generation enters retirement over the next decade and begins to draw from its investments, there is expected to be a marked decline in such growth.

Pension funds can be grouped as public or private plans. The largest public plan is the Federal Old Age and Disability Insurance Program (Social Security). It is a pay-as-you-go system in which current workers' contributions pay for the benefits to the current recipients.[12] The other public pension funds are those sponsored by state and local governments. Private pension plans are those sponsored by employers, groups, and individuals.

There are two general types of pension plans: a defined-benefit plan and a defined-contribution plan. A ***defined-benefit plan*** promises employees a specified benefit when they retire. The benefit is usually determined by a formula. For example, the employee's annual benefit during her retirement might be based on a specified percentage (for example, 2%) times the average salary over her last five years ($75,000) times her years of service (30 years): (.02)($75,000)(30) = $45,000. The funding of defined-benefit plans is the responsibility of the employer. Financial problems can arise when pension funds are underfunded and the company goes bankrupt. As a result, over the last two decades most new plans are structured as ***defined-contribution plans***. These plans specify what the employee will contribute to the plan, instead of what the plan will pay. At retirement, the benefits are equal to the contributions the employee has made and the returns earned from investing them. The employee's contributions to the fund are usually a proportion of his income, often with a proportion of that contribution made by the employer. An insurance company, bank trust department, or investment company often acts as the trustee and investment manager of the fund's assets. In many defined-contribution plans, the employee is allowed to determine the general allocation between equity, bonds, and money market securities in his individual accounts. Unfortunately, some companies have pension plans that encourage employees to invest exclusively in their own stock. As the Enron collapse painfully showed, this lack of diversification can lead to employees not only losing their jobs but also their pension investments if the company goes bankrupt.

To pension contributors, pension funds represent long-term investments through intermediaries. As of 2006, private funds sponsored by employers, groups, and individuals were one of the largest institutional investors in equity, with about 72% of their total investments of $6.2 trillion going to equity (46% in equity and 26% in mutual fund shares), 6% in government securities, and 5% in corporate and foreign bonds. In 2008, the value of private pension assets declined to $5.4 trillion, with equity representing 51% (30% equity and 22% mutual fund shares). In 2008, public funds sponsored by state and local governments had invested assets valued at over $3 trillion, with 38% in equity, 6% in mutual funds, 9% in corporate bonds, 10% in federal agency and Treasury securities, and 26% in credit market instruments (see Federal Reserve Flow of Fund Accounts, Table L118; www.federalreserve.gov/releases/z1/Current/).

Pension members are not taxed on their contributions, but they do pay taxes on benefits when they are paid out. Pension funds in the United States are governed by the 1974 ***Employee Retirement Income Security ACT (ERISA)***. ERISA requires prudent management of the fund's investments and requires that all private plans be fully funded; that is, that the assets and income cover all promised benefits. The act also ensures transferability of plans when employees change jobs, specifies disclosure requirements, and defines the minimum vesting requirements for determining eligibility. In 1974, Congress also created the ***Pension Benefit Guaranty Corporation*** (***PBGC*** or ***Penny Benny***) to provide insurance for employee benefits. Similar to the FDIC, Penny Benny is a government agency that insures pension benefits up to a limit if a company goes bankrupt and has an underfunded pension plan. It operates by charging pension plans a premium and it can borrow funds from the Treasury. In 2008, Penny Benny paid benefits to over 700,000 retirees of failed pension plans.[13]

In addition to employee and institutional pension plans, retirement plans for U.S. individuals can also be set up through ***Keough plans*** and ***individual retirement accounts*** (***IRAs***). In accordance with the Self-Employed Individual Tax Retirement Act of 1962, self-employed people can contribute up to 20% of their net earnings to a Keough plan (retirement account) with the contribution being tax deductible from gross income. The Pension Reform Act of 1978 updated the 1962 act to permit individual retirement accounts (IRAs). Subsequent legislation in 1981 and 1982, in turn, expanded the eligibility for creating tax-deferred accounts to include most individuals. The Small Business Protection Act of 1996 created a simplified retirement plan for businesses with 100 or fewer workers. In addition to company-sponsored and group-sponsored pensions, bank trust departments, insurance companies, and investment companies offer and manage individual retirement accounts and Keough plans. For small accounts, these institutions often combine the accounts in a ***commingled fund***, instead of managing each account separately. A commingled fund is similar to a mutual fund. For accounting purposes, individuals setting up accounts are essentially buying shares in the fund at their NAV and when they withdraw funds they are selling essential shares at their NAV. Like mutual funds, insurance companies and banks offer a number of commingled funds, such as money market funds, stock funds, and bond funds.

WEB INFORMATION

- Life Insurance Fact Book: www.acli.com/ACLI/Tools/Industry+Facts
- Federal Reserve Flow of Fund Accounts: www.federalreserve.gov/releases/z1/Current/

Go to PDF for Flow of Fund Tables by sector and security type:

  - See Table L117 for life insurance assets.
  - See Table L116 for property and casualty insurance assets.
  - See Table L117-L119 for pension fund assets.
  - Go to "Data Download Program" to download series to Excel.

- For insurance industry trends: www.riskandinsurance.com.
- For quality ratings and evaluation of insurers: www.bestweek.com
- For pension fund information and updates: www.ifebp.org
- For Social Security fund information: www.ssa.gov
- Pension Benefit Guarantee Corporation: www.pbgc.gov

9.7 CONCLUSION

Just like the markets for corporate and government securities, the intermediary financial markets offer borrowers and investors a wide array of instruments for financing and investing: from short-term securities, such as CDs, BAs, and shares in money market funds, to intermediate- and long-term instruments, such as mortgage-backed securities, mutual fund shares, ETFs, pension plans, annuities, and GICs.

KEY TERMS

acceptance financing
accepting banks
authorized participant
balanced funds
bank notes
banker's acceptances (BAs)
bank investment contracts (BICs)
bear and bull CD
brokered deposits
bullet contract
closed-end fund
collateralized debt obligations (CDOs)
collateralized mortgage obligations (CMOs)
commingled fund
creation basket
creation unit
deferred annuities
defined-benefit plan
defined-contribution plans
employee Retirement Income Security ACT (ERISA)
event of default
event of default notice
exchange-traded funds (ETFs)
fixed-period annuity
floating-rate CD (FRCD)
forward CD
global funds
growth funds
guaranteed investment contract (GIC)
hedge funds
income funds
index funds
individual retirement accounts (IRAs)
investment-grade loans
jumbo CD
Keough plans
last survivor's annuity
leveraged loans
life annuity
load funds
money market funds
mortgage-backed securities (MBSs)
net asset values (NAV)
no-load funds
nonprime CD
open-end funds (Mutual funds)
Pension Benefit Guaranty Corporation (PBGC or Penny Benny)
prime CD
real estate investment trust (REIT)
redeemable trust certificates
rising-rates CD
rollover CD (Roly-poly CD)
securitization
separate account contract
stable value investments
term CD
term life policy
unit investment trust
universal life policies
variable annuity
vulture funds
whole life policy

PROBLEMS AND QUESTIONS

1. Define the following:
   a. Prime CDs
   b. Nonprime banks
   c. Yankee CDs
   d. Eurodollar CDs
   e. Floating-rate CDs
   f. Bear and bull CDs
   g. Rising-rates CDS
   h. Forward CDs
2. Describe the primary market for CDs. Who are the some of the major investors in CDs?
3. Explain the history of the secondary CD market. In your explanation bring out the significance of a positively sloped yield curve that is not expected to change.
4. What are bank notes? How do they differ from medium-term notes issued by corporations?
5. Define banker's acceptance, acceptance financing, and accepting banks.
6. A U.S. oil exploration company drilling in the Gulf of Mexico wants to purchase $20 million of drilling equipment from a German tool manufacturing company. Once the transaction agreement is complete, the drilling equipment will be shipped to the U.S. company's drilling assembly operation facility in Houston. Shipping time is expected to take 30 days. Both companies agree to finance the transaction with a banker's acceptance in which the U.S. company's bank will guarantee the U.S. company's payment of $20 million with a letter of credit to be sent to the German company's bank. Upon notification of the receipt of the letter of credit, the German company will ship the equipment and take payment from its bank. Explain how the rest of the transactions between the U.S. and German companies would take place and how the banker's acceptance would be created.
7. Describe banker's acceptances as a security, the secondary market for such securities, and some of the principal participants in the market.
8. Define mortgage-backed security and explain how they are constructed. What is the primary risk that investors in MBSs are subject to?
9. Define and explain the distinguishing features of the following funds:
   a. Open-end fund
   b. Closed-end fund
   c. Real estate investment trust
10. Define a unit-investment trust. Explain how a financial institution would set up a unit-investment trust with 10-year T-bonds with $100 million par value selling at par as the underlying securities and with 100,000 certificates created.

11. Explain how hedge funds are structured. What types of investments do they make?

12. List the principal classifications of bond funds.

13. What are the primary investment objectives of the following: municipal bond fund, corporate bonds index fund, and money market fund?

14. Explain how an ETF is constructed. Include in your explanation the tracking methods, creation basket, authorized participant, and the issuance of ETF shares.

15. Explain why the price of an ETF should trade close to the underlying NAV.

16. Define:

   a. Annuities
   b. Life annuity
   c. Last survivor's annuity
   d. Fixed-period annuity
   e. Variable annuity
   f. Deferred annuities

17. Define a guaranteed investment contract (GIC). List some of its features of the generic GIC.

18. What would an investor/policyholder receive from investing $1 million in a six-year GIC paying 6% interest compounded semiannually?

19. How does a guaranteed investment contract differ from an investment in a zero-coupon debenture?

20. What makes GICs an attractive investment for pensions and other institutional investors?

21. What is a separate account GIC contract?

22. Define the following terms:

   a. Bank investment contracts
   b. Stable value investment
   c. Bullet contract
   d. Window GIC
   e. Floating-rate GIC

23. What is the difference between a defined-benefit pension plan and a defined-contribution plan?

24. Comment on how well the investments of pension plans are diversified.

25. Define the following:

   a. Employee Retirement Income Security ACT (ERISA)
   b. Pension Benefit Guaranty Corporation (PBGC or Penny Benny)
   c. Individual retirement accounts (IRAs)

26. Explain how financial institutions manage small IRA accounts as commingled funds.

WEB EXERCISES

1. Compare the historical yields on U.S. CDs to the yields on other money market securities. Go to www.federalreserve.gov/releases/h15/data.htm and download the historical yields to Excel.
2. Study some of the recent intermediate securities added to Moody's watch list:
   - Asset-backed securities
   - Residential MBSs
   - Collateral debt obligations
   - Leveraged finance
   - Syndicated loans
     - Go to www.moodys.com.
     - Search for "WATCHLIST."
3. Moody's provides information on default rates, ratings changes, and other credit information. Examine the default rates for syndicated bank loans and structured financed securities by going to their site. To access Moody's study of historical default rates:
   - Go to www.moodys.com.
   - Search for "Rating Methodologies and Performance."
   - "Historical Performance"
   - Look for:
     - Syndicated bank loans
     - Structured finance default studies
4. Go to the FINRA site to find information on fixed-income mutual funds: Go to www.finra.org/index.htm, "Sitemap," "Mutual Funds," and "Fixed Income."
5. Go to Yahoo!'s "Advanced Fund Screener" to search and find information on fixed-income mutual funds. Yahoo! Fund Screener: http://screen.yahoo.com/funds.html.
6. Go to the *Wall Street Journal* site and examine the following bonds and their yields:
   - Exchange-traded corporate bonds
   - Mortgage-backed securities
   - Syndicated loans
   - Guaranteed investment contracts
     - http://online.wsj.com/public/us
     - Click "Market Data" tab.
     - Click "Bond Rates and Credit Market."
7. Find the current money market rates by going to the *Wall Street Journal* site:
   - http://online.wsj.com/public/us
   - Click "Market Data" tab.
   - Click "Bond Rates and Credit Market."
   - Click "Money Rate."

8. Go to the *Wall Street Journal* site and use their ETF search screener to find information on fixed-income ETFs:
   - http://online.wsj.com/public/us
   - Click "Market Data" tab.
   - Click "ETF" tab.
   - Click "ETF Screener."
   - Fixed-Income
9. Go to the *Wall Street Journal* site and use their mutual fund search screener to search and find information on taxable and tax-exempt mutual funds:
   - http://online.wsj.com/public/us
   - Click "Market Data" tab.
   - Click "Mutual Fund" tab.
   - Click "Fund Screener."
   - Fixed-Income; Taxable and Tax-Exempt
10. Go to Yahoo! Finance and use their screener to find information on a real estate investment trust:
    - http://screen.yahoo.com/funds.html
    - Use Stock Screener to find REITs.
11. Learn more about real estate investment trusts by going to www.nareit.com.
12. Information and news on investment funds can be found at the Investment Company Institute Web site and from the Investment Company Institute Facts Book. Learn more about funds by going to the Investment Company Institute: www.ici.org.
13. Study some of the recent trends in open and closed-end funds and ETFs by going to the Investment Company Institute Facts Book: www.icifactbook.org/index.html.
14. Learn more about mutual fund ratings by going to the Web sites of Morningstar, Lipper Analytical Services, and Quickens: www.morningstar.com; www.lipperweb.com; http://investing.quicken.com/investing/.
15. Learn more about money market funds by going to www.ibcdata.com.
16. Insurance companies and pension funds manage a large proportion of the economy's assets. Their assets and liabilities and flow information can be found at the Federal Reserve Flow of Funds Accounts. Investigate their current balance sheets by going to the Federal Reserve Web site:
    - Federal Reserve Flow of Funds Accounts: www.federalreserve.gov/releases/z1/Current/
    - Go to PDF for Flow of Funds tables by sector and security type.
    - See Table L117 for life insurance assets.
    - See Table L116 for property and casualty insurance assets.
    - See Table L117-L119 for pension fund assets.
    - Go to "Data Download Program" to download series to Excel.
17. Find information on Social Security Fund assets, benefits, and other information by going to the Social Security Fund Web site: www.ssa.gov.

18. Go to the Social Security Fund Web site: www.ssa.gov.
Click "Estimate Your Retirement Benefits" tab to estimate your retirement income.

19. The Pension Benefit Guarantee Corporation faces major financial issues with the default of a number of defined-benefit plans. Examine their current financial condition by going to

- www.pbgc.gov
- Click "FAQ" tab.
- Understanding the Financial Conditions of the Pensions Insurance Programs: www.pbgc.gov/media/key-resources-for-the-press/content/page15247.html#qi1

NOTES

1. In the 1980s, it was possible for large investors to avoid such risk by investing in ***brokered deposits*** that spread the investment across a number of CDs, each with denominations of $100,000 or less. For example, a large investor with $1 million would go to a broker who would break the $1 million into 10 units of $100,000 each and then buy ten $100,000 CDs at 10 different banks. Since each of the deposits was $100,000, they each would qualify for insurance by the FDIC. In 1984, The FDIC passed regulations banning brokered deposits. The ban was challenged in federal court and overturned. The Federal Deposit Insurance Corporation Improvement Act (FDICIA) of 1991, though, limited FDIC's insurance on brokered deposits to those established for pension plans at well-capitalized banks.
2. Some smaller negotiable CDs are held by smaller investors. Introduced by Merrill Lynch in 1982, these CDs are sold by brokerage firms to smaller investors who can sell them before maturity, often back to the brokerage firm who stands ready to buy them.
3. Investment in a Eurodollar CD can be fully invested by the foreign subsidiary bank, whereas a similar investment in a domestic CD cannot because part of the CD proceeds must be used to satisfy reserve requirements.
4. It should be noted that when bank loans are held in a portfolio they must be marked to market. This can be difficult if a market is not transparent. Reuters Loan Pricing Corporation's Loan Trade Data Base (Reuter's LPC) provides institutional investors with dealer quotes. S&P Leveraged Commentary and Data (LCD) also has developed a leveraged loan index that can be used to determine fair values of loans.
5. Although it is generally true that the number of shares of a closed fund are fixed, such funds occasionally issue new shares either through a public offering or through a share dividend, which is sometimes offered to shareholders who are given an option of receiving either cash or new shares based on the NAV of the fund if the dividend or interest income is reinvested. Also, some funds occasionally go into the market and purchase their own shares.
6. Investment funds are regulated under a number of federal laws: the security acts of 1933 and 1934 require disclosure of funds and specify anti-fraud rules; the Security Act of 1940 requires that all funds be registered; the Investment Advisers Act of 1940 regulates fund advisors. In addition, the SEC rules require that funds publish detailed information on directors and that there be independence of the directors.
7. ETFs contract with third parties to calculate a real-time estimate of an ETF's current value, often called the Intraday Indicative Value (IIV). The IIVs are disseminated at

regular intervals during the trading day. Investors, in turn, can observe any discrepancies between the ETF's share price and its IIV during the trading day. When a gap exists between the ETF share price and its IIV, investors may decide to trade in either the ETF share or the underlying securities that the ETF holds in its portfolio in order to attempt to capture a profit. This trading helps to narrow any discrepancy between the price of the ETF share and its IIV. For more information, see *Investment Company Fact Book*: www.icifactbook.org/fb_sec3.html#what.

8. Approximately 10% of property and casualty insurance policies are reinsured. Reinsurance refers to the allocation of the policy to other insurers.
9. The health insurance programs of insurance companies compete with the nonprofit Blue Cross and Blue Shield programs. Blue Cross covers hospital and Blue Shield covers doctors' services. The government also provides Medicare (coverage of elderly) and Medicaid (coverage for people on welfare).
10. Health insurance has been a political issue for many years. In 1996, Congress passed laws making it more difficult for insurance companies to refuse to insure people with a preexisting condition. Since insurance is exempt from federal regulation by the McCarran-Ferguson Act of 1945, most of the changes in health insurance and health care laws are enacted at the state level. At the time of this writing, though, Congress passed and the President signed a national health care law.
11. Using actuarial tables, life insurance companies can predict with a relatively high degree of accuracy when death benefits would have to be paid.
12. With the large baby-boom generation starting to reach retirement, the government forecasts that the Social Security Trust Fund (built from excess payroll taxes) will be depleted by 2040 (some economist predict this will happen as early as 2020). After that, social security taxes will cover only 75% of the benefits. Compounding the problem is that the investments of the Social Security Trust Fund are in government securities (financing the deficit) that would have to be redeemed. For information on the Social Security Fund Assets, go to www.ssab.gov; www.ssab.gov/oact/stats.
13. Penny Benny is facing a severe funding crisis. Currently, there are many defined-benefit plans facing problems with funding obligations as a result of longer life spans, higher medical costs, and weaker economic conditions. Many firms with defined-benefit plans are also at a competitive disadvantage to other companies. Before their bankruptcy, General Motors' profit margin per car was estimated to be 0.5%; without pension and retiree health cost, their margins would have been 5.5%. For a further analysis of the pension fund crisis facing some U.S. companies, see Mishkin and Eakins, *Financial Markets and Institutions*, 6th ed., pp. 585–587. For information on the funding crisis facing Penny Benny, go to www.pbgc.gov and www.pbgc.gov/media/key-resources-for-the-press/content/page15247.html#qi1.

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by R. Stafford Johnson

CHAPTER 10

International Debt Securities

10.1 INTRODUCTION

Before the 1980s, the U.S. financial markets were larger than the markets outside the United States. With the growth of world business, this is no longer the case. Today, American corporations, banks, and institutional investors have increasingly tapped the international money and capital markets to raise or invest short-term and long-term funds, just as non-U.S. borrowers and investors have historically tapped the U.S. market to raise and invest funds. Similarly, global investment has been growing over the last 20 years as many investors have found diversification benefits by including international securities in their portfolios. In this chapter, we examine the characteristics and markets for international debt securities.

10.2 INTERNATIONAL DEBT SECURITIES

In general, a fixed-income investor looking to internationally diversify his bond portfolio has several options. First, he might buy a bond of a foreign government or foreign corporation that is issued in the foreign country or traded on that country's exchange. These bonds are referred to as ***domestic bonds***. Secondly, the investor might be able to buy bonds issued in a number of countries through an international syndicate. Such bonds are known as ***Eurobonds***. Finally, the investor might be able to buy a bond of a foreign government or corporation being issued or traded in his own country. These bonds are called ***foreign bonds***. If the investor were instead looking for short-term foreign investments, his choices would similarly include buying short-term domestic securities such as CP, CDs, and Treasuries issued in those countries, Eurocurrency CDs issued by Eurobanks, and foreign money market securities issued by foreign corporations and governments in their local countries. Similarly, a domestic financial institution or non-financial multinational corporation looking to raise funds may choose to do so by selling debt securities or by borrowing in his own financial markets, the foreign markets, or the Eurobond or Eurocurrency markets.

The markets where domestic, foreign, and Euro securities are issued and traded can be grouped into two categories—the ***internal bond market*** and the ***external bond market***. The internal market, also called the ***national market***, consists of the trading of both domestic bonds and foreign bonds; the external market, also called the ***offshore market***, is where Eurobonds and Euro deposits are bought and sold. There are some bonds that are sold in both the external and internal markets; these bonds are referred to as ***global bonds***. The market for just international securities can be divided into the ***foreign bond and security markets*** (those traded on the national market) and ***Eurobonds and Eurocurrency markets*** (those traded on external market). The international debt market can be further classified in terms of the international capital market for intermediate-term and long-term funds and an international money market for short-term funds.[1]

10.3 THE FOREIGN BOND MARKET

A foreign bond market refers to that market in which the bonds of issuers not domiciled in that country are sold and traded. For example, the bonds of a German company issued in the United States or traded in the U.S. secondary market would be part of the U.S. foreign bond market. Foreign bonds are sold in the currency of the local economy. They are also subject to the regulations governing all securities traded in the national market and sometimes to special regulations and disclosure requirements governing foreign borrowers.[2]

Foreign bonds have been issued and traded on national markets for centuries. For example, U.S. bonds sold in London in the 19th century financed a large proportion of the U.S. railroad system. In the United States, foreign bonds are referred to as ***Yankee bonds***; in Japan, they are called ***Samurai bonds***; in Spain they are called ***Matador bonds***; in the United Kingdom, they are nicknamed ***Bulldog bonds***; and in the Netherlands they are called ***Rembrandt bonds***. In the United States, Yankee bonds are registered with the SEC, and like other U.S. bonds, they typically pay interest semiannually. The Yankee market tends to be dominated by sovereign government issues or issues guaranteed by sovereign governments. With the growth in the Eurobond market, the Yankee market did experience declines in the1980s, but has since rebounded.

10.4 THE EUROBOND MARKET

A Eurobond is a bond issued outside the country in whose currency it is denominated. For example, a U.S. company might sell a bond denominated in U.S. dollars throughout Europe. Eurobonds are very popular debt instruments. Many multinational corporations finance many of their global operations by selling Eurobonds. Eurobonds are also a source of intermediate and long-term financing of sovereign governments and supranationals (e.g., the World Bank). Russia, for example, raised $4 billion in 1997 through the sale of Eurobonds. Currently, about 80% of the new issues in the international bond market are Eurobonds. In fact, the Eurobond market currently exceeds in size the U.S. bond market as a source of new funds.

Origin

In the 1950s and early 1960s, New York was the most accessible market for corporations to raise capital. As a result, many foreign companies issued dollar-denominated bonds in the United States. The popularity of the Yankee bond market began to decline starting in 1963 when the U.S. government imposed the Interest Equalization

Tax (IET) on foreign securities purchased by U.S. investors. The tax was aimed at reducing the interest-rate difference between higher yielding foreign bonds and lower yielding U.S. bonds. Predictably, it led to a decline in the Yankee bond market. It also contributed, though, to the development of the Euobond market as more foreign borrowers began selling dollar-denominated bonds outside the United States. The IET was repealed in 1974.

The Eurobond market also benefited in the 1970s from a U.S. foreign withholding tax that imposed a 30% tax on interest payments made by U.S. firms to foreign investors. There was a tax treaty, though, that exempted the withholding tax on interest payments from any Netherlands Antilles subsidiary of a U.S. incorporated company to non-U.S. investors. This tax-treaty led to many U.S. firms issuing dollar-denominated bonds in the Eurobond market through financial subsidiaries in the Netherlands Antilles. During this time, Germany also imposed a withholding tax on German DM-denominated bonds held by nonresidents. Even though the U.S and other countries with withholding taxes granted tax credits to their residents when they paid foreign taxes on the incomes from foreign security holdings, the tax treatments were not always equivalent. In addition, many tax-free investors, such as pension funds, could not take advantage of the credit (or could, but only after complying with costly filing regulations). As a result, during the 1970s and early 1980s, Eurobonds were often more attractive to foreign investors and borrowers than foreign bonds. The growth of the Eurobond market was also aided in the late 1970s by the investments of oil-exporting countries that had large dollar surpluses. From 1963 to 1984, the Eurobond market grew from a $75 million market with a total of seven Eurobonds issues to an $80 billion market with issuers that included major corporations, supranationals, and governments. In 1984, the United States and Germany rescinded their withholding tax laws on foreign investments and a number of other countries followed their lead by eliminating or relaxing their tax. Even with this trend, though, the Eurobond market had already been established and would continue to remain a very active market, growing from an $80 billion market in new issues in 1984 to a $525 billion market by 1990 and to a $1.4 trillion market by 1998.

Underwriting Process

The Eurobond market is handled through a multinational syndicate consisting of international banks, brokers, and dealers.[3] A corporation or government wanting to issue a Eurobond will usually contact a multinational bank that will form a syndicate of other banks, dealers, and brokers from different countries.[4] The members of the syndicate usually agree to underwrite a portion of the issue, which they usually sell to other banks, brokers, and dealers. The multinational makeup of the syndicate allows the issue to be sold in many countries. Approximately one-third of the issues are for more than $300 million and one-fourth are for $100 million or less. The major investors in the market are institutional investors and corporations.

Market makers handle the secondary market for Eurobonds. Many of them are the same dealers that are part of syndicate that helped underwrite the issue. An investor who wants to buy or sell an existing Eurobond can usually contact several market makers in the international OTC market to get several bid-ask quotes before selecting the best one. Although most secondary trading of Eurobonds occurs in the

OTC market, many Eurobonds are listed on organized exchanges in Luxembourg, London, and Zurich. These listings are done primarily to accommodate investors from countries that prohibit (or at one time did prohibit) institutional investors from acquiring securities that are not listed. In the early years of the Eurobond market, clearing was done by physical delivery of the securities and payment was by check. In the late 1960s, two clearing firms were established: Euroclear and Cedel. These firms consisted of international banks and investment companies and they were linked to other clearing systems.

Characteristics

Currency Denomination The generic Eurobond is a straight bond paying an annual fixed interest and having an intermediate- or long-term maturity.[5] There are several different currencies in which Eurobonds are sold with the major currency denominations being the U.S. dollar, yen, and the euro.[6] Companies tend to denominate their bonds in those currencies they receive from operations. Dollar-denominated Eurobonds are the largest currency segment, currently comprising about 50% of the market. Some Eurobonds are also valued in terms of a portfolio of currencies, sometimes referred to as a currency cocktail.

Credit Risk Compared to U.S. corporate bonds, Eurobonds have fewer protective covenants, making them an attractive financing instrument to corporations, but riskier to bond investors. Eurobonds differ in term of their default risk and are rated in terms of quality ratings.

Maturities The maturities on Eurobonds vary. Euro notes have intermediate terms (two to 10 years) and Eurobonds have long terms (10 to 30 years). There is also short-term Europaper or Euro CP. Like some CP issues, Europaper issues, as well as some Euro notes, are often secured by lines of credit. The credit lines are sometimes set up through ***note issuance facilities (NIFs)*** of international banks, also called ***revolving underwriting facilities (RUFS)***. These facilities provide credit lines in which borrowers can obtain funds up to a maximum amount by issuing short-term and intermediate-term paper over the term of the line.

Euro-Medium Term Notes There is a growing market for ***Euro-medium term notes*** (***Euro-MTNs)***. Like regular MTNs, they are offered to investors as a series of notes with different maturities. In addition, Euro-MTN programs also offer different currencies and are not subject to national regulations. They are sold though international syndicates and also through offshore trusts (offshore centers are discussed later) set up by banks, investment banks, and banking groups.

Nonregistered One feature of Eurobonds that has served to differentiate them from U.S. bonds is that many are issued as bearer bonds.[7] Although this feature of Eurobonds provides confidentiality, it has created some problems in countries such as the United States, where regulations require that security owners be registered on the books of issuers. However, to accommodate U.S. investors, the SEC allows U.S. investors to purchase these bonds after they are "seasoned" (sold for a period of time). Thus, U.S. investors are locked out of initial offerings of Eurobonds, but are active in acquiring them in the secondary market. The fact that U.S. investors are locked out of the primary market does not affect U.S. borrowers from issuing Eurobonds. In 1984, U.S. corporations were allowed to issue bearer bonds directly to non-U.S. investors, another factor that contributed to the growth of this market.

Other Features Like many securities issued today, Eurobonds often are sold with many innovative features. There are ***dual-currency Eurobonds***, for example, that pay the coupon interest in one currency and the principal in another, and ***option-currency Eurobonds*** that offer investors a choice of currency. A sterling/Canadian dollar bond, for instance, gives the holder the right to receive interest and principal in either currency. A number of Eurobonds have special conversion features or warrants attached to them. One type of convertible is a dual-currency bond that allows the holder to convert the bond into stock or another bond that is denominated in another currency. For example, the Toshiba Corporation sold a bond denominated in Swiss francs that could be converted into shares of Toshiba stock at a set yen/SF exchange rate. Some of the warrants sold with Eurobonds include those giving the holder the right to buy stock, additional bonds, currency, or gold. There are also floating-rate Eurobonds with rates often tied to the LIBOR and floaters with the rate capped. The Eurobond market has also issued zero-discount bonds, and at one time, the market offered perpetual Eurobonds with no maturities. Finally, there are ***step-up*** and ***step-down bonds***. These bonds have coupon rates that adjust to changes in quality ratings: a rating upgraded results in a lowering of the coupon rate and a downgrade triggers an increase in the rate.

Global Bonds

A global bond is both a foreign bond and a Eurobond. Specifically, it is issued and traded as a foreign bond (being registered in a country) and also it is sold through a Eurobond syndicate as a Eurobond. The first global bond issued was a 10-year, $1.5 billion bond issue sold by the World Bank in 1989. This bond was registered and sold in the United States (Yankee bond) and also in the Eurobond market. Currently, U.S. borrowers dominate the global bond market, with an increasing number of these borrowers being U.S. federal agencies.

10.5 NON-U.S. DOMESTIC BONDS

Bonds sold in a national market by companies, agencies, or intermediaries domiciled in that country are referred to as domestic bonds. In the preceding chapters many of the domestic bonds traded in the United States were examined. There are, of course, many countries whose corporations, governments, and financial institutions offer bonds that are attractive to U.S. and other foreign investors. For foreign investors, usually the most important factor for them to consider is that their price, interest payments, and principal are denominated in a different currency. This currency component exposes them to exchange-rate risk and affects their returns and overall risk.

Foreign investors who buy domestic bonds also will find differences from country to country in how the bonds are issued and regulated. In a number of countries, banks, instead of investment bankers, underwrite new bonds. In Germany, for example, there has been a long history of no separation between commercial and investment banking. Many banks in Germany have acted as security underwriters and as brokers and dealers in the secondary market, trading existing bonds and stocks through an interbank market. In Europe, though, the ***Single European Act*** did permit banks and financial institutions in the European Economic Community (EEC), as well as those entering the EEC, to offer a wide variety of the same banking and security services. This act has led to standardization in the EEC. Japan, like the United States until 2000, had a history of separating its commercial and investment banking activities. In Japan, brokerage houses such as Nikko, Normura, and Yamaichi broker and underwrite bonds and other securities. In the secondary market, some countries trade bonds exclusively on exchanges, whereas others, such as the United States, Japan, and the United Kingdom, trade bonds on both the exchanges and through market makers on an OTC market.

Bonds sold in different countries also differ in terms of whether they are sold as either registered bonds or bearer bonds. A foreign investor buying a domestic bond may also be subject to special restrictions. These can include special registrations, exchange controls, and foreign withholding taxes. Finally, domestic bonds in other countries differ in their innovations. For example, the British government issues a bond, also referred to as a ***gilt***, that has a short-term maturity that can be converted to a bond with a longer maturity. They also issue a gilt that does not mature, although it can be redeemed after a specified date.

Non-U.S. Government Bond Markets

The largest government bond markets after the United States and the United Kingdom are the markets in Japan followed by Germany, France, and Italy. Japanese government bonds (JGBs) include intermediate and long-term bonds; ***German government bonds (Bunds)*** have original maturities from eight to 30 years, and their notes (***Bobls***) have original maturities of five years; ***French government long-term bonds (OATs)*** have original maturities of 30 years and their notes (***BTAN***) have original maturities of between two and five years; Italian government issues include a variety of securities: five-, 10-, and 30-year fixed-rate and seven-year floating-rates, and two-year zero coupon issues. Like the U.S. government's Treasury Inflation Protection Securities (TIPS), sovereign governments also issue inflation-linked bonds, referred to as ***linkers***.

European Covered Bond Market

Residential and commercial mortgage loans and some public sector loans made by European banks are often packaged and used as the collateral for the issuance of ***covered bonds***. Like securitized assets, covered bond investors have a claim on the underlying collateral, referred to as the ***cover pool***. In creating covered bonds, though, the bank that originates the loans will hold them unless it becomes insolvent. If insolvency occurs, then the mortgage loan and commercial loan assets are separated from the other assets of the bank. In contrast, the securitization of residential and commercial mortgages in the United States often involves the bank selling the loans to a special purpose vehicle (SPV) when the mortgage-backed securities are created;

this separates the loans from the bank's balance sheet at the beginning and not when there is insolvency. The SPV will then issue the MBS securities. Moreover, covered bond holders, unlike MBS holders, have recourse to the issuing bank. Covered bonds also differ from MBSs in that the composition of the covered pool can change over time, whereas the portfolio of mortgages backing an MBS issue is fixed. Finally, covered bonds have single maturity dates, whereas MBSs often have tranches with different maturities, and covered bonds are issued in different currencies.

The covered bond market is one of the largest sectors of the European financial market. Currently, there are 20 European countries with a covered bond market. The largest of these markets is the German mortgage-backed market—the ***Pfandbriefe market***. In 2007, Bank of America issued the first covered bond in the United States.

10.6 EMERGING MARKET DEBT

Over the last two decades, ***emerging market debt*** has become a popular addition to global bond portfolios. Emerging markets include Latin America, Eastern Europe, Russia, and a number of Asian countries, and their sovereign debt includes Eurobonds, bonds they offer and trade domestically, performing loans that are tradable, and Brady bonds (sovereign bonds issued in exchange for rescheduled bank loans). The opening of markets and the privatization of companies in Russia and Eastern Europe along with the economic reforms in Latin America have enhanced the profit potential of many emerging economies and with that the expected rates of return on their securities. At the same time, such debt is subject to considerable risk. Much of the risk germane to emerging market securities comes from concerns over changes in political, social, and economic conditions (referred to as ***cross-border risk***) and ***sovereign risk*** in which the government is unable, or in some cases unwilling, to service its debt. Some of the more recent sovereign debt crises of note occurred in Latin America in the 1980s, Venezuela in 1994, East Asia in 1994, Mexico in 1995, and Russia in 1998.

One of the more popular emerging debt securities is the ***Brady bond***. Named after U.S. Treasury Secretary Nicholas Brady, these bonds were issued by a number of emerging countries in exchange for rescheduled bank loans. The bonds were part of a U.S. government program started in 1989 to address the Latin American debt crisis of the 1980s. The plan allowed debtor countries to exchange their defaulted bank debt for Brady bonds or restructured loans at lower rates. In return for this debt relief, the countries agreed to accept economic reforms proposed by the International Monetary Funds. Although there was some variation, the basic Brady plan offered creditor banks two choices for the nonperforming loans of emerging countries they were carrying: (1) a discount bond issued below par (e.g., 50% or 65% of par) in exchange for the original loan or a discount bond paying a floating rate tied to the LIBOR in exchange for fewer bonds than the original loan; (2) a bond issued at par and paying a below-market coupon in exchange for the original face value of the loan. The principal on a Brady bond was secured by U.S. Treasury securities (initial bonds were secured by special zero-coupon Treasuries and later ones by Treasury strips), and the interest was backed by investment-grade bonds, with the guarantee rolled forward from one interest payment to the next if the collateral was not used (this is known as a rolling interest guarantee). All Brady bonds were callable and some gave bondholders a value recovery option, giving them the right to recover some of the debt if certain events occurred such as an increase in gross domestic product or energy prices.

The first country to accept a Brady plan was Mexico, who used it in 1989 to restructure its approximately $50 billion in foreign debt to commercial banks.[8] Seventeen countries with significant debt repayment problems took advantage of the Brady plan. As of 1999, the total Brady debt was approximately $114 billion, with Brazil, Mexico, Venezuela, and Argentina accounting for approximately 73% of the debt. When they were introduced, the initial holders of Brady bonds were the creditor banks. With the principal and interest guarantees and the potentially high returns, the bonds were attractive investments to hedge funds, global bond funds, growth funds, and emerging market funds. As a result, many banks sold their Brady bonds to non-bank institutional investors who, in turn, became some of the primary holders.[9]

Today, emerging market government debt outstanding is over $450 billion, of which Brady bond debt still outstanding is about $45 billion. A significant proportion of the emerging market government debt is denominated in dollars and issued as Eurobonds or global bonds.

10.7 GLOBAL FIXED-INCOME MUTUAL FUNDS AND ETFs

In Chapter 9, we examined mutual funds and the growing market for exchange-traded funds. Not surprising, there are a number of mutual funds with a global focus. There is also an increasing number of equity, commodity, and fixed-income ETFs and exchange-traded products being offered outside the United States, as well as a number of U.S. ETFs that are tied to foreign bond market indexes. The fixed-income exchange-traded products vary from ETFs that are tied to foreign bond indexes to those linked to emerging markets. Exhibit 10.1 gives a sample of international mutual funds downloaded from Yahoo! and Table 10.1 shows a partial listing of global ETFs accessed from Bloomberg.

10.8 FOREIGN BOND RISK AND SOVEREIGN BOND RATINGS

The credit analysis of foreign bonds issued by corporations and sovereign governments needs to take into account the same issues of any bond (fundamental ratio analysis, financial soundness, industry analysis, and indentures examination). In addition, the analysis also needs to consider ***cross-border risk***: risk due to changes in political, social, and economic conditions in countries where the bonds are issued or where the company is incorporated. In the case of sovereign foreign debt, especially the debt of emerging markets, analysis needs to include an examination of sovereign risk: the risk that the government is unable or unwilling (due to political changes) to service its debt.[10] Some of the key areas of inquiry in a credit analysis of sovereign or private issuers of debt from an emerging market country relate to the following fundamental issues:

- Size and diversification of the country's exports. Countries that specialize in exporting only a few products may be more susceptible to recessions.

EXHIBIT 10.1 International Fixed-Income Investment Funds

Dryden Global Total Return A
The investment seeks total return made up of current income and capital appreciation. The fund invests at least 65% of total assets in income-producing debt securities of U.S. and foreign corporations and governments, supranational organizations, semigovernmental entities or government agencies, authorities or instrumentalities, investment-grade U.S. or foreign mortgages and mortgage-related securities, and U.S. or foreign short-term and long-term bank debt securities or bank deposits.

Evergreen International Bond A
The investment seeks capital appreciation and current income. The fund invests at least 80% of assets in debt securities including obligations of foreign governments or corporate entities or supranational agencies denominated in various currencies. It invests up to 35% of assets in the debt securities below investment-grade. The fund invests in at least three countries or supranational agencies. It invests up to 5% of assets in debt obligations or similar securities denominated in the currencies of developing countries.

GMO Currency Hedged Intl Bond III
The investment seeks total return. The fund invests in fixed-income securities included in the fund's benchmark and in securities with similar characteristics. It invests in global interest rate, currency, and emerging country debt markets. The fund invests at least 80% of assets in bond investments. It generally attempts to hedge at least 75% of its net foreign currency exposure into U.S. dollars.

Goldman Sachs Global Income A
The fund normally invests at least 80% of assets in fixed-income securities. It may invest more than 25% in the securities of corporate and governmental issuers located in Canada, Germany, Japan and the United Kingdom as well as in the securities of U.S issuers. The fund may also invest up to 10% of assets in emerging countries. It enters into foreign currency transactions. It must invest at least 30% of assets in securities dominated in U.S. dollars.

PIMCO Foreign Bond (USD-Hedged) B
The investment seeks maximum real return. The fund normally invests at least 80% of assets in fixed-income instruments that are tied to foreign countries, representing at least three foreign countries, which may be represented by forwards or derivatives such as options, future contracts, or swap agreements. It may invest all of its assets in derivative instruments, such as options, futures contracts, or swap agreements, or in mortgage- or asset-backed securities.

Pioneer Global Aggregate Bond C
The fund invests normally at least 80% of net assets in debt securities of issuers located throughout the world. It invests at least 40% of net assets in issuers located outside of the United States. The fund may invest up to 20% of net assets in debt securities rated below investment grade or, if unrated, are of equivalent credit quality as determined by Pioneer.

BlackRock Emerging Market Debt BlackRock
The investment seeks maximum long-term total return. The fund invests primarily in a global portfolio of fixed-income securities and derivatives of any maturity of issuers located in emerging markets that may be denominated in any currency (on a hedged or unhedged basis). The fund normally invests at least 80% of its assets in fixed-income securities issued by governments or their political subdivisions.

(continued)

EXHIBIT 10.1 *(Continued)*

Federated International High Income C
The investment seeks a high level of current income and capital appreciation is a secondary objective. The fund primarily invests in emerging market debt securities. It may invest in securities of any duration and does not limit the amount it may invest in securities rated below investment grade. The fund may invest in derivative contracts. It may also purchase shares of exchange-traded funds .

Morgan Stanley Inst Emerg Mkts Debt H
The investment seeks high total return. The fund invests primarily in fixed-income securities of government and government-related issuers and, to a lesser extent, of corporate issuers in emerging-market countries. It invests at least 80% of assets in debt securities of issuers located in emerging-market countries.

TCW Emerging Markets Income I
The investment seeks high total return from current income and capital appreciation. The fund invests at least 80% of assets in debt securities issued or guaranteed by companies, financial institutions and government entities in emerging market countries, or debt securities denominated in the currency of an emerging market country. It generally invests in at least four emerging market countries.

Source: Yahoo Fund Screener: http://screen.yahoo.com/funds.html.

- Political stability: Strength of the legal system, amount of unemployment, and distribution of wealth.
- History of meeting debt obligations.
- Balance of payments ratios: Country's total debt-to-export ratio.
- Economic factors: Inflation, growth in gross domestic product, interest rates, and unemployment.
- Susceptibility of the country's economy and exports to changes in economic conditions in industrialized countries.

Moody's, Standard & Poor's, and Fitch assign quality ratings to sovereign debt. In evaluating a sovereign government bond issue, these rating agencies consider not only economic and cross-border risk, but also exchange-rate risk for foreign currency-defaulted bonds (e.g., dollar-denominated or euro-denominated bonds). The latter risk relates to the ability of a foreign government to purchase the foreign currency to meet its debt obligations. Table 10.2 shows Moody's historical cumulative default rates for sovereign debt, and Exhibit 10.2 lists a number of sovereign default case since 1983 along with Moody's summary comments about the cases.

10.9 EUROCURRENCY MARKET

The ***Eurocurrency market*** is the money market equivalent of the Eurobond market. It is a market in which funds are intermediated (deposited or loaned) outside the country of the currency in which the funds are denominated. For example, a certificate of deposit denominated in dollars offered by a subsidiary of a U.S. bank incorporated in the Bahamas is a Eurodollar CD. Similarly, a loan made in yen from

TABLE 10.1 International Fixed-Income ETFs

Name	Parent Company Name	Bloomberg Objective
CLAL FIN BAT-MABAT TEL BD 20	Clal Finance Batucha Investment Managment Ltd/Israel	Region Fund-Geo Focused-Debt
HAREL SAL TEL-BOND 40	Harel Finance	Sector Fund-Debt
CLAL FIN BAT-MABAT GILON	Clal Finance Batucha Investment Managment Ltd/Israel	Govt/Agency-Short/Intermed
CLAL FIN-GALIL 2-5	Clal Finance Batucha Investment Managment Ltd/Israel	Government/Corporate
GILON GOVERNMENT BONDS	Index Tracking Certificates Ltd	Government/Agency-not Maturity dependent
INDEX SAL-GALIL BONDS 2-5Y	Index Tracking Certificates Ltd	Government/Corporate
CLAL FIN BAT-MABAT TEL BD 60	Clal Finance Batucha Investment Managment Ltd/Israel	Region Fund-Geo Focused-Debt
CLAL FIN BAT-MABAT TEL BD 40	Clal Finance Batucha Investment Managment Ltd/Israel	Region Fund-Geo Focused-Debt
HADAS GALIL BONDS 2-5Y	Hadas Indices Ltd/Israel	Government/Corporate
CLAL FIN BAT-MABAT SH SH 5+	Clal Finance Batucha Investment Managment Ltd/Israel	Government/Corporate
ISHARES CDN DEX ALL CORPORAT	iShares/Canada	Corporate/Preferred-not Rating dependent
HAREL SAL TEL-BOND 20	Harel Finance	Sector Fund-Debt
HAREL SAL TEL-BOND 60	Harel Finance	Sector Fund-Debt
ISHARES EBREXX GOV GE DE	Barclays Global Investors Deutschland AG	Government/Corporate
DB X-TR IBX EUR SOV EUROZON	DB x-trackers II	Government/Agency-not Maturity dependent
ISHARES GLOBAL GOV BOND	Barclays Global Investors Ireland Ltd/Ireland	Government/Corporate
DB X-TR II TRX CROSSOVER 5 Y	DB x-trackers II	Govt/Agency-Intermediate Term
EASYETF ITRAXX EUROPE MAIN	EasyETF/France	Index Fund-Debt
ISHARES BAR EURO TREAS BD-€	Barclays Global Investors Ireland Ltd/Ireland	Region Fund-Geo Focused-Debt
ISHARES EBREXX GOVGO 2.5-5.5	Barclays Global Investors Deutschland AG	Government/Corporate
CASAM ETF EUROMTS BROAD 7-10	Credit Agricole Asset Management-ETF/France	Govt/Agency-Intermediate Term
ETFLAB DB EUROGOV GERMAN 1-3	ETFlab Investment GmbH/Germany	Govt/Agency-Short Term

(continued)

TABLE 10.1 *(Continued)*

Name	Parent Company Name	Bloomberg Objective
ISHARES EBREXX GVG 5.5-10.5	Barclays Global Investors Deutschland AG	Government/Corporate
ETFLAB IBOXX LIQUID 1-3	ETFlab Investment GmbH/Germany	Government/Corporate
ETFLAB IBOXX LIQUID 3-5	ETFlab Investment GmbH/Germany	Government/Corporate
CASAM ETF EUROMTS BROAD 1-3	Credit Agricole Asset Management-ETF/France	Govt/Agency-Short Term
ISHARES BAR EUR GOV BD 5-7-€	Barclays Global Investors Ireland Ltd/Ireland	Region Fund-Geo Focused-Debt
ISHARES BARC EU BD EX F1-5 £	Barclays Global Investors Ireland Ltd/Ireland	Corporate/Preferred-not Rating dependent
ETFLAB DB EUROGOV GERMAN 10+	ETFlab Investment GmbH/Germany	Govt/Agency-Long Term
VENEZUELAN DOMESTIC DEBT FND	SEQUOIAN Sociedad de corretaje de valores CA/Venezuela	Govt/Agency-Short Term

Source: Used with permission of Bloomberg Finance LP. Bloomberg Copyright Clearance Center, 2009.

a bank located in the United States would be an American-yen loan. In both cases, the Eurodollar deposit and the American-yen loan represent intermediation occurring in the Eurocurrency market. Even though the intermediation occurs in many cases outside Europe, the Euro prefix usually remains. An exception is the Asian dollar market. This market includes banks in Asia that accept deposits and make loans in foreign currency; this market is sometimes referred to separately as the Asian dollar market.

Today the total amount of Eurocurrency deposits is estimated to be in excess of $3 trillion. The actual size of the market, though, is difficult to determine because of the lack of regulation and disclosure. By most accounts, though, it is one of the largest financial markets. The underlying reason for this is that Eurocurrency loan and deposit rates are often better than the rates on similar domestic loans and deposits because of the differences that exist in banking and security laws among countries. Foreign lending or borrowing, regardless of what currency it is denominated in and what country the lender or borrower is from, is subject to the rules, laws, and customs of the foreign country where the deposits or loans are made. Thus, a U.S. bank offering a CD through its foreign subsidiary located in the Bahamas (maybe in the form of a P.O. box) would be subject to the Bahamian laws with respect to reserve requirements, taxes on deposits, anonymity of the depositor, and the like. Accordingly, if a country's banking laws are less restrictive, then it is possible for a foreign bank or a foreign subsidiary of a bank to offer more favorable rates on its loans and deposits than it could in its own country by simply intermediating the deposits and loans in that country.[11] Thus, the absence of reserve requirements or regulations on rates paid on deposits in the Bahamas, for example, makes it possible

TABLE 10.2 Sovereign Default Rate: Moody's Historical Cumulative Default Rates, 1983–2007

Year	1	2	3	4	5	6	7	8	9	10
Aaa	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Aa	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A	0.00%	0.52%	1.09%	1.73%	2.44%	3.20%	3.20%	3.20%	3.20%	3.20%
Baa	0.00%	0.52%	1.09%	1.73%	2.44%	3.20%	3.20%	3.20%	3.20%	3.20%
Ba	0.89%	1.95%	3.78%	5.86%	8.13%	9.80%	12.01%	14.49%	16.49%	18.42%
B	2.80%	5.77%	6.90%	8.72%	10.51%	12.68%	14.50%	16.07%	18.08%	20.83%
Caa-C	22.54%	26.79%	32.42%	32.42%	32.42%	32.42%	32.42%	32.42%	32.42%	32.42%
Investment	0.00%	0.11%	0.22%	0.34%	0.48%	0.62%	0.62%	0.62%	0.62%	0.62%
Speculative	2.65%	4.64%	6.36%	8.25%	10.23%	12.01%	13.99%	16.06%	17.97%	20.05%
All	0.78%	1.43%	2.01%	2.63%	3.28%	3.86%	4.39%	4.91%	5.37%	5.82%

Source: Moody's, www.moodys.com.

EXHIBIT 10.2 Moody's Rated Sovereign Bond Default Cases since 1983: Default Date, Country, Total Defaulted, Debt ($ millions), and Moody's Comments

- Jul-98, Venezuela, $270: Default on domestic currency bonds in 1998, although the default was cured within a short period of time.
- Aug-98, Russia, $72,709: Missed payments first on local currency Treasury obligations. Later a debt service moratorium was extended to foreign currency obligations issued in Russia but mostly held by foreign investors. Subsequently, failed to pay principal on MINFIN III foreign currency bonds. Debts were restructured in Aug 1999 and Feb 2000.
- Sep-98, Ukraine, $1,271: Moratorium on debt service for bearer bonds owned by anonymous entities. Only those entities willing to identify themselves and convert to local currency accounts were eligible for debt repayments, which amounted to a distressed exchange.
- Jul-99, Pakistan, $1,627: Pakistan missed an interest payment in Nov 1998 but cured the default subsequently within the grace period (within four days). Shortly thereafter, it defaulted again and resolved that default via a distressed exchange which was completed in 1999.
- Sep-00, Peru, $4,870: Peru missed payment on its Brady bonds but subsequently paid approximately $80 million in interest payments to cure the default, within a 30-day period.
- Nov-01, Argentina, $82,268: Declared it would miss payment on foreign debt in November 2001. Actual payment missed on Jan 3, 2002. Debt was restructured through a distressed exchange offering where the bondholders received haircuts of approximately 70%.
- Jun-02, Moldova, $145: Missed payment on the bond in June 2001 but cured default shortly thereafter. Afterwards, it began gradually buying back its bonds, but in June 2002, after having bought back about 50% of its bonds, it defaulted again on remaining $70 million of its outstanding issue.
- May-03, Uruguay, $5,744: Contagion from Argentina debt crisis in 2001 led to a currency crisis in Uruguay. To restore debt-sustainability, Uruguay completed a distressed exchange with bondholders that led to extension of maturity by five years.
- Apr-05, Dominican Republic, $1,622: After several grace period defaults (missed payments cured within the grace period), the country executed an exchange offer in which old bonds were swapped for new bonds with a five-year maturity extension, but the same coupon and principal.
- Dec-06, Belize, $242: Belize announced a distressed exchange of its external bonds for new bonds due in 2029 with a face value of $546.8. The new bonds are denominated in U.S. dollars and provide for step-up coupons that have been set at 4.25% per annum for the first three years after issuance. When the collective action clause in one of Belize's existing bonds is taken into account, the total amount covered by this financial restructuring represents 98.1% of the eligible claim

Source: © Moody's Investors Service, Inc. and/or its affiliates. Reprinted with permission. All Rights Reserved. *Source:* Moody's: www.Moodys.com: www.moodys.com/moodys/cust/research/MDCdocs/17/2007100000482445.pdf?frameOfRef=corporate.

for the rates on Bahamian Eurodollar loans to be lower than U.S. bank loans and the rates on their deposits to be higher.

Brief History of the Eurocurrency Market

The origin of the Eurocurrency market is more political than economic. It started in the 1950s when the Soviet Union maintained large dollar deposits in banks in the United States in order to participate in world trade. However, poor political relations, as well as U.S. claims on the Soviet Union originating from the Lend-Lease Policy, led to fears by the Soviet Union that the U.S. government could expropriate their deposits. As a result, the USSR, with the aid of some U.S. banks, transferred their dollar deposits to banks in Paris and London, thus creating the first modern-day Eurodollar deposit. Subsequently, the increase in international trade, the rise of multinationals, the emergence of the dollar as an international reserve currency under the old Bretton Woods exchange rate system, and the policy of some governments to maintain dollar deposits led to a substantial increase in the amount of Eurodollar deposits abroad during the 1960s.

During the early 1960s, most of the Eurodollar deposits were in foreign banks that, in turn, used the deposits to make dollar loans to many U.S. companies, directly competing with U.S. banks. In 1963 there were only a few U.S. banks with foreign operations in Europe, and these banks were there principally to facilitate their corporate customers' international business. In the mid-1960s, though, U.S. banks began to go after Eurodollar deposits and loans by establishing foreign subsidiaries. What brought the American banks to Europe en masse was probably not so much the loss of business to European banks as it was the opportunity around Federal Reserve regulations. Specifically, with lower or no reserve requirements and no regulations governing the maximum rates payable on time deposits, U.S. banks, by offering Eurodollar deposits and loans, now had a way of offering their customers better rates on loans and higher rates on time deposits. By 1969 there were an estimated 40 American banks with branches abroad, lending approximately $14 billion in Eurodollars. This market, in turn, grew, despite a crisis in 1973, to a $270 billion market in 1974.

In the late 1970s, many oil-exporting countries used the Eurodollar market, depositing large dollar deposits. Some of these petrodollars were used to make loans to oil-importing counties, leading the dollar deposits from oil revenues to be recycled. By the 1980s, the Eurodollar market had become the second largest market in the world, extending beyond Europe and intermediating in currencies other than the dollar. Accordingly, the market gave rise to the offshore banking centers in such areas as Nassau, Singapore, Luxembourg, and Kuwait. These areas had less-restrictive banking laws and thus became a place for intermediation between both foreign lenders and foreign borrowers.

Current Market

Currently, the Eurocurrency market consists of a number of large banks, referred to as Eurobanks, corporations, and governments. The Eurobanks are the foundation of the market, offering various types of loans and deposits. Governments and companies use the market to deposit currencies, as well as to obtain loans to finance

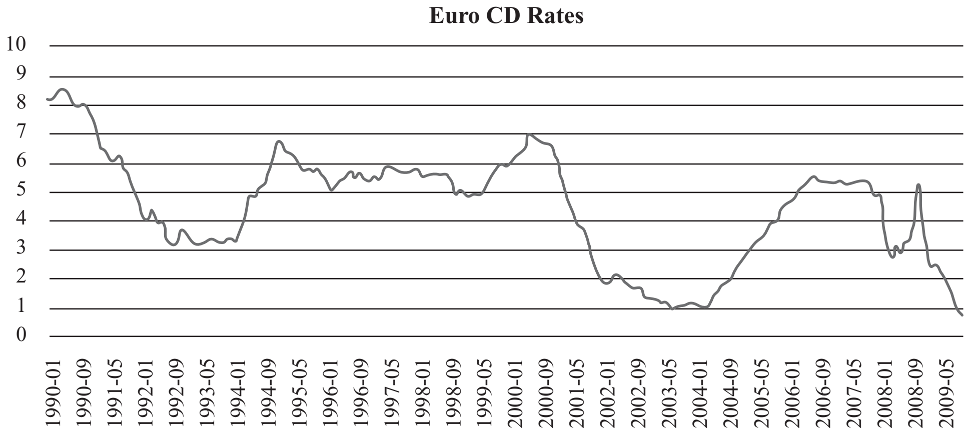

FIGURE 10.1 Six-Month Euro CD Rate, 1990–2009
Source: Federal Reserve: www.federalreserve.gov/releases/h15/data.htm.

assets, infrastructures, and even balance of payment deficits. Eurocurrency market transactions are large deposits and loans, usually $1million or more.

For large investors, the market offers two types of instruments: Eurocurrency CDs and primary deposits. As noted previously, because of favorable regulations in the offshore centers, the rates on the CDs are usually higher than comparable domestic CDs (see Figure 10.1).

The maturities on Eurocurrency CDs range from one day to several years, with the most common maturities being one, three, six and 12 months. For longer term CDs, the rates can be either fixed or variable. Eurocurrency CDs can also take the form of ***tap CDs***; these are CDs issued in single amounts to finance a specific Eurodollar loan. Also there exist the ***tranche CDs***, which are of a smaller denomination ($10,000), often offered to the public through a broker or an underwriter.

Primary deposits are time deposits with negotiated rates and short-term maturities. Once deposited, these deposits, in turn, are often sold and bought as part of the Eurocurrency's interbank deposit network. In the interbank market, many deposits are bought by large Eurobanks who use the proceeds to make large short-term loans. For example, suppose the ABC Company of Cincinnati wanted to invest $20 million excess cash from its operations for 30 days in the Nassau branch of the Midwest Bank of Cincinnati. To initiate this, the treasurer of ABC would call Midwest Bank's Eurocurrency trader in Cincinnati to get a quote on its Nassau bank's 30-day deposit rate. (Note, the Nassau branch of the Midwest bank is likely to be staffed in Cincinnati, with its physical presence in Nassau being nothing more than the Nassau incorporation papers and other documents in a lawyer's office.) The treasurer would give ABC a quote based on similar Eurocurrency rates. If acceptable, a 30-day Eurocurrency deposit would be created with ABC transferring its cash account to Midwest Bank, who would set up the Nassau account by recording it in its Nassau books in Cincinnati. Now unless the bank has an immediate need for the $20 million, the trader would invest the funds through the interbank market. This might involve selling the deposit to a London Eurobank who might be arranging a 30-day, $100 million loan to a Japanese company to finance its inventory of computer equipment purchased in New York.[12] If Midwest Bank and the Eurobank agree, then Midwest would transfer the $20 million Eurodollar deposit to the London Eurobank.

The rate paid on funds purchased by large London Eurobanks in the interbank market is called the ***London interbank bid rate (LIBID)***, whereas the rate on funds offered for sale by London Eurobanks is the London interbank offer rate (LIBOR). As pointed out earlier, the average LIBOR among London Eurobanks is a rate commonly used to set the rate on bank loans, deposits, and floating-rate notes and loans. The LIBOR can vary from overnight rates to 30-day ones. There are also similar rates for other currencies (e.g., Sterling LIBOR) and areas (e.g., Paris interbank offer rate, PIBOR, or Singapore interbank offer rate, SIBOR).

In addition to being an important source of funds for banks, the Eurocurrency market is also an important funding source for corporations and governments. The loans to corporations and governments can be either short or intermediate term, ranging in maturity from overnight to 10 years. The loans with maturities of over one year are sometimes called Eurocredits or Eurocredit loans. Since Euro deposits are short term, Eurobanks often offer Eurocredits with a floating rate tied to the LIBOR. Eurocurrency loans also vary is terms of some of their other features: Many take the form of lines of credit; some require an LOC instead of detailed covenants; loans can be either fixed or floating; loans can be in different currencies and have different currency clauses; many of the larger loans are provided by a syndicate of Eurobanks.

10.10 THE FOREIGN CURRENCY MARKET AND EXCHANGE-RATE RISK

When investors buy securities in the external market, they are subject to the same risk incurred when they buy domestically. They are also exposed to two additional risk factors—political risk and exchange-rate risk. ***Political risk*** is the uncertainty of converting international security holdings on unfavorable economic terms because of an unexpected change in laws or customs. For investors, political risk can result from a government placing restrictions on currency conversion, freezing assets, or imposing taxes. Such risk is greater in developing countries with unstable political regimes. ***Exchange-rate risk*** is the uncertainty over changes in the exchange rate. When an investor purchases a foreign security, she usually has to buy foreign currency, and when she receives income and principal from the security, or sells it, she usually has to convert the proceeds back to her own currency. Such investments are subject to the possibility that the exchange rates may change in the foreign currency market. Similarly, when a corporation sells a security or borrows funds in a currency that must be converted to finance operations in a different country, the company is subject to exchange-rate risk.

The Foreign Currency Market

The international buying and selling of goods, services, and assets creates a market in which individuals, businesses, and governments trade currencies. Most of the currency trading takes place in the ***interbank foreign exchange market***. This market consists primarily of major banks that act as currency dealers, maintaining inventories of foreign currencies to sell to or buy from their customers (corporations,

TABLE 10.3 Select Spot and Forward Quotes, October 28, 2009

Country/Currency	Close 9/28/09 in US$	Close 9/27/09 in US$
Canada dollar	0.9272	0.938
1-mos forward	0.9272	0.938
3-mos forward	0.9272	0.9379
6-mos forward	0.927	0.9375
Japan yen	0.011014	0.010883
1-mos forward	0.011015	0.01089
3-mos forward	0.01102	0.01089
6-mos forward	0.011028	0.0109
Switzerland franc	0.9744	0.978
1-mos forward	0.9746	0.978
3-mos forward	0.975	0.9785
6-mos forward	0.9757	0.9793
UK pound	1.6377	1.6373
1-mos forward	1.6374	1.637
3-mos forward	1.6369	1.6365
6-mos forward	1.6364	1.636
Euro	1.4717	1.4791
Australian Dollar	0.8872	0.9146
China yuan	0.1465	0.1464

Source: Wall Street Journal, http://online.wsj.com/public/us.

governments, or regional banks). The banks are linked by a sophisticated telecommunication system and operate by maintaining accounts with each other, enabling them to trade currency simply by changing computerized book entries in each other's accounts.

Transactions occurring in the interbank foreign currency market can include both spot and forward trades. In the spot market, currencies are delivered immediately (book entries changed); in the forward market, the price for trading or exchanging the currencies (***forward rate***) is agreed upon in the present, with the actual delivery or exchange taking place at a specified future date. The price of foreign currency or the exchange rate is defined as the number of units of one currency that can be exchanged for one unit of another. Table 10.3 shows selected spot and forward exchange rates and Table 10.4 shows Chicago Mercantile futures rates downloaded from the *Wall Street Journal* Web site on October 29, 2009. The rates are quoted in terms of the U.S. dollar price per unit of foreign currency (FC). For example, on October 28, 2009 the spot exchange rate (E_0) for the British pound (BP) was \$1.6377/BP and the one-month and three-month forward exchange rates ($E^f{}_0$) were \$1.6374/BP and \$1.6369/BP.

Exchange-Rate Risk

Investors buying foreign securities in national or offshore markets denominated in foreign currency are subject to changes in exchange rates that, in turn, can affect the rates of return they can obtain from their investments. For example, suppose a U.S. investor bought a French corporation's zero-coupon bond denominated in euros that

TABLE 10.4 Currency Futures Quotes, Chicago Mercantile Exchange, October 28, 2009

Contract	Expiration Month	Last Futures Price ($/FC)	Open	High	Low
EURO	Dec '09	1.472	1.4719	1.4761	1.4681
EURO	Mar '10	1.4722	1.4717	1.4755	1.4692
BRITISH POUND	Dec '09	1.648	1.6377	1.6505	1.6334
BRITISH POUND	Mar '10	1.649	1.6406	1.649	1.6388
SWISS FRANC	Dec '09	0.9763	0.9745	0.9777	0.9723
SWISS FRANC	Mar '10	0.9766	0.9766	0.9766	0.9766
AUSTRALIAN DOLLAR	Dec '09	0.9029	0.8945	0.9035	0.8904
AUSTRALIAN DOLLAR	Mar '10	0.894	0.8867	0.8941	0.8833
JAPANESE YEN	Dec '09	1.0973	1.1029	1.1084	1.096
JAPANESE YEN	Mar '10	1.0988	1.1031	1.1049	1.0988
CANADIAN DOLLAR	Dec '09	0.9292	0.9259	0.9306	0.924
CANADIAN DOLLAR	Mar '10	0.9263	0.9247	0.93	0.9242

Source: Wall Street Journal, http://online.wsj.com/public/us.

promised to pay €1,000 one year later for €900 when both the $/€ spot and one-year forward exchange rates were $1.4717/€ (or €0.6795/$). The U.S. investor's total dollar investment would therefore be $1,324.53.

$$\text{Dollar investment} = (\$1.4717/€)(€900) = \$1,324.53$$

One year later, the U.S. investor would receive €1,000 principal on the maturing French bonds for an 11.11% rate of return in euros [= (€1,000 – €900)/€900]. The rate the investor would earn on his dollar investment, though, would depend on the spot $/€ exchange rate. For example, if the $/€ spot exchange rate decreased (a dollar appreciation) by 15% from $1.4717/€ to $1.250945/€, the investor would lose 5.5% in dollars:

$$\text{Rate} = \frac{(\$1.250945/€)(1,000€)}{\$1,324.53} - 1 = -.055$$

The example illustrates that when investors purchase foreign securities they must take into account not only the risk germane to the security, but also the risk that exchange rates will move to an unfavorable level. A similar admonishment applies to borrowers who sell securities or procure loans in currencies that are converted to finance operations in other currencies.

It should be noted that the forward exchange market makes it possible for investors to hedge their investments and loans against exchange-rate risk. In the above case, for example, suppose when she purchased the French bond, the U.S. investor had entered into a forward contract to sell 1,000 euros one year later at the forward rate of $1.4717/€. At the end of the year, the investor would be sure of converting €1,000 into $1,471.70. Thus, even if the $/€ spot rate fell by 15%, the investor would still be able to earn 11.11% [= $1,471.70/$1,324.53) – 1] from her dollar investment. Thus, by entering a forward contract to sell foreign currency the investor is able to profit from her bond investment.

As we will examine in Parts 4 and 5, there are other ways investors, as well as borrowers, can hedge against exchange risk (futures, options, and swaps). Using these tools, in turn, allows investors and borrowers to focus on the choice of securities and the type of funding.

In general, the decision to invest in either foreign or domestic securities depends on the rates earned on investments in different countries and the expectation of future exchange rates relative to the exchange rates that the investor can lock in using a futures or forward contract. An important and useful relationship to guide investors in such decisions is the interest rate-parity relation.

10.11 CONCLUSION

Just like the markets for corporate and government securities, the international debt security markets offer borrowers and investors a wide array of instruments for financing and investing: from short-term securities, such as Eurocurrency CDs to intermediate- and long-term instruments, such as foreign bonds, Eurobonds, global bonds, covered bonds, emerging market debt securities, and ETFs with a global exposure. These debt securities, along with the many different types of corporate and government securities, characterize a debt market of both depth and breadth.

WEB INFORMATION

- *Wall Street Journal* site
  Fixed-income ETF with a global strategy:
  - http://online.wsj.com/public/us
  - Click "Market Data" tab.
  - Click "ETF" tab.
  - Click "ETF Screener."
  - Fixed-Income

  Global Bonds
  - http://online.wsj.com/public/us
  - Click "Market Data" tab.
  - Click "Bond Rates and Credit Market."
  - Click "Global Government Bonds."

  Money Rates
  - http://online.wsj.com/public/us
  - Click "Market Data" tab.
  - Click "Bond Rates and Credit Market."
  - Click "Money Rate."

  Current Currency Quotes
  - http://online.wsj.com/public/us
  - Click "Market Data" tab.
  - Click "Currency" tab.

  Mutual Funds
  - Click "Mutual Fund" tab.
  - International Fund Screener

- European Covered Bond Council, European Covered Bond Facts Book: http://ecbc.hypo.org/Content/Default.asp?PageID=313.
- German Covered Bond Market: www.pfandbrief.org.
- Eurobonds, LIBOR, spot and forward exchange rates: www.fxstreet.com. Euro-denominated and yen-denominated bond yields can be found by going to "Rates and Charts," and "Bond Yields."
- British Bankers Association Site: BBA, LIBOR, and other information: www.bba.org.uk.
- LIBOR Rates: www.bbalibor.com.
- Information on how LIBOR is used for benchmarking: www.bbalibor.com/bba/jsp/polopoly.jsp?d=1627.
- Moody's : www.moodys.com
  - Search for "Rating Methodologies and Performance."
  - Historical Performance
  - Examine Sovereign Defaults
- Bank for International Settlement: www.bis.org.
- Brady debt and bonds: www.bradynet.com.
- Emerging markets: www.securities.com.
- Country statistics: www.worldbank.org/data.
- Information on world indexes, exchange rates, and other international data: www.bloomberg.com.
- FINRA foreign or global fixed-income mutual fund: www.finra.org/index.htm, "Sitemap," "Mutual Funds," and "Fixed Income."
- Information on global fixed-income mutual funds: Yahoo's "Advanced Fund Screener": http://screen.yahoo.com/funds.html.
- World indexes, exchange rates, and other international data: www.bloomberg.com.
- Historical trends in Euro CD rates: www.federalreserve.gov/releases/h15/data.htm
- Euro-denominated and yen-denominated bond yields can be found by going to www.fxstreet.com, "Rates and Charts," and "Bond Yields."
- Historical foreign exchange rates and balance of payments: www.research.stlouisfed.org/fred2.
- European Central Bank and European Monetary Union: www.euro.gov.uk, www.ecb.int, and www.cepr.org.
  - International Monetary Fund: www.imf.org.

KEY TERMS

Aztec bonds
Bobls
Brady bond
BTAN
bulldog bonds
cover pool
covered bonds
cross-border risk
domestic bonds
dual-currency Eurobonds

emerging market debt
Euro-medium term notes (Euro-MTNs)
Eurobonds
Eurocurrency market
exchange-rate risk
external bond market
foreign bonds
foreign bond and securities markets
forward rate
French government long-term bonds (OATs)
German government bonds (Bunds)
global bonds
gilt
interbank foreign exchange market
internal bond market
linkers
London interbank offer rate (LIBOR)
matador bonds
national market
note issuance facilities (NIFs)
offshore market
option-currency Eurobonds
Pfandbriefe market
political risk
Rembrandt bonds
revolving underwriting facilities (RUFs)
samurai bonds
Single European Act
sovereign risk
step-up and step-down bonds
tap CD
Tranche CD
Yankee bonds

PROBLEMS AND QUESTIONS

1. Define the following markets:
   a. Eurobond
   b. Foreign bond
   c. Internal market or national market
   d. External or offshore market
   e. Global bond
2. List some of the popular foreign bonds and their names.
3. Explain the significance of the Interest Equalization Tax and Foreign Withholding Tax in contributing to the growth of the Eurobond market.
4. Explain how Eurobonds are issued in the primary market through a syndicate.
5. Describe the secondary market for Eurobonds.
6. Explain the following features associated with Eurobonds:
   a. Currency denomination
   b. Nonregistered (implication for U.S. investors and issuers)
   c. Credit risk
   d. Maturities
   e. Dual currency clauses
   f. Option-currency clause
7. How does the U.S. securities law requiring the registration of bond investors apply to the issuing of nonregistered Eurobonds by corporations? How does the law apply to U.S. investors?
8. What are some differences foreign investors find when they buy domestic bonds?

9. Define the following:
   a. Emerging market debt
   b. Cross-border risk
   c. Sovereign risk
   d. Brady bonds
   e. Brady plan
10. Define the Eurocurrency market. What is the fundamental factor contributing to the growth of this market?
11. Comment on how the following events impacted the historical development of the Eurocurrency market:
    a. USSR
    b. Fixed exchange-rate system,
    c. U.S. banks circumventing U.S. bank laws
    d. Petrodollars
    e. Offshore centers
12. What is the interbank Eurocurrency market?
13. Define the London interbank bid rate (LIBID) and London interbank offer rate (LIBOR).
14. List some of the features that characterize Eurocurrency loans.
15. Describe the Interbank Foreign Exchange Market.

WEB EXERCISES

1. Study the historical trends in Eurodollar CD rates and compare them to U.S. CD rates. Go to www.federalreserve.gov/releases/h15/data.htm and download the historical yields on Eurodollar CDs and CDs to Excel.
2. Examine the impact of exchange rates on dollar investment returns. Calculate the dollar return you would earn by investing in a selected foreign-denominated bond:
   - Go to www.research.stlouisfed.org/fred2.
   - Click “Exchange Rates” tab, “By Country” tab.
   - Download data to Excel.
   - Example: For Euro series, click EXUSEU and then click “Download Data” to send to Excel.
3. Information on exchange rates, yields on Eurobonds, and interest rate actions by central banks can be found on www.fxstreet.com. At the site, examine the following:
   - Spot exchange rates
   - Forward rates
   - LIBOR
   - Other rates

4. Study some of the sovereign bonds that have been added to Moody's watch list:
   - Go to www.moodys.com.
   - Search for "WATCHLIST."
5. Moody's provides information on default rates, ratings changes, and other credit information. Examine some of their information as it is related to sovereign debt. To access Moody's study of historical default rates:
   - Go to www.moodys.com.
   - Search for "Rating Methodologies and Performance."
   - Historical Performance
   - Examine some of the following:
     - Sovereign cumulative default rate
     - Sovereign default cases
6. The London Interbank Offer Rate, LIBOR, is an important benchmark rate. It is used to determine the rates on many floating-rate bonds and loans. Find out more about the LIBOR by going to www.bba.org.uk.
7. Determine the recent LIBOR by going to www.bbalibor.com.
8. After the 2008 financial crisis in which investors saw defaults in subprime MBSs, analysts began discussing the merits of European covered bonds. Examine this security by going to Council of European Covered Bond Facts Book:
   - http://ecbc.hypo.org/Content/Default.asp?PageID=313.
9. Go to the FINRA site to find information on a foreign or global fixed-income mutual fund:
   - Go to www.finra.org/index.htm, "Sitemap," "Mutual Funds," and "Fixed Income."
   - Look for emerging market debt funds, global income funds, and global debt funds.
10. Go to Yahoo's Advanced Fund Screener to find information on global fixed-income mutual funds. Yahoo! Advanced Fund Screener: http://screen.yahoo.com/funds.html
11. Go to the *Wall Street Journal* site and examine foreign government bonds and their yields:
    - http://online.wsj.com/public/us
    - Click "Market Data" tab.
    - Click "Bond Rates and Credit Market."
    - Click "Global Government Bonds."
    - Click "Bond Rates and Credit Market."
    - Click "Money Rate."
12. Find the current money market rates such as the Euro CP rate and LIBOR by going to the *Wall Street Journal* site:
    - http://online.wsj.com/public/us
    - Click "Market Data" tab.
    - Click "Bond Rates and Credit Market."
    - Click "Money Rate."

13. Go to *Wall Street Journal* site and use their ETF Search Screener to find information on fixed-income ETF with a global strategy:

- http://online.wsj.com/public/us
- Click "Market Data" tab.
- Click "ETF" tab.
- Click "ETF Screener."
- Fixed-Income

14. Find the current currency quotes by going to the *Wall Street Journal* site:

- http://online.wsj.com/public/us
- Click "Market Data" tab.
- Click "Currency" tab.

NOTES

1. Grouping bonds in terms of trading blocs is also a popular way to classify bonds. Today the popular blocs are the dollar bloc (United States, Canada, Australia, and New Zealand), the European bloc (Euro zone market bloc and non-Euro zone bloc), the Japanese bloc, and the emerging markets bloc.
2. Possible restrictions imposed by regulatory authorities on foreign bonds relate to disclosure of information and reporting, the bond's structure, and minimum and maximum issue sizes. With the growth of the foreign bond market, many of these restrictions have been eliminated or relaxed.
3. Eurobonds issued with only one underwriter are becoming more common. Such issues are referred to as bought deals.
4. The underwriting of Eurobonds is subject to some underwriting risk. Often the lead or managing underwriter has to handle any issues not sold by second-tier and third-tier underwriters. The underwriting agreement specifies the terms of the issue, including the price before the issue is sold; this is referred to as a bought deal. The increased underwriting risk is often reflected by a bigger spread.
5. The bond-equivalent yield (BEY) for a bond with annual coupon payments is

$$BEY = 2[(1 + \text{Annual YTM})^{1/2} - 1]$$

6. The central bank of a country can protect its currency from being used. For example, Japan prohibited the yen from being used for Eurobond issues of its corporations until 1984.
7. Some Eurobonds, such as those issued by sovereign countries, are sold as registered bonds.
8. ***Aztec bonds*** preceded the Mexican Brady bonds. These bonds were created in 1988 to redress J. P. Morgan's nonperforming loans to Mexico. J. P. Morgan accepted 30% reduction in the face value amount of its loan in return for a floating rate tied to LIBOR + 1.625%. The bonds were paid off eight years after they were issued.
9. Moody's generally rates an emerging country's Brady bonds with the country's sovereign foreign debt.
10. For a more detailed discussion of the credit analysis of emerging market debt see Jane Brauer, "Emerging Markets Debt," *The Handbook of Fixed-Income Securities*, 7th edition, editor, F. Fabozzi, 441–470. Discussions of country analysis can also be in many international financial texts; for example, see Jeff Madura, *International Financial Management*.

11. For example, if the United States reserve requirement were 5% on time deposits for a certain size bank, while no requirements existed in the Bahamas, then a U.S. bank, by accepting a domestic deposit, could only loan out 95% of the deposit, earning 95% of the loan rate, in contrast to a Bahamian deposit in which 100% of the deposit could be loaned out to earn the full amount of the loan rate. In a competitive market for deposits and loans, the rates on the Bahamian loans and deposits would have to be made more favorable, since a depositor or borrower would prefer his own country.
12. Midwest Bank might also call an international money broker in New York or London. The broker, in turn, would provide information and make arrangements. The information could be in the form of bid and offer quotes on other Eurocurrency deposits.

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Bond Evaluation, Selection, and Management, Second Edition
by R. Stafford Johnson

CHAPTER 11

Residential Mortgages and Mortgage-Backed Securities

11.1 INTRODUCTION

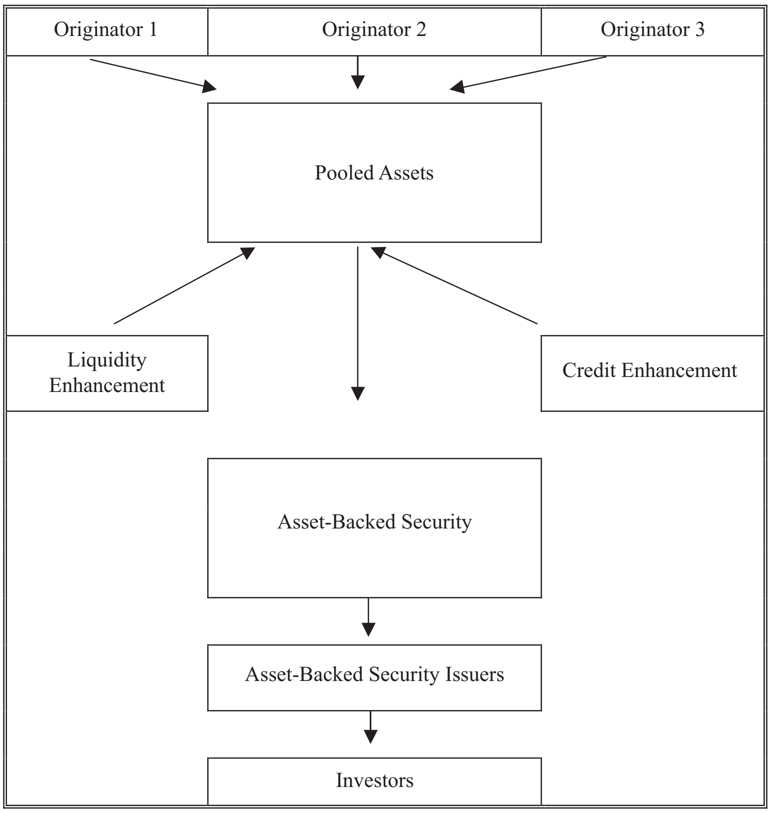
One of the most innovative developments to occur in the securities markets over the last three decades has been the securitization of assets. ***Securitization*** involves creating a new security backed by a large number of assets (e.g., mortgages or accounts receivables) that have been grouped into a pool. A trustee, such as a financial institution or government agency, often holds the pool of assets that serve as the collateral for the new securities. The new securities are then sold to investors as ***asset-backed securities***. The most common type of asset-backed security is a ***pass-through***, a security that has the cash flows from the pool of assets pass through the trustee before being disbursed to the asset-backed security holders.

The securitization process starts when an ***originator***, who owns the assets, sells them to a ***conduit*** (e.g., government agency or bank) that assembles the pool of assets. The conduit/issuer then creates a security backed by the assets; the asset-backed security or pass-through is then sold to investors. As noted, the securitization process often involves a third-party trustee, who not only holds the securities but also ensures that the issuer complies with the terms underlying the asset-backed security. Further, many securitized assets are backed by credit enhancements, such as a guarantee from the conduit or a third party against the default on the underlying assets (see Exhibit 11.1).

The most common types of asset-backed securities are those secured by mortgages, automobile loans, credit card receivables, and home equity loans. By far the largest type and the one in which the process of securitization has been most extensively applied is mortgages. Asset-backed securities formed with mortgages are called ***mortgage-backed securities (MBSs)*** or ***mortgage pass-throughs***. These securities entitle the holder to the cash flow from a pool of mortgages. Typically, the issuer of a MBS buys a portfolio or pool of mortgages of a certain type from mortgage originators, such as a commercial banks, savings and loans, or mortgage bankers. The issuer finances the purchases of the mortgages through the sale of the mortgage pass-throughs, which have a claim on the mortgage portfolio's cash flow. The mortgage originators usually agree to continue to service the loans, passing the payments on to the MBS holders.

In this chapter we examine the construction and characteristics of agency residential MBSs, and in Chapter 12, we examine nonagency residential MBSs, commercial

EXHIBIT 11.1 Securitization Process



MBSs, and other asset-backed securities. As we will see, the characteristics and value of such securities ultimately depend on the characteristics of the underlying asset. We begin our analysis with an overview of residential mortgage loans.

11.2 RESIDENTIAL MORTGAGE LOANS

A mortgage is a loan secured by a specific real estate property, typically the one being acquired by the borrower. Real estate property can be either residential or nonresidential. Residential includes houses, condominiums, and apartments; it is classified as either single-family or multiple-family. Nonresidential includes commercial and agricultural property. In a standard mortgage, the lender will place a lien against the property. A ***lien*** is a public record attached to the title of the property that specifies the lender has the right to sell the property if the owner defaults.

Most mortgages originate from commercial banks, savings and loans, other thrifts, or mortgage bankers.[1] The mortgage originator underwrites the loan, processes the necessary documents, conducts credit checks, evaluates the property, sets up the loan contracts and terms, and provides the funds. Mortgages differ in terms of their maturity, interest rate (fixed or adjustable), security, credit quality, and prepayment.

Maturity

Many residential mortgages have original maturities of 30 years, with shorter maturities of 10, 15, or 20 years also popular. The majority of mortgages are fully amortized, meaning each month the mortgage payment includes both a payment of interest on the mortgage balance and a payment of principal, with the total number of monthly payments being such that the loan is retired at maturity. There are some mortgages structured as ***balloon loans***. With a balloon loan, the borrower makes payments based on an amortized schedule, such as 30 years (or just interest-only payments), for a specified period (e.g., three to five years), at which time the total loan balance becomes due. If the borrower's credit profile has not deteriorated, then the lender will typically refinance the debt.[2]

Fixed-Rate and Adjustable-Rate Mortgages

Mortgage loans are either fixed rate (FRM), with the rate fixed for the life of the mortgage, or adjustable rate, in which the rate is reset periodically based on some prespecified rate or index. For an ***adjustable-rate mortgage (ARM)***, the monthly payments are recalculated at each specified reset date. At each reset date, the rate is set equal to a reference rate plus a margin (spread over the reference rate or index). The reference rate could be a market-determined rate, such as the average T-bill rate, LIBOR, the Constant Maturity Treasury (CMT) rate, or a 12-month Moving Treasury Average (MTA), or the reference rate could be an index, such as the National or Federal Home Loan Calculated Cost of Fund Index (COFI). Some ARMs also place a periodic cap or floor on the reset rate, limiting the amount the rate can increase or decrease each period, and some have a lifetime cap or floor, specifying the maximum or minimum rate on the mortgage over its term to maturity. Both FRM and ARM can include ***discount points*** (or ***points***). Points are interest payments made at the beginning of the loan. For example, the borrower might agree to pay 1% (one point) of the loan at the beginning in return for a reduced interest rate.

Other Mortgage Types

Since the late 1970s, other types of mortgage loans have been introduced. These include ***graduated payment mortgages (GPMs)***, which start with low monthly payments in earlier years and then gradually increase. One type of GRM is the ***2/28 ARM***, often referred to as the ***teaser loan***. These loans fix the rate for two years and then increase them significantly. There are also ***reset mortgages***, which allow the borrower to renegotiate the terms of the mortgage at specified future dates, and ***interest-only mortgages (IO)***, in which only the interest is paid for a specified period (***lockout period***), after which the loan is fully amortized for the remaining life of the loan. Between fixed-rate mortgages and ARMs, there are also ***hybrid ARMs*** in which the rate is fixed for the early years of the mortgage (e.g., five years) and then reset to an ARM. Exhibit 11.2 summarizes the different types of mortgage loans.

EXHIBIT 11.2 Mortgage Types and Terms

1. **Conventional Mortgage:** Mortgage loan not guaranteed by the government or federal agency.
2. **Insured Mortgage**: Mortgage guaranteed by the Federal Housing Administration or Department of Veterans Affairs.
3. **Private Mortgage Insurance** (PMI) **Mortgage:** Conventional mortgage insured by a private mortgage insurer.
4. **Adjustable-Rate Mortgage** (ARM): Mortgage whose rates are tied to the rates on another security or index and adjusted periodically.
5. **Graduated Payment Mortgage** (GPM): Mortgage that starts with low monthly payments in earlier years and then gradually increases.
6. **Reset Mortgage**: Mortgage that allows the borrower to renegotiate the terms of the mortgage at specified future dates.
7. **Interest-Only Mortgage** (IO): Mortgage in which only the interest is paid for a specified period (lockout period), after which the loan is fully amortized for the remaining life of the loan.
8. **Hybrid Adjustable-Rate Mortgage**: Mortgage in which the rate is fixed for the early years of the mortgage (e.g., five years) and then reset to an ARM.
9. **Shared-Appreciation Mortgage:** Mortgage in which the lender provides a low interest rate in exchange for a share in the appreciation of the real estate.
10. **Equity Participation Mortgage**: Mortgage in which the lender accepts a lower down-payment or lower monthly payments in exchange for a share in the appreciation of the property.
11. **Second Mortgage:** Loan secured by a second lien against the property.
12. **Reverse Annuity Mortgage**: Mortgage that has an increasing balance in which the lender advances periodic funds (usually on a monthly basis) to the owner/borrower. The loan comes due when the property is sold.
13. **2/28 ARM or Teaser Loan:** An ARM that starts at a very low rate for the first two years and then is reset at a significantly higher rate.
14. **Stretch Loan**: Loan that allows borrowers to commit more than 50% of their gross monthly income to the monthly mortgage payment.
15. **Stated-Income Loan:** Mortgage loan that allows borrowers to state their income without verification.
16. **Piggyback Loan:** Two loans consisting of a first mortgage and a second loan (usually a second mortgage) that starts at origination. The second loan is used to finance the down payment.
17. **Conforming Limit Loan:** Mortgages guaranteed by Fannie Mae and Freddie Mac have limits on the loan balance, referred to as conforming limits. As of January 2008, the maximum for one-family homes in the lower 48 was $417,000 and $801,950 for four-family homes. (The 2008 Economic Stimulus Act gave Fannie Mae and Freddie Mac temporary authority to purchase mortgages with loan balances exceeding the conforming limits.)
18. **Jumbo Loan:** Loan greater than the conforming limits set by Fannie Mae and Freddie Mac.

Mortgage Payment

The monthly payment on a mortgage, p, is found by solving for the p that makes the present value of all scheduled payments equal to the mortgage balance, F_0. That is:

$$F_0 = \sum_{t=1}^{M} \frac{p}{(1+(R^A/12))^t}$$

$$F_0 = p\left[\frac{1-1/(1+(R^A/12))^M}{R^A/12}\right] \tag{11.1}$$

$$p = \frac{F_0}{\left[\frac{1-1/(1+(R^A/12))^M}{R^A/12}\right]}$$

where F_0 = Face value of the loan
R^A = Annualized interest rate
p = Monthly payment
M = Maturity in months

Thus, the monthly payment on a \$100,000, 30-year, 9% fixed rate mortgage would be \$804.62:

$$p = \frac{\$100{,}000}{\left[\frac{1-1/(1+(.09/12))^{360}}{.09/12}\right]} = \$804.62$$

The \$804.62 payment applies toward both the interest and principal. After the monthly payment p has been made, the principal balance at the end of month t is

$$F_t = F_{t-1} + [(R^A/12)F_{t-1}] - p \tag{11.2}$$

and the interest payment for month $t-1$ is

$$\text{Interest payment} = (R^A/12)(F_{t-1}) \tag{11.3}$$

Table 11.1 shows the schedule of interest and principal payments on the \$100,000, 30-year, 9% mortgage for selected months, and Figure 11.1 shows the payments of scheduled interest and principal payments over the life of the mortgage. The figure highlights the pattern that in the early life of the mortgage most of the monthly payments go toward paying interest, whereas in the later life of the mortgage the payments are applied more toward the payment of the principal.

Suppose the \$100,000, 30-year mortgage were an ARM with an annual reset. For an ARM, the monthly payments are adjusted at the reset dates, with the new payment, p, calculated based on the balance, the remaining term, and the new reset rate. This process of resetting payments is known as ***recasting***. For example, at the beginning of month 13, the balance is \$99,316.80 and there are 29 years or

TABLE 11.1 Cash Flow from Fixed-Rate Mortgage: $M = 30$ Years (360 Months), Rate = 9%

Period	Balance	Interest	Principal	Payment
1	$100,000.00	$750.00	$54.62	$804.62
2	$99,945.38	$749.59	$55.03	$804.62
3	$99,890.35	$749.18	$55.45	$804.62
4	$99,834.90	$748.76	$55.86	$804.62
5	$99,779.04	$748.34	$56.28	$804.62
6	$99,722.76	$747.92	$56.70	$804.62
263	$55,698.95	$417.74	$386.88	$804.62
264	$55,312.07	$414.84	$389.78	$804.62
265	$54,922.28	$411.92	$392.71	$804.62
266	$54,529.58	$408.97	$395.65	$804.62
267	$54,133.93	$406.00	$398.62	$804.62
268	$53,735.31	$403.01	$401.61	$804.62
269	$53,333.70	$400.00	$404.62	$804.62
270	$52,929.08	$396.97	$407.65	$804.62
360	$798.63	$5.99	$798.63	$804.62

248 months remaining on the mortgage. If the reset rate were 10%, then the monthly payment would increase from $804.62 to $876.45:

$$p = \frac{\$99{,}316.80}{\left[\dfrac{1 - 1/(1 + (.10/12))^{348}}{.10/12}\right]} = \$876.45$$

Next, suppose the $100,000, 30-year, 9% mortgage were an interest-only mortgage for five years and then was set at the 9% note rate for the remaining 25 years. For the first five years (the lockout period), the monthly mortgage payments would be $750 [= (.09/12)($100,000)]. Starting in year 5, the mortgage payment would be

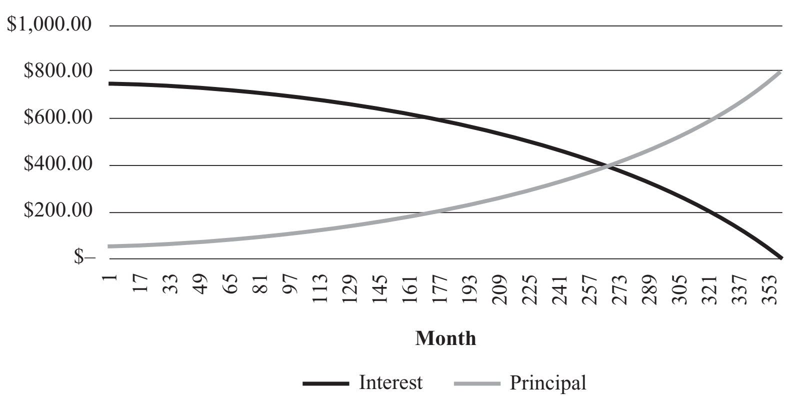


FIGURE 11.1 Monthly Interest and Principal Payments

set to equal the payment on a fully amortized loan for 25 years at 9%. This would result in the monthly payment increase of 11.89% from $750 to $839.20:

$$p = \frac{\$100{,}000}{\left[\dfrac{1 - 1/(1 + (.09/12))^{300}}{.09/12}\right]} = \$839.20$$

Finally, suppose the $100,000, 9% mortgage were a balloon loan with payments based on a 30-year amortization schedule and with the loan balance due at the end of five years (or the beginning of year 6). The borrower in this case would pay $804.62 each month and would owe $94,903.32 at the balloon date.

Mortgage Guarantees

In addition to the property securing the mortgage, many mortgages are also insured against default by the borrower. Two federal agencies, the ***Federal Housing Administration (FHA)*** and the ***Department of Veterans Affairs (VA)*** provide mortgage insurance to qualified borrowers.[3] FHA and VA mortgages typically require smaller down payments than conventional mortgages. The Federal Housing Administration is under the Department of Housing and Urban Development (HUD). HUD also administers the VA loan guarantee program that comes under the auspices of the U.S. Department of Veteran Affairs. FHA- and VA-insured mortgages are referred to as ***insured mortgages***. Mortgage loans not guaranteed by these agencies are called ***conventional mortgages***. Many conventional loans, though, are insured by ***private mortgage insurance*** companies (***PMIs***), such as the Mortgage Guarantee Investment Corporation (MGIV) or the PMI Group Inc. In addition to PMI-insured mortgages there are also many conventional loans that are not initially insured, but later receive insurance when they are pooled with other mortgages to back an MBS issue. Conventional loans that are packaged by Fannie Mae and Freddie Mac are ones that meet the underwriting guidelines of Fannie Mae and Freddie Mac (per type of property, loan-to-value ratio, credit score, and documentation). These loans are referred to as ***conforming loans***.

Credit Quality

Mortgage loans can be classified as prime, subprime, or alternative-A loans. A ***prime loan*** is considered a high-quality loan where the borrower is deemed to have a strong income and credit history sufficient to make the loan payments, as well as a sufficient equity-to-property value such that the lender would be able to cover the mortgage balance in case there was a default and the lender was forced to sell the property. A ***subprime loan***, on the other hand, is considered low quality, where the borrower is at higher risk of default and where the equity-to-property value is low, or where the mortgage has a secondary claim (e.g., a second mortgage). ***Alternative-A loans*** (or an ***Alt-A-loans***) are mortgage loans somewhere between prime and subprime. They are considered to have almost prime quality, but have some factors that tend to increase their credit risk.

In assessing credit quality, lenders consider measures such as the ***payment-to-income (PTI)*** and ***loan-to-value ratios (LTV)***. A ***front PTI*** ratio is the monthly payments (including property taxes and insurance) to the borrower's monthly gross income, whereas a ***back PTI*** is the ratio of monthly payments of the mortgage plus other monthly debt obligations (e.g., car loans and credit card payments) to the borrower's monthly income. Front and back PTI ratios measure the ability of the borrower to make monthly payments. For a mortgage loan to be considered prime, it typically has to have a front ratio of 28% or less and a back ratio of 36% or less. The LTV, in turn, is the ratio of the amount of the loan to the market or appraised value of the property. The lower the LTV ratio, the greater the protection the lender has to recover the loan if the borrower defaults and the lender has to sell the repossessed property. In addition to PTI and LTV ratios, lenders also look at credit ratings and credit scores computed from statistically based models. Most of these ratings models are patterned after the one developed by Fair, Isaac, and Company (FICO). The models calculate a credit risk index or score for most borrowers based on such factors as the borrower's payment history, current debt, types of credit held, and number of credit inquiries. The borrower's credit score is commonly referred to as the ***FICO score***. Today, the FICO score is one of the important inputs used to classify prime and sub-prime loans. Prime loans tend to have a FICO score of 600 or higher, with the average being 742; front and back PTIs of 28% and 26%, respectively; and LTV ratios of less than 95%. In contrast, a subprime loan typically has a FICO score below 660, with the average being 624, and higher PTI and LTV ratios. An Alt-A loan, in turn, could have scores and ratios between the prime and subprime; for example, high FICO scores, but low PTI and LTV ratios.

Before securitization, most potential home buyers who did not meet strict qualification standards were denied loans. As a result, most mortgages were considered prime. In 2000, the Mortgage Bankers Association estimated that 70% of all loans were prime conventional, 20% were FHA, 8% were VA, and 2% were subprime. With the combination of the growth in MBSs, the push by Congress to increase home ownership, and the introduction of innovative mortgage loans such as teasers, stretch loans, piggyback loans, and stated income loans (see Exhibit 11.2), subprime mortgages accelerated from 2000 to 2006. In 2006, the Mortgage Bankers Association estimated that 70% of all mortgage loans were conventional, with 17% of those being subprime. As a rule, if property values increase, subprime borrowers are in a position to sell their properties and pay off their loans. Unfortunately, the real estate market, which had been accelerating since 2001, cooled in 2006 when a number of the innovative loans were reset at higher rates that many borrowers could not make. This led to defaults, bankruptcies, the decline in property values, and ultimately the collapse of the subprime market and the beginning of the financial crisis of 2008.

Prepayment

Prepayment is the amount of payment made in excess of the monthly mortgage payment. Total prepayment occurs when the entire balance is paid off before maturity. This can be the result of the borrower refinancing the loan or selling the property. Most mortgages have a ***due-on-sale*** requirement in which the mortgage balance must be paid when the property is sold. Prepayment can also be for only part of the balance. This is known as a ***curtailment.*** The borrower's right to prepay a loan in total or partially is an option that benefits the borrower. Some mortgages impose a prepayment penalty to minimize prepayment or to lower the prepayment cost to the lender of originating new loans at lower interest rates.

11.3 MORTGAGE PORTFOLIO

After creating a number of mortgages, the mortgage originator ends up with a mortgage loan portfolio which he may hold or sell to an agency or financial institution. The holder of the mortgage portfolio is subject to credit risk if the mortgages are not insured by FHA and VA, liquidity risk, given that the portfolio is large and lacking divisibility, and ***prepayment risk***.

Prepayment Risk

For the holder of a mortgage portfolio, prepayment creates an uncertainty concerning the portfolio's cash flows. For example, if a bank has a pool of mortgages with a weighted average mortgage rate of 9% and mortgage rates, in turn, decrease in the market to 7%, then the bank's mortgage portfolio is likely to experience significant prepayment as borrowers refinance their loans. The option borrowers have to prepay makes it difficult for the lender to predict future cash flows or to determine the value of the portfolio. A number of prepayment models have been developed to try to predict the cash flows from a portfolio of mortgages. Most of these models estimate the prepayment rate, referred to as the ***prepayment speed*** or simply ***speed***, in terms of four factors: refinancing incentive, seasoning (the age of the mortgage), monthly factors, and prepayment burnout.

The refinancing incentive is the most important factor influencing prepayment. If mortgage rates decrease below the mortgage loan rate, borrowers have a strong incentive to refinance. This incentive increases during periods of falling interest rates, with the greatest increases occurring when borrowers determine that rates have bottomed out. The refinancing incentive can be measured by the difference between the mortgage portfolio's weighted average rate, referred to as the ***weighted average coupon rate (WAC)*** or ***weighted average loan rate*** and the refinancing rate (R^{ref}). A study by Goldman, Sachs, and Company found that the annualized prepayment speed, referred to as the ***conditional prepayment rate (CPR)***, is greater the larger the positive difference between the WAC and R^{ref}. The study reported that when $\text{WAC} - R^{\text{ref}} = 0$ (known as the current coupon), FHA and VA mortgages prepay at a rate of approximately 6%, and conventional mortgages prepay at approximately 9%. The study also found that prepayment rates decrease slightly when mortgage rates are at a discount ($\text{WAC} < R^{\text{ref}}$) and the refinancing rate is increasing relative to the mortgage rate. In such cases, prepayment is primarily due to new home purchases. In contrast, Goldman, Sachs, and Company found that prepayment rates increase significantly when mortgage rates are at a premium ($\text{WAC} > R^{\text{ref}}$) and the refinancing rate is decreasing relative to the mortgage rate. For example, when the difference between the WAC and R^{ref} is between 3% and 4%, the prepayment rate for conventional mortgages equals approximately 50% of the outstanding pool, and for FHA/VA mortgages the rate equals 40%.

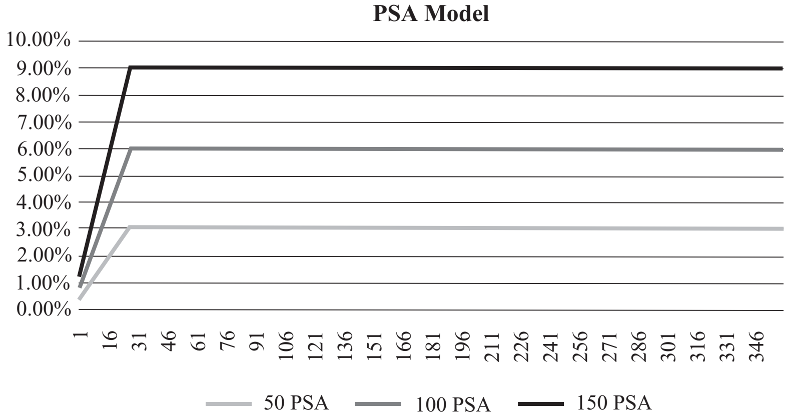


FIGURE 11.2 PSA Prepayment Model

A second factor determining prepayment is the age of the mortgage, referred to as ***seasoning***. Prepayment tends to be greater during the early part of the loan, and then stabilize after about three years. Figure 11.2 depicts a commonly referenced seasoning pattern known as the ***PSA model (Public Securities Association)***. In the standard PSA model, known as 100 PSA, the CPR starts at .2% for the first month and then increases at a constant rate of .2% per month to equal 6% at the 30th month; then after the 30th month the CPR stays at a constant 6%. Thus for any month t, the CPR is

$$\begin{aligned} \text{CPR} &= .06\left(\frac{t}{30}\right), \ \text{if } t \leq 30, \\ \text{CPR} &= .06, \ \text{if } t > 30 \end{aligned} \tag{11.4}$$

Note that the CPR is quoted on an annual basis. The monthly prepayment rate, referred to as the ***single monthly mortality rate (SMM)***, can be obtained given the annual CPR by using the following formula:

$$\text{SMM} = 1 - [1 - \text{CPR}]^{1/12} \tag{11.5}$$

The 100 PSA model is often used as a benchmark. The actual aging pattern will differ depending on whether the mortgage pool is current (WAC = R^{ref}), at a discount (WAC < R^{ref}), or at premium (WAC > R^{ref}). Analysts often refer to the applicable pattern as being a certain percentage of the PSA. For example, if the pattern is described as being 200 PSA, then the prepayment speeds are twice the 100 PSA rates, and if the pattern is described as 50 PSA, then the CPRs are half of the 100 PSA rates (see Figure 11.2). Thus, a current mortgage pool described by a 100 PSA would have an annual prepayment rate of 2% after 10 months (or a monthly prepayment rate of SMM = .00168), and a premium pool described as a 150 PSA would have a 3% CPR (or SMM = .002535) after 10 months.

In addition to the effect of seasoning, mortgage prepayment rates are also influenced by the month of the year, with prepayment tending to be higher during the summer months. ***Monthly factors*** can be taken into account by multiplying the CPR by the estimated monthly multiplier to obtain a monthly-adjusted CPR. PSA provides estimates of the monthly multipliers. Finally, many prepayment models also try to capture what is known as the ***burnout factor***. The burnout factor refers to the tendency for premium mortgages to hit some maximum CPR and then level off. For example, in response to a 2% decrease in refinancing rates, a pool of premium mortgages might peak at a 40% prepayment rate after one year, then level off at approximately 25%.

In addition to the refinancing incentives, seasoning, monthly adjustments, and burnout factors, there are other factors that can influence the pool of mortgages: secular variations (variations due to different locations such as California or New York mortgages), types of mortgages (e.g., FRM or ARM, single-family or multiple-family, residential or commercial, etc.), and the original terms of the mortgage (30 years or 15 years). With these myriad factors influencing prepayment, analysts have found that estimating the cash flows from a pool of mortgages is significantly more difficult than estimating the cash flows of other fixed-income securities.

Estimating a Mortgage Pool's Cash Flow with Prepayment

The cash flow from a portfolio of insured mortgages consists of the interest payments, scheduled principal, and prepaid principal. Consider a bank that has a pool of current fixed-rate insured mortgages that are worth \$100 million, yield a WAC of 8%, and have a weighted average maturity (WAM) of 360 months. For the first month, the portfolio would generate an aggregate mortgage payment of \$733,765:

$$p = \frac{\$100{,}000{,}000}{\left[\frac{1 - 1/(1 + (.08/12))^{360}}{.08/12}\right]} = \$733{,}765$$

From the \$733,765 payment, \$666,667 would go toward interest and \$67,098 would go toward the scheduled principal payment:

$$\text{Interest} = \left(\frac{R^A}{12}\right) F_0 = \left(\frac{.08}{12}\right) \$100{,}000{,}000 = \$666{,}667$$

$$\text{Scheduled principal payment} = p - \text{Interest} = \$733{,}765 - \$666{,}667 = \$67{,}098$$

The projected first month prepaid principal can be estimated with a prepayment model. Using the 100 PSA model, the monthly prepayment rate (SMM) for the first month ($t = 1$) is equal to 0.0001668:

$$\text{CPR} = \left(\frac{1}{30}\right) .06 = .002$$

$$\text{SMM} = 1 - [1 - .002]^{1/12} = .0001668$$

Given the prepayment rate, the projected prepaid principal in the first month is found by multiplying the balance at the beginning of the month minus the scheduled principal by the SMM. Doing this yields a projected prepaid principal of \$16,671 in the first month:

$$\text{Prepaid principal} = \text{SMM}[F_0 - \text{Scheduled principal}]$$

$$\text{Prepaid principal} = .00016682[\$100{,}000{,}000 - \$67{,}098] = \$16{,}671$$

Thus, for the first month, the mortgage portfolio would generate an estimated cash flow of \$750,435 and a balance at the beginning of the next month of \$99,916,231:

$$\text{CF} = \text{Interest} + \text{Scheduled principal} + \text{prepaid principal}$$

$$\text{CF} = \$666{,}666 + \$67{,}098 + \$16{,}671 = \$750{,}435$$

$$\text{Beginning balance for month 2} = F_0 - \text{Scheduled principal} - \text{prepaid principal}$$

$$\text{Beginning balance for month 2} = \$100{,}000{,}000 - \$67{,}098 - \$16{,}671 = \$99{,}916{,}231$$

In the second month ($t = 2$), the projected payment would be \$733,642 with \$666,108 going to interest and \$67,534 to scheduled principal. Using the 100 PSA model, the estimated monthly prepayment rate is 0.000333946, yielding a projected prepaid principal in month 2 of \$33,344:

$$\text{CPR} = \left(\frac{2}{30}\right).06 = .004$$

$$\text{SMM} = 1 - (1 - .004)^{1/12} = .000333946$$

$$\text{Prepaid principal} = .000333946(\$99{,}916{,}231 - \$67{,}534) = \$33{,}344$$

Thus, for the second month, the mortgage portfolio would generate an estimated cash flow of \$766,986 and have a balance at the beginning of month 3 of \$99,815,353:

$$\text{CF} = \$666{,}108 + \$67{,}534 + \$33{,}344 = \$766{,}986$$

$$\text{Beginning balance for month 3} = \$99{,}916{,}231 - \$67{,}534 - \$33{,}344 = \$99{,}815{,}353$$

Table 11.2 summarizes the mortgage portfolio's cash flow for the first two months and other selected months. In examining the table, two points should be noted: First, starting in month 30 the SMM remains constant at 0.005143; this reflects the 100 PSA model's assumption of a constant CPR of 6% starting in month 30; second, the projected cash flows are based on a static analysis in which rates

TABLE 11.2 Projected Cash Flows: Mortgage Portfolio = $100,000,000, WAC = 8%, WAM = 360 Months, Prepayment = 100 PSA

Period	Balance $100,000,000	Interest	p	Scheduled Principal	SMM	Prepaid Principal	Cash Flow
1	$100,000,000	$666,667	$733,765	$67,098	0.0001668	$16,671	**$750,435**
2	$99,916,231	$666,108	$733,642	$67,534	0.0003339	$33,344	**$766,986**
3	$99,815,353	$665,436	$733,397	$67,961	0.0005014	$50,011	**$783,409**
4	$99,697,380	$664,649	$733,029	$68,380	0.0006691	$66,664	**$799,694**
5	$99,562,336	$663,749	$732,539	$68,790	0.0008372	$83,294	**$815,833**
6	$99,410,252	$662,735	$731,926	$69,191	0.0010055	$99,892	**$831,817**
7	$99,241,170	$661,608	$731,190	$69,582	0.0011742	$116,449	**$847,639**
23	$94,291,147	$628,608	$703,012	$74,405	0.0039166	$369,010	**$1,072,023**
24	$93,847,732	$625,652	$700,259	$74,607	0.0040908	$383,607	**$1,083,866**
25	$93,389,518	$622,597	$697,394	$74,798	0.0042653	$398,017	**$1,095,411**
26	$92,916,704	$619,445	$694,420	$74,975	0.0044402	$412,234	**$1,106,653**
27	$92,429,495	$616,197	$691,336	$75,140	0.0046154	$426,250	**$1,117,586**
28	$91,928,105	$612,854	$688,146	$75,292	0.0047909	$440,059	**$1,128,204**
29	$91,412,755	$609,418	$684,849	$75,430	0.0049668	$453,653	**$1,138,502**
30	$90,883,671	$605,891	$681,447	$75,556	0.0051430	$467,027	**$1,148,475**
31	$90,341,088	$602,274	$677,943	$75,669	0.0051430	$464,236	**$1,142,179**
32	$89,801,183	$598,675	$674,456	$75,781	0.0051430	$461,459	**$1,135,915**
110	$54,900,442	$366,003	$451,112	$85,109	0.0051430	$281,916	**$733,028**
111	$54,533,417	$363,556	$448,792	$85,236	0.0051430	$280,028	**$728,820**
112	$54,168,153	$361,121	$446,484	$85,363	0.0051430	$278,148	**$724,632**
113	$53,804,641	$358,698	$444,188	$85,490	0.0051430	$276,278	**$720,466**
114	$53,442,873	$356,286	$441,903	$85,617	0.0051430	$274,417	**$716,320**
115	$53,082,839	$353,886	$439,631	$85,745	0.0051430	$272,565	**$712,195**
357	$496,620	$3,311	$126,231	$122,920	0.0051430	$1,922	**$128,153**
358	$371,778	$2,479	$125,582	$123,103	0.0051430	$1,279	**$126,861**
359	$247,395	$1,649	$124,936	$123,287	0.0051430	$638	**$125,574**
360	$123,470	$823	$124,293	$123,470	0.0051430	$0	**$124,293**

$$p = \frac{\text{Balance}}{\left[\dfrac{1 - [1/(1+(.08/12))]^{\text{Remaining Periods}}}{(.08/12)}\right]}$$

Interest = (.08/12)(Balance)

Scheduled principal = p − (.08)Balance

Prepaid principal = SMM[Beginning balance − Scheduled principal]

are assumed fixed over the time period. A more realistic model would incorporate interest rate changes and corresponding different prepayment speeds.

Default Risk

A portfolio of residential mortgages, insured by FHA,VA, or as a mortgage pool by Ginnie Mae, is subject to prepayment risk but not default risk. Such mortgages are referred to as ***agency mortgages.*** In contrast, conventional prime and subprime residential mortgages are subject to default risk, as well as mortgages insured by

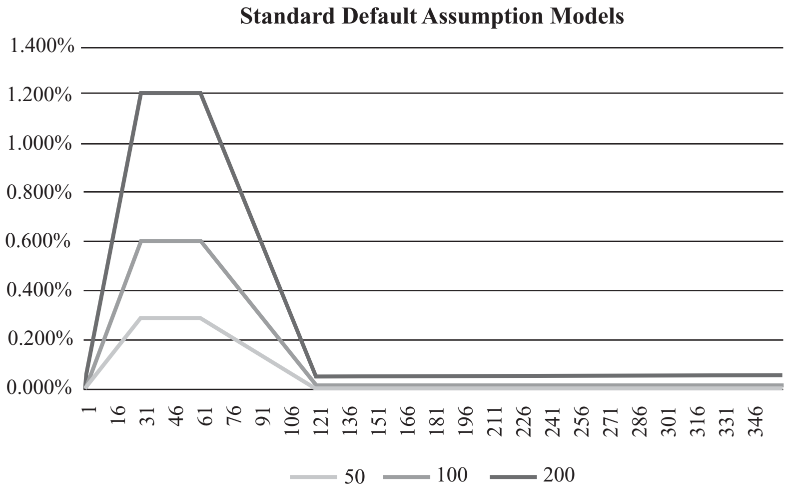


FIGURE 11.3 PSA Default Loss Model

private insurers where the risk has simply been transferred. A portfolio of these ***nonagency mortgages*** is therefore subject to both prepayment and default risk.

In estimating the cash flows from a portfolio of nonagency mortgages, the expected default losses need to be taken into account. In addition to its prepayment models, the Public Service Association also provides models for estimating standardized default rates for different mortgages. Figure 11.3 shows the PSA's ***standardized default assumption (SDA)*** model for a 30-year mortgage—the ***100 SDA*** model. In this default rate model, the annualized default rate (ADR) starts at 0.02% in the first month and then increases at a constant rate of 0.02% per month to reach 0.60% at the 30th month; from month 30 to month 60, the ADR remains constant at 0.60%; from month 61 to month 120, the ADR declines from 0.60% to .03%; from month 120 on, the ADR remains constant at 0.03%.

In calculating monthly cash flows, monthly default rates (MDR) can be obtained given the annual default rates by using the following formula:

$$MDR = 1 - [1 - ADR]^{1/12} \tag{11.6}$$

The estimated monthly loss due to default is equal to the MDR times the difference between the balance minus the monthly scheduled principal payments:

$$\begin{aligned}\text{Monthly default loss} = \text{MDR}[&\text{Beginning balance} \\ &- \text{Monthly scheduled principal payment}]\end{aligned}$$

The 100 SDA model is used like the PSA prepayment model as a benchmark in evaluating different types of prime and subprime mortgages. To reflect greater or lower default risk, analysts will adjust the baseline SDA model to be equal to a certain percentage of the 100 SDA model. For example, if the pattern is described as being 200 SDA, then ADRs will be twice the 100 SDA rates, and if the pattern is described as 50 SDA, then the ADRs will be set equal to half of the 100 PSA rates (see Figure 11.3). Thus, a current mortgage pool described by a 200 SDA would start with an annualized default rate of 0.04% in the first month and then would increase at a constant rate of 0.04% per month to reach 1.20% at the 30th month; from month 30 to month 60, the ADR would remain constant at 1.20%; from month 61 to month 120, the ADR would declines from 1.20% to 0.06%; from month 120 on, the ADR would remain constant at 0.06%.

To see the applicability of the SDA model, consider again the bank with a pool of current fixed-rate mortgages worth \$100 million, with WAC of 8% and WAM of 360 months. If there were no default or prepayment, the portfolio would generate a mortgage payment of \$733,765, with \$666,667 going toward interest and \$67,098 toward the scheduled principal payment:

$$p = \frac{\$100{,}000{,}000}{\left[\dfrac{1 - 1/(1 + (.08/12))^{360}}{.08/12}\right]} = \$733{,}765$$

$$\text{Interest} = \left(\frac{R^A}{12}\right) F_0 = \left(\frac{.08}{12}\right) \$100{,}000{,}000 = \$666{,}667$$

$$\text{Scheduled principal payment} = p - \text{Interest} = \$733{,}765 - \$666{,}667 = \$67{,}098$$

If the portfolio, though, consisted of mortgages that were characterized by the 100 SDA model, then in the first month the monthly loss due to default would be \$1,665.70 and the balance at the beginning of the next month would be \$99,916,231:

$$\text{Annual default rate} = \text{ADR} = .0002$$

$$\text{MDR} = 1 - [1 - .0002]^{1/12} = .00001666819$$

$$\text{Monthly default loss} = \text{MDR[Beginning balance} - \text{Scheduled principal]}$$

$$\text{Monthly default loss} = (.00001666819)(\$100{,}000{,}000 - \$67{,}097.91]$$

$$\text{Monthly default loss} = \$1{,}665.70$$

$$\text{Beginning balance for month 2} = F_0 - \text{Scheduled principal} - \text{Default loss}$$

$$\text{Beginning balance for month 2} = \$100{,}000{,}000 - \$67{,}098 - \$1{,}665.70$$

$$\text{Beginning balance for month 2} = \$99{,}931{,}236$$

As expected, the losses are miniscule in the beginning. By month 30, though, the monthly default loss is \$48,670 and the balance is \$97,152,847; if there were no default loss, the principal would have been \$97,861,164.

Table 11.3 summarizes the mortgage portfolio's cash flow for the first month and other selected months. In examining the table, several points should be noted. First, starting in month 30 and going to month 60, the MDR remains constant

TABLE 11.3 Projected Cash Flows with Default Loss
Mortgage Portfolio = $100,000,000
WAC = 8%, WAM = 360 months
100 SDA Model

1 Period	2 Balance	3 *p*	4 Interest	5 Scheduled Principal	6 Monthly Default Rate	7 Default Loss	8 Cumulative Loss	9 Cummaltive Default Rates
1	$100,000,000	$733,765	$666,667	$67,098	0.00001667	$1,666	$1,666	0.0017%
2	$99,931,236	$733,752	$666,208	$67,544	0.00003334	$3,329	$4,995	0.0050%
3	$99,860,363	$733,728	$665,736	$67,992	0.00005001	$4,991	$9,986	0.0100%
4	$99,787,380	$733,691	$665,249	$68,442	0.00006669	$6,650	$16,636	0.0166%
5	$99,712,287	$733,642	$664,749	$68,894	0.00008337	$8,307	$24,944	0.0249%
6	$99,635,086	$733,581	$664,234	$69,347	0.00010006	$9,962	$34,906	0.0349%
7	$99,555,777	$733,508	$663,705	$69,803	0.00011674	$11,614	$46,520	0.0465%
23	$98,001,040	$730,672	$653,340	$77,332	0.00038414	$37,617	$454,554	0.4546%
24	$97,886,091	$730,392	$652,574	$77,818	0.00040088	$39,210	$493,764	0.4938%
25	$97,769,063	$730,099	$651,794	$78,305	0.00041762	$40,798	$534,562	0.5346%
26	$97,649,960	$729,794	$651,000	$78,794	0.00043437	$42,382	$576,944	0.5769%
27	$97,528,784	$729,477	$650,192	$79,285	0.00045112	$43,961	$620,905	0.6209%
28	$97,405,538	$729,148	$649,370	$79,778	0.00046787	$45,536	$666,441	0.6664%
29	$97,280,224	$728,807	$648,535	$80,272	0.00048462	$47,105	$713,546	0.7135%
30	$97,152,847	$728,454	$647,686	$80,768	0.00050138	$48,670	$762,216	0.7622%
31	$97,023,409	$728,088	$646,823	$81,266	0.00050138	$48,605	$810,821	0.8108%
32	$96,893,538	$727,723	$645,957	$81,766	0.00050138	$48,540	$859,360	0.8594%
60	$93,069,494	$717,576	$620,463	$97,113	0.00050138	$46,615	$2,190,995	2.1910%
61	$92,925,767	$717,216	$619,505	$97,711	0.00049342	$45,803	$2,236,798	2.2368%
62	$92,782,253	$716,862	$618,548	$98,314	0.00048546	$44,994	$2,281,793	2.2818%
63	$92,638,944	$716,514	$617,593	$98,921	0.00047750	$44,188	$2,325,981	2.3260%
120	$84,568,276	$706,172	$563,789	$142,383	0.00002500	$2,111	$3,588,684	3.5887%
121	$84,423,782	$706,154	$562,825	$143,329	0.00002500	$2,107	$3,590,791	3.5908%
358	$2,589,160	$874,586	$17,261	$857,325	0.00002500	$43	$3,904,869	3.9049%
359	$1,731,791	$874,564	$11,545	$863,019	0.00002500	$22	$3,904,891	3.9049%
360	$868,751	$874,542	$5,792	$868,751	0.00002500	$0	$3,904,891	3.9049%

TABLE 11.4 Cumulative Default Rates for 100, 200, and 300 SDA Models
Mortgage Portfolio = $100,000,000
WAC = 8%, WAM = 360 Months

SDA/Month	30	60	120	360	Total Loss
100	0.76%	2.19%	3.59%	3.90%	$3,904,891
200	1.52%	4.34%	7.06%	7.67%	$7,668,089
300	2.28%	6.46%	10.42%	11.29%	$11,294,229

at 0.00050138; this reflects the 100 SDA model's assumption of a constant ADR of 0.6% from month 30 to month 60. During this period, the monthly default losses decrease slightly from the $48,671 in month 30 to $46,615 in month 60. Second, from month 60 to month 120, the monthly default rates decrease at a greater rate, causing the monthly default losses to fall from $46,615 in month 60 to $2,111 in month 120. From month 120 on, though, there is slight decrease in rates, causing the monthly losses to decrease from $2,111 in month 120 to only $21.79 in month 359.

The total default loss for the 30-year, 8% mortgage portfolio with a total principal of $100,000,000 is $3,904,890 using the 100 SDA model; this equates to a cumulative loss rate for the period of 3.9% (cumulative default losses /$100,000,000). Columns 7, 8, and 9 in Table 11.3 show the monthly default and cumulative default losses and rates, and Table 11.4 and Figure 11.4 show the cumulative default rates for the mortgage portfolio given different SDAs of 100, 200, and 300. As shown, the cumulative default rates for the 100 SDA model are 0.76% after 30 months, 2.19% after 60 months, 3.59% after 120 months, and 3.9% after 360 months. For the riskier mortgage portfolio characterized by the 200 SDA, the total loss for the period is $7,668,089 and the cumulative default rates are 1.52% after 30 months, 4.34% after 60, 7.06% after 120, and 7.67% after 360 months. Finally, the riskier 300 SDA model has total losses of $11,294,229, with cumulative default rates after 30, 60, 120, and 360 months of 2.28%, 6.46%, 10.42%, and 11.29% —the 300 SDA model might be an example of a mortgage portfolio of subprime mortgages.

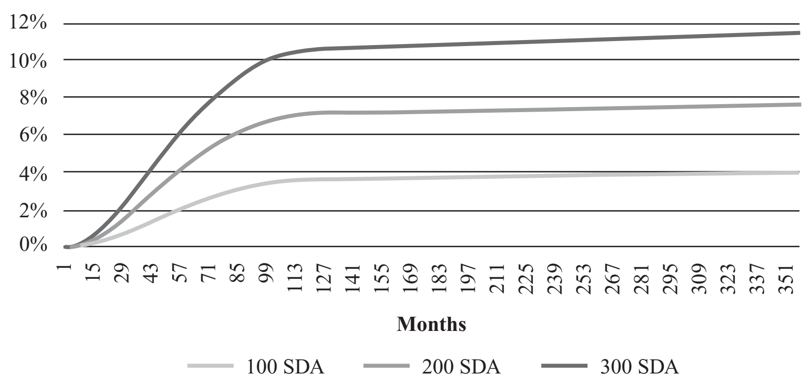


FIGURE 11.4 Cumulative Default Rates for 100, 200, and 300 SDA Models

11.4 MORTGAGE-BACKED SECURITIES

A mortgage originator with a pool of mortgages has the option of holding the portfolio, selling it, or selling it to be used to securitize an MBS issue or deal. Depending on the types of mortgages, the originator who sells mortgages to become a securitized asset can sell them to one of the three agencies (Fannie Mae, Ginnie Mae, or Freddie Mac) or to a private-sector conduit.[4] Residential MBSs created by one of the agencies are collectively referred to as ***agency MBSs***, and those created by private conduits are called ***nonagency MBSs*** or ***private labels***. As discussed previously, residential mortgages can be divided into prime and subprime mortgages. Prime mortgages include those that are both conforming (meet the agency's underwriting standards) and nonconforming but still meeting credit quality standards. In contrast, subprime mortgages include those with low credit ratings. Typically, agency residential MBSs are created from conforming loans. In more recent periods, though, agency MBS issues backed by pools of lower quality mortgages were issued. All other mortgages that are securitized are nonagency MBSs. After the mortgages are sold to an agency or private conduit, the originator typically continues to service the loan for a service fee (that is, collect payments, maintain records, forward tax information, and the like). The service fee is typically a fixed percentage (25 to 100 basis points) of the outstanding balance. The originator can also sell the servicing to another party. Investors who buy the MBSs receive a pro rata share from the cash flow of the pool of mortgages.

Agency MBSs

Ginnie Mae Mortgage-Backed Securities Ginnie Mae (Government National Mortgage Association, GNMA) is a true federal agency. As such, the MBSs that it guarantees are backed by the full faith and credit of the U.S. government. Ginnie Mae MBSs are put together by a lender/originator (bank, thrift, or mortgage banker), who presents a block of mortgages that meet Ginnie Mae's underwriting standards. If Ginnie Mae finds them in order, it will issue a guarantee and assign a pool number that identifies the MBS that is to be issued. The lender will then transfer the mortgages to a trustee, and then issue the pass-through securities as a Ginnie Mae pass-through security. Ginnie Mae, therefore, provides the guarantee, but does not issue the Ginnie Mae MBS. Thus, different from the standard MBS that is issued by the other agencies or a conduit, Ginnie Mae MBSs are issued by the lenders. The mortgages underlying Ginnie Mae MBSs can be grouped into one of two Ginnie Mae MBS programs: Ginnie Mae I and Ginnie Mae II. The Ginnie Mae I program consists of MBSs backed by single-family and multifamily mortgage loans that have a fixed note rate and are sold by only one issuer. The Ginnie Mae II program consists of just single-family mortgage loans that can have either fixed or adjustable rates and have multiple issuers. The minimum denomination on a Ginnie Mae pass-through is $25,000 and the minimum pool is $1 million.

Fannie Mae and Freddie Mac Mortgage-Backed Securities As we examined in Chapter 7, Fannie Mae and Freddie Mac are government-sponsored enterprises (GSEs) initially created to provide a secondary market for mortgages. Today, their activities include not only the buying and selling of mortgages, but also creating and guaranteeing mortgage-backed pass-through securities, as well as buying MBSs. Both GSEs are regulated by the Office of Federal Housing Enterprise Oversight (OFHEO) and both were placed in conservatorship in September 2008. Prior to being placed in conservatorship, the Fannie Mae and Freddie Mac MBSs were guaranteed by each of the companies, but not the government. As part of the banking bailout in 2008, though, government backing was provided to their MBSs.

Freddie Mac issues MBSs that it refers to as participation certificates (PCs).[5] Freddie Mac and Fannie Mae have regular MBSs (also called cash PCs), which are backed by a pool of conforming mortgages that they have purchased from mortgage originators. They also offer a pass-thoughs formed through their Guarantor/Swap Program. In this program, mortgage originators can swap mortgages for a FMLMC pass-through. Unlike Ginnie Mae, Fannie Mae's and Freddie Mac's MBSs are formed with more heterogeneous mortgages. The minimum denomination on a Freddie Mac and Fannie Mae pass-through is $100,000 and their mortgage pools range up to several hundred million dollars.

Nonagency MBSs

Nonagency pass-throughs or private labels are sold by commercial banks, investment banks, other thrifts, and mortgage bankers. As noted, nonagency pass-throughs are often formed with prime or subprime nonconforming mortgages. Larger issuers of nonagency MBSs include Citigroup, Bank of America, and GE Capital Mortgage. Their pass-throughs are often guaranteed against default through external credit enhancements, such as the guarantee of a corporation or a bank letter of credit or by private insurance from a monoline insurer. Many are also guaranteed internally through the creation of senior and subordinate classes of bonds with different priority claims on the pool's cash flows in case some of the mortgages in the pool default. The more subordinate claims sold relative to the senior claims, the more secure the senior claims. Because of their credit risk, nonagency MBSs are rated by Moody's and Standard & Poor's, and, unlike ***agency pass-throughs***, they must be registered with the SEC when they are issued. Finally, most financial entities that issue private-label MBSs or derivatives of MBSs are legally set up so that they do not have to pay taxes on the interest and principal that passes through them to their MBS investors. The requirements that MBS issuers must meet to ensure tax-exempt status are specified in the Tax Reform Act of 1983 in the section on trusts referred to as ***real estate mortgage investment conduits (REMIC)***. Private-label MBS issuers who comply with these provisions are sometimes referred to as REMICs.

Nonagency residential MBSs differ fundamentally from agency MBSs in that their cash flows are subject to default risk, whereas agency MBSs with their government and agency guarantees are considered default free. Nonagency residential MBSs are examined in more detail in Chapter 12.[6]

11.5 FEATURES OF MORTGAGE-BACKED SECURITIES

Cash Flows

Cash flows from MBSs are generated from the cash flows from the underlying pool of mortgages, minus servicing and other fees. Typically, fees for constructing, managing,

TABLE 11.5 Projected Cash Flows from an Agency MBS Issue
Mortgage Portfolio = $100,000,000, WAC = 8%, WAM = 355
PT Rate = 7.5%, Prepayment: 150 PSA

Period	Balance $100,000,000	Interest	p	Scheduled Principal	SMM	Prepaid Principal	Principal	Cash Flow
1	$100,000,000	$625,000	$736,268	$69,601	0.0015125	$151,147	$220,748	$845,748
2	$99,779,252	$623,620	$735,154	$69,959	0.0017671	$176,194	$246,153	$869,773
3	$99,533,099	$622,082	$733,855	$70,301	0.0020223	$201,148	$271,449	$893,531
4	$99,261,650	$620,385	$732,371	$70,627	0.0022783	$225,990	$296,617	$917,002
5	$98,965,033	$618,531	$730,702	$70,936	0.0025350	$250,701	$321,637	$940,168
6	$98,643,396	$616,521	$728,850	$71,227	0.0027925	$275,262	$346,489	$963,011
20	$91,641,550	$572,760	$684,341	$73,398	0.0064757	$592,971	$666,369	$1,239,128
21	$90,975,181	$568,595	$679,910	$73,408	0.0067447	$613,101	$686,510	$1,255,105
22	$90,288,672	$564,304	$675,324	$73,399	0.0070144	$632,804	$706,204	$1,270,508
23	$89,582,468	$559,890	$670,587	$73,370	0.0072849	$652,066	$725,436	$1,285,327
24	$88,857,032	$555,356	$665,702	$73,321	0.0075563	$670,873	$744,194	$1,299,550
25	$88,112,838	$550,705	$660,671	$73,253	0.0078284	$689,211	$762,463	$1,313,169
26	$87,350,375	$545,940	$655,499	$73,164	0.0078284	$683,243	$756,406	$1,302,346
27	$86,593,968	$541,212	$650,368	$73,075	0.0078284	$677,322	$750,397	$1,291,609
28	$85,843,572	$536,522	$645,277	$72,986	0.0078284	$671,448	$744,434	$1,280,957
29	$85,099,137	$531,870	$640,225	$72,897	0.0078284	$665,621	$738,519	$1,270,388
30	$84,360,619	$527,254	$635,213	$72,809	0.0078284	$659,840	$732,649	$1,259,903
31	$83,627,969	$522,675	$630,240	$72,721	0.0078284	$654,106	$726,826	$1,249,501
32	$82,901,143	$518,132	$625,307	$72,632	0.0078284	$648,416	$721,049	$1,239,181
33	$82,180,094	$513,626	$620,411	$72,544	0.0078284	$642,772	$715,317	$1,228,942
100	$44,933,791	$280,836	$366,433	$66,874	0.0078284	$351,237	$418,111	$698,947
101	$44,515,680	$278,223	$363,564	$66,793	0.0078284	$347,965	$414,758	$692,981
102	$44,100,923	$275,631	$360,718	$66,712	0.0078284	$344,718	$411,430	$687,061
103	$43,689,493	$273,059	$357,894	$66,631	0.0078284	$341,498	$408,129	$681,188
200	$16,163,713	$101,023	$166,983	$59,225	0.0078284	$126,073	$185,298	$286,321
201	$15,978,416	$99,865	$165,676	$59,153	0.0078284	$124,623	$183,776	$283,641
353	$148,527	$928	$50,171	$49,181	0.0078284	$778	$49,958	$50,887
354	$98,569	$616	$49,778	$49,121	0.0078284	$387	$49,508	$50,124
355	$49,061	$307	$49,388	$49,061	0.0078284	$0	$49,061	$49,368

$$\text{Monthly payment} = p = \frac{\text{Balance}}{\left[\dfrac{1 - [1/(1 + (.08/12))]^{\text{Remaining periods}}}{(.08/12)}\right]}$$

Interest = (.075/12)(Balance)
Scheduled principal = p − (.08)Balance
Prepaid principal = SMM[Beginning balance − Scheduled principal]

and servicing the underlying mortgages (also referred to as the mortgage collateral) and the MBSs are equal to the difference between the WAC associated with the mortgage pool and the MBS ***pass-through (PT) rate*** (also called ***pass-through coupon rate***) that is paid to the MBS investors. Table 11.5 shows the monthly cash flows for an agency MBS issue constructed from a mortgage pool with a current balance of $100 million, a WAC of 8%, a WAM of 355 months, and a PT rate of 7.5%. The monthly fees implied on the MBS issue are equal to .04167% = (8% – 7.5%)/12 of the monthly balance.

The cash flow for the MBS issue shown in Table 11.5 differs in several respects from the cash flows for the $100 million mortgage pool shown in Table 11.2.

First, the MBS issue has a WAM of 355 months and an assumed prepayment speed equal to 150% of the standard PSA model, compared to 360 months and an assumption of 100% PSA for the pool.[7] As a result, the first month's CPR for the MBS issue reflects a five-month seasoning in which $t = 6$, and a speed that is 1.5 times greater than the 100 PSA. For the MBS issue, this yields a first month SMM of .0015125 and a constant SMM of .0078284 starting in month 25. Secondly, for the mortgage pool, a WAC of 8% is used to determine the mortgage payment, scheduled principal, and interest; on the MBS issue, though, the WAC of 8% is only used to determine the mortgage payment and scheduled principal, while the PT rate of 7.5% is used to determine the interest. Finally, the MBS is an agency issue and therefore has cash flows that have no default losses, which is similar to the cash flows shown in Table 11.2, but not Table 11.3 that shows default losses.

Price Quotes

Investors can acquire newly issued MBSs from the agencies, originators, or dealers specializing in specific pass-throughs. There is also a secondary MBS market consisting of dealers who operate in the OTC market as part of the ***Mortgage-Backed Security Dealers Association***. These dealers form the core of the secondary market for the trading of existing pass-throughs. The Mortgage-Backed Securities Clearing Corporation (MBSCC) automates the trading of these MBSs.

Mortgage pass-throughs are normally sold in denominations ranging from \$25,000 to \$250,000, although some privately-placed issues are sold with denominations as high as \$1 million. There are mortgage-backed mutual funds and ETFs, which provide individual investors with an opportunity to invest in a lower-denominated MBS fund share. The prices of MBSs are quoted as a percentage of the underlying mortgages' balance. The mortgage balance at time t, F_t, is usually calculated by the servicing institution and is quoted as a proportion of the original balance, F_0. This proportion is referred to as the ***pool factor***, pf:

$$pf_t = \frac{F_t}{F_0} \tag{11.7}$$

For example, suppose a Fannie Mae MBS, backed by a mortgage pool with an original par value of \$100 million, is currently priced at 95-16 (the fractions on MBSs are quoted like Treasury bonds and notes in terms of 32nds), with a pool factor of .92. An institutional investor who purchased \$10 million of the MBS when it was first issued (and had a pool fact of $pf = 1$) would now have securities valued at \$8.786 million that are backed by mortgages that are worth \$9.2 million:

$$\text{Par value remaining} = (\$10{,}000{,}000)(.92) = \$9{,}200{,}000$$

$$\text{Market value} = (\$9{,}200{,}000)(.9550) = \$8{,}786{,}000$$

The market value of \$8.786 million represents a clean or flat price that does not include accrued interest. If the institutional investor were to sell the MBS, the accrued interest (ai) would need to be added to the flat price to determine the invoice price. The normal practice is to determine accrued interest based on the time period between the settlement date (SD) (usually two days after the trade date) and the first day of the month, M_0. If the coupon rate on the Fannie Mae MBS held by the institutional investor were 5% and the time period between SD and M_0 were 20 days, then the accrued interest would be $25,556:

$$ai_t = \frac{SD - M_0}{30}\frac{WAC}{12}F_t$$

$$ai_t = \left(\frac{20}{30}\right)\left(\frac{.05}{12}\right)\$9{,}200{,}000 = \$25{,}556$$

Extension Risk and Average Life

Like other fixed-income securities, the value of an MBS is determined by the MBS's future cash flow, maturity, credit risk, liquidity, and other features germane to fixed-income securities. In contrast to other bonds, though, MBSs are also subject to prepayment risk. As discussed earlier, the mortgage borrower's option to prepay makes it difficult to estimate the cash flow from the MBS. The prepayment risk associated with an MBS is primarily a function of interest rates. If interest rates decrease, then the prices of MBSs, like the prices of all bonds, increase as a result of lower discount rates. However, the decrease in rates will also augment prepayment speed, causing the earlier cash flow of the mortgages to be larger which, depending on the level of rates and the maturity remaining, could also contribute to increasing an MBS's price. In contrast, if interest rates increase, then the prices of MBSs will decrease as a result of higher discount rates and possibly the smaller earlier cash flow resulting from lower prepayment speeds.

The effect of an interest rate increase in lowering the price of the bond by decreasing the value of its cash flow is known as ***extension risk***. Extension risk can be described in terms of the relationship between interest rates and the MBS's ***average life***. The average life of an MBS is the weighted average of the security's time periods, with the weights being the periodic principal payments (scheduled and prepaid principal) divided by the total principal:

$$\text{Average life} = \frac{1}{12}\sum_{t=1}^{T} t\left(\frac{\text{Principal received at } t}{\text{Total principal}}\right)$$

For example, the average life for the MBS issue described in Table 11.5 is 9.18 years:

$$\text{Average life} = \frac{1}{12}\left(\frac{1(\$220{,}748) + 2(\$246{,}153) + \cdots + 355(\$49{,}061)}{\$100{,}000{,}000}\right) = 9.18 \text{ years}$$

The average life of an MBS depends on prepayment speed. For example, if the PSA speed of the $100 million MBS issue were to increase from 150 to 200, the MBS's average life would decrease from 9.18 to 7.55, reflecting greater principal payments in the earlier years; in contrast, if the PSA speed were to decrease from 150 to 100, then the average life of the MBS would increase to 11.51. For MBSs, prepayment risk can be evaluated in terms of how responsive an MBS's average life is to changes in prepayment speeds:

$$\text{Prepayment risk} = \frac{\Delta\text{Average life}}{\Delta\text{PSA}}$$

Thus, an MBS with an average life that did not change with PSA speeds, in turn, would have stable principal payments over time and would be absent of prepayment risk. Moreover, one of the more creative developments in the security market industry over the last two decades has been the creation of derivative securities formed from MBSs that have different prepayment risk characteristics, including some that are formed that have average lives that are invariant to changes in prepayment rates. The most popular of these derivatives are collateralized mortgage obligations and stripped MBSs.

11.6 COLLATERALIZED MORTGAGE OBLIGATIONS AND STRIPS

To address the problems of prepayment risk, many MBS issuers began to offer ***collateralized mortgage obligations (CMOs)***. Introduced in the mid-1980s, these securities are formed by dividing the cash flow of an underlying pool of mortgages or an MBS issue into several classes, with each class having a different claim on the mortgage collateral and with each sold separately to different types of investors. The different classes making up a CMO are called ***tranches*** or bond classes. There are two general types of CMO tranches—sequential-pay tranches and planned amortization class tranches.

Sequential-Pay Tranches

A CMO with sequential-pay tranches, called a ***sequential-pay CMO***, is divided into classes with different priority claims on the collateral's principal. The tranche with the first priority claim has its principal paid entirely before the next priority class, which has its principal paid before the third class, and so on. Interest payments on most CMO tranches are made until the tranche's principal is retired.

An example of a sequential-pay CMO is shown in Table 11.6. This CMO consists of three tranches, A, B, and C, formed from the collateral making up the $100 million agency MBS described in Table 11.5. In terms of the priority disbursement rules, tranche A receives all principal payment from the collateral until its principal of $50 million is retired. No other tranche's principal payments are disbursed until the principal on A is paid. After tranche A's principal is retired, all principal payments from the collateral are then made to tranche B until its principal of $30 million is retired. Finally, tranche C receives the remaining principal that is equal to its par value of $20 million. Even though the principal is paid sequentially, each tranche does receive interest each period equal to its stated pass-through rate (7.5%) times its outstanding balance at the beginning of each month.

Given the different possible prepayment speeds, the actual amount of principal paid each month and the time it will take to pay the principal on each tranche is uncertain. Table 11.6 shows the cash flow patterns on the three tranches based

TABLE 11.6 Cash Flows from Sequential-Pay CMO
Collateral: Balance = $100m, WAM = 355 Months, WAC = 8%, PT Rate = 7.5%,
Prepayment: 150 PSA, Tranches: A: $50 million, B = $30 million, C = $20 million

	Par = $100m		Rate = 7.5%	A: Par = $50m		Rate = 7.5%	B: Par =$30m		Rate = 7.5%	C: Par = $20m		Rate = 7.5%
Period	Collateral			Tranche A			Tranche B			Tranche C		
Month	Balance	Interest	Principal	Balance	Interest	Principal	Balance	Principal	Interest	Balance	Principal	Interest
	$100,000,000			$50,000,000			$30,000,000			20000000	0	
1	$100,000,000	$625,000	$220,748	$50,000,000	$312,500	$220,748	$30,000,000	0	$187,500	$20,000,000	0	$125,000
2	$99,779,252	$623,620	$246,153	$49,779,252	$311,120	$246,153	$30,000,000	0	$187,500	$20,000,000	0	$125,000
3	$99,533,099	$622,082	$271,449	$49,533,099	$309,582	$271,449	$30,000,000	0	$187,500	$20,000,000	0	$125,000
4	$99,261,650	$620,385	$296,617	$49,261,650	$307,885	$296,617	$30,000,000	0	$187,500	$20,000,000	0	$125,000
5	$98,965,033	$618,531	$321,637	$48,965,033	$306,031	$321,637	$30,000,000	0	$187,500	$20,000,000	0	$125,000
85	$51,626,473	$322,665	$471,724	$1,626,473	$10,165	$471,724	$30,000,000	0	$187,500	$20,000,000	0	$125,000
86	$51,154,749	$319,717	$467,949	$1,154,749	$7,217	$467,949	$30,000,000	0	$187,500	$20,000,000	0	$125,000
87	$50,686,799	$316,792	$464,204	$686,799	$4,292	$464,204	$30,000,000	0	$187,500	$20,000,000	0	$125,000
88	$50,222,595	$313,891	$460,488	$222,595	$1,391	$222,595	$30,000,000	$237,893	$187,500	$20,000,000	0	$125,000
89	$49,762,107	$311,013	$456,802	0	0	0	$29,762,107	$456,802	$186,013	$20,000,000	0	$125,000
90	$49,305,305	$308,158	$453,144	0	0	0	$29,305,305	$453,144	$183,158	$20,000,000	0	$125,000
91	$48,852,161	$305,326	$449,515	0	0	0	$28,852,161	$449,515	$180,326	$20,000,000	0	$125,000
92	$48,402,646	$302,517	$445,915	0	0	0	$28,402,646	$445,915	$177,517	$20,000,000	0	$125,000
178	$20,650,839	$129,068	$222,016	0	0	0	$650,839	$222,016	$4,068	$20,000,000	0	$125,000
181	$19,990,210	$124,939	$216,625	0	0	0	0	0	0	$19,990,210	$216,625	$124,939
182	$19,773,585	$123,585	$214,856	0	0	0	0	0	0	$19,773,585	$214,856	$123,585
183	$19,558,729	$122,242	$213,101	0	0	0	0	0	0	$19,558,729	$213,101	$122,242
184	$19,345,627	$120,910	$211,360	0	0	0	0	0	0	$19,345,627	$211,360	$120,910
353	$148,527	$928	$49,958	0	0	0	0	0	0	$148,527	$49,958	$928
354	$98,569	$616	$49,508	0	0	0	0	0	0	$98,569	$49,508	$616
355	$49,061	$307	$49,061	0	0	0	0	0	0	$49,061	$49,061	$307

Tranche	Maturity	Window	Average Life
A	88 Months	87 Months	3.69 years
B	179 Months	92 Months	10.71 years
C	355 Months	176 Months	20.59 years
Collateral	355 Months	355 Months	9.18 years

PSA	Collateral	Tranche A	Tranche B	Tranche C
50	14.95	7.53	19.4	26.81
100	11.51	4.92	14.18	23.99
150	9.18	3.69	10.71	9.18
200	7.55	3.01	8.51	17.46
300	5.5	2.26	6.03	12.82

on a 150% PSA prepayment assumption. As shown, the first month cash flow for tranche A consists of a principal payment (scheduled and prepaid) of $220,748 and an interest payment of $312,500 [= (.075/12)($50,000,000) = $312,500]. In month 2, tranche A receives an interest payment of $311,120 based on the balance of $49,779,252 and a principal payment of $246,153. Based on the assumption of a 150% PSA speed, it takes 88 months before A's principal of $50 million is retired. During the first 88 months, the cash flows for tranches B and C consist of just the interest on their balances, with no principal payments made to them. Starting in month 88, tranche B begins to receive the principal payment. Tranche B is paid off in month 180, at which time principal payments begin to be paid to tranche C. Finally, in month 355 tranche C's principal is retired.

Features of Sequential-Pay CMOs By creating sequential-pay tranches, issuers of CMOs are able to offer investors maturities, principal payment periods, and average lives different from those defined by the underlying mortgage collateral. For example, tranche A in our example has a maturity of 88 months (7.33 years) compared to the collateral's maturity of 355 months; tranche B's maturity is 180 months (15 years); tranche C's maturity is 355 months (29.58 years). Each tranche also has a larger cash flow during the periods when their principal is being retired. The period between the beginning and ending principal payment is referred to as the ***principal pay-down window***. Tranche A has a window of 87 months, B's window is 92 months, and C's window is 176 months (see table at the bottom of Table 11.6). CMOs with certain size windows and maturities often are attractive investments for investors who are using cash flow-matching strategies. Moreover, issuers of CMOs are able to offer a number of CMO tranches with different maturities and windows by simply creating more tranches.

Finally, each of the tranches has an average life that is either shorter or longer than the collateral's average life of 9.18 years. With a 150 PSA model, tranche A has an average life of 3.69 years, B has an average life of 10.71 years, and C has a life of 20.59 years. In general, a CMO tranche with a lower average life is less susceptible to prepayment risk. Such risk, though, is not eliminated. As noted earlier, if prepayment speed decreases, an MBS's average life will increase, resulting in lower-than-projected early cash flow and therefore lower returns. In the table at the bottom of Table 11.6, the average lives for the collateral and the three tranches are shown for different PSA models. Note that the average life of each of the tranches still varies as prepayment speed changes.

Accrual Tranche Most sequential-pay CMOs have an ***accrual bond class***. Such a tranche, also referred to as the ***Z bond***, does not receive current interest but has it deferred. The Z bond's current interest is used to pay down the principals on the other tranches, increasing their speeds and reducing their average lives. For example, suppose in our illustrative sequential-pay CMO example we make tranche C an accrual tranche in which its interest of 7.5% is to be paid to the earlier tranches and its principal of $20 million and accrued interest is to be paid after tranche B's principal has been retired (see Table 11.7). Since the accrual tranche's current interest of $125,000 is now used to pay down the other classes' principals, the other tranches now have lower maturities and average lives. For example, as shown in Table 11.7 the principal payment on tranche A is $345,748 in the first month ($220,748 of

TABLE 11.7 Cash Flows From Sequential-Pay CMO With Z Tranche
Collateral: Balance = $100m, WAM = 355 Months, WAC = 8%, PT Rate = 7.5%,
Prepayment: 150 PSA, Tranches: A: $50M, B = $30M, C = $20M

	Par = $100m	Rate = 7.5%		A: Par = $50m	Rate = 7.5%		B: Par = $30m	Rate = 7.5%		Z: Par = $20m	Rate = 7.5%	
Period	Collateral			A			B			Z		
Month	Balance $100,000,000	Interest	Principal	Balance $50,000,000	Interest	Principal	Balance $30,000,000	Principal	Interest	Bal.+Cum Int $20,000,000	Principal	Interest
1	$100,000,000	$625,000	$220,748	$50,000,000	$312,500	$345,748	$30,000,000	0	$187,500	$20,000,000	0	0
2	$99,779,252	$623,620	$246,153	$49,654,252	$310,339	$371,153	$30,000,000	0	$187,500	$20,125,000	0	0
3	$99,533,099	$622,082	$271,449	$49,283,099	$308,019	$396,449	$30,000,000	0	$187,500	$20,250,000	0	0
4	$99,261,650	$620,385	$296,617	$48,886,650	$305,542	$421,617	$30,000,000	0	$187,500	$20,375,000	0	0
5	$98,965,033	$618,531	$321,637	$48,465,033	$302,906	$446,637	$30,000,000	0	$187,500	$20,500,000	0	0
68	$60,253,239	$376,583	$540,668	$1,878,239	$11,739	$665,668	$30,000,000	0	$187,500	$28,375,000	0	0
69	$59,712,571	$373,204	$536,352	$1,212,571	$7,579	$661,352	$30,000,000	0	$187,500	$28,500,000	0	0
70	$59,176,219	$369,851	$532,069	$551,219	$3,445	$551,219	$30,000,000	$105,850	$187,500	$28,625,000	0	0
71	$58,644,150	$366,526	$527,821	0	0	0	$29,894,150	$652,821	$186,838	$28,750,000	0	0
72	$58,116,329	$363,227	$523,605	0	0	0	$29,241,329	$648,605	$182,758	$28,875,000	0	0
122	$36,470,935	$227,943	$350,111	0	0	0	$1,345,935	$475,111	$8,412	$35,125,000	0	0
123	$36,120,824	$225,755	$347,292	0	0	0	$870,824	$472,292	$5,443	$35,250,000	0	0
125	$35,429,038	$221,431	$341,719	0	0	0	0	0	0	$35,429,038	$341,719	$221,431
126	$35,087,319	$219,296	$338,966	0	0	0	0	0	0	$35,087,319	$338,966	$219,296
354	$98,569	$616	$49,508	0	0	0	0	0	0	$98,569	$49,508	$616
355	$49,061	$307	$49,061	0	0	0	0	0	0	$49,061	$49,061	$307

Tranche	Window	Average Life
A	69 Months	3.06 Years
B	54 Months	8.23 Years

scheduled and projected prepaid principal and $125,000 of Z's interest); in contrast, the principal is only $220,748 when there is no Z bond (see Table 11.6).

As a result of the Z bond, tranche A's window is reduced from 87 months to 69 months and its average life from 3.69 years to 3.06 years (see table at the bottom of Table 11.7).

Floating-Rate Tranches

In order to attract investors who prefer variable-rate securities, CMO issuers often create floating-rate and inverse floating-rate tranches. The monthly coupon rate on the floating-rate tranche is usually set equal to a reference rate such as the London Interbank Offer Rate, LIBOR, whereas the rate on the inverse floating-rate tranche is determined by a formula that is inversely related to the reference rate. An example of a sequential-pay CMO with floating and inverse floating tranches is shown in Table 11.8. The CMO is identical to our preceding CMO, except that tranche B has been replaced with a floating-rate tranche, FR, and an inverse floating-rate tranche, IFR. The par values of the FR and IFR tranches are equal to the par value of tranche B, with the FR tranche's par value of $22.5 million representing 75% of B's par value of $30 million, and the IFR's par value of $7.5 million representing 25% of B's par value. The rate on the FR tranche, R_{FR}, is set to the LIBOR plus 50 basis points, with the maximum rate permitted being 9.5%; the rate on the IFR tranche, R_{IFR}, is determined by the following formula:

$$R_{IFR} = 28.5 - 3\text{LIBOR}$$

This formula ensures that the weighted average coupon rate (WAC) of the two tranches will be equal to the coupon rate on tranche B of 7.5%, provided the LIBOR is less than 9.5%. For example, if the LIBOR is 8%, then the rate on the FR tranche is 8.5%, the IFR tranche's rate is 4.5%, and the weight average PT rate (WPTR) of the two tranches is 7.5%:

$$\begin{aligned} \text{LIBOR} &= 8\% \\ R_{FR} &= \text{LIBOR} + 50\text{bp} = 8.5\% \\ R_{IFR} &= 28.5 - 3\text{LIBOR} = 4.5\% \\ \text{WPTR} &= .75\,R_{FR} + .25\,R_{IFR} = 7.5\% \end{aligned}$$

TABLE 11.8 Sequential-Pay CMO With Floaters

Tranche	Par Value	PT Rate
A	$50,000,000	7.5%
FR	$22,500,000	LIBOR +50bp
IFR	$7,500,000	28.3 – 3 LIBOR
Z	$20,000,000	7.5%
Total	$100,000,000	7.5%

Notional Interest-Only Class

Each of the fixed-rate tranches in the previous CMOs has the same coupon rate as the collateral PT rate of 7.5%. Many CMOs, though, are structured with tranches that have different rates. When CMOs are formed this way, an additional tranche, known as a ***notional interest-only (IO) class,*** is often created. This tranche receives the excess interest on the other tranches' principals, with the excess rate being equal to the difference in the collateral's PT rate minus the tranches' PT rates. To illustrate, a sequential-pay CMO with a Z bond and notional IO tranche is shown in Table 11.9. This CMO is identical to our previous CMO with a Z bond, except that each of the tranches has a coupon rate lower than the collateral rate of 7.5% and there is a notional IO class. The notional IO class receives the excess interest on each tranche's remaining balance, with the excess rate based on the collateral rate of 7.5%. In the first month, for example, the IO class would receive interest of $87,500:

$$\text{Interest} = \left(\frac{.075 - .06}{12}\right)\$50{,}000{,}000 + \left(\frac{.075 - .065}{12}\right)\$30{,}000{,}000$$

$$\text{Interest} = \$62{,}500 + \$25{,}000 = \$87{,}500$$

In the table, the IO class is described as paying 7.5% interest on a notional principal of $15,333,333. This notional principal is determined by summing each tranche's notional principal. A tranche's notional principal is the number of dollars that makes the return on the tranche's principal equal to 7.5%. Thus, the notional principal for tranche A is $10,000,000 [= (.075 − .06)$50,000,000/.075], for B,

TABLE 11.9 Sequential-Pay CMO With Notional IO Tranche
Collateral: Balance = $100m, WAM = 355 Months, WAC = 8%, PT Rate = 7.5%, Prepayment = 150 PSA, Tranches: A: $50m, B = $30m, Z = $20m, Notional IO = $15.333333m

	Collateral:	Par = $100m	Rate = 7.5%	Tranche A:	Par = $50m	Rate = 6%	
Period		Collateral			A		
Month	Balance $100,000,000	Interest	Principal	Balance $50,000,000	Interest 0.06	Principal	Notional Interest 0.015
1	$100,000,000	$625,000	$220,748	$50,000,000	$250,000	$345,748	$62,500
2	$99,779,252	$623,620	$246,153	$49,654,252	$248,271	$371,153	$62,068
3	$99,533,099	$622,082	$271,449	$49,283,099	$246,415	$396,449	$61,604
4	$99,261,650	$620,385	$296,617	$48,886,650	$244,433	$421,617	$61,108
5	$98,965,033	$618,531	$321,637	$48,465,033	$242,325	$446,637	$60,581
70	$59,176,219	$369,851	$532,069	$551,219	$2,756	$551,219	$689
71	$58,644,150	$366,526	$527,821	0	0	0	0
72	$58,116,329	$363,227	$523,605	0	0	0	0
122	$36,470,935	$227,943	$350,111	0	0	0	0
123	$36,120,824	$225,755	$347,292	0	0	0	0
124	$35,773,533	$223,585	$344,494	0	0	0	0
125	$35,429,038	$221,431	$341,719	0	0	0	0
126	$35,087,319	$219,296	$338,966	0	0	0	0
127	$34,748,353	$217,177	$336,235	0	0	0	0
353	$148,527	$928	$49,958	0	0	0	0
354	$98,569	$616	$49,508	0	0	0	0
355	$49,061	$307	$49,061	0	0	0	0

\$4,000,000 [= (.075 − .065)\$30,000,000/.075], and for Z, \$1,333,333 [= (.075 − .07)\$20,000,000/.075], yielding a total notional principal of \$15,333,333.

Planned Amortization Class

Sequential-pay-structured CMOs provide investors with different maturities and average lives. As noted earlier, though, they are still subject to prepayment risk. A CMO with a ***planned amortization class*** (***PAC***), though, is structured such that there is virtually no prepayment risk. In a PAC-structured CMO, the underlying mortgages or MBSs (i.e., the collateral) are divided into two general tranches: the PAC (also called the PAC bond) and the ***support class*** (also called the ***support bond*** or the ***companion bond***). The two tranches are formed by generating two monthly principal payment schedules from the collateral; one schedule is based on assuming a relatively low PSA speed, whereas the other is obtained by assuming a relatively high PSA speed. The PAC bond is then set up so that it will receive a monthly principal payment schedule based on the minimum principal from the two principal payments. Thus, the PAC bond is designed to have no prepayment risk provided the actual prepayment falls within the minimum and maximum assumed PSA speeds. The support bond, on the other hand, receives the remaining principal balance and is therefore subject to prepayment risk.

To illustrate, suppose we form PAC and support bonds from the \$100 million collateral that we used to construct our sequential-pay tranches (underlying MBS = \$100 million, WAC = 8%, WAM = 355 months, and PT rate = 7.5%). To generate the minimum monthly principal payments for the PAC, assume a minimum speed of 100 PSA, referred to as the ***lower collar***, and a maximum speed of 300 PSA, called the ***upper collar***. Table 11.9 shows the principal payments (scheduled and

Tranche B: Par = \$30m Rate = 6.5%				Tranche Z: Par = \$20m Rate = 7%				Notional Par = \$15.333m
B				Z				
Balance 30,000,000	Principal	Interest 0.065	Notional Interest 0.01	Balance 20,000,000	Principal 0	Interest 0.07	Notional Interest 0.005	Notional Total CF
\$30,000,000	\$0	\$162,500	\$25,000	\$20,000,000	0	0	0	\$87,500
\$30,000,000	\$0	\$162,500	\$25,000	\$20,125,000	0	0	0	\$87,068
\$30,000,000	\$0	\$162,500	\$25,000	\$20,250,000	0	0	0	\$86,604
\$30,000,000	\$0	\$162,500	\$25,000	\$20,375,000	0	0	0	\$86,108
\$30,000,000	\$0	\$162,500	\$25,000	\$20,500,000	0	0	0	\$85,581
\$30,000,000	\$105,850	\$162,500	\$25,000	\$28,625,000	0	0	0	\$25,689
\$29,894,150	\$652,821	\$161,927	\$24,912	\$28,750,000	0	0	0	\$24,912
\$29,241,329	\$648,605	\$158,391	\$24,368	\$28,875,000	0	0	0	\$24,368
\$1,345,935	\$475,111	\$7,290	\$1,122	\$35,125,000	0	0	0	\$1,122
\$870,824	\$472,292	\$4,717	\$726	\$35,250,000	0	0	0	\$726
\$398,533	\$398,533	\$2,159	\$332	\$35,375,000	−\$54,038	\$206,354	\$14,740	\$15,072
0	0	0	0	\$35,429,038	\$341,719	\$206,669	\$14,762	\$14,762
0	0	0	0	\$35,087,319	\$338,966	\$204,676	\$14,620	\$14,620
0	0	0	0	\$34,748,353	\$336,235	\$202,699	\$14,478	\$14,478
0	0	0	0	\$148,527	\$49,958	\$866	\$62	\$62
0	0	0	0	\$98,569	\$49,508	\$575	\$41	\$41
0	0	0	0	\$49,061	\$49,061	\$286	\$20	\$20

prepaid) for selected months at both collars. The fourth column in the table shows the minimum of the two payments. For example, in the first month the principal payment is $170,085 for the 100 PSA and $374,456 for the 300 PSA; thus, the principal payment for the PAC would be $170,085. In examining the table, note that for the first 98 months the minimum principal payments come from the 100 PSA model, and from month 99 on, the minimum principal payment come from the 300 PSA model. Based on the 100-300 PSA range, a PAC bond can be formed that would promise to pay the principal based on the minimum principal payment schedule shown in Table 11.9.

The support bond would receive any excess monthly principal payment. The sum of the PAC's principal payments is $63,777,030. Thus, the PAC can be described as having a par value of $63,777,030, a coupon rate of 7.5%, a lower collar of 100 PSA, and an upper collar of 300 PSA. The support bond, in turn, would have a par value of $36,222, 970 ($100,000,000 – $63,777,030) and pay a coupon of 7.5% (see Table 11.9).

As noted, the objective in creating a PAC bond is to eliminate prepayment risk. In this example, the PAC bond has no risk as long as the actual prepayment speed is between 100 and 300. This can be seen by calculating the PAC's average life given different prepayment rates. The table at the bottom of Table 11.9 shows the average lives for the collateral, PAC bond, and support bond for various prepayment speeds ranging from 50 PSA to 350 PSA. As shown, the PAC bond has an average life of 6.98 years between 100 PSA and 300 PSA; its average life does change, though, when prepayment speeds are outside the 100-300 PSA range. In contrast, the support bond's average life changes as prepayment speed changes. In fact, changes in the support bond's average life due to changes in speed are greater than the underlying collateral's responsiveness.

Other PAC-Structured CMOs

The PAC and support bond underlying a CMO can be divided into different classes. Often the PAC bond is divided into several sequential-pay tranches, with each PAC having a different priority in principal payments over the other. Each sequential-pay PAC, in turn, will have a constant average life if the prepayment speed is within the lower and upper collars. In addition, it is possible that some PACs will have ranges of stability that will increase beyond the actual collar range, expanding their effective collars.

In addition to a sequential structure, a PAC-structured CMO can also be formed with PAC classes having different collars; in fact, some PACs are formed with just one PSA rate. These PACs are referred to as ***targeted amortization class (TAC) bonds***. Finally, different types of tranches can be formed out of the support bond class. These include sequential-pay, floating and inverse-floating rate, and accrual bond classes.

Given the different ways in which CMO tranches can be formed, as well as the different objectives of investors, perhaps it is not surprising to find PAC-structured CMOs with as many as 50 tranches. In the mid-1990s, the average number of tranches making up a CMO was 23.[8]

TABLE 11.9 PAC and Support Bonds
PAC formed 100 AND 300 PSA MODEL
Collateral: Balance = $100m, WAM = 355 Months, WAC = 8%, PT Rate = 7.5%, Prepayment = 150 PSA

	PAC					Collateral				Support			
	Low PSA Pr	High PSA Pr	Min. Principal	Int	CF	Balance	Interest	Principal	CF	Principal	Balance	Interest	CF
Period Month	100	300		0.075		$100,000,000				Col Pr – PAC Pr		0.075	
1	$170,085	$374,456	$170,085	$398,606	$568,692	$100,000,000	$625,000	$220,748	$845,748	$50,662	$36,222,970	$226,394	$277,056
2	$187,135	$425,190	$187,135	$397,543	$584,678	$99,779,252	$623,620	$246,153	$869,773	$59,018	$36,172,308	$226,077	$285,095
3	$204,125	$475,588	$204,125	$396,374	$600,499	$99,533,099	$622,082	$271,449	$893,531	$67,324	$36,113,290	$225,708	$293,032
4	$221,048	$525,572	$221,048	$395,098	$616,147	$99,261,650	$620,385	$296,617	$917,002	$75,568	$36,045,966	$225,287	$300,856
5	$237,895	$575,064	$237,895	$393,716	$631,612	$98,965,033	$618,531	$321,637	$940,168	$83,742	$35,970,398	$224,815	$308,557
98	$381,871	$386,139	$381,871	$135,237	$517,108	$45,780,181	$286,126	$424,898	$711,025	$43,028	$24,142,190	$150,889	$193,916
99	$380,032	$379,499	$379,499	$132,851	$512,349	$45,355,283	$283,471	$421,491	$704,962	$41,993	$24,099,163	$150,620	$192,613
100	$378,204	$372,970	$372,970	$130,479	$503,449	$44,933,791	$280,836	$418,111	$698,947	$45,141	$24,057,170	$150,357	$195,498
101	$376,384	$366,552	$366,552	$128,148	$494,700	$44,515,680	$278,223	$414,758	$692,981	$48,205	$24,012,029	$150,075	$198,281
102	$374,575	$360,242	$360,242	$125,857	$486,099	$44,100,923	$275,631	$411,430	$687,061	$51,188	$23,963,824	$149,774	$200,962
201	$235,460	$61,932	$61,932	$19,312	$81,245	$15,978,416	$99,865	$183,776	$283,641	$121,844	$12,888,435	$80,553	$202,396
202	$234,395	$60,806	$60,806	$18,925	$79,731	$15,794,640	$98,716	$182,266	$280,982	$121,460	$12,766,592	$79,791	$201,251
203	$233,336	$59,699	$59,699	$18,545	$78,244	$15,612,374	$97,577	$180,768	$278,345	$121,069	$12,645,131	$79,032	$200,101
204	$232,283	$58,611	$58,611	$18,172	$76,783	$15,431,606	$96,448	$179,282	$275,729	$120,671	$12,524,062	$78,275	$198,946
205	$231,235	$57,542	$57,542	$17,806	$75,348	$15,252,325	$95,327	$177,807	$273,134	$120,265	$12,403,392	$77,521	$197,786
206	$230,193	$56,492	$56,492	$17,446	$73,938	$15,074,517	$94,216	$176,344	$270,560	$119,852	$12,283,127	$76,770	$196,622
354	$124,660	$2,559	$2,559	$32	$2,591	$98,569	$616	$49,508	$50,124	$46,948	$93,517	$584	$47,533
355	$124,203	$2,493	$2,493	$16	$2,509	$49,061	$307	$49,061	$49,368	$46,568	$46,568	$291	$46,859
			Par = $63,777,030							Par = $36,222,970			

PSA	Collateral	Average Life PAC	Life Support
50	14.95	7.90	21.50
100	11.51	6.98	19.49
150	9.18	6.98	13.05
200	7.55	6.98	8.55
250	6.37	6.98	5.31
300	5.50	6.98	2.91
350	4.84	6.34	2.71

Stripped Mortgage-Backed Securities

In the mid-1980s, FNMA introduced ***stripped mortgage-backed securities***. In general, stripped MBSs consist of two classes: a ***principal-only (PO) class*** and an ***interest-only (IO) class***. As the names imply, the PO class receives only the principal from the underlying mortgages, whereas the IO class receives just the interest.

In general, the return on a PO MBS is greater with greater prepayment speed. For example, a PO class formed with $100 million of mortgages (principal) and priced at $75 million would yield an immediate return of $25 million if the mortgage borrowers prepaid immediately. Since investors can reinvest the $25 million, this early return will have a greater return per period than a $25 million return that is spread out over a longer period. Because of prepayment, the price of a PO MBS tends to be more responsive to interest rate changes than an option-free bond. That is, if interest rates are decreasing, then like the price of most bonds, the price of a PO MBS will increase. In addition, the price of a PO MBS is also likely to increase further because of the expectation of greater earlier principal payments as a result of an increase in prepayment caused by the lower rates. In contrast, if rates are increasing, the price of a PO MBS will decrease as a result of both lower discount rates and lower returns from slower principal payments. Thus, like most bonds, the prices of PO MBSs are inversely related to interest rates, and, like other MBSs with embedded principal prepayment options, their prices tend to be more responsive to interest rate changes.

Cash flows from an I0 MBS come from the interest paid on the mortgage portfolio's principal balance. In contrast to a PO MBS, the cash flows and the returns on an IO MBS will be greater, the slower the prepayment rate. For example, if the mortgages underlying a $100 million, 7.5% MBS with PO and IO classes were paid off in the first year, then the IO MBS holders would receive a one-time cash flow of $7.5 million [= (.075)($100,000,000)]. If $50 million of the mortgages were prepaid in the first year and the remaining $50 million in the second year, then the IO MBS investors would receive an annualized cash flow over two years totaling $11.25 million [= (.075) ($100,000,000) + (.075)($100,000,000 – $50,000,000)]; if the mortgage principal is paid down $25 million per year, then the cash flow over four years would total $18.75 million [= (.075)($100m) + (.075)($100m – $25m) + (.075)($75m – $25m) + (.075)($50m – $25m)]. Thus, IO MBSs are characterized by an inverse relationship between prepayment speed and returns: the slower the prepayment rate, the greater the total cash flow on an IO MBS. Interestingly, if this relationship dominates the price and discount rate relation, then the price of an IO MBS will vary directly with interest rates.

Examples of a PO MBS and an IO MBS are shown in Table 11.10. The stripped MBSs are formed from the collateral described in Table 11.2 (Mortgage = $100 million, WAC = 8%, PT Rate = 8%, WAM = 360, and PSA = 100).

The table at the bottom of Table 11.10, in turn, shows the values of the collateral, PO MBS, and IO MBS for different discount rate and PSA combinations of 8% and 150, 8.5% and 125, and 9% and 100. As shown in the table, the IO MBS is characterized by a direct relation between its value and rate of return.

Note that issuers can form IO and PO classes not only with MBSs, but also with CMOs. For example, one of the tranches of the PAC-structured CMOs or sequential-structured CMOs discussed in the preceding sections could be divided into an IO

TABLE 11.10 Projected Cash Flows for Stripped PO and IO
Collateral: Mortgage Portfolio = $100 million, WAC = 8%, WAM = 360 Months, Prepayment: 100% PSA

	Collateral						Stripped	
Period Month	Balance $100,000,000	Interest	Scheduled Principal	Prepaid Principal	Total Principal	CF	PO	IO
1	$100,000,000	$666,667	$67,098	$16,671	$83,769	$750,435	$83,769	**$666,667**
2	$99,916,231	$666,108	$67,534	$33,344	$100,878	$766,986	$100,878	**$666,108**
3	$99,815,353	$665,436	$67,961	$50,011	$117,973	$783,409	$117,973	**$665,436**
4	$99,697,380	$664,649	$68,380	$66,664	$135,044	$799,694	$135,044	**$664,649**
5	$99,562,336	$663,749	$68,790	$83,294	$152,084	$815,833	$152,084	**$663,749**
100	$58,669,646	$391,131	$83,852	$301,307	$385,159	$776,290	$385,159	**$391,131**
101	$58,284,486	$388,563	$83,977	$299,326	$383,303	$771,866	$383,303	**$388,563**
200	$27,947,479	$186,317	$97,308	$143,234	$240,542	$426,858	$240,542	**$186,317**
201	$27,706,937	$184,713	$97,453	$141,996	$239,449	$424,162	$239,449	**$184,713**
358	$371,778	$2,479	$123,103	$1,279	$124,382	$126,861	$124,382	**$2,479**
359	$247,395	$1,649	$123,287	$638	$123,925	$125,574	$123,925	**$1,649**
360	$123,470	$823	$123,470	0	$123,470	$124,293	$123,470	**$823**

	Price	Sensitivity		
Discount Rate	PSA	Value of PO	Value of IO	Value of Collateral
8.00%	150	$54,228,764	$47,426,196	$101,654,960
8.50%	125	$49,336,738	$49,513,363	$98,850,101
9.00%	100	$44,044,300	$51,795,188	$95,799,488

class and a PO class. Such tranches are referred to as ***CMO strips***. CMOs can also be formed from PO MBSs. These CMOs are called ***PO-collateralized CMOs***.

11.7 EVALUATING MORTGAGE-BACKED SECURITIES

Like all securities, MBSs can be evaluated in terms of their characteristics. With MBSs, such an evaluation is more complex because of the difficulty in estimating cash flows due to prepayment. Two approaches are used to evaluate MBS and CMO tranches: yield analysis and vector analysis.

Yield Analysis

Yield analysis involves calculating the yields on MBSs or CMO tranches given different prices and prepayment speed assumptions or alternatively calculating the values on MBSs or tranches given different rates and speeds. For example, suppose an institutional investor is interested in buying a MBS issue described by the collateral in Table 11.5. This MBS issue has a par value of $100 million, WAC = 8%, WAM = 355 months, and a PT rate of 7.5%. The value, as well as average life, maturity, duration, and other characteristics of this security, would depend on the rate the investor requires on the MBS and the prepayment speed she estimates. If the investor's required return on the MBS is 9% and her estimate of the PSA speed is 150, then she would value the MBS issue at $93,702,142. At that rate and speed, the MBS would have an average life of 9.18 years. Whether a purchase of the MBS issue at $93,702,142 to yield 9% represents a good investment depends, in part, on rates for other securities with similar maturities, durations, and risk, and in part, on how good the prepayment rate assumption is. For example, if the investor felt that the prepayment rate should be the 100 PSA and her required rate with that level of prepayment is 9%, then she would price the MBS issue at $92,732,145 and the average life would be 11.51 years. In general, for many institutional investors the decision on whether or not to invest in a particular MBS or tranche depends on the price the institution can command. For example, based on an expectation of a 100 PSA, our investor might conclude that a yield of 9% on the MBS would make it a good investment. In this case, the investor would be willing to offer no more than $92,732,145 for the MBS issue.

One common approach used in conducting a yield analysis is to generate a matrix of different yields by varying the prices and prepayment speeds. Table 11.11 shows the different values for our illustrative MBS given different combinations of required rates and prepayment speeds. Using this matrix, an investor could determine, for a given price and assumed speed, the estimated yield, or determine, for a given speed and yield, the price. Using this approach, an investor can also evaluate for each price the average yield and standard deviation over a range of PSA speeds.

Vector Analysis

One of the limitations of the above yield analysis is the assumption that the PSA speed used to estimate the yield is constant during the life of the MBS; in fact, such an analysis is sometimes referred to as ***static yield analysis***. In practice, prepayment

TABLE 11.11 Cash Flow Analysis
Mortgage Portfolio = $100M, WAC = 8%, WAM = 355 Months, PT Rate = 7.5

Rate/PSA	50	100	150
	Value	*Value*	*Value*
7%	$106,039,631	$105,043,489	$104,309,207
8%	$98,251,269	$98,526,830	$98,732,083
9%	$91,442,890	$92,732,145	$93,702,142
10%	$85,457,483	$87,554,145	$89,146,871
Average Life	14.95	11.51	9.18
	Vector Month Range: PSA	*Vector* Month Range: PSA	*Vector* Month Range: PSA
	1–50: 200	1–50: 200	1–50: 200
	51–150: 250	51–150: 300	51–150: 150
	151–250: 150	151–250: 350	151–250: 100
	251–355: 200	251–355: 400	251–355: 50
Rate	*Value*	*Value*	*Value*
7%	$103,729,227	$103,473,139	$104,229,758
8%	$98,893,974	$98,964,637	$98,756,370
9%	$94,465,328	$94,794,856	93,826,053
10%	$90,395,704	$90,929,474	89,364,229

speeds change over the life of an MBS as interest rates change in the market. To address this, a more dynamic yield analysis, known as ***vector analysis***, can be used. In applying vector analysis, PSA speeds are assumed to change over time. In the above case, a matrix of values for different rates can be obtained for different PSA vectors formed by dividing the total period into a number of periods with different PSA speeds assumed for each period. A vector analysis example is also shown in Table 11.11. In Chapter 14, we will examine how binomial interest rate models can be used to identify vectors to value MBSs.

11.8 CONCLUSION

Up until the mid-1970s most mortgages originated when savings and loans, commercial banks, and other thrifts borrowed funds or used their deposits to provide loans, possibly later selling the resulting instruments in the secondary market to Fannie Mae, Freddie Mac, or Ginnie Mae. To a large degree, residential real estate until then was financed by individual deposits, with little financing coming from institutional investors. In an effort to attract institutional investors' funds away from corporate bonds and other securities, as well as to minimize their poor hedge, financial institutions began to sell mortgage-backed securities. Over time, these securities were structured in different ways (as PACS, POs, IOs, etc.) to make them more attractive to different types of investors. Today, MBSs have becoming one of the most popular securities held by institutional investors, competing with a number of different types of bonds for inclusion in the portfolios of institutional investors. More significantly, they have revolutionized the way in which real estate is financed.

MBSs represent the largest and most extensively developed asset-backed security. Since 1985, a number of other asset-backed securities have been developed. The most common types are asset-backed securities backed by nonagency residential mortgages, commercial mortgages, automobile loans, credit card receivables, and home equity loans. These asset-backed securities are examined in the next chapter.

WEB INFORMATION

1. MBS price information:
   Wall Street Journal

   Go to http://online.wsj.com/public/us, "Market Data," "Bonds, Rates, & Credit Markets," and "Mortgage-Backed Securities, CMO."

   Investinginbonds.com

   MBS Price Index: The Merrill Lynch Mortgage-Backed Securities (MBS) Index is a statistical composite tracking the overall performance of the mortgage-backed securities market over time. The index includes U.S. dollar-denominated 30-year, 15-year and balloon pass-through mortgage securities.
   - Go to http://investinginbonds.com/; click "MBS/ABS Market At-A-Glance."
2. For information on the mortgage industry, statistics, trends, and rates, go to www.mbaa.org.
3. For information on mortgage rates in different geographical areas, go to www.interest.com.
4. For historical 30-year mortgage rates, go to http://research.stlouisfed.org/fred2 and click on "Interest Rates."
5. Agency information:
   - Fannie Mae: www.fanniemae.com
   - Ginnie Mae: www.ginniemae.gov
   - Freddie Mac: www.freddiemac.com
6. Agency MBS prospectus and other information:
   - Fannie Mae information and prospectus:
     Use Advance Search to find an MBS and its pool number:

   Go to Fannie Mae: www.fanniemae.com, "Site Map," "Mortgage-Backed Securities," and "More Search Options."

   Or go to http://sls.fanniemae.com/slsSearch/Home.do.

   Use pool number to find information on Fannie Mae MBSs.

   Information includes prospectus and common pool information.

   - Ginnie Mae information and prospectus:
     Go to Ginnie Mae: www.ginniemae.gov.
     To find pool number, go to "Multiple Issue Pool Number" found under "Issuer."

To find prospectus on MBS, go to "REMIC Offering Circulars" under "Investors."
Or go to www.ginniemae.gov/investors/prospectus.asp?Section=Investors

- Freddie Mac information on types of MBS:
  www.freddiemac.com
  Go to "Mortgage Securities."

7. Office of Federal Housing Enterprise Oversight: www.ficc.com
8. Rating Agencies:
   - www.moodys.com
   - www.standardandpoors.com
   - http://reports.fitchratings.com
9. For Moody's information on MBSs:
   - www.moodys.com
   - Search for "Structured Finance."
   - Search for "Historical Performance" and look for Structured Finance Default Studies.

KEY TERMS

2/28 ARM or teaser loan
100 SDA
accrual bond class
adjustable-rate mortgage (ARM)
agency CMOs
agency MBSs
agency mortgages
agency pass-throughs
Alt-A-loan
asset-backed security
average life
back PTI
balloon loan
burnout factor
conduit
conventional loan
CMO strips
collateralized mortgage obligations (CMOs)
conditional prepayment rate (CPR)
conforming loan
conventional loan
conventional mortgage
curtailment
Department of Veterans Affairs (VA)
discount points or points
due-on-sale
extension risk
Federal Housing Administration (FHA)
FICO score
front PTI
fully modified pass-through
graduated payment mortgages (GPMs)
hybrid ARM
insured mortgage
interest-only (IO) class
interest-only (IO) mortgages
lien
loan-to-value ratio (LTV)
lockout period
lower collar
modified pass-through
monthly factors
mortgage-backed securities (MBSs)
mortgage-backed security dealers association
mortgage bankers
mortgage pass-throughs
nonagency mortgages
nonagency MBSs
notional interest-only (IO) class
originator
pass-through
pass-through rate or coupon rate
payment-to-income ratio (PTI)
planned amortization class (PAC)
PO-collateralized CMOs

pool factor
prepayment risk
prepayment speed or speed
prime loan
principal pay-down window
principal-only (PO) class
private-label CMOs
private labels
private mortgage insurance (PMI)
PSA model (Public Securities Association)
real estate mortgage investment conduits (REMICs) recasting
reset mortgages
seasoning
securitization
sequential-pay CMO
single monthly mortality rate (SMM)
standard default assumption (SDA)
static yield analysis
stripped mortgage-backed securities
subprime loan
support class or companion bond
targeted amortization class (TAC) bonds
tranches
upper collar
vector analysis
weighted average coupon rate (WAC)
weighted average loan term
weighted average maturity (WAM)
whole-loan CMOs
Z bond

PROBLEMS AND QUESTIONS

1. Suppose ABC Bank has a fixed-rate mortgage portfolio with the following features:
   - Mortgage portfolio balance = $100,000,000
   - Weighted average coupon rate (WAC) = 8%
   - Weighted average maturity (WAM) = 360 months
   - Estimated prepayment speed = 150 PSA

   Complete the table:

Item	Month 1	Month 2
Balance	100,000,000	
Interest	666,667	
p	733,765	
Scheduled principal		
CPR		
SMM		
Prepaid principal		
Total principal		
Cash flow		

2. Suppose ABC Bank in Question 1 sells mortgage-backed securities backed by its $100 million portfolio of fixed-rate mortgages with the MBS having the following features:
   - Mortgage collateral = $100,000,000
   - Weighted average coupon rate (WAC) = 8%

- Weighted average maturity (WAM) = 360 months
- Estimated prepayment speed = 150 PSA
- MBS pass-through rate = PT rate = 7%
- ABC will service the mortgage portfolio.

a. Show in the table the first two months of cash flows going to the MBS investors.

Item	Month 1	Month 2
Balance	100,000,000	
Interest	583,333	
p	733,765	
Scheduled principal		
CPR		
SMM		
Prepaid principal		
Total principal		
Cash flow		

b. What compensation would ABC receive for servicing the mortgages?

3. Suppose the standard (100) prepayment profile for 10-year (120 month) conventional mortgages is one in which the CPR starts at zero and increases at a constant rate of .2% per month for 20 months to equal 4% at the 20th month; then after the 20th month the CPR stays at a constant 4%.

a. Show graphically the 100% prepayment profile.
b. In the same graph, show the prepayment profile for speeds of 200%, 150%, and 50% of the standard.
c. In the table below, show the first month's cash flow for an MBS portfolio of 10-year mortgages with the following features:

- Mortgage collateral = $50,000,000
- Weighted average coupon rate (WAC) = 9%
- Weighted average maturity (WAM) = 120 months
- Estimated prepayment speed = 200 PSA
- MBS pass-through rate = PT Rate = 8.5%

Item	Month 1
Balance	
Interest	
p	
Scheduled principal	
CPR	
SMM	
Prepaid principal	
Total principal	
Cash flow	

4. Explain some of the factors that determine the prepayment speed on a mortgage portfolio.

5. Define agency pass-throughs and describe some of their features.
6. Define private-label pass-throughs and describe some of their features.
7. What is the market value (clean price) of an 8% MBS issue backed by a mortgage pool with an original par value of $100 million, if its price is quoted at 105-16 with a pool factor of .95?
8. Explain how interest rate changes affect a MBS differently than an option-free bond.
9. Explain the relationship between extension risk, prepayment risk, and average life.
10. What was the primary motivation behind the creation of MBS derivatives in the 1980s?
11. Explain how the following CMOs are constructed and their features:
    a. Sequential-pay tranche
    b. Sequential-pay tranches with an accrual bond tranche
    c. Floating-rate and inverse floating-rate tranches
    d. Notional IO tranche
    e. PAC and support bonds
    f. Sequential-pay PAC
12. Explain the interest rate and value relation for a principal-only stripped MBS and an interest-only stripped MBS.
13. Suppose interest-only and principal-only stripped MBSs are formed from the following mortgage collateral:
    - Mortgage collateral = $100,000,000
    - Weighted average coupon rate (WAC) = 8%
    - Weighted average maturity (WAM) = 360 months
    - Estimated prepayment speed = 150 PSA
    - MBS pass-through rate = PT rate = 7.5%

    Complete the table:

Period	Collateral	Collateral	Collateral	Collateral	Collateral	Stripped PO	Stripped IO
Month	Balance	Interest	Scheduled Principal	Prepaid Principal	Total Principal	Cash Flow	Cash Flow
1	100,000,000						
2							

14. Given the following mortgage collateral and sequential-pay CMO:
    - Mortgage collateral = $100,000,000
    - Weighted average coupon rate (WAC) = 8%
    - Weighted average maturity (WAM) = 360 months
    - Estimated prepayment speed = 150 PSA
    - MBS pass-through rate = PT rate = 7.5%

- Tranche A receives all principal payment from the collateral until its principal of \$50 million is retired.
- Tranche B receives its principal of \$50 million after A's principal is paid.
- Tranche B receives interest each period equal to its stated coupon rate of 7.5% times its outstanding balance at the beginning of each month.

a. Complete the table:

Month	Collateral Balance	Collateral Interest	Collateral Principal	A Balance	A Interest	A Principal	B Balance	B Principal	B Interest
1	100,000,000								
92	50,324,347	314,527	460,885	324,347	2,027	324,347	50,000,000	136,538	312,500
93	49,863,462	311,647	457,196						

b. Suppose the CMO has a PT rate of 7% on Tranche A, a rate of 6.5% on Tranche B, and a notional principal. What would Tranche A's and Tranche B's interest receipts be in the first month? What would the notional principal tranche's cash flow be in the first month? What would be the quoted principal on the notional principal tranche?

15. Given the following mortgage collateral and sequential-pay CMO:

- Mortgage collateral = \$100,000,000
- Weighted average coupon rate (WAC) = 8%
- Weighted average maturity (WAM) = 360 months
- Estimated prepayment speed = 150 PSA
- MBS pass-through rate = PT rate = 7.5%
- Tranche A receives all principal payment from the collateral until its principal of \$50 million is retired.
- Tranche B receives its principal of \$25 million after A's principal is paid.
- Tranche B receives interest each period equal to its stated coupon rate of 7.5% times its outstanding balance at the beginning of each month.
- Tranche Z is an accrual bond that receives its principal of \$25 million after B's principal is paid

a. Complete the table:

Month	Collateral Balance	Collateral Interest	Collateral Principal	A Balance	A Interest	A Principal	B Balance	B Principal	B Interest	Z Balance	Z Principal	Z Interest
1	100,000,000	625,000	92,116									
2	99,907,884	624,424	117,586									
70	61,458,838	384,118	549,863	677,588	4,235	677,588	25,000,000	28,525	156,250	35,781,250	0	0
71	60,908,975	380,681	545,475	0	0	0	24,971,475	701,725	156,071	35,937,500	0	0
111	42,171,175	263,570	395,531	0	0	0	0	0	0	42,171,175	395,531	263,570

b. Suppose the CMO has a PT rate of 7% on Tranche A, a PT rate of 6.5% on Tranche B (Tranche Z's rate stays at 7.5%), and a notional principal tranche. What would Tranche A's and B's interest receipts be in the first month? What would the notional principal tranche's cash flow be in the first month? What would be the quoted principal on the notional principal tranche?

16. Given the following mortgage collateral and PAC:

- Mortgage collateral = $100,000,000
- Weighted average coupon rate (WAC) = 8%
- Weighted average maturity (WAM) = 360 months
- Estimated prepayment speed = 150 PSA
- MBS pass-through rate = PT rate = 7.5%
- PAC formed from the collateral with a lower collar of 100 and upper collar of 300
- Support bond receiving the residual principal

Complete the table:

Month	Collateral Balance	Collateral Interest	Collateral Principal	PAC Low PSA Principal	PAC High PSA Principal	PAC Minimum Principal	Support Principal
1	100,000,000						

17. What is yield analysis?

18. Explain the difference between static yield analysis and vector analysis.

19. Suppose ABC Bank has a fixed-rate mortgage portfolio with the following features:

- Mortgage portfolio balance = $100,000,000
- Weighted average coupon rate (WAC) = 8%
- Weighted average maturity (WAM) = 360 months
- Default loss profile = 100 SDA
- No prepayment

Complete the table:

Item	Month 1	Month 2
Beginning principal balance	100,000,000	
Interest	666,667	
p	733,765	
Scheduled principal	67,098	
Annual default rate		
Monthly default rate		
Dollar default loss		
Ending balance: Beginning balance – Default loss		
Cash flow		

20. Using the MBS Collateral with Default Loss Excel program, create an Excel table for the mortgage collateral described in Question 19. In your table, you may want to hide many of the rows and some of the columns. Do keep columns for period, balance, interest, scheduled principal, cumulative default loss, cumulative default rates, and cash flow. Using the program, find the following:

a. The cumulative default losses and default rates as a proportion of the principal for SDAs of 100, 200, and 300.

b. The cumulative default rates after months 30, 60, 120, and 360 for SDAs of 100, 200, and 300.

21. Using the MBS collateral Excel program, create an Excel table for the following agency MBS.

- Mortgage collateral = $50,000,000
- WAC = 7%
- PT rate = 6.5
- WAM = 350
- Seasoning = 10
- PSA = 75%

Note: In your table, you may want to hide many of the rows and some of the columns. Do keep columns for period, balance, interest, scheduled principal, prepaid principal, and cash flow.

22. Using the MBS collateral program, determine for the MBS in Question 21 the values and average lives for the following yield analysis matrix:

Discount Rate/PSA	**50**	**150**
	Value	Value
5%		
6%		
7%		
8%		
Average Life		

23. Using the MBS collateral Excel program, create an Excel table for a principal-only stripped MBS and interest-only stripped MBS formed from the MBS described in Question 21. In your table, hide many of the rows and hide all columns *except* the ones for period, collateral balance, collateral interest, collateral principal, cash flow for PO strip, and cash flow for IO strip.

24. Using the MBS collateral program:

a. Determine the values for the following yield analysis matrix for the PO and IO strip MBSs in Question 23.

PSA/Discount Rate	**50 PO**	**50 IO**	**150 PO**	**150 IO**
	Value	Value	Value	Value
5%				
6%				
7%				
8%				

b. Given PSA speeds increase as rates decrease, determine the values for the IO strip MBS for the following discount rate and PSA pairs.

Discount Rate	**PSA**	**Value of IO Strip**
5%	200	
6%	150	
7%	100	
8%	50	

c. Comment on the interest rate and value relation you observe.

25. Given the following MBS:

- Mortgage collateral = \$100,000,000
- Weighted average coupon rate (WAC) = 6%
- Weighted average maturity (WAM) = 180 months
- Standard (100%) prepayment model
- Number of periods to fixed CPR = 15
- Fixed CPR = .06
- Seasoning = 0
- MBS pass-through rate = PT rate = 5.5%

Determine the values and average lives for the following discount rate and prepayment speed pairs:

Discount Rate, Speed (as % of standard)	Value	Average Life
5%, 200		
6%, 150		
7%, 100		
8%, 50		

26. Given the following sequential-pay CMO with a notional IO tranche formed from the MBS in Question 25:

- Tranche A receives all principal payments first from the collateral until its principal of \$50 million is retired. Its interest is 5.5%
- Tranche B receives its principal of \$25 million after A's principal is paid. Its PT rate is 5%.
- Tranche C receives its principal of \$25 million after B's principal is paid. Its PT rate is 5%.
- Notional IO tranche receives the residual interest.

a. Using the MBSseqNIO Excel program, create an Excel table for the CMO. Assume the estimated PSA is 150. In your table, hide many of the rows and hide all columns except the ones for the period, the balance, interest, and principal for the collateral and each tranche, and the cash flows for the notional class.

b. Using the MBSseqNIO Excel program, determine the average lives and windows for the collateral and each tranche:

	Window	Average Life
Collateral		
Tranche A		
Tranche B		
Tranche C		

c. What is the principal for the notional IO tranche?

27. Given the following mortgage collateral and PAC:

- Mortgage collateral = \$100,000,000
- Weighted average coupon rate (WAC) = 7%
- Weighted average maturity (WAM) = 350 months
- Seasoning = 10 months

- Estimated prepayment speed = 150 PSA
- MBS pass-through rate = PT rate = 6.5%
- PAC formed from the collateral with a lower collar of 100 and upper collar of 300
- Support bond receiving the residual principal

a. Using the MBSpac Excel program, create an Excel table for the CMO. In your table, hide many of the rows and hide all columns except for the following: period, the balance, interest, and principal for the collateral, the interest, lower collar principal, upper collar principal, PAC principal, and cash flow for the PAC bond, and principal, interest, and cash flow for the support bond.

b. Using the MBSpac Excel program, determine the average life for collateral, PAC bond, and support bond given the PSA speeds shown in the table:

PSA	Collateral	PAC	Support
	Average Life	Average Life	Average Life
50			
100			
150			
200			
250			
300			
350			

c. Comment on the PAC's average life given the different PSA speeds.

WEB EXERCISES

1. Price and yield quotes on some MBSs can be found at the *Wall Street Journal* site. Go to the site and check the current price and yield information:
   - Go to http://online.wsj.com/public/us, "Market Data," "Bonds, Rates, & Credit Markets," and "Mortgage-Backed Securities, CMO."
2. The Merrill-Lynch Price Index can be found at Investinginbonds.com. Go to the site and check the current MBS index:
   - Go to http://investinginbonds.com/; click "MBS/ABS Market At-A-Glance."
3. Study the prospectus and common pool information on a Fannie Mae MBS by going to the Fannie Mae site and downloading a prospectus.
   - Use "Advanced Search" to find an MBS and its pool number:
   - Go to Fannie Mae: www.fanniemae.com, "Site Map," "Mortgage-Backed Securities," and "More Search Options."
   - Or go to http://sls.fanniemae.com/slsSearch/Home.do.
   - Use the pool number to find information on a Fannie Mae MBS.
   - Download the prospectus and common pool information.

4. Study the prospectus and common pool information on a Ginnie Mae MBS by going to the Ginnie Mae site and downloading a prospectus.
   - Go to Ginnie Mae: www.ginniemae.gov.
   - To find the pool number, go to "Multiple Issue Pool Number" under "Issuer."
   - To find prospectus on MBS, go to "REMIC Offering Circulars" under "Investors."
   - Or go to www.ginniemae.gov/investors/prospectus.asp?Section=Investors.
5. Study the key terms and information of some Freddie Mac MBSs:
   - Go to www.freddiemac.com.
   - Click "Mortgage Securities."
6. SIFMA provides a review of conditions and trends in MBSs. Go to their site and examine recent conditions:
   - Go to www.investinginbonds.com.
   - Click "MBS/ABS Market At-A-Glance."
7. Moody's provides information on default rates, ratings changes, and other credit information on MBSs. Examine Moody's study of historical default rates on MBSs:
   - Go to www.moodys.com.
   - Search for "Historical Performance" and look for "Structured Finance Default Studies."
8. Study the functions of the Depository Trust and Clearing Corporation by going to www.ficc.com and clicking "Mortgage-Backed Securities."

NOTES

1. ***Mortgage bankers*** are dealers, not bankers, who either provide mortgage loans or purchase them, holding them for a short period before selling them to a financial institution.
2. Many mortgages in the 1930s were balloon loans that the lender would not renew or the borrower defaulted. To redress the mortgage collapse of the 1930s, the government implemented a recovery program in which it assumed delinquent balloon loans and allowed the borrower to repay the loans over a longer period.
3. The Farmers Home Administration (FmHA) also provides mortgage insurance to qualified borrowers for agricultural-related real estate.
4. As noted in Chapter 7, Fannie Mae and Freddie Mac are government-sponsored enterprises (GSE), whereas Ginnie Mae is a federal agency. In our discussion of MBSs, we will refer to all three as being agencies.
5. Agencies can guarantee both interest and principal on a pass-through at the time payments are due (called a ***fully modified pass-through***) or they will guarantee both interest and principal but not at the time payments are due. For example, principal payments could be guaranteed to be paid one year later. This is called a ***modified pass-through***.
6. For nonagency MBSs, a prepayment benchmark profile is provided by the issuer in the prospectus. It is called the prospectus prepayment schedule.
7. After an MBS is issued, the WAM of the underlying pool will change. The remaining number of months to maturity of the mortgage collateral is sometimes referred to as the weighted average remaining maturity (WARM).

8. PAC-structured CMOs, as well as sequential-pay CMOs, are issued by agencies and financial institutions. By definition, CMOs issued by Fannie Mae, Ginnie Mae, and Freddie Mac are called ***agency CMOs***. CMOs issued by nonagencies in which the collateral consists of mortgage-backed securities that are guaranteed by one of the federal agencies are called ***private-label CMOs***. Finally, CMOs formed with a pool of unsecured mortgages or MBSs are called ***whole-loan CMOs***.

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CHAPTER 12

Nonagency Residential MBSs, Commercial MBSs, and Other Asset-Backed Securities

12.1 INTRODUCTION

Agency MBSs represent the largest and most extensively developed asset-backed security. Since 1985, a number of other asset-backed securities have been developed. The most common types are nonagency residential MBSs, ***commercial MBSs***, and asset-backed securities backed by automobile loans, credit card receivables, and home equity loans. These asset-backed securities are structured as pass-throughs and many have prepayment tranches. Different from agency MBSs, though, the collateral backing these asset-backed securities is subject to credit and default risk.

12.2 NONAGENCY MORTGAGE-BACKED SECURITIES

MBSs created by one of the agencies are collectively referred to as ***agency MBSs***, and those created by private conduits are called ***nonagency MBSs*** or ***private labels.*** As discussed in Chapter 11, agency residential MBSs are created from conforming loans. All other mortgages that are securitized are nonagency MBSs. Nonagency residential MBSs can, in turn, be classified as either ***prime MBSs***, where the underlying mortgages are all prime, or ***subprime MBSs***, where the underlying mortgage pool consists of subprime mortgages. In grouping the different types of securitized assets (residential mortgages, commercial mortgages, and other assets), nonagency subprime MBSs are typically grouped with asset-backed securities and not mortgage-backed securities.

Default Loss and Credit Tranches

Nonagency MBSs or nonagency CMOs are subject to default losses. As we saw in Section 11.3, a portfolio of 30-year, 8% mortgages with a 100 standard default assumption (SDA) has a cumulative default rate after 120 months of 3.59% and one with a 300 SDA has a cumulative default rate of 10.42% (see Table 11.4).

Different from agency MBSs, investors of nonagency MBSs need to take into account the expected default losses in determining the credit spread for pricing such securities.

TABLE 12.1 Senior-Subordinated Structured MBS

Bond Class	Tranche	Principal	Credit Ratings
Senior	1	$400 million	AAA
Subordinate	2	$40 million	AA
Subordinate	3	$20 million	A
Subordinate	4	$10 million	BBB
Subordinate	5	$10 million	BB
Subordinate	6	$10 million	B
Subordinate	7	$10 million	Not Rated

MBS conduits also address credit risk on nonagency MBSs by providing credit enhancements designed to absorb the expected losses from the underlying mortgage pool resulting from defaults. For nonagency MBSs or CMOs, credit enhancements include senior-subordinate structures, excess spreads, overcollateralization, and monoline insurance.

Senior-Subordinate Structures

An MBS issue with a ***senior-subordinate structure*** is formed with two general bond classes: a senior bond class and a subordinated bond class, with each class consisting of one or more tranches. Table 12.1 shows a $500 million senior-subordinate structured MBS with one senior bond class with a principal of $400 million and six subordinate or junior classes with a total principal of $100 million.

For this MBS issue, the default losses are absorbed first by Tranche 7 (starting at the bottom) and ascend up. Thus, if losses on the collateral are less than $10 million, then only Tranche 7 will experience a loss; if losses are $30 million, then Tranches 7, 6, and 5 will realize losses. The senior-subordinated structured MBS spreads the credit risk amongst the bond classes. This is referred to as ***credit tranching***. The rules for the distribution of the cash flows that include the distribution of losses are referred to as the ***cash flow waterfalls*** or simply ***waterfalls***. Because of the different levels of default risk, each of the subordinate tranches created in a senior-subordinate structured MBS are separately rated by Moody's or Standard & Poor's, with the lower tranches receiving lower ratings.

In evaluating the credit risk of the bond classes of a senior-subordinated structured MBS, the more subordinate claims that are issued relative to the senior claims, the more secured the senior claims. The proportion of the mortgage balance of the senior bond class to the total mortgage deal is referred to as ***senior interest*** (initial senior interest = $400m/$500m = .80); whereas the proportion of the mortgage balance of the subordinated bond classes to the total mortgage deal is referred to as ***subordinate interest*** (initial subordinate interest = $100m/$500m = .20). The greater the subordinate interest, the greater the level of credit protection for the senior bond.

Over the life of the MBS deal, the level of credit protection will change as principal is prepaid. In general, with prepayment, senior interest will increase and the subordinate interest will decrease over time. Because of this, most senior-subordinate

TABLE 12.2 Shifting Interest Schedule

Years after Issuance	Shifting Interest Percentage
1–5	100%
6	70%
7	60%
8	40%
9	20%
10	10%
After 10	0

structured MBS deals have a ***shifting interest schedule*** designed to maintain the credit protection for the senior bond class. The schedule is used to determine the allocation of prepayment that goes to the senior and subordinate tranches. Table 12.2 gives an example of a shifting interest schedule.

In determining the allocation to the senior holders, their percentage of prepayment is equal to their initial senior interest (for example, 80% = $400m/$500m) plus the shifting interest (based on the schedule) times the subordinate interest (20%) [= ($100m/$500m)]:

$$\begin{aligned}&\text{Senior prepayment percentage} = \text{Initial senior interest percent}\\&\quad + (\text{Shifting interest proportion})(\text{Initial subordinate interest})\end{aligned}$$

Based on the above schedule, 100% of the prepayment would go to the senior class for the first five years [= 80% + (1)(20%) = 100%]; 94% in year 6 [= 80% + (.70)(20%)], and 92% in year 7, and so on. After year 10, the allocation of principal between senior and subordinate classes would match their initial senior and subordinate interest proportions of 80% and 20%. The shifting-interest schedule from 100% to 70% in year 6, to 60% in year 7, to finally 0% after year 10 is known as a ***step-down provision;*** such a provision allows for reductions in the credit support over time.

In many senior-subordinated structured MBS deals, provisions are included that allow for changes in the shifting interest schedule if credit conditions related to the underlying collateral deteriorate. Typically, the provisions prohibit the step-down provision in the shifting interest schedule from occurring if certain performance measures are not met. For example, if the cumulative default losses exceed a certain limit of the original balance or if the 60-day delinquency rate exceeds a specified proportion of the current balance, then step-downs would not be allowed.

Other Credit Enhancements

Two other internal credit enhancements that can be found on nonagency MBSs are excess interest and overcollateralization. ***Excess interest*** (or ***excess spread***) is the interest from the collateral that is not being used to pay MBS investors and fees (mortgage servicing and administrative services). The excess spread can be used to offset any losses. If the excess interest is retained, it can be accumulated in an account and used to offset future default losses. When this is done, the excess interest can be set up similar to a notional IO class, with the proceeds going to a reserve account and paid out to IO holders at some future date if there is an excess. ***Overcollateralization*** is having the par value of the collateral exceed the value of the MBS issued. For example, if the MBS issue of $500 million had $550 million in collateral, the $50 million excess would then be used to absorb default losses.

Senior-subordinate structures, excess interest, and overcollateralization are internal credit enhancement tools. Some nonagency MBSs also have external credit enhancements in the form of insurance provided by monoline insurance companies such as the Finance Guarantee Insurance Corporation, the Capital Markets Insurance Corporation, or the Financial Security Assurance Company. The guarantees provided by monoline insurers, in turn, shift the default risk to the insurer.

12.3 COMMERCIAL MORTGAGE-BACKED SECURITIES

Commercial Mortgage Loans

Real estate property can be either residential or nonresidential. Residential includes houses, condominiums, and apartments; it is classified as either single-family or multiple-family. Nonresidential includes commercial and agricultural property. Commercial real estate loans are for income-producing properties. They are used to finance the purchase of the property or to refinance an existing one. Commercial property can include:

- Shopping centers
- Shopping strips
- Multifamily apartment buildings
- Industrial properties
- Warehouses
- Hotels
- Health care facilities

In contrast to residential mortgages where the interest and principal payments come from borrowers' income-generating ability or wealth, commercial mortgage loans come from income produced from the property. As such, commercial mortgage loans are referred to as ***non-recourse loans***. Lenders in assessing the credit quality of commercial loans look at the ***debt-to-service ratio*** [= Rental income – Operating expenses)/Interest payments] and the ***loan-to-value ratio,*** where value is equal to the present value of expected cash flows or the appraised value.

Commercial mortgage loans also differ from residential mortgage loans in that they typically have prepayment protection. Such protection can take the form of prepayment penalties, provisions prohibiting prepayment for a specified period, and defeasance. The latter is an agreement whereby the borrower agrees to invest funds in risk-free securities in an amount that would match the cash flows of a prepayment schedule. Finally, unlike residential mortgage loans in which the principal is amortized over the life of the loan, commercial mortgage loans are typically balloon loans. At the balloon date, the borrower is therefore obligated to pay the remaining balance. This is typically done by refinancing. As a result, the lender is subject to ***balloon risk***: the risk that the borrower will not be able to make the balloon payment because they cannot refinance or sell the property at a price that will cover the loan. With many commercial property loans, there is a ***special servicer*** who takes over the loan when default is imminent. These servicers have the responsibility to try to modify the loan terms to avert default.

Commercial Mortgage-Backed Assets

A commercial mortgage-backed security (CMBS) is a security backed by one or more commercial mortgage loans. Some CMBSs are backed by Fannie Mae, Freddie Mac, and Ginnie Mae. These agency CMBSs are limited to multifamily mortgages and health care facilities. Most CMBSs are private labels formed by either a single borrower with many properties or by a conduit with multiple borrowers. Similar to nonagency residential MBSs, many CMBSs have ***credit tranches*** (senior-subordinated structures), credit enhancements (overcollaterialization, excess interests, and monoline insurance) and prepayment tranches (sequential-pay, PACs, notional interest-only, floaters, etc.).

One feature common to residential and commercial mortgage-backed securities is ***cross-collateralization***: property used to secure one loan is also used to secure the other loans in the pool. Cross-collateralization prevents the MBS investors/lenders from calling the loan if there is a default, provided there is sufficient cash flows from the other loans to cover the loan's default loss. Such protection is called ***cross-default protection***. Unlike residential MBSs that tend to be formed with a larger number of homogeneous mortgages, commercial MBSs can be formed with a fewer number of loans and with some loans being more important to the pool than others. As a result, commercial MBSs often have less cross-default protection. To redress this, some commercial MBSs include a ***property release provision*** that requires the borrower of a commercial loan to pay a premium (e.g. 105% of par) if the property is removed from the pool. The provision is aimed at averting potential deterioration in the overall credit quality of the collateral when the best property in the pool is prepaid.

As noted, CMBSs can be formed from a single borrower with multiple properties. These deals are often set up by large real estate developers who use commercial MBSs as a way to finance or refinance their numerous projects: shopping malls, office buildings, hotels, apartment complexes, and the like. The other type of commercial MBS deal is one in which there are multiple borrowers or originators with the MBS set up through a conduit—a ***conduit deal***. When the deal has one large borrower or property combined with a number of smaller borrowers, the deal is referred to as a ***fusion conduit deal***. Conduit deals are often structured by large banks such as Bank of America, Wells Fargo, or JPMorgan Chase. Moreover, it is not uncommon for the conduit deal to be used to finance properties totaling as much as $1 billion, with as many as 200 property loans, varying in type (office, shopping center, warehouses, etc.), geographical distributions, and credit enhancements. With such large deals, there are different servicing levels. For example, there may be subservicing by the local originators who are required to collect payments and maintain records, a master servicer responsible for overseeing the commercial MBS deal, and a special servicer responsible for taking action if a loan becomes past due.

Commercial MBS investors include institutional investors. These investors, in turn, evaluate a commercial MBS issue not only in terms of the issue's general sensitivity to economic conditions and interest rates, but also assess each income-producing property on an ongoing basis.

12.4 ASSET-BACKED SECURITIES

Asset-backed securities (ABSs) are securities created from securitizing pools of loans other than residential prime mortgage loans and commercial loans; as noted, residential subprime MBSs are included in the ABS category. Loans used to create ABSs include home equity loans, credit card receivables, home improvement loans, trade receivables, franchise loans, small business loans, equipment leases, operating assets, and subprime mortgages. Like most securitized assets, ABSs can be structured with different prepayment and credit tranches and can include different credit enhancements. The three most common types of ABSs are those backed by automobile loans, credit card receivables, and home equity loans.

Automobile Loan-Backed Securities

Automobile loan-backed securities are often referred to as ***CARs*** (***certificates of automobile receivables***). They are issued by the financial subsidiaries of auto manufacturing companies, commercial banks, and finance companies specializing in auto loans. The automobile loans underlying these securities are similar to mortgages in that borrowers make regular monthly payments that include interest and a scheduled principal. Also like mortgages, automobile loans are characterized by prepayment. For such loans, prepayment can occur as a result of car sales, trade-ins, repossessions, wrecks, and refinancing when rates are low. CARs differ from MBSs in that they have shorter maturities, their prepayment rates are less influenced by interest rates than mortgage prepayment rates, and they are subject to greater default risk.

The prepayment for auto loans is typically measured in terms of the ***absolute prepayment speed (APS)***. APS measures prepayment as a percentage of the original collateral amount, instead of the prior period's balance. The relation between APS and the monthly prepayment rate (single monthly mortality rate maturity, SMM) is

$$\text{SMM} = \frac{\text{ABS}}{1 - (\text{ABS})(\text{M} - 1)}$$

where M = month. For example, if the absolute prepayment speed is 2%, then the monthly prepayment rate in month 25 is 3.8462%

$$\text{SMM} = \frac{\text{ABS}}{1 - (\text{ABS})(\text{M} - 1)}$$

$$\text{SMM} = \frac{.02}{1 - (.02)(25 - 1)} = .038462$$

A large part of auto manufacturers' sales are sold from installment sales contracts, with the company's credit department (often a financial subsidiary) making administrative decisions on extending credit, setting underwriting standards, originating loans, and then later servicing the loans. Automobile loan-backed securities are often created from these loans and typically issued by ***special purpose vehicles (SPVs)*** created by the manufacturer or its financial subsidiary; the financial subsidiary may also be set up as a special purpose vehicle. For example, a car manufacturer might have $500 million of installment loans resulting from monthly car sales. The manufacturer could set up (or may already have set up) an SPV to sell the installment loans for $500 million cash. The SPV would then sell the $500 million in securities backed by the loans as ABSs.

Instead of securitizing the installment loans as ABSs through an SPV, the auto manufacturer could have alternatively raised $500 million by issuing corporate notes, either as a debenture or collateralized by the installment loans. If the manufacturer were to default, though, all of its creditors would be able to go after all of its assets. If the manufacturer sells the installment loans to its SPV, though, the SPV owns the loans/assets and not the manufacturer. Thus, if the manufacturer were forced into bankruptcy, its creditors would not be able to recover the installment loans of the SPV. Thus, when the SPV issues ABSs, the investors only look at the credit risk associated with the installment loans and not the manufacturer. As a result, by financing with securitization via an SPV, the ABS issue often has a better credit rating and a lower rate than the manufacturer's notes.

It should be noted that, in practice, the manufacturer often uses a ***two-step securitization*** process whereby it first sells the loans to its financial subsidiary (an intermediate SPV) who then sells the loans to the SPV who creates the ABS. This two-step securitization process is done to ensure that the transaction is considered a true sale for tax purposes.[1]

In addition to the use of SPVs, ABSs also are characterized by having a number of features: credit tranches, overcollateralization, excess interest, sequential-pay tranches, and derivative positions. Figure 12.1 shows an example of an ABS deal of a representative U.S. auto manufacturer's financial subsidiary in which car loans are securitized.[2] The key features of the deal include:

1. Car loans totaling $1.1 billion purchased from the car manufacturer's financial subsidiary by a special purpose vehicle.
2. $1 billion of CARs (auto loan-backed securities) issued (overcollateralization).
3. A senior-subordinated structure consisting of $800 million senior class bonds (A) and $200 million subordinate class bonds (B, C, and D).
4. Bond classes Aa (A1a, 2a, A3a) are fixed rate.
5. Bond classes Ab (A1b and A2b) are floating rate.
6. Senior bond classes A are sequential pay: 1, 2, and 3.
7. Principal amount for senior fixed rate is $600 million:
   - A1a = $300 million
   - A2a = $200 million
   - A3a = $100 million
8. Principal amount for senior floating rate is $200 million
   - A1b = $100 million
   - A2b = $100 million

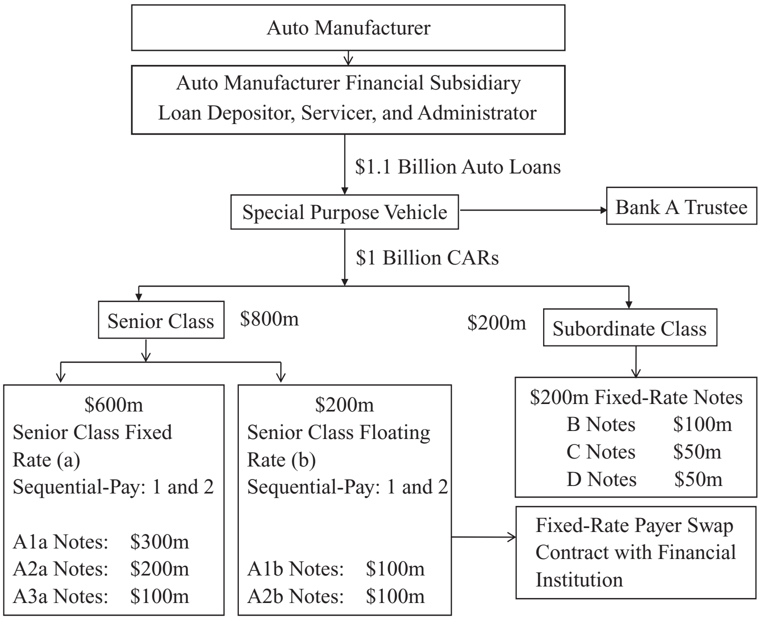


FIGURE 12.1 Auto Loan-Backed Security Deal

9. Principal amount for subordinate fixed rate is $200 million:
   - B = $100 million
   - C = $50 million
   - D = $50 million
10. The SPV entered into an interest rate swap with a financial institution for each of the floating rate bonds to fix the rate (swaps are discussed in Chapter 18).
11. Each month the cash flows from the collateral are used to pay the service fee and the payments to the swap.
12. Bank A is the Trustee.

Home Equity Loan-Backed Securities

Home equity loan-backed securities are referred to as ***HELs***. They are similar to MBSs in that they pay a monthly cash flow consisting of interest, scheduled principal, and prepaid principal. In contrast to mortgages, the home equity loans securing HELs tend to have a shorter maturity and different factors influencing their prepayment rates. The home equity loans forming the pool backing a HEL issue are also subject to default. Like nonagency MBSs, commercial MBSs, and CARs, HELs deals are often structured with different prepayment tranches, credit tranches, and credit enhancements.

Credit Card Receivable-Backed Securities Credit card receivable-backed securities are commonly referred to as ***CARDs*** (***certificates of amortized revolving debt***). Securitized assets formed with home equity loans, residential mortgages, and auto loans are backed by loans that are amortized. ABSs with amortizing assets are sometime referred to as ***self-liquidating structures***. In contrast, CARDs investors do not receive an amortized principal payment as part of their monthly cash flow. That is, the credit card receivables backing a CARD are nonamortizing loans where there is not a schedule of periodic principal payments. As such, prepayment does not apply for a pool of credit card receivable loans.

Credit cards are issued by banks (VISA and MasterCard), retailers, and global payment and travel companies (American Express). Credit card borrowers usually make a minimum principal payment in which, if the payment is less than the interest on the debt, the shortfall is added to the principal balance, and if it is greater, it is used to reduce the balance. The cash flow from a pool of card receivables comes from finance charges (interest charges based on unpaid balance), principal collected, and fees. The CARDs formed from a pool of credit card receivables are often structured with two periods. In one period, known as the ***lockout period*** (or ***revolving period***), all principal payments made on the receivables are retained and either reinvested in other receivables or invested in other securities. When new assets are added to an ABS deal, the structure is called a ***revolving structure***. In the other period, known as the ***principal-amortization period*** (or ***amortizing period***), all current and accumulated principal payments are distributed to the CARD holders.

In structuring an ABS secured by credit card receivables, the issuer often sets up a ***master trust*** where the credit card accounts meeting certain eligibility requirements are pledged. The master trust is very large, including millions of credit card accounts totaling billions of dollars. Numerous credit card deals or series are then issued from the master trust. Each series is, in turn, identified by a year and a number:

2007-1	2008-1	2009-1
2007-2	2008-2	2009-2
2007-3	2008-3	2009-3
2007-4	2008-4	
2007-5		

Each series has a lockout period where, as noted, the principal payments made by the credit card borrowers are retained by the trustee and reinvested in additional receivables or securities. During the lockout period, the cash flow to CARD investors comes from finance charges and fees. This period can last a number of years. The lockout period is followed by the principal amortizing period when principal received by the trustee is paid to CARD investors. There can also be an early amortizing provision in some series that requires early amortization of principal if certain events occur.

In evaluating a CARD series, investors often monitor the ***monthly payment rate (MPR)***: the monthly payment of finance charges, fees, and principal repayment from the credit card receivable portfolio (e.g., $50 million) as a percentage of the credit card debt outstanding (e.g., $500 million; MPR = 10%). For a CARD series with low or declining MPRs, there is a chance there may not be sufficient cash to pay off the principal. If there is an early amortization provision, an MPR falling below a threshold MPR would be the trigger for early amortization. Other important rate measures for evaluating CARDs include

- ***Gross portfolio yield***: Finance charges and fees collected as a proportion of the credit card debt outstanding.
- ***Charge-offs***: The accounts charged off as uncollectable as a proportion of the credit card debt outstanding.

- ***Net portfolio yield***: Gross profit yield minus charge-offs as a proportion of the credit card debt outstanding; this is the return CARD holders receive.
- ***Delinquency rate***: Proportion of receivables that are past due—30, 60, or 90 days.

Like many ABSs, CARDs are characterized by having a number of features. Figure 12.2 shows an example of a CARD deal of a representative credit card issuer. Key features of the series:

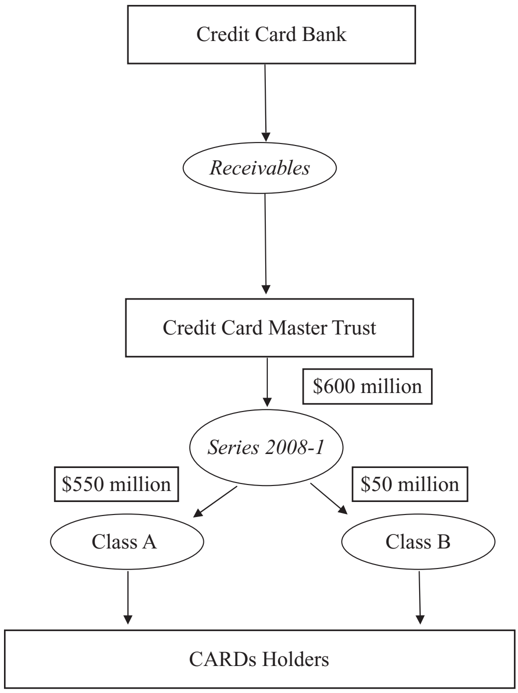
1. The issuing entity is the credit card issuer's master trust.
   - The depositor is the card issuer's finance corporation.
   - The sponsors and originators are the credit card issuer's bank.
   - The service is the credit card company.
   - The CARD is identified as 2008 Series 1.
2. Total CARDs issue is $600 million.
3. There is a senior-subordinate structure with $550 million issued to the Senior A class and $50 million to Subordinate B class.
4. Interest payment to each class is equal to the monthly LIBOR + spread.
5. The final payment date of the series is anticipated to be 2015.
6. Principal collected during the lockout period is to be used to invest in additional receivables. Principal is to be accumulated in a principal funding account.
7. In January 2012, the Trust will begin accumulating collection of receivables for principal repayment and begin distributing principal to Bond Class A and Bond Class B.
8. Early amortization is triggered if MPR for any three consecutive months is less than a specified base level.
9. If the collection of receivables is less than expected, principal may be delayed.

12.5 COLLATERALIZED DEBT OBLIGATIONS

As explained in Chapter 9, ***Collateralized Debt Obligations*** (**CDOs**) are securities backed by a diversified pool of one or more fixed-income assets or derivatives. Assets from which CDOs are formed include investment grade corporate bonds, asset-backed securities, high-yield corporate bonds, leveraged bank loans, distressed debt, residential mortgage-backed securities, commercial loans, commercial mortgage-backed securities, real estate investment trusts, municipal bonds, and emerging market bonds. The issuance of CDOs grew from the 1990s to 2007, but stopped in 2008 in the aftermath of the 2008 financial crisis. There are still, though, a number of issues outstanding.

CDO deals are set up with a collateral manager who is responsible for purchasing the debt obligation and managing the portfolio of debt obligations. CDOs can vary in terms of their objectives. There are four types of CDOs:

1. ***Cash flow CDOs*** that make periodic payment of interest and principal.
2. ***Market value CDOs*** that are characterized by total returns generated from the collateral: interest income, capital gains, and principal.



Provisions:

1. Interest payment to each class is equal to the monthly LIBOR + spread.
2. The final payment date of the series is anticipated to be 2015.
3. Principal collected during the lockout period is to be used to invest in additional receivables. Principal is to be accumulated in a "principal funding account."
4. In January 2012, the Trust will begin accumulating collection of receivables for principal repayment and begin distributing principal to Bond Class A and Bond Class B.
5. Early amortization is triggered if MPR for any three consecutive months is less than 10%.
6. If the collection of receivables is less than expected, principal may be delayed.

FIGURE 12.2 Credit Card-Backed Security Deal

3. ***Synthetic CDOs*** that are formed with derivatives.
4. ***Balance sheet CDOs*** consisting of bank loans in which the objective is to sell or remove the loans from the balance sheet.

Before the financial crisis of 2008, synthetic CDOs were one of the fastest growing segments of the CDO market. A common structure for a synthetic CDO was the issuance of the CDOs to finance the purchase of high-quality bonds with the CDO manager then entering into credit default swap contracts as the seller to enhance the return; that is, from the swap position the fund would receive premiums for providing default protection against a bond or bond portfolio. Credit default swaps are examined in Chapter 22.

Example

As with most asset-backed securities, CDOs are often structured with different tranches and credit enhancements. As an example, Table 12.3 shows a CDO with four tranches to be backed by a $200 million collateral investment consisting of fixed-rate, investment-grade bonds with a par value of $200 million, a weighted average maturity of five years, and yielding a return of 200 basis points over the five-year T-notes. The CDO's four tranches consist of

- A senior A1 tranche with a par value of $100 million, paying a fixed rate equal to the five-year T-note rate plus 150 basis points.
- A senior A2 tranche with a par value of $60 million and paying a floating rate equal to LIBOR plus 100 basis points.
- A subordinate B Tranche with a par value of $20 million and paying a fixed rate equal to the five-year T-note rate plus 200 basis points.
- A subordinate/equity tranche with a par value of $20 million that receives the excess return: return from collateral minus returns paid to the other tranches.

Since Tranche A2 pays a floating rate and the underlying collateral is to consist of fixed-rate bonds, the CDO deal allows the manager to take a derivative position to fix the rate on the A2 tranche. In this deal, the manager enters an interest rate swap contract to pay a fixed rate of 6% on a $60 million notional principal in return for the receipt of a floating rate payment equal to the LIBOR on a $60 million notional principal. As will be explained in more detail in Chapter 20, this interest rate swap contract when combined with the floating rate loan obligation on Tranche A2 serves to fix the rate on the tranche at 7% (see middle portion of Table 12.3).

If the initial investment of collateral were in investment-grade bonds yielding 8% when five-year Treasuries were yielding 6%, then the CDO deal would be expected to yield an excess return of $2.7 million in the first year, with the $2.7 million going to the subordinate/equity tranche (see the lower table of Table 12.3). As a rule, managers in structuring a cash flow CDO will estimate the expected return to the subordinate/equity tranche investors, as well as the return and risk of the tranches to determine the feasibility of the CDO deal.

CDO Restrictions

Restrictions are imposed on what the collateral manager can do. In the above example, the manager was required to invest the collateral in investment-grade bonds with an average maturity of five years. In general, the restrictions on CDOs include constraints on the payment of interest and principal to the CDO investors, the credit management of the portfolio, and the lengths of investment periods. For example, rules for the distribution of interest and principal could specify that the manager distribute all interest and principal to senior tranches but restrict the payment of

TABLE 12.3 Collateralized Debt Obligation Deal

Tranche	Par	Coupon	Coupon Rate
Senior A1	$100m	Fixed	5-year T-note Rate + 150bp
Senior A2	$60m	Floating	LIBOR + 100bp
Junior B	$20m	Fixed	5-year T-note Rate + 200bp
Subordinate/Equity	$20m	–	–

Collateral Requirements:
Investment-grade bonds
Weight average maturity of 5 years
Average quality rating of A

Swap
Manager will enter interest rate swap contracts to fix the rate on the A2 tranche.

Senior-Subordinate Structure:
Tranche B is subordinate to A1 and A2

Initial Collateral Investment:
$200 million investment in investment-grade portfolio yielding 8%
T-note rate at time of initial investment of 6%
Initial spread on portfolio of 200 basis points

Initial Swap Agreement:
CDO manager agrees to pay 6% on $60 million notional principal in return for a payment of LIBOR on $60 million.

Tranche A2	Pay LIBOR + 100 basis point	− (LIBOR + 1%)
Swap	Pay 6%	− 6%
Swap	Receive LIBOR	+ LIBOR
Net	Pay 6% + 1%	− 7%

Projected First-Year Cash Flow

Interest from collateral = (.08)($200m)	$16m
Payment to A1 tranche: ($100m)(.06 + .015)	− $7.5m
Payment to A2 tranche: (LIBOR + .01)($60m)	− (LIBOR + .01)($60m)
Interest paid to swap counterparty: (.06)($60M) = $3.6m	−$3.6m
Interest received from swap counterparty: (LIBOR)($60m)	+ (LIBOR)($60m)
Payment to B Tranche: (.06 + .02)($20m)	− $1.6m
Net	$2.7m
Payment to Subordinate/Equity Tranche	$2.7m

principal to subordinate tranches if certain credit conditions are not met (e.g. a coverage ratio not being met). There could also be credit restrictions that prohibit the manager from making certain investments if the asset fails to meet certain quality tests as it relates to the collateral's diversification, maturity, and average credit quality.[3] Such restriction are often specified in terms of a ***par value test*** that requires that the value of the underlying collateral be equal to a certain percentage (e.g., 110%) of the par value of the CDOs or the par value of the senior CDO class. If the collateral value were to drop below the par value test, then the manager would be required to take certain actions such as making all principal payments to senior tranche holders. Similarly, the restriction might be defined in terms of an ***interest coverage test*** that requires the collateral's return to meet interest payments.

Most CDO deals also have an early termination requirement if an ***event of default*** occurs. Such an event relates to conditions that could significantly impact the performance of the collateral. This could include a failure to comply with certain coverage ratios, the bankruptcy of an issuing credit, or the departure of the collateral management team. Many of the CDOs that were based on subprime mortgage loans resulted in the CDOs issuing ***events of default notices***.

12.6 CONCLUSION

The securitization process, the creation of prepayment tranches like PACs, the use of credit tranches such as senior-subordinate structures, the structuring of ABSs for credit cards, home equity loans, receivables, and other assets, and the formation and management of CDOs formed with interest rate swaps, credit default swaps, and different tranches all point to the innovativeness that has characterized the financial community over the last 20 years. These innovations, in turn, have revolutionized the way real estate property, accounts receivables, car loans, and other assets are financed. Moreover, like securitized assets, all of the fixed-income securities that we have examined over the last seven chapters demonstrate how well the financial system is able to create securities needed to finance the myriad investments of corporations, governments, intermediaries, and consumers. In the next chapter, we begin Part 3 where we examine how investment and portfolio managers select and analyze these various fixed-income securities.

KEY TERMS

absolute prepayment speed (APS)
agency MBSs
balance sheet CDOs
balloon risk
CARDs (Certificates of amortized revolving debt)
CARs (Certificates of automobile receivables)
cash flow CDOs
cash flow waterfalls (or waterfalls)
charge-offs
collateralized debt obligations (CDOs)
commercial MBS
conduit deal
credit tranches
credit tranching
cross-collateralization
cross-collateralization protection

cross-default protection
debt-to-service ratio
delinquency rate
event of default
events of default notices
excess spread (or excess interest)
fusion conduit deal
gross portfolio yield
home equity loan-backed securities (HELs)
interest coverage test
loan-to-value ratio
lockout period (or revolving period)
market value CDOs
master trust
monthly payment rate (MPR)
net portfolio yield
nonagency MBSs (or private labels)
non-recourse loans
overcollateralization
par value test
prime MBS
principal-amortization period (or amortizing period)
property release provision
revolving structure
self-liquidating structure
senior interest
senior-subordinate structure
shifting interest schedule
special purpose vehicles
special servicer
step-down provision
subordinate interest
subprime MBS synthetic CDOs
two-step securitization

WEB INFORMATION

1. ABS Price Index Information:
   Merrill-Lynch ABS Index: The Merrill Lynch Fixed-Rate Asset-Backed Index is a statistical composite tracking the overall performance of the fixed-rate asset-backed securities (ABS) market over time. The index includes U.S. dollar-denominated ABSs having a fixed coupon, a minimum amount outstanding of $25 million, and an investment-grade credit rating of BBB or higher. Go to http://investinginbonds.com/; click "MBS/ABS Market At-A-Glance."
2. Agency Information:
   - Fannie Mae: www.fanniemae.com
   - Ginnie Mae: www.ginniemae.gov
   - Freddie Mac: www.freddiemac.com
3. Rating Agencies:
   - www.moodys.com
   - www.standardandpoors.com
   - http://reports.fitchratings.com
4. For Moody's information on ABSs, Commercial MBSs, and CDOs:
   - www.moodys.com
   - Structured Finance
   - Look for ABS, Commercial MBS, Residential MBS, and CDO/Derivatives.
   - For each category, search for Watchlist, Credit Card Data Base (for ABS), Indices, and Special Reports.

(*continued*)

(*Continued*)

5. For Moody's information on default losses on ABSs, Commercial MBSs, and CDOs:
   - www.moodys.com
   - Search for "Study of historical default rates for corporate bonds."
   - Search for "Historical Performance" and look for Structured Finance Default Studies.

PROBLEMS AND QUESTIONS

1. Suppose the collateral backing a nonagency MBS consists of a fixed-rate residential mortgage portfolio with the following features:
   - Mortgage portfolio balance = $500,000,000
   - Weighted average coupon rate (WAC) = 6%
   - Weighted average maturity (WAM) = 360 months
   - No prepayment

   Using the MBS Collateral with Default Loss Excel program, determine the following:

   a. The cumulative default rates after months 30, 60, 120, and 360 for SDAs of 100, 200, and 300.

   b. Graph the cumulative default rates from the beginning to maturity (360th month) for SDAs of 100, 200, and 300.

2. Suppose a conduit was structuring a senior-subordinate structured MBS deal based on the collateral described in Question 1. Determine the allocations of the $500 million principal for the senior and subordinate classes that the conduit would need if he wanted to eliminate the projected default losses based on SDAs of 100, 200, and 300.

3. The table below shows a $300 million senior-subordinate structured MBS with one senior bond class and four subordinate classes:

   Senior-Subordinated Structured MBS

Bond Class	Tranche	Principal	Credit Ratings
Senior	1	$250 million	AAA
Subordinate	2	$30 million	AA
Subordinate	3	$10 million	A
Subordinate	4	$5 million	BBB
Subordinate	5	$5 million	B

   Answer the following:

   a. What is the senior interest?

   b. What is the subordinate interest?

   c. If the default losses on the collateral totaled $17 million, what tranches would absorb the loss?

4. Determine the percentage of prepayment that would go to the senior and subordinate classes described in Question 3 given the following shifting-interest schedule:

Years after Issuance	Shifting Interest Percentage
1–7	100%
8	80%
9	60%
10	40%
After 10	0%

5. Explain what is meant by a step-down provision as it relates to the shifting-interest schedule in Question 4.

6. Explain how excess interest is set up with an interest-only account to provide a credit enhancement to a nonagency MBS.

7. Overcollateralization is used as a credit enhancement for ABSs. Explain what impact the 2008 financial crisis had on the collateral on nonagency MBSs and MBSs backed by subprime mortgages.

8. Explain some of the features that characterize commercial mortgages.

9. Explain the similarities and differences between commercial and residential MBSs.

10. Explain the two types of commercial MBS deals.

11. What is the important principle analysts must adhere to when examining commercial MBS deals?

12. XYZ Inc. manufactures and sells machine tools with a large part of its sales coming from installment sales contracts. XYZ's credit department makes decisions on extending credit, originating loans, and servicing them. XYZ has $500 million installment sales contracts. It also would like to raise $500 million in new funds. Explain how XYZ would set up a special purchase vehicle to securitize its sales contracts as an alternative to issuing a $500 million debt issue. What are the advantages of SPV securitization by an SPV over issuing debt?

13. What is the main structural difference between an ABS formed with automobile loans, home equity loans, or residential mortgage loans and one formed with credit card receivables?

14. How is the payment of principal on an ABS backed by credit card receivables typically structured?

15. How do ABSs backed by automobile loans (CARs) differ from MBSs?

16. Suppose the estimated absolute prepayment speed (APS) on a pool of automobile loans is 2.5%. What would be the monthly prepayment rate for the mortgage pool in month 15?

17. ABC is setting up a CDO with four tranches to be backed by $300 million collateral investment consisting of fixed-rate, investment-grade bonds with a par value of $300 million, a weighted average maturity of 10 years, and yielding a return expected to pay 200 basis points over the 10-year T-notes. The CDO is to have four tranches:

- A senior A1 tranche with a par value of $150 million, paying a fixed rate equal to the 10-year T-note rate plus 150 basis points.
- A senior A2 tranche with a par value of $100 million and paying a floating rate equal to LIBOR plus 100 basis points.
- A subordinate B tranche with a par value of $20 million and paying a fixed rate equal to the 10-year T-note rate plus 200 basis points.
- A subordinate/equity tranche with a par value of $30 million that receives the excess return: return from collateral minus returns paid to the other tranches.

The managers will also enter a swap contract as a fixed-rate payer to fix the rate on Tranche A2.

Questions:

a. Suppose the manager enters an interest rate swap contract where the manager agrees to pay a fixed rate of 5% on a $100 million notional principal in return for the receipt of a floating rate payment equal to the LIBOR on a $100 million notional principal. Show in a table format how the interest rate swap, when combined with the floating rate loan obligation on Tranche A2, serves to fix the rate on the tranche at 6%.

b. If the manager expected the initial investment of collateral to be in investment-grade bonds yielding 7% when Treasuries were yielding 5%, determine the expected first-year cash flows to the collateral, each tranche, the swaps, and the equity/subordinate class.

18. Explain how a synthetic CDO uses a credit default swap combined with an investment in high-quality bonds.

19. Short-Answer Questions:

a. What is cross-collateralization and how does it relate to cross-default protection?

b. What is a fusion conduit deal? What protective provisions are often included with such deals?

c. What is the difference between gross portfolio yield and net portfolio yield on a pool of credit card receivables?

d. What is a par value test and how is it used?

e. What is meant by an event of default in a CDO?

WEB EXERCISES

1. The Merrill-Lynch Price Index for ABSs can be found at Investinginbonds.com. Go to the site and check the current ABS index:

- Go to http://investinginbonds.com/; click "MBS/ABS Market At-A-Glance."

2. Moody's provides information on ABSs, Commercial MBSs, and CDOs. Study the trends for these securities by going to Moody's.
   - Go to www.moodys.com.
   - Look for "Structured Finance."
   - Look for ABS, Commercial MBS, Residential MBS, or CDO/Derivatives.
   - For each category, go to Watchlist, Credit Card Data Base (for ABSs), Indices, and Special Reports.
3. Moody's provides information on default rates, ratings changes, and other credit information on ABSs. Study the historical default rates and losses of ABSs by going to Moody's.
   - Go to www.moodys.com.
   - Search for "Historical Performance," and look for Structured Finance Default Studies.

NOTES

1. If the manufacturer's financial subsidiary is considered a wholly owned subsidiary, then it may only be allowed to engage in purchasing, owning, and selling receivables.
2. This example is based on the ABS deal issued by DaimlerChrysler Auto Trust 2007. The deal is described in Fabozzi, *Bond Markets, Analysis, and Strategies*, 2009.
3. Moody's has a measure they use to determine the diversity of the collateral's assets based on the geographical or sector distribution of the bonds. Standard & Poor's and Fitch rate the credit quality of a pool of debt obligation by a weighted average ratings factor (WARF).

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Bond Evaluation, Selection, and Management, Second Edition
by R. Stafford Johnson

PART Three

Bond Strategies and the Evaluation of Bonds with Embedded Options

CHAPTER 13

Bond Investment Strategies

13.1 INTRODUCTION

The volatile interest rate environment characterizing the last three decades led to the introduction of many different types of fixed-income securities. As we saw in the last seven chapters, there are myriad corporate, government, intermediary, international, and securitized debt securities, with different collateral and security arrangements, embedded option features, and rate-payment terms. Concomitant with this increase in the number of securities over this time period has been the development of bond investment management strategies. Prior to the 1960s, bond investment strategies could be classified as either active or passive, with active being primarily limited to speculative strategies and passive being simple buy-and-hold strategies. Today, strategies include active and passive, as well as hybrid strategies that combine elements of both, and the number of strategies under these groupings is extensive.

Active strategies involve taking speculative positions in which the primary objective is to obtain an abnormal return. This might include taking a long position in longer duration bonds in anticipation of a decrease in long-term rates, investing in Treasury securities based on the expectation of the Fed implementing an expansionary monetary policy, or investing in lower quality bonds in hopes of future economic growth. Active strategies also include defensive strategies in which the objective is to protect the value of a bond investment. For example, to minimize the potential decrease in a bond fund's value from an expected increase in interest rates, a fund manager might reallocate more of the fund's holdings toward lower duration bonds and away from higher duration ones. Institutional investors, hedge funds, and individual investors all use active strategies. In the case of a hedge fund, the fund itself might be defined by the strategy. For example, Long-Term Capital Management was a hedge fund that could be defined by one of its active strategies of trying to profit from a narrowing of a yield spread. On the other hand, an institutional investor such as a pension or a life insurance company might use an active approach only in the initial investment of its funds, and then afterwards follow a passive or hybrid investment-style approach. Finally, institutional fund managers might use an active approach to change the allocation of their existing funds from one bond group to another in order to increase the expected return or as a defensive strategy to protect the value of the fund. This latter active strategy involves liquidating one bond group and simultaneously purchasing another. Such strategies are referred to as ***bond swaps***.

A ***passive strategy*** is one in which no change of position is necessary once the bonds are selected, or in the case of investing new funds, the investment strategy is not changed once it is set up. Prudence and practicality, though, usually dictate at least minimal monitoring and change. Life insurance companies, deposit institutions, and pensions that have a primary objective of ensuring that there are sufficient funds to meet future liabilities typically use passive strategies. Some investment companies that manage bond portfolios and some mutual funds try to construct bond portfolios whose returns over time replicate those of some specified bond index. Such a strategy is known as ***indexing***. This is a passive strategy where once the fund is constructed or the investment strategy is defined, it is usually not changed.

Hybrid strategies consist primarily of immunization positions. Recall, in Chapter 5 we defined immunization as a strategy of minimizing market risk by selecting a bond or bond portfolio with a duration that matched the investor's horizon date. Fund managers of pensions and insurance companies that have future liabilities whose amounts and times of payments are known often employ immunization. The discussion of bond immunization in Chapter 5 suggested that such a strategy is a passive one of simply matching duration to the horizon date. In practice, though, immunization requires frequent changing or rebalancing of the bond portfolio, and as such can be characterized as having both passive and active management styles. In addition to immunization, other hybrid strategies are contingent immunization and combination matching, both of which have active and passive elements.

In this chapter we extend our analysis from the evaluation of bond securities to the selection of bonds by examining the various active, passive, and hybrid investment strategies. We begin by first examining some of the popular active selection strategies, including trading strategies based on anticipated interest rate changes, credit strategies, and valuation approaches. This is followed by an analysis of two passive strategies: cash flow matching and indexing. Finally, we conclude the chapter by examining bond immunization strategies.

13.2 ACTIVE INVESTMENT STRATEGIES

In our evaluation of bond characteristics in Part 1, we identified a number of fundamental relationships. For example, in our discussion of bond price relationships in Chapter 2, we noted that the greater a bond's maturity or the lower its coupon rate, the greater its price sensitivity to interest rate changes (or equivalently the greater its duration). In Chapter 5, we discussed several empirical studies that presented evidence showing a positive relationship between credit spreads and the state of the economy; specifically, the studies showed that the spread between the yields on low and high quality bonds widens in periods of economic downturn and narrows in periods of economic growth. We also noted in Chapter 5 that the spread between yields on callable and noncallable bonds tends to widen in high-interest rate periods and narrow in low-rate periods. Many active bond strategies are, in turn, predicated on these fundamental bond relations. Some of the more popular ones are interest-rate expectations strategies based on anticipated changes in interest rates or yield curve shifts, credit strategies based on credit analysis and economic forecast, and valuation strategies based on determining the fundamental values of bonds to identify mispriced ones. Each of these strategies can be applied as an approach for investing initial funds or as a bond swap in which two or more bond positions are simultaneously changed in order to change the allocation of a bond portfolio.

Interest Rate Anticipation Strategies

If a bond investor expected interest rates to decrease across all maturities by the same number of basis points (i.e., a parallel shift in the yield curve), she could attain greater expected returns by purchasing bonds with longer durations, or if she is managing a bond fund by reallocating her portfolio, by selling shorter-duration bonds and buying longer-duration ones. In contrast, if a bond manager expected the yield curve to shift up, she could minimize her exposure to market risk by changing her investments or portfolio to include more bonds with shorter durations. Active strategies of selecting bonds or bond portfolios with specific durations based on interest rate expectations are referred to as ***rate-anticipation strategies*** and when they involve simultaneously selling and buying bonds with different durations they are referred to as a ***rate-anticipation swap***.[1]

Rate-Anticipation Swaps When interest rates are expected to decrease across all maturities, a bond fund manager, as just noted, could increase the value of her fund by lengthening the portfolio's duration. This could be done with a rate-anticipation swap in which the manager sells her lower duration bonds and buys higher duration ones. By doing this, the portfolio's value would be more sensitive to interest rate changes and as a result would subject the manager to a higher return-risk position, providing greater upside gains in value if rates decrease but also greater losses in value if rates decrease.

In contrast, when interest rates are expected to increase, a bond fund manager could use a rate-anticipation swap to try to preserve the value of the fund. In this case, the objective would be to shorten the portfolio's duration by selling longer duration bonds and buying shorter ones. One way to shorten the fund's duration is for the manager to buy ***cushion bonds***. A cushion bond is a callable bond with a coupon that is significantly above the current market rate. The bond has the features of a high coupon yield and with its embedded call option a market price that is lower than a comparable noncallable bond. For example, suppose a bond manager had a fund consisting of 10-year, 10% option-free bonds valued at 113.42 per $100 par to yield 8%. Also suppose that there were comparable 10-year, 12% coupon bonds callable at 110 that were trading in the market at a price close to their call price (note, callable bonds cannot trade at prices higher than their call prices). If the manager expected rates to increase, he could cushion the negative price impact on the fund's value by selling some of his option-free bonds and buying the higher coupon, callable bonds—the cushion bonds. The swap of his existing bonds (priced at 113.42) for the cushion bonds (priced at 110) would, in turn, provide him an immediate gain in income plus he would receive higher coupon income in the future. Thus, the interest rate swap of option-free bonds for cushion bonds provides some value preservation. Note: A callable bond has a lower duration than a noncallable one with the same maturity (how to estimate the duration of bonds with embedded options is discussed in Chapter 15).

The 10-year cushion bond with its call feature and higher coupon rate has a relatively lower duration than the 10-year option-free bond; thus, the swap of cushion bonds for option-free bonds in this example represents a switch of longer duration bonds for shorter ones.

Yield Curve Shifts and Strategies Interest rate anticipation strategies require not only forecasting general interest rate movements, but also changes in the term structure of rates. Some rate-anticipation strategies are based on estimating the type of yield curve shift. Three types of yield curve shifts occur with some regularity: parallel shifts, shifts with twists, and shifts with humpedness. In a ***parallel shift***, yields on all maturities change by the same magnitude (see Figure 13.1). A ***twist***, on the other hand, is a non-parallel shift. It implies either a flattening or steepening of the yield curve. As shown in Figure 13.1, if there is a flattening, the spread between long-term and short-term rates decreases; if there is a steepening, the spread increases. A shift with ***humpedness*** is also a non-parallel shift in which short-term and long-term rates change by different magnitudes than intermediate rates. An increase in both short- and long-term rates relative to intermediate rates is referred to as a ***positive butterfly***, and a decrease is known as a ***negative butterfly***.

Given the different types of yield curve shifts, investors actively managing bond portfolios will pursue different strategies based on their yield curve forecast. There are three general types of yield curve strategies used by active bond investors: bullet, barbell, and ladder. The ***bullet strategy*** is implemented by constructing a portfolio concentrated in one maturity area. For example, a bullet strategy consisting of a portfolio of long-term bonds could be formed if there were an expectation of a downward shift in the yield curve with a twist such that long-term rates were expected to decrease more than short-term.[2] Similarly, if investors expected a simple downward parallel shift in the yield curve, a bullet strategy with longer duration bonds would also yield the greater returns than an investment strategy in intermediate or short-term bonds if the expectation turns out to be correct. The ***barbell strategy*** is one in which investments are concentrated in both the short-term and long-term bonds. This strategy could be profitable for an investor who is forecasting a negative butterfly yield curve shift. Finally, the ***ladder strategy*** is constructed with equal allocations in each maturity group.

In implementing an active strategy based on a forecasted shift in the yield curve, there is always the question of how to determine the correct yield curve strategy. One method for identifying the appropriate strategy is to use ***total return analysis*** or ***horizon analysis*** that was broached in Chapter 2. In this approach, potential returns from several yield-curve strategies are evaluated for a number of possible interest rate changes over different horizon periods to identify the best strategy. A problem using this type of analysis is included as one of the end-of-the-chapter problems.

Credit Strategies

Active credit investment strategies consist of quality swaps and credit analysis strategies. A ***quality swap*** is a strategy of moving from one quality group to another in anticipation of a change in economic conditions. A credit analysis strategy, in turn, involves a credit analysis of corporate, municipal, or foreign bonds in order to identify potential changes in default risk. This information is then used to identify bonds to include or exclude in a bond portfolio or bond investment strategy.

Quality Swap In a quality swap, investors try to profit from expected changes in yield spreads between different quality sectors. Quality swaps often involve a ***sector rotation*** in which more funds are allocated to a specific quality sector in anticipation

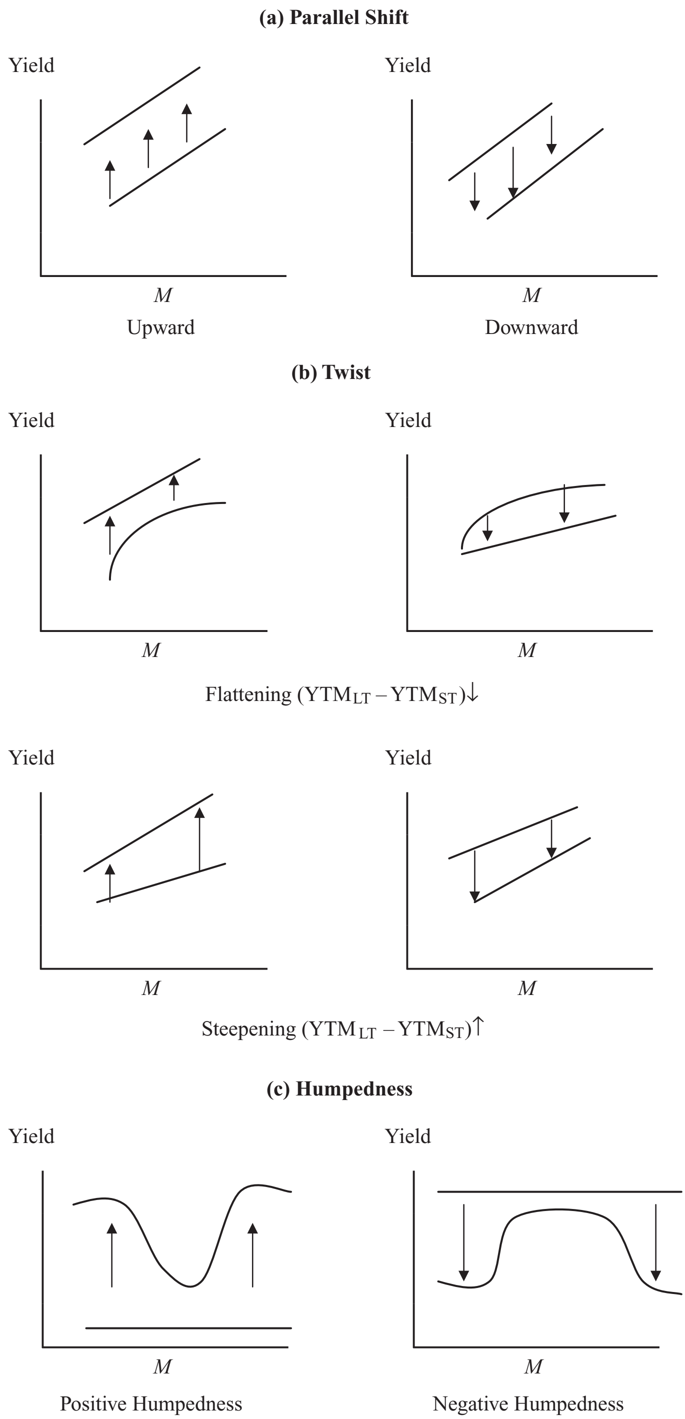

FIGURE 13.1 Yield Curve Shifts

of a price change. For example, suppose a bond fund manager expected a recession accompanied by a flight to safety in which the demand for higher quality bonds would increase and the demand for lower quality ones would decrease. To profit from this expectation, the manager could change the allocation of her bond fund by selling some of her low-quality ones and buying more high-quality bonds. On the other hand, suppose the economy were in a recession, but the bond manager believed that it was near its trough and that economic growth would follow. To capitalize from this expectation, the manager could tilt her bond portfolio toward lower quality bonds by selling some of her higher quality bonds and buying lower quality ones.

In addition to sector rotations, quality swaps can also be constructed to profit from anticipated changes in yield spreads between quality sectors. Recall the studies of default risk presented Chapter 5 that showed how quality yield spreads have tended to widen during periods of economic recession and narrow during periods of economic expansion. If the economy were at the trough of a recession and were expected to grow in the future, speculators or a hedge fund might anticipate a narrowing in the spread between lower and higher quality bonds. To exploit this, they could form a quality swap by taking a long position in lower quality bonds and a short position in higher quality bonds with similar durations. Whether rates increase or decrease, speculators would still profit from these positions, provided the quality spread narrows. For example, if rates increase but the quality spread narrows because of economic growth, then the percentage decrease in the price of lower quality bonds would be less than the percentage decrease in the price of higher quality bonds. As a result, the capital gain from the short position in the higher quality bonds would dominate the capital loss from the long position in the lower quality bonds. Similarly, if rates decrease but the quality spread narrows, then the percentage increase in the price for the lower quality bonds would be greater than the percentage increase for the higher quality bonds. In this case, the capital gain from the long position in lower quality bonds would dominate the capital loss from the short position in the higher quality bonds.[3]

Credit Analysis The objective of a credit analysis strategy is to determine expected changes in default risk. If changes in quality ratings of a bond can be projected prior to an upgrade or downgrade announcement by Moody's, Standard & Poor's, or Fitch (if the market is efficient, it may be necessary to project the change before the announcement), bond investors can realize gains by buying bonds they project will be upgraded, and they can avoid losses by selling or not buying bonds they project will be downgraded. The potential gains from effective credit analysis can be realized more from high-yield bonds than investment-grade bonds. Over the last two decades, the spread between low investment-grade bonds and Treasuries has ranged from 150 basis points (bp) to over 1,000 bp. At the same time, though, the default risk on such bonds has been relatively high. In a study on cumulative default rates of corporate bonds, Douglass and Lucas found the five-year cumulative default rate for B-rated bonds was approximately 24% and the 10-year cumulative default rate was approximately 36%; for CCC-rated bonds they found the cumulative default rates to be approximately 46% and 57% for five years and 10 years, respectively. In contrast, Douglass and Lucas found the five-year and 10-year cumulative default rates for A-rated bonds were only .53% and .98%, and for BBB-rated, the rates were 2.4% and 3.67%. The Douglass and Lucas study, as well as several Moody's studies on cumulative default rates that were examined in Chapter 5, show there is high degree of default risk associated with low-quality bonds. The studies also suggest, though, that with astute credit analysis there are significant gains possible by being able to forecast upgrades and significant losses that can avoided by projecting downgrades. In fact, the strategy of many managers of high-yield bond funds or funds that have some of their investments allocated to high-yield bonds is to develop effective credit analysis models so that they can identify bonds with high yields and high probabilities of upgrades to include in their portfolios, as well as identify bonds with high probabilities of downgrades to exclude from their fund. Credit analysis can be done through basic fundamental analysis of the bond issuer and the indenture and with statistical-based models, such as a multiple discriminate model.

Fundamental Credit Analysis: Many large institutional investors and banks have their own credit analysis departments to evaluate bond issues in order to determine the abilities of companies, municipalities, and foreign issuers to meet their contractual obligations, as well as to determine the possibility of changes in a bond's quality ratings and therefore a change in its price.

Corporate Issues: The credit analysis for corporate bonds often includes the following types of examination:

- ***Industrial Analysis:*** Assessment of the growth rate of the industry, stage of industrial development, cyclicality of the industry, degree of competition, industry and company trends, government regulations, and labor costs and issues.
- ***Fundamental Analysis:*** Comparison of the company's financial ratios with other firms in the industry and with the averages for bonds based on their quality ratings. Ratios often used for analysis include (1) interest coverage (EBIT/interest), (2) leverage (long-term debt/total assets), (3) cash flow (net income + depreciation + amortization + depletion + deferred taxes) as a proportion of total debt (cash flow/debt), and (4) return on equity.
- **Asset and Liability Analysis:** Determination of the market values of assets and liabilities, age and condition of plants, working capital (current assets minus current liabilities), intangible assets and liabilities (e.g., unfunded pension liabilities), and foreign currency exposure.
- ***Indenture Analysis:*** Analysis of protective covenants, including a comparison of covenants with the industry norms.[4]

Municipal Analysis: In analyzing the creditworthiness of municipals (general obligations, GOs, and revenue bonds), the two most important areas of examination are indenture analysis and economic and credit analysis. Some of the key questions related to indenture analysis were examined Chapter 8. The important areas of inquiry in credit and economic analysis relate to debt burden, fiscal soundness, overall economic climate, and red flags.[5]

- ***Debt Burden:*** This analysis involves assessing the total debt burden of the municipal issuer. For GOs, debt burden should include determining the total debt outstanding, including moral obligation bonds, leases, and unfunded pension liabilities. The assessment of the debt burden should include ratios based on debt per capita, historical averages, and debt per capita ratios relative to other areas. For revenue bonds, debt burden should also focuses on relevant coverage ratios relating the debt on the revenue bond to user charges, earmarked revenue, lease rental, and the like.

- ***Fiscal Soundness:*** The objective of this analysis is to determine the issuer's ability to meet obligations. For GOs, the areas of inquiry can include: What are the primary sources of revenue? Is the issuer dependent on any one particular source of revenue? How well has the budget been managed over the last several years? Are there any large liabilities or contracts? Is there a dependence on short-term debt? For revenue bonds, relevant questions relate to the soundness of the project or operation being financed (e.g., power plant) or the source of income (e.g., rents, student fees, and the like), and the quality of the guarantee.
- ***Overall Economic Climate:*** General economic analysis includes examining fundamentals such as growth rates for income, population, and property values. It also should determine the status of the largest property values and employers.
- ***Red Flags:*** Some of the negative indicators suggesting greater credit risk are decreases in population, unemployment increases, decreases in the number of building permits, actual revenue levels consistently falling below projections, declines in property values, loss of large employers, use of debt reserves, and declines in debt coverage ratios. For revenue bonds, additional red flags could include cost overruns on projects, schedule delays, and frequent rate or rental increases.

International Debt: As explained in Chapter 10, the credit analysis of foreign bonds issued by corporations needs to take into account the same issues of any corporate bond (fundamental ratio analysis, financial soundness, industry analysis, and indentures examination). In addition, the analysis also needs to consider ***cross-border risk***: risk due to changes in political, social, and economic conditions in countries where the bonds are issued or where the company is incorporated. In the case of sovereign foreign debt, especially the debt of emerging markets, analysis needs to include an examination of sovereign risk: the risk that the government is unable or unwilling (due to political changes) to service its debt.

Multiple Discriminate Analysis: Multiple discriminate analysis is a statistical technique that can be used to forecast default or changes in credit ratings. When applied to credit analysis, the model estimates a bond's credit score or index, S_i, to determine its overall credit quality. The score is based on a set of explanatory variables, X_i, and estimated weights or coefficients, c_i, measuring the variables' relative impact on the bond's overall credit quality:

$$S_i = c_0 + c_1 X_1 + c_2 X_2 + \cdots + c_n X_n$$

For corporate bonds, possible explanatory variables include the financial ratios (interest coverage, leverage, and cash flows), as well as the corporation's capitalization level, profitability (operating margins), and variability (variance of profitability ratio).[6]

One way to apply multiple discriminate analysis is to compute and then rank the credit quality scores of a number of bonds. To do this requires estimating the coefficient c_i (possibly using a cross-sectional regression technique) and then determining the current ratios for the companies to be analyzed. Given the c_i and X_i values, each company's current credit quality score S_i can be computed using the preceding equation. Once the scores are estimated, then the bonds can be ranked in the order of their scores to assess each bond's relative default risk. Discriminate analysis can also be used to forecast a change in default risk. In this case, the expected future financial ratios of each company are estimated and then used in the above equation to determine the company's future score or expected change in score.

High-Yield Bond Funds As noted earlier, credit analysis is an important tool for managing high-yield funds. Successful funds have fund managers that are able to identify those low-quality bonds that have the potential for being upgraded and therefore should be included in the fund and those bonds that are in jeopardy of being downgraded and therefore should be excluded. It should be noted that today the management of high-yield bond funds has become quite challenging given the many types of high-yield bonds. In the 1980s, many high-yield funds consisted only of privately placed junk bonds of corporations who issued them to finance their mergers. Today many lower quality corporate bonds are sold with special provisions. As we discussed in Chapters 6, these bonds include income bonds, reset notes, payment-in-kind bonds, convertibles, putable notes, extendable bonds, bonds with warrants, and credit-sensitive bonds. In addition, as we saw in Chapter 8 and 10, there are also many lower quality municipals and foreign bonds with different features. The successful performance of today's high-yield funds therefore requires not only effective credit analysis of the company, government, or country issuer, but also a careful analysis of the bond's indenture and its special security provisions.

Chapter 11 Funds A special type of high-yield fund is the ***Chapter 11 fund***: a fund consisting of the bonds of bankrupt or distressed companies. Such bonds consist of issues of corporations who are going through a bankruptcy process or those that are in distress, but have not yet filed. The general strategy is to buy bonds whose prices have plummeted as a result of a filing (or on information that indicates a filing is imminent) but where there is a good expectation that there will be a successful reorganization or possible asset sale that will lead in the future to an increase in the debt's value or to the replacement of the debt with a more valuable claim. Chapter 11 funds are sometimes set up as a hedge fund in which large investors buy, through the fund, a significant block of debt of a specific bankrupt company, giving them some control in the reorganization. The funds are also set up as so-called ***vulture funds*** that invest in the securities of a number of bankrupt firms. Just like any high-yield fund, the success of Chapter 11 funds depends on the ability of the fund managers to conduct an effective credit-type analysis. In this case, though, the analysis often involves studying the feasibility of the reorganization plan submitted out of the bankruptcy process or trying to project the type of plan that will be submitted.[7]

Fundamental Valuation Strategies

A common approach to stock selection is fundamental analysis. The objective of fundamental stock analysis is to determine a stock's equilibrium price or intrinsic value. By doing this, fundamentalists hope to profit by purchasing stocks they estimate to be underpriced (a stock whose market price is below the intrinsic value) and selling or shorting stocks they determine to be overpriced. The objective of fundamental bond analysis is the same as that of fundamental stock analysis. It involves determining a bond's intrinsic value and then comparing that value with the bond's market price. The active management of a bond portfolio using a fundamental strategy, in turn, involves buying bonds that are determined to be underpriced and selling or avoiding those determined to be overpriced.

A bond fundamentalist often tries to determine a bond's intrinsic value by estimating the required rate for discounting the bond's cash flows. This rate, R, depends on the current level of interest rates as measured by the risk-free rate on a Treasury with the same maturity as the bond in question, R_f, and the bond's characteristics (maturity, option features, and quality risk) and the risk premiums or yield spreads associated with those characteristics: default risk premium (DRP), liquidity premium (LP), and option-adjusted spread (OAS):

$$R = R_f + \text{DRP} + \text{LP} + \text{OAS}$$

Fundamentalists use various models, such as regressions, multiple discriminate analysis, and the option pricing models, to estimate the various spreads. They also use binomial interest rate trees (discussed in Chapters 14 and 15) to estimate the values of bonds with embedded option features.

A variation of fundamental bond strategies is a ***yield pickup swap***. In a yield pickup swap, investors or arbitrageurs try to find bonds that are identical, but for some reason are temporarily mispriced, trading at different yields. When two identical bonds trade at different yields, abnormal return can be realized by going long in the underpriced (higher yield) bond and short in the overpriced (lower yield) bond, then closing the positions once the prices of the two bonds converge. It is important to note that to profit from a yield pickup swap, the bonds must be identical. It could be the case that two bonds appear to be identical, but are not. For example, two bonds with the same durations, default ratings, and call features may appear to be identical when, in fact, they have different marketability characteristics that explain the observed differences in their yields.

The strategy underlying a yield pickup swap can be extended from comparing different bonds to comparing a bond with a portfolio of bonds constructed to have the same features. For example, suppose a portfolio consisting of an AAA quality, 10-year, 10% coupon bond and an A quality, five-year, 5% coupon bond is constructed such that it has the same cash flows and features as, say, an AA quality, 7.5-year, 7.5% coupon bond. If an AA quality, 7.5-year, 7.5% coupon bond does not have the same yield as the portfolio, then an arbitrageur or speculator could form a yield pickup swap by taking opposite positions in the portfolio and the bond. A fundamentalist could also use this methodology for identifying underpriced bonds: buying all AA quality, 10-year, 7.5% coupon bonds with yields exceeding the portfolio formed with those features.

The yield-pickup strategy also can be applied to comparing a bond and a portfolio of strip securities with identical features. In fact, the dealer strategies we discussed in Chapter 7 of purchasing a T-note or federal agency security, stripping it, and selling the stripped securities or purchasing a portfolio of stripped securities, bundling them, and selling them as a coupon bond, can be considered yield-pickup strategies of going long and short in two identical positions that are not equally priced.

Other Active Strategies

In the above discussion of active strategies we identified three bond swaps: rate-anticipation swaps, quality swaps, and yield pickup swaps. In addition to these swaps, two other swaps that should be noted are tax swaps and swaps of callable and noncallable bonds.

Tax Swap In a *tax swap*, an investor sells one bond and purchases another in order to take advantage of the tax laws. For example, suppose a bond investor purchased $10,000 worth of a particular bond and then sold it after rates decreased for $15,000, realizing a capital gain of $5,000 and also a capital gains tax liability. One way for the investor to negate the tax liability would be to offset the capital gain with a capital loss. If the investor were holding bonds with current capital losses of, say, $5,000, he could sell those to incur a capital loss to offset his gain. Except for the offset feature, though, the investor may not otherwise want to sell the bond; for example, he might want to hold the bond because he expects an upgrade in the bond's quality rating. If this were the case, then the investor could execute a bond swap in which he sells the bond needed for creating a capital loss and then uses the proceeds to purchase a similar, though not identical, bond. Thus, the tax swap allows the investor to effectively hold the bond he wants, while still reducing his tax liability. It should be noted that for the capital loss to be tax deductible, the bond purchased in the tax swap cannot be identical to the bond sold; if it were, then the swap would represent a wash sale that would result in the IRS disallowing the deduction.[8] In contrast to the IRS's wash sales criterion on stocks, though, the wash sale criterion used for bonds does permit the purchase of comparable bonds that have only minor differences.

Callable/Noncallable Bond Swaps During periods of high interest rates, the spread between the yields on callable and noncallable bonds is greater than during periods of relatively low interest rates. Accordingly, if investors expected the spread between callable and noncallable bonds to narrow, they could capitalize by forming a ***callable/noncallable bond swap***, short in the callable bond and long in the noncallable one. To effectively apply this bond swap requires investors to forecast changes in the spread. Similar swaps can also be extended to bonds with and without other option features, such as putable and nonputable bonds.

13.3 PASSIVE BOND MANAGEMENT STRATEGIES

The objectives underlying passive management strategies vary from a simple buy-and-hold approach of investing in bonds with specific maturities, coupons, and quality ratings with the intent of holding the bonds to maturity; to forming portfolios with returns that mirror the returns on a bond index; to constructing portfolios that ensure there are sufficient funds to meet future liabilities. Here we look at two passive strategies: indexing and cash flow matching.[9] Indexing strategies involve constructing bond portfolios that are highly correlated with a specified bond index. These strategies are applicable for investment funds whose performances are evaluated on a period-by-period basis. Cash-flow matching strategies involve constructing bond portfolios with cash flows that will meet future liabilities. They are liability management strategies applicable to insurance companies, deposit institutions, and pension funds that have cash outlays that must be made at specific times.

Indexing

Bond indexing involves constructing a bond portfolio whose returns over time replicate the returns of a bond index. Indexing is a passive strategy, often used by investment fund managers who believe that actively managed bond strategies do not outperform bond market indexes.

The first step in constructing a bond index fund is to select the appropriate index. Bond indexes can be either general, such as the Barclays Aggregate or the Merrill-Lynch Composite, or specialized, such as Barclay's Global Government Bond Index (see Table 9.3). Also, some investment companies offer their own customized indexes specifically designed to meet certain investment objectives. After selecting the index, the next step is to determine how to replicate the index's performance. One approach is to simply purchase all of the bonds comprising the index in the same proportion that they appear in the index. This is known as ***pure bond indexing*** or the ***full-replication approach***. This approach would result in a perfect correlation between the bond fund and the index. However, with some indexes consisting of as many as 5,000 bonds, the transaction costs involved in acquiring all of the bonds is very high. An alternative to selecting all bonds is to use only a sample.[10] By using a smaller-size portfolio, the transaction costs incurred in constructing the index fund would be smaller. However, with fewer bonds, there may be more credit risk and also less than perfect positive correlation between the index and the index fund. The difference between the returns on the index and the index fund are referred to as ***tracking errors***. Using a sample is subject to tracking errors.

When a sample approach is used, the index fund can be set up using an optimization approach to determine the allocation of each bond in the fund such that it minimizes the tracking error. Another approach is to use a ***cell matching*** strategy. With this approach, the index is decomposed into cells, with each cell defining a different mix of features of the index (duration, credit rating, sector, etc.). For example, a bond index might be described as having two durations ($D > 5$ years and $D < 5$ years), two sectors (corporate and Treasury), and two quality ratings (AAA, AA). These features can be broken into eight unique types of cells, C_i ($2 \times 2 \times 2 = 8$):

$C_1 = D < 5$, AAA, Corp

$C_2 = D < 5$, AAA, Treasury

$C_3 = D < 5$, AA, Corp

$C_4 = D < 5$, AA, Treasury

$C_5 = D > 5$, AAA, Corp

$C_6 = D > 5$, AAA, Treasury

$C_7 = D > 5$, AA, Corp

$C_8 = D > 5$, AA, Treasury

Given the cells, the index fund is constructed by selecting bonds to match each cell and then allocating funds to each type of bond based on each cell's allocation.

TABLE 13.1 Duration/Sector and Duration/Quality Cell Matching

Sector	Percentage of Value	Duration
Treasury	20%	4.50
Federal Agency	10%	3.25
Municipals	15%	5.25
Corporate Industry	15%	6.00
Corporate Utility	10%	6.25
Corporate Foreign	10%	5.55
Sovereign	10%	5.75
Asset-Backed	10%	6.25
	100%	Weighted Average = 5.29
Quality Sector	**Percentage of Value**	**Duration**
AAA	60%	5.25
AA	15%	5.35
A	10%	5.25
BBB	5%	5.65
BB	5%	5.25
B	5%	5.30
	100%	Weighted Average = 5.29

Given the number of possible attributes describing an index, cell matching can be quite complex. For example, three duration classes, three sectors, and three quality ratings give rise to 27 cells. To minimize the number of constraints, one approach is to base the cell identification on just two features, such as the durations and sectors or the durations and quality ratings. A duration/sector index is formed by matching the amounts of the index's durations that make up each of the various sectors. This requires estimating the duration for each sector comprising the index (e.g., Treasury, federal agency, corporate industry, corporate utility, corporate foreign, sovereign, and asset-backed) and determining each sector's percentage of value to the index. If 20% of an index's value consists of Treasury securities with the Treasuries having an estimated portfolio duration of 4.5, then the index portfolio being constructed would consist of 20% of Treasuries with an average duration of 4.5 (see top of Table 13.1). Instead of sectors, duration matching could be done with quality sectors. This would require determining the percentages of value and average durations of each quality-rating group making up the index (see bottom of Table 13.1).

It should be noted that an important feature of any bond portfolio or index is its call option exposure. In constructing an index portfolio, an index's call exposure can be difficult to replicate. One approach is to decompose each cell further into callable and noncallable sectors; another is to form duration/sector or duration/maturity cells as just described with the duration estimated using an option-adjusted technique (described in Chapter 15).

Whether the index fund is formed with the population of all bonds encompassing the specified index or a sample, the objective of indexing is still to replicate the performance of the index. A variation of straight indexing is ***enhanced bond indexing***. This approach allows for minor deviations of certain features and some active management in order to attain a return better than the index. Usually the deviations are in quality ratings or sectors, and not in durations, and they are based on some active management strategy. For example, a fund indexed primarily to the Merrill-Lynch composite but with more weight given to lower quality bonds based on an expectation of an improving economy would be an enhanced index fund combining indexing and sector rotation.

Cash Flow Matching: Dedicated Portfolios

Liabilities of financial institutions can vary. Some liability amounts and timing are known with certainty (for example, a CD obligation of a bank); for others the amount is predictable, but not the timing (e.g., life insurance policy); and in others both the amount and the time are unknown (e.g., property insurance or pension obligations). In the latter two cases, the law of large numbers makes it possible for actuaries to make reasonably accurate forecasts of the future cash outlays. Given projected cash outlays, the objective of the investment manager is to obtain a sufficient return from investing the premiums, deposits, or pension contributions, while still meeting the projected liabilities. Among the most popular approaches used in liability management strategies are cash flow matching and bond immunization (discussed in Section 13.4).

A ***cash flow matching strategy***, also referred to as a ***dedicated portfolio strategy***, involves constructing a bond portfolio with cash flows that match the outlays of the liabilities. For example, a pension fund forming a cash flow matching strategy to meet projected liabilities of $1 billion, $3 billion, and $4 billion for each of the next three years would need to construct a bond portfolio with the same, or approximately the same, cash flows.

One method that can be used for cash flow matching is to start with the final liability for time T and work backwards. For the last period, one would select a bond with a principal (F_T) and coupon (C_T) that matches the amount of that final liability (L_T):

$$L_T = F_T + C_T$$
$$L_T = F_T(1 + \mathrm{C}^{\mathrm{R0}})$$

where C^{R0} is the coupon rate (C_T/F_T). To meet this liability, one could buy $L_T/(1 + C^{\mathrm{R0}})$ of par value of bonds maturing in T periods. Since these bonds' coupons will also be paid in earlier periods, they can be used to reduce the liabilities in each of the earlier periods. Thus, to match the liability in period $T - 1$, one would need to select bonds with a principal of F_{T-1} and coupon C_{T-1} (or coupon rate of $C^{\mathrm{R1}} = C_{T-1}/F_{T-1}$) that is equal to the projected liability in period $T - 1$ (L_{T-1}) less the coupon amount of C_T from the T-period bonds selected:

$$L_{T-1} - C_T = F_{T-1} + C_{T-1}$$
$$L_{T-1} - C_T = F_{T-1}(1 + \mathrm{C}^{\mathrm{R1}})$$

TABLE 13.2 Cash Flow Matching Case

Bonds	Coupon Rate	Par	Yield	Market Value	Liability	Year
3-Year	5%	100	5%	100	$4,000,000	3
2-year	5%	100	5%	100	$3,000,000	2
1-year	5%	100	5%	100	$1,000,000	1

1	2	3	4	5	6
Year	**Total Bond Values**	**Coupon Income**	**Maturing Principal**	**Liability**	**Ending Balance (3) + (4) – (5)**
1	$7,128,820	$356,441	$ 643,559	$1,000,000	0
2	$6,485,261	$324,263	$2,675,737	$3,000,000	0
3	$3,809,524	$190,476	$3,809,524	$4,000,000	0

To meet this liability, one could buy $(L_{T-1} - C_T)/(1 + C^{R1})$ worth of bonds maturing in $T - 1$ periods. The C_{T-1} coupons paid on these bonds, as well as the first bonds (C_T) would likewise be used to reduce liabilities in all earlier periods. Thus, to meet the liability in period $T - 2$, the next bonds to be selected would have a principal and coupon in which

$$L_{T-2} - C_T - C_{T-1} = F_{T-2} + C_{T-2}$$

$$L_{T-2} - C_T - C_{T-1} = F_{T-2}(1 + C^{R2})$$

For this liability, one could buy $(L_{T-2} - C_T - C_{T-1})/(1 + C^{R2})$ worth of par value of bonds maturing in $T - 2$ periods.

A simple cash-flow matching case is presented in Table 13.2. The table shows the matching of liabilities of $4 million, $3 million, and $1 million in years 3, 2, and 1 with three-year, two-year, and one-year bonds each paying 5% annual coupons and selling at par. The $4 million liability at the end of year 3 is matched by buying $3,809,524 worth of three-year, 5% annual coupon bonds trading at par: $3,809,524 = $(L_3 /(1 + C^{R0})$ = $4,000,000/1.05). At the end of year 3, the bonds will pay a principal of $3,809,524 and interest of $190,476 [(.05)($3,809,524)] that match the $4 million liability. The $3 million liability at the end of year 2 is matched by buying $2,675,737 of two-year, 5% annual coupon bonds trading at par: $2,675,737 = $(L_2 - C_3)/(1 + C^{R1})$ = ($3,000,000 – $190,476)/1.05. At the end of year 2, these two-year bonds will pay a principal of $2,675,737 and coupon interest of $133,787; this amount combined with the interest of $190,476 from the original three-year bond will meet the $3 million liability of year 2. Finally, the $1million liability at the end of year 1 is matched by buying $643,559 of a one-year, 5% annual coupon bond trading at par: $643,559 = $(L_1 - C_3 - C_2)/(1 + C^{R2})$ = ($1,000,000 – $190,476 – $133,787)/1.05. At the end of year 1, these one-year bonds will pay principal of $643,559 and coupon interest of $32,178; this principal and interest plus the interest of $190,476 and $133,787 from the original three-year and two-year bonds will meet the $1 million liability of year 2.

With cash flow matching the basic goal is to simply build a portfolio that will provide a stream of payments from coupons, sinking funds, and maturing principals that will match the liability payments. A dedicated portfolio strategy is subject to some minor market risk given that some cash flows may need to be reinvested forward. It also can be subject to default risk if lower quality bonds are purchased. The biggest risk with cash flow matching strategies, though, is that the bonds selected to match forecasted liabilities may be called, forcing the investment manager to purchase new bonds yielding lower rates. To minimize such risk, one can look for noncallable bonds, deep discount bonds, or zero-coupon stripped securities. There are also option and hedging strategies that can be implemented to hedge the risk of embedded call options.

13.4 BOND IMMUNIZATION STRATEGIES

Classical Immunization

An alternative to cash flow matching strategies for pensions, insurance companies, and thrifts is to apply immunization strategies to liability management. In Chapter 5, we defined immunization as a strategy of minimizing market risk by selecting a bond or bond portfolio with a duration equal to the horizon date. For liability management cases, the liability payment date is the liability's duration. Thus, immunization can be described as a duration-matching strategy of equating the duration of the bond to the duration of the liability. As we examined in Chapter 5, when a bond's duration is equal to the liability's duration, the direct reinvestment effect, in which the interest earned from reinvesting the bond's cash flows changes directly with interest rate changes, and the inverse price effect, in which the bond's price changes inversely to interest rate changes, exactly offset each other. As a result, the total return from the investment (TR), or the value of the investment at the horizon or liability date, does not change because of an interest rate change.

The foundation for bond immunization strategies comes from a 1952 article by F. M. Redington.[11] He argued that a bond investment position could be immunized against interest rate changes by matching durations of the bond and the liability. To illustrate, consider a pension fund with a single liability of \$1,352 due in 3.5 years. Assuming a flat yield curve at 10%, the pension fund could immunize its investment against market risk by purchasing a bond with a duration of 3.5 years (using Macaulay's measure), priced at \$968.50 [= $\$1{,}352/(1.10)^{3.5}$]. This could be done by buying a four-year, 9% annual coupon bond with a principal of \$1,000.[12] This bond has both a duration of 3.5 years and is worth \$968.50, given a yield curve at 10%. If the pension fund buys this bond, then any parallel shift in the yield curve in the very near future would have price and interest rate effects that exactly offset each other. As a result, the cash flow or ending wealth at year 3.5, referred to as the ***accumulation value*** or ***target value***, would be \$1,352 (see Table 13.3).

Note that in addition to matching duration, immunization also requires that the initial investment of assets purchased be equal to or greater than the present value of the liability using the current YTM as a discount factor. In this example, the present value of the \$1,352 liability is \$968.50 [= $\$1{,}352/(1.10)^{3.5}$], which equals the current value of the bond and implies a 10% rate of return.

TABLE 13.3 Duration-Matching and Maturity-Matching

Ending Values at 3.5 Years Given Different Interest Rates for 4 Year, 9% Annual Coupon Bond with Duration of 3.5 and 10% Annual Coupon Bond with Maturity of 3.5 Years

Duration = 3.5 **Time (yr)**	**6%**	**10%**	**11%**
1	\$ 90(1.06)2.5 = \$98.22	\$ 90(1.10)2.5 = \$114.21	\$ 90(1.11)2.5 = \$116.83
2	90(1.06)1.5 = \$104.11	90(1.10)1.5 = \$103.83	90(1.11)1.5 = \$105.25
3	90(1.06).5 = \$92.66	90(1.10).5 = \$94.39	90(1.11).5 = \$94.82
3.5	1090/(1.06).5 = \$1058.70	1090/(1.10).5 = \$1039.27	1090/(1.11).5 = \$1034.58
Target Value	\$1352.00	\$1352.00	\$1352.00
Total Return from \$968.50	10%	10%	10%

Maturity = 3.5 Years **Time (yr)**	**6%**	**10%**	**11%**
1	\$ 100(1.06)2.5 = \$109.13	\$ 100(1.10)2.5 = \$126.91	\$ 100(1.11)2.5 = \$129.81
2	100(1.06)1.5 = \$115.68	100(1.10)1.5 = \$115.37	100(1.11)1.5 = \$116.95
3	100(1.06).5 = \$102.96	100(1.10).5 = \$104.88	100(1.11).5 = \$105.36
3.5	1050 = \$1050.00	1050 = \$1050.00	1050 = \$1050.00
Target Value	\$1378.00	\$1397.00	\$1402.00
Total Return from \$1,000	9.59%	10%	10.135%

Note:
The 4-year, 9% bond with duration of 3.5 is initially priced at \$968.50 and has a total return of 10% for each scenario:
Total Return = $[\$1{,}352/\$968.50]^{1/3.5} - 1]$
The 3.5 year, 10% bond is priced at \$1,000 and has a target value and total return that varies with each scenario:
Total Return = $[\text{Target Value}/\$1{,}000]^{1/3.5} - 1]$

Redington's duration-matching strategy is sometimes referred to as ***classical immunization***. Again, it works by having offsetting price and reinvestment effects. In contrast, a maturity-matching strategy where a bond is selected with a maturity equal to the horizon date has no price effect and therefore no way to offset the reinvestment effect. This can be seen in Table 13.3 where, unlike the duration-matched bond, a 10% annual coupon bond with maturity of 3.5 years and initially priced at $1,000 has different ending values and total returns given different interest rates.

Rebalancing

In a 1971 study, Fisher and Weil compared duration-matched immunization positions with maturity-matched ones under a number of interest rate scenarios. They found that even though the duration-matched positions were closer to their initial YTM than the maturity-matched strategies, they were not absent of market risk. Two reasons they offered for the presence of market risk with classical immunization were that the shifts in yield curves were not parallel and that immunization only works when the duration of assets and liabilities are equal at all times. To achieve immunization, Fisher and Weil point out that the duration of the bond or portfolio must be equal to the remaining time in the horizon period.

Since the durations of assets and liabilities change with both time and yield changes, immunized positions require active management, called ***rebalancing***, to ensure that the duration of the portfolio is always equal to the remaining time to horizon. Thus, a bond and liability that currently have the same durations will not necessarily be equal as time passes and rates change. For one, the duration of a coupon bond declines more slowly than the terms to maturity. In our earlier example, our four-year, 9% bond with a Maculay duration of 3.5 years (modified duration of 3.2) when rates were 10%, one year later would have a duration of 2.77 years (modified duration of 2.5) with no change in rates. Secondly, duration changes with interest rate changes. Specifically, there is an inverse relation between interest rates and duration: duration increasing as rates decrease and increasing as rates increase.

Maintaining an immunized position when the bond's duration is no longer equal to the duration of the liability or remaining horizon period requires resetting the bond position such that the durations are again matched. This rebalancing could be done by selling the bond and buying a new one with the correct match, adding a bond to form a portfolio that will have the correct portfolio duration, investing the bond's cash flows differently, or perhaps taking a futures, options, or swap position. In practice, an important consideration for a bond manager immunizing a position is how frequently the position must be rebalanced. Greater transaction costs incurred from frequent rebalancing must be weighed against having a position exposed to less market risk.

In addition to rebalancing the asset position over time and in response to interest rate changes, bond managers also have to decide whether to immunize with a bond or a portfolio of bonds. For a single liability, immunization can be attained with a focus strategy or a barbell strategy. In a ***focus strategy***, a bond is selected with a duration that matches the duration of the liability or a bullet approach is applied where a portfolio of bonds is selected with all the bonds close to the desired duration. For example, if the duration of the liability is four years, one could select a bond with a four-year duration or form a portfolio of bonds with durations of four and five years. In a barbell strategy, the duration of the liability is matched with a bond portfolio with durations more at the extremes. Thus, for a duration liability of four years, an investor might invest half of her funds in a bond with a two-year duration and half in a bond with a six-year duration. The problem with the barbell strategy is that it may not immunize the position if the shift in the yield curve is not parallel.

Immunizing Multiple-Period Liabilities

For multiple-period liabilities, bond immunization strategies can be done either by matching the duration of each liability with the appropriate bond or bullet bond portfolio or by constructing a portfolio with its duration equal to the weighted average of the durations of the liabilities ($D^P{}_L$). For example, if a pension fund had multiple liabilities of $100 million each in years 4, 5, and 6, it could either invest in three bonds, each with respective durations of 4 years, 5 years, and 6 years, or it could invest in a bond portfolio with duration equal to 5 years:

$$D_L^P = \frac{\$100 \text{ m}}{\$300 \text{ m}} 4 \text{ yrs} + \frac{\$100 \text{ m}}{\$300 \text{ m}} 5 \text{ yrs} + \frac{\$100 \text{ m}}{\$300 \text{ m}} 6 \text{ yrs} = 5 \text{ yrs}$$

The latter approach is relatively simple to construct, as well as to manage. However, studies have shown that matching the portfolio's duration of assets with the duration of the liabilities does not always immunize the positions.[13] Thus, for multiple-period liabilities, the best approach is generally considered to be one of immunizing each liability. As with single liabilities, this also requires rebalancing each immunized position.

The costs of setting up and managing a matching immunization strategy with rebalancing applied to multiple liabilities must be weighed against having a position exposed to market risk. In some cases, a bond manager may find that a cash flow matching strategy with little or no rebalancing is preferable to an immunization strategy that requires frequent rebalancing.[14] In fact, some managers combine cash flow matching and immunization strategies. Known as ***combination matching*** or ***horizon matching***, these strategies consist of using cash flow matching strategies for early liabilities and an immunization strategy for longer term liabilities.

Surplus Management and Duration Gap Analysis

The major users of immunization strategies are pensions, insurance companies, and commercial banks and thrifts. Pensions and life insurance companies use multiple-period immunization to determine the investments that will match a schedule of forecasted payouts. Insurance companies, banks and thrifts, and other financial corporations also use immunization concepts for ***surplus management***. Surplus management refers to managing the surplus value of assets over liabilities. This surplus can be measured as ***economic surplus***, defined as the difference between the market value of the assets and the present value of the liabilities. Thus, a pension with a bond portfolio currently valued at \$200 million and liabilities with a present value of \$180 million would have an economic surplus of \$20 million. Whether the \$20 million surplus is adequate depends, in part, on the ***duration gap***: the difference in the duration of assets and the duration of the liabilities. If the duration of the assets exceeds the duration of the liabilities, then the economic surplus will vary inversely to interest rates: increasing if rates fall and decreasing if rates rise. For example, if the duration of the bond portfolio is seven years and the duration of the liabilities is five years, a decrease in rates by 100 basis points would augment the value of bond portfolio from \$200 million to approximately \$214 million [= \$200 million (1.07)] and increase the present value of the liabilities from \$180 million to approximately \$189 million [= \$180 million (1.05)], causing the economic surplus to increase from \$20 million to \$25 million. However, if rates were to increase by 100 basis points, then the surplus would decrease from \$20 million to approximately \$15 million:

$$\text{Economic surplus} = \$200 \text{ million } (1 - .07) - \$180 \text{ million } (1 - .05)$$
$$\text{Economic surplus} = \$15 \text{ million}$$

On the other hand, if the duration of the bond portfolio is less than the duration of the liabilities, then the surplus value will vary directly with interest rates. Finally, if the durations of assets and liabilities are equal (an immunized position), then the surplus will be invariant to rate changes.

In addition to its use by pensions and insurance companies, duration gap analysis is also used by banks to determine changes in the market value of the institution's net worth to changes in interest rates.[15] With gap analysis, a bank's asset sensitivity and liability sensitivity to interest rate changes is found by estimating Macaulay's duration for the assets and liabilities and then using the formula for modified duration to determine the percentage change in value to a percentage change in interest rates:

$$\%\Delta P = -(\text{Macaulay's duration})\ (\Delta R/(1+R)$$

As an example, Table 13.4 shows the amounts and durations of a commercial bank's assets and liabilities. The bank has assets and liabilities each equal to \$150 billion with a weighted Macaulay duration of 2.88 years on its assets and a weighted duration of 1.467 on its liabilities given an interest rate level of 10%. The bank's positive duration gap of 1.413 suggests an inverse relation between changes in rates and net worth. For example, if interest rate were to increase from 10% to 11%, the bank's asset value would decrease by 2.62% and its liabilities by 1.33%, resulting in a decrease in the bank's net worth of \$1.93 billion:

$$\%\Delta P = -\ (\text{Macaulay's duration})\ (\Delta R/(1+R)$$
$$\text{Assets: } \%\Delta P = -\ (2.88)\ (.01/1.10) = -.0262$$
$$\text{Liabilities: } \%\Delta P = -\ (1.467)\ (.01/1.10) = -.0133$$
$$\text{Change in net worth} = (-.0262)(\$150 \text{ billion}) - (-.0133)(\$150 \text{ billion})$$
$$\text{Change in net worth} = -\$1.93 \text{ billion}$$

TABLE 13.4 Duration Gap Analysis of a Bank

Assets	Amount in Billions	Macaulay Duration	Weighted Duration	Liabilities	Amount in Billions	Macaulay Duration	Weighted Duration
Reserves	$10	0.0	0.000	Demand deposits	$15	1.0	0.100
Short-term securities	$15	0.5	0.050	Nonnegotiable deposits	$15	0.5	0.050
Intermediate securities	$20	1.5	0.200	Certificates of deposit	$35	0.5	0.117
Long-term securities	$20	5.0	0.667	Fed funds	$5	0.0	0.000
Variable-rate mortgages	$10	0.5	0.033	Short-term borrowing	$40	0.5	0.133
Fixed-rate mortgages	$25	6.0	1.000	Intermediate-term borrowing	$40	4.0	1.067
Short-term loans	$20	1.0	0.133				
Intermediate loans	$30	4.0	0.800				
Total	$150		2.883	Total	$150		1.467

On the other hand, if rates were to decrease from 10% to 9%, then the bank's net worth would increase by $1.93 billion. Thus, with a positive duration gap, an increase in rates would result in a loss in the bank's capital, and a decrease in rate would cause the bank's capital to increase. If the bank's duration gap had been negative, then a direct relation would exist between the bank's net worth and interest rates, and if the gap were zero, then its net worth would be invariant to interest rate changes. As a tool, duration gap analysis helps the bank's management ascertain the degree of exposure that its net worth has to interest rate changes.

Contingent Immunization

Developed by Leibowitz and Weinberger, ***contingent immunization*** is an enhanced immunization strategy that combines active management to achieve higher returns and immunization strategies to ensure a floor.[16] In a contingent immunization strategy, a client of an investment management fund agrees to accept a potential return below an immunized market return. The lower potential return is referred to as the ***target rate*** and the difference between the immunized market rate and the target rate is called the ***cushion spread***. The acceptance of a lower target rate means that the client is willing to take an end-of-the period investment value, known as the ***minimum target value***, which is lower than the fully immunized value. This acceptance, in turn, gives the management fund some flexibility to pursue an active strategy.

As an example, suppose an investment management fund sets up a contingent immunization position for a client who has just placed $1 million with them and who has an investment horizon of 3.5 years. Furthermore, suppose that the yield curve is currently flat at 10% and that even though the investment fund can obtain an immunized rate of 10% (for example, it could buy a four-year, 9% annual coupon bond trading at 10%), the client agrees to a lower immunization rate of 8% in return for allowing the fund to try to attain a higher rate using some active strategy. By accepting a target rate of 8%, the client is willing to accept a minimum target value of $1,309,131 at the 3.5-year horizon date:

$$\text{Minimum target value} = \$1{,}000{,}000(1.08)^{3.5} = \$1{,}309{,}131$$

The difference between the client's investment value (currently $1 million) and the present value of the minimum target value is the management fund's ***safety margin*** or cushion. The initial safety margin in this example is $62,203:

$$\text{Safety margin} = \text{Investment value} - \text{PV(Minimum target value)}$$

$$\text{Safety margin} = \$1{,}000{,}000 - \$1{,}309{,}131/(1.10)3.5 = \$62{,}203$$

As long as the safety margin is positive, the management fund will have a cushion and can therefore pursue an active strategy. For example, suppose the fund expected long-term rates to decrease in the future and invested the client's funds in 10-year, 10% annual coupon bonds trading at par (YTM = 10%). If rates in the future decreased as expected, then the value of the investment and the safety margin would increase; if rates increased, though, the value of the investment and safety margin would decrease. Moreover, if rates increased to the point that the investment value was equal to the present value of the minimum target value (that is, where the safety margin is zero), then the management fund would be required to immunize the investment position. For example, suppose one year later the yield curve shifted down, as the management fund was hoping, to 8% (continue to assume a flat yield curve). The value of the investment (value of the original 10-year bonds plus coupons) would now be \$1,224,938:

$$\text{Bond value} = \sum_{t=1}^{9} \frac{10}{(1.08)^t} + \frac{100}{(1.08)^9} = 112.4938$$
$$\text{Investment value} = \frac{112.4938}{100}(\$1{,}000{,}000) + (.10)(\$1{,}000{,}000) = \$1{,}224{,}938$$

The present value of the minimum target value (MTV) would be \$1.08 million:

$$\text{PV(MTV)} = \frac{\$1{,}309{,}131}{(1.08)^{2.5}} = \$1{,}080{,}000$$

and the safety margin would be \$144,938:

$$\text{Safety margin} = \$1{,}224{,}938 - \$1{,}080{,}000 = \$144{,}938$$

Thus, the downward shift in the yield curve has led to an increase in the safety margin from \$62,203 to \$144,938. At this point, the investment management fund could maintain its position in the original 10-year bond or take some other active position. Note that if the management fund immunized the client's position when rates were at 8% and the safety margin was positive, it would be able provide the client with a rate of return for the 3.5-year period that exceeded the initial immunization rate of 10%. For example, if the fund sold the bonds and reinvested the proceeds and coupons in bonds with durations of 2.5 years and a yield of 8%, it would be able to lock in a total return of 11.96% for the 3.5-year period:

$$TR_{3.5} = \left[\frac{\$1{,}224{,}938(1.08)^{2.5}}{\$1{,}000{,}000}\right]^{1/3.5} - 1 = .1196$$

Suppose after one year, though, the yield curve shifted up to 12.25% instead of down to 8%. At 12.25%, the value of investment would be only \$981,245 and the present value of the minimum target value would be \$980,657, leaving the fund with a safety margin that is close to zero (\$588):

$$\text{Bond value} = \sum_{t=1}^{9} \frac{10}{(1.1225)^t} + \frac{100}{(1.1225)^9} = 88.1245$$
$$\text{Investment value} = \frac{88.1245}{100}(\$1{,}000{,}000) + (.10)(\$1{,}000{,}000) = \$981{,}245$$

$$PV(MTV) = \frac{\$1{,}309{,}131}{(1.1225)^{2.5}} = \$980{,}657$$

$$\text{Safety margin} = \$981{,}245 - \$980{,}657 = \$588$$

The investment management fund now would be required to immunize the portfolio. This could be done by selling the bond and reinvesting the proceeds plus the coupon (total investment of $981,245) in bonds with durations of 2.5 years and yielding the current rate of 12.25%. Doing this would yield a value of $1,309,916, which is approximately equal to the minimum target value of $1,309,131 at the end of the period, and the target rate of 8%:

$$TR_{3.5} = \left[\frac{\$981{,}245(1.1225)^{2.5}}{\$1{,}000{,}000}\right]^{1/3.5} - 1 = .08$$

Table 13.5 summarizes the investment values, present values of the minimum target value, safety margins, and total return for various interest rates. As shown

TABLE 13.5 Investment Value, Present Value of the Minimum Target Value, and Safety Margin after One Year Given Different Rates

Interest Rate	Investment Value	PV (Minimum Target Value)	Safety Margin	Total Return
0.0800	$1,224,937.76	$1,079,999.91	$144,937.85	0.1196
0.0850	$1,191,785.94	$1,067,600.48	$124,185.46	0.1145
0.0900	$1,159,952.47	$1,055,399.45	$104,553.02	0.1095
0.0950	$1,129,376.42	$1,043,392.74	$85,983.68	0.1047
0.1000	$1,100,000.00	$1,031,576.39	$68,423.61	0.1000
0.1050	$1,071,768.38	$1,019,946.55	$51,821.83	0.0954
0.1100	$1,044,629.52	$1,008,499.44	$36,130.09	0.0909
0.1150	$1,018,534.03	$997,231.39	$21,302.65	0.0865
0.1200	$993,435.00	$986,138.81	$7,296.20	0.0823
0.1225	$981,245.15	$980,657.23	$587.93	0.0802
0.1250	$969,287.88	$975,218.21	−$5,930.32	0.0781
0.1275	$957,557.98	$969,821.33	−$12,263.35	0.0761
0.1300	$946,050.35	$964,466.17	−$18,415.82	0.0741
0.1350	$923,682.17	$953,879.36	−$30,197.19	0.0701
0.1400	$902,145.13	$943,454.54	−$41,309.41	0.0663

Investment value = Value of nine-year, 10% bond with face value of $1million plus $100,000 coupon interest
PV (Minumum target value) = $1,309,131/(1 + Rate)
Safety margin = Investment value – PV (Minimum target value)
Trigger rate = 12.25%
$[TR = (\text{Investment value } (1 + \text{Rate})^{2.5}/\$1{,}000{,}000)]^{1/3.5} - 1$

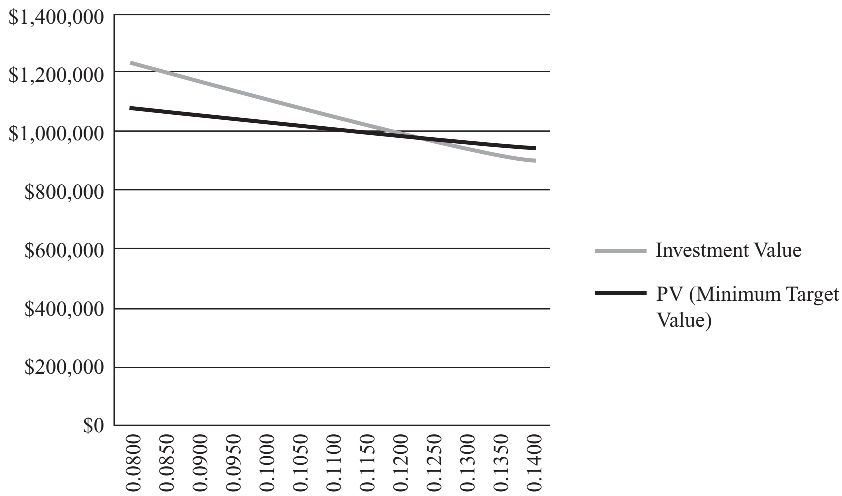

FIGURE 13.2 Contingent Immunization

in Table 13.5, for rates below 12.25%, safety margins are positive and the total returns for the 3.5-year period are above the 8% target rate if the position were immunized; for rates above 12.25%, though, safety margins are negative and the total returns are less than 8%. Thus, 12.25% is the trigger rate for immunizing the position when there are 2.5 years left. The relation between the investment values and the present values of the minimum target value given different rates is also shown graphically in Figure 13.2. The difference between the two graphs in the figure shows the safety margins and the point of intersection of the graphs defines the trigger rate at 12.25%.

In general, the contingent immunization strategy provides investors with a return-risk opportunity that is somewhere between those provided by active and fully immunized strategies. In practice, setting up and managing contingent immunization strategies is more complex than this example suggests. Safety margin positions must be constantly monitored to ensure that if the investment value decreases to the trigger point it will be detected and the immunization position implemented. In addition, active positions are more detailed, nonparallel shifts in the yield curve need to be addressed, and if the immunization position is implemented, it will need to be rebalanced.

13.5 CONCLUSION

In this chapter we have extended our analysis of bonds from evaluation to selection and management. As with all investment strategies, the method of selecting bonds or portfolios depends on the objective of the investor. Active strategies can be pursued to obtain abnormal returns. These include trying to profit from forecasting yield curve shifts, taking positions in different quality bonds in anticipation of a narrowing or a widening of the quality yield spread, identifying mispriced bonds, or taking positions in identical bonds that are not equally priced. Active strategies can also be used to set up defensive positions in order to protect the value of a portfolio. For example, moving to lower duration bonds, such as a cushion bond, when rates are expected to increase or tilting a bond portfolio more toward higher quality bonds when economic slowdowns are anticipated. For investors who must meet future liability requirements or obtain maximum returns subject to risk constraints, passive strategies such as cash flow matching or indexing, or hybrid strategies such as immunization or contingent immunization strategies, can be used.

WEB INFORMATION

- For financial information on securities, market trends, and analysis, see:
  - www.finance.yahoo.com
  - www.hoovers.com
  - www.bloomberg.com
  - www.businessweek.com
  - www.ici.org
  - http://seekingalpha.com
  - http://bigcharts.marketwatch.com
  - www.morningstar.com
  - http://free.stocksmart.com
  - http://online.wsj.com/public/us
- For information on bond funds:
  - *Wall Street Journal* site: http://online.wsj.com/public/us
    - Click "Market Data" tab.
    - Click "Mutual Fund" tab.
    - Use Screener.
  - FINRA
    - www.finra.org/index.htm
    - "Sitemap," "Mutual Funds," and "Fixed Income"
  - Yahoo!
    - http://screen.yahoo.com/funds.html
    - Fund Screener
  - Yield Curves
    - Investinginbonds: http://investinginbonds.com/
    - Click "Government Market-At-A-Glance."
  - Bloomberg: www.bloomberg.com
    - Click "Market Data" and "Rates and Bonds."
  - *Wall Street Journal*: http://online.wsj.com/public/us
    - Click "Market Data" and "Bonds, Rates & Credit Markets" tabs.
    - Click "Main: Overview, Intraday Quotes, Links to All" tab to find interactive chart to study yields over different time periods.

- FINRA: Current yield curves and yield curves one year earlier for Treasuries and different quality corporate and municipal bonds are found on the FINRA site.
  - Go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds."
  - On U.S. Treasury yield curve, click "View All."
  - On "Current" tab, click one year on the drop-down menu; on "Search," click quality rating you want to explore on the drop-down menu.

KEY TERMS

accumulation value
active strategies
barbell strategy
bond swaps
bullet strategy
callable/noncallable bond swap
cash flow matching strategy
cell matching
Chapter 11 fund
classical immunization
combination matching
contingent immunization
cross-border risk
cushion bonds
cushion spread
debt burden
dedicated portfolio strategy
duration gap
economic surplus
enhanced bond indexing
fiscal soundness
focus strategy
full-replication approach
fundamental analysis
horizon analysis
horizon matching
humpedness
indenture analysis
indexing
industrial analysis
international debt
ladder strategy
minimum target value
multiple discriminant analysis
municipal analysis
negative butterfly
overall economic climate
parallel shift
passive strategy
positive butterfly
pure bond indexing
quality swap
rate-anticipation strategies
rate-anticipation swap
rebalancing
red flags
safety margin
sector rotation
surplus management
target rate
target value
tax swap
total return analysis
tracking errors
twist
vulture funds
yield pickup swap

PROBLEMS AND QUESTIONS

1. The yield curve for A-rated bonds is presently flat at a promised YTM of 10%. You own an A-rated, five-year, 10% coupon bond with annual coupon payments. You expect rates to decrease over the next year and would like to take advantage of your expectation with a rate-anticipation swap. The bond you are considering substituting is an A-rated, 10-year, 10% coupon with annual coupon payments.
   a. In the table below, evaluate your rate anticipation swap by comparing your current bond with the substitute candidate given the following scenario: The yield curve will shift down one year from now from 10% to 9%. Assume the coupon date is one year from now.

	Current Bond: 5 Yr, 10% Coupon Bond	**Substitute Bond: 10 Yr, 10% Coupon Bond**
Current value		
Current Macaulay duration		
Coupons		
Interest on interest		
Bond price one year later		
Dollar return one year later		
One-year TR		

   b. In the table below, evaluate your rate anticipation swap given the following scenario: The yield curve will shift up one year from now from 10% to 11%. Assume the coupon date is one year from now.

	Current Bond: 5 Yr, 10% Coupon Bond	**Substitute Bond: 10 Yr, 10% Coupon Bond**
Current value		
Current Macaulay duration		
Coupons		
Interest on interest		
Bond price one year later		
Dollar return one year later		
One-year TR		

   c. Comment on your rate-anticipation swap.

2. You manage a fund in which you currently have $5 million invested in AA-rated, 15-year, 7% coupon bonds with semiannual coupon payments and currently priced to yield 6%. Interest rates have been decreasing over the last several years and you believe that they are near a trough and will increase over the next year. Currently, the yield curve for AA-rated bonds is flat at 6%. Given your expectation, you are considering a rate-anticipation swap. The bond you

are considering substituting is an AA-rated, three-year, 10% callable bond with semiannual coupon payments and priced at its call price of 110.

a. In the table below, evaluate your rate-anticipation swap by comparing your current bond with the substitute candidate given the following scenario: The yield curve will shift up one year from now from 6% to 7%. Assume the coupon date is one year from now.

	Current Bond: 15 Yr, 7% Coupon Bond	**Substitute Bond: 3 Yr, 10% Coupon Bond**
Current value per 100 face value		110
Coupons		
Interest on interest		
Bond price one year later		
Dollar return one year later		
One-year TR		

b. Comment on your rate-anticipation swap. What term is used to describe the three-year, 10% bond?

3. Given a current flat yield curve for AAA bonds at 6% and the following bonds:

Bond	**Quality**	**Maturity**	**Annual Coupon (Coupons Paid Annually)**	**Current Price**	**YTM**	**Macaulay Duration**
A	AAA	5 years	6%	100	6%	4.46
B	AAA	11 years	6%	100	6%	8.36
C	AAA	20 years	6%	100	6%	12.16

a. What is the portfolio duration of a barbell portfolio formed with equal allocation to Bonds A and C? How does the barbell portfolio's duration compare with a bullet portfolio consisting of Bond B?

b. In the table below, calculate (using Excel) each bond's and the barbell portfolio's values and dollar returns one year later given the parallel shifts in the yield curve shown in the table. What differences do you observe between the barbell portfolio and the bullet portfolio formed with Bond B? What bond or portfolio would you select if you expected a significant downward shift in the yield curve? What bond or portfolio would give you the greatest protection in value if you expected a significant upward shift in the yield curve? Comment on your findings.

Yield Curve Change in BP	Value A B C	Return A B C	Return Barbell Bullet	Difference
200				
150				
100				
50				
25				
0				
−25				
−50				
−100				
−150				
−200				

c. Suppose yield curve shifts are characterized by a flattening where for each change in Bond B (intermediate bond), Bond A increases 25 bp more and Bond C decreases by 25 bp less:

$$\Delta y_A = \Delta y_B + 25\text{bp}$$
$$\Delta y_C = \Delta y_B - 25\text{bp}$$

In the table below, calculate using Excel each bond's and the barbell portfolio's values and dollar returns one year later given the yield changes in Bond B shown in the table. What differences do you observe between the barbell portfolio and bullet portfolio formed with Bond B? How do the differences with the twist compare to the differences with the parallel shifts?

Yield Change for B in BP	Value A B C	Return A B C	Return Barbell Bullet	Difference
200				
150				
100				
50				
25				
0				
−25				
−50				
−100				
−150				
−200				

4. Suppose you are a strategist for a hedge fund. Your research indicates that the quality spread for BBB bonds and AAA bonds is 150 basis points in periods of economic slowdown and only 100 bp in periods of economic expansion. Currently, the economy is in a recession, and one- and two-year zero coupon bonds for AAA and BBB bonds are trading at 6% and 7.5%. Leading economic indicators, though, strongly point to the economy hitting its trough relatively soon and then expanding over the next year.

a. Given the economic growth forecast, construct a quality spread for your hedge fund formed by going short in one of the bonds with the proceeds used to purchase the other. Assume perfect divisibility.
b. Show what your hedge fund's profit or loss could be if you closed your position a year later and the economy were growing and yields on AAA bonds had increased to 7% and the quality yield spread narrowed to 100 basis points as you predicted. Assume a flat yield curve and annual compounding.
c. Show what your hedge fund's position would be if yields on AAA bonds had instead decreased to 5%, but the quality yield spread was still 100 BP as you predicted.
d. What would happen to your profit or loss if the yield spread widened instead of narrowed?

5. Suppose an arbitrageur for a hedge fund finds two identical bonds trading at different YTMs: Bond A, an AA-rated, 10-year, option-free, 10% annual coupon bond trading at par, and Bond B, an AA-rated, 10-year, option-free, 10% annual coupon bond trading to yield 10.25%. What are the prices of each bond? What swap strategy would you recommend to the arbitrageur? What is the risk in this strategy?

6. Comment on the objective of many strategies based on credit analysis.

7. List the factors that should be considered in conducting a credit analysis of a general obligation bond and explain some of the ways of measuring them.

8. List the factors that should be considered in conducting a credit analysis of a revenue bond, and explain some of the ways of measuring them.

9. List some of the important factors that should be considered in conducting a credit analysis of a corporate bond.

10. Briefly explain the policy objectives of high-yield bond funds and Chapter 11 funds.

11. Explain the fundamental objective of fundamental bond analysis.

12. Explain how a tax swap is used to take advantage of the tax laws.

13. Explain the differences in bond indexing using the full-replication approach and a sample approach.

14. Suppose a bond index consists of municipal and corporate bonds, has durations ranging from 1 to 10, and has quality ratings ranging from B to AAA. Decompose the index into cells based on three durations ranges ($D < 4$; $4 \leq D \leq 7$; $D > 7$), two quality ratings (investment grade and speculative grade), and the two sectors. Explain how you would construct a bond index portfolio using the cells.

15. Given the information on the composition of a bond index in the table:
a. Explain how you would construct a bond index portfolio using duration/sector and duration/quality sector approaches.
b. Define enhanced bond indexing.
c. How would you apply enhanced bond indexing to your bond index portfolio if you expected a slow economy to improve and grow in the near future?

Sector	Percentage of Value	Duration
Treasury	30%	4.50
Federal agency	5%	3.25
Municipals	10%	5.25
Corporate	35%	6.25
Sovereign	10%	5.75
Asset-backed	10%	6.25
	100%	
Quality Sector	**Percentage of Value**	**Duration**
AAA	40%	5.25
AA	25%	5.35
A	20%	5.25
BBB	5%	5.65
BB	5%	5.25
B	5%	5.30
	100%	

16. How are call features handled in constructing a bond index portfolio?

17. Suppose an investment management fund has the following liabilities for the next four years:

Year	Liability
1	$2 million
2	$12 million
3	$7 million
4	$10 million

a. Construct a dedicated portfolio from 6% coupon bonds with different maturities that will match the liabilities. Assume the applicable yield curve is flat at 6% and coupon payments are annual.

b. Show in the table below that the coupon income and maturing principal each year match the liabilities.

1	2	3	4	5	6
Year	Total Bond Value Outstanding	Coupon Income	Maturing Principal	Liability	Ending Balance (3) + (4) – (5)
1					
2					
3					
4					

18. What is the major risk associated with a cash flow-matching strategy? How can the risk be minimized?

19. A 10-year, 5% coupon bond making annual payments has a Macaulay duration of eight years if the bond is priced at 92.64 per 100 face value to yield 6%. Show how classical immunization works by showing the target value and total return for an eight-year horizon period are approximately the same given different interest rate changes. Specifically, assume there is a one-time shift in the yield curve to 4% and to 8% just after the investment is made. Assume the yield curve is flat and use annual compounding.

20. Suppose your horizon period is six years and you are considering the following investments:

    AAA-Rated, 6% coupon bond with annual coupon payments, maturity of 6 years, and Macaulay duration of 5.21

    AAA-Rated, 5% coupon bond with annual coupon payments, maturity of 7 years, priced at 94.42 to yield 6%, and Macaulay duration of 6.04

    Suppose the applicable yield curve is flat at 6%.

    a. Determine the target value at your horizon date and the total return for a classical duration-matching strategy and for a maturity-matching strategy given the following interest rate scenarios:
    - The yield curve shifts down to 4% just after you buy the bond and stays there until you reach your horizon date.
    - The yield curve stays at 6%.
    - The yield curve shifts up to 8% just after you buy the bond and stays there until you reach your horizon date.

    b. Comment on the difference between a classical duration-matching strategy and a maturity-matching strategy.

    c. Determine the target value at your horizon date and the total return for the duration matching strategy given the yield curve shifts after two years to 4% and 8%, instead of immediately after you buy the bond. What is the duration of your bond after two years at 4% and 8%? Does it match your remaining horizon? Comment on your findings.

21. In a 1971 study, Fisher and Weil demonstrated that even though duration-matched positions were closer to their initial YTM than maturity-matched strategies, they were not absent of market risk. What reasons did they offer for the presence of market risk with classical immunization and what did they recommend as a method for achieving immunization?

22. Explain how initially immunized positions lose their immunization and how they can be rebalanced.

23. Explain the alternative ways in which multiple-period liabilities can be immunized.

24. What is a combination matching strategy? When is it used?

25. ABC Trust manages a pension fund. The assets of the fund are in a bond portfolio currently worth $500 million and with an average duration of 6. The present value of the pension's liabilities is $450 million and the average duration of the liabilities is 10.
    a. What is the pension's economic surplus?
    b. What would happen to the economic surplus if interest rates were to increase by100 basis points?
    c. What would happen to the economic surplus if interest rates were to decrease by 100 basis points?
    d. How could the fund minimize the impact the interest rate changes have on its economic surplus?

26. Suppose you set up a contingent immunization strategy for a $50 million fund you are managing. Suppose the horizon date for the fund is four years, the immunization rate is 10%, the minimum target rate is 8%, the yield curve is flat at 10%, and all investment cash flows are annual.
    a. What is the current minimum target value and safety margin?
    b. Suppose you invest in a 15-year, 10% annual coupon bond selling at par. What would be the value of your fund and your safety margin if a year later rates were at 8% on all maturities? What would your contingent immunization strategy be in this case? What would your total return be if you immunized?
    c. Suppose you invest in a 15-year, 10% annual coupon bond selling at par. What would be the value of your fund and your safety margin if a year later rates were at 11.97% on all maturities? What would your contingent immunization strategy be in this case? What would your total return be if you immunized?

27. What are some of the practical considerations required to effectively manage a contingent immunization position?

WEB EXERCISES

1. Go to www.federalreserve.gov/releases/h15/data.htm. The site contains data on a number of different bond yields. Go to "Historical Data" and compare the annual historical yields on 10-year Treasury bonds to Moody's Aaa and Baa from 1976 to the present. Copy the data to Excel and make a chart so that the data is easier to analyze.
    - Identify points when Treasury yields are at a trough and when they are at a peak. Comment on interest rate anticipation strategies that bond investors in hindsight could have implemented either to earn higher returns or to protect their bond positions.
    - Examine the risk premiums on Aaa and Baa bonds to determine if they widen during periods of economic slowdown (such as in the late 1970s and early 1980s and in the late 1980s and early 1990s, 2000, and 2008) and narrow during periods of economic growth (mid 1980s, 1990s, 2001–2006). Comment on how sector rotation and quality swap strategies could have been used during these periods.

2. Corporate bonds on Moody's watch list or those undergoing changes can be found by going to www.moodys.com and looking for "Corporate Finance" and then "Ratings Actions" and "Watchlist." Take one of the companies on Moody's watch list or with a recent ratings change and do an analysis of it and its sector with information obtained from:
   - www.hoovers.com
   - www.businessweek.com
   - www.ici.org
   - http://seekingalpha.com
   - http://bigcharts.marketwatch.com
   - www.morningstar.com
3. Select a municipal bond from www.investinginbonds.com ("Municipal Bonds At-A-Glance") and then go to Moody's to study its credit history and profile:
   - www.moodys.com
   - Enter CUSIP on "Quick Search" and click "Go."
   - Review the official statement of the bond. To access the statement, go to http://emma.msrb.org/ and enter the municipal's CUSIP in the "Muni Search Box."
4. Select a state and local government bond on Moody's watch list or with a recent rating change and do an analysis of it with information obtained from the Bureau of Economic Analysis Web site: www.bea.gov. At the site, go to "Regional" data. You may want to look at employment data by going to www.economagic.com. At the site, click "Browse by Region" or click "Bureau of Labor Statistics and States." You may also find information on a state's financing by going to the census site: www.census.gov/govs/estimate/index.html and www.census.gov/govs/www/financegen.html.
5. Go to the FINRA or Yahoo! sites to find information on fixed-income mutual funds. Summarize some of their policy statements:
   - FINRA: www.finra.org/index.htm, "Sitemap," "Mutual Funds," and "Fixed Income."
   - Yahoo! Fund Screener: http://screen.yahoo.com/funds.html.
6. Form a high-yield bond portfolio (with 10 to 15 bonds) using Yahoo's bond screener. Describe your portfolio in terms of the portfolio's YTM (approximate), average quality rating, maturity, and duration:
   - http://screen.yahoo.com/bonds.html
7. Form an investment-grade bond portfolio (with 10 to 15 bonds) using Yahoo's bond screener. Describe your portfolio in terms of the portfolio's approximate YTM, average quality rating, maturity, and duration:
   - http://screen.yahoo.com/bonds.html
8. Form a Treasury bond portfolio (with 10 to 15 bonds) using Yahoo's bond screener. Describe your portfolio in terms of the portfolio's average YTM, maturity, and duration:
   - http://screen.yahoo.com/bonds.html

9. Form a Treasury and agency bond portfolio (with 10 to 15 bonds) using FINRA's bond screener. Describe your portfolio in terms of the portfolio's average YTM, maturity, and duration:
   - www.finra.org/index.htm

NOTES

1. In Chapter 18, we will describe how rate-anticipation swaps can be implemented using futures contracts.
2. Note that concentrating in one maturity group does not mean constructing a portfolio with a portfolio duration corresponding to that maturity. For example, if an investor expected five-year bond rates to decrease, but not short-term or long-term rates, she could profit by investing in a bond with duration of five years, but she could incur losses from a portfolio of short- and long-term bonds with a portfolio duration of five years.
3. Note that instead of taking positions in different quality bonds, speculators alternatively can form a quality swap by taking positions in futures contracts on bonds with different quality ratings (e.g., opposite positions in a Treasury bond futures contract and a municipal bond index contract). Constructing bond swaps with futures is examined in Chapter 18.
4. For a more detailed analysis covering the guidelines in the credit analysis of corporate credits, see Fabozzi, "Credit Analysis for Corporate Bonds," *The Handbook of Fixed-Income Securities*, 7th ed., ed. F. Fabozzi, 733–777.
5. For a more detailed analysis covering the guidelines in the credit analysis of municipals, see Feldstein and Grant, "Guidelines in the Credit Analysis of General Obligation and Revenue Municipal Bonds," *The Handbook of Fixed-Income Securities*, 7th ed., ed. F. Fabozzi, 800–824.
6. For additional discussion of credit analysis models, see Altman and Scott, *Investing in Junk Bonds*, and Reilly and Brown, *Investment Analysis and Portfolio Management*, 629–631.
7. For more discussion on investing in the debt securities of bankrupt firms, see Jane Howe, "Investing in Chapter 11 and Other Distressed Companies," *The Handbook of Fixed-Income Securities*, 6th ed., ed. F. Fabozzi, 469–489.
8. Another type of tax swap involves switching between high- and low-coupon bonds to take advantage of different tax treatments applied to capital gains and income. This swap can be used if the tax rate on capital gains differs from the tax rate on income. If it does, then an investor might find it advantageous to swap a low-coupon bond for a high-coupon bond with the same duration.
9. One of the most well-known approaches to portfolio construction is the use of the Markowitz portfolio model. While most of the applications of the Markowitz model are to stocks, it can be used for constructing bond portfolios. For a discussion of the use of the Markowitz model for bond portfolios, see Elton and Gruber (1995).
10. The study by McEnally and Boardman (discussed in Chapter 5) suggests that the maximum diversification benefits from a bond portfolio are realized with a portfolio consisting of approximately 20 bonds.
11. F. M. Redington, "Review of the Principles of Life–Office Foundation," *Journal of the Institute of Actuaries* 78 (1952): 286–340.
12. This example is similar to the one presented in Section 5.4.
13. See Bierwag, Kaufman, and Tuevs (1982).
14. For an analysis of the cost and benefits of cash flow matching and immunization, see Fong and Vasicek (1986).

15. In addition to gap analysis, banks also conduct income gap analysis in which they look at the impact of interest rate changes on the income received and paid on their rate-sensitive assets and liabilities.
16. Martin Leibowitz and Alfred Weinberger, "Contingent Immunization–Part I: Risk Control Procedures," *Financial Analyst Journal* 38, November-December 1982: 17–32; Martin Leibowitz and Alfred Weinberger, "Contingent Immunization–Part II: Problem Areas," *Financial Analyst Journal* 39, January-February 1983: 35–50.

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CHAPTER 14

Binomial Interest Rate Trees and the Valuation of Bonds with Embedded Options

14.1 INTRODUCTION

An option is a right to buy or sell a security at a specific price on or possibly before a specified date. As we noted in Chapter 5, many bonds have a call feature giving the issuer the right to buy back the bond from the bondholder. In addition to callable bonds, there are also putable bonds, giving the bondholder the right to sell the bond back to the issuer, sinking fund bonds in which the issuer has the right to call the bond or buy it back in the market, mortgage- and asset-backed securities with prepayment options on the underlying collateral, and convertible bonds that give the bondholder the right to convert the bond into a specified number of shares of stock.

The inclusion of option features in a bond contract makes the evaluation of such bonds more difficult. A 10-year, 10% callable bond issued when interest rates are relatively high may be more like a three-year bond given that a likely interest rate decrease would lead the issuer to buy the bond back. Determining the value of such a bond requires taking into account not only the value of the bond's cash flow, but also the value of the call option embedded in the bond. One way to capture the impact of a bond's option feature on its value is to construct a model that incorporates the random paths that interest rates follow over time. Such a model allows one to value a bond's option at different interest rate levels. One such model is the ***binomial interest rate tree***. Patterned after the binomial option pricing model, this model assumes that interest rates follow a binomial process in which in each period the rate is either higher or lower. In this chapter, we examine how to evaluate bonds with option features using a binomial interest rate tree approach. We begin by defining a binomial tree for spot rates and then showing how the tree can be used to value a ***callable bond***. After examining the valuation of a callable bond, we then show the valuation of putable bonds, bonds with sinking funds, convertible bonds, and mortgage-backed securities with prepayment options. In this chapter, we focus on defining the binomial tree and explaining how it can be used to value bonds with embedded options; in the next chapter, we take up the more technical subject of how the tree can be estimated.

14.2 BINOMIAL INTEREST RATE MODEL

A binomial model of interest rates assumes that a spot rate of a given maturity follows a binomial process where in each period it has either a higher or lower rate. For example, assume that a one-period, riskless spot rate (S) follows a process in which in each period the rate is equal to a proportion u times its beginning-of-the-period value or a proportion d times its initial value, where u is greater than d. After one period, there would be two possible one-period spot rates: $S_u = uS_0$ and $S_d = dS_0$. If the proportions u and d were constant over different periods, then after two periods there would be three possible rates. That is, as shown in Figure 14.1, after two periods the one-period spot rate can either equal $S_{uu} = u^2S_0$, $S_{ud} = udS_0$, or $S_{dd} = d^2S_0$. Similarly, after three periods, the spot rate could take on four possible values: $S_{uuu} = u^3S_0$, $S_{uud} = u^2dS_0$, $S_{udd} = ud^2S_0$, and $S_{ddd} = d^3S_0$.

To illustrate, suppose the current one-period spot rate is 10%, the upward parameter u is 1.1 and the downward parameter d is .95. As shown in Figure 14.2, the two possible one-period rates after one period are 11% and 9.5%, the three possible one-period rates after two periods are 12.1%, 10.45%, and 9.025%, and the four possible rates after three periods are 13.31%, 11.495%, 9.927%, and 8.574%.

Valuing a Two-Period Bond

Given the possible one-period spot rates, suppose we wanted to value a bond that matures in two periods. Assume that the bond has no default risk or embedded option features and that it pays an 8% coupon each period and a $100 principal at maturity. Since there is no default or call risk, the only risk an investor assumes in buying this bond is market risk. This risk occurs at time period one. At that time, the original two-period bond will have one period to maturity where there is a certain payoff of $108. We don't know, though, whether the one-period rate will be 11% or 9.5%. If the rate is 11%, then the bond would be worth $B_u = 108/1.11 = 97.297$; if the rate is 9.5%, the bond would be worth $B_d = 108/1.095 = 98.630$. Given these two possible values in period 1, the current value of the two-period bond can be found by calculating the present value of the bond's expected cash flow in period 1.

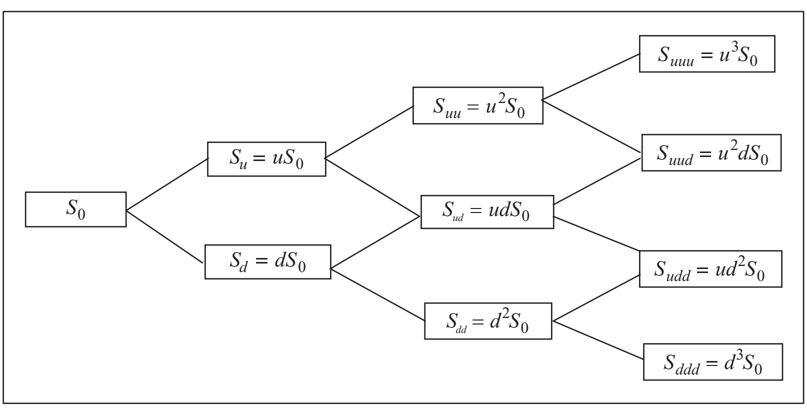


FIGURE 14.1 Binomial Tree of One-Period Spot Rates

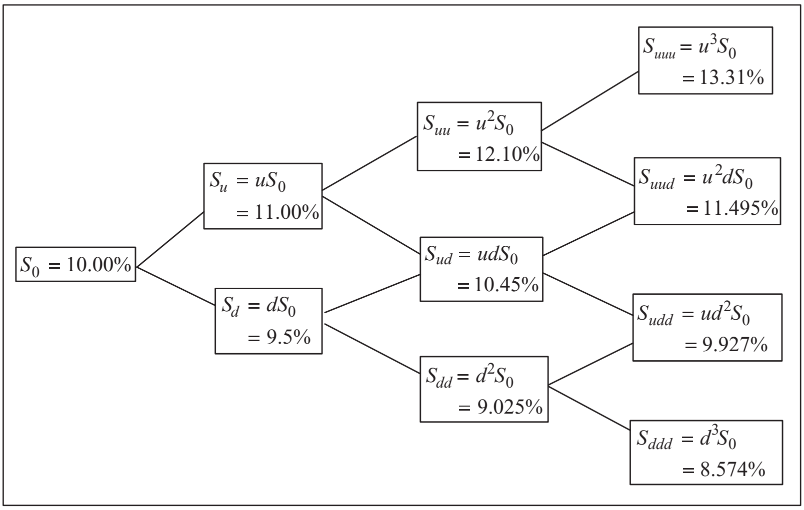


FIGURE 14.2 Binomial Tree

If we assume that there is an equal probability (q) of the one-period spot rate being higher ($q = .5$) or lower ($1 - q = .5$), then the current value of the two-period bond (B_0) would be 96.330 (see Figure 14.3):

$$B_0 = \frac{q\,[B_u + C] + (1 - q)\,[B_d + C]]}{1 + S_0}$$

$$B_0 = \frac{.5\,[97.297 + 8] + .5\,[98.630 + 8]}{1.10} = 96.330$$

Now suppose that the two-period, 8% bond has a call feature that allows the issuer to buy back the bond at a call price (CP) of 98. Using the binomial tree approach, this call option can be incorporated into the valuation of the bond by determining at each node in period 1 whether or not the issuer would exercise his right to call. The issuer will find it profitable to exercise whenever the bond price is above the call price (assuming no transaction or holding costs). This is the case when the one-period spot rate is 9.5% in period 1 and the bond is priced at 98.630. The price of the bond in this case would be the call price of 98. It is not profitable, however, for the issuer to exercise the call at the spot rate of 11% when the bond is worth 97.297; the value of the bond in this case remains at 97.297.[1] In general, since the bond is only exercised when the call price is less than the bond value, the value of the callable bond in period 1 is therefore the *minimum* of its call price or its binomial value:

$$B_t^C = \text{Min}[B_t, \text{CP}]$$

Rolling the two callable bond values in period 1 of 97.297 and 98 to the present, we obtain a current price of 96.044:

$$B_t^C = \frac{.5\,[97.297 + 8] + .5\,[98 + 8]}{1.10} = 96.044$$

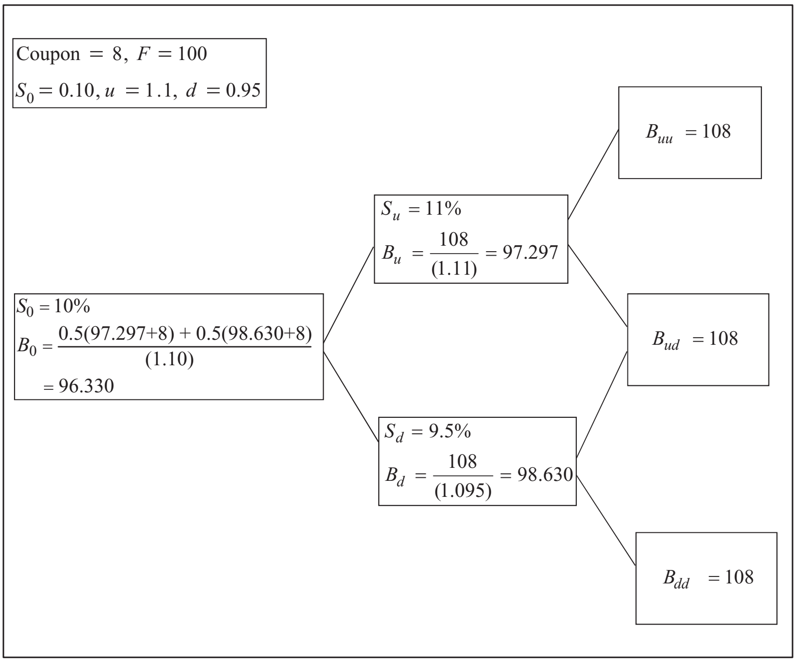


FIGURE 14.3 Value of Two-Period Option-Free Bond

As we should expect, the bond's embedded call option lowers the value of the bond from 96.330 to 96.044. The value of the callable bond in terms of the binomial tree is shown in Figure 14.4. Note, at each of the nodes in period 1, the value of the callable bond is determined by selecting the minimum of the binomial bond value or the call price, and then rolling the callable bond values to the current period.

Instead of using a price constraint at each node, the price of the callable bond can alternatively be found by determining the value of the call option at each node, ${V_t}^C$, and then subtracting that value from the noncallable bond value (${B_t}^C = {B_t}^{NC} - {V_t}^C$). In this two-period case, the values of the call option are equal to their intrinsic values, IV (or exercise values). The intrinsic value is the maximum of ${B_t}^{NC} - \text{CP}$ or zero:

$$V_t^C = \text{Max}[B_t^{NC} - \text{CP}, 0]$$

As shown in Figure 14.4, the two possible call values in period 1 are zero and .63 and the corresponding callable bond values are 97.297 and 98—the same values obtained using the minimum price constraint approach. The value of the call option in the current period is equal to the present value of the expected call value in period 1. In this case, the current value is 0.2864:

$$V_0^C = \frac{.5[0] + .5\,[.630]}{1.10} = .2864$$

Subtracting the call value of 0.2864 from the noncallable bond value of 96.330, we obtain a callable bond value of 96.044 (see Figure 14.4)—the same value obtained using the constraint approach.

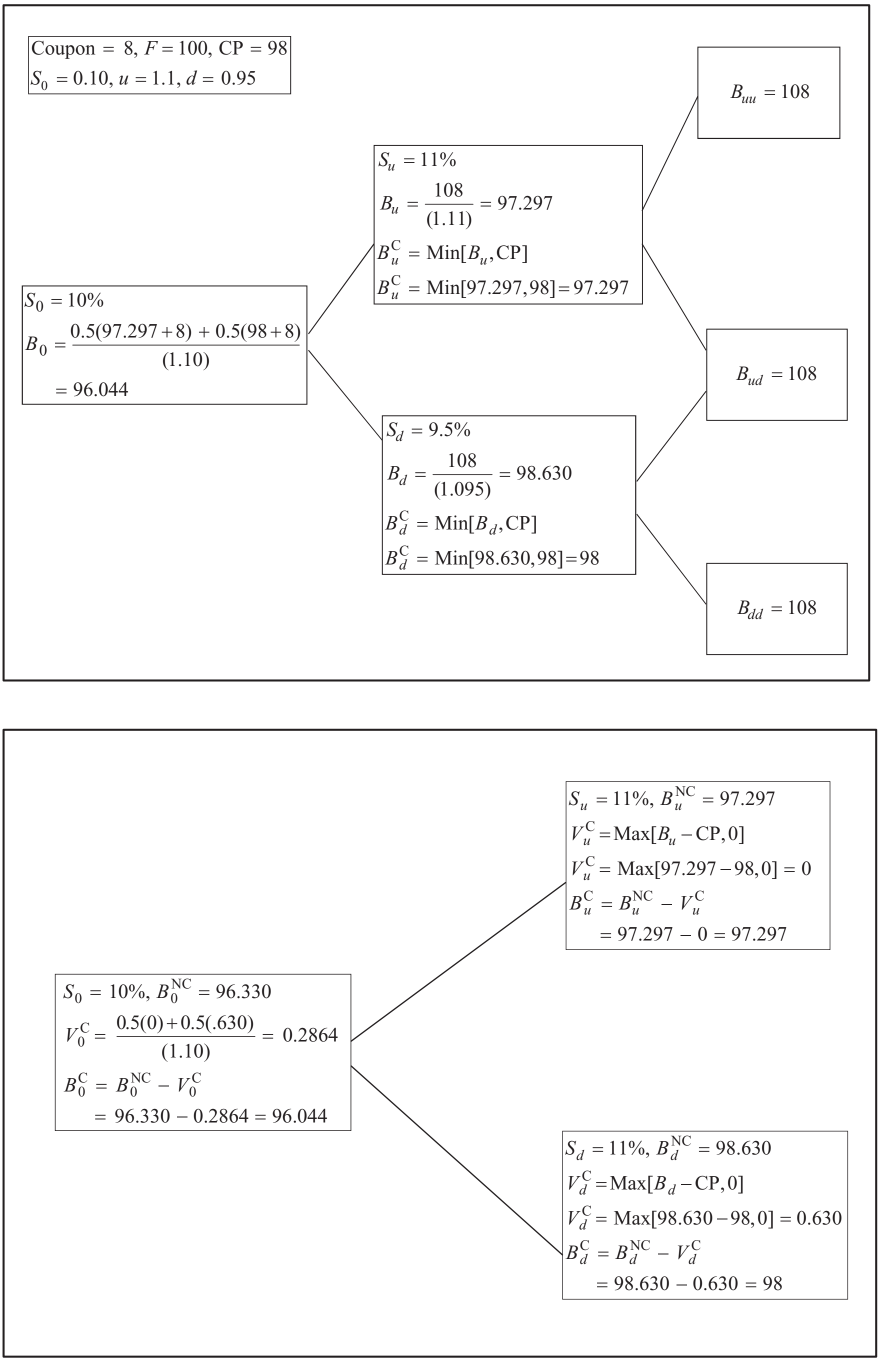


FIGURE 14.4 Value of Two-Period Callable Bond

Valuing a Three-Period Bond

The binomial approach to valuing a two-period bond requires only a one-period binomial tree of one-period spot rates. If we want to value a three-period bond, we in turn need a two-period interest rate tree. For example, suppose we wanted to value a three-period, 9% coupon bond with no default risk or option features. In this case, market risk exists in two periods: period 2, where there are three possible spot rates, and period 1, where there are two possible rates. To value the bond, we first determine the three possible values of the bond in period 2 given the three possible spot rates and the bond's certain cash flow next period (maturity). As shown in Figure 14.5, the three possible values in period 2 are $B_{uu} = 109/1.121 = 97.2346$, $B_{ud} = 109/1.1045 = 98.6872$, and $B_{dd} = 109/1.09025 = 99.9771$. Given these values, we next roll the tree to the first period and determine the two possible values there. Note, in this period the values are equal to the present values of the expected cash flows in period 2; that is:

$$B_u = \frac{.5[97.2346 + 9] + .5\,[98.6872 + 9]}{1.11} = 96.3612$$
$$B_d = \frac{.5\,[98.6872 + 9] + .5\,[99.9771 + 9]}{1.095} = 98.9335$$

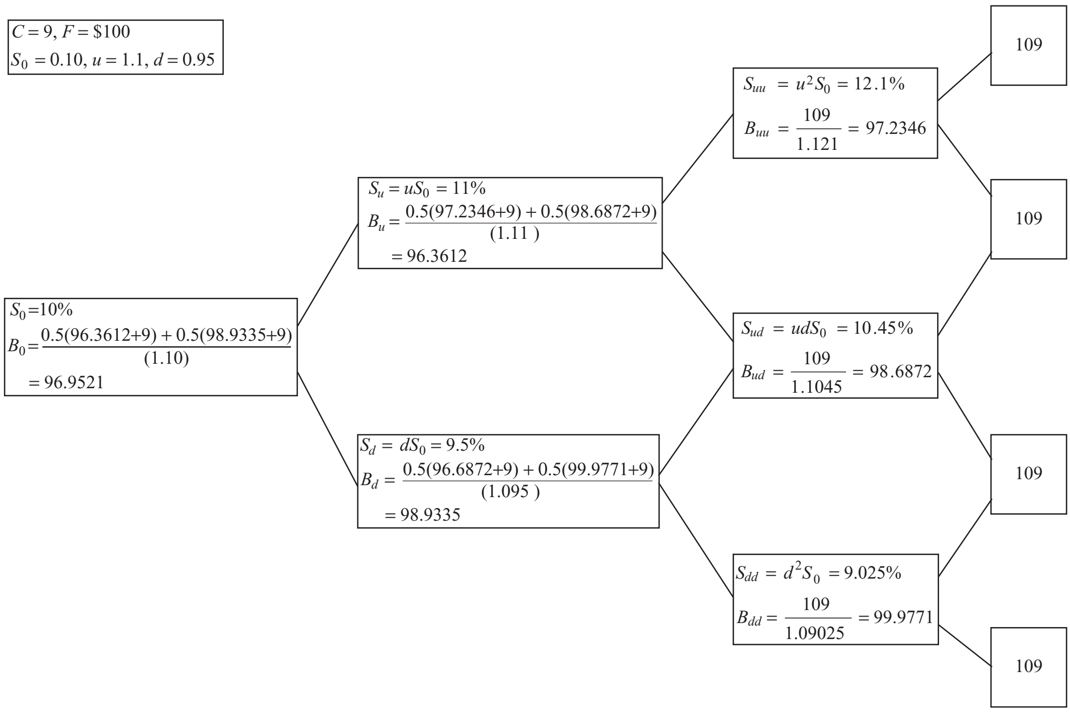
Finally, using the bond values in period 1, we roll the tree to the current period where we determine the value of the bond to be 96.9521:

$$B_0 = \frac{.5\,[96.3612 + 9] + .5\,[98.9335 + 9]]}{1.10} = 96.9521.$$

If the bond is callable, we can determine its value by first comparing each of the noncallable bond values with the call price in period 2 (one period from maturity) and taking the minimum of the two as the callable bond value. We next roll the callable bond values from period 2 to period 1 where we determine the two bond values at each node as the present value of the expected cash flows, and then for each case we select the minimum of the value we calculated or the call price. Finally, we roll those two callable bond values to the current period and determine the callable bond's price as the present value of period 1's expected cash flows.

Figure 14.6 shows the binomial tree value of the three-period, 9% bond given a call feature with a CP = 98. Note, at the two lower nodes in period 2, the bond would be called at 98 and therefore the callable bond price would be 98; at the top node, the bond price of 97.2346 would prevail. Rolling these prices to period 1, the present values of the expected cash flows are 96.0516 at the 11% spot rate and 97.7169 at the 9.5% rate. Since neither of these values is less than the CP of 98, each represents the callable bond value at that node. Rolling these two values to the current period, we obtain a value of 96.2584 for the three-period callable bond.

The alternative approach to valuing the callable bond is to determine the value of the call option at each node and then subtract that value from the noncallable value to obtain the callable bond's price. However, different from our previous two-period case, when there are three periods or more, we need to take into account that prior



Period	Path 1	Geometric Mean	CF	PV
1	0.100000	0.1000000	9	8.18182
2	0.095000	0.0974972	9	7.47198
3	0.090250	0.0950761	109	83.00296
				98.65676

Period	Path 2	Geometric Mean	CF	PV
1	0.100000	0.1000000	9	8.18182
2	0.095000	0.0974972	9	7.47198
3	0.104500	0.0998265	109	81.93208
				97.58588

Period	Path 3	Geometric Mean	CF	PV
1	0.100000	0.1000000	9	8.18182
2	0.110000	0.1049887	9	7.37101
3	0.104500	0.1048258	109	80.82489
				96.37771

Period	Path 4	Geometric Mean	CF	PV
1	0.100000	0.1000000	9	8.18182
2	0.110000	0.1049887	9	7.37101
3	0.121000	0.1103002	109	79.63523
				95.18806

Weighted average value = (98.65676 + 97.58588 + 96.37771 + 95.18806)/4 = 96.9521

FIGURE 14.5 Value of Three-Period Option-Free Bond

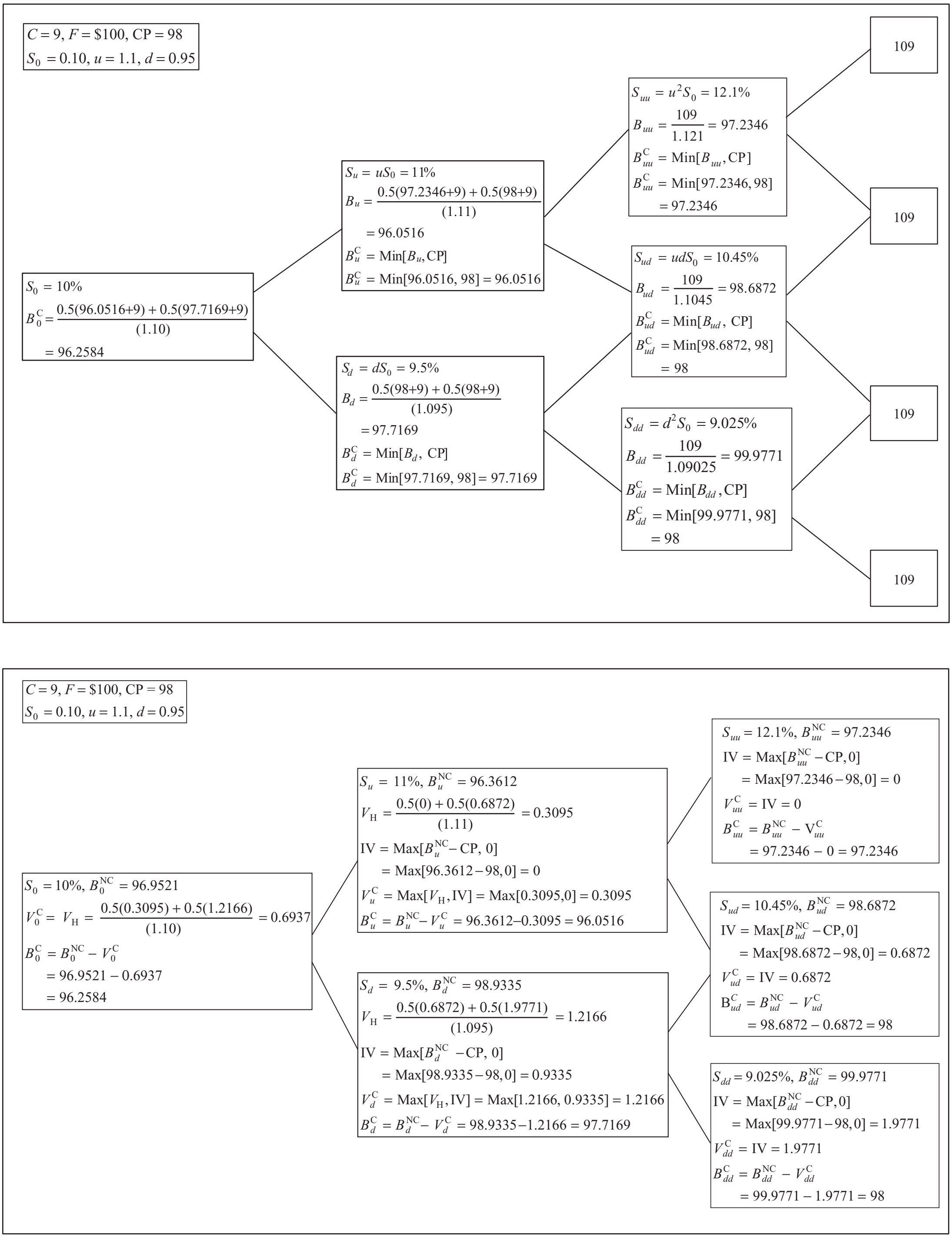


FIGURE 14.6 Value of Three-Period Callable Bond

to maturity the bond issuer has two choices: She can either exercise the option or she can hold it for another period. The exercising value, IV, is

$$\text{IV} = \text{Max}[B_t^{NC} - \text{CP}, 0]$$

whereas the value of holding, V_H, is the present value of the expected call value next period:

$$V_H = \frac{qV_u^C + (1-q)V_d^C}{1+S}$$

If V_H exceeds IV, the issuer will hold the option another period and the value of the call in this case will be the holding value. In contrast, if IV is greater than V_H, then the issuer will exercise the call immediately and the value of the option will be IV. Thus, the value of the call option is equal to the maximum of IV or V_H:

$$V^C = \text{Max}[\text{IV}, \text{V}_H]$$

Figure 14.5 shows this valuation approach applied to the three-period callable bond. Note, in period 2 the value of holding is zero at all three nodes since next period is maturity where it is too late to call. The issuer, though, would find it profitable to exercise in two of the three cases where the call price is lower than the bond values. The three possible callable bond values in period 2 are:

$$B_{uu}^C = B_{uu}^{NC} - V_{uu}^C = 97.2346 - \text{Max}[97.2346 - 98, 0] = 97.2346$$
$$B_{ud}^C = B_{ud}^{NC} - V_{ud}^C = 98.6872 - \text{Max}[98.6872 - 98, 0] = 98$$
$$B_{dd}^C = B_{dd}^{NC} - V_{dd}^C = 99.9771 - \text{Max}[99.9771 - 98, 0] = 98$$

In period 1, the noncallable bond price is greater than the call price at the lower node. In this case, the IV is 98.9335 − 98 = .9335. The value of holding the call, though, is 1.2166:

$$V_H = \frac{.5[\text{Max}[98.6872 - 98, 0]] + .5[\text{Max}[99.9771 - 98, 0]]}{1.095} = 1.2166$$

Thus, the issuer would find it more valuable to defer the exercise one period. As a result, the value of the call option is Max[IV, V_H] = Max[.8394, 1.2166] = 1.2166 and the value of the callable bond is 97.7169 (the same value we obtained using the price constraint approach):

$$B_d^C = B_d^{NC} - V_d^C = 98.9335 - 1.2166 = 97.7169$$

At the upper node in period 1 where the price of the noncallable is 96.3612, the exercise value is zero. The value of the call option in this case is equal to its holding value of 0.3095:

$$V_H = \frac{.5[\text{Max}[97.2346 - 98, 0]] + .5[\text{Max}[98.6872 - 98, 0]]}{1.11} = .3095$$

and the value of the callable bond is 96.0517 (the same value as the constraint one with some slight rounding):

$$B_u^C = B_u^{NC} - V_u^C = 96.3612 - .3095 = 96.0517$$

Finally, rolling the two possible option values of .3095 and 1.2166 in period 1 to the current period, we obtain the current value of the option of .6937 and the same callable bond value of 96.2584 that we obtained using the first approach:

$$V_0^C = \frac{.5[.3095] + .5[1.2166]}{1.10} = .6937$$

$$B_0^C = B_0^{NC} - V_0^C = 96.9521 - .6937 = 96.2584$$

Alternative Binomial Valuation Approach

In valuing an option-free bond with the binomial approach, we started at the bond's maturity and rolled the tree to the current period. An alternative but equivalent approach is to calculate the weighted average value of each possible path defined by the binomial process. This value is referred to as the ***theoretical value***.

To see this approach, consider again the three-period, 9% option-free bond valued with a two-period interest rate tree (Figure 14.5). For a two-period interest rate tree, there are four possible ***interest rate paths***. That is, to get to the second-period spot rate of 9.025%, there is one path (spot rate decreasing two consecutive periods); to get to 10.45%, there are two paths (decrease in the first period and increase in the second and increase in the first and decrease in the second); to get to 12.1% there is one path (increase two consecutive periods). Given the three-period bond's cash flows of 9, 9, and 109, the value or equilibrium price of each path is obtained by discounting each of the cash flows by their appropriate one-, two-, and three-period spot rates:

$$B_0^{\text{Path } i} = \frac{CF_1}{(1 + S_1^{\text{Path } i})} + \frac{CF_2}{(1 + S_2^{\text{Path } i})^2} + \frac{CF_3}{(1 + S_3^{\text{Path } i})^3}$$

The t-period spot rate is equal to the geometric average of the current and expected one-period spot rates. For example, for Path 1 (path with two consecutive decreases in rates), its one-period rate is $S_t = S_1 = 10\%$, its two-period rate is $S_2 = 9.74972\%$ (geometric average of $S_0 = 10\%$ and $S_d = 9.5\%$), and its three-period rate is $S_3 = 9.50761\%$ (geometric average of $S_0 = 10\%$, $S_d = 9.5\%$, and $S_{dd} = 9.025\%$).

Discounting the three-period bond's cash flows by these rates yields a value for Path 1 of 98.65676:

$$B_0^{\text{Path } 1} = \frac{9}{(1.10)} + \frac{9}{(1.0974972)^2} + \frac{109}{(1.0950761)^3} = 98.65676$$

The periodic spot rates and bond values for each of the four paths are shown at the bottom of Figure 14.5. Given the path values, the bond's weighted average value is obtained by summing the weighted values of each path with the weights being the probability of attaining that path. For a two-period interest rate tree with a probability of the rate increasing in one period being $q = .5$, the probability of attaining each path is .25. Using these probabilities, the three-period bond's weighted average value or theoretical value is equal to 96.9521—the same value we obtained earlier by rolling the bond's value from maturity to the current period.

14.3 VALUING BONDS WITH OTHER OPTION FEATURES

In addition to call features, bonds can have other embedded options such as a put option, a stock convertibility clause, or a sinking fund arrangement in which the issuer has the option to buy some of the bonds back either at their market price or at a call price. The binomial tree can be easily extended to the valuation of bonds with these embedded option features.

Putable Bond

A ***putable bond***, or put bond, gives the holder the right to sell the bond back to the issuer at a specified exercise price (or put price), PP. In contrast to callable bonds, putable bonds benefit the holder: If the price of the bond decreases below the exercise price, then the bondholder can sell the bond back to the issuer at the exercise price. From the bondholder's perspective, a put option provides a hedge against a decrease in the bond price. If rates decrease in the market, then the bondholder benefits from the resulting higher bond prices, and if rates increase, then the bondholder can exercise, giving her downside protection. Given that the bondholder has the right to exercise, the price of a putable bond will be equal to the price of an otherwise identical nonputable bond plus the value of the put option ($V_0{}^P$):

$$B_0^P = B_0^{NP} + V_0^P$$

Since the bondholder will find it profitable to exercise whenever the put price exceeds the bond price, the value of a putable bond can be found using the binomial approach by comparing bond prices at each node with the put price and selecting the *maximum* of the two, Max[B_t, PP]. The same binomial value can also be found by determining the value of the put option at each node and then pricing the putable bond as the value of an otherwise identical nonputable bond plus the value of the put option. In using the second approach, the value of the put option will be the maximum of either its intrinsic value (or exercising value), IV = Max[PP – B_t,0], or its holding value (the present value of the expected put value next period). In most cases, though, the put's intrinsic value will be greater than its holding value.

To illustrate, suppose the three-period, 9% option-free bond in our previous example had a put option giving the bondholder the right to sell the bond back to the issuer at an exercise price of PP = 97 in periods 1 or 2. Using the two-period tree of one-period spot rates and the corresponding bond values for the option-free bond (Figure 14.5), we start, as we did with the callable bond, at period 2 and investigate each of the nodes to determine if there is an advantage for the holder to exercise. In all three of the cases in period 2, the bond price exceeds the exercise price; thus, there are no exercise advantages in this period and each of the possible prices of the putable bond are equal to their non-putable values and the values of each of the put options are zero (see Figure 14.7). In period 1, though, it is profitable for the holder to exercise when the spot rate is 11%. At that node, the value of the non-putable bond is 96.3612, compared to PP = 97; thus the value of putable bond is its exercise price of 97:

$$B_u^P = \text{Max}[96.3612, 97] = 97$$

The putable bond price of 97 can also be found by subtracting the value of the put option from the price of the non-putable bond. The value of the put option at this node is 0.6388,

$$V_u^P = \text{Max}[\text{IV}, V_H] = \text{Max}[97 - 96.3612, 0] = .6388$$

Thus, the value of the putable bond is

$$B_u^P = B_u^{NP} + V_u^P$$
$$B_u^P = 96.3612 + .6388 = 97$$

At the lower node in period 1, it is not profitable to exercise nor is there any holding value of the put option since there is no exercise advantage in period 2. Thus, at the lower node, the non-putable bond price prevails. Rolling the two putable bond values in period 1 to the present, we obtain a current value of the putable bond of 97.2425:

$$B_0^P = \frac{.5[97 + 9] + .5[98.9335 + 9]}{1.10} = 97.2425$$

This value also can be obtained using the alternative approach by computing the present value of the expected put option value in period 1 and then adding that to the current value of the non-putable bond. With possible exercise values of .6388 and 0 in period 1, the current put option value is 0.2904:

$$V_0^P = \frac{.5[.6388] + .5[0]}{1.10} = 0.2904$$

Thus:

$$B_0^P = B_0^{NP} + V_0^P = 96.9521 + 0.2904 = 97.2425$$

The inclusion of the put option in this example causes the bond price to increase from 96.9521 to 97.2425, reflecting the value the put option has to the bondholder.

Sinking-Fund Bonds

Many bonds have sinking fund clauses specified in their indenture requiring that the issuer make scheduled payments into a fund or buy up a certain proportion of the bond issue each period. Often when the sinking fund agreement specifies an orderly

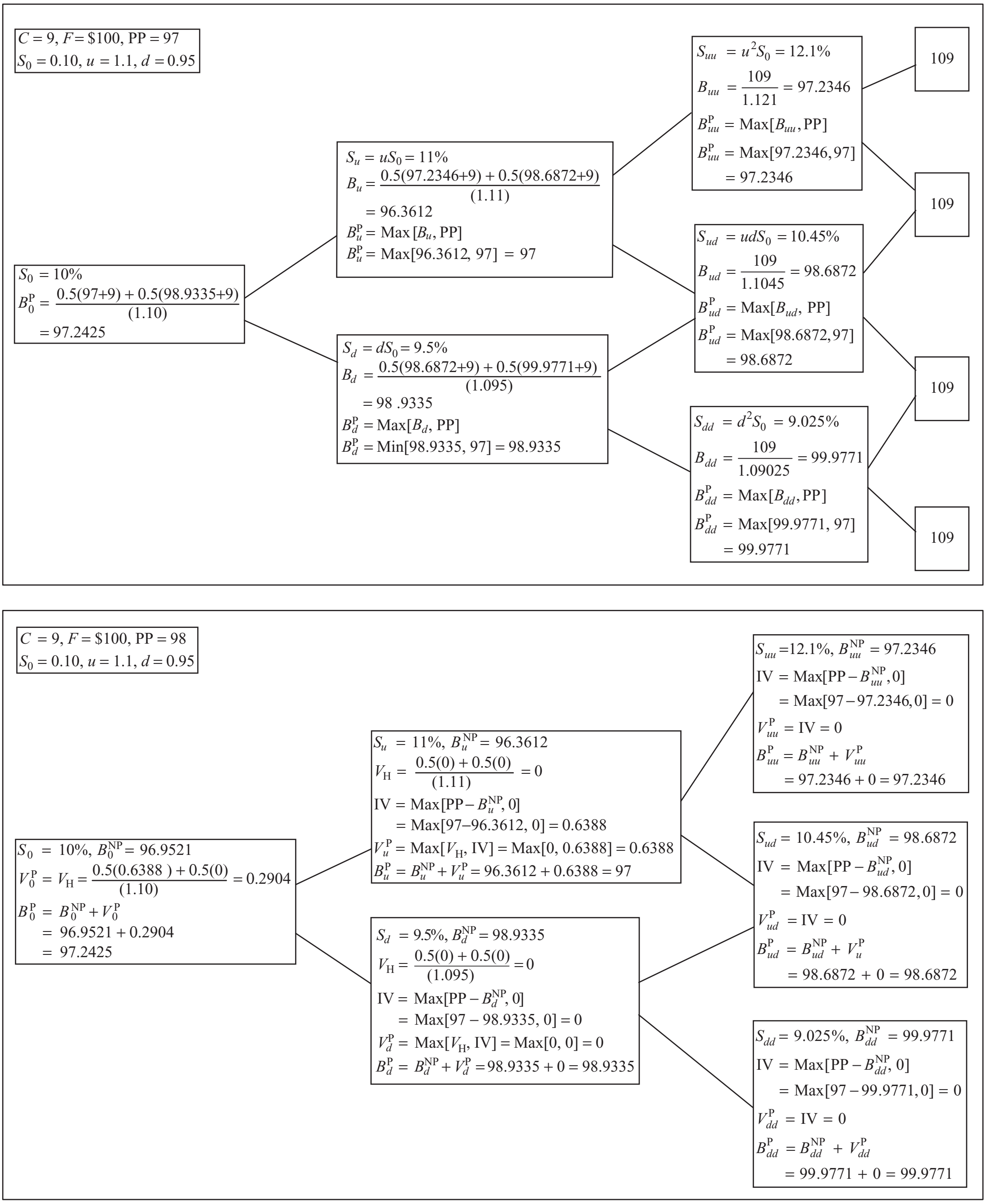


FIGURE 14.7 Value of Putable Bond

retirement of the issue, the issuer is given an option of either purchasing the bonds in the market or calling the bonds at a specified call price. This option makes the sinking fund valuable to the issuer. If interest rates are relatively high, then the issuer will be able to buy back the requisite amount of bonds at a relatively low market price; if rates are low and the bond price high, though, then the issuer will be able to buy back the bonds on the call option at the call price. Thus, a ***sinking fund bond*** with this type of call provision should trade at a lower price than an otherwise identical non-sinking fund bond.

Similar to callable bonds, a sinking fund bond can be valued using the binomial tree approach. To illustrate, suppose a company issues a $15 million, three-period bond with a sinking fund obligation requiring that the issuer sink $5 million of face value after the first period and $5 million after the second, with the issuer having an option of either buying the bonds in the market or calling them at a call price of 98. Assume the same interest rate tree and bond values characterizing the three-period, 9% noncallable described in Figure 14.5 apply to this bond without its sinking fund agreement. With the sinking fund, the issuer has two options: At the end of period 1, the issuer can buy $5 million worth of the bond either at 98 or at the bond's market price, and at the end of period 2, the issuer has another option to buy $5 million worth of the bond either at 98 or the market price. As shown in Figure 14.8, the value of the period 1 option (in terms of $100 face value) is ${V_0}^{SF(1)} = 0.4243$ and the value of the period 2 call option is ${V_0}^{SF(2)} = 0.6937$. Note, since the sinking fund arrangement requires an immediate exercise or bond purchase at the specified sinking fund dates, the possible values of the sinking fund's call features at those dates are equal to the intrinsic values. This differs from the valuation of a standard callable bond where a holding value is also considered in determining the value of the call option.

Since each option represents one-third of the issue, the value of the bond's sinking fund option is

$$V_0^{SF} = (1/3)(.4243) + (1/3)(.6937) = 0.3727$$

and the value of the sinking fund bond is 96.5794 per $100 face value:

$$B_0^{SF} = B_0^{NSF} - V_0^{SF}$$
$$B_0^{SF} = 96.9521 - 0.3727 = 96.5794$$

Thus, the total value of the $15 million face value issue is $14,486,910:

$$\text{Issue value} = \frac{96.5794}{100}\$15{,}000{,}000 = \$14{,}486{,}910$$

or

$$\text{Issue value} = \left(\frac{96.9521}{100}\right)\$15{,}000{,}000 - \left(\frac{0.3727}{100}\right)\$15{,}000{,}000$$
$$= \$14{,}486{,}910$$

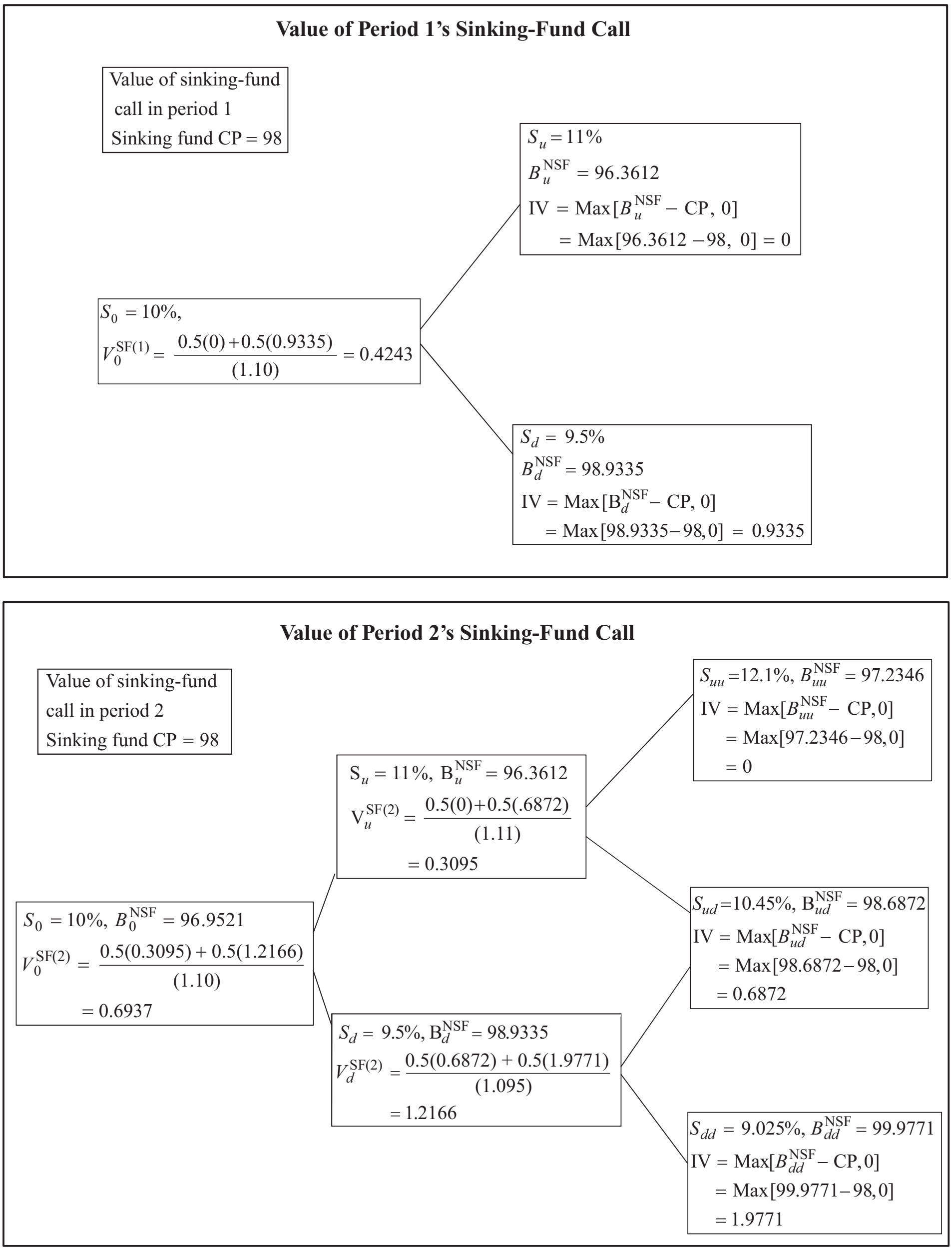


FIGURE 14.8 Value of Sinking Fund Call

Like a standard callable bond, a sinking fund provision with a call feature lowers the value of an otherwise identical non-sinking fund bond.

14.4 CONVERTIBLE BOND

A ***convertible bond*** gives the holder the right to convert the bond into a specified number of shares of stock. Convertibles are often sold as a subordinate issue, with the conversion feature serving as a bond sweetener. To the investor, convertible bonds offer the potential for a high rate of return if the company does well and its stock price increases, while providing some downside protection as a bond if the stock declines. Convertibles are usually callable, with the convertible bondholder usually having the right to convert the bond to stock if the issuer does call.

Convertible Bond Terms

Suppose our illustrative three-period, 9% bond were convertible into four shares of the underlying company's stock. The conversion features of this bond include its conversion ratio, conversion value, and straight debt value. The ***conversion ratio (CR)*** is the number of shares of stock that can be converted when the bond is tendered for conversion. The conversion ratio for this bond is four. The ***conversion value (CV)*** is the convertible bond's value as a stock. At a given point in time, the conversion value is equal to the conversion ratio times the market price of the stock (P_t^S):

$$\text{CV}_t = (\text{CR})P_t^S$$

If the current price of the stock were 92, then the bond's conversion value would be CV = (4)($92) = $368.[2] Finally, the ***straight debt value (SDV)*** is the convertible bond's value as a nonconvertible bond. This value is obtained by discounting the convertible's cash flow by the discount rate on a comparable nonconvertible bond.

Minimum and Maximum Convertible Bond Prices

Arbitrageurs ensure that the minimum price of a convertible bond is the greater of either its straight debt value or its conversion value:

$$\text{Min}\,B_t^{\text{CB}} = \text{Max}[\text{CV}_t, \text{SDV}_t]$$

If a convertible bond is priced below its conversion value, arbitrageurs could buy it, convert it to stock, and then sell the stock in the market to earn a riskless profit. Arbitrageurs seeking such opportunities would push the price of the convertible up until it is at least equal to its CV. Similarly, if a convertible is selling below its SDV, then arbitrageurs could profit by buying the convertible and selling it as a regular bond.

In addition to a minimum price, if the convertible is callable, the call price at which the issuer can redeem the bond places a maximum limit on the convertible. That is, the issuer will find it profitable to buy back the convertible bond once its price is equal to the call price. Buying back the bond, in turn, frees the company to sell new stock or bonds at prices higher than the stock or straight debt values associated with the convertible. Thus, the maximum price of a convertible is the call price. The actual price that a convertible will trade for will be at a premium above its minimum value but below its maximum.

Valuation of Convertibles Using Binomial Trees

The valuation of a convertible bond with an embedded call is more difficult than the valuation of a bond with just one option feature. In the case of a callable convertible bond, one has to consider not only the uncertainty of future interest rates, but also the uncertainty of stock prices. A rate decrease, for example, may not only increase the convertible's SDV and the chance the bond could be called, but if the rate decrease is also associated with an increase in the stock price, it may also increase the conversion value of the convertible and the chance of conversion. The valuation of convertibles therefore needs to take into account the random patterns of interest rates, stock prices, and the correlation between them.

To illustrate the valuation of convertibles, consider a three-period, 10% convertible bond with a face value $1,000 that can be converted to 10 shares of the underlying company's stock (CR = 10). To simplify the analysis, assume the bond has no call option and no default risk, that the current yield curve is flat at 5%, and that the yield curve will stay at 5% for the duration of the three periods (i.e., no market risk). In this simplified world, the only uncertainty is the future stock price. Like interest rates, suppose the convertible bond's underlying stock price follows a binomial process where in each period it has an equal chance it can either increase to equal u times its initial value or decrease to equal d times the initial value, where $u = 1.1$, $d = 1/1.1 = .9091$, and the current stock price is $92. The possible stock prices resulting from this binomial process are shown in Figure 14.9, along with the convertible bond's conversion values.

Since spot rates are assumed constant, the value of the convertible bond will only depend on the stock price. To value the convertible bond, we start at the maturity date of the bond. At that date, the bondholder will have a coupon worth 100 and will either convert the bond to stock or receive the principal of $1,000. At the top stock price of $122.45, the convertible bondholder would exercise her option, converting the bond to ten shares of stock. The value of the convertible bond, B^{CB}, at the top node in period 3 would therefore be equal to its conversion value of $1,224.50 plus the $100 coupon:

$$B_{uuu}^{CB} = \text{Max}[\text{CV}_t, F] + C$$
$$B_{uuu}^{CB} = \text{Max}[1224.50, 1{,}000] + 100$$
$$B_{uuu}^{CB} = 1324.50$$

Similarly, at the next stock price of $101.20, the bondholder would also find it profitable to convert; thus, the value of the convertible in this case would be its conversion value of $1,012 plus the $100 coupon. At the lower two stock prices in period 3 of $83.64 and $69.12, conversion is worthless; thus, the value of the convertible bond is equal to the principal plus the coupon: $1,100.

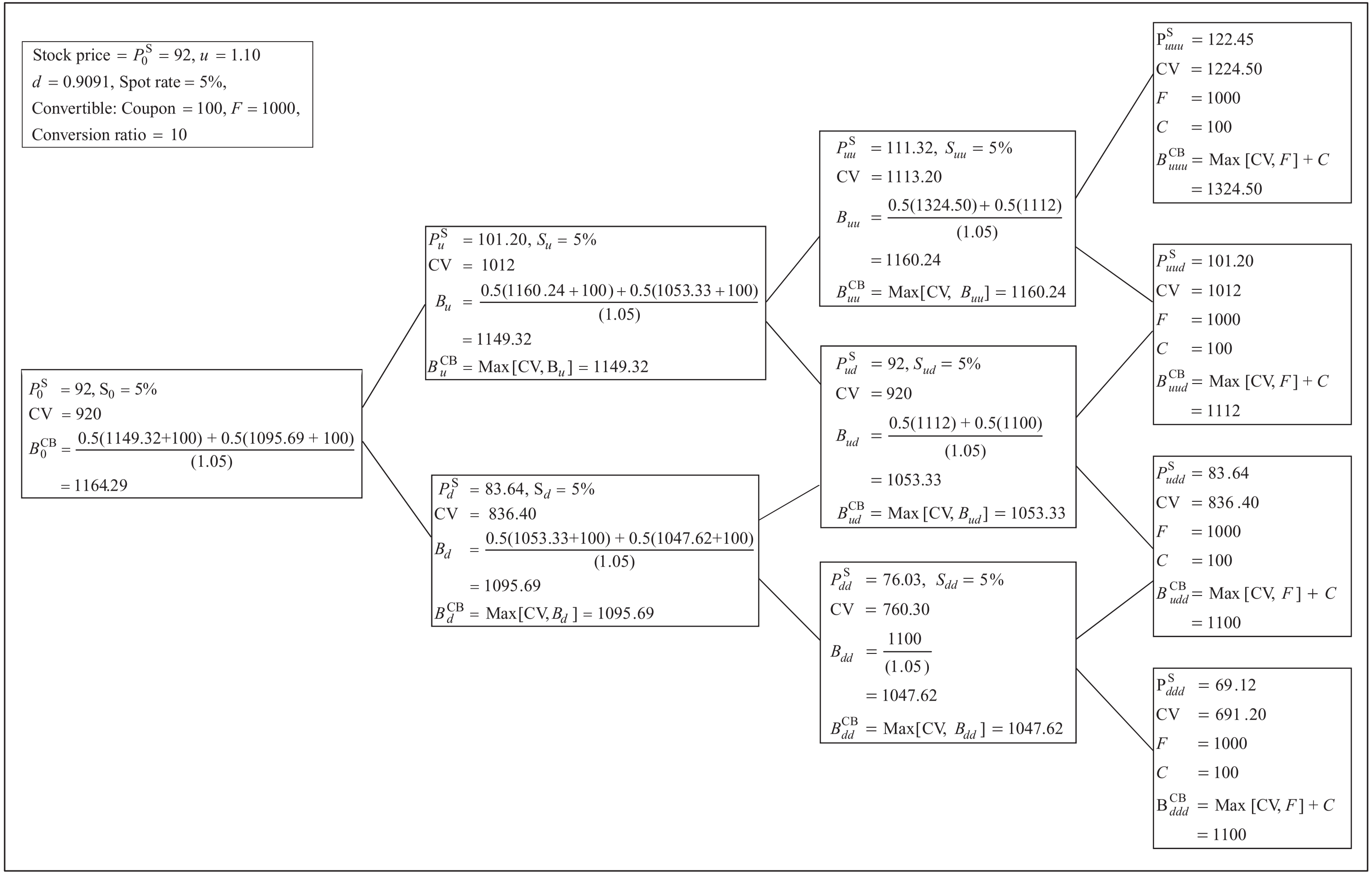


FIGURE 14.9 Value of Convertible Bond

In period 2, at each node the value of the convertible bond is equal to the maximum of either the present value of the bond's expected value at maturity or its conversion value. At all three stock prices, the present values of the bond's expected values next period are greater than the bond's conversion values, including at the highest stock price; that is, at $P^S{}_{uu}$ = \$111.32, the CV is \$1,113.20 compared to the convertible bond value of \$1,160.24; thus the value of the convertible bond is \$1,160.24:

$$B_{uu} = \frac{.5[1324.50] + .5[1112]}{1.05} = 1160.24$$

$$B^{CB}_{uu} = \text{Max}[B_{uu}, \text{CV}] = [1160.24, 1113.20] = 1160.24$$

Thus, in all three cases, the values of holding the convertible bond are greater than the conversion values. Similarly, the two possible bond values in period 1 (generated by rolling the three convertible bond values in period 2 to period 1) also exceed their conversion values. Rolling the tree to the current period, we obtain a convertible bond value of \$1,164.29. As we would expect, this value exceeds both the convertible bond's current conversion value of \$920 and its SDV of \$1,136.16 (assuming a 5% discount rate):

$$\text{SDV} = \frac{\$100}{(1.05)} + \frac{\$100}{(1.05)^2} + \frac{\$1{,}100}{(1.05)^3} = \$1{,}136.16$$

As noted, the valuation of a convertible becomes more complex when the bond is callable. With callable convertible bonds, the issuer will find it profitable to call the convertible prior to maturity whenever the price of the convertible is greater than the call price. However, when the convertible bondholder is faced with a call, she usually has the choice of either tendering the bond at the call price or converting it to stock. Since the issuer will call whenever the call price exceeds the convertible bond price, he is in effect forcing the holder to convert. By doing this, the issuer takes away the bondholder's value of holding the convertible, forcing the convertible bond price to equal its conversion value.

To see this, suppose the convertible bond is callable in periods 1 and 2 at a CP = \$1,100. At the top stock price of \$111.32 in period 2, the conversion value is \$1,113.20 (see Figure 14.10). In this case, the issuer can force the bondholder to convert by calling the bond. The call option therefore reduces the value of the convertible from \$1,160.24 to \$1,113.20. At the other nodes in period 2, neither conversion by the bondholders nor calling by the issuer is economical; thus the bond values prevail. In period 1, the call price of \$1,100 is below the bond value (\$1,126.92), but above the conversion value (\$1,012). In this case, the issuer would call the bond and the holder would take the call instead of converting. The value of the callable convertible bond in this case would be the call price of \$1,100. At the lower node, calling and converting are not economical and thus the bond value of \$1,095.69 prevails. Rolling period 1's upper and lower convertible bond values to the current period, we obtain a value for the callable convertible bond of \$1,140.80, which is less than the noncallable convertible bond value of \$1,164.29 and greater than the straight debt value of a noncallable bond of \$1,136.16.

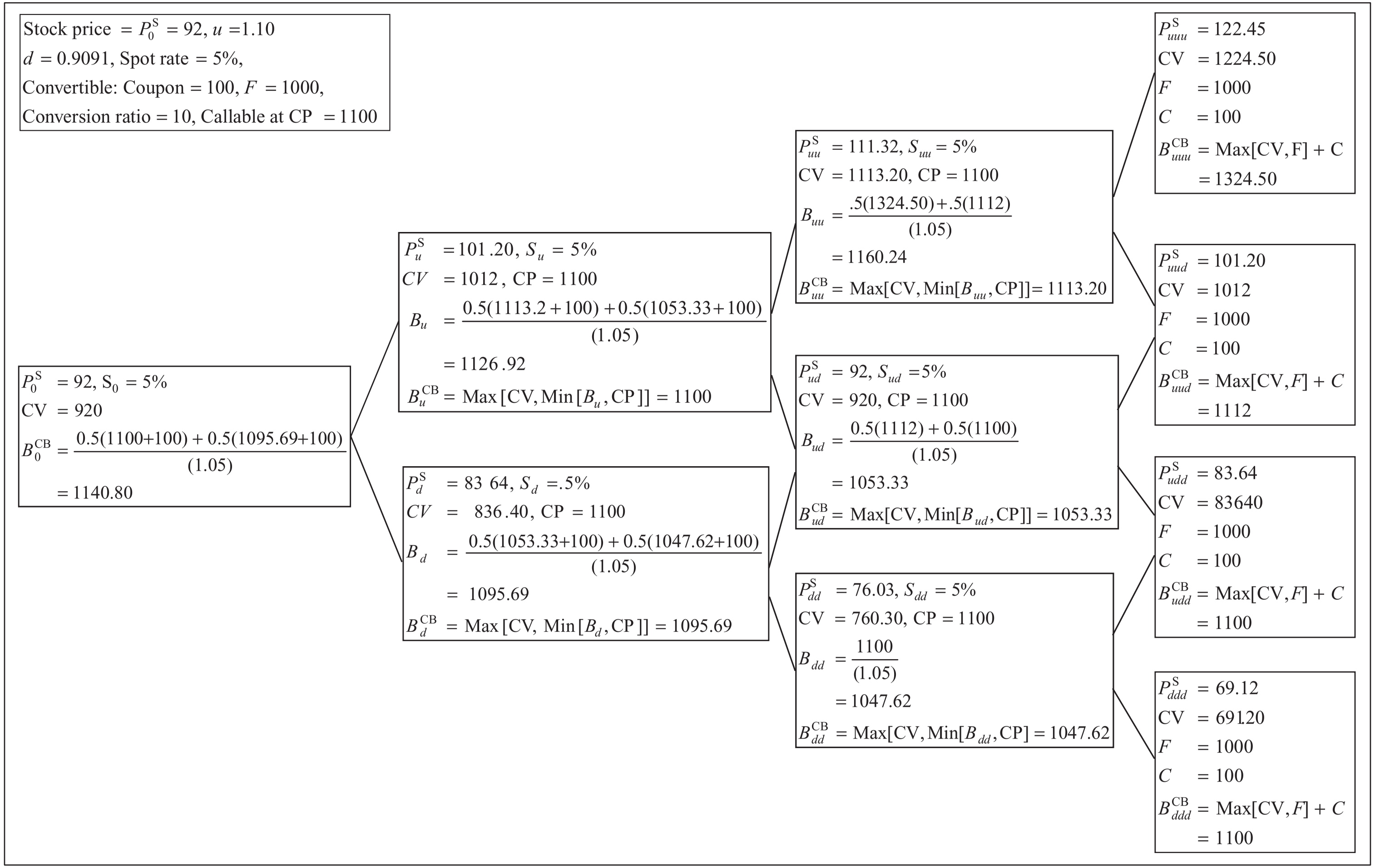


FIGURE 14.10 Value of Convertible Bond with Call

In the above two cases, we assumed for simplicity that the yield curve remained constant at 5% for the period. As noted, the complexity of valuing convertibles is taking into account the uncertainty of two variables—stock prices and interest rates.[3] A simple way to model such behavior is to use correlation or regression analysis to first estimate the relationship between a stock's price and the spot rate, and then either with a binomial model of spot rates identify the corresponding stock prices or with a binomial model of stock prices identify the corresponding spot rates. For example, suppose using regression analysis we estimated the following relationship between the stock in our above example and the one-period spot rate:

$$S_t = .16 - .001 P_t^S$$

Using this equation, the corresponding spot rates associated with the stock prices from the three-period tree would be

P_t^S	S_t
111.32	4.87%
101.20	5.90%
92.00	6.80%
83.64	7.64%
76.03	8.40%

Figure 14.11 shows the binomial tree of stock prices along with their corresponding spot rates. Given the rates and stock prices, the methodology for valuing the convertible bond is identical to our previous analysis. Again, we start at maturity where we value the bond at each node as the maximum of either its conversion value or face value. Given these values, we then move to period 2, where we first determine the present values of period 3's expected bond values and coupons using the spot rates we have estimated. We then compare each of those values with the call price and the conversion value. As we noted in the previous case, if the call price exceeds the convertible bond value, then the issuer will call the bond, and the convertible bond's price will reflect what is more profitable for the bondholder: accept the call and receive the call price or convert to stock. As in our previous case, at the top node in period 2, the call price of $1,100 is less than the convertible bond value of $1,161.68. In this case, the issuer will call the bond and the holder will find it more profitable to convert; that is, the conversion value of $1,113.20 exceeds the call price of $1,100. Thus, the convertible bond would be equal to the conversion value. The other two convertible bond prices in period 2 are less than the call price, implying the issuer would not exercise; the prices also are greater than the conversion value, implying the holder would not convert. Thus, the values of the convertibles in these two cases are equal to the present values of their expected cash flows for the next period. In period 1, the call price is less than the bond value ($1,108.96) and greater than the conversion value ($1,012) at the top node. In this case the issuer would call and the bondholder would find it better to accept the call instead of converting; thus, the convertible bond price at this node would be the call price of $1,100. Rolling the tree to the current period with this value and the lower node value, we obtain a

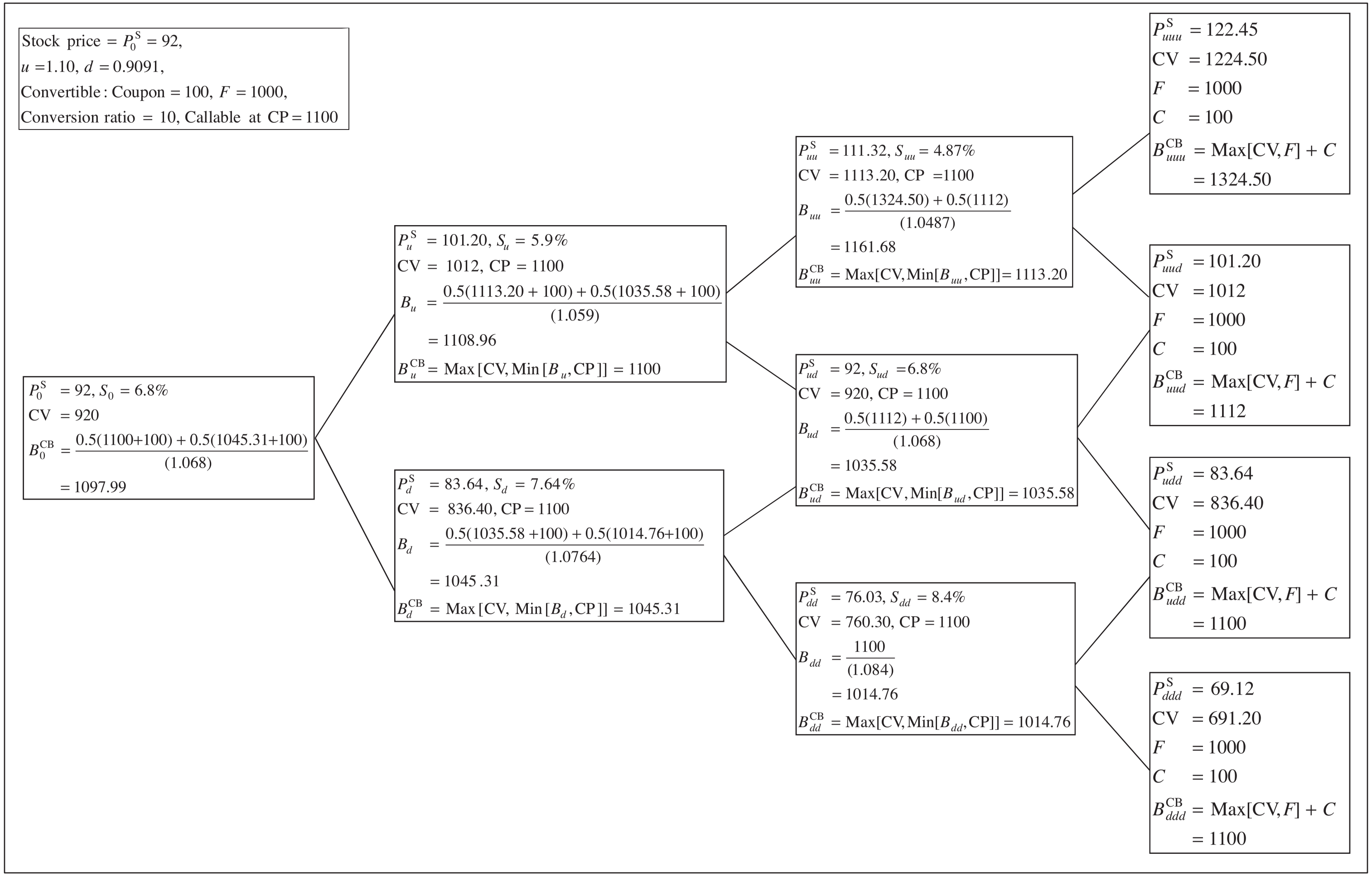


FIGURE 14.11 Value of Convertible Bond with Call and Different Spot Rates

convertible bond value of $1,097.99. This value is lower than the previous case in which we assumed a constant yield curve at 5%.

It should be noted that modeling a bond with multiple option features and influenced by the random patterns of more than one factor is more complex in practice than the simple model described here. The preceding model is intended only to provide some insight into the dynamics involved in valuing a bond with embedded convertible and call options given different interest rate and stock price scenarios.

14.5 VALUING MORTGAGE-BACKED SECURITIES

The valuation of mortgage-backed securities (MBSs) and other asset-backed securities is more complex because of the difficulty in estimating cash flows due to the prepayment options of the mortgage borrowers. One common approach used to determine the possible values of an MBS is vector analysis. As explained in Chapter 11, vector analysis involves generating a matrix of MBS values based on different discount rates and PSA vectors. Each PSA vector, in turn, is obtained by dividing the total number of periods to maturity into a number of periods with different PSA speeds. One way to estimate different PSA vectors is to use a binomial interest rate tree. Using the different vectors to value a security represents a Monte Carlo simulation approach to valuation.

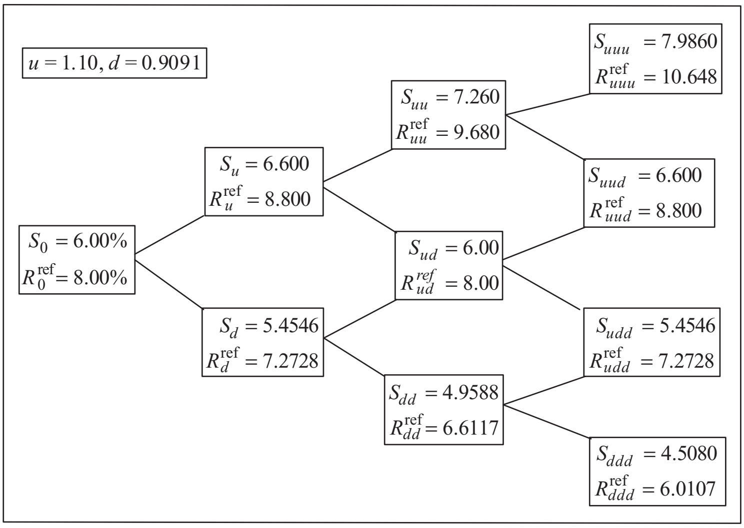
Monte Carlo Simulation

Monte Carlo simulation involves generating a set of cash flows for an MBS, ABS, or CMO tranche based on simulated future interest rates. From the cash flows, the value of the MBS can be determined given the assumed rates and an assumed speed. The simulation involves first generating a number of interest rate paths, next estimating the cash flow for each path based on a prepayment model that is dependent on the assumed interest rates, third determining the present values of each path's cash flow, and last calculating the average value and standard deviation of the distribution of values from the assumed paths. The average value is referred to as the theoretical value.

Interest Rate Paths

The first step in the simulation process is to generate interest rate paths. This can be done by using a binomial interest rate tree model. Typically, the trees are generated for monthly spot rates and for mortgage refinancing rates, with the length of each period being one month and with the number of periods equaling the maturity of the MBS or tranche (e.g., 360 months). From these trees, thousands of interest rate paths can be generated. The figure in Table 14.1 shows a simpler (and more manageable) three-period interest rate tree defined for a one-year spot rate, S_t, and a mortgage refinancing rate with a maturity between 7 and 10 years, $R^{\text{ref}}{}_t$, with the length of each period being one year. Both the one-year spot rates and the refinancing rates shown in the table are derived by assuming that in each period the rates will either increase to equal a proportion $u = 1.1$ of the beginning period's rate or decrease to equal a proportion $d = .9091$ of that rate. With this three-period binomial process,

TABLE 14.1 Binomial Tree for Spot and Refinancing Rates



Path 1	Path 2	Path 3	Path 4
6.0000%	6.0000%	6.0000%	6.0000%
5.4546	5.4546	5.4546	6.6000
4.9588	4.9566	6.0000	6.0000
4.5080	5.4546	5.4546	5.4546
Path 5	**Path 6**	**Path 7**	**Path 8**
6.0000%	6.0000%	6.0000%	6.0000%
5.4546	6.6000	6.6000	6.6000
6.0000	6.0000	7.2600	7.2600
6.6000	6.6000	6.6000	7.9860

there are four possible rates at the end of the third period for the spot and refinancing rates, and as shown in Table 14.1, there are eight possible interest rate paths. For example, to get to the third-period spot rate of 4.508%, there is one path (rates decreasing three consecutive periods); to get to rate 5.4546% there are three paths (decrease in the first period, decrease in the second period, and increase in the third; decrease in the first, increase in the second, and decrease in the third; increase in the first and then decrease in the second and third); to get to rate 6.6%, there are also three paths; to get to rate 7.986%, there is one path.

Estimating Cash Flows The second step is to estimate the cash flow for each interest rate path. The cash flow depends on the prepayment rates assumed. As explained in Chapter 11, most analysts use a prepayment model in which the conditional prepayment rate (CPR) is determined by the seasonality of the mortgages, and by a refinancing incentive that ties the interest rate paths to the proportion of the mortgage collateral prepaid. To illustrate, consider an MBS formed from a mortgage pool with a par value of \$1 million, weighted average coupon (WAC) of 8%, and weighted average maturity of 10 years. To fit this example to the three-year binomial

TABLE 14.2 Cash Flows from MBS: Par Value = \$1Million, WAC = 8%, WAM = 10 Yrs, PT Rate = 8%

Year	Balance	p	Interest	Scheduled Principal	Cash Flow
1	\$1,000,000	\$149,029	\$80,000	\$69,029	\$149,029
2	\$930,971	\$149,029	\$74,478	\$74,552	\$149,029
3	\$856,419	\$149,029	\$68,513	\$80,516	\$149,029
4	\$775,903	\$149,029	\$62,072	\$86,957	\$837,975

Balloon at End of Fourth Year, Annual Cash Flows, and No Prepayment

$$\begin{aligned} \text{Balloon} &= \text{Balance(yr4)} - \text{Sch.prin(yr4)} \\ &= \$775{,}903 - \$86{,}957 = \$688{,}946 \\ CF_4 &= \text{Balloon} + p \\ &= \$688{,}946 + \$149{,}029 = \$837{,}975 \\ CF_4 &= \text{Balance(yr4)} + \text{Interest} \\ &= \$775{,}903 + \$62{,}072 = \$837{,}975 \end{aligned}$$

tree, assume that the mortgages in the pool all make annual cash flows (instead of monthly); all have a balloon payment at the end of year 4; and the pass-through rate on the MBS is equal to the WAC of 8%. Thus, the mortgage pool can be viewed as a four-year asset with a principal payment made at the end of year 4 that is equal to the original principal less the amount paid down. As shown in Table 14.2, if there were no prepayments, then the pool would generate cash flow of \$149,029 each year and a balloon payment of \$688,946 at the end of year 4.

Such a cash flow is, of course, unlikely given prepayment. A simple prepayment model to apply to this mortgage pool is shown in Table 14.3. The model assumes that if the WAC of 8% is less than the current refinancing rate ($X = \text{WAC} - R^{ref} < 0$), the annual CPR will be equal to 5%. On the other hand, if the WAC of 8% is equal to or greater than the refinancing rate ($X = \text{WAC} - R^{ref} \geq 0$), the model assumes that the CPR will exceed 5% and that it will increase within certain ranges as the premium ($X = \text{WAC} - R^{ref}$) increases. Finally, for simplicity the model posits

TABLE 14.3 Prepayment Model

Range $X = \text{WAC} - R^{ref}$	CPR
$X \leq 0$	5%
$0.0\% < X \leq 0.5\%$	10%
$0.5\% < X \leq 1.0\%$	20%
$1.0\% < X \leq 1.5\%$	30%
$1.5\% < X \leq 2.0\%$	40%
$2.0\% < X \leq 2.5\%$	50%
$2.5\% < X \leq 3.0\%$	60%
$X > 3.0\%$	70%

that the relationship between the CPRs and the range of rates is the same in each period; that is, there is no seasoning factor.

With this prepayment model, cash flows can be generated for the eight interest rate paths. These cash flows are shown in Table 14.4. The cash flows for Path 1 (the path with three consecutive decreases in rates) consist of \$335,224 in year 1 (Interest = \$80,000, Scheduled principal = \$69,029.49, and Prepaid principal = \$186,194.10, reflecting a CPR of .20), \$324,764 in year 2, with \$205,540 being prepaid principal (CPR = .30), \$257,259 in year 3, with \$173,802 being prepaid principal (CPR = .40), and \$281,560 in year 4. The year 4 cash flow with the balloon payment is equal to the principal balance at the beginning of the year and the 8% interest on that balance. In contrast, the cash flows for Path 8 (the path with three consecutive interest rate increases) are smaller in the first three years and larger in year 4, reflecting the low CPR of 5% in each period.

Valuing Each Path Like any bond, an MBS or CMO tranche should be valued by discounting the cash flows by the appropriate risk-adjusted spot rates. For an MBS or CMO tranche, the risk-adjusted spot rate, z_t, is equal to the riskless spot rate, S_t, plus a risk premium. If the underlying mortgages are insured against default, then the risk premium would only reflect the additional return needed to compensate investors for the prepayment risk they are assuming. As noted in Chapter 5, this premium is referred to as the option-adjusted spread (OAS). If we assume no default risk, then the risk-adjusted spot rate can be defined as

$$z_t = S_t + k_t$$

where: k = OAS, and the value of each path can be defined as

$$V_i^{\text{Path}} = \sum_{M=1}^{T} \frac{\text{CF}_M}{(1+z_M)^M} = \frac{\text{CF}_1}{1+z_1} + \frac{\text{CF}_2}{(1+z_2)^2} + \frac{\text{CF}_3}{(1+z_3)^3} + \cdots + \frac{\text{CF}_T}{(1+z_T)^T}$$

where $i = i$th path
z_M = spot rate on bond with M-year maturity.
T = maturity of the MBS

For this example, assume the option-adjusted spread is 2% greater than the one-year, riskless spot rates shown in Table 14.1. From these current and future one-year spot rates, the current one-year, two-year, three-year, and four-year equilibrium spot rates can be obtained for each path by using the geometric mean:

$$z_M = ((1+z_{10})(1+z_{11}) \cdots (1+z_{1,M-1}))^{1/M} - 1$$

Thus, the set of spot rates z_1, z_2, z_3, and z_4 needed to discount the cash flows for Path 1 would be:

$$z_1 = .08$$

$$z_2 = ((1+z_{10})(1+z_{11}))^{1/2} - 1 = ((1.08)(1.074546))^{1/2} - 1 = .07727$$

TABLE 14.4 Cash Flow Analysis of an MBS

Pathl	1	2	4	5	6	7	8	9	10	11	12	13	14	15
Year	R^{rel}	Balance	WAC	Interest	PVIF	p	Sch. Prin.	CPR	Prepaid Pri	CF	$z_{1,t-1}$	Z_{to}	Value	Prob.
1	0.072728	1000000	0.08	80000	6.710081	149029.5	69029	0.20	186194	335224	0.080000	0.080000	310392	0.5
2	0.066117	744776	0.08	59582	6.246888	119223.6	59641	0.30	205540	324764	0.074546	0.077270	279846	0.5
3	0.060107	479594	0.08	38368	5.746639	83456.51	45089	0.40	173802	257259	0.069588	0.074703	207255	0.5
4		260703	0.08	20856						281560	0.065080	0.072289	212972	
												Value =	1010465	0.125
Path 2	1	2	4	5	6	7	8	9	10	11	12	13	14	15
Year	R^{rel}	Balance	WAC	Interest	PVIF	p	Sch. Prin.	CPR	Prepaid Pri	CF	$z_{1,t-1}$	Z_{to}	Value	Prob.
1	0.072728	1000000	0.08	80000	6.710081	149029.5	69029	0.20	186194	335224	0.080000	0.080000	310392	0.5
2	0.066117	744776	0.08	59582	6.246888	119223.6	59641	0.30	205540	324764	0.074546	0.077270	279846	0.5
3	0.072728	479594	0.08	38368	5.746639	83456.51	45089	0.20	86901	170358	0.069588	0.074703	137245	0.5
4		347604	0.08	27808						375413	0.074546	0.074664	281461	
												Value =	1008945	0.125
Path 3	1	2	4	5	6	7	8	9	10	11	12	13	14	15
Year	R^{rel}	Balance	WAC	Interest	PVIF	p	Sch. Prin.	CPR	Prepaid Pri	CF	$z_{1,t-1}$	Z_{to}	Value	Prob.
1	0.072728	1000000	0.08	80000	6.710081	149029.5	69029	0.20	186194	335224	0.080000	0.080000	310392	0.5
2	0.080000	744776	0.08	59582	6.246888	119223.6	59641	0.05	34257	153480	0.074546	0.077270	132253	0.5
3	0.072728	650878	0.08	52070	5.746639	113262.4	61192	0.20	117937	231200	0.080000	0.078179	184465	0.5
4		471749	0.08	37740						509489	0.074546	0.077270	378301	
												Value =	1005411	0.125
Path 4	1	2	4	5	6	7	8	9	10	11	12	13	14	15
Year	R^{rel}	Balance	WAC	Interest	PVIF	p	Sch. Prin.	CPR	Prepaid Pri	CF	$z_{1,t-1}$	Z_{to}	Value	Prob.
1	0.088000	1000000	0.08	80000	6.710081	149029.5	69029	0.05	46549	195578	0.080000	0.080000	181091	0.5
2	0.080000	884422	0.08	70754	6.246888	141578	70824	0.05	40680	182258	0.086000	0.082996	155393	0.5
3	0.072728	772918	0.08	61833	5.746639	134499.1	72666	0.20	140050	274550	0.080000	0.081996	216742	0.5
4		560202	0.08	44816						605018	0.074546	0.080129	444494	
												Value =	997720	0.125

(*continued*)

TABLE 14.4 *(Continued)*

Path 5	1	2	4	5	6	7	8	9	10	11	12	13	14	15
Year	R^{rel}	Balance	WAC	Interest	PVIF	p	Sch. Prin.	CPR	Prepaid Pri	CF	$z_{1,t-1}$	Z_{to}	Value	Prob.
1	0.072728	1000000	0.08	80000	6.710081	149029.5	69029	0.20	186194	335224	0.080000	0.080000	310392	0.5
2	0.080000	744776	0.08	59582	6.246888	119223.6	59641	0.05	34257	153480	0.074546	0.077270	132253	0.5
3	0.088000	650878	0.08	52070	5.746639	113262.4	61192	0.05	29484	142747	0.080000	0.078179	113892	0.5
4		560202	0.08	44816						605018	0.086000	0.080129	444494	
												Value =	1001031	0.125
Path 6	1	2	4	5	6	7	8	9	10	11	12	13	14	15
Year	R^{rel}	Balance	WAC	Interest	PVIF	p	Sch. Prin.	CPR	Prepaid Pri	CF	$z_{1,t-1}$	Z_{to}	Value	Prob.
1	0.088000	1000000	0.08	80000	6.710081	149029.5	69029	0.05	46549	195578	0.080000	0.080000	181091	0.5
2	0.080000	884422	0.08	70754	6.246888	141578	70824	0.05	40680	182258	0.086000	0.082996	155393	0.5
3	0.088000	772918	0.08	61833	5.746639	134499.1	72666	0.05	35013	169512	0.080000	0.081996	133820	0.5
4		665240	0.08	53219						718459	0.086000	0.082996	522269	
												Value =	992574	0.125
Path 7	1	2	4	5	6	7	8	9	10	11	12	13	14	15
Year	R^{rel}	Balance	WAC	Interest	PVIF	p	Sch. Prin.	CPR	Prepaid Pri	CF	$z_{1,t-1}$	Z_{to}	Value	Prob.
1	0.088000	1000000	0.08	80000	6.710081	149029.5	69029	0.05	46549	195578	0.080000	0.080000	181091	0.5
2	0.096000	884422	0.08	70754	6.246888	141578	70824	0.05	40680	182258	0.086000	0.082996	155393	0.5
3	0.088000	772918	0.08	61833	5.746639	134499.1	72666	0.05	35013	169512	0.092600	0.086188	132277	0.5
4		665240	0.08	53219						718459	0.086000	0.086141	516247	
												Value =	985008	0.125
Path 8	1	2	4	5	6	7	8	9	10	11	12	13	14	15
Year	R^{rel}	Balance	WAC	Interest	PVIF	p	Sch. Prin.	CPR	Prepaid Pri	CF	$z_{1,t-1}$	Z_{to}	Value	Prob.
1	0.088000	1000000	0.08	80000	6.710081	149029.5	69029	0.05	46549	195578	0.080000	0.080000	181091	0.5
2	0.096000	884422	0.08	70754	6.246888	141578	70824	0.05	40680	182258	0.086000	0.082996	155393	0.5
3	0.106480	772918	0.08	61833	5.746639	134499.1	72666	0.05	35013	169512	0.092600	0.086188	132277	0.5
4		665240	0.08	53219						718459	0.099860	0.089590	509741	
												Value =	978502	0.125
												Wt. Value	$997,457	

$$z_3 = ((1 + z_{10})(1 + z_{11})(1 + z_{12}))^{1/3} - 1$$
$$= ((1.08)(1.074546)(1.069588))^{1/3} - 1 = .074703$$
$$z_4 = ((1 + z_{10})(1 + z_{11})(1 + z_{12})(1 + z_{13}))^{1/4} - 1$$
$$= ((1.08)(1.074546)(1.069588)(1.06508))^{1/4} - 1 = .072289$$

Using these rates, the value of the MBS following Path 1 is \$1,010,465:

$$V_1^{\text{Path}} = \frac{\$335{,}224}{1.08} + \frac{\$324{,}764}{(1.07727)^2} + \frac{\$257{,}259}{(1.074703)^3} + \frac{\$281{,}560}{(1.072289)^4} = \$1{,}010{,}465$$

The spot rates and values of each of the eight paths are shown in columns 13 and 14 in Table 14.4.

Theoretical Value In a Monte Carlo simulation, the ***theoretical value of the MBS*** is defined as the average of the values of all the interest rate paths:

$$\overline{V} = \frac{1}{N}\sum_{i=1}^{N} V_i^{\text{path}}$$

In this example, the theoretical value of the MBS issue is \$997,457 or 99.7457% of its par value (see bottom of Table 14.4).

The theoretical value along with the standard deviation of the path values are useful measures in evaluating an MBS or CMO tranche relative to other securities. A MBS's theoretical value can also be compared to its actual price to determine if the MBS is over- or underpriced. For example, if the theoretical value is 98% of par and the actual price is at 96%, then the mortgage security is underpriced, "\$2 cheap," and if it is priced at par, then it is considered overpriced, "\$2 rich."

Option-Adjusted Spread and Other Parameters

Instead of determining the theoretical value of the MBS or tranche given a path of spot rates and option-adjusted spreads, analysts can use a Monte Carlo simulation to estimate the mortgage security's rate of return given its market price. Since the security's rate of return is equal to a riskless spot rate plus the OAS (assuming no default risk), many analysts use the simulation to estimate just the OAS. From the simulation, the OAS is determined by finding that OAS that makes the theoretical value of the MBS equal to its market price. This spread can be found by iteratively solving for the k that satisfies the following equation:

$$\text{Market price} = \frac{1}{N}\left(\left[\sum_{M=1}^{T}\frac{\text{CF}_{(1)M}}{(1 + S_{(1)M} + k)^M}\right] + \left[\sum_{M=1}^{T}\frac{\text{CF}_{(2)M}}{(1 + S_{(2)M} + k)^M}\right] + \cdots + \left[\sum_{M=1}^{T}\frac{\text{CF}_{(N)M}}{(1 + S_{(N)M} + k)^M}\right]\right)$$

where N = number of paths

In addition to estimating the theoretical value, OAS, and standard deviation, a Monte Carlo simulation can be used to estimate the average life of each path, and from that the mean and standard deviation of the average life of all the paths.

14.6 CONCLUSION

In 1985, Merrill Lynch introduced the liquid yield option note (LYON). The LYON was a zero-coupon bond, convertible into the issuer's stock, callable, with the call price increasing over time, and putable with the put price increasing over time. The LYON is a good example of how innovative the investment community can be in structuring debt instruments with option clauses. As we saw in Chapters 11 and 12, this type of innovation is quite extensive in the construction of mortgage- and asset-backed securities. Fixed-income securities with embedded options, though, are difficult to value. In this chapter, we have examined how a binomial model that identifies the possible random paths that interest rates follow over time can be used to value bonds with embedded options. In the next chapter, we take up the more technical subject of how the tree can be estimated.

WEB INFORMATION

FINRA

- Go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds."
- To find bonds with embedded options, use the bond search: Click "Corporate" tab, "Advanced Bond Search," and then select call, put, and/or convertible.

Yahoo.com

- To find convertible bond funds, go to to http://finance.yahoo.com/funds and click "Fund Screener."

KEY TERMS

binomial interest rate tree
callable bond
conversion price
conversion ratio (CR)
conversion value (CV)
convertible bond
interest rate paths
Monte Carlo simulation
putable bond
sinking-fund bond
straight debt value (SDV)
theoretical value
theoretical value of the MBS

PROBLEMS AND QUESTIONS

1. Given a current one-period spot rate of $S_0 = 10\%$, upward and downward parameters of $u = 1.1$ and $d = .9091$, and probability of the spot rate increasing in one period of $q = .5$:
   a. Generate a two-period binomial tree of spot rates.
   b. Using the binomial interest rate tree from Question 1.a, determine the value of a two-period, option-free 9% coupon bond with $F = 100$.
   c. Using the binomial interest rate tree from Question 1.a, determine the value of the 9% bond assuming it is callable at a call price of CP = 99. Use the minimum constraint approach.
   d. Using the binomial interest rate tree, show at each node the call option values of the callable bond (CP = 99). Given your call option values, determine the values at each node of the callable bond as the difference between the option-free values found in Question 1.b and the call option values. Do your callable bond values match the ones you found in Question 1.c?
   e. Comment on values of your call options being equal to the present value of the interest savings the issuer realizes from refunding the bond at lower rates.
   f. Using the binomial interest rate tree, determine the value of the bond assuming it is putable in periods one and two at a put price of PP = 99. Use the maximum constraint approach.
   g. Using the binomial interest rate tree, show at each node the put option values of the putable bond (PP = 99). Given your put option values, determine the values at each node of the putable bond as the sum of the option-free bond values found in Question 1.b and the put option values. Do your putable bond values match the ones you found in Question 1.f?

2. Given a current one-period spot rate of $S_0 = 10\%$, $u = 1.1$, $d = .9091$, and $q = .5$, determine the value of the two-period, option-free 9% coupon bond in Question 1 as the weighted average value of the possible paths defined by the binomial process. Does the value match the value you obtained in Question 1.b?

3. Suppose a corporation issues a two-period, 9% coupon bond with the face value of the issue worth \$9 million. Suppose the issue has a sinking fund obligation requiring the company to sink \$3 million in period 1, with the company having the option to either buy the bonds in the market or call them at CP = 99 per \$100 face value. Using the same interest rate tree you generated in Question 1, calculate the value of the sinking fund bond.

4. Given an ABC convertible bond with $F = \$1{,}000$, maturity of two periods, periodic coupon rate of 5%, conversion ratio of CR = 10, and an underlying stock with a current price of \$100, $u = 1.05$, $d = .952381$, and $q = .5$, calculate the value of the bond using a binomial tree of stock prices. Assume no call on the bond and a flat yield curve at 5% that is not expected to change.

5. Given a current one-period spot rate of $S_0 = 5\%$, upward and downward parameters of $u = 1.1$, $d = 1/1.1$, and a probability of the spot rate increasing in one period of $q = .5$:
   a. Generate a two-period binomial tree of spot rates.

b. Using a binomial tree approach, calculate the value of a three-period, option-free bond paying a 5% coupon per period and with face value of 100.
c. Using the binomial tree, calculate the value of the bond given it is callable with a call price of CP = 100.
d. Using the tree, calculate the value of the bond given it is putable with a put price of PP = 100.

6. Given a current one-period spot rate of $S_0 = 5\%$, $u = 1.1$, $d = 1/1.1$, and $q = .5$, determine the value of the three-period, option-free 5% coupon bond in Question 5 as the weighted average value of the possible paths defined by the binomial process. Does the value match the value you obtained in Question 5.b?

7. Given a current one-period spot rate of $S_0 = 5\%$, $u = 1.1$, $d = 1/1.1$, and $q = .5$, determine the values of three bonds stripped from the three-period, option-free 5% coupon bond in Question 5: a one-period zero-coupon bond paying $F = 5$ at maturity, a two-period zero-coupon bond paying $F = 5$ at maturity, and a three-period zero-coupon bond paying $F = 105$ at maturity. Does the sum of the values of the three strips equal the value you obtained for the three-period 5% coupon bond in Question 5.b?

8. Suppose a corporation issues a $9 million, three-period, 5% coupon bond with a sinking fund obligation requiring the company to sink $3 million in period 1 and $3 million in period 2, with the company having the option to either buy the bonds in the market or call them at CP = 100. Using the same interest rate tree you generated in Question 5, calculate the value of the sinking fund bond.

9. Given the following features of the XYZ convertible bond:

   Coupon rate (annual) = 10% (annual compounding)
   Face value = F = $1,000
   Maturity = 10 years
   Callable at $1,100
   YTM on a comparable, nonconvertible bond = 12%
   Conversion ratio = 10 shares
   Current stock price = S_0 = $90

   Calculate the following:

   a. XYZ's conversion price
   b. XYZ's conversion value
   c. XYZ's straight debt value
   d. Minimum price of the convertible
   e. The arbitrage strategy if the price of the convertible were $880

10. Given an ABC convertible bond with $F = \$1,000$, maturity of three periods, CR = 10, current stock price of $100, and $u = 1.1$, $d = .95$, and $q = .5$ on the stock:

    a. Calculate the value of the bond using a binomial tree of stock prices. Assume no call on the bond and a flat yield curve at 10% that is not expected to change.
    b. Calculate the value of the bond using a binomial tree of stock prices. Assume the bond is callable at CP = $1,200 and a flat yield curve at 10% that is not expected to change.

11. Given a current annual spot rate of $S_0 = 6\%$, upward and downward parameters on the spot rate of $u = 1.2$, $d = 1/1.2$, and $q = .5$:

- **a.** Determine the value of a three-year, option-free, 6% annual coupon bond with $F = \$100$.
- **b.** Assuming the bond is callable at a call price of 98, determine the values at each node of the embedded call option.
- **c.** Determine the value of the callable bond.

12. Excel Problems: The Excel program called "Binomial Bond Valuation" can be downloaded from the Web site that is associated with this book. These programs can be used to price callable and putable bonds based on u and d inputs or mean and variance values (discussed in Chapter 15). This problem and Problem 13 should be done using the programs.
Given a binomial interest rate tree with the following features: $S_0 = 6\%$, length of the tree = .5 years, and upward and downward parameters for .5 years of $u = 1.0488$ and $d = .9747$ and $q = .5$, determine the values of the following bonds:

- **a.** The value of an option-free bond with maturity of 10 years, annual coupon of $C = 6$, semiannual payments, and $F = 100$.
- **b.** The value of a callable bond with maturity of 10 years, annual coupon of $C = 6$, semiannual payments, $F = 100$, and call price of 100.
- **c.** The value of a putable bond with maturity of 10 years, annual coupon of $C = 6$, semiannual payments, $F = 100$, and put price of 100.
- **d.** The value of an option-free bond with maturity of 20 years, annual coupon of $C = 6$, semiannual payments, and $F = 100$.
- **e.** The value of a callable bond with maturity of 20 years, annual coupon of $C = 6$, semiannual payments, $F = 100$, and call price of 100.
- **f.** The value of a putable bond with maturity of 20 years, annual coupon of $C = 6$, semiannual payments, $F = 100$, and put price of 100.

13. Using the Excel program "Binomial Bond Valuation," generate the price-yield curves for (a) an option-free bond with a maturity of 10 years, annual coupon of $C = 6$, semiannual payments, and $F = 100$ and (b) a callable bond with the same features and a call price 100. Determine the values for annual spot rates of 4%, 4.5%, 5%, 5.5%, 6%, 6.5%, 7%, 7.5%, and 8%. Assume the applicable binomial interest rate tree for the bonds has the following features: length of the tree = .5 years, and upward and downward parameters for .5 years of $u = 1.1$, $d = .9091$, and $q = .5$. Comment on the differences between the two bonds price-yield curves.

14. Using a Monte Carlo simulation approach, determine the theoretical value of an MBS issue with a face value of \$1,000,000, WAC = 8%, WAM = 10 years, PT rate = 8%, *annual* cash flows (instead of the standard monthly), and a balloon at the end of the *second* year.
Assume:

- The current one-year spot rate is $S_0 = 6\%$ and the current refinancing rate is $R_0^{ref} = 8\%$.
- The future spot and refinancing rates can both be described by a *two*-period binomial interest rate where $u = 1.1$, $d = 1/1.1$, the length of the period is one year, and $q = .5$.

- The following prepayment model applies:

$$CPR = 20\% \text{ if } (WAC - Rref) > 0$$

$$CPR = 5\% \text{ if } (WAC - Rref) \leq 0$$

- A risk-adjusted one-year spot rate is equal to the spot rate plus an option-adjusted spread of 2%: $S^{RA} = S + 2\%$.

WEB EXERCISES

1. Evaluate several bonds with call and/or put options using the FINRA site:
   - Go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds."
   - To find bonds with embedded options, use the bond search: click "Corporate" tab, "Advanced Bond Search," and then select call, put, and/or convertible.
2. Evaluate several convertible bonds found using the FINRA site:
   - Go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds."
   - To find convertible bonds, click "Corporate" tab, "Advanced Bond Search," and then select convertible.
3. Go to Yahoo! Finance to find some convertible bond funds to evaluate.
   - Go to http://finance.yahoo.com/funds and click "Fund Screener."

NOTES

1. In this case, the issuer could buy the bond back at 98 financed by issuing a one-year bond at 9.5% interest. One period later the issuer would owe 98(1.095) = 107.31; this represents a savings of 108 – 107.31 = 0.69. Note, the value of that savings in period 1 is .69/1.095 = 0.63, which is equal to the difference between the bond price and the call price: 98.630 − 98 = .63.
2. Another convertible bond term is its conversion price. The ***conversion price*** is the bond's par value divided by the conversion ratio: F/CR.
3. Looking historically at stock prices and general interest rates, there have been some periods where a negative correlation between stock prices and interest rates has been observed: In the high interest rate periods of the late 1970s, early 1980s, and the early 1990s, stock prices were relatively low, and in the mid-1980s and mid-1990s interest rates were relatively low whereas stock prices were comparatively high. This negative correlation, though, is certainly not always the case.

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CHAPTER 15

Estimating the Binomial Tree

15.1 INTRODUCTION

In the last chapter, we examined how the binomial interest rate tree can be used to price debt securities with call and put options, sinking fund agreements, convertible clauses, and prepayment options. We did not address, though, the more fundamental problem of how to estimate the tree. In this chapter, we examine two models that have been used for estimating binomial interest rate trees. Before we describe these models, though, we first need to look at how we can subdivide the tree so that the assumed binomial process is defined in terms of a realistic length of time, with a sufficient number of possible rates at maturity.

15.2 SUBDIVIDING THE BINOMIAL TREE

The binomial model is more realistic when we subdivide the periods to maturity into a number of subperiods. That is, as the number of subperiods increases, the length of each period becomes smaller, making the assumption that the spot rate will either increase or decrease more plausible, and the number of possible rates at maturity increases, which again adds realism to the model. For example, suppose the three-period bond in our illustrative example in Chapter 14 were a three-year bond. Instead of using a three-period binomial tree, where the length of each period is a year, suppose we evaluate the bond using a six-period tree with the length of each period being six months. If we do this, we need to divide the one-year spot rates and the annual coupon by two, adjust the u and d parameters to reflect changes over a six-month period instead of one year (this adjustment will be discussed in the next section), and define the binomial tree of spot rates for five periods, each with a length of six months. Figure 15.1 shows a five-period binomial tree for one-year spot rates with the length of each period being six months, u =1.0488, and d = .9747. Given the spot rates in the figure, the binomial value of a three-year, 9% bond would be determined by using a $45 coupon (reflecting the accrued interest) and discounting at a six-month rate (annual spot rate divided by two) at each node.

If we wanted to value the bond every quarter, then we would need an 11-period tree of spot rates with the length of each period being three months and with the

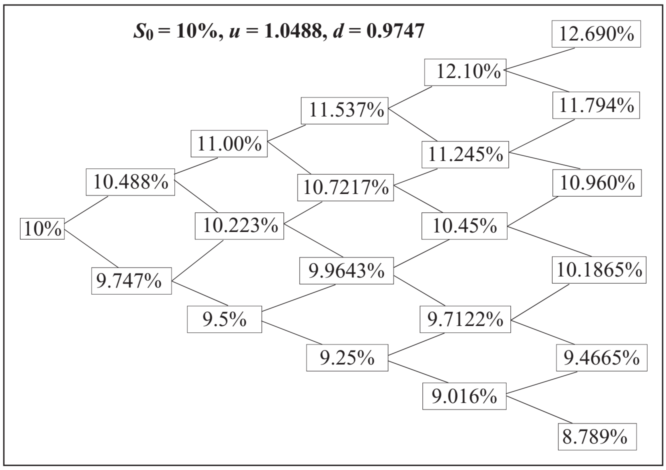


FIGURE 15.1 Five-Period Tree of Spot Rates

annual rates divided by four and u and d adjusted to reflect movements over three months. In general, let

h = length of the period in years

n = number of periods of length h defining the maturity of the bond, where n = (maturity in years)/h

Thus, a three-year bond evaluated over quarterly periods ($h = 1/4$ of a year), would have a maturity of n = (3 years)/(1/4 years) = 12 periods and would require a binomial interest rate tree with $n - 1 = 12 - 1 = 11$ periods. To evaluate the bond over monthly periods ($h = 1/12$ of a year), the bond's maturity would be n = (3 years)/(1/12 years) = 36 periods and would require a 35-period binomial tree of spot rates; for weekly periods ($h = 1/52$), the bond's maturity would be 156 periods of length one week, and we would need a 155-period tree of spot rates. Thus, by subdividing we make the length of each period smaller, which makes the assumption of only two possible rates at the end of one period more plausible, and we increase the number of possible rates at maturity.

15.3 ESTIMATING THE BINOMIAL TREE

In practice, determining the value of a bond using a binomial tree requires that we be able to estimate the random process that spot rates follow. There are two general approaches to estimating binomial interest rate movements. The first [models derived by Rendleman and Bartter (1980) and Cox, Ingersoll, and Ross (1985)] is to estimate the u and d parameters based on estimates of the spot rate's mean and variability. The estimating formulas for u and d are obtained by solving for the u and d values that make the mean and variance of spot rates resulting from a binomial process equal to their estimated values. Bond values obtained using this approach can then be compared to actual bond prices to determine if the bond is mispriced. Binomial interest rate models that generate a process of interest rate movements without a constraint that the model's price matches the market equilibrium price (the price obtained by discounting the bond's cash flows by spot rates) are sometimes referred to as ***equilibrium models***. The second approach [models derived by Black, Derman, and Toy (1990), Ho and Lee (1986), and Heath, Jarrow, and Morton (1992)] is to calibrate the binomial tree to the current spot yield curve and to the interest rate's volatility. This approach is analogous to estimating the implied variance used in option pricing. This calibration approach solves for the spot rate that satisfies a variability condition and a price condition that ensures that the binomial tree is consistent with the term structure of current spot rates. Since the binomial tree is calibrated to current spot rates, the ***calibration model*** yields bond values for option-free bonds that are equal to the equilibrium prices. Given the tree's rates, the model is then used to value bonds with embedded option features.

The formulas for estimating u and d are obtained by solving mathematically for the u and d values that make the statistical characteristics (mean and variance) of the spot rate's logarithmic return equal to the characteristics' estimated values. In addition, the variability condition constraining the calibration model also comes from the formulas for u and d. As background to understanding the u and d formulas, let us first examine the probability distribution that characterizes a binomial process.

Probability Distribution Resulting from a Binomial Process

In the last section, we assumed a simple binomial approach where in each period the one-period spot rate would either increase to equal a proportion u times its initial rate or decrease to equal a proportion d times the initial rate, with the probability of the increase in one period being $q = .5$. At the end of n periods, this binomial process yields a distribution of $n + 1$ possible spot rates (e.g., for $n = 3$, there are four possible rates: $S_{uuu} = u^3 S_0$, $S_{uud} = u^2 d S_0$, $S_{udd} = ud^2\, S_0$, and $S_{ddd} = d^3 S_0$). This distribution, though, is not normally distributed since spot rates cannot be negative (i.e., we normally do not have negative interest rates). However, the distribution of spot rates can be converted into a distribution of logarithmic returns, g_n, where:

$$g_n = \ln\left(\frac{S_n}{S_0}\right)$$

This distribution can take on negative values and can be normally distributed if $q = .5$. Figure 15.2 shows the binomial distributions of spot rates for $n = 1$, 2, 3, and 4 periods and their corresponding logarithmic returns for the case in which $u = 1.1$, $d = .95$, $S_0 = 10\%$, and $q = .5$. As shown in the figure, when $n = 1$, there are two possible spot rates of 11% and 9.5%, with respective logarithmic returns of .0953 and −.0513:

$$g_u = \ln\left(\frac{uS_0}{S_0}\right) = \ln u = \ln 1.1 = .0953$$

$$g_d = \ln\left(\frac{dS_0}{S_0}\right) = \ln d = \ln .95 = -.0513$$

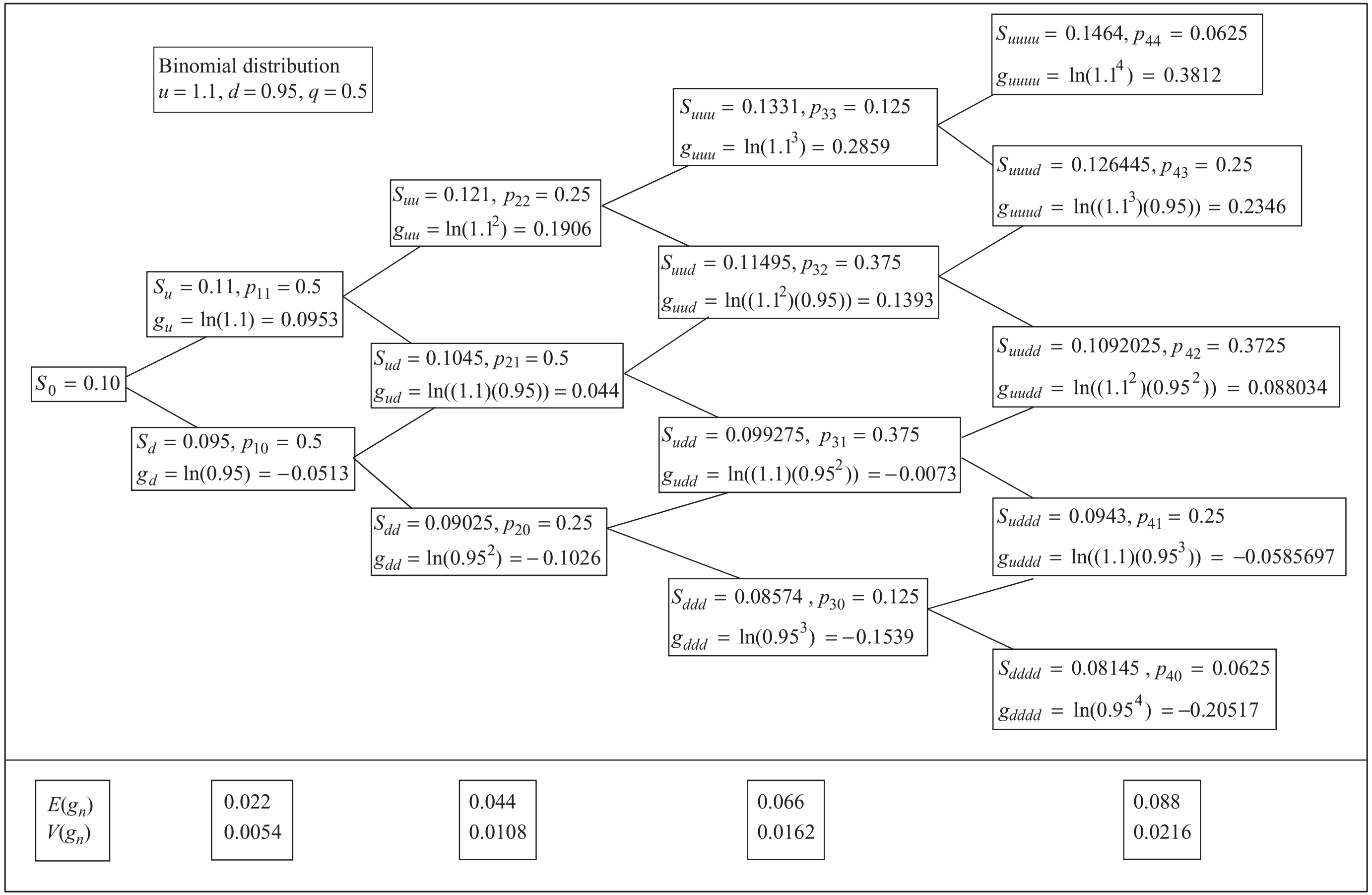


FIGURE 15.2 Binomial Distribution of Logarithmic Returns

When $n = 2$, there are three possible spot rates of 12.1%, 10.45%, and 9.025% with corresponding logarithmic returns of

$$g_{uu} = \ln\left(\frac{u^2 S_0}{S_0}\right) = \ln u^2 = \ln(1.1^2) = .1906$$

$$g_{ud} = \ln\left(\frac{udS_0}{S_0}\right) = \ln ud = \ln[(1.1)(.95)] = .044$$

$$g_{dd} = \ln\left(\frac{d^2 S_0}{S_0}\right) = \ln(d^2) = \ln(.95^2) = -.1026$$

When $n = 3$, there are four possible spot rates of 13.31%, 11.495%, 9.9275%, and 8.574%, with logarithmic returns of

$$g_{uuu} = \ln\left(\frac{u^3 S_0}{S_0}\right) = \ln(u^3) = \ln(1.1^3) = .2859$$

$$g_{uud} = \ln\left(\frac{u^2 d S_0}{S_0}\right) = \ln(u^2 d) = \ln[(1.1^2)(.95)] = .1393$$

$$g_{udd} = \ln\left(\frac{ud^2 S_0}{S_0}\right) = \ln(ud^2) = \ln[(1.1)(.95^2)] = -.0073$$

$$g_{ddd} = \ln\left(\frac{d^3 S_0}{S_0}\right) = \ln(d^3) = \ln(.95^3) = -.1539$$

The probability of attaining any one of these rates is equal to the probability of the spot rate increasing j times in n periods, p_{nj}. That is, the probability of attaining spot rate 10.45% in period 2 is equal to the probability of the spot rate increasing one time ($j = 1$) in two periods ($n = 2$), p_{21}. In a binomial process this probability can be found using the following formula:[1]

$$p_{nj} = \frac{n!}{(n-j)!\,j!} q^j (1-q)^{n-j}$$

Thus after two periods, the probability of the spot rate equaling 19.06% is $p_{22} = .25$, 10.45% is $p_{21} = .5$, and 9.025% is $p_{20} = .25$. Using these probabilities, the expected value and the variance of the distribution of logarithmic returns after two periods would be equal to $E(g_2) = 4.4\%$ and $V(g_2) = .0108$:

$$E(g_n) = .25(.1906) + .5(.0440) + .25(-.1026) = .044$$

$$V(g_n) = .25[.1906 - .044]^2 + .5[.044 - .044]^2 + .25[-.1026 - .044]^2 = .0108$$

The mean and variance for each of the four distributions are shown at the bottom of Figure 15.2. In examining each distribution's mean and variance, note that as the number of periods increases, the expected value and variance increase by a multiplicative factor such that $E(g_n) = nE(g_1)$ and $V(g_n) = nV(g_1)$. Also, note that the expected value and the variance are also equal to

$$E(g_n) = nE(g_1) = n[q \ln u + (1 - \mathrm{q})\ln d]$$

$$V(g_n) = nV(g_1) = nq(1 - q)[\ln (u/d)]^2$$

15.4 *u* AND *d* ESTIMATION APPROACH

Solving for *u* and *d*

Given the features of a binomial distribution, the formulas for estimating u and d are found by solving for the u and d values that make the expected value and the variance of the binomial distribution of the logarithmic return of spot rates equal to their respective estimated parameter values under the assumption that $q = .5$ (or equivalently that the distribution is normal). If we let μ_{e} and V_{e} be the estimated mean and variance of the logarithmic return of spot rates for a period equal in length to n periods, then our objective is to solve for the u and d values that simultaneously satisfy the following equations:

$$nE(g_1) = n[q \ \ln \ u + (1 - q) \ln d] = \mu_{\mathrm{e}}$$

$$nV(g_1) = nq(1 - q)[\ln (u/d)]^2 = V_{\mathrm{e}}$$

If $q = .5$, then the formula values for u and d that satisfy the two equations are

$$u = \mathrm{e}^{\sqrt{V_{\mathrm{e}}/n} + \mu_{\mathrm{e}}/n}$$

$$d = \mathrm{e}^{-\sqrt{V_{\mathrm{e}}/n} + \mu_{\mathrm{e}}/n}$$

In terms of our example, if the *estimated* expected value and variance of the logarithmic return were $\mu_{\mathrm{e}} = .044$ and $V_{\mathrm{e}} = .0108$ for a period equal in length to $n = 2$, then using the above equations, u would be 1.1 and d would be .95:

$$u = \mathrm{e}^{\sqrt{.0108/2} + .044/2} = 1.1$$

$$d = \mathrm{e}^{-\sqrt{.0108/2} + .044/2} = .95$$

Annualized Mean and Variance

In order to facilitate the estimation of u and d for a number of bonds with different maturities, it is helpful to use an annualized mean and variance ($\mu^{\mathrm{A}}{}_{\mathrm{e}}$ and $V^{\mathrm{A}}{}_{\mathrm{e}}$). Annualized parameters are obtained by simply multiplying the estimated parameters of a given length by the number of periods of that length that make up a year. For example, if quarterly data is used to estimate the mean and variance ($\mu^{\mathrm{q}}{}_{\mathrm{e}}$ and $V^{\mathrm{q}}{}_{\mathrm{e}}$), then we simply multiply those estimates by four to obtain the annualized parameters ($\mu^{\mathrm{A}}{}_{\mathrm{e}} = 4\mu^{\mathrm{q}}{}_{\mathrm{e}}$ and $V^{\mathrm{A}}{}_{\mathrm{e}} = 4V^{\mathrm{q}}{}_{\mathrm{e}}$). Thus, if the estimated quarterly mean and variance were .022 and .0054, then the annualized mean and variance would be .088 and .0216, respectively.[2] Note, when the annualized mean and variance are used, then these parameters must by multiplied by the proportion h, defined earlier as the time of the period being analyzed expressed as a proportion of a year, and n is not needed since h defines the length of tree's period:

$$u = e^{\sqrt{hV_e^A} + h\mu_e^A}$$
$$d = e^{-\sqrt{hV_e^A} + h\mu_e^A}$$

If the annualized mean and variance of the logarithmic return of one-year spot rates were .044 and .0108, and we wanted to evaluate a three-year bond with six-month periods (h = 1/2 of a year), then we would use a six-period tree to value the bond [n = (3 years)/(1/2) = 6 periods] and u and d would be 1.1 and .95:

$$u = e^{\sqrt{(1/2).0108} + (1/2).044} = 1.1$$
$$d = e^{-\sqrt{(1/2).0108} + (1/2).044} = .95$$

If we make the length of the period monthly (h = 1/12), then we would value the three-year bond with a 36-period tree and u and d would be equal to 1.03424 and .9740:[3]

$$u = e^{\sqrt{(1/12).0108} + (1/12).044} = 1.03424$$
$$d = e^{-\sqrt{(1/12).0108} + (1/12).044} = .9740$$

Estimating μ_e^A and V_e^A Using Historical Data

To estimate u and d requires estimating the mean and variance: μ_e and V_e. The simplest way to do this is to estimate the parameters using the average mean and variance from a historical sample of spot rates. As an example, historical quarterly one-year spot rates over 13 quarters are shown in Table 15.1. The 12 logarithmic returns are calculated by taking the natural log of the ratio of spot rates in one period to the rate in the previous period (S_t/S_{t-1}). From this data, the historical quarterly logarithmic mean return and variance are:

$$\mu_e = \frac{\sum_{t=1}^{12} g_t}{12} = \frac{0}{12} = 0$$

$$V_e = \frac{\sum_{t=1}^{12} [g_t - \mu_e]^2}{11} = \frac{.046297}{11} = .004209$$

Multiplying the historical quarterly mean and variance by 4, we obtain an annualized mean and variance, respectively, of 0 and .016836. Given the estimated annualized mean and variance, u and d can be estimated once we determine the number of periods to subdivide (see bottom of Table 15.1).

Binomial Tree Excel Program

To determine the price of a callable or putable bond for a binomial interest rate tree generated by u and d estimates:

1. Calculate u and d given the estimates of $\mu^A{}_e$ and $V^A{}_e$.

TABLE 15.1 Estimating Mean and Variance with Historical Data

Quarter	Spot rate, S_t	$S_t/S_t - 1$	$g_t = \ln(S_t/S_t)$	$(g_t - \mu_e)^2$
Y1.1	10.6%	–	–	–
Y1.2	10.0%	.9434	−.0583	.003395
Y1.3	9.4%	.9400	−.0619	.003829
Y1.4	8.8%	.9362	−.0659	.004350
Y2.1	9.4%	1.0682	.0660	.004350
Y2.2	10.0%	1.0638	.0619	.003829
Y2.3	10.6%	1.0600	.0583	.003395
Y2.4	10.0%	.9434	−.0583	.003395
Y3.1	9.4%	.9400	−.0619	.003829
Y3.2	8.8%	.9362	−.0660	.004350
Y3.3	9.4%	1.0682	.0660	.004350
Y3.4	10.0%	1.0638	.0619	.003829
Y4.1	10.6%	1.0600	.0583	.003395
			0	.046297
			$\mu_e = 0$	$V_e^q = \frac{.046297}{11} = .004209$

$\mu_e^A = 4\mu_e^q = 4(0) = 0$; $V_e^A = 4V_e^q = 4(.004209) = .016836$

Length	h	u	d
Year	1	1.1385	.8783
Quarter	1/4	1.0670	.9372
Month	1/12	1.0382	.9632

2. Generate a binomial tree of spot rates for $n - 1$ periods of size h, where n is the number of periods to maturity on the bond.
3. Starting at maturity (or the end of period n), set the value of the bond equal to its terminal value (face value plus coupon).
4. Using the procedure of starting at maturity and rolling the tree to the present, determine the price of the bond at each node with the constraint that the price of the callable (putable) bond be the minimum (maximum) of its binomial value or call price (put price).

The procedure for determining the price of a callable or putable bond is rather cumbersome, especially when there are a number of periods to maturity. The recursive procedure, though, easily lends itself to computer programming. The Excel programs called "Binomial Bond Valuation" can be downloaded from the Web site that is associated with this book. These programs can be used to price callable and putable bonds based on u and d inputs or $\mu^A{}_e$ and $V^A{}_e$ values. Exhibit 15.1 shows the output from the program from pricing a three-year, 10% callable bond with a face value and call price of 100, a current spot rate of 10%, annualized mean for the spot rate's logarithmic return of .022, annualized variance for the spot rate's

EXHIBIT 15.1 Binomial Callable Bond Prices

Three-year, 10% callable bond with a face and call price of 100, current spot rate of 10%, annualized mean for the spot rate's logarithmic return of 0.022, annualized variance for the spot rate's logarithmic return of 0.0054, and a binomial interest rate model with quarterly steps (h = 4).

Binomial Tree Bond Call

Data Input	
Maturity (M or n)	3
Length of Periods (h)	0.25000000
Spot Rate (S0)	10.00%
Face Value (F)	$ 100.00
Coupon	$ 10.00
Call Price (CP)	$ 100.00
Distribution	Variance-Mean
Upward (u)	1.0000
Downward (d)	1.0000
OR	
Variance	0.005400
Mean	0.022000
Probability (q)	0.50

Data Used to Calculate Tree	
Maturity [Periods]	12
N Periods (n)	11
Periodic Spot Rate	2.50%
Periodic Coupon Rate	2.50%
Period Upward (u)	1.0431
Period Downward (d)	0.9692
Variance	0.000000
Mean	0.000000

Path \ Period	0	1	2	3	4	5	6	7	8	9	10	11	12
0	102.50	98.58	97.49	96.71	96.20	95.92	95.87	96.02	96.34	96.83	97.44	98.15	98.88
1	102.50	98.85	98.00	97.42	97.10	96.99	97.09	97.37	97.81	98.36	98.97	99.55	
2	102.50	99.10	98.47	98.10	97.95	98.00	98.24	98.63	99.11	99.59	99.95		
3	102.50	99.33	98.91	98.72	98.74	98.94	99.27	99.65	99.95	100.00			
4	102.50	99.55	99.33	99.31	99.47	99.73	99.97	100.00	100.00				
5	102.50	99.75	99.71	99.83	99.99	100.00	100.00	100.00					
6	102.50	99.94	100.00	100.00	100.00	100.00	100.00						
7	102.50	100.00	100.00	100.00	100.00	100.00							
8	102.50	100.00	100.00	100.00	100.00								
9	102.50	100.00	100.00	100.00									
10	102.50	100.00	100.00										
11	102.50	100.00											
12	102.50												

logarithmic return of .0054, and a binomial interest rate model with quarterly steps ($h = 1/4$).

15.5 CALIBRATION MODEL

A binomial interest rate tree generated using the u and d estimation approach is constrained to have an end-of-the-period distribution with a mean and variance that matches the analyst's estimated mean and variance. The tree is not constrained, however, to yield a bond price that matches its equilibrium value. As a result, analysts using such models need to make additional assumptions about the risk premium in order to explain the bond's equilibrium price. In contrast, calibration models are constrained to match the current term structure of spot rates and therefore yield bond prices that are equal to their equilibrium values.[4]

The calibration model generates a binomial tree by first finding spot rates that satisfy a variability condition between the upper and lower rates. Given the variability relation, the model then solves for the lower spot rate that satisfies a price condition in which the bond value obtained from the tree is consistent with the equilibrium bond price given the current spot yield curve.

Variability Condition

In our derivation of the formulas for u and d, we assumed that the distribution of the logarithmic return of spot rates was normal. This assumption also implies the following relationship between the upper and lower spot rate:

$$S_u = S_d e^{2\sqrt{V_e/n}}$$

That is, from the binomial process we know:

$$S_u = uS_0$$
$$S_d = dS_0$$

Therefore:

$$\frac{S_u}{u} = S_0 = \frac{S_d}{d}$$
$$S_u = S_d \frac{u}{d}$$

Substituting the equations for u and d, we obtain:

$$S_u = S_d \frac{e^{\sqrt{V_e/n} + \mu_e/n}}{e^{-\sqrt{V_e/n} + \mu_e/n}} = S_d e^{2\sqrt{V_e/n}}$$

or in terms of the annualized variance:

$$S_u = S_d e^{2\sqrt{h V_e^A}}$$

Thus, given a lower rate of 9.5% and an annualized variance of .0054, the upper rate for a one-period binomial tree of length one year ($h = 1$) would be 11%:

$$S_u = 9.5\% e^{2\sqrt{.0054}} = 11\%$$

If the current one-year spot rate were 10%, then these upper and lower rates would be consistent with the upward and downward parameters of $u = 1.1$ and $d = .95$. This variability condition would therefore result in a binomial tree identical to the one shown in Figure 15.2.

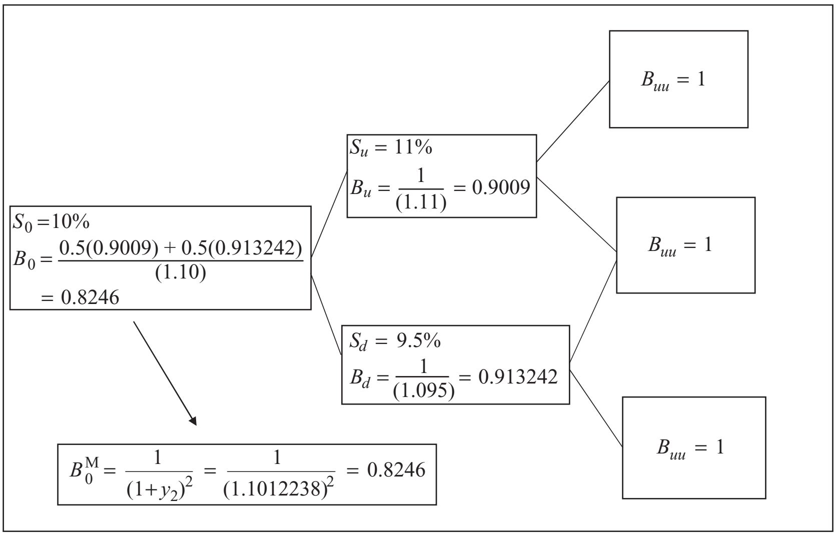
Price Condition

One of the problems with using just a variability condition (or equivalently just u and d estimates) is that it does not incorporate all of the information. As we discussed in Chapter 4, the yield curve reflects not only the supply and demand for bonds of different maturity segments, but also the expectations of investors about future interest rates. Thus, in addition to the variability relation between upper and lower spot rates, the calibration model also tries to generate a binomial tree that is consistent with the current yield curve's spot rates. This is done by solving for a lower spot rate that satisfies the variability relation and also yields a bond price that is equal to the equilibrium bond price. To see this, suppose the current yield curve has one-, two-, and three-year spot rates of $y_1 = 10\%$, $y_2 = 10.12238\%$, and $y_3 = 10.24488\%$, respectively. Furthermore, suppose that we estimate the annualized logarithmic mean and variance to be .048167 and .0054, respectively. Using the u and d approach, a one-period tree of length one year would have up and down parameters of $u = 1.12936$ and $d = .975$. Given the current one-period spot rate of 10%, the tree's possible spot rate would be $S_u = 11.2936\%$ and $S_d = 9.75\%$. These rates, though, are not consistent with the existing term structure. That is, if we value a two-year zero-coupon with a face value of \$1 using this tree, we obtain a value of 0.82258 that, given the two-year spot rate of 10.12238%, differs from the equilibrium price on the two-year zero discount bond of $B_0{}^M = .8246$:

$$B_0 = \frac{.5B_u + .5B_d}{1 + S_0} = \frac{.5[1/1.112936] + .5[1/1.0975]}{1.10} = .82258$$

$$B_0^M = \frac{1}{(1 + y_2)^2} = \frac{1}{(1.1012238)^2} = .8246$$

Thus, the tree generated from our estimates of u and d is not consistent with the current interest rate structure, nor is it consistent with the market's expectation of future rates given that expectations are incorporated into the term structure. To



S_d of 9.5% can be found algebraically by solving for the S_d that makes the binomial bond value equal to the equilibrium price. That is, solve for S_d where:

$$\frac{1}{(1+y_2)^2} = \frac{0.5B_u + 0.5B_d}{1+S_0}$$

$$\frac{1}{(1+y_2)^2} = \frac{0.5[1/(1+S_u)] + 0.5[1/(1+S_d)]}{1+S_0}$$

$$\frac{1}{(1+y_2)^2} = \frac{0.5[1/(1+S_d e^{2\sqrt{hV_e^A}})] + 0.5[1/(1+S_d)]}{1+S_0}$$

$$\frac{1}{(1.1012238)^2} = \frac{0.5[1/(1+S_d e^{2\sqrt{0.0054}})] + 0.5[1/(1+S_d)]}{1.10}$$

FIGURE 15.3 Calibration of Binomial Tree to Two-Period Zero-Coupon Bond with $F = 1$

make our tree consistent with the term structure, we need to find the value such that when

$$S_u = S_d e^{2\sqrt{hV_e^A}} = S_d e^{2\sqrt{.0054}}$$

the value of the two-year bond obtained from the tree is equal to the current equilibrium price of a two-year zero discount bond. This can be done for a one-period tree algebraically (see bottom of Figure 15.3) or iteratively: trying different S_d values until we find that value that equates the binomial price to the equilibrium price. In this case, solving iteratively for S_d yields a rate of 9.5%; that is, at $S_d = 9.5\%$, we have a binomial tree of one-year spot rates of $S_u = 11\%$ and $S_d = 9.5\%$ that simultaneously satisfies our variability condition and price condition; that is, the rate is consistent with the estimated volatility of .0054 and the current yield curve with one-year and two-year spot rates of 10% and 10.12238% (see Figure 15.3).

It should be noted that the lower rate of 9.5% represents a decline from the current rate of 10%, which is what we tend to expect in a binomial process. This is because we have calibrated the binomial tree to a relatively flat yield curve. If we had calibrated the tree to a positively sloped yield curve, then it is possible that both rates next period could be greater than the current rate; although the upper rate will be greater than the lower. For example, if the current two-year spot rate were 10.5% instead of 10.1022385%, then the equilibrium price of a two-year bond would be .8189 and the S_d and S_u values that calibrate the tree to this price and a variability of .0054 would be 10.20066% and 11.8156%. By contrast, if we had calibrated the tree to a negatively sloped curve, then it is possible that both rates next period could be lower than the current one.

Two-Period Binomial Tree

Given our estimated one-year spot rates after one period of 9.5% and 11%, we can now move to the second period and determine the tree's three possible spot rates using a similar methodology. The variability condition follows the same form as the one period; that is:

$$S_{ud} = S_{dd} e^{2\sqrt{h V_e^A}}$$

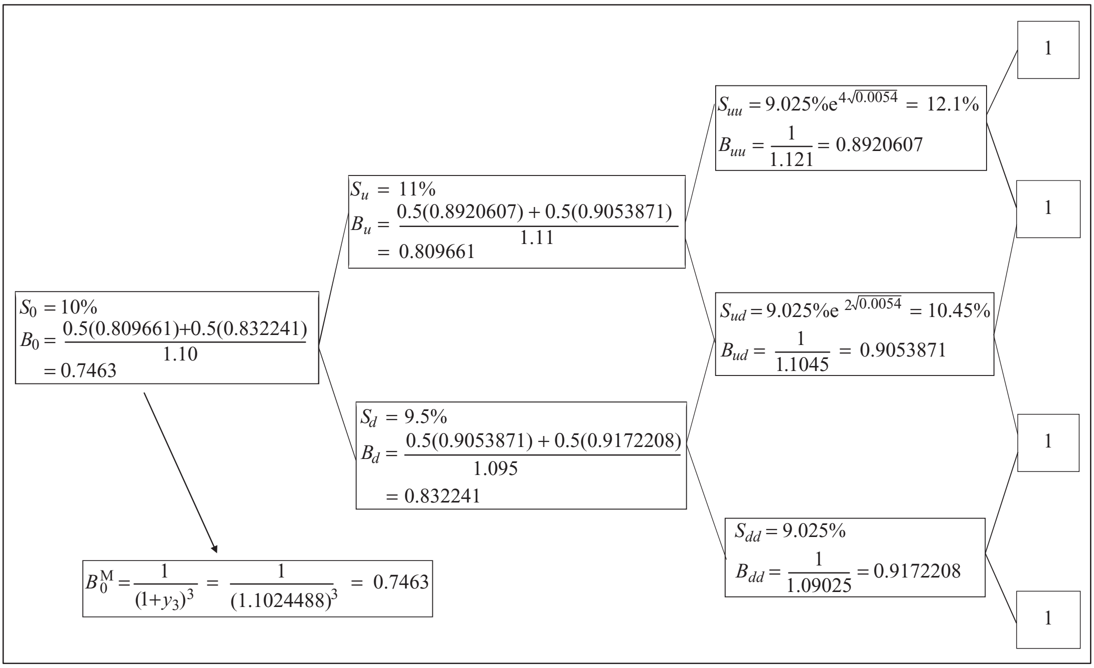
$$S_{uu} = S_{ud} e^{2\sqrt{h V_e^A}} = S_{dd} e^{4\sqrt{h V_e^A}}$$

Similarly, the price condition requires that the binomial value of a three-year zero-coupon bond be equal to the equilibrium price. Analogous to the one-period case, this condition is found by solving for the lower rate S_{dd} that, along with the above variability conditions and the rates for S_u and S_d obtained previously, yields a value for a three-year zero-coupon bond that is equal to the price on a three-year zero coupon bond yielding 10.24488%. Using an iterative approach, we find that a lower rate of $S_{dd} = 9.025\%$ yields a binomial value that is equal to the equilibrium price of the three-year bond of .7463 (see Figure 15.4).

The two-period binomial tree is obtained by combining the upper and lower rates found for the first period with the three rates found for the second period (see Figure 15.5). This yields a tree that is consistent with the estimated variability condition and with the current term structure of spot rates. To grow the tree, we continue with this same process. For example, to obtain the four rates in period 3, we solve for the S_{ddd} that, along with the spot rates found previously for periods one and two and the variability relations, yields a value for a four-year zero-coupon bond that is equal to the equilibrium price.

Valuation of Coupon Bonds

One of the features of using a calibrated tree to determine bond values is that the tree will yield prices that are equal to the bond's equilibrium price; that is, the price



The solution for S_{dd} can be mathematically stated as one of finding S_{dd} where the binomial price of a three-year bond is equal to its equilibrium price. That is, find S_{dd} where:

$$\frac{1}{(1+y_2)^3} = \frac{0.5B_u + 0.5B_d}{1+S_0}$$

$$\frac{1}{(1+y_2)^3} = \frac{0.5\left[\dfrac{0.5B_{uu} + 0.5B_{ud}}{1+S_u}\right] + 0.5\left[\dfrac{0.5B_{ud} + 0.5B_{dd}}{1+S_d}\right]}{1+S_0}$$

$$\frac{1}{(1.1024488)^3} = \frac{0.5\left[\dfrac{0.5[1/(1+S_{dd}e^{4\sqrt{0.0054}})] + 0.5[1/(1+S_{dd}e^{2\sqrt{0.0054}})]}{1.11}\right] + 0.5\left[\dfrac{0.5[1/(1+S_{dd}e^{2\sqrt{0.0054}})] + 0.5[1/(1+S_{dd})]}{1.095}\right]}{1.10}$$

FIGURE 15.4 Calibration of Binomial Tree to Three-Period Zero-Coupon Bond with $F = 1$

obtained by discounting cash flows by spot rates. For example, the value of a three-year, 9% option-free bond using the tree we just derived is 96.9521 (this is the illustrative example from Chapter 14; see Figure 15.6). This value is also equal to the equilibrium bond price obtained by discounting the bond's periodic cash flows at the spot rates of 10%, 10.12238% and 10.24488%:

$$B_3^M = \frac{9}{1.10} + \frac{9}{(1.1012238)^2} + \frac{109}{(1.1024488)^3} = 96.9521$$

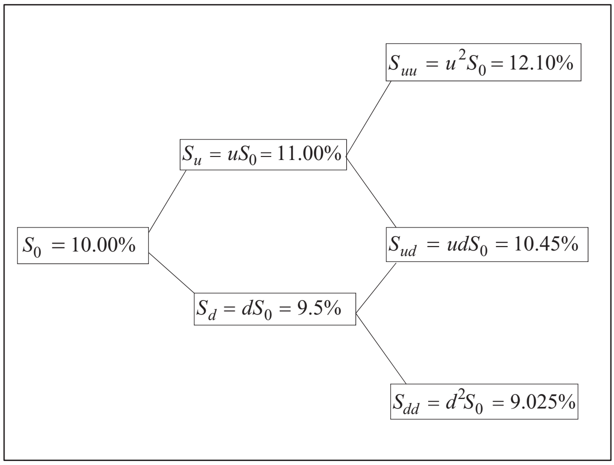


FIGURE 15.5 Calibrated Binomial Tree

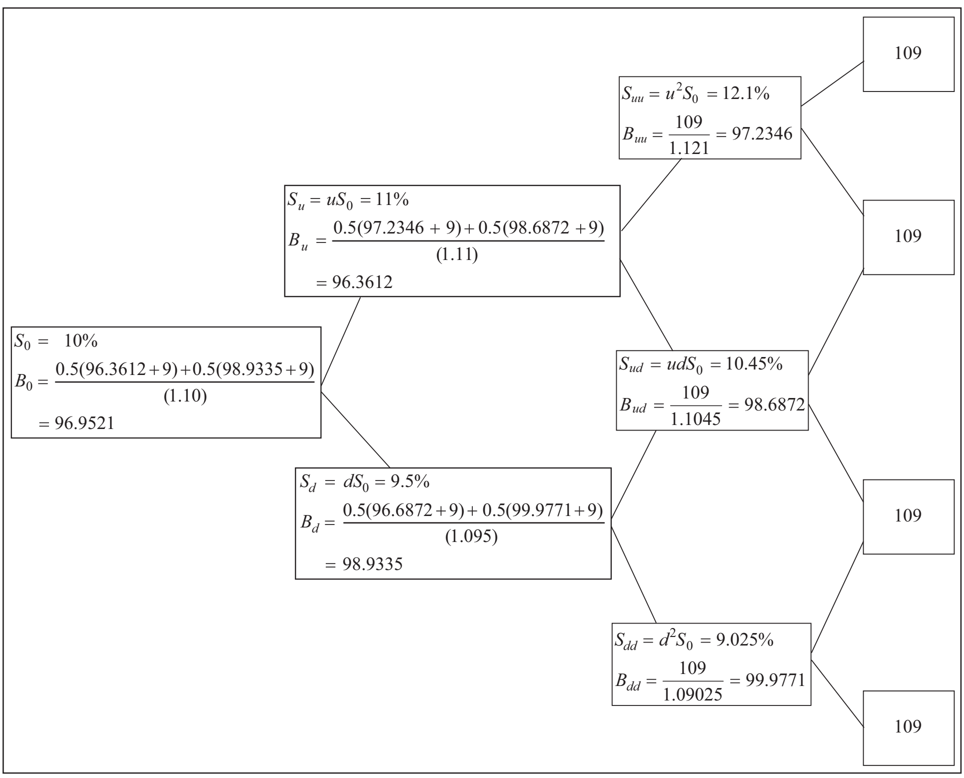


FIGURE 15.6 Value of Three-Period Option-Free 9% Bond Using the Calibrated Binomial Tree in Figure 15.5: $C = 9$, $F = 100$

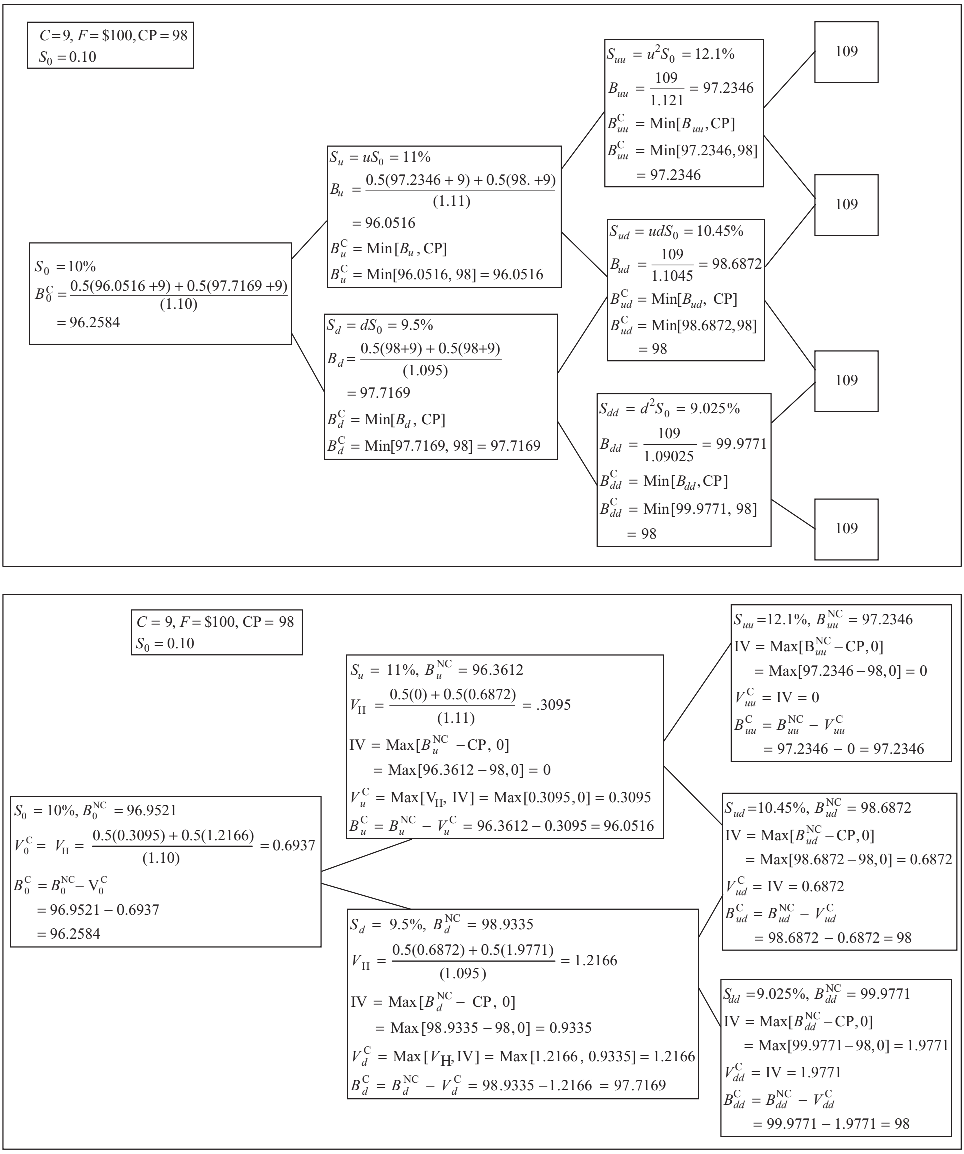


FIGURE 15.7 Value of Callable Bond

This feature should not be too surprising since we derived the tree by calibrating it to current spot rates. Nevertheless, one of the features of the calibrated tree is that it yields values on option-free bonds that are equal to the bond's equilibrium price. The primary purpose of generating the tree, though, is to value bonds with embedded options. In this example, if the three-year bond were callable at 98 (our Chapter 14 example), then its value would be 96.2584 (see Figure 15.7).

Option-Free Features

One of the features of the calibration model is that it prices a bond equal to its equilibrium price. Recall, a bond's equilibrium price is an arbitrage-free price. That is, if the market does not price the bond at its equilibrium value, then arbitrageurs would be able to realize a riskless return either by buying the bond, stripping it into a number of zero discount bonds, and selling them, or by buying a portfolio of zero discount bonds, bundling them into a coupon bond, and selling it. In general, a security can be valued by arbitrage by pricing it to equal the value of its replicating portfolio: a portfolio constructed so that it has the same cash flows. The replicating portfolio of a coupon bond, in turn, is the portfolio of zero discount bonds. Thus, one of the important features of the calibration model is that it yields prices on option-free bonds that are arbitrage free. In addition to satisfying an arbitrage-free condition on option-free bonds, the calibration model also values a bond's embedded options as arbitrage-free prices. Appendix F provides an example of this condition for the case of the three-year, 9% callable bond.

With the option values equal to their replicating portfolio values, the calibration model has the feature of pricing embedded options equal to their arbitrage-free prices. Because of this feature and the feature of pricing option-free bonds equal to their equilibrium prices, the calibration model is referred to as an ***arbitrage-free model***.[5] The calibration model presented here is the Black-Derman-Toy model. In addition to being arbitrage free, its major attribute is that it captures the volatility and drift in rates that are dependent on the current level of interest rates. Other calibration models have been developed that differ in terms of the assumptions they make about the evolution of interest rates. The Ho-Lee model, for example, assumes interest rates each period are determined by the previous rate plus or minus an additive rather than multiplicative random shock. The Black-Karasinski model, in turn, is characterized by a mean reversion process in which short-term rates revert to a central tendency. Each of these models, though, is characterized by the property that if their assumption about the evolution of rates is correct, the model's bond and embedded option prices are supported by arbitrage. This arbitrage-free feature of the calibration model is one of the main reasons that many practitioners favor this model over the equilibrium model based on estimating u and d.

15.6 OPTION-ADJUSTED SPREAD

Once we have derived a binomial tree, we can then use it to value any bond with embedded option features. In addition to valuation, the tree also can be used to estimate the option spread (the difference in yields between a bond with option features and an otherwise identical option-free bond), as well as the duration and convexity of bonds with embedded option features.

The simplest way to estimate the option spread is to estimate the YTM for a bond with an option given the bond's values as determined by the binomial model, then subtract that rate from the YTM of an otherwise identical option-free bond. For example, in the previous example the value of the three-year, 9% callable was 96.2584, whereas the equilibrium price of the noncallable was 96.9521. Using these prices, the YTM on the callable is 10.51832% and the YTM on the noncallable is 10.2306, yielding an option spread of .28772%:

$$\text{Option-free bond: } 96.9521 = \frac{9}{1+\text{YTM}} + \frac{9}{(1+\text{YTM})^2} + \frac{109}{(1+\text{YTM})^3}$$
$$\Rightarrow \text{YTM}^{NC} = 10.2306\%$$
$$\text{Callable bond :} 96.2584 = \frac{9}{1+\text{YTM}} + \frac{9}{(1+\text{YTM})^2} + \frac{109}{(1+\text{YTM})^3}$$
$$\Rightarrow \text{YTM}^{C} = 10.51832\%$$
$$\text{Option spread} = \text{YTM}^{C} - \text{YTM}^{NC}$$
$$= 10.51832\% - 10.2306\% = .28772\%$$

One of the problems with using this approach to estimate the spread is that not all of the possible cash flows of the callable bond are considered. In three of the four interest rate scenarios, for example, the bond could be called, changing the cash flow pattern from three periods of 9, 9, and 109 to two periods of 9 and 107. An alternative approach that addresses this problem is the ***option-adjusted spread (OAS) analysis***.

OAS analysis solves for the option spread (k) that makes the average of the present values of the bond's cash flows from all of the possible interest rate paths equal to the bond's market price. The first step in this approach is to specify the cash flows and spot rates for each path. In the case of the three-year bond valued with a two-period binomial interest rate tree, there are four possible paths:

	Path 1		**Path 2**		**Path 3**		**Path 4**	
Time	**S_1**	**CF**	**S_1**	**CF**	**S_1**	**CF**	**S_1**	**CF**
0	.10	–	.10	–	.10	–	.10	–
1	.0950	9	.095	9	.11	9	.11	9
2	.09025	107	.1045	107	.1045	107	.121	9
3	–		–		–		–	109

Given the four paths, we next determine the appropriate two-year spot rates (y_2) and three-year rates (y_3) to discount the cash flows. These rates can be found using the geometric mean and the one-year spot rates from the tree; that is:

Path 1	Path 2
$y_1 = .10$	$y_1 = .10$
$y_2 = [(1.10)(1.095)]^{1/2} - 1 = .097497$	$y_2 = [(1.10)(1.095)]^{1/2} - 1 = .097497$
$y_3 = [(1.10)(1.095)(1.09025)]^{1/3} - 1$	$y_3 = [(1.10)(1.095)(1.1045]^{1/3} - 1$
$= .095076$	$= .099826$

Path 3	Path 4
$y_1 = .10$ $y_2 = [(1.10)(1.11)]^{1/2} - 1 = .104989$ $y_3 = [(1.10)(1.11)(1.1045]^{1/3} - 1$ $= .104826$	$y_1 = .10$ $y_2 = [(1.10)(1.11)]^{1/2} - 1 = .104989$ $y_3 = [(1.10)(1.11)(1.121]^{1/3} - 1$ $= .110300$

Given a discount rate equal to the spot rate plus the spread, k, the final step is to solve for the k that makes the average present value of the paths equal to the callable bond's market price, $B^M{}_0$; that is:

$$B_0^M = (1/4)\left\{\begin{array}{l} \left[\dfrac{9}{(1+.10+k)} + \dfrac{107}{(1+.097497+k)^2}\right] \\ + \left[\dfrac{9}{(1+.10+k)} + \dfrac{109}{(1+.097497+k)^2}\right] \\ + \left[\dfrac{9}{(1+.10+k)} + \dfrac{109}{(1+.104989+k)^2}\right] \\ + \left[\begin{array}{l} \dfrac{9}{(1+.10+k)} + \dfrac{9}{(1+.104989+k)^2} \\ + \dfrac{109}{(1+.110300+k)^3} \end{array}\right] \end{array}\right\}$$

Note, if the market price is equal to the binomial value we obtained using the calibration model, then the option spread, k, is equal to zero. This reflects the fact that we have calibrated the tree to the yield curve and have considered all of the possibilities. In practice, though, we do not expect the market price to equal the binomial value. If the market price is below the binomial value, then k will be positive. For example, if the market priced the three-period bond at 94.6097, then the OAS (k) would be 2%. Many analysts in trying to identify mispriced bonds use the OAS approach to estimate k instead of comparing the market price with the binomial value.

15.7 DURATION AND CONVEXITY

Like our earlier measure of bond value, we defined a bond's duration (modified and Macaulay durations) and convexity without factoring in its embedded option features. Bonds with call options are sometimes said to have negative convexity, meaning that the bond's duration moves inversely with rate changes. When a bond has a call option, a rate decrease can lead to an early call, which shortens the life of the bond and lowers its duration, whereas an interest rate increase tends to lengthen the expected life of the bond, causing its duration to increase. Thus, the option features of a bond can have a significant impact on the bond's duration, as well as its convexity.

The duration and convexity of bonds with embedded option features can be estimated using a binomial tree and the effective duration and convexity measures defined in Chapter 5:

$$\text{Effective duration} = \frac{B_- - B_+}{2(B_0)(\Delta y)}$$

$$\text{Effective convexity} = \frac{B_+ + B_- - 2B_0}{(B_0)(\Delta y)^2}$$

where B_- = price associated with a small decrease in rates and
B_+ = price associated with small increase in rates.

The binomial tree calibrated to the yield curve can be used to estimate B_0, B_- and B_+. First, the current yield curve and calibrated tree can be used to determine B_0; next, B_- can be estimated by allowing for a small equal decrease in each of the spot yield curve rates (e.g. 10 basis points) and then using the tree calibrated to the new rates to find the price; finally, B_+ can be estimated in a similar way by allowing for a small equal increase in the yield curve's rates and then estimating the bond price using the tree calibrated to these higher rates.

15.8 A NOTE ON THE BLACK-SCHOLES OPTION PRICING MODEL

Before finishing our analysis of binomial interest rate models and their use in valuing bonds with embedded options, it should be noted that an approximate value of the embedded option features of a bond can also be estimated using the well-known ***Black-Scholes Option Pricing Model (B-S OPM)***—a model commonly used in pricing options. The B-S formula for determining the equilibrium price of an embedded call or put option is:

$$V_0^C = B_0 N(d_1) - XN(d_2)e^{-R_f T}$$

$$V_0^P = X(1 - N(d_2))e^{-R_f T} - B_0(1 - N(d_1))$$

$$d_1 = \frac{\ln(B_0/X) + (R_f + .5\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

where X = call price (CP) or put price (PP)
σ^2 = variance of the logarithmic return of the bond price $= V[\ln(B_n/B_0)]$
T = time to expiration expressed as a proportion of a year
R_f = continuously compounded annual risk-free rate [if simple annual rate is R, the continuously compounded rate is $\ln(1 + R)$]

$N(d)$ = cumulative normal probability; this probability can be looked up in a standard normal probability table or by using the following formula:
$N(d) = 1 - n(d)$, for $d < 0$
$N(d) = n(d)$, for $d > 0$

where $n(d) = 1 - .5[1 + .196854\ (|d|) + .115194\ (|d|)^2 + .0003444\ (|d|)^3 + .019527(|d|)^4]^{-4}$
$|d|$ = absolute value of d

For example, suppose a three-year, noncallable bond with a 10% annual coupon is selling at par ($F = 100$). A callable bond that is identical in all respects except for its call feature should sell at 100 minus the call price. In this case, suppose the call feature gives the issuer the right to buy the bond back at any time during the bond's life at an exercise price of 115. Assuming a risk-free rate of 6% and a variability of $\sigma = .10$ on the noncallable bond's logarithmic return, the call price using the Black-Scholes model would be 8.95:

$$V_0^C = B_0 N(d_1) - XN(d_2)e^{-R_f T}$$

$$V_0^C = 100(.62519) - 115(.55772)e^{-(.06)(3)} = 8.95$$

$$d_1 = \frac{\ln(100/115) + (.06 + .5(.10^2)3}{.10\sqrt{3}} = .31892$$

$$d_2 = .31892 - .10\sqrt{3} = .14571$$

$$N(d_1) = N(.31892) = .62519$$

$$N(d_2) = N(.14571) = .55772$$

Thus, the price of the callable bond is 91.05:

Price of callable bond = Price of noncallable bond − Call premium
Price of callable bond = 100 − 8.95 = 91.05

It should be noted that the B-S OPM assumes interest rates are constant. For most bonds, though, the change in their value is due to interest rate changes. Thus, the use of the B-S OPM to value the call option embedded in a bond should be viewed only as an approximation.

WEB INFORMATION

Fisher Black received a Ph.D. in Applied Math from Harvard University in 1964. In 1971, he taught at the University of Chicago and later at the MIT Sloan School of Management. In 1985, he joined Goldman Sachs.

The Nobel Prize is not given posthumously, so it was not awarded to Black in 1997 when his co-author Myron Scholes received the honor for their landmark work on option pricing along with Robert C. Merton, another

(*continued*)

(*Continued*)

pioneer in the development of valuation of stock options. In the announcement of the award that year, the Nobel committee prominently mentioned Black's key role.

Black has also received recognition as the coauthor of the Black-Derman-Toy interest-rate derivatives model, which was developed for in-house use by Goldman Sachs in the 1980s but eventually published.

Biography:

http://en.wikipedia.org/wiki/Fischer_Black

Major works:

http://cepa.newschool.edu/het/profiles/black.htm

Myron S. Scholes is chairman of Platinum Grove Asset Management, an alternative investment fund specializing in liquidity provision services to the global wholesale capital markets. Professor Scholes is the Frank E. Buck Professor of Finance Emeritus at the Stanford University Graduate School of Business and a director of the Chicago Mercantile Exchange, Dimensional Fund Investors mutual funds.

Professor Scholes is widely known for his seminal work in options pricing, capital markets, tax policies and the financial services industry. For his options pricing work, he was awarded the Alfred Nobel Memorial Prize in Economic Sciences in 1997.

Biography:

www.msri.org/about/governance/trustees/TrusteesInfo/
400006218/show_trustees

Stephen A. Ross is the Franco Modigliani Professor of Finance and Economics at the MIT Sloan School of Management. Ross received his doctorate of economics from Harvard University and has taught at the University of Pennsylvania, Yale School of Management, and MIT.

In addition to being the codiscoverer of the binomial model for pricing derivatives, Stephen Ross is also known as the inventor of the Arbitrage Pricing Theory and contributor to the creation of the Cox-Ingersoll-Ross model for interest rate dynamics. He is a cofounder of Roll & Ross Asset Management Corporation.

Biography:

http://en.wikipedia.org/wiki/Stephen_Ross (economist)

http://sloancf.mit.edu/vpf/detail-if.cfm?in_spseqno=226&co_list=F

Major works of Stephen A. Ross:

http://cepa.newschool.edu/het/profiles/ross.htm

Working papers:

http://papers.ssrn.com/sol3/cf_dev/AbsByAuth.cfm?per_id=26373

15.9 CONCLUSION

In the last two chapters, we have examined how binomial interest rate trees can be constructed and used to value bonds with call and put options, sinking fund arrangements, convertibility clauses, and prepayment options. Given that many bonds, as well as other debt contracts, have embedded option features, the binomial tree provides both a useful and practical approach to the valuation of debt securities. In this chapter we have focused on introducing how the tree can be estimated. Bonds with multiple option features and ones that are subject to the uncertainties of more than one factor, such as callable convertible bonds or LYONs, are more difficult to evaluate in that may require more complex binomial models. Each, though, can be evaluated by extending the basic binomial model presented here.

KEY TERMS

arbitrage-free model
Black-Scholes option pricing model (B-S OPM)
calibration model
equilibrium model
option-adjusted spread (OAS) analysis

PROBLEMS AND QUESTIONS

1. Explain how subdividing the number of periods to expiration makes the binomial interest rate tree more realistic.
2. Assume a one-period spot rate follows a binomial process, is currently at $S_0 = 5\%$, $u = 1.02$, $d = 1/1.02$, and the probability of the spot rate increasing in one period is $q = .5$.
   a. Show with a binomial tree the spot rates, logarithmic returns, and probabilities after one period, two periods, and three periods.
   b. What are the spot rate's expected logarithmic return and variance for each period?
   c. Define the properties of a binomial distribution.
   d. Verify that the u and d formulas yield the u and d values of 1.02 and 1/1.02 given the logarithmic return's mean and variance after three periods.
3. Explain the methodology used for deriving the formulas for u and d.
4. Comment on the arbitrage-free features of valuing a bond using the equilibrium model.
5. Suppose a spot rate has the following probability distribution of possible rates after four months:

Annualized Spot Rate (%)	Probability
6.623	.0625
6.332	.2500
6.054	.3750
5.788	.2500
5.534	.0625

   a. Calculate the spot rate's expected logarithmic return and variance. Assume the current rate is 6%.

b. Calculate the spot rate's annualized variance and mean.

c. What are the spot rate's u and d values for a period of length one month (h = length of the period in years = 1/12), one week (h = 1/52), and one day (h = 1/360)?

d. Suppose the spot rate's mean is equal to zero, what are the rate's u and d values for the periods of lengths one month, week, and day? Comment on the importance of the mean in calculating u and d when n is large.

6. Suppose a spot rate has the following rates over the past 13 quarters:

Quarter	Annualized Spot Rate (%)
y 1.1	5.5
y 1.2	5.0
y 1.3	4.7
y 1.4	4.4
y 2.1	4.7
y 2.2	5.0
y 2.3	5.4
y 2.4	5.0
y 3.1	4.7
y 3.2	4.4
y 3.3	4.7
y 3.4	5.0
y 4.1	5.5

a. Calculate the spot rate's average logarithmic return and variance.

b. What is the rate's annualized mean and variance?

c. Calculate the spot rate's up and down parameters for periods with the following lengths:

(1) One quarter (h = length in years = 1/4)
(2) One month (h = 1/12)
(3) One week (h = 1/52)
(4) One day (h = 1/360)

7. Excel Problem: The following problem should be done using the Excel program Binominal Bond Valuation.xls.
Suppose the current spot rate in Question 6 is at 5%. Using the estimated spot rate's mean and variance calculated in Question 6.b determine the value of a five-year, 5% option-free bond (F = 100) using a binomial tree with monthly steps (h = 1/12). Determine the value of the bond given it is callable with a call price of 100.

8. Excel Problem: The following problems should be done using the Excel programs Binomial Bond Valuation.xls.
Given the following:

- Current spot = 0.08
- Annualized mean for the spot rate's logarithmic return of .022
- Annualized variance for the spot rate's logarithmic return of .0054
- Binomial interest rate tree with monthly steps

Determine the values of the following:

a. Five-year, 8% option-free bond, with $F = 100$
b. Five-year, 8% callable bond ($F = 100$) with call price = 100
c. Five-year, 8% putable bond ($F = 100$) with put price = 100

9. Explain the methodology for estimating a binomial tree using the calibration model. Comment on the arbitrage-free features of this approach.

10. Given a variability of $\sigma = \sqrt{hV_e^A} = .10$ and current one- and two-period spot rates of $y_1 = .07$ and $y_2 = .0804$:

a. Generate a one-period binomial interest rate tree using the calibration model. (Hint: try $S_d = .08148$).
b. What do the values of the upper and lower spot rates relative to the current spot rate of 7% tell you about the structure of interest rates?
c. Using the calibrated tree, determine the equilibrium price of a two-period, option-free, 10.5% coupon bond ($F = 100$).
d. Does the binomial tree price the 10.5% option-free bond equal to the bond's equilibrium price? Comment on this feature of the calibration model.
e. Using the tree, calculate the value of a two-period, 10.5% bond ($F = 100$) callable in period 1 at CP = 101.

11. Using the calibrated binomial tree from Question 10 ($S_0 = .07$, $S_d = 8.148\%$ and $S_u = 9.9952\%$), answer the following:

a. Show in a binomial tree the following values at each node:
i. The values of an option-free, one-period, 10.5% coupon bond ($F = 100$).
ii. The values of an option-free, two-period, 10.5% coupon bond ($F = 100$).
iii. The values of an embedded call option on a two-period, 10.5% callable bond with the call price equal to CP = 101 and callable in period 1.
iv. The values of a two-period, 10.5% coupon bond callable at CP = 101 in period 1.

b. Construct a portfolio with the one-period and two-period 10.5% option-free bonds that replicates the period 1 up and down values of the embedded call option on the two-period, 10.5% callable bond (hint: try $n_1 = -.70463$ and $n_2 = .701457$). What is the current value of the replicating portfolio? Does the current value of your replicating portfolio match the current value of the callable bond's embedded call option? Comment on the arbitrage-free features of the calibration model. For more insight into this question, see Appendix F.

12. Given a variability of $\sigma = \sqrt{hV_e^A} = .10$ and current one-, two-, and three-period spot rates of $y_1 = .07$, $y_2 = .0804$, and $y_3 = .0904952$:

a. Generate a two-period binomial interest rate tree using the calibration model. (Hint: try $S_d = .08148$ from Problem 10 and $S_{dd} = .0906$).
b. Using the calibrated tree, determine the equilibrium price of a three-period, 10.5% option-free bond ($F = 100$).
c. Does the binomial tree price the 10.5% option-free bond equal to the bond's equilibrium price?

d. Using the calibrated tree, calculate the value of a three-period, 10.5% bond ($F = 100$) callable at CP = 101 in periods 1 and 2.
e. Based on the option-free and callable bond values you determined in 12.b and 12.d, estimate the option-adjusted spread.

13. Explain how a spread is estimated using the option-adjusted spread analysis.

14. Using the option-adjusted spread analysis, what is the option-adjusted spread for the three-period, 10.5% callable bond in Question 12 if it is priced to equal its equilibrium price of 103.30? What is the spread if the market prices the bond at 102.80?

15. Explain the methodology used to estimate a bond's duration and convexity with the calibration model.

16. Given the following information on a callable bond:
    - Coupon rate = 10% (annual), with payments made annually
    - Face value = F = $1,000
    - Maturity = 5 years
    - Callable at $1,100
    - YTM on a similar noncallable bond = 10%
    - Annualized standard deviation of the noncallable bond's logarithmic return = .15
    - Continuously compounded annual risk-free rate = 5%

    Questions:

    a. What is the value of the noncallable bond?
    b. Using the B-S OPM, what is the value of the callable bond's call feature to the issuer? Use the B-S Excel program found on the Web site.
    c. What is the value of the callable bond?

17. Given the following information on a putable bond:
    - Coupon rate = 10% (annual), with payments made annually
    - Face value = F = $1,000
    - Maturity = 5 years
    - Putable at $950
    - YTM on a similar nonputable bond = 10%
    - Annualized standard deviation of the nonputable bond's logarithmic return = .15
    - Continuously compounded annual risk-free rate = 5%

    Questions:

    a. What is the value of the nonputable bond?
    b. Using the B-S OPM, what is the value of the putable bond's put feature to the holder? Use the B-S Excel program found on the Web site.
    c. What is the value of the putable bond?

WEB EXERCISES

1. Calculate the variance of the logarithmic return using historical data on T-bill or CP yields from the Federal Reserve. Go to www.federalreserve.gov/releases

and go to "Monthly Releases" and "Interest Rates," and click on "Historical Data." Copy and paste the data to Excel. Given the data, convert the yields to logarithmic returns, and then find the standard deviation.

NOTES

1. $n!$ (Read as n factorial) is the product of all numbers from 1 to n; also $0! = 1$.
2. Note that the annualized standard deviation cannot be obtained simply by multiplying the quarterly standard deviation by four. Rather, one must first multiply the quarterly variance by four and then take the square root of the resulting annualized variance.
3. Note, in the equations for u and d, as n increases, the mean term in the exponent goes to zero quicker than the square root term. As a result, for large n (e.g., $n = 30$), the mean term's impact on u and d is negligible and u and d can be estimated as:

$$u = e^{\sqrt{V_e/n}} \quad \text{and} \quad d = e^{-\sqrt{V_e/n}} = 1/u$$

4. The u and d formulas derived here assume an interest rate process in which the variance and mean are stable and where the end-of-the-period distribution is symmetrical. Other models can be used to address cases in which these assumptions do not hold. Merton's mixed diffusion-jump model, for example, accounts for the possibilities of infrequent jumps in the underlying price or interest rate, and Cox and Ross's constant elasticity of variance model is applicable for cases in which the variance is inversely related to the underlying price or rate.
5. Students of option pricing may recall that arbitrage-free models can alternatively be priced using a risk-neutral pricing approach. When applied to bond pricing, this approach requires finding the pseudo probabilities that make a binomial tree of bond prices equal to the equilibrium price. The risk-neutral pricing approach is equivalent to the calibration approach.

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PART Four

Debt Derivatives: Futures and Options

Bond Evaluation, Selection, and Management, Second Edition
by R. Stafford Johnson

CHAPTER 16

Futures Contracts on Debt Securities: Fundamentals

16.1 INTRODUCTION

In the 1840s, Chicago emerged as a transportation and distribution center for agriculture products. Midwestern farmers transported and sold their products to wholesalers and merchants in Chicago, who often would store and later transport the products by either rail or the Great Lakes to population centers in the East. Partly because of the seasonal nature of grains and other agriculture products and partly because of the lack of adequate storage facilities, farmers and merchants began to use ***forward contracts*** as a way of circumventing storage costs and pricing risk. These contracts were agreements in which two parties agreed to exchange commodities for cash at a future date, but with the terms and the price agreed upon in the present. For example, an Ohio farmer in June might agree to sell his expected wheat harvest to a Chicago grain dealer in September at an agreed-upon price. This forward contract enabled both the farmer and the dealer to lock in the September wheat price in June. In 1848, the Chicago Board of Trade (CBOT) was formed by a group of Chicago merchants to facilitate the trading of grain. This organization subsequently introduced the first standardized forward contract, called a to-arrive contract. Later, it established rules for trading the contracts and developed a system in which traders ensured their performance by depositing good-faith money to a third party. These actions made it possible for speculators as well as farmers and dealers who were hedging their positions to trade their forward contracts. By definition, ***futures*** are marketable forward contracts. Thus, the CBOT evolved from a board offering forward contracts to the first organized exchange listing futures contracts—a futures exchange.

Since the 1840s, as new exchanges were formed in Chicago, New York, London, Singapore, and other large cities throughout the world, the types of futures contracts grew from grains and agricultural products to commodities and metals and finally to financial futures: futures on foreign currency, debt securities, and security indexes. Because of their use as a hedging tool by financial managers and investment bankers, the introduction of financial futures in the early 1970s led to a dramatic growth in futures trading, with the user's list reading as a who's who of major investment houses, banks, and corporations. The financial futures market formally began in 1972 when the Chicago Mercantile Exchange (CME) created the International Monetary Market (IMM) division, to trade futures contracts on foreign currency. In 1976, the CME extended its listings to include a futures contract on a Treasury bill.

The CBOT introduced its first futures contract in October of 1975 with a contract on the GNMA pass-through, and in 1977 they introduced the Treasury bond futures contract. The first cash-settled futures contract was introduced by the CME in 1981 with its contract on a three-month Eurodollar deposit. The Kansas City Board of Trade was the first exchange to offer trading on a futures contract on a stock index when it introduced the Value Line Composite Index contract (VLCI) in 1983. This was followed by the introduction of the S&P 500 futures contract by the CME and the NYSE index futures contract by the New York Futures Exchange (NYFE).

Whereas the 1970s marked the advent of financial futures, the 1980s saw the globalization of futures markets with the openings of the London International Financial Futures Exchange, LIFFE (1982), Singapore International Monetary Market (1986), Toronto Futures Exchange (1984), New Zealand Futures Exchange (1985), and Tokyo Financial Futures Exchange (1985). Table 16.1 lists the major exchanges trading futures and options and their Web sites. The increase in the number of futures exchanges internationally led to a number of trading innovations: electronic trading systems, 24-hour world-wide trading, and alliances between exchanges. Concomitant with the growth in futures trading on organized exchanges has been the growth in futures contracts offered and traded on the over-the-counter (OTC) market. In this market, dealers offer and make markets in more tailored-made forward contracts in currencies, indexes, and various interest-rate products. Today, the total volume of forward contracts created on the OTC market exceeds the volume of exchange-traded futures contracts. The combined growth in futures and forward contracts also created a need for more governmental oversight to ensure market efficiency and to guard against abuses. In 1974 the Commodity Futures Trading Commission, CFTC, was created by Congress to monitor and regulate futures trading, and in 1982 the National Futures Association, NFA, an organization of futures market participants, was established to oversee futures trading. Finally, the growth in futures markets led to the consolidation of exchanges. In 2006, the CME and the CBOT approved a deal in which the CME acquired the CBOT, forming the CME Group, Inc.

Definition

Formally, a forward contract is an agreement between two parties to trade a specific asset at a future date with the terms and price agreed upon today. A ***futures contract***, in turn, is a marketable forward contract, with marketability provided through futures exchanges that list hundreds of standardized contracts, establish trading rules, and provide for clearinghouses to guarantee and intermediate contracts. In contrast, forward contracts are provided by financial institutions and dealers, are less standardized and more tailor-made, are usually held to maturity, and unlike futures, they often do not require initial or maintenance margins. Both forward and futures contracts are similar to option contracts in that the underlying asset's price on the contract is determined in the present with the delivery and payment occurring at a future date. The major difference between these derivative securities is that the holder of an option has the right, but not the responsibility, to execute the contract (i.e., it is a contingent-claim security), whereas the holder of a futures or forward contract has an obligation to fulfill the terms of the contract. In this chapter, we examine the markets and fundamental uses of interest rate futures and forward contracts, and in Chapter 17 we examine the markets and fundamental uses

TABLE 16.1 Major Futures and Options Exchanges

U.S. Exchanges	
American Exchanges (AMEX)	www.amex.com
Chicago Board of Trade (CBOT)	www.CBT.com
Chicago Board of Options Exchange (CBOE)	www.cboe.com
Chicago Mercantile Exchange (CME)	www.cme.com
Coffee, Sugar, and Cocoa Exchange (NY)	www.csce.com
Commodity Exchange (COMEX) (NY)	www.nymex.com
Kansas City Board of Trade (KCBT)	www.kcbt.com
Mid-American Commodity Exchange (MidAm)	www.midam.com
Minneapolis Grain Exchange (MGE)	www.mgex.com
New York Cotton Exchange (NYCE)	www.nyce.com
New York Futures Exchange (NYFE)	www.nyfe.com
New York Mercantile Exchange (NYMEX)	www.nymex.com
Philadelphia Exchange (PHLX)	www.phlx.com
Non-U.S. Markets	
Amsterdam Exchanges (AEX)	www.aex.nl
Australian Stock Exchange (ASX)	www.asx.com
Brussels Exchange (BXS)	www.bxs.be
Bolsa de Mercadorias y Futuros, Brazil (BM&F)	www.bmf.com.br
Copenhagen Stock Exchange (FUTOP)	www.xcse.dk
Deutsche Termin Boerse, Germany (DTB)	www.exchange.de
Eurex (EUREX)	www.eurexchange.com
Hong Kong Futures Exchange (HKFE)	www.hkfe.com
London International Financial Futures and Options Exchange (LIFFE)	www.liffe.com
Marche a Terme International de France (MATIF)	www.matif.com
Marche des Options Negociables de Paris (MONEP)	www.monep.fr
MEFF Renta Fija And Variable, Spain (MEFF)	www.meff.es
Osaka Securities Exchange (OSA)	www.ose.or.jp
Singapore International Monetary Exchange (SIMEX)	www.simex.com.sg
Stockholm Options Exchange (SOM)	www.omgroup.com
Sydney Futures Exchange (SFE)	www.sfe.com.au
Toronto Stock Exchange (TSE)	www.tse.com
Alliances	
Eurex is an alliance of DTB, CBOT, and exchanges in Switzerland and Finland	www.eurexchange.com
GLOBEX is an alliance of CME, ME, MATIF, SIMEX and exchanges in Brazil and the Paris Bourse.	www.globexalliance.com
Euronext is an alliance of exchanges in Amsterdam, Brussels, and Paris	www.euronext.com

of options. With this background, in Chapter 18 we look at managing debt positions with exchange-traded futures and options, and in Chapter 19 we examine managing positions with OTC derivatives.

WEB INFORMATION

- CME Group site: www.cme.com.
- See Table 16.1. for a listing of derivative exchanges and their Web sites.
- Commodity Futures Trading Commission: www.cftc.gov.
- National Futures Association: www.nfa.futures.org.

16.2 THE MARKET AND CHARACTERISTICS OF FUTURES ON DEBT SECURITIES

Microstructure

Like other organized exchanges, futures exchanges are typically structured as membership organizations with a fixed number of seats and with the seat being a precondition for direct trading on the exchange. On most futures exchanges, there are two major types of futures traders/members: commission brokers and locals. Commission brokers buy and sell for their customers. They carry out most of the trading on the exchanges, serving the important role of linking futures traders. ***Locals,*** on the other hand, trade from their own accounts, acting as speculators or arbitrageurs. They serve to make the market operate more efficiently. Some exchanges also permit members to engage in ***dual trading.*** Under dual trading rules, a broker is allowed to fill orders for customers as well as trade for his own account as long as the customer's order is given priority.[1]

The mode of trading on futures exchanges in the United States, London (LIFFE), Paris (MATIF), Sydney (SFE), Singapore (SIMEX), and other locations still takes place the way it did over 100 years ago on the CBOT with brokers and dealers going to a pit and using the ***open outcry*** method to trade. In this system, orders are relayed to the floor by runners or by hand signals to a specified trading pit. The order is then offered in open outcry to all participants in the pit, with the trade being done with the first person to respond.

While the open-outcry system is still used on the major exchanges, electronic trading systems are being used by the physical exchanges in the United States, London, Paris, and Sydney. The CME and CBOT developed with Reuters (the electronic information service company) the ***GLOBEX*** trading system. This is a computerized order-matching system with an international network linking member traders. Similarly, SFE offers after-hours trading through their SYCOM system, and LIFFE offers such trading through their Automated Pit Trading (APT) system. In addition to dual systems, since 1985 all new derivative exchanges have been organized as electronic exchanges. The German exchange [Deutsche Termin Boerse (DTB)] and Stockholm Option Market (SOM), for example, were both set up as screen-based trading systems. Most of these electronic trading systems are order-driven systems in which customer orders (bid and ask prices and size) are collected and matched by a computerized matching system. This contrasts with a price-driven system such as the one used on the Swiss Options and Futures Exchange (SOFE) in which dealers provide bid and ask quotes and make markets.

In addition to linking futures traders, the futures exchanges also make contracts more marketable by standardizing contracts, providing continuous trading, establishing delivery procedures, and providing 24-hour trading through exchange alliances.

Standardization The futures exchanges provide standardization by specifying the grade or type of each asset and the size of the underlying asset. Exchanges also specify how contract prices are quoted. For example, the contract prices on ***Eurodollar futures*** are quoted in terms of an index equal to one hundred minus a discount yield, and a T-bond is quoted in terms of dollars and 1/32s of a T-bond with a face value of $100.

Continuous Trading Many physical exchanges use market makers or specialists to ensure a continuous market. On many futures exchanges, continuous trading also is provided, but not with market makers or specialists assigned by the exchange to deal in a specific contract. Instead, futures exchanges provide continuous trading through locals who are willing to take temporary positions in one or more futures. These exchange members fall into one of three categories: ***scalpers***, who offer to buy and sell simultaneously, holding their positions for only a few minutes and profiting from a bid-ask spread; ***day traders***, who hold positions for less than a day; ***position traders***, who hold positions for as long as a week before they close. Collectively, these exchange members make it possible for the futures markets to provide continuous trading.

Price and Position Limits Without market makers and specialists to provide an orderly market, futures exchanges are allowed to impose price limits as a tool to stop possible destabilizing price trends from occurring. When done, the exchanges specify the maximum price change that can occur from the previous day's settlement price. The price of a contract must be within its daily price limits, unless the exchange intervenes and changes the limit. When the contract price hits its maximum or minimum limit, it is referred to as being limited up or limited down. In addition to price limits, futures exchanges also can set position limits on many of their futures contracts. This is done as a safety measure both to ensure sufficient liquidity and to minimize the chances of a trader trying to corner a particular asset.

Delivery Procedures Only a small number of contracts lead to actual delivery. As we will discuss in Section 16.3, most futures contracts are closed prior to expiration. Nevertheless, detailed delivery procedures are important to ensure that the contract prices on futures are determined by the spot price on the underlying asset and that the futures price converges to the spot price at expiration. The exchanges have various rules and procedures governing the deliveries of contracts and delivery dates. The date or period in which delivery can take place is determined by the exchange. When there is a delivery period, the party agreeing to sell has the right to determine when the asset will be delivered during that period.

Alliances and 24-Hour Trading In addition to providing off-hour trading via electronic trading systems, 24-hour trading is also possible by using futures exchanges that offer trading on the same contract. The CME, LIFFE, and SIMEX all offer identical contracts on 90-day Eurodollar deposits. This makes it possible to trade the contract in the United States, Europe, and the Far East. Moreover, these exchanges have alliance agreements making it possible for traders to open a position in one market and close it in another. A similar alliance exists between SFE, CBOT, and LIFFE on U.S. T-bond contracts.

Types of Interest Rate Futures

Table 16.2 describes the features of various interest rates futures contracts traded on the CBOT, CME, LIFFE, and other exchanges. One of the first financial futures contracts listed on the CME was the T-bill contract. Today, the three most popular interest rate futures contracts are T-bonds, T-notes, and Eurodollar deposits.

T-Bill Futures *T-bill futures* contracts call for the delivery (***short position***) or purchase (***long position***) of a T-bill with a maturity of 91 days and a face value of \$1 million. Futures prices on T-bill contracts are quoted in terms of an index. This index, I, is equal to 100 minus the annual percentage discount rate, R_D, for a 90-day T-bill:

$$I = 100 - R_D(\%)$$

Given a quoted index value or discount yield, the actual contract price on the T-bill futures contract is:

$$f_0 = \frac{100 - R_D\%(90/360)}{100}\$1{,}000{,}000 \tag{16.1}$$

Note, the index is quoted on the basis of a 90-day T-bill with a 360-day year. This implies that a one-point move in the index would equate to a \$2,500 change in the futures price. The implied yield to maturity (YTM_f) on a T-bill that is delivered on the contract is often found using 365 days and the actual maturity on the delivered bill of 91 days. For example, a T-bill futures contract quoted at a settlement index value of 95.62 ($R_D = 4.38\%$) would have a futures contract price (f_0) of \$989,050 and an implied YTM_f of 4.515%:

$$f_0 = \frac{100 - 4.38(90/360)}{100}\$1{,}000{,}000 = \$989{,}050$$

and

$$YTM_f = \left[\frac{F}{f_0}\right]^{365/91} - 1$$

$$YTM_f = \left[\frac{\$1{,}000{,}000}{\$989{,}050}\right]^{365/91} - 1 = .04515$$

TABLE 16.2 Select Interest Rate Futures Contracts

Contract	Exchange	Contract Size	Delivery Month	Delivery
Treasury bond	CBOT	T-bond with $100,000 face value (or multiple of that)	Mar/June/ Sept/Dec	T-bonds with an invoice price that is equal to the futures settlement price times a conversion factor plus accrued interest.
Five-year Treasury note	CBOT	T-note with $100,000 face value (or multiple of that)	Mar/June/ Sept/Dec	T-notes that have maturity of no more than five years and three months. Invoice price is equal to the futures settlement price times a conversion factor plus accrued interest.
Treasury note	CBOT	T-note with $100,000 face value (or multiple of that)	Mar/June/ Sept/Dec	T-notes maturing at least 6 1/2 years, but no more than 10 years. Invoice price is equal to the futures settlement price times a conversion factor plus accrued interest.
Three-month Treasury bill	CME	$1,000,000	Mar/June/ Sept/Dec	Delivery can be made on three successive business days. The first delivery day is the first day on which a 13-week T-bill is issued.
Three-month Eurodollar	CME	$1,000,000	Mar/June/ Sept/Dec	Cash settlement.
One-month LIBOR	CME	$3,000,000	All calendar months	Cash settlement; settlement price based on a survey of participants in London Interbank Eurodollar market.
Municipal bond index	CBOT	$1,000 times the closing value of the *Bond Buyer*™ municipal bond index (a price of 95 means a contract size of $95,000)	Mar/June/ Sept/Dec	Cash settlement; settlement price based on *Bond Buyer*™ municipal bond index value at expiration.
Three-month Euroyen	SIMEX	100,000,000 yen	Mar/June/ Sept/Dec	Cash settlement.
Ten-year Japanese government bond index	TSE	100,000,000 yen face value	Mar/June/ Sept/Dec	Exchange-listed Japanese government bond having a maturity of seven years or more but less than 11 years.
Long gilt	LIFFE	50,000 British pounds	Mar/June/ Sept/Dec	Delivery may be any gilt with 15 to 25 years to maturity.
Three-month sterling interest rate	LIFFE	500,000 British pounds	Mar/June/ Sept/Dec	Cash settlement; settlement price based on the three-month sterling deposit rate being offered to prime banks.

Eurodollar Futures Contract As noted in Chapter 10, a Eurodollar deposit is a time deposit in a bank located or incorporated outside the United States. A Eurodollar interest rate is the rate that one large international bank is willing to lend to another large international bank. The average rate paid by a sample of London Eurobanks is known as the London Interbank Offer Rate (LIBOR). The LIBOR is higher than the T-bill rate, and as noted in Chapter 10, it is used as a benchmark rate on bank loans and deposits.

The CME's futures contract on the Eurodollar deposit calls for the delivery or purchase of a Eurodollar deposit with a face value of \$1 million and a maturity of 90 days. Like T-bill futures contracts, Eurodollar futures are quoted in terms of an index equal to 100 minus the annual discount rate, with the actual contract price found by using Equation (16.1). For example, given a settlement index value of 95.09 on a Eurodollar contract, the actual futures price would be \$987,725:

$$f_0 = \frac{100 - 4.91(90/360)}{100}\$1{,}000{,}000 = \$987{,}725$$

The major difference between the Eurodollar and T-bill contracts is that Eurodollar contracts have cash settlements at delivery, whereas T-bill contracts call for the actual delivery of the instrument. When a Eurodollar futures contract expires, the cash settlement is determined by the futures price and the settlement price. The settlement price or expiration futures index price is 100 minus the average three-month LIBOR offered by a sample of designated Eurobanks on the expiration date:

$$\text{Expiration futures price} = 100 - \text{LIBOR}$$

In addition to the CME's Eurodollar futures, there are also a number of other contracts traded on interest rates in other countries. For example, there are Euroyen contracts traded on the CME and the Singapore Exchange, Euroswiss contracts traded on the LIFFE, and Euribor contracts (three-month LIBOR contract for the euro) traded on the LIFFE and Marche a Terme International de France, MATIF.

T-Bond Futures Contracts The most heavily traded long-term interest rate futures contract is the CBOT's T-bond contract. The contract calls for the delivery or purchase of a T-bond with a maturity of at least 15 years. The CBOT has a ***conversion factor*** to determine the actual price received by the seller. The futures contract is based on the delivery of a T-bond with a face value of \$100,000. To ensure liquidity, any T-bond with a maturity of 15 years is eligible for delivery, with a conversion factor used to determine the actual price of the deliverable bond. Since ***T-bond futures*** contracts allow for the delivery of a number of T-bonds at any time during the delivery month, the CBOT's delivery procedure on such contracts is more complicated than the procedures on other futures contracts. The T-bond futures contract delivery procedure is described in Appendix G.

T-bond futures prices are quoted in dollars and 32nds for T-bonds with a face value of \$100. Thus, if the quoted price on a T-bond futures was 106-14 (i.e., 106 14/32 or 106.437), the price would be \$106,437 for a face value of \$100,000. The actual price paid on the T-bond or revenue received by the seller in delivering the bond on the contract is equal to the quoted futures price times the conversion factor, CFA, on the delivered bond plus any accrued interest:

$$\text{Seller's revenue} = (\text{Quoted futures price})(\text{CFA}) + \text{Accrued interest}$$

Thus, at the time of delivery, if the delivered bond has a CFA of 1.3 and accrued interest of $2 and the quoted futures price is 94-16, then the cash received by the seller of the bond and paid by the futures purchaser would be $124.85 per $100 face value:

$$\text{Seller's revenue} = (94.5)(1.3) + 2 = 124.85$$

T-Note Futures Contracts T-note contracts are similar to T-bond contracts, except that they call for the delivery of any T-note with maturities between 6 1/2 and 10 years; the five-year T-note contracts are also similar to T-bond and T-note contracts except that they require delivery of the most recently auctioned five-year T-note. Both contracts, though, have delivery procedures similar to T-bond contracts.

Forward Contracts—Forward Rate Agreements (FRA) Forward contracts for interest rate products are private, customized contracts between two financial institutions or between a financial institution and one of its clients. Interest rate forward contracts predate the establishment of interest rate futures markets. A good example of an interest rate forward product is a ***forward rate agreement (FRA)***.[2] This contract requires a cash payment or provides a cash receipt based on the difference between a realized spot rate such as the LIBOR and a pre-specified rate. For example, the contract could be based on a specified rate of $R_k = 6\%$ (annual) and the three-month LIBOR (annual) in five months and a notional principal, NP (principal used only for calculation purposes) of $10 million. In five months, the payoff would be

$$\text{Payoff} = (\$10{,}000{,}000)\frac{[\text{LIBOR} - .06]\,(91/365)}{1 + \text{LIBOR}(91/365)}$$

If the LIBOR at the end of five months exceeds the specified rate of 6%, the buyer of the FRA (or long position holder) receives the payoff from the seller; if the LIBOR is less than 6%, the seller (or short position holder) receives the payoff from the buyer. Thus, if the LIBOR were at 6.5%, the buyer would be entitled to a payoff of $12,267 from the seller; if the LIBOR were at 5.5%, the buyer would be required to pay the seller $12,297. Note that the terminology is the opposite of futures. In Eurodollar or T-bill futures, the party with the long position hopes rates will decrease and prices will go up, whereas the short position holder hopes that rates will increase and prices will go down.

In general, an FRA that matures in T months and is written on an M-month LIBOR rate is referred to as a $T \times (T{+}M)$ agreement. Thus, in this example the FRA is a 5×8 agreement. At the maturity of the contract (T), the value of the contract, V_T is

$$V_T = NP\,\frac{[\text{LIBOR} - R_k]\,(M/365)}{1 + \text{LIBOR}(M/365)}$$

FRAs originated in 1981 amongst large London Eurodollar banks that used these forward agreements to hedge their interest rate exposure. Today, FRAs are offered by banks and financial institutions in major financial centers and are often written for the bank's corporate customers. They are customized contracts designed to meet the needs of the corporation or financial institution.[3]

WEB INFORMATION

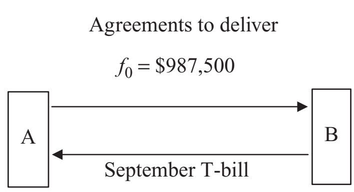
Current prices on futures contracts on Eurodollar, T-bill and other futures can be obtained by going to www.cme.com and clicking on "Quotes" in "Market Data" and then clicking on "Interest Rate Products." For T-bonds, T-notes, and other futures, go to www.cbt.com and click on "Quotes and Data."

16.3 THE NATURE OF FUTURES TRADING AND THE ROLE OF THE CLEARINGHOUSE AND MARGINS

Futures Positions

A futures holder can take one of two positions on a futures contract: a long position (or futures purchase) or a short position (futures sale). In a long futures position, the holder agrees to buy the contract's underlying asset at a specified price, with the payment and delivery to occur on the expiration date (also referred to as the delivery date); in a short position, the holder agrees to sell an asset at a specific price, with delivery and payment occurring at expiration.

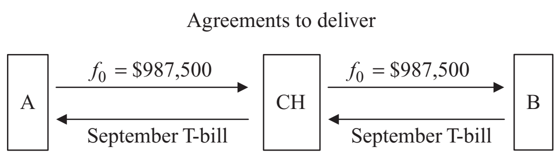
To illustrate how positions are taken, suppose in June, Speculator A believes that the Federal Reserve will expand its open market purchases over the next three months, leading to lower interest rates and higher prices on T-bills. With hopes of profiting from this expectation, suppose Speculator A decides to take a long position in a September T-bill futures contract and instructs her broker to buy one September futures contract listed on the CME (one contract calls for the purchase of a T-bill with \$1 million face value and maturity of 91 days). To fulfill this order, suppose A's broker finds a T-bill broker representing Speculator B, who believes that an expanding economy and tighter monetary policy in the ensuing months will push short-term rates up and prices down and as such is wanting to take a short position in the September T-bill contract. After hearing bid and ask quotes, suppose the brokers agree to a price on the September contract for their clients that is equal to the CME index of 95 ($R_D = 5\%$) or $f_0 = \$987{,}500$. In terms of futures positions, Speculator A would have a long position in which she agrees to buy a 91-day T-bill with a face value of \$1million for \$987,500 from Speculator B at the delivery date in September, and Speculator B would have a short position in which he agrees to sell a 91-day T-bill to Speculator A at the delivery date in September:



If both parties hold their contracts to delivery, their profits or losses would be determined by the price of the T-bill on the spot market. For example, suppose the Fed does engage in expansionary monetary policy, causing the spot discount yield on T-bills to fall to $R_D = 4\%$ at the time of the expiration date on the September futures contracts. At 4%, the spot price (S) on a T-bill with $1 million face value would be $990,000. Accordingly, Speculator A would be able to buy a 91-day T-bill on her September futures contract at $987,500 from Speculator B, and then sell the bill for $990,000 on the spot market to earn a profit of $2,500. On the other hand, to deliver a T-bill on the September contract, Speculator B would have to buy the security on the spot market for $990,000, and then sell it on the futures contract to Speculator A for $987,500, resulting in a $2,500 loss.

Clearinghouse

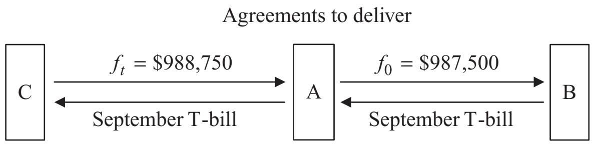
To provide contracts with marketability, futures exchanges use clearinghouses. The exchange clearinghouse is an adjunct of the exchange. It consists of clearinghouse members who guarantee the performance of each party of the transaction and act as intermediaries by breaking up each contract after the trade has taken place. Thus, in the above example, the clearinghouse (CH) would come in after Speculators A and B have reached an agreement on the price of a September T-bill, becoming the effective seller on A's long position and the effective buyer on B's short position:



Once the clearinghouse has broken up the contract, then A's and B's contracts would be with the clearinghouse. The clearinghouse, in turn, would record the following entries in its computers:

	Clearinghouse record:
1.	Speculator A agrees to buy September T-bill at $987,500 from the clearinghouse.
2.	Speculator B agrees to sell September T-bill at $987,500 to the clearinghouse.

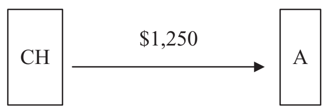
The intermediary role of the clearinghouse makes it easier for futures traders to close their positions before expiration. To see this, suppose that in June, short-term interest rates drop, leading speculators such as C to want to take a long position in the September T-bill contract. Seeing a profit potential from the increased demand for long positions in the September contract, suppose Speculator A agrees to sell a September T-bill futures contract to Speculator C for $988,750 ($R_D = 4.5\%$ and Index = 95.5). Upon doing this, Speculator A now would be short in the new September contract, with Speculator C having a long position, and there now would be two contracts on September T-bills. Without the clearinghouse intermediating, the two contracts can be described as follows:



After the new contract between A and C has been established, the clearinghouse would step in and break it up. For Speculator A, the clearinghouse's records would now show the following:

Clearinghouse records for Speculator A:
1. Speculator A agrees to BUY September T-bill from the clearinghouse for $987,500.
2. Speculator A agrees to SELL September T-bill to the clearinghouse for $988,750.

Thus

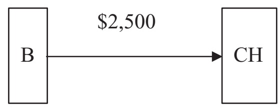


The clearinghouse accordingly would close Speculator A's positions by paying her $1,250 at expiration. Since Speculator A's short position effectively closes her position, it is variously referred to as a ***closing***, ***reversing out***, or ***offsetting position*** or simply as an offset. Thus, the clearinghouse makes it easier for futures contracts to be closed prior to expiration.

The expense and inconvenience of delivery causes most futures traders to close their positions instead of taking delivery. As the delivery date approaches, the number of outstanding contracts, referred to as ***open interest***, declines, with only a relatively few contracts still outstanding at delivery. Moreover, at expiration (T), the contract prices on futures contracts established on that date (f_T) should be equal (or approximately equal for some contracts) to the prevailing spot price on the underlying asset (S_T). That is, at expiration, $f_T = S_T$. If f_T does not equal S_T at expiration, an arbitrage opportunity would exist. Arbitrageurs could take a position in the futures contract and an opposite position in the spot market. For example, if the September T-bill futures contracts were trading at $990,000 on the delivery date in September and the spot price on T-bills were trading at $990,500, an arbitrageur could go long in the September contract, take delivery by buying the T-bill at $990,000 on the futures contract, then sell the bill on the spot at $990,500 to earn a risk-free profit of $500. The arbitrageur's efforts to take a long position, though, would drive the contract price up to $990,500. On the other hand, if f_T exceeds $990,500, then an arbitrageur would reverse their strategy, pushing f_T down to $990,500. Thus, at delivery, arbitrageurs will ensure that the price on an expiring contract is equal to the spot price. As a result, closing a futures contract with an offsetting position at expiration will yield the same profits or losses as purchasing (selling) the asset on the spot and selling (buying) it on the futures contract.

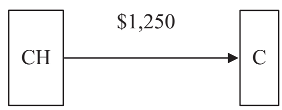
Returning to our example, suppose near the delivery date on the September contract the spot T-bill price and the price on the expiring September futures contracts are $990,000 ($R_D$ = 4% or index = 96). To close his existing short contract, Speculator B would need to take a long position in the September contract, while to offset her existing long contract, Speculator C would need to take a short position. Suppose Speculators B and C take their offsetting positions with each other on the expiring September T-bill contract priced at $f_T = S_T = \$990{,}000$. After the clearinghouse breaks up the new contract, Speculator B would owe the clearinghouse \$2,500 and Speculator C would receive \$1,250 from the clearinghouse:

	Clearinghouse records for Speculator B:
1.	Speculator B agrees to SELL September T-bill to CH for \$987,500.
2.	Speculator B agrees to BUY September T-bill from CH at \$990,000.



And,

	Clearinghouse records for Speculator C:
1.	Speculator C agrees to BUY September T-bill at \$988,750.
2.	Speculator C agrees to SELL September T-bill for \$990,000.



To recapitulate, in this example, the contract prices on September T-bill contracts went from \$987,500 on the A and B contract, to \$988,750 on the A and C contract, to \$990,000 on the B and C contract at expiration. Speculators A and C each received \$1,250 from the clearinghouse, whereas Speculator B paid \$2,500 to the clearinghouse; the clearinghouse with a perfect hedge on each contract received nothing (other than clearinghouse fees attached to the commission charges), and no T-bill was actually purchased or delivered.

Margin Requirements

Since a futures contract is an agreement, it has no initial value. Futures traders, however, are required to post some security or good faith money with their brokers. Depending on the brokerage firm, the customer's margin requirement can be satisfied either in the form of cash or cash equivalents.

Futures contracts have both initial and maintenance margin requirements. The ***initial (or performance) margin*** is the amount of cash or cash equivalents that must be deposited by the investor on the day the futures position is established. The futures trader does this by setting up a margin (or commodity) account with the broker and depositing the required cash or cash equivalents. The amount of the margin is determined by the margin requirement, defined as a proportion (m) of the contract value (usually 3% to 5%). For example, if the initial margin requirement is 5%, then Speculators A and B in our example would be required to deposit \$49,375 in cash or cash equivalents in their commodity accounts as good faith money on their September futures contracts:

$$m[\text{Contract Value}] = .05[\$987{,}500] = \$49{,}375$$

At the end of each trading day, the futures trader's account is adjusted to reflect any gains or losses based on the settlement price on new contracts.[4] In our example, suppose the day after Speculators A and B established their respective long and short positions, the settlement index value on the September T-bill was 95.5 ($f_t = 988{,}750$, $R_D = 4.5$). The value of A's and B's margin accounts would therefore be

$$\text{A: Account value} = \$49{,}375 + (\$988{,}750 - \$987{,}500) = \$50{,}625$$

$$\text{B: Account value} = \$49{,}375 + (\$987{,}500 - \$988{,}750) = \$48{,}125$$

With a lower rate and higher futures price, A's long position has increased in value by \$1,250 and B's short position has decreased by \$1,250. When there is a decrease in the account value, the futures trader's broker has to exchange money through the clearing firm equal to the loss on the position to the broker and clearinghouse with the gain. This process is known as ***marking to market***. Thus in our case, B's broker and clearing firm would pass on \$1,250 to A's broker and clearing firm.

To ensure that the balance in the trader's account does not become negative, the brokerage firm requires a margin to be maintained by the futures traders.[5] The ***maintenance (or variation) margin*** is the amount of additional cash or cash equivalents that futures traders must deposit to keep the equity in their commodity account equal to a certain percentage (e.g., 75%) of the initial margin value. If the maintenance margin requirement were set equal to 100% of the initial margin, then Speculators A and B would need to keep the equity values of their accounts equal to \$49,375. If Speculator B did not deposit the required margin immediately, then he would receive a ***margin call*** from the broker instructing him to post the required amount of funds. If Speculator B did not comply with the margin call, the broker would close the position.

Points on Margin Requirements

Several points should be noted in describing margin requirements. First, the marking to market of futures contracts effectively settles the futures contract daily. Each day, the futures holder's gain or loss is added to or subtracted from the holder's account to bring the value of the position back to zero. Once marking to market has occurred, there are no outstanding balances. On the CME, clearing members' exchange payments in a day can range from \$100 million to over \$2.5 billion. The purpose of this stringent settlement system is to reduce the chance of default.

Second, the minimum levels of initial and maintenance margins are set by the exchanges, with the brokerage firms allowed to increase the levels. Margin levels are determined by the variability of the underlying asset and can vary by the type of trader. The margins for hedgers are less than those for speculators.

Third, the maintenance margin requirement on futures requires constant management of one's account. With daily resettlement, futures traders who are undermargined have to decide each day whether to close their positions and incur losses or post additional collateral; similarly, those who are overmargined must decide each day whether or not they should close. One way for an investor to minimize the management of her futures position is to keep her account overmargined by depositing more cash or cash equivalents than initially required or by investing in one of a number of ***futures funds***. A futures fund pools investors' monies and uses them to set up futures positions. Typically, a large percentage of the fund's money is invested in money market securities. Thus, the funds represent overmargined futures positions.

Fourth, maintaining margin accounts can be viewed as part of the cost of trading futures. In addition to margin requirements, transaction costs are also involved in establishing futures positions. Such costs include broker commissions, clearinghouse fees, and the bid-ask spread. On futures contracts, commission fees usually are charged on a per contract basis and for a round lot (i.e., the fee includes both opening and closing the position), and the fees are negotiable. The clearinghouse fee is relatively small and is collected along with the commission fee by the broker. The bid-ask spreads are set by locals and represent an indirect cost of trading futures.

Finally, the margin requirements and clearinghouse mechanism that characterize futures exchanges also serve to differentiate them from customized forward contracts on debt and interest rate positions written by banks and investment companies. Forward contacts are more tailor-made contracts, usually do not require margins, and the underlying asset is typically delivered at maturity instead of closed; they are, though, less marketable than exchange-traded futures.

16.4 FUTURES HEDGING

Futures markets provide corporations, financial institutions, and others with a tool for hedging their particular spot positions against adverse price movements, for speculating on expected spot price changes, and for creating synthetic debt and investment positions with better rates than direct positions. Of these uses, the most extensive one is hedging.

Two hedging positions exist: long hedge and short hedge. In a ***long hedge*** (or hedge purchase), a hedger takes a long position in a futures contract to protect against an increase in the price of the underlying asset or commodity. Long hedge positions on debt securities are used by money-market managers, fixed-income managers, and dealers to lock in their costs on future securities purchases. In a ***short hedge***, a hedger takes a short futures position to protect against a decrease in the price of the underlying asset. In contrast to long hedging, short hedge positions are used by bond and money market managers, investment bankers, and dealers who are planning to sell securities in the future; by banks and other intermediaries to lock in the rates they pay on future deposits; and by corporate treasurers and other borrowers who want to lock in the future rates on their loans or who want to fix the rates on the variable rate loans.

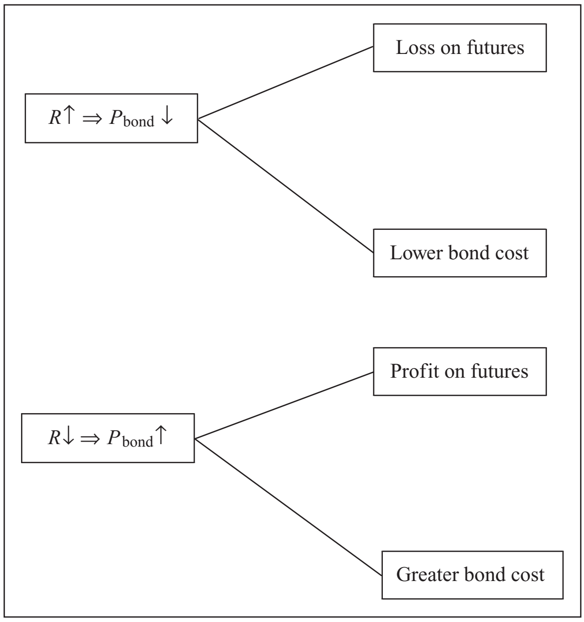


FIGURE 16.1 Long Hedge: Hedging Bond Purchase with Long Position in Interest Rate Futures

Long Hedge

A long position in an interest rate futures contract can be used by money market and fixed-income managers to lock in the purchase price on a future investment. As illustrated in Figure 16.1, if interest rates at the future investment date are lower, then the price on the fixed-income securities will be higher, and as a result, the cost of buying the securities will be higher. With a long futures position, though, the manager would be able to profit when he closes his long futures position. With the profit from the futures, the manager would be able to defray the additional cost of purchasing the higher-priced fixed-income securities. In contrast, if rates are higher, the cost of securities will be lower, but the manager would have to use part of the investment cash inflow to cover losses on his futures position. In either interest rate scenario, though, the manager would find he can purchase approximately the same number of securities given his hedged position.

Long Hedge Example: Future Eurodollar Investment To illustrate a long hedge position, consider the case of a money market manager who is expecting in September a cash inflow of $9,875,000, which he plans to invest in a 90-day jumbo certificate of deposit, CD, with a face value of $10 million. Fearing that short-term rates could decrease (causing CD prices to increase), suppose the manager goes long in 10 September Eurodollar futures trading at $R_D = 5\%$ or $f_0 = \$987{,}500$. Given equal spot and expiring futures prices at expiration, the manager will find that any additional costs of buying the jumbo CD above the $9,875,000 price on the spot market will be offset by a profit from his futures position; while on the other hand, any benefits from the costs of the CD being less than the $9,875,000 price would be

TABLE 16.3 Long Hedge Example

Initial Position: Long in 10 September Eurodollar futures contracts at $R_D = 5$ (index = 95, $f_0 = \$987,500$) to hedge \$9,875,000 CD investment in September.

Positions	6%	5%	4%
(1) September spot RD	6%	5%	4%
(2) September spot and futures price	\$985,000	\$987,500	\$990,000
(3) Cost of \$10m face value 90-day CD	\$9,850,000	\$9,875,000	\$9,900,000
(4) Profit on futures	(\$25,000)	0	\$25,000
Net costs: Row (3) – row (4)	\$9,875,000	\$9,875,000	\$9,875,000

Profit on futures = 10 (Spot price – \$987,500)

negated by losses on the Eurodollar futures position. As a result, the manager's costs of buying CDs on the spot and closing his futures position would be \$9,875,000.

The money market manager's long hedge position is shown in Table 16.3. In the table, the third row shows three possible costs of buying the \$10 million face value CD at the September delivery date of \$9,850,000, \$9,875,000 and \$9,900,000 given settlement LIBORs of 6%, 5%, and 4%. The fourth row shows the profits and losses from the long futures position in which the offset position has a contract or cash settlement price (f_T) equal to the spot price (S_T). The last row shows the net costs of \$9,875,000 resulting from purchasing the CDs and closing the futures position. Thus, if the spot Eurodollar discount rate is at 6% at the September delivery date, the manager would pay \$9,850,000 for the jumbo CD and \$25,000 to the clearinghouse to close his futures positions (i.e., the agreement to buy 10 contracts at \$987,500 per contract and the offsetting agreement to sell at \$985,000 means the manager must pay the clearinghouse \$25,000); if the spot Eurodollar rate is 4%, then the manager will have to pay \$9,900,000 for the CD, but will be able to finance part of that expenditure with the \$25,000 received from the clearinghouse from closing (i.e., agreement to buy 10 contracts at \$987,500 and the offsetting agreement to sell at \$990,000 means the clearinghouse will pay the manager \$25,000).

Short Hedge

Short hedges are used when corporations, municipal governments, financial institutions, dealers, and underwriters are planning to sell bonds or borrow funds at some future date and want to lock in the rate. As illustrated in Figure 16.2, if interest rates are higher at the time the fixed-income securities are sold (or the loan starts), then the price on the fixed-income securities will be lower, and as a result, the revenue from selling the fixed-income securities will be less (or the rate on the loan is higher). With a short futures-hedged position, though, the security seller (or borrower) would be able to profit when he closes his short position by going long in a lower priced expiring futures contract. With the profit from the futures, the seller would be able to offset the lower revenue from selling the securities (or defray the additional interest cost of the loan). In contrast, if rates decrease, the revenue from selling the securities at higher prices will be greater (or loan interest cost lower), but the security seller will have to use part of investment cash inflow (interest savings) to cover losses on

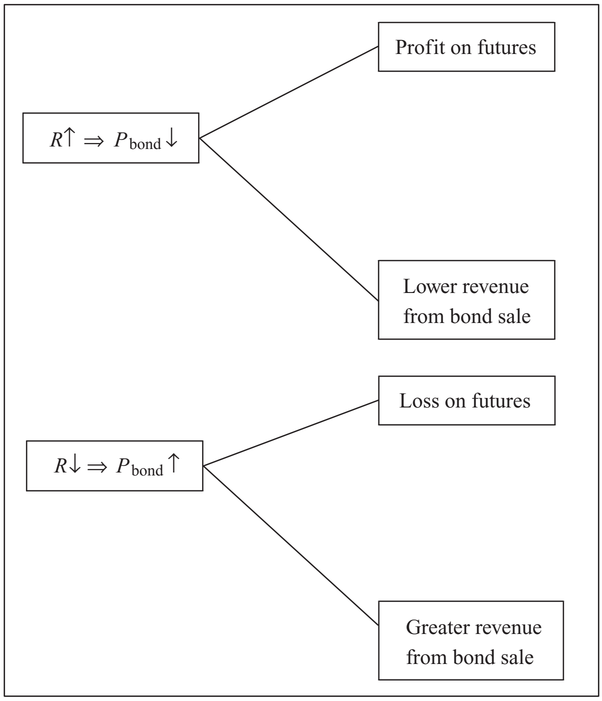


FIGURE 16.2 Short Hedge: Hedging Bond Sale with Short Position in Interest Rate Futures

his futures position. In either interest rate scenario, though, the manager will find less revenue variation from selling securities and closing his futures (or effective rate paid on loans), given his hedged position.

Short Hedge Example: Future T-Bond Sale To illustrate how a short hedge works, consider the case of a fixed-income manager who in July anticipates needing cash in September that she plans to obtain by selling 10 6% T-bonds, each with a face value of $100,000 and currently trading at par. Suppose that the September T-bond futures contract is trading at 100, and at the time of the anticipated September sale, the T-bonds will be at a coupon date with a maturity of exactly 15 years and no accrued interest at that date. If the manager wants to lock in a September selling price on her T-bonds of $100,000 per bond, she could go short in 10 September T-bond futures contracts. At the September expiration, if the ***cheapest-to-deliver bond*** is the 15-year, 6% coupon bond with a conversion factor of 1, then the treasurer would receive $1 million in revenue at delivery from selling her T-bonds on the spot market and closing the futures contract by going long in the expiring September contract trading at price equal to the spot price on the 15-year, 6% T-bond. This can be seen in Table 16.4. In the table, the second row shows three revenue amounts from selling the 10 T-bonds at three possible spot T-bond prices of 95, 100, and 105; the third row shows the profits and losses from the futures position, and the last row shows the hedged revenue from aggregating both positions. For example, at 95, the manager receives only $950,000 from selling her 10 bonds. This lower revenue,

TABLE 16.4 Short Hedge Example

Initial Position: Short in 10 September T-bond futures contracts at $f_0 = 100$ to hedge a September sale of 10 T-bonds.
At the delivery expiration date, the 10 T-bonds each have a maturity of 15 years, no accrued interest, and can be delivered on the futures contracts with a conversion factor of 1.

Positions	95	100	105
(1) September spot and futures price	$95,000	$100,000	$105,000
(2) Revenue from sale of 10 T-bonds	$950,000	$1,000,000	$1,050,000
(3) Profit on futures	$50,000	0	($50,000)
Net revenue: Row (2) + Row (3)	$1,000,000	$1,000,000	$1,000,000

Profit on futures = 10 ($100,000 – Spot price)

though, is offset by $50,000 profit from her futures position; that is, the agreement to sell 10 September T-bonds for $100,000 per bond is closed with an agreement to buy 10 expiring September T-bonds futures for $95,000 per bond, resulting in a $50,000 receipt from the clearinghouse. On the other hand, if the manager is able to sell her 10 bonds for $105,000 per bond, she also will have to pay the clearinghouse $50,000 to close the futures position. Thus, regardless of the spot price, the manager receives $1,000,000 from selling the bonds and closing the futures positions.

Hedging Risk

The above examples represent perfect hedging cases in which certain revenues or costs can be locked in at a future date. In practice, perfect hedges are the exception and not the rule. There are three types of hedging risk that preclude one from obtaining a zero risk position: ***quality risk, timing risk***, and ***quantity risk***.

Quality risk exists when the commodity or asset being hedged is not identical to the one underlying the futures contract. The manager in our long hedge example, for instance, may be planning to invest in commercial paper instead of a Eurodollar CD. In such hedging cases, futures contracts written on a different underlying asset are often used to hedge the spot asset. In this case, the manager could use a Eurodollar futures contract to hedge the CP purchase. Similarly, a portfolio manager planning to buy corporate bonds in the future might hedge the acquisition by going long in T-bond futures. These types of hedges are known as ***cross hedges***. Unlike ***direct hedges*** in which the future's underlying assets are the same as the assets being hedged, cross-hedges minimize price risk, but do not eliminate it.[6] Cross hedging cases are presented in Chapter 18.

Timing risk occurs when the delivery date on the futures contract does not coincide with the date the hedged asset needs to be purchased or sold. For example, timing risk would exist in our long hedging example if the manager needed to invest in Eurodollar CDs on the first of September instead of at the futures' expiration at the end of the September. If the spot asset is purchased or sold at a date that differs from the expiration date on the futures contract, then the price on the futures (f_t) and the spot price (S_t) will not necessarily be equal. The difference between the futures and spot price is called the ***basis*** (B_t). The basis tends to narrow as expiration nears, converging to zero at expiration ($B_T = 0$). Prior to expiration, the basis can vary, with greater variability usually observed the longer the time is to expiration. Given this ***basis risk***, the greater the time difference between buying or selling the hedged asset and the futures' expiration date, the less perfect the hedge. To minimize timing risk or basis risk, hedgers often select futures contracts that mature before the hedged asset is to be bought or sold but as close as possible to that date. For very distant horizon dates, though, hedgers sometimes follow a strategy known as ***rolling the hedge forward***. This hedging strategy involves taking a futures position, then at expiration closing the position and taking a new one. Finally, because of the standardization of futures contracts, futures hedging is subject to quantity risk.

The presence of quality, timing, and quantity risk means that pricing risk cannot be eliminated totally by hedging with futures contracts. As a result, the objective in hedging is to try to minimize risk. Several hedging models try to achieve this objective: naive-hedge, price-sensitivity, minimum variance, and utility-based hedging models. Two of these models and their applications are examined in Chapter 18.

16.5 FUTURES PRICING

The underlying asset price on a futures contract is governed primarily by the spot price of the underlying asset. Theoretically, the relationship between the spot price and the futures or forward price can be explained by the ***carrying-cost model*** (or cost of carry model). In this model, arbitrageurs ensure that the equilibrium forward price is equal to the net costs of carrying the underlying asset to expiration. The model is used to explain what determines the equilibrium price on a forward contract. However, if short-term interest rates are constant, the carrying-cost model can be extended to pricing futures contracts.

In terms of the carrying-cost model, the price difference between futures and spot prices can be explained by the costs and benefits of carrying the underlying asset to expiration. For futures on debt securities, the carrying costs include the financing costs of holding the underlying asset to expiration, and the benefits include the coupon interest earned from holding the security.

Pricing a T-Bill Futures Contract

To illustrate the carrying-cost model, consider the pricing of a T-bill futures contract.[7] With no coupon interest, the underlying T-bill does not generate any benefits during the holding period and the financing costs are the only carrying costs. In terms of the model, the equilibrium relationship between the futures and spot price on the T-bill is

$$f_0 = S_0(1 + R_f)^T \tag{16.2}$$

where f_0 = contract price on the T-bill futures contract
T = time to expiration on the futures contract

S_0 = current spot price on a T-bill identical to the T-bill underlying the futures (maturity = M = 91 and F = \$1 million) except it has a maturity of 91 + T

R_f = risk-free rate or repo rate

$S_0(1 + R_f)^T$ = financing costs of holding a spot T-bill

If Equation (16.2) does not hold, an arbitrage opportunity occurs. The arbitrage strategy is referred to as a ***cash-and-carry arbitrage*** and involves taking opposite positions in the spot and futures contracts. For example, suppose in June there is a September T-bill futures contract expiring in 70 days that is trading when the 161-day spot T-bill and the 70-day repo or risk-free rate are trading at the following price:

- 161-day T-bill is priced at \$97.5844 per \$100 face to yield 5.7%
- 70-day risk-free rate or repo rate is at 6.38%

Using the carrying-cost model, the equilibrium price of the September T-bill futures contract is f_0 = 987,487 or \$98.74875 per \$100 par value:

$$f_0 = S_0(1 + R_f)^T$$

$$f_0 = 97.5844(1.0638)^{70/365} = 98.74875$$

where:

$$S_0 = \frac{100}{(1.057)^{161/365}} = 97.5844$$

If the market price on the T-bill futures contract were not equal to 98.74875, then a cash-and-carry arbitrage opportunity would exist. For example, if the T-bill futures price is at $f_0{}^M$ = 99, an arbitrageur could earn a risk-free profit of \$2,512.50 per \$1 million face value or 0.25125 per \$100 face value (99 – 98.74875) at the expiration date by executing the following strategy:

1. Borrow \$97.5844 at the repo (or borrowing) rate of 6.38%, and then buy a 161-day spot T-bill for $S_0(161)$ = 97.5844
2. Take a short position in a T-bill futures contract expiring in 70 days at the futures price of $f_0{}^M$ = 99

At expiration, the arbitrageur would earn \$0.25125 per \$100 face value (\$2,512.50 per \$1 million par) when she

1. Sells the T-bill on the spot futures contract at 99
2. Repays the principal and interest on the loan of $97.5844(1.0638)^{70/365}$ = 98.74875:

$$\pi_T = f_0^M - f_0^*$$

$$\pi_T = 99 - 97.5844(1.0638)^{70/365}$$

$$= 99 - 98.74875 = .25125$$

$$\pi_T = \frac{0.25125}{100}(\$1{,}000{,}000) = \$2{,}512.50$$

In addition to the arbitrage opportunity when the futures is overpriced at 99, a money-market manager currently planning to invest for 70 days in a T-bill at 6.38% also could benefit with a greater return by creating a synthetic 70-day investment by buying a 161-day bill and then going short at 99 in the T-bill futures contract expiring in 70 days. For example, using the above numbers, if a money market manager were planning to invest 97.5844 for 70 days, she could buy a 161-day bill for that amount and go short in the futures at 99. Her return would be 7.8%, compared to 6.38% from the 70-day spot T-bill:

$$R = \left[\frac{99}{97.5844}\right]^{365/70} - 1 = .078$$

Both the arbitrage and the investment strategies involve taking short positions in the T-bill futures. These actions would therefore serve to move the price on the futures down toward 98.74875.

If the market price on the T-bill futures contract is below the equilibrium value, then the cash-and-carry arbitrage strategy is reversed. In our example, suppose the futures were priced at 98. In this case, an arbitrageur would go long in the futures, agreeing to buy a 91-day T-bill 70 days later, and would go short in the spot T-bill, borrowing the 161-day bill, selling it for 97.5844, and investing the proceeds at 6.38% for 70 days. Seventy days later (expiration), the arbitrageur would buy a 91-day T-bill on the futures for 98 (${f^M}_0$), use the bill to close her short position, and collect 98.74875 (f_0^* from her investment), realizing a cash flow of \$7,487.50 or \$0.74875 per \$100 par:

$$\pi_T = f_0^* - f_0^M$$

$$\pi_T = 97.5844(1.0638)^{70/365} - 98$$

$$= 98.74875 - 98 = .74875$$

$$\pi_T = \frac{.78475}{100}(\$1{,}000{,}000) = \$7{,}487.50$$

In addition to this cash-and carry arbitrage, if the futures price is below 98, a money manager currently holding 161-day T-bills also could obtain an arbitrage by selling the bills for 97.5844, investing the proceeds at 6.38% for 70 days, and then going long in the T-bill futures contract expiring in 70 days. Seventy days later, the manager would receive 98.74875 from the investment and would pay 98 on the futures to reacquire the bills for a cash flow of \$0.74875 per \$100 par.

Other Equilibrium Conditions Implied by the Carrying-Costs Model

For T-bill futures as well as Eurodollar futures contracts, the equilibrium condition defined by the carrying-cost model in Equation (16.2) can be redefined in terms of the following equivalent conditions: (1) the rate on a spot T-bill (or actual repo rate) is equal to the rate on a synthetic T-bill (or implied repo rate); (2) the rate implied on the futures contract is equal to the implied forward rate.

Equivalent Spot and Synthetic T-Bill Rates As illustrated in the above example, a money market manager planning to invest funds in a T-bill for a given short-term horizon either can invest in the spot T-bill or construct a synthetic T-bill by purchasing a longer term T-bill, then locking in its selling price by going short in a T-bill futures contract. In the preceding example, the manager either could buy a 70-day spot T-bill yielding a 6.38% rate of return and trading at $S_0 = 98.821$:

$$S_0 = 100/(1.0638)^{70/365} = 98.821$$

or could create a long position in a synthetic 70-day T-bill by buying the 161-day T-bill trading at $S_0 = 97.5844$, and then locking in the selling price by going short in the T-bill futures contract expiring in 70 days. If the futures price in the market exceeds the equilibrium value as determined by the carrying-cost model ($f_0^M > f_0^*$), then the rate of return on the synthetic T-bill (R_{syn}) will exceed the rate on the spot; in this case, the manager should choose the synthetic T-bill. As we saw, at a futures price of 99, the manager earned a rate of return of 7.8% on the synthetic, compared to only 6.38% from the spot. On the other hand, if the futures price is less than its equilibrium value ($f_0^M < f_0^*$), then R_{syn} will be less than the rate on the spot; in this case, the manager should purchase the spot T-bill.

Note that in an efficient market, money managers will drive the futures price to its equilibrium value as determined by the carrying-cost model. When this condition is realized, R_{syn} will be equal to the rate on the spot and the money manager would be indifferent to either investment. In our example, this occurs when the market price on the futures contract is equal to the equilibrium value of 98.74875. At that price, R_{syn} is equal to 6.38%.

$$R_{syn} = \left[\frac{98.74875}{97.5844}\right]^{365/70} - 1 = .0638$$

Thus, if the carrying-cost model holds, the rate earned from investing in a spot T-bill and the rate from investing in a synthetic T-bill will be equal.

Implied and Actual Repo Rates The rate earned from the synthetic T-bill is commonly referred to as the ***implied repo rate***. Formally, the implied repo rate is defined as the rate where the arbitrage profit from implementing the cash-and-carry arbitrage strategy is zero:

$$\pi = f_0 - S_0(1 + R_f)^T$$

$$0 = f_0 - S_0(1 + R_f)^T$$

$$R = \left[\frac{f_0}{S_0}\right]^{1/T} - 1$$

The actual repo rate is the one we use in solving for the equilibrium futures price in the carrying-cost model; in our example, this was the rate on the 70-day T-bill (6.38%). Thus, the equilibrium condition that the synthetic and spot T-bills be equal can be stated equivalently as the equality between the actual and the implied repo rates.

Implied Forward and Futures Rates The other condition implied by the carrying-cost model is the equality between the rate implied by the futures contract, YTM_f, and the implied forward rate, R_I, first explained in Chapter 2:

$$\text{Implied futures rate} = \text{Implied forward rate}$$

$$\text{YTM}_f = R_I$$

$$\left[\frac{F}{f_0}\right]^{365/91} - 1 = \left[\frac{S(T)}{S(T+91)}\right]^{365/91} - 1$$

where F = face value on the spot T-bill
$T + 91$ = maturity of the spot T-bill

The right-hand side of the above equation is the implied forward rate. This rate is determined by the current spot prices on T-bills maturing at T and at $T + 91$. In our illustrative example, the implied forward rate is 5.18%:

$$R_I = \left[\frac{S(70)}{S(161)}\right]^{365/91} - 1 = \left[\frac{98.821}{97.5844}\right]^{365/91} - 1 = .0518$$

The left-hand side of the equation is the rate implied on the futures contract. If an investor purchases a 91-day T-bill on the futures contract at the equilibrium price, then the implied futures rate will be equal to the implied forward rate. In terms of our example, if $f_0 = 98.74875$, then the implied futures rate will be 5.18%:

$$\text{YTM}_f = \left[\frac{F}{f_0^*}\right]^{365/91} - 1 = \left[\frac{100}{98.74875}\right]^{365/91} - 1 = .0518$$

Recall from Chapter 2, the implied forward rate is the interest rate attained at a future date that is implied by current rates. This rate can be attained by a locking-in strategy consisting of a short position in a shorter term bond and a long position in a longer term one. In terms of our example, the implied forward rate on a 91-day T-bill investment to be made 70 days from the present, $R_I(91,70)$, is obtained by

1. Selling short the 70-day T-bill at 98.821 (or equivalently borrowing 98.821 at 6.38%).
2. Buying $S_0(T)/S_0(T+91) = S_0(70)/S_0(161) = 98.821/97.5844 = 1.01267$ issues of the 161-day T-bill.

3. Paying 100 at the end of 70 days to cover the short position on the maturing bond (or the loan).
4. Collecting 1.01267(100) at the end of 161 days from the long position.

This locking-in strategy would earn an investor a return of $101.267, 91 days after the investor expends $100 to cover the short sale; thus, the implied forward rate on a 91-day investment made 70 days from the present is 1.267%, or annualized, 5.18%:

$$R_I(91,70) = \left[\frac{\$101.267}{\$100}\right]^{365/91} - 1 = .0518$$

If the futures price does not equal its equilibrium value, then the implied forward rate will not be equal to the implied futures rate, and an arbitrage opportunity will exist from the cash-and-carry arbitrage strategy.

In summary, we have three equivalent equilibrium conditions governing futures prices on T-bill and Eurodollar futures contracts: (1) the futures price is equal to the costs of carrying the underlying spot security; (2) the rate on the spot is equal to the rate on the synthetic security (or the implied repo rate is equal to the actual repo rate); (3) the implied rate of return on the futures contract is equal to the implied forward rate.

Equilibrium T-Bond Futures Price

Because of the uncertainty over the T-bond or T-note to be delivered and the time of the delivery created by the delivery procedure, the pricing of a T-bond and T-note futures contracts are more complex than the pricing of T-bill or Eurodollar futures contracts. Appendix G explains the pricing of T-bond futures using the carrying-cost model.

16.6 CONCLUSION

During the 1980s, many countries experienced relatively sharp swings in interest rates. Because of their hedging uses, the market for interest rate futures grew dramatically during this period. Currently, the most popular interest rate futures are T-bond contracts, T-note contracts, and Eurodollar contracts, which have similar features to the T-bill contract. In this chapter, we have examined the characteristics and hedging uses of these contracts. In the next chapter, we examine the market and some of the uses of options on debt securities. Given this foundation, in Chapter 18 we will examine how both derivatives are used to manage fixed-income positions.

KEY TERMS

basis
basis risk
carrying-cost model
cash-and-carry arbitrage
cheapest-to-deliver bond
closing

conversion factor
cross hedges
day traders
direct hedges
dual trading
Eurodollar futures
forward contracts
forward rate agreement (FRA)
futures
futures contract
futures funds
GLOBEX
implied repo rate
initial (or performance) margin
locals
long hedge
long position
marking to market
maintenance (or variation) margin
margin call
offsetting position
open interest
open outcry
position traders
quality risk
quantity risk
reversing out
rolling the hedge forward
scalpers
short hedge
short position
T-bill futures
T-bond futures
timing risk

PROBLEMS AND QUESTIONS

1. Explain the differences between forward and futures contracts.
2. Define and explain the functions provided by futures exchanges.
3. Explain why the price on an expiring futures contract must be equal or approximately equal to the spot price on the contract's underlying asset.
4. What is the major economic justification of the futures market?
5. Define price limits and explain why they are used by the exchanges.
6. Calculate the actual futures prices and implied futures YTM for the following three T-bill futures contracts:

T-Bill Futures Contract	IMM Index
March	93.764
June	93.3092
September	91.8607

7. Suppose you took a short position in a June Eurodollar futures at $R_D = 5.5\%$. Determine the futures settlement prices and your position's profits and losses given the following LIBOR at the June futures' expiration: 4.75%, 5.00%, 5.25%, 5.5%, 5.75%, 6%, and 6.25%. Determine your profits and losses if you had taken a long position in the June contract at $R_D = 5.5\%$.
8. Suppose you took a long position in a September T-bill futures priced at IMM index 95.5. What would be your profit or loss on the position if the price of a spot 91-day T-bill were trading at YTM of 5% (actual/365day-count convention) at the September expiration?

9. Suppose you were long in a June T-bond futures contract at 92-16. What would you have to pay at the futures expiration for a delivered T-bond if the bond's conversion factor was 1.2 and the accrued interests on the deliverable bond were $1.50 per $100 face value?

10. Suppose you were short in a September T-bond futures contract at 93-16. What would your profit or loss be at the September expiration if the cheapest deliverable bond you could purchase in the market were a 15-year, 7% T-bond trading at 115 (clean price) that had accrued interest of $2 and a conversion factor of 1.25?

11. Define a forward rate agreement (FRA). Provide your own example of an FRA.

12. Given an FRA with the following terms:

a. Notional principal = $20 million
b. Reference rate = LIBOR
c. Contract rate = R_k = .05 (annual)
d. Time period = 90 days
e. Day-count convention = Actual/365

Show in a table the payments and receipts for long and short positions on the FRA given possible spot LIBORs at the FRA's expiration of 4%, 4.5%, 5%, 5.5%, and 6%.

13. Explain the similarities and differences between an FRA tied to the LIBOR and a Eurodollar futures contract.

14. Explain how the clearinghouse would record the futures trades in a–d. Include the clearinghouse's payments and receipts needed to close each position.

a. Mr. A buys a September T-bond futures contract from Ms. B for $95,000 on June 20.
b. Mr. D buys a September T-bond futures contract from Mr. E for $94,500 on June 25.
c. Ms. B buys a September T-bond futures from Mr. D for $94,250 on June 28.
d. Mr. E buys a September T-bond futures from Mr. A for $96,000 on July 3.

15. Suppose on March 1 you take a long position in a June T-bill futures contract at R_D = 5%.

a. How much cash or risk-free securities would you have to deposit to satisfy an initial margin requirement of 5%?
b. Calculate the values of your equity account on the following days, given the following discount yields:

March 2	5.1%
March 3	5.2%
March 4	5.0%
March 5	4.8%
March 8	4.7%
March 9	5.0%

c. If the maintenance margin requirement specifies keeping the value of the equity account equal to 100% of the initial margin requirement each day, how much cash would you need to deposit in your commodity account each day?

16. Ms. Hunter is a money market manager. In July, she anticipates needing cash in September that she plans to obtain by selling 10 \$1 million face-value T-bills she currently holds. At the time of the anticipated September sale, the T-bills will have a maturity of 91 days. Suppose there is a September T-bill futures contract trading a discount yield of $R_D = 6\%$.

a. If Ms. Hunter is fearful that short-term interest rates could increase, how could she lock in the selling price on her T-bills?

b. Show in a table Ms. Hunter's net revenue at the futures' expiration date from closing the futures position and selling her 10 T-bills at possible discount yields of 5%, 6%, and 7%. Assume no quality, quantity, or timing risk.

17. Suppose Ms. Hunter anticipates a cash inflow of \$9.875 million in September that she plans to invest in 10 \$1million face-value T-bills with a maturity of 91 days. Suppose there is a September T-bill futures contract trading at a discount yield of $R_D = 5\%$.

a. If Ms. Hunter is fearful that short-term interest rates could decrease, how could she lock in the purchase price on her T-bills?

b. Show in a table Ms. Hunter's net costs at the futures' expiration date from closing the futures position and buying her 10 T-bills at possible discount yields of 4%, 5%, and 6%. Assume no quality, quantity, or timing risk.

18. Cagle Manufacturing forecasts a cash inflow of \$10 million in two months that it is considering investing in a Sun National Bank CD for 90 days. Sun National Bank's jumbo CD pays a rate equal to the LIBOR. Currently such rates are yielding 5.5% (annual rate with an actual/365 day-count convention). Cagle is concerned that short-term interest rates could decrease in the next two months and would like to lock in a rate now. As an alternative to hedging its investment with Eurodollar futures, Sun National suggests that Cagle hedge with a Forward Rate Agreement (FRA).

a. Define the terms of the FRA that would effectively hedge Cagle's futures CD investment.

b. Show in a table the payoffs that Cagle and Sun National would pay or receive at the maturity of the FRA given the following LIBORs: 5%, 5.25%, 5.5%, 5.75%, and 6%.

c. Show in a table Cagle's cash flows from investing the \$10M cash inflow plus or minus the FRA receipts or payments at possible LIBORs of 5%, 5.25%, 5%, 5.75%, and 6%. What is the hedged rate of return Cagle would earn from its \$10 million investment?

19. Suppose there is a Eurodollar futures contract listed on the IMM that expires at the same time as the FRA in Question 18.

a. What would Cagle's FRA equivalent positions be using the Eurodollar futures? What would Cagle's cash flows be from the equivalent Eurodollar futures if the LIBOR were 5% and 6% at expirations?

b. What would Sun National Bank's FRA equivalent positions be using the Eurodollar futures? What would Sun National's cash flows be from the equivalent Eurodollar futures if the LIBOR were 5% and 6% at expirations?
c. Explain how Sun Bank could hedge its FRA in Question 18 by taking a position in the Eurodollar futures contract. Using a table, evaluate their hedge position (net position in Eurodollar plus FRA) at possible LIBORs at the FRA and Eurodollar futures expiration of 5%, 5.25%, 5.5%, 5.75%, and 6%.

20. Briefly comment on the following:
a. The importance of the delivery procedure on futures contracts, even though most futures contracts are closed by offsetting positions.
b. The advantages and disadvantages of price limits.
c. The benefits of futures funds.
d. The role of locals in ensuring a continuous futures market.
e. The basis and its relationship to the time to expiration.
f. Rolling the hedge.

21. Short-Answer Questions:
a. What was the primary factor that contributed to the dramatic growth in futures trading over the last twenty years?
b. What is a hedge called in which the asset underlying the futures contract is not the same as the asset being hedged?
c. A bond manager who hedged her expected sale of 10-year, AA bonds in early September with a CBOT T-bond futures would be subject to what types of hedging risks?
d. How much cash or risk-free securities would a speculator have to deposit in a commodity account if he goes long in one September T-bond futures contract at 95 and the initial margin requirement is 5%?
e. Who ensures that the price on an expiring futures contract is equal or approximately equal to its spot price?
f. What is the number of futures contracts outstanding at a given point in time called?
g. How does a physical futures market provide continuous trading without market makers or specialists?
h. What is the trading referred to when exchange members trade for both their clients and themselves?
i. What is the actual price on a T-bill futures contract, if its quoted IMM index price is 92?
j. What is the implied YTM on a September T-bill futures contract that is trading at 93 (IMM index price)?
k. What is the expiration futures price per $100 face value on an expiring Eurodollar futures contract if the three-month LIBOR is 5% on the expiration day?
l. What is the major difference between Eurodollar and T-bill futures contracts?
m. How much would the actual Eurodollar futures price change, if the IMM index moves one point?
n. What is the implied futures YTM on a March T-bill futures contract trading at 93.75 (IMM index)?

22. Given (1) a 121-day spot T-bill trading at 98.318 to yield 5.25%; (2) a 30-day risk-free rate of 5.15%; (3) a T-bill futures contract with an expiration of T = 30 days:

a. What is the equilibrium T-bill futures price and its implied futures YTM (annualized)?

b. Explain what a money market manager planning to invest funds for 30 days should do if the price on the T-bill futures were trading at 98.8. What rate would the manager earn?

c. Explain the arbitrage a money market manager could execute if she were holding a 121-day T-bill and the T-bill futures were trading at 98.

23. In the table below, the IMM index prices on three T-bill futures contracts with expirations of 91, 182, and 273 days are shown.

T-Bill Contract	Days to Expiration	IMM Index
March	91	93.7640
June	182	93.3092
September	273	91.8607

a. Calculate the actual futures prices and the YTMs (annualized) on the futures.

b. Given the spot 182-day T-bill is trading at annualized YTM of 6.25%, what is the implied 91-day repo rate?

c. If the carrying cost model holds, what would be the price of a 91-day spot T-bill?

d. What would be the equilibrium price on the March contract if the actual 91-day repo rate were 4.75%? What strategy would an arbitrageur pursue if the IMM index price were at 93.764?

WEB EXERCISES

1. Determine the recent prices on listed interest rate futures contracts at the CME Group site: www.cme.com.

2. Determine the carrying-cost value of a listed Eurodollar futures contract. For current market quotes and information, go to www.cme.com and www.wsj.com/free.

NOTES

1. As a matter of security law, dual traders are not allowed to trade on their own accounts when they are about to trade for their clients. With advance knowledge of a client's position, a dual trader could profit by taking a favorable position before executing the client's order. This type of price manipulation is known as front running.
2. To avoid exchanging principals, the FRA evolved from forward-forward contracts in which international banks would enter an agreement for a future loan at a specified rate.

3. Different from financial futures, FRAs are contracts between two parties and therefore are subject to the credit risk of either party defaulting.
4. On futures contracts, the settlement price is determined by the clearinghouse officials and is based on the average price of the last several trades of the day.
5. Clearinghouse members are also required to maintain a margin account with the clearinghouse. This is known as a clearing margin.
6. Cross-hedging can occur when an entire group of assets or liabilities are hedged by one type of futures contract; this is referred to as macro hedging. Micro hedging, on the other hand, occurs when each individual asset or liability is hedged separately.
7. Note: T-bill futures are often delisted. However, the more popular Eurodollar futures contracts are similar to T-bills except for their cash settlement features.

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CHAPTER 17

Interest Rate Options Contracts: Fundamentals

17.1 INTRODUCTION

Like the futures market, the options market in the United States can be traced back to the 1840s when options on corn meal, flour, and other agriculture commodities were traded in New York. These options contracts gave the holders the right, but not the obligation, to purchase or to sell a commodity at a specific price on or possibly before a specified date. Like futures contracts, options made it possible for farmers or agriculture dealers to lock in future prices. In contrast to commodity futures trading, though, the early market for commodity options trading was relatively thin. The market did grow marginally when options on stocks began trading on the over-the-counter (OTC) market in the early 1900s. This market began when a group of investment firms formed the Put and Call Brokers and Dealers Association. Through this association, an investor who wanted to buy an option could do so through a member who either would find a seller through other members or would sell (write) the option himself.

The OTC option market was functional, but suffered because it failed to provide an adequate secondary market. In 1973, the Chicago Board of Trade formed the Chicago Board Options Exchange (CBOE). The CBOE was the first organized options exchange for the trading of options. Just as the CBOT had served to increase the popularity of futures, the CBOE helped to increase the trading of options by making the contracts more marketable.

Since the creation of the CBOE, organized stock exchanges, such as the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), the Philadelphia Stock Exchange (PHLX), and the Pacific Stock Exchange (PSE); most of the organized futures exchanges; and many security exchanges outside the United States also began offering markets for the trading of options. As the number of exchanges offering options increased, so did the number of securities and instruments with options written on them. Today, options contracts exist not only on stocks but also on foreign currencies, security indexes, futures contracts, and of particular interest here, debt and interest rate-sensitive securities.

In this chapter, we continue our discussion of derivative debt securities by examining options contracts on interest-sensitive securities. In this chapter, we define some of the common option terms, examine the fundamental option strategies, and identify some of the important factors that determine the price of an option.

WEB INFORMATION

Information on the CBOE: www.cboe.com.
See Exhibit 16.1 for links to the Web sites of other exchanges.

17.2 OPTIONS TERMINOLOGY

Spot Options

By definition, an option is a security that gives the holder the right to buy or sell a particular asset at a specified price on, or possibly before, a specific date. Depending on the parties and types of assets involved, options can take on many different forms. Certain features, however, are common to all options. First, with every options contract there is a right, but not an obligation, to either buy or sell. Specifically, by definition a ***call*** is the right to buy a specific asset or security, whereas a ***put*** is the right to sell. Every options contract has a buyer who is referred to as the option ***holder*** (who has a ***long position*** in the option). The holder buys the right to ***exercise*** or evoke the terms of the option claim. Every option also has a seller, often referred to as the option ***writer*** (and having a ***short position***), who is responsible for fulfilling the obligations of the option if the holder exercises. For every option there is an option price, exercise price, and exercise date. The price paid by the buyer to the writer when an option is created is referred to as the ***option premium*** (call premium and put premium). The ***exercise price*** or ***strike price*** is the price specified in the options contract at which the asset or security can be purchased (call) or sold (put). Finally, the ***exercise date*** is the last day the holder can exercise. Associated with the exercise date are the definitions of European and American options. A ***European option*** is one that can be exercised only on the exercise date, whereas an ***American option*** can be exercised at any time on or before the exercise date.

Futures Options

Options contracts on stocks, debt securities, foreign currency, and indexes are sometimes referred to as ***spot options*** or options on actuals. This reference is to distinguish them from ***options on futures*** contracts (also called options on futures, futures options, and commodity options). A futures option gives the holder the right to take a position in a futures contract. Specifically, a call option on a futures contract gives the holder the right to take a long position in the underlying futures contract when she exercises, and requires the writer to take the corresponding short position in the futures. Upon exercise, the holder of a futures call option in effect takes a long position in the futures contract at the *current* futures price and the writer takes the short position and pays the holder via the clearinghouse the difference between the current futures price and the exercise price. In contrast, a put option on a futures option entitles the holder to take a short futures position and the writer the long position. Thus, whenever the put holder exercises, she in effect takes a short futures position at the current futures price and the writer takes the long position and pays the holder via the clearinghouse the difference between the exercise price and the current futures price. Like all option positions, the futures option buyer pays an option premium for the right to exercise, and the writer, in turn, receives a credit when he sells the option and is subject to initial and maintenance margin requirements on the option position.

In practice, when the holder of a futures call option exercises, the futures clearinghouse will establish for the exercising option holder a long futures position at the futures price equal to the exercise price and a short futures position for the assigned writer. Once this is done, margins on both positions will be required and the position will be marked to market at the current settlement price on the futures. When the positions are marked to market, the exercising call holder's margin account on her long position will be equal to the difference between the futures price and the exercise price, $f_t - X$, whereas the assigned writer will have to deposit funds worth $f_t - X$ to satisfy his maintenance margin on his short futures position. Thus, when a futures call is exercised, the holder takes a long position at f_t with a margin account worth $f_t - X$; if she were to immediately close the futures she would receive cash worth $f_t - X$ from the clearinghouse. The assigned writer, in turn, is assigned a short position at f_t and must deposit $f_t - X$ to meet his margin. If the futures option is a put, the same procedure applies except that holder takes a short position at f_t (when the exercised position is marked to market), with a margin account worth $X - f_t$, and the writer is assigned a long position at f_t and must deposit $X - f_t$ to meet his margin.

The current U.S. market for futures options began in 1982 when the Commodity Futures Trading Commission (CFTC) initiated a pilot program in which it allowed each futures exchange to offer one option on one of its futures contracts. In 1987, the CFTC gave the exchanges permanent authority to offer futures options. Currently, the most popular futures options are the options on the financial futures on the S&P 500, T-bond, T-note, Eurodollar deposit, and the major foreign currencies. In addition to options on financial futures contracts, futures options also are available on gold, precious metals, agriculture commodities, and energy products.[1]

It should be noted that spot options and futures options are equivalent if the options and the futures contracts expire at the same time, the carrying-costs model holds, and the options are European. There are, though, several factors that serve to differentiate the two contracts. First, since many futures contracts are relatively more liquid than their corresponding spot security, it is usually easier to form hedging or arbitrage strategies with futures options than with spot options. Second, futures options often are easier to exercise than their corresponding spot. For example, to exercise an option on a T-bond futures contract, one simply assumes the futures position, whereas exercising a spot T-bond option requires an actual purchase or delivery. Finally, most futures options are traded on the same exchange as their underlying futures contract, whereas most spot options are traded on exchanges different from their underlying securities. This, in turn, makes it easier for futures options traders to implement arbitrage and hedging strategies than spot options

WEB INFORMATION

For market information and prices on futures options, go to
CME: www.cme.com.

17.3 MARKETS AND TYPES OF INTEREST RATE OPTIONS

Markets

Many different types of interest rate options are available on both the OTC market and the organized futures and options exchanges in Chicago, London, Singapore and other major cities. Exchange-traded interest rate options include both futures options and spot options. On the U.S. exchanges, the most heavily traded options are the CME's and CBOT's futures options on T-bonds, T-notes, and Eurodollar contracts. The CBOE, AMEX, and PHLX have offered options on actual Treasury securities and Eurodollar deposits. These spot options, however, proved to be less popular than futures options and have been delisted. A number of non-U.S. exchanges, though, do list options on actual debt securities, typically government securities. For example, options listed on the European Options Exchange are all spot options. Exhibit 16.1 lists futures and options exchanges with their Web sites.

In addition to exchange-traded options, there is also a large OTC market in debt and interest-sensitive securities and products in the United States and a growing OTC market outside the United States. Currently, security regulations in the United States prohibit off-exchange trading in options on futures. All U.S. OTC options are therefore options on actuals. The OTC markets in and outside the United States consist primarily of dealers who make markets in the underlying spot security, investment banking firms, and commercial banks. OTC options are primarily used by financial institutions and nonfinancial corporations to hedge their interest rate positions. The options contracts offered in the OTC market include spot options on Treasury securities, LIBOR-related securities, and special types of interest rate products, such as interest rate calls and puts, caps, floors, and collars.

Types of Interest Rate Options

As noted, on the organized exchanges the most heavily traded exchange-traded options are futures options on T-bonds, T-notes, T-bills, and Eurodollar contracts. On the OTC market, the most popular interest rate options include options on spot Treasury securities and caps and floors.

T-Bond Futures Options The CBT offers trading on futures options on T-bonds and T-notes with maturities of 10 years, five years, and two years, as well contracts on the Municipal Bond Index, a mortgage-backed bond contract, and other financial futures. The premiums on the call and put options contracts on the T-bond and T-note futures are quoted as a percentage of the face value of the underlying bond or note. For example, a buyer of May 109 T-bond futures call trading at 2–60 (or 2 60/64 = 2.9375) would pay $2,937.50 for the option to take a long position in the May T-bond futures at an exercise price of $109,000. If long-term rates were to subsequently drop, causing the May T-bond futures price to increase to $f_t = 113$, then the holder, upon exercising, would have a long position in the May T-bond futures contract and a margin account worth

\$4,000. If she closed her futures contract at 113, she would have a profit of \$1,062.50:

$$\text{Value of margin} = \frac{f_t - X}{100}(\$100{,}000) = \left[\frac{113 - 109}{100}\right]\$100{,}000 = \$4{,}000$$

$$\pi = \$4{,}000 - \$2{,}937.50 = \$1{,}062.50$$

By contrast, if long-term rates were to stay the same or increase, then the call would be worthless and the holder would simply allow it to expire, losing the \$2,937.50 premium.

Eurodollar and T-Bill Futures Options The CME offers trading on short-term interest rate futures options on Eurodollar deposits, 30-day LIBOR contracts, and, when listed, T-bill futures. The maturities of the options correspond to the maturities on the underlying futures contracts, and the exercise quotes are based on the system used for quoting the futures contracts. Thus, the exercise prices on the Eurodollar and T-bill contracts are quoted in terms of an index (I) equal to 100 minus the annual discount yield: $I = 100 - R_D$. The option premiums are quoted in terms of an index point system. For T-bills and Eurodollars, the dollar value of an option quote is based on a \$25 value for each basis point underlying a \$1 million T-bill or Eurodollar. The actual quotes are in percents; thus a 1.25 quote would imply a price of \$25 times 12.5 basis points: (\$25)(12.5) = \$312.50. In addition, for the closest maturing month, the options are quoted to the nearest quarter of a basis point; for other months, they are quoted to the nearest half of a basis point. For example, the actual price on a March Eurodollar call with an exercise price of 94.5 quoted at 5.92 is \$1,481.25. The price is obtained by rounding the 5.92 quoted price to 5.925, converting the quote to basis points (multiply by 10), and multiplying by \$25: (5.925)(10)(\$25) = \$1,481.25. A 10.30 quote on a 94.5 April call indicates a call price (10.30)(10)(\$25) = \$2,575 [or simply, (10.30)(\$250) = \$2,575].

An investor buying the 94.5 March call would therefore pay \$1,481.25 for the right to take a long position in the CME's \$1 million March Eurodollar futures contract at an exercise price of \$986,250:

$$X = \frac{100 - (100 - 94.5)(90/360)}{100}\$1{,}000{,}000 = \$986{,}250$$

If short-term rates were to subsequently drop, causing the March Eurodollar futures price to increase to an index price of 95.5 ($R_D = 4.5$ and $f_t = [[100 - 4.5(90/360)]/100](\$1{,}000{,}000) = \$988{,}750$), the holder, upon exercising, would have a long position in the CME March Eurodollar futures contract and a futures margin account worth \$2,500. If she closed the position at 95.5, she would realize a profit of \$1,018.75:

$$\begin{aligned}\text{Margin value} &= f_t - X = \$988{,}750 - \$986{,}250 = \$2{,}500\\ \text{Margin value} &= \$25\,(\text{Futures index} - \text{Exercise index}) = \$25[95.5 - 94.5](100)\\ &= \$2{,}500\\ \pi &= \$2{,}500 - \$1{,}481.25 = \$1{,}018.75\end{aligned}$$

If short-term rates were at $R_D = 5.5\%$ and stayed there or increased, then the call would be worthless and the holder would simply allow the option to expire, losing her $1,481.25 premium.

OTC Options

Although OTC options can be structured on almost any interest-sensitive position an investor or borrower may wish to hedge, U.S. Treasuries, LIBOR-related instruments, and mortgage-backed securities are often the underlying security. When spot options are structured on securities, terms such as the specific underlying security, its maturity and size, the option's expiration, and the delivery are all negotiated. For Treasuries, the underlying security is often a recently auctioned Treasury (on-the-run bond), although some selected existing securities (off-the-run securities) are used. Although the prices on OTC options tend to conform to the basic option pricing relation (discussed in Section 17.6), their bid-ask spreads tend to be larger than exchange-traded ones. The option maturities on OTC contracts can range from one day to several years, with many of the options being European.

OTC T-Bond and T-Note Options In the case of OTC spot T-bond or T-note options, OTC dealers often offer or will negotiate contracts giving the holder to right to purchase or sell a specific T-bond or T-note. For example, a dealer might offer a T-bond call option to a fixed income manager giving him the right to buy a specific T-bond maturing in the year 2016 and paying a 6% coupon with a face value of $100,000. Because the options contract specifies a particular underlying bond, the maturity of the bond, as well as its value, will be changing during the option's expiration period. For example, a one-year call option on a 15-year bond, if held to expiration, would be a call option to buy a 14-year bond. Note that in contrast, a spot T-bill options contract offered by a dealer on the OTC market usually calls for the delivery of a T-bill meeting the specified criteria (e.g., principal = $1 million, maturity = 91 days). With this clause, a T-bill option is referred to as a ***fixed deliverable bond***, and unlike specific-security T-bond options, T-bill options can have expiration dates that exceed the T-bill's maturity.

A second feature of a spot T-bond or T-note option offered or contracted on the OTC market is that the underlying bond or note can pay coupon interest during the option period. As a result, if the option holder exercises on a non-coupon paying date, the accrued interest on the underlying bond must be accounted for. For a T-bond or T-note option, this is done by including the accrued interest as part of the exercise price. Like futures options, the exercise price on a spot T-bond or T-note option is quoted as an index equal to a proportion of a bond with a face value of $100 (e.g., 95). If the underlying bond or note has a face value of $100,000, then the exercise price would be

$$X = \left[\frac{\text{Index}}{100}\right](\$100{,}000) + \text{Accrued interest}$$

Finally, the prices of spot T-bond and T-note options are typically quoted like futures T-bond options in terms of points and 32nds of a point. Thus, the price of a call option on a $100,000 T-bond quoted at 1 5/32 is $1,156.25 [= (1.15625/100)($100,000)].

Interest Rate Call

In addition to options contracts on specific securities, the OTC market also offers a number of interest-rate options products. These products are usually offered by commercial or investment banks to their clients. Two products of note are the interest rate call and the interest rate put. An ***interest rate call***, also called a ***caplet***, gives the buyer a payoff on a specified payoff date if a designated interest rate, such as the LIBOR, rises above a certain exercise rate, R_x. On the payoff date, if the rate is less than R_x, the interest rate call expires worthless; if the rate exceeds R_x, the call pays off the difference between the actual rate and R_x, times a notional principal, NP, times the fraction of the year specified in the contract. For example, given an interest rate call with a designated rate of LIBOR, $R_x = 6\%$, NP = $1 million, time period of 180 days, and day count convention of 180/360, the buyer would receive a $5,000 payoff on the payoff date if the LIBOR were 7%: (.07 −.06)(180/360) ($1,000,000) = $5,000.

Interest rate call options are often written by commercial banks in conjunction with futures loans they plan to provide to their customers. The exercise rate on the option usually is set near the current spot rate, with that rate often being tied to the LIBOR. For example, a company planning to finance a future $10 million inventory 60 days from the present by borrowing from a bank at a rate equal to the LIBOR + 100 basis points at the start of the loan could buy from the bank an interest rate call option with an exercise rate equal to, say, 8%, expiration of 60 days, and notional principal of $10 million. At expiration (60 days later) the company would be entitled to a payoff if rates were higher than 8%. Thus, if the rate on the loan were higher than 8%, the company would receive a payoff that would offset the higher interest on the loan.

Interest Rate Put

An ***interest rate put***, also called a ***floorlet***, gives the buyer a payoff on a specified payoff date if a designated interest rate is below the exercise rate, R_x. On the payoff date, if the rate is more than R_x, the interest rate put expires worthless; if the rate is less than R_x, the put pays off the difference between R_x and the actual rate times a notional principal, NP, times the fraction of the year specified in the contract. For example, given an interest rate put with a designated rate of LIBOR, $R_x = 6\%$, NP = $1 million, time period of 180 days, and day-count of 180/360, the buyer would receive a $5,000 payoff on the payoff date if the LIBOR were 5%: (.06 – .05)(180/360)($1,000,000) = $5,000.

A financial or nonfinancial corporation that is planning to make an investment at some future date could hedge that investment against interest rate decreases by purchasing an interest rate put from a commercial bank, investment banking firm, or dealer. For example, suppose that instead of needing to borrow $10 million, the previous company was expecting a net cash inflow of $10 million in 60 days from its operations and was planning to invest the funds in a 90-day bank CD paying the LIBOR. To hedge against any interest rate decreases, the company could purchase an interest rate put (corresponding to the bank's CD it plans to buy) from the bank with the put having an exercise rate of, say, 7%, expiration of 60 days, and notional principal of $10 million. The interest rate put would provide a payoff for the company if the LIBOR were less than 7%, giving the company a hedge against interest rate decreases.

Cap

A popular option offered by financial institutions in the OTC market is the ***cap***. A plain-vanilla cap is a series of European interest rate call options—a portfolio of caplets. For example, a 7%, two-year cap on a three-month LIBOR, with an NP of $100 million, provides, for the next two years, a payoff every three months of (LIBOR − .07)(.25)($100,000,000) if the LIBOR on the reset date exceeds 7%, and nothing if the LIBOR equals or is less than 7%. (Typically, the payment is not on the reset date, but rather on the next reset date three months later.) Caps are often written by financial institutions in conjunction with a floating-rate loan and are used by buyers as a hedge against interest rate risk. For example, a company with a floating-rate loan tied to the LIBOR could lock in a maximum rate on the loan by buying a cap corresponding to its loan. At each reset date, the company would receive a payoff from the caplet if the LIBOR exceeded the cap rate, offsetting the higher interest paid on the floating-rate loan; on the other hand, if rates decrease, the company would pay a lower rate on its loan while its losses on the caplet would be limited to the cost of the option. Thus, with a cap, the company would be able to lock in a maximum rate each quarter and still benefit with lower interest costs if rates decrease.

Floor

A plain-vanilla ***floor*** is a series of European interest rate put options—a portfolio of floorlets. For example, a 7%, two-year floor on a three-month LIBOR, with an NP of $100 million, provides, for the next two years, a payoff every three months of (.07 − LIBOR)(.25)($100,000,000) if the LIBOR on the reset date is less than 7%, and nothing if the LIBOR equals or exceeds 7%. Floors are often purchased by investors as a tool to hedge their floating-rate investments against interest rate declines. Thus, with a floor, an investor with a floating-rate security is able to lock in a minimum rate each period, while still benefiting from higher yields if rates increase.

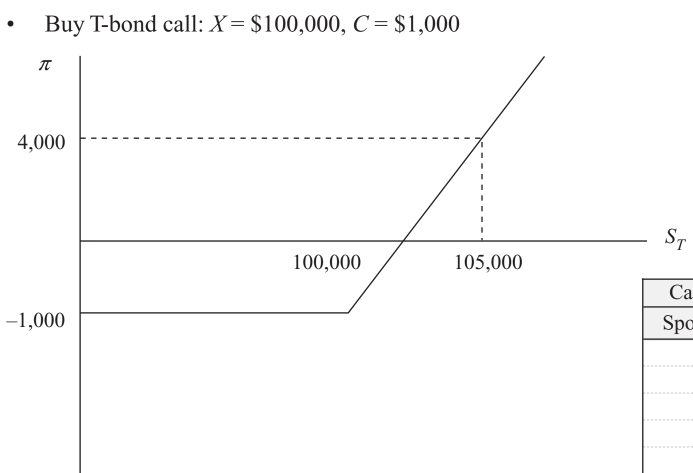
17.4 OPTION POSITIONS

Many types of options strategies with esoteric names such as straddles, strips, spreads, combinations, and so forth, exist. The building blocks for these strategies are four fundamental options strategies: call and put purchases and call and put writes. The features of these fundamental strategies can be seen by examining the relationship between the price of the underlying security and the possible profits or losses that would result if the option either is exercised or expires worthless.[2]

Fundament Spot Option Positions

Call Purchase To see the major characteristics of a call purchase, suppose an investor buys a spot call option on a 6% T-bond with a face value of $100,000, a 15-year maturity at the option's expiration, no accrued interest at the option's expiration date, and currently selling at par. Suppose the T-bond's exercise price (X) is $100,000 (quoted at 100) and the investor buys the options at a call premium of C_0 = $1,000 (quoted at 1). If the bond price reaches $105,000 at expiration, the holder would realize a profit of $4,000 by exercising the call to acquire the bond for $100,000, then selling the bond in the market for $105,000: a $5,000 capital gain minus the $1,000 premium. If the holder exercises at expiration when the bond is trading at $101,000, she will break even: The $1,000 premium will be offset exactly by the $1,000 gain realized by acquiring the bond from the option at $100,000 and selling in the market at $101,000. Finally, if the price of the bond is at the exercise price of $100,000 or below, the holder will not find it profitable to exercise, and as a result, she will let the option expire, realizing a loss equal to the call premium of $1,000. Thus, the maximum loss from the call purchase is $1,000.

The investor's possible profit/loss and bond price combinations can be seen graphically in Figure 17.1 and the accompanying table. In the graph, the profits/losses are shown on the vertical axis and the market prices of the spot T-bond, S, at expiration or when the option is exercised (signified as T: S_T) are shown along the horizontal axis. This graph is known as a ***profit graph***. The line from the ($100,000, −$1,000) coordinate to the ($105,000, $4,000) coordinate and beyond shows all the profit and losses per call associated with each bond price. The horizontal segment shows a loss of $1,000 that is equal to the premium paid when the option was purchased. Finally, the horizontal intercept is the break-even price ($101,000). The break-even price can be found algebraically by solving for the bond price at



Call purchase	
Spot price at T	Profit/Loss
90,000	–1,000
95,000	–1,000
100,000	–1,000
101,000	0
102,000	1,000
103,000	2,000
104,000	3,000
105,000	4,000
106,000	5,000

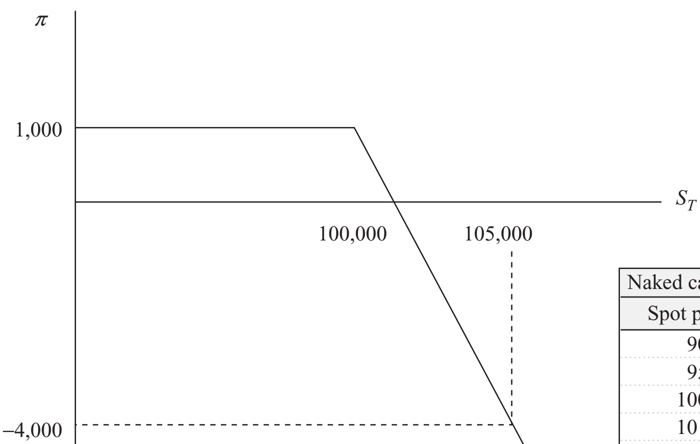
FIGURE 17.1 Call Purchase

the exercise date (S_T) in which the profit (π) from the position is zero. The profit graph with Figure 17.1 highlights two important features of call purchases. First, the position provides an investor with unlimited profit potential; second, losses are limited to an amount equal to the call premium.

Naked Call Write The second fundamental strategy involves the sale of a call in which the seller does not own the underlying security. Such a position is known as a ***naked call write***. To see the characteristics of this position, assume the same spot T-bond call option with an exercise price of \$100,000 and premium of \$1,000. The profits or losses associated with each bond price from selling the call are depicted in Figure 17.2. As shown, when the price of the bond is at \$105,000 at expiration, the seller suffers a \$4,000 loss when the holder exercises the right to buy the bond from the writer at \$100,000. Since the writer does not own the bond, he would have to buy it in the market at its market price of \$105,000, and then turn it over to the holder at \$100,000. Thus, the call writer would realize a \$5,000 capital loss, minus the \$1,000 premium received for selling the call, for a net loss of \$4,000. When the T-bond is at \$101,000, the writer will realize a \$1,000 loss if the holder exercises. This loss will offset the \$1,000 premium received. Thus, the break-even price for the writer is \$101,000—the same as the holder's. Finally, at a bond price of \$100,000 or less, the holder will not exercise, and the writer will profit by the amount of the premium, \$1,000.

As highlighted in the graph, the payoffs to a call write are just the opposite of a call purchase; that is: gains/losses for the buyer of a call are exactly equal to the losses/gains of the seller. Thus, in contrast to the call purchase, the naked call write position provides the investor with only a limited profit opportunity equal to the

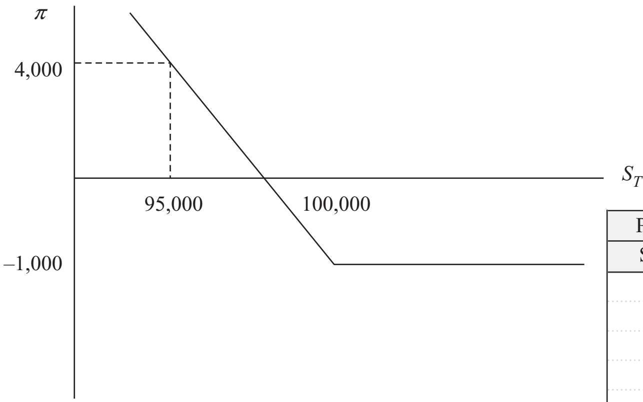
- Sell T-bond call: X = \$100,000, C = \$1,000



Naked call write	
Spot price at T	Profit/Loss
90,000	1,000
95,000	1,000
100,000	1,000
101,000	0
102,000	–1,000
103,000	–2,000
104,000	–3,000
105,000	–4,000
106,000	–5,000

FIGURE 17.2 Naked Call Write

- Buy T-bond put: $X = \$100,000$, $P = \$1,000$



Put purchase	
Spot price at T	Profit/Loss
90,000	9,000
95,000	4,000
100,000	–1,000
101,000	–1,000
102,000	–1,000
103,000	–1,000
104,000	–1,000
105,000	–1,000
106,000	–1,000

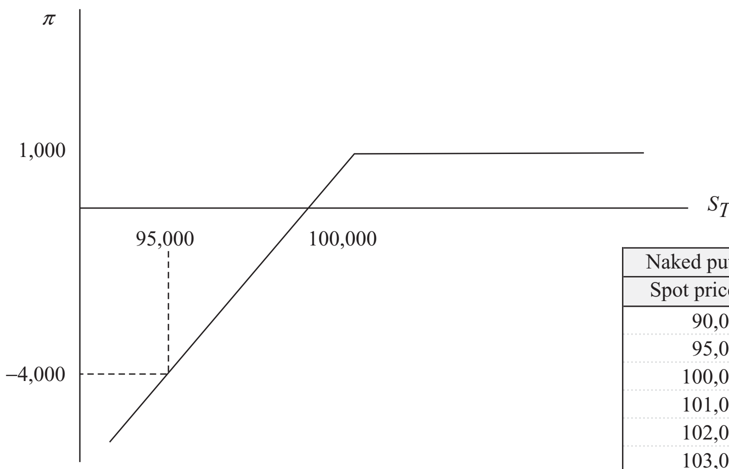
FIGURE 17.3 Put Purchase

value of the premium, with unlimited loss possibilities. Although this limited profit and unlimited loss feature of a naked call write may seem unattractive, the motivation for an investor to write a call is the cash or credit received and the expectation that the option will not be exercised. Like futures contracts, though, there are margin requirements on an option write position in which the writer is required to deposit cash or risk-free securities to secure the position.

Put Purchase Since a put gives the holder the right to sell the underlying security, profit is realized when the security's price declines. With a decline, the put holder can buy the security at a low price in the market, and then sell it at the higher exercise price on the contract. To see the features related to the put purchase position, assume the exercise price on a put option on the 6% T-bond is again $100,000 and the put premium (*P*) is $1,000. If the T-bond is trading at $95,000 at expiration, the put holder could purchase a 15-year, 6% T-bond at $95,000, then use the put contract to sell the bond at the exercise price of $100,000. Thus, as shown by the profit graph in Figure 17.3 and its accompanying table, at $95,000 the put holder would realize a $4,000 profit (the $5,000 gain from buying the bond and exercising minus the $1,000 premium). The break-even price in this case would be $99,000. Finally, if the T-bond is trading at $100,000 or higher at expiration, it will not be rational for the put holder to exercise. As a result, a maximum loss equal to the $1,000 premium will occur when the stock is trading at $100,000 or more (again assuming no accrued interest at expiration).

Thus, similar to a call purchase, a long put position provides the buyer with potentially large profit opportunities (not unlimited becuase the price of the security cannot be less than zero), while limiting the losses to the amount of the premium.

- Sell T-bond put: X = \$100,000, P = \$1,000



Naked put write	
Spot price at T	Profit/Loss
90,000	–9,000
95,000	–4,000
100,000	1,000
101,000	1,000
102,000	1,000
103,000	1,000
104,000	1,000
105,000	1,000
106,000	1,000

FIGURE 17.4 Naked Put Write

Unlike the call purchase strategy, the put purchase position requires the security price to decline before profit is realized.

Naked Put Write The exact opposite position to a put purchase (in terms of profit/loss and security price relations) is the sale of a put, defined as the naked put write. This position's profit graph is shown in Figure 17.4. Here, if the T-bond price is at \$100,000 or more at expiration, the holder will not exercise and the writer will profit by the amount of the premium, \$1,000. In contrast, if the T-bond decreases, a loss is incurred. For example, if the holder exercises at \$95,000, the put writer must buy the bond at \$95,000. An actual \$5,000 loss will occur if the writer elects to sell the bond and a paper loss if he holds on to it. This loss, minus the \$1,000 premium, yields a loss of \$4,000 when the market price is \$95,000. For this naked put write position, the break-even price in which the profit from the position is zero is \$99,000, the same as the put holder's.

Fundamental Futures Options Positions

The important characteristics of futures options can also be seen by examining the profit relationships for the fundamental call and put positions formed with these options. Figure 17.5 shows the profit and futures price relationship at expiration for the long call position on a Eurodollar futures contract. The call has an exercise price equal to 90 (index) or X = \$975,000, is priced at \$1,250 [quote of 5: (5)(10)(\$25) = \$1,250] and it is assumed the Eurodollar futures option expires at the same time as the underlying Eurodollar futures contract. The numbers shown in the figure reflect a case in which the holder exercises the call at expiration, if profitable, when

Call on Eurodollar futures:

- X = IMM 90 or X = \$975,000
- Premium quote = 5, C = \$1,250
- Futures and futures options have same expiration

R_D	$S_T = f_T$	$\pi_C = \text{Max}\,(f_T - 975{,}000, 0)$
		−1,250
10.5	973,750	−1,250
10.0	975,000	−1,250
9.5	976,250	0
9.0	977,500	1,250
8.5	978,750	2,500
8.0	980,000	3,750

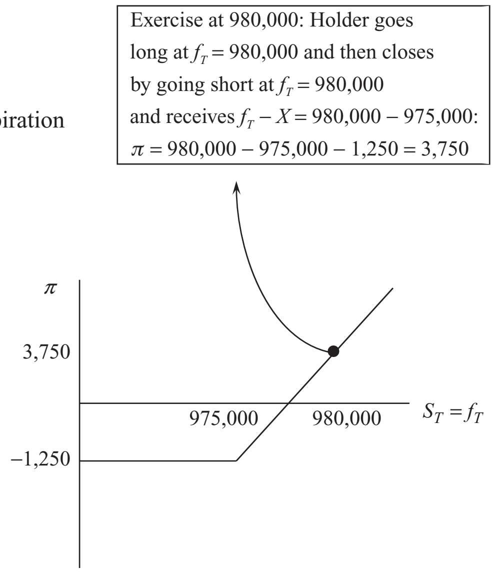


FIGURE 17.5 Futures Options Call on Eurodollars

the spot price is equal to the price on the expiring futures contract. For example, at $S_T = f_T$ = \$980,000, the holder of the 90 Eurodollar futures call would receive a cash flow of \$5,000 for a profit of \$3,750 (= \$5,000 – \$1,250). That is, upon exercising the holder would assume a long position in the expiring Eurodollar futures priced at \$980,000 and a futures margin account worth \$5,000 [$(f_T - X)$ = \$980,000 – \$975,000 = \$5,000]. Given we are at expiration, the holder would therefore receive \$5,000 from the expired futures position, leaving her with a profit of \$3,750. The opposite profit and futures price relation is attained for a naked call write position. In this case, if the Eurodollar futures is at \$975,00 or less, the writer of the futures call would earn the premium of \$1,250, and if $f_T >$ \$975,000, he, upon the exercise by the holder, would assume a short position at f_T and would have to pay $f_T - X$ to bring the margin on his expiring short position into balance.

Figure 17.6 shows a long position on the 90 Eurodollar futures put purchased at \$1,250. In the case of a put purchase, if the holder exercises when f_T is less than X, then he will have a margin account worth $X - f_T$ on an expiring short futures position. For example, if $S_T = f_T$ = \$970,000 at expiration, then the put holder upon exercising would receive \$5,000 from the expiring short futures ($X - f_T$ = \$975,000 – \$970,000), yielding a profit from her futures option of \$3,750. The put writer's position, of course, would be the opposite.

It should be noted that even though the technicalities on exercising futures options are cumbersome, the profits from closing a futures option at expiration still are equal to the maximum of either zero or the difference in $f_T - X$ (for calls) or $X - f_T$ (for puts), minus the option premium. Moreover, if the futures option and the underlying futures contract expire at the same time, as we assumed above, then $f_T = S_T$, and the futures option can be viewed simply as an option on the underlying spot security with the option having a cash settlement clause.

Put on Eurodollar futures:

- X = IMM 90 or X = \$975,000
- Premium quote = 5, P = \$1,250
- Futures and futures options have same expiration

R_D	$S_T = f_T$	$\pi_P = (975{,}000 - f_T, 0) - 1{,}250$
12.0	970,000	3,750
11.5	971,250	2,500
11.0	972,500	1,250
10.5	973,750	0
10.0	975,000	−1,250
9.5	976,250	−1,250
9.0	977,500	−1,250
8.5	978,750	−1,250
8.0	980,000	−1,250

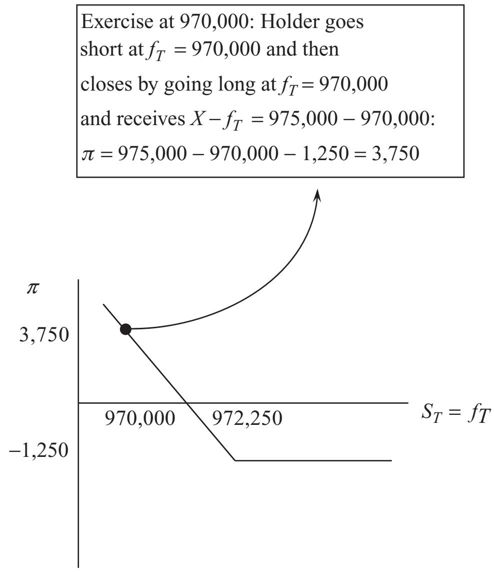


FIGURE 17.6 Futures Options Put on Eurodollars

Fundamental Interest Rate Call and Put Positions

The profit graphs for interest rate calls and puts can be defined in terms of the profit and interest rate relations for the option. Figure 17.7 shows the profit graph and table for an interest rate call with the following terms: Exercise rate = 6%, reference rate = LIBOR, NP = \$10 million, time period as proportion of a year = .25, and the cost of the option = \$12,500. As shown in the figure, if the LIBOR reaches 7.5% at expiration, the holder would realize a payoff of [(.075 – .06)(\$10,000,000)(.25) =] \$37,500 and a profit of \$25,000; if the LIBOR is 6.5%, the holder would break even with the \$12,500 payoff equal to the option's cost; if the LIBOR is 6% or less, there would be no payoff and the holder would incur a loss equal to the call premium of \$12,500. Just the opposite relationship between profits and rates exists for an interest rate put. Figure 17.8 shows the profit graph and table for an interest rate put with terms similar to those of the interest rate call.

Other Options Strategies

One of the important features of an option is that it can be combined with positions in the underlying security and other options to generate a number of different investment strategies. Two well-known strategies formed by combining option positions are ***straddles*** and ***spreads***.

Straddle A straddle purchase is formed by buying both a call and a put with the same terms—the same underlying security, exercise price, and expiration date. A straddle write, in contrast, is constructed by selling a call and a put with the same terms.

Exercise rate = 6%, Reference rate = LIBOR, NP = $10 million,
Period = 0.25 year, Option cost = $12,500

LIBOR	π
4.0%	–$12,500
5.0%	–$12,500
6.0%	–$12,500
6.5%	0
7.0%	$12,500
7.5%	$25,000
8.0%	$37,500

π = Max [LIBOR – 0.06, 0](.25)($10M) – $12,500

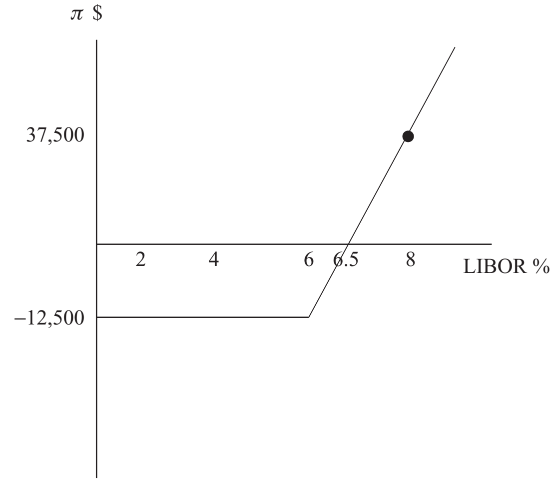


FIGURE 17.7 Interest Rate Call Option

In Figure 17.9, the profit graphs are shown for spot T-bond call, put, and straddle purchases in which both the call and the put have exercise prices of $100,000 and premiums of $1,000 and there is no accrued interest at expiration.[3] The straddle purchase shown in the figure is geometrically generated by vertically summing the profits on the call purchase position and put purchase position at each bond price. The resulting straddle purchase position is characterized by a V-shaped profit and

Exercise rate = 6%, Reference rate = LIBOR, NP = $10 million,
Period = 0.25 year, Option cost = $12,500

LIBOR	π
4.0%	$37,500
4.5%	$25,000
5.0%	$12,500
5.5%	0
6.0%	–$12,500
7.0%	–$12,500
8.0%	–$12,500

π = Max[0.06 – LIBOR, 0](0.25)($10M) – $12,500

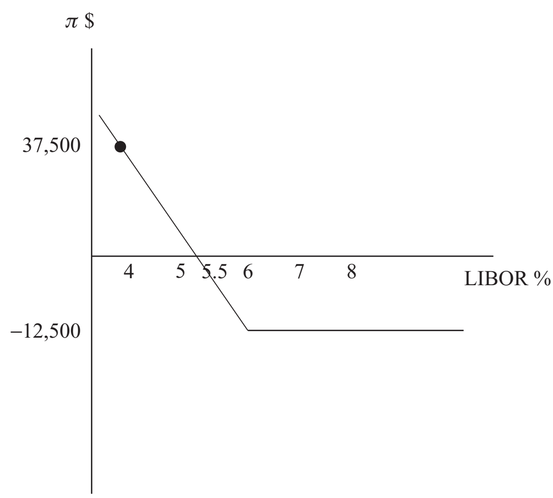
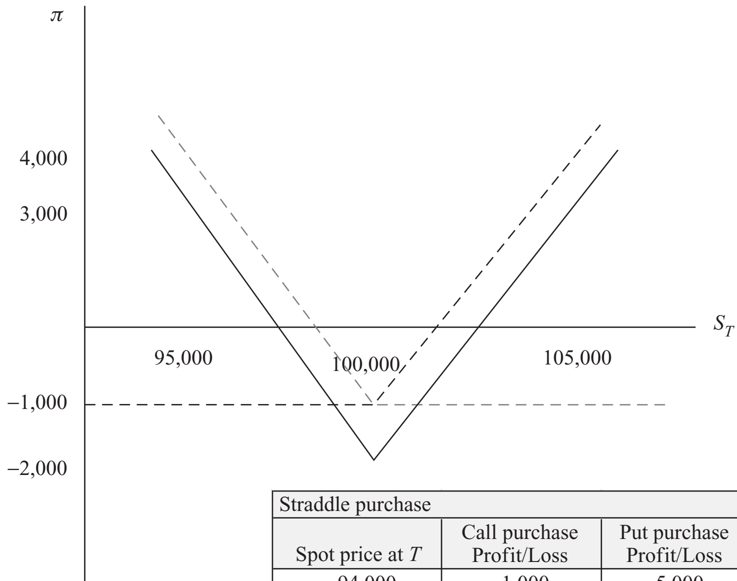


FIGURE 17.8 Interest Rate Put Option

- Buy 100 T-bond put for 1 and buy 100 T-bond call for 1:



Straddle purchase			
Spot price at T	Call purchase Profit/Loss	Put purchase Profit/Loss	Total Profit/Loss
94,000	–1,000	5,000	4,000
97,000	–1,000	2,000	1,000
98,000	–1,000	1,000	0
100,000	–1,000	–1,000	–2,000
102,000	1,000	–1,000	0
103,000	2,000	–1,000	1,000
106,000	5,000	–1,000	4,000

FIGURE 17.9 Straddle Purchase

spot price relation. Thus, the motivation for buying a straddle comes from the expectation of a large price movement in either direction. Losses on the straddle occur if the price of the underlying security remains stable, with the maximum loss being equal to the costs of the straddle ($2,000) and occurring when the bond price is equal to the exercise price. Finally, the straddle is characterized by two break-even prices ($98,000 and $102,000).

In contrast to the straddle purchase, a straddle write yields an inverted V-shaped profit graph. The seller of a straddle is betting against large price movements. A maximum profit equal to the sum of the call and put premiums occurs when the bond price is equal to the exercise price; losses occur if the bond price moves significantly in either direction.

Spread A spread is the purchase of one option and the sale of another on the same underlying security but with different terms: different exercise prices (***money spread***), different expirations (***time spread***), or both (***diagonal spread***). Two of the most popular time-spread positions are the ***bull spread*** and the ***bear spread***. A bull call spread is formed by buying a call with a certain exercise price and selling another call with a higher exercise price, but with the same expiration date. A bear call spread is the reversal of the bull spread; it consists of buying a call with a certain exercise

- Buy 100 T-bond call for 1 and sell 101 T-bond call for 0.75:

Bull Spread			
Spot price at T	100 call purchase at 1 Profit/Loss	101 call sale at 0.75 Total Profit/Loss	Profit/Loss
94,000	–1,000	750	–250
97,000	–1,000	750	–250
98,000	–1,000	750	–250
100,000	–1,000	750	–250
100,250	–750	750	0
101,000	0	750	750
102,000	1,000	–250	750
103,000	2,000	–1,250	750
106,000	5,000	–4,250	750

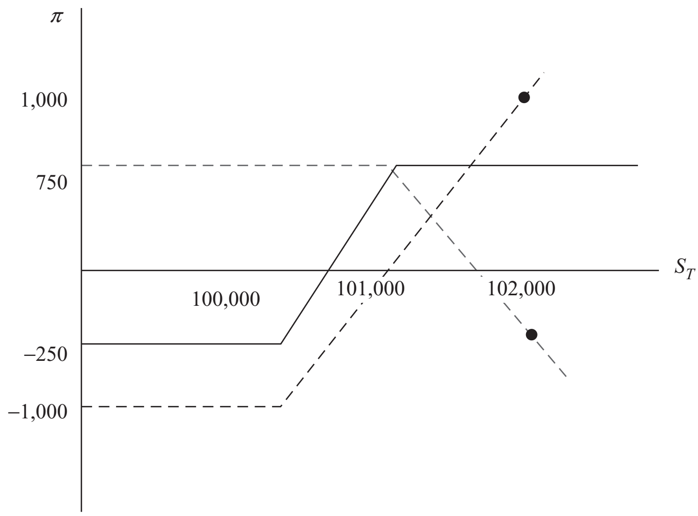


FIGURE 17.10 Bull Spread

price and selling another with a lower exercise price. (The same spreads also can be formed with puts.)

In Figure 17.10, the profit graph and table for a bull call spread strategy is shown. The spread is formed with the purchase of the 100 T-bond call (X = \$100,000) for 1 (C = \$1,000) and the sale of a 101 T-bond call (X = \$101,000) for 0.75 (C = \$750), with both options having the same underlying T-bond and expiration. The bull spread is characterized by losses limited to \$250 when the T-bond price is \$100,000 or less, limited profits of \$750 starting when the bond price hits \$101,000, and a break-even price of \$100,250.

A bear call spread results in the opposite profit and security price relation as the bull spread: limited profits occur when the security price is equal to or less than the lower exercise price and limited losses occur when the security price is equal to or greater than the higher exercise price.

Speculation

The above profit graphs illustrate how interest rate options can be used to hedge, as well as speculate, on movements in interest rates. A speculator who believes the

EXHIBIT 17.1 Different Option Positions

1. **Bull call spread:** Long in call with low X and short in call with high X.
2. **Bull put spread:** Long in put with low X and short in put with high X.
3. **Bear call spread:** Long in call with high X and short in call with low X.
4. **Bear put spread:** Long in put with high X and short in put with low X.
5. **Long butterfly spread:** Long in call with low X, short in two calls with middle X, and long in call with high X (similar position can be formed with puts).
6. **Short butterfly spread:** Short in call with low X, long in two calls with middle X, and short in call with high X (similar position can be formed with puts).
7. **Straddle purchase:** Long call and put with similar terms.
8. **Strip purchase:** Straddle with additional puts (e.g., long call and long two puts).
9. **Strap purchase:** Straddle with additional calls (e.g., long two calls and long put).
10. **Straddle sale:** Short call and put with similar terms (strip and strap sales have additional calls and puts).
11. **Money combination purchase:** Long call and put with different exercise prices.
12. **Money combination sale:** Short call and put with different exercise prices.

Federal Reserve System will lower long-term interest rates in the near future to stimulate the economy could profit (if her expectation is correct) by taking a long position in a T-bond futures call. As a speculative strategy, this long call position can be viewed as an alternative to a long position in a T-bond futures contract. When compared to the futures position, the call position provides a limited loss and unlimited profit, whereas the futures position provides an unlimited profit and loss profile. In contrast, if a speculator believes that long-term interest rates are going to increase in the near future, then she should take a long position in a T-bond futures put option. Similar positions could be taken in Eurodollar futures options by speculators who expect short-term rates to change.

Finally, between outright call and put positions, options can be combined in different ways to obtain various types of profit relations. Speculators who expect rates to increase in the future but don't want to assume the risk inherent in a put purchase position could form a bear call spread. In contrast, speculators who expect rates to be stable over the near term could, in turn, try to profit by forming a straddle write. Thus by combining different option positions, speculators can obtain positions that match their expectations and their desired risk-return preference. Exhibit 17.1 lists some of the other option positions. Exercises for generating profit tables and graphs to illustrate the features of different option positions are included as part of the end-of-the-chapter problems. In the next chapter, we will focus on how interest rate options are used for hedging positions.

17.5 OPTIONS TRADING—MICROSTRUCTURE

The primary objective of derivative exchanges offering options is to provide marketability to options contracts by linking brokers and dealers, standardizing contracts, establishing trading rules and procedures, guaranteeing and intermediating contracts through a clearinghouse, and providing continuous trading through electronic matching or with market makers, specialists, and locals.

Standardization

Similar to the futures exchanges, the options exchanges standardize contracts by setting expiration dates, exercise prices, and contract sizes on options. The expiration dates on options are defined in terms of an expiration cycle. For example, the March cycle has expiration months of March, June, September, and December. In a three-month option cycle, only the options with the three nearest expiration months trade at any time. Thus, as an option expires, the exchange introduces a new option. On many options contracts, the expiration day is the Saturday after the third Friday of the expiration month; the last day on which the expiring option trades, though, is Friday. For a number of futures options, the last trading day is the same as the futures delivery date. For these options, exercising at expiration would be essentially a cash settlement. For other futures options, the expiration date on the option can be one to two weeks before the futures delivery period. Exercising such options at their expiration date gives the holder a futures position with a one- or two-week expiration and a current margin position equal to the difference between the futures and exercise prices.

In addition to setting the expiration, the exchanges also choose the exercise prices for each option. Usually, at least three strike prices (sometimes as many as six) are associated with each option when an option cycle begins. Once an option with a specific exercise price has been introduced, it will remain listed until its expiration date. The exchange can, however, introduce new options with different exercise prices at any time.

Continuous Trading

As noted in Chapter 16, on the futures exchanges continuous trading is provided through locals who are willing to take temporary positions to make a market. Many of the options exchanges, though, use market makers and specialists to ensure a continuous market.

Clearinghouse and Options Clearing Corporation

As we discussed in Chapter 16, to make derivative contracts more marketable, derivative exchanges provide a clearinghouse (CH) or Options Clearing Corporation (OCC), as it is referred to on the option exchange. In the case of options, the CH intermediates each transaction that takes place on the exchange and guarantees that all option writers fulfill the terms of their options if they are assigned. In addition, the CH also manages option exercises, receiving notices and assigning corresponding positions to clearing members.

As an intermediary, the CH functions by breaking up each option trade. After a buyer and seller complete an option trade, the CH steps in and becomes the effective buyer to the option seller and the effective seller to the option buyer. At that point, there is no longer any relationship between the original buyer and seller. If the buyer of a futures call option, for example, decides to later exercise, her broker will notify the exchange's CH (the brokerage firm may well be the clearing member). Overnight, the CH will select a writer from its pool of option sellers on the exercised futures call option and assign that writer the obligation of fulfilling the terms of the exercise request. Before trading commences on the following day, the CH will establish a long futures position at a futures price equal to the exercise price for the exercising option holder and a short futures position for the assigned writer. Once this is done, margins on both positions will be required and the positions will be marked to market at the current settlement price on the futures. As noted earlier, when the positions are marked to market, the exercising call holder's margin account on his long position will be equal to the difference between the futures price and the exercise price, $f_t - X$, while the assigned writer will have to deposit funds in a futures margin account equal to $f_t - X$ to satisfy his maintenance margin on his short futures position. At this point, the futures positions are indistinguishable from any other futures. If the futures option is a put, the same procedure applies except that holder takes a short position at f_t (when the exercised position is marked to market), with a margin account worth $X - f_t$, and the assigned writer is assigned a long position at f_t and must deposit $X - f_t$ to meet his margin.[4]

By breaking up each options contract, the CH makes it possible for option investors to close their positions before expiration. If a buyer of an option later becomes a seller of the same option, or vice versa, the CH computer will note the offsetting position in the option investor's account and will therefore cancel both entries. For example, suppose in January, Investor A buys a March 95 T-bill futures call for 10 [$X = \$987{,}500$, $C = (10)(\$250) = \$2{,}500$] from Investor B. When the CH breaks up the contract, it records Investor A's right to exercise with the CH (i.e., the right to take a long T-bill futures position at X) and Investor B's responsibility to take a short futures position at X if a party long on the contract decides to exercise and the CH subsequently assigns B the responsibility. The transaction between A and B would lead to the following entry in the clearing firm's records:

	January Clearinghouse Records for March 95 T-Bill Futures Call
1.	Investor A has the *right* to exercise.
2.	Investor B has *responsibility*.

Suppose that in late January, 60 days before the expirations on the T-bill futures and futures options, short-term rates have decreased resulting in the following prices:

- The price on the spot 151-day T-bill is at \$988,000.
- The price on the March T-bill futures is priced at its carrying cost value of \$995,956 [$f_t = S_0(1+R_f)^T = \$988{,}000(1.05)^{(60/365)} = \$995{,}956$, where $R_f = 60$-day repo rate $= .05$].
- The price on the March 95 T-bill futures call is at \$9,000.

Seeing profit potential, suppose instead of exercising, Investor A decides to close her call position by selling a March 95 T-bill futures call at \$9,000 to Investor C. After the CH breaks up this contract, its records would have a new entry showing Investor A with the responsibility of taking a short position at $X = \$987{,}500$ if assigned. This entry, though, would cancel out Investor A's original entry giving her the right to take a long position at $X = \$987{,}500$:

	January Clearinghouse Records for March 95 T-Bill Futures Call	
1.	~~Investor A has the *right* to exercise~~	
2.	Investor B has *responsibility*	Closed
3.	Investor C has the *right* to exercise	
4.	~~Investor A has *responsibility*~~	

The CH would accordingly close Investor A's position. Thus, Investor A bought the call for \$2,500 and then closed her position by simply selling the call for \$9,000. Her call sale, in turn, represents an offsetting position and is referred to as an ***offset*** or ***closing sale***.

If a writer also wanted to close his position at this date, he could do so by simply buying a March 95 T-bill futures call. For example, suppose Investor B feared that rates could go lower and therefore decided to close his short position by buying a March 95 T-bill futures call at \$9,000 from Investor D. After this transaction, the CH would again step in, break up the contracts, and enter Investor B's and D's positions on its records. The CH's records would now show a new entry in which Investor B has the right to take a long position in the T-bill futures at \$987,500. This entry, in turn, would cancel Investor B's previous entry in which he had a responsibility to take a short position at \$987,500 if assigned. The offsetting positions (the right to buy and the obligation to sell) cancel each other and the CH computer system simply erases both entries.

	January Clearinghouse Records for March 95 T-Bill Futures Call	
1.	~~Investor B has *responsibility*~~	
2.	Investor C has the *right* to exercise	Closed
3.	~~Investor B has the *right* to exercise~~	
4.	Investor D has *responsibility*	

Since Investor B's second transaction serves to close his opening position, it is referred to as a ***closing purchase***. In this case, Investor B loses \$6,500 by closing: selling the call for \$2,500 and buying it back for \$9,000.

Margin Requirements

To secure the CH's underlying positions, exchange-traded options contracts have initial and maintenance margin requirements. Different from the margin requirements on futures contracts, the margin requirements on options only apply to the option writer. On most exchanges, the initial margin is the amount of cash or cash equivalents that must be deposited by the writer; some exchanges, such as the German exchange (DTM), do allow a third party guarantee to be used as a substitute for cash margins to secure the position. For written positions, the amount of margin required is equal to a certain percentage (e.g., 3% to 5%) of the exercise value of the contract. The CH sets the minimum initial margin requirement, with the brokerage firm allowed to increase it. In addition to the initial margin, the writer also has a maintenance margin requirement with the brokerage firm in which he has to keep the value of his account equal to a certain percentage of the initial margin value. Thus, if the value of the option position moves against the writer, he is required to deposit additional cash or cash equivalents to satisfy his maintenance requirement.

In discussing margin requirements for futures options, one should remember that there are two sets of margins: a margin requirement for the option writer and a futures margin requirement that must be met if the futures option is exercised. If the futures option is exercised, both the holder and writer must establish and maintain the futures margin positions, with the writer's margin position on the option now being replaced by his new futures position.

Types of Option Transactions

The CH provides marketability by making it possible for option investors to close their positions instead of exercising. In general, there are four types of trades investors of an exchange-traded option can make: opening, expiring, exercising, and closing transactions. The ***opening transaction*** occurs when investors initially buy or sell an option. An ***expiring transaction***, in turn, is allowing the option to expire: that is, doing nothing when the expiration date arrives because the option is worthless (out of the money). If it is profitable, a holder can exercise. Finally, holders or writers of options can close their positions with ***offsetting*** or ***closing transactions*** or orders.

As a general rule, option holders should close their positions rather than exercise. As we will discuss in Section 17.6, if there is some time to expiration, an option holder who sells her option will receive a price that exceeds the exercise value. Because of this, many exchange-traded options are closed.

OTC Options

In the OTC options market, interest rate options contracts are negotiable, with buyers and sellers entering directly into an agreement. Thus, the dealer's market provides options contracts that are tailor made to meet the specific needs of the holder or writer. The market, though, does not have a clearinghouse to intermediate and guarantee the fulfillment of the terms of the options contract, or market makers or specialists to ensure continuous markets; the options, therefore, lack marketability.

Since each OTC option has unique features, the secondary market is limited. Prior to expiration, holders of OTC options who want to close their position may be able to do so by selling their positions back to the original option writers or possibly to an OTC dealer who is making a market in the option. This type of closing is more likely to occur if the option writer is a dealer that can hedge an option position and also if the option is relatively standard (e.g., OTC option on a T-bond). Because of this inherent lack of marketability, the premiums on OTC options are higher than exchange-traded ones. For example, the bid-ask spread on an OTC T-bond is typically twice that of an exchange-traded T-bond futures option. Finally, since there is no CH to guarantee the option writer, OTC options also have different credit structures than exchange-traded options. Depending on who the option writer is, the contract may require initial and maintenance margins to be established.

17.6 OPTION PRICE RELATIONSHIPS

In our discussion of the fundamental option strategies, we treated the option premium as a given. The price of an option, though, is determined in the market and is a function of the time to expiration, the strike price, the security price, and the volatility of the underlying security. These factors and how the option price is determined form the basis of the option pricing model. The pricing of interest rate options using the binomial interest rate model is examined in Appendix H and the Black futures option model is explained in Appendix I. Here, we identify some of the factors that determine the prices of spot and futures interest rate options.

Call Price Relationships

Boundary Conditions and Time Value Premium The price of any option is constrained by certain boundary conditions. One of those boundary conditions is the intrinsic value. By definition, the ***intrinsic value*** (***IV***) of a call at a time prior to expiration (let t signify any time *prior* to expiration), or at expiration (T again signifies expiration date) is the maximum of the difference between the price of the underlying security or futures (S_t or f_t) and the exercise price or zero: IV = Max[$f_t - X$, 0] or Max[$S_t - X$, 0]. The intrinsic value can be used as a reference to define ***in-the-money, on-the-money*** and ***out-of-the-money*** calls. Specifically, an in-the-money call is one in which the price of the underlying security or futures contract exceeds the exercise price; as a result, its IV is positive. When the price of the security or futures is equal to the exercise price, the call's IV is zero and the call is said to be on the money. Finally, if the exercise price exceeds the security or futures price, the call would be out of the money and the IV would be zero:

Type	Spot Call	Futures Call
In-the-Money	$S_t > X => IV > 0$	$f_t > X => IV > 0$
On-the-Money	$S_t = X => IV = 0$	$f_t = X => IV = 0$
Out-of-the-Money	$S_t < X => IV = 0$	$f_t < X => IV = 0$

For an American futures option, the IV defines a boundary condition in which the price of a call has to trade at a value at least equal to its IV: $C_t \geq \text{Max}[f_t - X, 0]$. If this condition does not hold ($C_{.t} < \text{Max}[f_t - X, 0]$), an arbitrageur could buy the call, exercise, and close the futures position. For example, suppose a T-bill futures contract expiring in 182 days were trading at $987,862 (index = 95.1448) and a 95 T-bill futures call expiring in 182 days (X = $987,500) were trading at $100, below its IV of $362. Arbitrageurs could realize risk-free profits by (1) buying the call at $100, (2) exercising the call to obtain a margin account worth $f_t - X$ = $987,862 − $987,500 = $362 plus a long position in the T-bill futures contract priced $987,862, and (3) immediately closing the long futures position by taking an offsetting short position at $987,862. Doing this, arbitrageurs would realize a risk-free profit of $262. By pursuing this strategy, though, arbitrageurs would push the call premium up until it is at least equal to its IV of $f_t - X$ = $362 and the arbitrage profit is zero.

The above arbitrage strategy requires that the option be exercised immediately. Thus, the condition applies only to an American futures option. The boundary conditions for European futures, American spot, and European spot options are explained in a number of derivative texts (see Johnson, *Introduction to Derivatives*, Chapters 4 and 12).

The other component of the value of an option is the ***time value premium*** (***TVP***). By definition, the TVP of a call is the difference between the call's price and IV: TVP = C_t − IV. For example, if the 95 T-bill futures call expiring in 182 days (X = \$987,500) was trading at \$562 when the T-bill futures contract expiring in 182 days was trading at \$987,862 (index = 95.1448), the IV would be \$362 and the TVP would be \$200. It should be noted that the TVP decreases as the time remaining to expiration decreases.

Call Price Curve Graphically, the relationship between C_t, TVP, and IV is depicted in Figure 17.11.

In the figure, graphs plotting the call price and the IV (on the vertical axis) against the futures price (on the horizontal axis) are shown for the American 95 T-bill futures call option. The IV line shows the linear relationship between the IV and the futures price. The line emanates from a horizontal intercept equal to the exercise price. When the price of the futures is equal to or less than the exercise price, the IV is equal to zero; when the futures price exceeds the exercise price, the IV is positive and increases as the futures price increases. The IV line, in turn, serves as a reference for the call price curve (CC). The noted arbitrage condition dictates that the price of the call cannot trade (for long) at a value below its IV. Graphically, this means that the call price curve cannot go below the IV line. Furthermore, the IV line would be the call price curve if we are at expiration since the TVP = 0 and thus C_T = IV. The call price curve (CC) in Figure 17.11 shows the positive relationship between C_t and f_t. The vertical distance between the CC curve and the IV line, in turn, measures the TVP. The CC curve for a comparable call with a greater time to expiration would be above the CC curve, reflecting the fact that the call premium

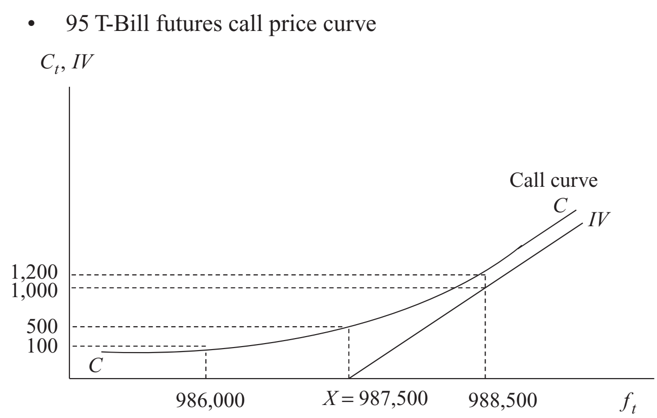


FIGURE 17.11 Call Price Curve

increases as the time to expiration increases. It should be noted that the slopes of the CC curves approach the slope of the IV line when the security price is relatively high (known as a ***deep in-the-money-call***), and it approaches zero (flat) when the price of the futures is relatively low (a ***deep out-of-the-money call***).

Variability The call price curve illustrates the positive relation between a call price and the underlying security or futures price and the time to expiration. An option's price also depends on the volatility of the underlying security or futures contract.

Since a long call position is characterized by unlimited profits if the security or futures increases but limited losses if it decreases, a call holder would prefer more volatility rather than less. Specifically, greater variability suggests, on the one hand, a given likelihood that the security will increase substantially in price, causing the call to be more valuable. On the other hand, greater volatility also suggests a given likelihood of the security price decreasing substantially. However, given that a call's losses are limited to just the premium when the security price is equal to the exercise price or less, the extent of the price decrease would be inconsequential to the call holder. Thus, the market will value a call option on a volatile security or contract more than a call on one with lower variability.

Put Price Relationships

Boundary Conditions Analogous to calls, the price of a put at a given point in time prior to expiration (P_t) also can be explained by reference to its IV, boundary conditions, and TVP. In the case of puts, the IV is defined as the maximum of the difference between the exercise price and the security or futures price or zero: IV = Max[$X - f_t$,0] or Max[$X - S_t$,0]. Similar to calls, in-the-money, on-the-money, and out-of-the-money puts are defined as:

Type	Spot Put	Futures Put
In-the-Money	$X > S_t => IV > 0$	$X > f_t => IV > 0$
On-the-Money	$X = S_t => IV = 0$	$X = f_t => IV = 0$
Out-of-the-Money	$X < S_t => IV = 0$	$X < f_t => IV = 0$

For an American futures option, the IV defines a boundary condition in which the price of the put has to trade at a price at least equal to its IV: $P_t \geq \text{Max}[X - f_t, 0]$. If this condition does not hold, an arbitrageur could buy the put, exercise, and close the futures position. For example, suppose a T-bill futures contract expiring in 182 days were trading at \$987,200 and a 95 T-bill futures put expiring in 182 days (X = \$987,500) were trading at \$100, below its IV of \$300. Arbitrageurs could realize risk-free profits by (1) buying the put at \$100, (2) exercising the put to obtain a margin account worth $X - f_t$ = \$987,500 − \$987,200 = \$300 plus a short position in the T-bill futures contract priced \$987,200, and (3) immediately closing the short futures position by taking an offsetting short position at \$987,200. Doing this, the arbitrageur would realize a risk-free profit of \$200. By pursuing this strategy, though,

- Put price curve

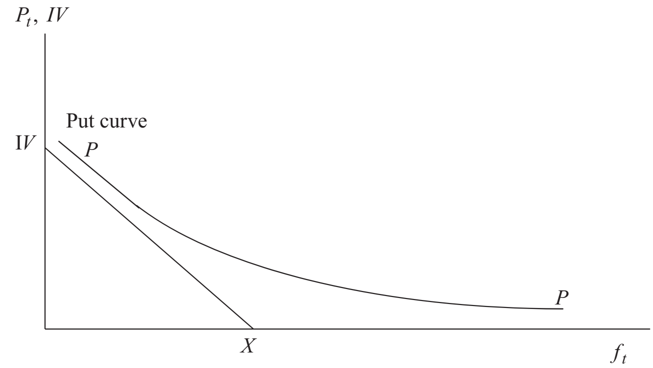


FIGURE 17.12 Put and Futures Price Relation

arbitrageurs would push the put premium up until it is at least equal to its IV of $X - f_t = \$300$ and the arbitrage profit is zero. The arbitrage strategies governing the boundary conditions for the European futures put options and the American and European spot put options are examined in a number of derivative texts.

Put Price Curve Similar to calls, the TVP for a put is defined as TVP = P_t – IV. The relation between the price of a put and the IV to the underlying futures price is shown in Figure 17.12. The figure shows a negatively sloped put-price curve (PP) and a negatively sloped IV line going from the horizontal intercept (where $f_t = X$) to the vertical intercept where the IV is equal to the exercise price when the futures is trading at zero (i.e., IV = X, when $f_t = 0$). The slope of the PP curve approaches the slope of the IV line for relatively low futures prices (***deep in-the-money puts***) and approaches zero for relatively large futures prices (***deep-out-of-the money puts***).

Variability Like calls, the price of a put option depends not only on the underlying security or futures price and time to expiration, but also on the volatility of the underlying security or futures contract. Since put losses are limited to the premium when the price of the underlying security or futures is greater than or equal to the exercise price, put buyers, like call buyers, will value puts on securities or futures with greater variability more than those with lower variability.

Closing As noted earlier, if there is some time to expiration, an option holder who sells her option will receive a price that exceeds the exercise value; that is, if the holder sells the option, she will receive a price that is equal to an IV plus a TVP; if she exercises, though, her exercise value is only equal to the IV. Thus, by exercising instead of closing she loses the TVP. For example, in our CH case, suppose Investor A sold her 95 T-bill futures call ($X = \$987{,}500$) at \$9,000 when the T-bill futures contract was trading at \$995,956; the \$9,000 price would be equal to an IV = \$8,456 and a TVP = \$544. If Investor A had exercised her futures call instead of simply closing it, she would have received a long T-bill futures position at \$995,956 and a margin account worth the IV of \$8,456. Thus, by exercising instead of selling the option, she lost the TVP. Thus, an option holder in most cases should close her position instead of exercising. There are some exceptions to the general rule of closing instead of exercising. For example, if an American call option on a security that was expected to pay a high coupon that exceeded the TVP on the option, then it would be advantageous to exercise the option instead of selling it.

Put–Call Parity Since the prices of options on the same security are derived from that security's price, it follows that the put and call prices are related. The relationship governing put and call prices is known as ***put–call parity***. The relation can be defined in terms of an arbitrage formed with a position known as a ***conversion***. For European spot options on debt securities with no coupons (e.g., T-bills and Eurodollar deposits), the conversion consists of (1) a long position in an underlying security that will have a maturity equal to the maturity of the option's underlying security (e.g., a T-bill that will have maturity of 91 days at the expiration on the spot T-bill option), and (2) a short position in a call and a long position in a put with the same exercise price and time to expiration. As shown below, the conversion yields a certain cash flow at expiration equal to the exercise price:

Closing Position	$S_T < X$	$S_T = X$	$S_T > X$
Long Bond	S_T	S_T	S_T
Long Put	$X - S_T$	0	0
Short Call	0	0	$-(S_T - X)$
Net	X	X	X

To preclude arbitrage, the risk-free conversion portfolio must be worth the same as a risk-free zero-coupon bond with a face value of X maturing at the same time as the option's expiration. Thus, in equilibrium:

$$P_0^e - C_0^e + S_0 = \frac{X}{(1 + R_f)^T}$$

The put–call parity condition on options on T-bonds, T-notes, and other debt securities paying interest is similar to options on zero-coupon bonds except that the accrued interest on the underlying bond is included. That is, at expiration the conversion will yield a risk-free cash flow equal to the exercise price plus the accrued interest. Thus, the equilibrium value of the conversion will equal the value of a risk-free bond with a face value of X plus the accrued interest:

$$P_0^e - C_0^e + S_0 = \frac{X + \text{Accrued interest}}{(1 + R_f)^T}$$

For European futures options, the conversion is formed with a long position in the futures contract and a long position in a put and a short position in a call on the futures contract. As shown below, if the options and the futures contracts expire at the same time, then the conversion would be worth $X - f_0$ at expiration, regardless of the price on the futures contract.

Closing Position	$f_T < X$	$f_T = X$	$f_T > X$
Long Futures	$f_T - f_0$	$f_T - f_0$	$f_T - f_0$
Long Futures Put	$X - f_T$	0	0
Short Futures Call	0	0	$-(f_T - X)$
Net	$X - f_0$	$X - f_0$	$X - f_0$

Since this position yields a risk-free return, in equilibrium its value would be equal to the present value of a risk-free bond with a face value of $X - f_0$ (remember the futures contract has no initial value). Thus:

$$P_0^e - C_0^e = \frac{X - f_0}{(1 + R_f)^T}$$

Note: If the carrying-cost model holds and the futures and options expire at the same time, then the equilibrium relation defining put–call parity for European futures options will be equal to the put–call parity for European spot options. This can be seen algebraically by substituting the carrying cost equation $S_0(1+R_f)^T$ for f_0 in the above equation. Finally, note that put–call parity is defined in terms of European options, not American.

17.7 CONCLUSION

In this chapter, we have provided an overview of interest rate options by defining option terms and markets, examining the fundamental option strategies, and describing the basic option pricing relations. Like interest rate futures, exchange-traded and OTC options can be used as a tool for speculating on interest rates and for managing different types of debt positions. Also like futures, the prices of interest rate options are governed by arbitrage forces. In the next chapter, we extend our analysis of these derivatives by examining how they are used to manage debt positions.

KEY TERMS

American option
bear spread
bull spread
call
cap
closing purchase
closing sale
conversion
deep in-the-money call
deep in-the-money put
deep out-of-the-money call
deep out-of-the-money put
diagonal spread
European option
exercise
exercise date

exercise price
expiring transaction
fixed deliverable bond
floor
holder
interest rate call (caplet)
interest rate put (floorlet)
in-the-money
intrinsic value (IV)
long position
money spread
naked call write
offset
offsetting or closing transaction
on-the-money
opening transaction
option premium
options on futures
out-of-the-money
profit graph
put
put–call parity
short position
spot options
spreads
straddles
strike price
time spread
time value premium (TVP)
writer

PROBLEMS AND QUESTIONS

Note: A number of the problems can be done in Excel by writing a program or using the Option Strategies Excel program available on the Web site.

1. Show graphically and in a table the profit and T-bond price relationships at expiration for the following positions on OTC T-bond options. In each case, assume that the T-bond spot call and put options each have exercise prices of \$100,000 and premiums of \$1,000, and that there is no accrued interest at expiration. Evaluate at spot T-bond prices of \$90,000, \$95,000, \$100,000, \$105,000, and \$110,000.
   - **a.** A straddle purchase formed with long positions in the T-bond call and put options.
   - **b.** A straddle write formed with short positions in T-bond call and put options.
   - **c.** A simulated long T-bond position formed by buying the T-bond call and selling the T-bond put.
   - **d.** A simulated short stock position formed by selling the T-bond call and buying the T-bond put.
   - **e.** A strip purchase formed with long positions in one T-bond call and two puts.
   - **f.** A strap write formed with short positions in two T-bond calls and one put.
2. Assume that there is an OTC T-bond spot call with an exercise price of \$100,000 and premium of \$1,000 and an OTC T-bond spot call option with an exercise price of \$101,000 and premium of \$500. Also assume the options expire at the same time and that there is no accrued interest at expiration. Show graphically and in a table the profit and T-bond price relationships at expiration for the following positions on the OTC T-bond options. Evaluate at spot T-bond prices of \$95,000, \$97,500, \$100,000, \$102,500, \$105,000, and \$107,500.
   - **a.** A bull call spread formed by buying the 100 T-bond call and selling the 101 T-bond call.
   - **b.** A bear call spread formed by buying the 101 T-bond call and selling the 100 T-bond call.

3. Assume that there is an OTC T-bond spot call with an exercise price of $100,000 and premium of $1,000, an OTC T-bond spot call option with an exercise price of $101,000 and premium of $500, and an OTC T-bond spot call option with an exercise price of $102,000 and premium of $250. Also assume the three options expire at the same time and that there is no accrued interest at expiration. Show graphically and in a table the profit and T-bond price relationships at expiration for the following positions on the OTC T-bond options. Evaluate at spot T-bond prices of $99,500, $99,750, $100,000, $100,250, $100,500, $100,750, $101,000, $101,250, $101,500, $101,750, $102,000, $102,250, and $102,500.
   a. A long butterfly spread formed by buying one 100 T-bond call, selling two 101 T-bond calls, and buying one 102 T-bond call.
   b. A short butterfly spread formed by selling one 100 T-bond call, buying two 101 T-bond calls, and selling one 102 T-bond call.

4. Show graphically and in a table the profit and T-bill futures price relationships at expiration for the following positions on T-bill futures options. In each case, assume that the T-bill futures call and put options each have exercise prices of $987,500 (IMM index = 95) and premiums of $1,250. Evaluate at spot discount yields at expiration of 6.5%, 6%, 5.5%, 5%, 4.5%, 4%, and 3.5%.
   a. A straddle purchase formed with T-bill futures call and put options.
   b. A straddle write formed with T-bill futures call and put options.
   c. A simulated long T-bill position formed by buying a T-bill futures call and selling a T-bill futures put.
   d. A simulated short T-bill position formed by selling a T-bill futures call and buying a T-bill futures put.

5. Show graphically and in a table the profit and LIBOR relationships at expiration for the following positions on interest rate options. In each case, assume that the interest rate call and put options each have exercise rates of 7%, a LIBOR reference rate, notional principals of $20 million, time period of .25 per year, and premiums of $25,000. Evaluate at spot discount yields at expiration of 5%, 5.5%, 6.0%, 6.5%, 7%, 7.5%, 8%, 8.5%, and 9.0%.
   a. An interest rate call purchase
   b. An interest rate put purchase
   c. An interest rate call sale
   d. An interest rate put sale

6. Explain what arbitrageurs would do if the price of an American T-bill futures call with an exercise price of $987,500 were priced at $900 when the underlying futures price was trading at $988,500. What impact would their actions have in the option market on the call's price? Would arbitrageurs follow the same strategy if the call option were European? If not, why?

7. Explain what arbitrageurs would do if the price of an American T-bill futures put with an exercise price of $987,500 were priced at $900 when the underlying futures price was trading at $986,500. What impact would their actions have in the option market on the put's price?

8. If the premium on an option increases, does that mean there is a greater demand for the option? Comment.

9. Explain the role and functions of the Option Clearing Corporation.

10. Suppose in February Ms. X sold a June 95 Eurodollar futures call contract to Mr. Z for 5, then later closed her position by buying a June 95 Eurodollar futures call for 6 from Mr. Y. Explain how the OCC would handle these contracts. Use actual prices and not the index values.

11. Explain the various types of option transactions.

12. Evaluate a long position in an OTC December 97 T-bond call option purchased at 1.6 (1 6/32). Evaluate in terms of the profit at T-bond index values at expiration of 95, 96, 97, 98, 99, 100, and 101. Assume no accrued interest. What is the break-even index price?

13. Evaluate a long position in an OTC December 97 T-bond put option purchased at 0.25 (0 25/32). Evaluate in terms of the profits at T-bond index values at expiration of 94, 95, 96, 97, 98, 99, and 100. Assume no accrued interest. What is the break-even price?

14. Show that the December 97 T-bond call and put options in Problems 12 and 13 conform to the put–call parity model. To show this, assume that the T-bond underlying the option currently is priced at 96, the December expiration is exactly .25 years from the present, and the annual risk-free rate on securities maturing in 90 days is 6.0154%.

15. Prove the following boundary conditions using an arbitrage argument. In your proof, show the initial positive cash flow when the condition is violated and prove there are no liabilities at expiration or when the positions are closed.

a. European futures call option: $C_t \geq \text{Max}[\text{PV}(f_t - X), 0]$
b. European futures put option: $P_t \geq \text{Max}[\text{PV}(X - f_t), 0]$
c. American spot call option: $C_t \geq \text{Max}[S_t - X, 0]$
d. American spot put option: $P_t \geq \text{Max}[X - S_t, 0]$
e. Put–call futures parity for European options: $P_t - C_t = (X - f_t)/(1 + R_f)^T$

16. Explain intuitively and with an example why call and put options are more valuable, the greater their underlying security's variability.

17. Explain why option holders should, in most cases, close their options instead of exercising. Under what condition would it be beneficial to exercise a call option early?

18. Short-Answer Questions:

a. What is the profit-loss characteristic that characterizes a naked call write?
b. What term is used to describe the number of outstanding options contracts?
c. What exchange listed then later delisted spot T-bill spot options?
d. A September 92 Eurodollar futures call option gives the holder the right to do what?
e. A September 92 Eurodollar futures put option gives the holder the right to do what?
f. What is the conversion strategy underlying the put–call futures parity model?
g. What conditions are necessary to make the spot options and futures options equivalent?

- **h.** True or false: If the futures options contract and the underlying futures contract expire at the same time, then the futures options can be viewed as an option on the underlying spot security with the option having a cash settlement clause.
- **i.** Define a cap and explain one of its uses.
- **j.** Define a floor and explain one of its uses.

WEB EXERCISES

1. Determine the recent prices on exchange options by going to www.cboe.com.
2. Determine the recent prices on listed futures options contracts by going to the CME Group site: www.cme.com.
3. Select several exchange-listed interest rate futures call and put options and determine their recent prices by going to www.wsj.com/free and clicking "Market Data Center" ("Commodity and Futures"). Using your options, evaluate some of the following strategies with a profit table and graph (use Excel or the Excel program Option Strategies that can be downloaded from the Web site): call purchase, put purchase, straddle purchase, and straddle sale.

NOTES

1. Before 1936, the U.S. futures exchanges offered futures options for a number of years. In 1936, though, the instruments were banned when U.S. security regulations were tightened following the 1929 stock market crash. Futures options have been available on foreign exchanges for a number of years.
2. Although many OTC options are exercised, most exchange-traded options are not exercised, but instead are closed by holders selling contracts and writers buying contracts. As a starting point in developing a fundamental understanding of options, though, it is helpful to first examine what happens if the option is exercised.
3. In many of our examples we assume calls and puts with the same terms are priced the same. We do this for simplicity. In most cases, though, calls and puts with the same terms are not priced equally. The relation between call and put prices is discussed in Section 17.6.
4. On the CBOT, assignment of exercise is random; on other exchanges it is based on first in and first out: the clearing member with the oldest written position will be assigned first.

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CHAPTER 18

Managing Fixed-Income Positions with Interest Rate Derivatives

18.1 INTRODUCTION[1]

Today, futures and options contracts on debt securities are used by banks and financial intermediaries to manage the maturity gaps between loans and deposits, by corporations and financial institutions to fix or cap the rates on future loans, and by fixed-income portfolio managers, money managers, investment bankers, and security dealers in locking in the future purchase or selling price on their fixed-income securities. In this chapter, we continue our analysis of the use of derivatives by examining how they are used in managing fixed-income positions. We begin by looking at how interest rate futures and options can be used for hedging future debt and bond investment positions against interest rate risk. We next look at how interest rate derivatives are used to create different speculative positions. We then conclude the chapter by examining how synthetic fixed-rate and floating-rate debt and investment positions are formed with derivatives. This chapter focuses on managing interest rate positions with exchange-traded futures options and futures. In Chapter 19, we extend the analysis of interest rate management to examining the uses of such OTC interest rate derivative products as caps, floors, and collars that are offered by financial institutions.

18.2 HEDGING FIXED-INCOME POSITIONS

A fixed-income manager planning to invest a future inflow of cash in high-quality bonds could hedge the investment against possible higher bond prices and lower rates by going long in T-bond futures contracts. If long-term rates were to decrease, the higher costs of purchasing the bonds would then be offset by profits from his T-bond futures positions. On the other hand, if rates increased, the manager would benefit from lower bond prices, but he would also have to cover losses on his futures position. Similar long hedging positions using T-bill or Eurodollar futures could also be applied by money managers who are planning to invest future cash inflows in short-term securities and want to minimize their risk exposure to short-term interest rate changes. In such hedging cases, if rates were to increase, a manager would benefit from lower bond prices, but he would also have to cover losses on his futures position. Thus, hedging future fixed-income investments with futures locks in a future price and return and therefore eliminates not only the costs of unfavorable price movements but also the benefits from favorable movements. However, by hedging with either exchange-traded futures call options on Treasury or Eurodollar contracts or with an OTC spot call options on a debt security, a hedger can obtain protection against adverse price increases while still realizing lower costs if security prices decrease.

For cases in which bond or money market managers are planning to sell some of their securities in the future, hedging can be done by going short in a T-bond, T-bill, or Eurodollar futures contract. If rates were higher at the time of the sale, the resulting lower bond prices and therefore revenue from the bond sale would be offset by profits from the futures position (just the opposite would occur if rates were lower). The hedge also can be formed by purchasing an exchange-traded futures put option on Treasury or Eurodollar contracts or an OTC spot put option on a debt security. This hedge would provide downside protection if bond prices decrease while earning increasing revenues if security prices increase.

Short hedging positions with futures and put options can be used not only by holders of fixed-income securities planning to sell their instruments before maturity, but also by bond issuers, borrowers, and debt security underwriters. A company planning to issue bonds or borrow funds from a financial institution at some future date, for example, could hedge the debt position against possible interest rate increases by going short in debt futures contracts, or cap the loan rate by buying an OTC put or exchange-traded futures put. Similarly, a bank that finances its short-term loan portfolio of one-year loans by selling 90-day CDs could manage the resulting maturity gap [maturity of the assets (one-year loans) not equal to the maturity of liabilities (90-day CDs)] by also taking short positions in Eurodollar futures or futures options. Finally, an underwriter or a dealer who is holding a debt security for a short period of time could hedge the position against interest rate increases by going short in an appropriate futures contract or by purchasing a futures put option.

Naive Hedging Model

The simplest model to apply to hedging a debt position is the ***naive hedging model***. For debt positions, a naive hedge can be formed by hedging each dollar of the face value of the spot position with one market-value dollar in the futures or options contract. For example, if a T-bond futures' price is at 90, then 100/90 = 1.11 futures contracts could be used to hedge each dollar of the face value of the bond. A naive hedge also can be formed by hedging each dollar of the market value of the spot position with one market-value dollar of the futures or futures option. Thus, if $98 were to be used to buy the above T-bond at some future date, then 98/90 = 1.089 futures contracts could be purchased to hedge the position.

Long Hedging Cases

Example 1: Hedging with a Long T-Bill Futures Contract Consider the case of a treasurer of a corporation who is expecting a $5 million cash inflow in June, which she is planning to invest in T-bills for 91 days. If the treasurer wants to lock in the yield on the T-bill investment, she could do so by going long in June T-bill futures contracts. For example, if the June T-bill contract were trading at the index price of

95, the treasurer could lock in a yield (YTM_f) of 5.1748% on a 91-day investment made at the futures' expiration date in June:

$$f_0(\text{June}) = \frac{100 - (5)(90/360)}{100}(\$1{,}000{,}000) = \$987{,}500$$

$$\text{YTM}_f = \left[\frac{\$1{,}000{,}000}{\$987{,}500}\right]^{365/91} - 1 = .051748$$

To obtain the 5.1748% yield, the treasurer would need to form a hedge in which she bought $n_f = 5.063291$ June T-bill futures contracts (assume perfect divisibility):

$$n_f = \frac{\text{Investment in June}}{f_0} = \frac{\$5{,}000{,}000}{\$987{,}500} = 5.063291 \text{ Long contracts}$$

At the June expiration date, the treasurer would close the futures position at the price on the spot 91-day T-bills. If the cash flow from closing is positive, the treasurer would invest the excess cash in T-bills; if it is negative, the treasurer would cover the shortfall with some of the anticipated cash inflow earmarked for purchasing T-bills. For example, suppose at expiration the spot 91-day T-bill were trading at a YTM of 4.5%, or $S_T = \$1{,}000{,}000/(1.045^{91/365}) = \$989{,}086$. In this case, the treasurer would realize a profit of $8,030.38 from closing the futures position:

$$\pi_f = [S_T - f_0]\, n_f$$

$$\pi_f = [\$989{,}086 - \$987{,}500]\, 5.063291$$

$$\pi_f = \$8{,}030.38$$

With the $8,030.38 profit on the futures, the $5 million inflow of cash (assumed to occur at expiration), and the spot price on the 91-day T-bill at $989,086, the treasurer would be able to purchase 5.063291 T-bills ($M = 91$ days and face value of $1 million):

$$\text{Number of 91-day T-bills} = \frac{\$5{,}000{,}000 + \$8{,}030.38}{\$989{,}086} = 5.063291$$

Ninety-one days later the treasurer would have $5,063,291, which equates to a rate of return from the $5 million inflow of 5.1748%—the rate that is implied on the futures contract:

$$\text{Rate} = \left[\frac{5.063291(\$1{,}000{,}000)}{\$5{,}000{,}000}\right]^{365/91} - 1 = .051748$$

On the other hand, if the spot T-bill rate were 5.5% at expiration, or $S_T = \$1{,}000{,}000/(1.055)^{91/365} = \$986{,}740$, the treasurer would lose $3,848 from closing the futures position: [$986,740 − $987,500]5.063291 = −$3,848. With the inflow of $5 million, the treasurer would need to use $3,848 to settle the futures position, leaving her only $4,996,152 to invest in T-bills. However, with the price of the

T-bill lower in this case, the treasurer would again be able to buy 5.063291 T-bills (\$4,996,152/\$986,740 = 5.063291), and therefore realize a 5.1748% rate of return from the \$5 million investment. Note that the hedge rate of 5.1748% occurs for any rate scenario.

Example 2: Hedging with a T-Bill Futures Call Suppose the treasurer expected higher short-term rates in June but was still concerned about the possibility of lower rates. To be able to gain from the higher rates and yet still hedge against lower rates, the treasurer could alternatively buy a June call option on a T-bill futures contract. For example, suppose there is a June T-bill futures call with an exercise price of \$987,500 (index = 95, $R_D = 5$) priced at \$1,000 (quote = 4; $C = (4)(\$250) = \$1{,}000$), with the June expiration (on both the underlying futures and futures option) occurring at the same time the \$5 million cash inflow is to be received. To hedge the 91-day investment with this call, the treasurer would need to buy 5 calls at a cost of \$5,000:

$$n_C = \frac{V_T}{X} = \frac{\$5{,}000{,}000}{\$987{,}500} \cong 5 \text{ calls}$$
$$\text{Cost} = n_C C = (5)(\$1{,}000) = \$5{,}000$$

If T-bill rates were lower at the June expiration, then the treasurer would profit from the calls and could use the profit to defray part of the cost of the higher-priced T-bills. As shown in Table 18.1, if the spot discount rate on T-bills is 5% or less, the treasurer will be able to buy 5.06 spot T-bills (assume perfect divisibility) with the

TABLE 18.1 Hedging a \$5 Million Cash Flow in June with June T-Bill Futures Calls

1 Spot Discount Rates	2 Spot Price = Futures Price at T	3 Call Cash Flow	4 Hedged Investment Funds	5 Number of Bills	6 YTM
R_D %	$S_T = f_T$		\$5,000,000 + Col 3	Col (4)/Col(2)	
3.75	\$990,625	\$15,625	\$5,015,625	5.06	0.0516
4.00	\$990,000	\$12,500	\$5,012,500	5.06	0.0516
4.25	\$989,375	\$ 9,375	\$5,009,375	5.06	0.0516
4.50	\$988,750	\$ 6,250	\$5,006,250	5.06	0.0517
4.75	\$988,125	\$ 3,125	\$5,003,125	5.06	0.0517
5.00	\$987,500	\$ 0	\$5,000,000	5.06	0.0517
5.25	\$986,875	\$ 0	\$5,000,000	5.07	0.0544
5.50	\$986,250	\$ 0	\$5,000,000	5.07	0.0571
5.75	\$985,625	\$ 0	\$5,000,000	5.07	0.0598
6.00	\$985,000	\$ 0	\$5,000,000	5.08	0.0625
6.25	\$984,375	\$ 0	\$5,000,000	5.08	0.0652

Cash flow = $5[\text{Max}(f_T - \$987{,}500), 0]$
YTM = $[(\text{Number of bills})(\$1{,}000{,}000)]/\$5{,}000{,}000]^{365/91} - 1$

$5 million cash inflow and the cash flow from the futures calls, locking in a YTM for the next 91 days of approximately 5.16% on the $5 million investment (this excludes the cost of the calls). On the other hand, if T-bill rates are higher, then the treasurer would benefit from lower spot prices while her losses on the call would be limited to just the $5,000 costs of the calls. For spot discount rates above 5%, the treasurer would be able to buy more T-bills the higher the rates, resulting in higher yields as rates increase. Thus, for the cost of the call options, the treasurer is able to establish a floor by locking in a minimum YTM on the $5 million June investment of approximately 5.16%, with the chance to earn a higher rate if short-term rates increase.

Example 3: Hedging a Future 182-Day T-Bill Investment Suppose in the preceding example, the treasurer was planning to invest the expected $5 million June cash inflow in T-bills for a period of 182 days instead of 91 days, and again wanted to lock in the investment rate. Since the underlying T-bill on a futures contract has a maturity of 91 days, not 182, the treasurer would need to take two long futures positions: one position expiring at the end of 91 days (the June contract) and the other expiring at the end of 182 days (the September contract). By purchasing futures contracts with expirations in June and September, the treasurer would have the equivalent of one June T-bill futures contract on a T-bill with 182-day maturity.

The implied futures rate of return earned on a 182-day investment made in June, YTM_f(June, 182), is equal to the geometric average of the implied futures rate on the contract expiring in June, YTM_f(June, 91), and the implied futures rate on the contract expiring in September, YTM_f(Sept, 91):

$$\text{YTM}_f(\text{June, 182}) = \left[(1 + \text{YTM}_f(\text{June, 91}))^{91/365} \times (1 + \text{YTM}_f(\text{Sept, 91}))^{91/365}\right]^{365/182} - 1$$

In this example, if the index on the June T-bill contract is at 95 and the index on a September T-bill contract is at 95.2, then the implied futures rate on each contract's underlying T-bill would be 5.1748% and 5.38865%, respectively, and the implied futures rate on the182-day investment made in June would be 5.28167%:

$$f_0(\text{June}) = \frac{100 - (5)(90/360)}{100}(\$1{,}000{,}000) = \$987{,}500$$

$$\text{YTM}_f(\text{June}) = \left[\frac{\$1{,}000{,}000}{\$987{,}500}\right]^{365/91} - 1 = .051748$$

$$f_0(\text{Sept}) = \frac{100 - (5.2)(90/360)}{100}(\$1{,}000{,}000) = \$987{,}000$$

$$\text{YTM}_f(\text{Sept}) = \left[\frac{\$5{,}000{,}000}{\$987{,}000}\right]^{365/91} - 1 = .0538865$$

$$\text{YTM}_f(\text{June, 182}) = \left[(1.051748)^{91/365}(1.0538865)^{91/365}\right]^{365/182} - 1 = .0528167$$

To actually lock in the 182-day rate for the $5 million investment, the treasurer would need to purchase 5.06329 June contracts and 5.065856 September contracts (again, assume perfect divisibility). That is, using a naive hedging model, the required hedging ratios would be:

$$n_f(\text{June}) = \frac{\$5{,}000{,}000}{\$987{,}500} = 5.06329 \text{ Long contracts}$$

$$n_f(\text{Sept}) = \frac{\$5{,}000{,}000}{\$987{,}000} = 5.065856 \text{ Long contracts}$$

At the June expiration date, the treasurer would close both contracts and invest the cash inflows plus (or minus) the futures profit (costs) in 182-day T-bills. By doing this, the treasurer in effect would be creating a June futures contract on a T-bill with a maturity of 182 days. If the equilibrium pricing model governing futures (discussed in Chapter 16) holds, then the treasurer will earn a rate of return on the 182-day investment equal to 5.28167%.

Managing the Maturity Gap

Hedging with a Short Eurodollar Futures Contract An important use of short hedges is in minimizing the interest rate risk that financial institutions are exposed to when the maturity of their assets does not equal the maturity of their liabilities—the ***maturity gap***. As an example, consider the case of a small bank with a maturity gap problem in which its short-term loan portfolio has an average maturity greater than the maturity of the CDs that it is using to finance its loans. Specifically, suppose in June, the bank makes loans of $100 million, all with maturities of 180 days. To finance the loans, though, suppose the bank's customers prefer 90-day CDs to 180-day CDs and, as a result, the bank sells $100 million worth of 90-day CDs at a rate equal to the current LIBOR of 5%. Ninety days later (in September) the bank would owe $\$101{,}210{,}311 = \$100{,}000{,}000(1.05)^{90/365}$; to finance this debt, the bank would have to sell $101,210,311 worth of 90-day CDs at the LIBOR at that time. In the absence of a hedge, the bank would be subject to interest rate risk. If short-term rates increase, the bank would have to pay higher interest on its planned September CD sale, lowering its interest spread; if rates decrease, the bank's spread would increase.

Suppose the bank is fearful of higher rates in September and decides to minimize its exposure to market risk by hedging its $101,210,311 CD sale in September with a September Eurodollar futures contract trading at an index value of 95. To hedge the liability, the bank would need to go short in 102.491454 September Eurodollar futures (assume perfect divisibility):

$$f_0(\text{Sept}) = \frac{100 - (5)(90/360)}{100}(\$1{,}000{,}000) = \$987{,}500$$

$$n_f = \frac{\$101{,}210{,}311}{\$987{,}500} = 102.491454 \text{ Short Eurodollar contracts}$$

TABLE 18.2 Hedging a Maturity Gap

(1) September LIBOR	R	.045	.055
(2) September spot and expiring futures price	$S_T = f_T = \$1{,}000{,}000/(1+R)^{90/365}$	\$989,205	\$986,885
(3) Profit on futures	$\pi_f = 102.491454[\$987{,}500 - f_T]$	–\$174,748	\$63,032
(4) Debt on June CD	$\$100{,}000{,}000(1.05)^{90/365}$	\$101,210,311	\$101,210,311
(5) Total funds to finance	Row (4) – Row (3)	\$101,385,059	\$101,147,279
(6) Debt at end of period	$[\text{Row }(5)](1+R)^{90/365}$	102,491,433	102,491,462
(7) Rate paid for 180-day period	$[(\text{Row }(6))/\$100{,}000{,}000]^{365/180}$	5.117%	5.117%
	(Allow for slight rounding differences)		

At a futures price of \$987,500, the bank would be able to lock in a rate on its September CDs of 5.23376%. With this rate and the 5% rate it pays on its first CDs, the bank would pay 5.117% on its CDs over the 180-day period:

$$YTM_f(\text{Sept}) = \left[\frac{\$1{,}000{,}000}{\$987{,}500}\right]^{365/90} - 1 = .0523376$$

$$YTM_{180} = \left[(1.05)^{90/365}(1.0523376)^{90/365}\right]^{365/180} - 1 = .05117$$

That is, when the first CDs mature in September, the bank will issue new 90-day CDs at the prevailing LIBOR to finance the \$101,210,311 first CD debt plus (minus) any loss (profit) from closing its September Eurodollar futures position. If the LIBOR in September has increased, the bank will have to pay a greater interest on the new CD, but it will realize a profit from its futures contracts, decreasing the amount of funds it needs to finance at the higher rate. On the other hand, if the LIBOR is lower, the bank will have lower interest payments on its new CDs, but it will also incur a loss on its futures position and therefore will have more funds that need to be financed at the lower rates. The impact that rates have on the amount of funds needed to be financed and the rate paid on them will exactly offset each other, leaving the bank with a fixed debt amount when the September CDs mature in December. This can be seen in Table 18.2, where the bank's December liability (the liability at end of the 180-day period) is shown to be approximately \$102.4914 million given September LIBOR scenarios of 4.5% and 5.5% (this will be true at any rate). Note, the debt at the end of 180 days of \$102.4914 million equates to a 180-day rate for the period of 5.117%:

$$R = \left[\frac{\$102.4914 \text{ million}}{\$100 \text{ million}}\right]^{365/180} - 1 = .05117$$

Hedging with a Eurodollar Futures Put Instead of hedging its future CD sale with Eurodollar futures, the bank could alternatively buy put options on Eurodollar futures. By hedging with puts, the bank would be able to cap the maximum rate it pays on its September CD. For example, suppose the bank decides to hedge its September CD sale by buying September Eurodollar futures puts with expirations coinciding with the maturity of its September CD, exercise price of 95 ($X = \$987{,}500$), and a premium of 2 (multiplier = \$250). With the September debt from the June CD of \$101,210,311, the bank would need to buy 102.491454 September Eurodollar futures puts (assume perfect divisibility) at a total cost of \$51,246 to cap the rate it pays on its September CD:

$$n_P = \frac{\$101{,}210{,}311}{\$987{,}500} = 102.491454 \text{ Contracts}$$

$$\text{Cost} = (102.49154)(2)(\$250) = \$51{,}246$$

If the LIBOR at the September expiration is greater than 5%, the bank will have to pay a higher rate on its September CD, but it will profit from its Eurodollar futures put position, with the put profits being greater, the higher the rate. The put profit would serve to reduce part of the \$101,210,311 funds the bank would need to pay on its maturing June CD. This reduction would, in turn, offset the higher rate it would have to pay on its September CD. As shown in Table 18.3, if the LIBOR is at discount yield higher than 5%, then the bank would be able to lock in a debt obligation 90 days later of \$102,491,454 for an effective 180-day rate of 5.1% (this excludes the cost of the puts). On the other hand, if the rate is less than or equal to 5%, then the bank would be able to finance its \$101,210,311 debt at lower rates. As a result, for lower rates the bank would realize a lower debt obligation 90 days later and therefore a lower rate paid over the 180-day period. Thus, for the cost of the puts, hedging the maturity gap with puts allows the bank to lock in a maximum rate on its debt obligation, with the possibility of paying lower rates if interest rates decrease.

18.3 CROSS HEDGING

Price-Sensitivity Model

The preceding examples represent perfect hedging cases in which certain revenues or costs can be locked in at a future date. Whether it is a commodity, currency, equity position, or a fixed-income position, the presence of quality, timing, and quantity risks makes perfect hedges the exception and not the rule. In practice, hedging risk cannot be eliminated totally by hedging with futures contracts. As a result, the objective in hedging is to try to minimize risk. If the debt position to be hedged has a futures contract with the same underlying asset, such as in the above hedging cases, then a naive hedge usually will be effective in reducing interest rate risk. Many debt positions, though, involve securities and interest rate positions in which a futures contract on the underlying security does not exist. In such cases, an effective cross hedge needs to be determined to minimize the price risk in the underlying spot position. Two commonly used models for cross hedging are the regression model and

TABLE 18.3 Hedging Maturity Gap with Eurodollar Futures Puts

1 LIBOR%	2 Spot and Futures Price	3 Put Cash Flow	4 Sept Debt on June CD	5 Sept Funds Needed Col (4) – Col (3)	6 December Debt Obligation [Col (5)] $(1+\text{LIBOR})^{(90/365)}$	7 June to Dec Hedged Rate $[\text{Col (6)}/\$100\text{m}]^{(365/180)} - 1$
3.50	$991,553	$ 0	$101,210,311	$101,210,311	$102,072,483	0.042
3.75	$990,964	$ 0	$101,210,311	$101,210,311	$102,133,222	0.044
4.00	$990,376	$ 0	$101,210,311	$101,210,311	$102,193,850	0.045
4.25	$989,790	$ 0	$101,210,311	$101,210,311	$102,254,368	0.046
4.50	$989,205	$ 0	$101,210,311	$101,210,311	$102,314,778	0.047
4.75	$988,623	$ 0	$101,210,311	$101,210,311	$102,375,078	0.049
5.00	$988,042	$ 0	$101,210,311	$101,210,311	$102,435,270	0.050
5.25	$987,462	$3,852	$101,210,311	$101,206,459	$102,491,454	0.051
5.50	$986,885	$63,040	$101,210,311	$101,147,271	$102,491,454	0.051
5.75	$986,309	$122,053	$101,210,311	$101,088,258	$102,491,454	0.051
6.00	$985,735	$180,893	$101,210,311	$101,029,418	$102,491,454	0.051
6.25	$985,163	$239,560	$101,210,311	$100,970,751	$102,491,454	0.051
6.50	$984,592	$298,055	$101,210,311	$100,912,256	$102,491,454	0.051

the price-sensitivity model. In the ***regression model***, the estimated slope coefficient of the regression equation is used to determine the hedge ratio. The coefficient, in turn, is found by regressing the spot price on the bond to be hedged against the futures price.

The second hedging approach is to use the ***price-sensitivity model*** developed by Kolb and Chiang (1981) and Toevs and Jacobs (1986). This model has been shown to be relatively effective in reducing the variability of debt positions. The model determines the number of futures contracts that will make the value of a portfolio consisting of a fixed-income security and an interest rate futures contract invariant to small changes in interest rates. The optimum number of futures contracts that achieves this objective is

$$n_f = \frac{\text{Dur}_S}{\text{Dur}_f} \frac{V_0}{f_0} \frac{(1 + \text{YTM}_f)^T}{(1 + \text{YTM}_S)^T} \tag{18.1}$$

where Dur_S = duration of the bond being hedged
Dur_f = duration of the bond underlying the futures contract (for T-bond futures this would be the cheapest-to-deliver bond)
V_0 = current value of bond to be hedged
YTM_S = yield to maturity on the bond being hedged
YTM_f = yield to maturity implied on the futures contract

For options hedging, the number of options (calls for hedging long positions and puts for short positions) using the price-sensitivity model is:

$$n_{\text{options}} = \frac{\text{Dur}_S}{\text{Dur}_{\text{option}}} \frac{V_0}{X} \frac{(1 + \text{YTM}_{\text{option}})^T}{(1 + \text{YTM}_S)^T}$$

where $\text{Dur}_{\text{option}}$ = duration of the bond underlying the options contract.

Example: Hedging a Bond Portfolio with T-Bond Futures Puts

Suppose a bond portfolio manager is planning to liquidate part of his portfolio in September. The portfolio he plans to sell consists of investment grade bonds with a weighted average maturity of 15.25 years, face value of $10 million, weighted average yield of 8%, portfolio duration of 10, and current value of $10 million. Suppose the manager would like to benefit from lower long-term rates that he expects to occur in the future but would also like to protect the portfolio sale against the possibility of a rate increase. To achieve this dual objective, the manager could buy an OTC spot or exchange-traded futures put on a T-bond. Suppose there is a September 95 (X = $95,000) T-bond futures put option trading at $1,156 with the cheapest-to-deliver T-bond on the put's underlying futures being a bond with a current maturity of 15.25 years, duration of 9.818, and currently priced to yield 6.0%. Using the price-sensitivity model, the manager would need to buy 81 puts at a cost of \$93,636 to hedge his bond portfolio:

$$n_p = \frac{\text{Dur}_S}{\text{Dur}_p}\frac{V_0}{X}\frac{(1+\text{YTM}_p)^T}{(1+\text{YTM}_S)^T}$$

$$n_p = \frac{10}{9.818}\frac{\$10{,}000{,}000}{\$95{,}000}\frac{(1.06)^{15.25}}{(1.08)^{15.25}} \cong 81$$

$$\text{Cost} = (81)(\$1{,}156) = \$93{,}636$$

Suppose that in September, long-term rates were higher, causing the value of the bond portfolio to decrease from \$10 million to \$9.1 million and the price on September T-bond futures contracts to decrease from 95 to 86. In this case, the bond portfolio's \$900,000 loss in value would be partially offset by a \$635,364 profit on the T-bond futures puts: $\pi = 81(\$95{,}000 - \$86{,}000) - \$93{,}636 = \$635{,}364$. The manager's hedged portfolio value would therefore be \$9,735,364; a loss of 2.6% in value (this loss includes the cost of the puts) compared to a 9% loss in value if the portfolio were not hedged. On the other hand, if rates in September were lower, causing the value of the bond portfolio to increase from \$10 million to \$10.5 million and the prices on the September T-bond futures contracts to increase from 95 to 100, then the puts would be out of the money and the loss would be limited to the \$93,636 costs of the put options. In this case, the hedged portfolio value would be \$10,406,365—a 4.06% gain in value compared to the 5% gain for an unhedged position.

Note: If the manager were more certain that long-term rates would increase in the future, then he could minimize interest rate risk by alternatively going short in T-bond futures and using the price-sensitivity model to determine the number of contracts he needed to effectively hedge his position.

18.4 SPECULATING WITH INTEREST RATE DERIVATIVES

Although interest rate derivatives are extensively used for hedging, they are also frequently used to speculate on expected interest rate changes. A long futures or call position can be taken when interest rates are expected to fall and a short futures or put position can be taken when rates are expected to rise. In the case of futures, speculating on interest rate changes by taking an outright or naked futures position represents an alternative to buying or short-selling a bond on the spot market. Because of the risk inherent in such outright futures positions, though, some speculators form intracommodity and intercommodity spreads instead of taking a naked position. A futures spread is formed by taking long and short positions on different futures contracts simultaneously. Two general types of spreads exist: intracommodity and intercommodity. An ***intracommodity spread*** is formed with futures contracts on the same asset but with different expiration dates; an ***intercommodity spread*** is formed with two futures contracts with the same expiration but on different assets.

Intracommodity Spread

An intracommodity spread is often used to reduce the risk associated with a pure outright position. Consistent with the carrying-cost model, the prices on more distant futures contracts (T_2) are more price sensitive to changes in the spot price, S, than near-term futures (T_1):

$$\frac{\%\Delta f_{T_2}}{\%\Delta S} > \frac{\%\Delta f_{T_1}}{\%\Delta S}$$

Thus, a speculator who expects the interest rate on long-term bonds to decrease in the future could form an intracommodity spread by going long in a longer term T-bond futures contract and short in a shorter term one. This type of intracommodity spread will be profitable if the expectation of long-term rates decreasing occurs. That is, the increase in the T-bond price resulting from a decrease in long-term rates will cause the price on the longer term T-bond futures to increase more than the shorter term one. As a result, a speculator's gains from his long position in the longer term futures will exceed his losses from his short position. If rates rise, though, losses will occur on the long position; these losses will be offset partially by profits realized from the short position on the shorter term contract. On the other hand, if a bond speculator believes rates will increase but does not want to assume the risk inherent in an outright short position, he could form a spread with a short position in a longer term contract and a long position in the shorter term one. Note that in forming a spread, the speculator does not have to keep the ratio of long-to-short positions one-to-one, but instead could use any ratio (two-to-one, three-to-two, etc.) to obtain his desired return-risk combination.

Intercommodity Spread

Intercommodity spreads consist of long and short positions on futures contracts with the same expirations, but with different underlying assets. Recall, in Chapter 13 we defined two active bond strategies: the ***rate-anticipation swap*** and the ***quality swap***. These swap strategies can be set up as intercommodity spreads formed with different debt security futures.

Consider the case of a speculator who is forecasting a general decline in interest rates across all maturities (i.e., a downward parallel shift in the yield curve). Since bonds with greater maturities are more price sensitive to interest rate changes than those with shorter maturities, the speculator could set up a rate-anticipation swap by going long in a longer term bond with the position partially hedged by going short in a shorter term one. Instead of using spot securities, the speculator alternatively could form an intercommodity spread by going long in a T-bond futures contract that is partially hedged by a short position in a T-note (or T-bill) futures contract. On the other hand, if an investor were forecasting an increase in rates across all maturities, instead of forming a rate-anticipation swap with spot positions, she could go short in the T-bond futures contract and long in the T-note (or T-bill). Forming spreads with T-note and T-bond futures is sometimes referred to as the ***NOB strategy*** (***notes over bonds***).

Another type of intercommodity spread is a quality swap formed with different futures contracts on bonds with different default risk characteristics. For example, a spread formed with futures contracts on a T-bond and a Municipal Bond Index (MBI), or contracts on T-bills and Eurodollar deposits. Like the quality swap formed with spot positions that were discussed in Chapter 13, profits from these futures spreads are based on the ability to forecast a narrowing or a widening of the spread between the yields on the underlying bonds. For example, in an economic recession the demand for lower default-risk bonds often increases relative to the demand for higher default-risk bonds. If this occurs, then the default risk spread (lower-grade bond yields minus higher-grade bond yields) would tend to widen. A speculator forecasting an economic recession could, in turn, profit from an anticipated widening in the risk premium by forming an intercommodity spread consisting of a long position in a T-bond futures contract and a short position in an MBI contract. Similarly, since Eurodollar deposits are not completely risk free, where T-bills are, a spreader forecasting riskier times (and therefore a widening of the spread between Eurodollar rates and T-bill rates) could go long in the T-bill futures contract and short in the Eurodollar futures contract. A spread with T-bills and Eurodollars contracts is referred to as a ***TED spread***.

Speculating with Interest Rate Options

As explained in Chapter 17, one of the important features of options is that they can be combined with positions in the underlying security and other options to generate a number of different investment strategies. In the case of speculating on interest rates using options, a speculator who expects long-term interest rates to decrease in the near future could profit (if her expectation is correct) by taking a long position in a T-bond futures call. As a speculative strategy, this long call position can be viewed as an alternative to a long position in a T-bond futures contract. In contrast, if a speculator expects long-term interest rates to increase in the near future, then she should take a long position in a T-bond futures put option. Similar positions could be taken in Eurodollar or T-bill futures options by speculators who expect short-term rates to change.

Between outright call and put positions, options can be combined in different ways to obtain various types of profit relations. As we noted in Chapter 17, speculators who expect rates to increase in the future but don't want to assume the risk inherent in a put purchase position could form a bear call spread. In contrast, speculators who expect rates to be stable over the near term could, in turn, try to profit by forming a straddle write. Thus, by combining different option positions, speculators can obtain positions that match their expectation and their desired risk-return preference.

Managing Asset and Liability Positions

Interest rate derivatives can also be used by financial and nonfinancial corporations to alter the exposure of their balance sheets to interest rate changes. The change can be done for speculative purposes (increasing the firm's exposure to interest rate changes) or for hedging purposes (reducing exposure). As an example, consider the case of a bond fund that manages its bond portfolio against Barclays' aggregate government/corporate index. Suppose the fund expects interest rates to decrease in the coming year across all maturities. To outperform the index, suppose the fund would like to lengthen the duration of its bond fund relative to the index's duration. The fund could do this by swapping some of its shorter term Treasuries in its portfolio for a longer term one. Given that longer term (higher-duration) bonds are more price sensitive to interest rate changes, the bond fund would find an interest rate decrease across all maturities would cause the value of its bond portfolio to increase proportionally more than the index if it made the swap. However, instead of increasing the duration of its bond portfolio by changing the fund's allocation from long-term to short-term Treasuries, the fund alternatively could take a long position in T-bond futures contracts. If rates, in turn, were to decrease across all maturities as expected, then the fund would realize not only an increase in the value of its bond portfolio, but also a profit from its long futures position; on the other hand, if rates were to increase, then the fund would see not only a decrease in the value of its bond portfolio but also losses on its futures position. Thus, by adding futures to its fund, the fund would be changing its bond portfolio's exposure to interest rates by effectively increasing its duration.

Instead of increasing its balance sheet's exposure to interest rate changes, a company may choose to reduce it. For example, if the above bond fund expected interest rates to increase, it could reduce its bond portfolio's duration by taking a short position in an interest rate futures contract. This action would be similar to the bond portfolio hedging example discussed in Section 18.3.

Using derivatives to change the exposure of an asset or liability to interest rates, exchange rates, or other market parameters without changing the original composition of the assets and liabilities is referred to as ***off-balance sheet restructuring***. It should be noted that companies need to guard against unplanned actions that might change their hedging positions to speculative ones. Many companies have traders hired to manage their risk exposure to interest rates. As time goes by, some traders (often under the illusion they can beat the market) take more speculative positions, often causing a transformation of the company's treasury department into a de facto profit center.

18.5 SYNTHETIC DEBT AND INVESTMENT POSITIONS

There are some cases in which the rates on debt and investment positions can be improved by creating synthetic positions with futures and other derivative securities such as swaps. These cases involve creating a synthetic fixed-rate loan by combining a floating-rate loan with short positions in Eurodollar contracts and creating a synthetic floating-rate loan by combining a fixed-rate loan with long positions in Eurodollar contracts. Similar synthetic fixed-rate and floating-rate investment positions can also be formed.

Synthetic Fixed-Rate Loan

A corporation wanting to finance its operations or its capital expenditures with fixed-rate debt has a choice of either a direct fixed-rate loan or a synthetic fixed-rate loan formed with a floating-rate loan and short positions in Eurodollar futures contracts, whichever is cheaper. Consider the case of a corporation that can obtain a \$10 million fixed-rate loan from a bank at 9.5% or alternatively can obtain a one-year, floating-rate loan from a bank. In the floating-rate loan agreement, suppose the loan starts on date 9/20 at a rate of 9.5% and then is reset on 12/20, 3/20, and 6/20 to equal the spot LIBOR (annual) plus 250 basis points divided by four: (LIBOR+.025)/4.

To create a synthetic fixed-rate loan from this floating-rate loan, the corporation could go short in a series of Eurodollar futures contracts—a ***Eurodollar strip***. For this case, suppose the company goes short in a series of 10 contracts expiring at 12/20, 3/20, and 6/20 and trading at the following prices:

T	**12/20**	**3/20**	**6/20**
Index	93.5	93.75	94
f_0	\$983,750	\$984,375	\$985,000

The locked-in rates obtained using Eurodollar futures contracts are equal to 100 minus the index plus the basis points on the loan:

$$\text{Locked-in rate} = [100 - \text{Index}] + \text{bp}/100$$
$$12/20 : R_{12/20} = [100 - 93.5] + 2.5 = 9\%$$
$$3/20 : R_{3/20} = [100 - 93.75] + 2.5 = 8.75\%$$
$$6/20 : R_{6/20} = [100 - 94] + 2.5 = 8.5\%$$

For example, suppose on date 12/20, the settlement LIBOR is 7%, yielding a settlement index price of 93 and a closing futures price of \$982,500. At that rate, the corporation would realize a profit of \$12,500 [= (10)(\$1,250)] from its 10 short positions on the 12/20 futures contract:

$$f_T = \frac{(100 - (100 - 93)(90/360)\ (\$1{,}000{,}000)}{100} = \$982{,}500$$
$$\text{Profit on 12/20 contract} = (10)\ (\$983{,}750 - \$982{,}500) = \$12{,}500$$

At the 12/20 date, though, the new interest that the corporation would have to pay for the next quarter would be set at \$237,500:

12/20 interest = [(LIBOR + .025)/4](\$10,000,000)

12/20 interest = [(.07 +.025)/4](\$10,000,000)

12/20 interest = \$237,500

Subtracting the futures profit of \$12,500 from the \$237,500 interest payment (and ignoring the time value factor), the corporation's hedged interest payment for the next quarter is \$225,000. On an annualized basis, this equates to a 9% interest on a \$10 million loan, the same rate as the locked-in rate:

$$\text{Hedged rate} = \frac{4(\$225{,}000)}{\$10{,}000{,}000} = .09$$

On the other hand, if the 12/20 LIBOR were 6%, then the quarterly interest payment would be only \$212,500 (= [(.06 + .025)/4](\$10,000,000)] = \$212,500). This gain to the corporation, though, would be offset by a \$12,500 loss on the futures contract (i.e., at 6%, f_T = \$985,000, yielding a loss on the 12/20 contract of 10(\$983,750 − \$985,000) = −\$12,500). As a result, the total quarterly debt of the company again would be \$225,000 [= \$212,500 + \$12,500]. Ignoring the time value factor, the annualized hedged rate the company pays would again be 9%. Thus, the corporation's short position in the 12/20 Eurodollar futures contract at 93.5 enables it to lock in a quarterly debt obligation of \$225,000 and a 9% annualized borrowing rate.

Given the other locked-in rates, the one-year fixed rate for the corporation on its floating-rate loan hedged with the Eurodollar futures contracts would be 8.9369%:

$$\text{Synthetic fixed rate} = \left[(1.095)^{.25}(1.09)^{.25}(1.0875)^{.25}(1.085)^{.25}\right]^1 - 1 = .089369$$

Thus, the corporation would gain by financing with a synthetic fixed-rate loan at 8.9369% instead of a direct fixed-rate loan at 9.5%.

Synthetic Floating-Rate Loan

A synthetic floating-rate loan is formed by borrowing at a fixed rate and taking a long position in a Eurodollar or T-bill futures contract. For example, suppose the corporation in the preceding example had a floating-rate asset it wanted to finance and wanted a floating-rate loan instead of a fixed one. It could take the one offered by the bank of LIBOR plus 250 bp or it could form a synthetic floating-rate loan by borrowing at a fixed rate for one year and going long in a series of Eurodollar futures expiring at 12/20, 3/20, and 6/20. The synthetic loan will provide a lower rate than the direct floating-rate loan if the fixed rate is less than 9%. For example, suppose the corporation borrows at a fixed rate of 8.5% for one year with interest payments made quarterly at dates 12/20, 3/20, and 6/20 and then goes long in the series of Eurodollar futures to form a synthetic floating-rate loan. On date 12/20, if the settlement LIBOR were 7% (settlement index price of 93 and a closing futures price of \$982,500), the corporation would lose \$12,500 [= (10)(\$982,500 – \$983,750)] from its 10 long positions on the 12/20 futures contracts and would pay \$212,500 on its fixed-rate loan [(.085/4)(\$10,000,000) = \$212,500]. The company's effective annualized rate would be 9% ([4(\$212,500 + \$12,500)]/\$10,000,000 = .09), which is .5% less than the rate paid on the floating-rate loan (LIBOR + 250BP = 7% + 2.5% = 9.5%). If the settlement LIBOR were 6%, though (settlement index price of 94 and a closing futures price of \$985,000), the corporation would realize a profit of \$12,500 [= (10)(\$985,000 − \$983,750)] from the 10 long positions on the 12/20 futures contracts and would pay \$212,500 on its fixed-rate loan. Its effective annualized rate would be 8% [(4)($212,500 − $12,500)/$10,000,000 = .08)], which again is .5% less than the rate on the floating-rate loan (LIBOR + 250bp = 6% + 2.5% = 8.5%).

Synthetic Investments

Futures can also be used on the asset side to create synthetic fixed- and floating-rate investments. An investment company setting up a three-year unit investment trust offering a fixed rate could invest funds either in three-year fixed-rate securities or a synthetic one formed with a three-year floating-rate note tied to the LIBOR and long positions in a series of Eurodollar futures, whichever yields the higher rate. By contrast, an investor looking for a floating-rate security could alternatively consider a synthetic floating-rate investment consisting of a fixed-rate security and a short Eurodollar strip. Several problems at the end of this chapter deal with constructing these synthetic investments.

18.6 USING OPTIONS TO SET A CAP OR FLOOR ON A CASH FLOW

The above cases involved creating a fixed or floating rate on the cash flow of an asset or liability. When there is a series of cash flows, such as a floating-rate loan or an investment in a floating-rate note, a strip of interest rate options can be used to place a cap or a floor on the cash floors. For example, a company with a one-year floating-rate loan starting in September at a specified rate and then reset in December, March, and June to equal the spot LIBOR plus bp, could obtain a cap on the loan by buying a series of Eurodollar futures puts expiring in December, March, and June. At each reset date, if the LIBOR exceeds the discount yield on the put, the higher LIBOR applied to the loan will be offset by a profit on the nearest expiring put, with the profit increasing the greater the LIBOR; if the LIBOR is equal to or less than the discount yield on the put, the lower LIBOR applied to the loan will only be offset by the limited cost of the put. Thus, a strip of Eurodollar futures puts used to hedge a floating-rate loan places a ceiling on the effective rate paid on the loan.

In the case of a floating-rate investment, such as a floating-rate note tied to the LIBOR or a bank's floating rate loan portfolio, a minimum rate or floor can be obtained by buying a series of Eurodollar futures calls, with each call having an expiration near the reset date on the investment. If rates decrease, the lower investment return will be offset by profits on the calls; if rates increase, the only offset will be the limited cost of the calls. Several end-of-the chapter problems are included that involve using futures call and put options to cap a loan and set a floor on an investment.

18.7 CONCLUSION

Introduced during the volatile interest rate periods of the 1970s and 1980s, interest rate derivatives have become one of the most popular derivative contracts. In this chapter, we have examined the applications of exchange-traded futures and options. These derivatives can be used by (1) financial institutions to manage the maturity gap between their assets and liabilities; (2) financial and nonfinancial corporations to fix the rates on their floating-rate loans, to create synthetic fixed-rate or floating-rate debt and investment positions, or to set a cap on a floating-rate loan or a floor on a floating-rate note; and (3) fixed-income managers, money market managers, and dealers to lock in the future purchase or selling price of their fixed-income securities. In the next chapter, we will continue the analysis of interest rate derivatives by examining the use of tailor-made OTC interest rate derivative products that are also widely used for fixed-income management.

KEY TERMS

Eurodollar strip
intercommodity spread
intracommodity spread
maturity gap
naive hedging model
notes over bonds (NOB) strategy
off-balance sheet restructuring
price-sensitivity model
quality swap
rate-anticipation swap
regression model
TED spread

PROBLEMS AND QUESTIONS

1. In June, the Kendall Money Market Fund forecast a September cash inflow of \$18 million that it plans to invest for 91 days in T-bills. The fund is uncertain about future short-term interest rates and would like to lock in the rate on the September investment with T-bill futures contracts. Currently, September T-bill contracts are trading at 93 (IMM index).
   a. What is the implied YTM on the September T-bill futures contract?
   b. How many September contracts does Kendall need to lock in the implied futures YTM (assume perfect divisibility)?
   c. Assuming the fund's \$18 million cash inflow comes at the same time as the September futures contract's expiration, show how the fund's futures-hedged T-bill purchase yields the same rate from an \$18 million investment as the implied YTM on the futures. Evaluate at spot T-bill rates at the futures' expiration of 6.5% and 8.5%.
2. Suppose the Kendall Money Market Fund in Question 1 expects interest rates to be higher in September when it plans to invest its \$18 million cash flow in 91-day T-bills, but is worried that rates could decrease. Suppose there is a September T-bill futures call contract with an exercise price of 93 (IMM index), trading at 5, and expiring at the same time as the September T-Bill futures contract.
   a. How many September T-bill futures call options does Kendall need in order to lock in a minimum rate on its investments (do not assume perfect divisibility)? What is the cost?

b. Assuming the fund's $18 million cash inflow comes at the same time as the September T-bill futures call contract expires, use the table below to determine the fund's option-hedged T-bill yield for possible spot discount rates, at the option's expiration date, of 6%, 6.25%, 6.5%, 7%, 7.5%, and 8%.

1	2	3	4	5	6
Spot Discount Rates	Spot Price = Futures Price	Call Profit/Loss	Hedged Investment Funds	Number of Bills	YTM
6.00					
6.25					
6.50					
7.00					
7.50					
8.00					

3. Bryce National Bank is planning to make a $10 million short-term loan to Midwest Mining Company. In the loan contract, Midwest agrees to pay the principal and an interest of 12% (annual) at the end of 180 days. Since Bryce National sells more 90-day CDs than 180-day CDs, it is planning to finance the loan by selling a 90-day CD now at the prevailing LIBOR of 8.25%, and then 90 days later (mid-September) selling another 90-day CD at the prevailing LIBOR. The bank would like to minimize its exposure to interest rate risk on its future CD sale by taking a position in a September Eurodollar futures contract trading at 92 (IMM index).

a. How many September Eurodollar futures contracts would Bryce National Bank need to effectively hedge its September CD sale against interest rate changes? Assume perfect divisibility.

b. Determine the total amount of funds the bank would need to raise on its CD sale 90 days later if the LIBOR is 7.5% and if it is 9% (assume futures are closed at the LIBOR). What would the bank's debt obligations be at the end of the 180-day period? What is the bank's effective rate for the entire 180-day period?

4. Suppose Bryce National Bank in Question 3 makes a $10 million, 180-day loan to Midwest Mining Company with the loan financed by selling a 90-day CD now at the prevailing LIBOR of 8.25% and then 90 days later (mid-September) selling another 90-day CD at the prevailing LIBOR. Suppose, though, the bank would like to minimize its exposure to interest rate risk on its future CD sale but would also like to benefit if CD rates decrease. Suppose there is a September Eurodollar futures put with an exercise price of 92 trading on the CME at 2.

a. How many September Eurodollar futures puts would Bryce National Bank need in order to effectively hedge its September CD sale against interest rate changes? Assume perfect divisibility.

b. Assume that the Eurodollar futures are closed at the LIBOR and the Eurodollar futures and futures options expire at the same time as the Bank's first CD. Using the table below, determine the total amount of funds the bank would need to raise on its September CD, the bank's debt obligations at the end of the 180-day period, and the bank's hedged rates for the entire 180-day period given the following LIBORs at the options and first CD maturity date: 7%, 7.5%, 8%, 8.5%, 9%, 9.5%, and 10%.

1	2	3	4	5	6	7
LIBOR	Spot and Futures Price	Put Profit	Debt on Sept-Issued CD	Sept Funds Needed	Dec Debt Obligation	Hedged Rate for 180 Days
.07						
.075						
.08						
.085						
.09						
.10						

5. Mr. Devine is a fixed-income portfolio manager for Stacy Investments. He forecasts a cash outflow of $10 million in June and plans to sell his baseline bond portfolio. The fund currently is worth $10 million, has an A quality rating, duration of seven years, weighted average maturity of 15 years, annual coupon rate of 10.25%, and YTM of 10.25% (note: the fund is selling at its par value). Suppose Mr. Devine is afraid that long-term interest rates could increase and decides to hedge his June sale by taking a position in June T-bond futures contracts when the June T-bond contract is trading at 80-16 and the T-bond most likely to be delivered on the contract has a YTM of 9.5%, maturity of 15 years, and a duration of nine years.
   a. Using the price-sensitivity model, explain how Mr. Devine could hedge his June bond portfolio sale against interest rate risk.
   b. Suppose long-term interest rates increase over the period such that at the June expiration Mr. Devine's baseline portfolio (A-rated, 10.25% coupon rate, 15-year maturity, and seven-year duration) is trading at 96 of par and the price on the expiring June T-bond contract (f_T) is 76. Determine Mr. Devine's revenue from selling his baseline bond portfolio, his profit on the futures contracts, and his total revenue.
6. As an alternative to a nine-month, 10% fixed-rate loan for $10 million, the Zuber Beverage Company is considering a synthetic fixed-rate loan formed with a $10 million floating-rate loan from First National Bank and a Eurodollar strip. The floating-rate loan has a maturity of 270 days (.75 of a year), starts on December 20th, and the rate on the loan is set each quarter. The initial quarterly rate is equal to 9.5%/4, the other rates are set on 3/20 and 6/20 equal to one fourth of the annual LIBOR on those dates plus 100 basis points: (LIBOR % + 1%)/4. On December 20th, the Eurodollar futures contract expiring on 3/20 is trading at 91 (IMM index), the contract expiring on 6/20 is trading at 92, and the time separating each contract is .25/year.
   a. Explain how Zuber could use the strip to lock in a fixed rate. Calculate the rate the Zuber Company could lock in with a floating-rate loan and Eurodollar futures strip.

b. Calculate and show in a table the company's quarterly interest payments, futures profits, hedged interest payments (interest minus futures profit), and hedged rate for each period (12/20, 3/20, and 6/20) given the following rates: LIBOR = 10% on 3/20 and LIBOR = 9% on 6/20.

7. Given the following CME Eurodollar futures put options:
   - March Eurodollar futures put with exercise price of 91 (IMM index) selling at 2.
   - June Eurodollar futures put with exercise price of 91 (IMM index) selling at 2.1.

   a. Explain how the Zuber Beverage Company in Question 6 could attain a cap on its floating-rate loan with these options. What is the cost of the put strip?

   b. Show in the table below the company's quarterly interest payments, option cash flow, hedged interest payments (interest minus option cash flow), and hedged rate as a proportion of a $10 million loan (do not include option cost) for each period (12/20, 3/20, and 6/20) given the following rates: LIBOR = 10% on 3/20 and LIBOR = 9% on 6/20. Assume the interest reset dates and option expiration dates coincide.

1	2	3	4	5	6	7	8	9
Date	LIBOR	Futures and Spot Price	Put Cash Flow at Option Expiration	Value of Put CF at Payment Date	Loan Interest on Payment Date	Hedged Debt	Hedged Rate	Unhedged Rate
12/20								
3/20								
6/20								
9/20								

8. XSIF Trust is planning to invest $10 million for one year. As an alternative to a one-year fixed-rate note paying 8.5%, XSIF is considering a synthetic investment formed by investing in a Second National Bank one-year floating-rate note (FRN) paying LIBOR plus 100 basis points and taking a position in a Eurodollar futures strip. The FRN starts on 12/20 at 9% (LIBOR = 8%) and is then reset the next three quarters on 3/20, 6/20, and 9/20. On December 20th, the Eurodollar futures contract expiring on 3/20 is trading at 91 (IMM index), the contract expiring on 6/20 is trading at 92, and the contract expiring on 9/20 is trading at 92.5; the time separating each contract is .25/year and the reset dates on the floating-rate note and the expiration dates on the futures expiration are the same.

   a. Explain how XSIF Trust could use a strip to lock in a fixed rate. Calculate the rate XSIF could lock in with a floating-rate note and Eurodollar futures strip.

   b. Calculate and show in a table XSIF's quarterly interest receipts, futures profits, hedged interest return (interest plus futures profit), and hedged rate for each period (12/20, 3/20, 6/20, and 9/20) given the following rates: LIBOR = 9.5% on 3/20, LIBOR = 9% on 6/20 and LIBOR = 7% on 9/20.

9. Given the following CME Eurodollar futures call options:
   - June Eurodollar futures call with exercise price of 93 (IMM index) selling at 2.
   - September Eurodollar futures call with exercise price of 93 (IMM index) selling at 2.1.
   - December Eurodollar futures call with exercise price of 93 (IMM index) selling at 2.2.

   a. Explain how XSIF Trust in Question 8 could attain a floor on its floating-rate note with the options. What is the cost of the options?

   b. Show in the table below XSIF's quarterly interest receipts, option cash flow, hedged interest revenue (interest plus option cash flow), and hedged rate as a proportion of a $10 million investment (do not include option cost) for each period given the following rates: LIBOR = 7.5% on 3/20, 7% on 6/20, 6.5% on 9/20, and 6% on 12/20. Assume the interest reset dates and option expiration dates coincide.

1	2	3	4	5	6	7	8	9
Date	LIBOR	Futures and Spot Price	Call Cash Flow at Option Expiration	Value of CF at Payment Date	Interest Receipt on Payment Date	Hedged Income	Hedged Rate	Unhedged Rate
3/20								
6/20								
9/20								
12/20								
3/20								

10. Explain the types of spreads bond speculators could use given the following cases:

    a. The yield curve is expected to shift down with rates for bonds with differing maturities decreasing by roughly the same percentage.

    b. Even though the economy is growing, leading economic indicators augur for an economic recession.

    c. Even though the economy is in recession, leading economic indicators point to an economic expansion.

WEB EXERCISES

1. Read more about CME interest rate derivatives by going to www.cme.com.
2. Make a forecast for future intermediate or long-term interest rates and then form an intracommodity spread with a listed T-bond or T-note futures contract based on your forecast. For price quotes on futures, go to www.wsj.com/free (Market Data Center, "Commodities and Futures").

NOTE

1. Some of the material in this chapter draws from Johnson, *Introduction to Derivatives*, 2009.

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CHAPTER 19

Managing Fixed-Income Positions with OTC Derivatives

19.1 INTRODUCTION[1]

Today, there is a large OTC market in debt and interest-sensitive securities and products in the United States and a growing OTC market outside the United States. OTC derivatives are primarily used by financial institutions and nonfinancial corporations to manage their interest rate positions. As discussed in Chapters 16 and 17, the derivative contracts offered in the OTC market include spot options and forward contracts on Treasury securities, LIBOR-related securities such as forward rate agreements, and special types of interest rate products, such as interest rate calls and puts, caps, floors, and collars. OTC interest rate derivative products are typically private, customized contracts between two financial institutions or between a financial institution and one of its clients. In this chapter, we continue that analysis of the management of fixed-income positions by examining the hedging uses of OTC interest rate derivatives. We begin by looking at the hedging uses of forward rate agreements, interest rate calls, interest rate puts, caps, and floors. With this background, we then look at how caps and floors are used to manage interest rate positions. We finish the chapter by looking at hedging foreign currency-denominated debt positions with currency options.

19.2 HEDGING WITH OTC DERIVATIVES

Forward Rate Agreements

As explained in Chapter 16, a ***forward rate agreement (FRA)*** is a contract requiring a cash payment or providing a cash receipt based on the difference between a realized spot rate such as the LIBOR and a prespecified rate.[2] For example, the contract could be based on a specified rate of $R_k = 6\%$ (annual) and the three-month LIBOR (annual) in five months and a notional principal, NP of \$10 million. In five months, the payoff would be

$$\text{Payoff} = (\$10{,}000{,}000)\frac{[\text{LIBOR} - .06]\,(91/365)}{1 + \text{LIBOR}(91/365)}$$

If the LIBOR at the end of five months exceeds the specified rate of 6%, the buyer of the FRA (or long position holder) receives the payoff from the seller; if the LIBOR is less than 6%, the seller (or short position holder) receives the payoff from the buyer. In general, an FRA that matures in T months and is written on an M-month LIBOR rate is referred to as a $T \times (T + M)$ agreement. Thus, in this example the FRA is a 5×8 agreement. At the maturity of the contract (T), the value of the contract, V_T is

$$V_T = \text{NP}\frac{[\text{LIBOR} - R_k]\,(M/365)}{1 + \text{LIBOR}(M/365)}$$

Today, FRAs are offered by banks and financial institutions in major financial centers and are often offered to the bank's corporate customers who want to lock in the rate on a future loan or investment.

Example: Hedging the Rate on a Future CD Investment with an FRA Suppose Kendall Manufacturing forecasts a cash inflow of $10 million in two months that it is considering investing in a Sun National Bank CD for 90 days. Sun National Bank's jumbo CD pays a rate equal to the LIBOR. Currently such rates are yielding 5.5%. Kendall is concerned that short-term interest rates could decrease in the next two months and would like to lock in a rate now. As an alternative to hedging its investment with Eurodollar futures, Sun suggests that Kendall hedge with a short forward rate agreement with the following terms:

- FRA would mature in two months (T) and would be written on a 90-day (three-month) LIBOR $[T \times (T + M) = 2 \times 5]$ agreement
- NP = $10 million
- Contract rate = $R_k = 5.5\%$
- Day count convention = 90/365
- Kendall would take the short position on the FRA, receiving the payoff from Sun National if the LIBOR were less than $R_k = 5.5\%$.
- Sun National would take the long position on the FRA, receiving the payoff from Kendall if the LIBOR were greater than $R_k = 5.5\%$.

Table 19.1 shows Kendall's FRA receipts or payments and cash flows from investing the $10 million cash inflow plus or minus the FRA receipts or payments at possible LIBORs of 5%, 5.25%, 5%, 5.75%, and 6%. As shown, Kendall is able to earn a hedged rate of return of 5.5% from its $10 million investment:

$$\text{Payoff} = (\$10{,}000{,}000)\frac{[\text{LIBOR} - .055]\,(90/365)}{1 + \text{LIBOR}(90/365)}$$

CF at CD Maturity = CD Investment (1 + LIBOR(90/365))
Hedged Rate = [(CF at maturity/$10 million) – 1](365/90)

Interest Rate Call

As explained in Chapter 17, an interest rate call, also called a ***caplet***, gives the buyer a payoff on a specified payoff date if a designated interest rate rises above a certain exercise rate, R_X.

TABLE 19.1 Hedging CD Investment with Forward Rate Agreement

LIBOR	Sun National Payoff	Kendall Payoff	Kendall CD Investment $10m + FRA Payoff	CF at CD Maturity	Hedged Rate
0.0500	−$12,178.62	$12,178.62	$10,012,178.62	$10,135,616	0.0550
0.0525	−$6,085.60	$6,085.60	$10,006,085.60	$10,135,616	0.0550
0.0550	$0.00	$0.00	$10,000,000.00	$10,135,616	0.0550
0.0575	$6,078.21	−$6,078.21	$9,993,921.79	$10,135,616	0.0550
0.0600	$12,149.03	−$12,149.03	$9,987,850.97	$10,135,616	0.0550

That is, if the rate exceeds R_X, the call pays off the difference between the actual rate and R_X, times a notional principal, NP, times the fraction of the year specified in the contract; if the rate is less than R_X, the interest rate call expires worthless.

Interest rate call options are often written by commercial banks in conjunction with future loans they plan to provide to their customers. The exercise rate on the option usually is set near the current spot rate, with that rate often being tied to the LIBOR. For example, a company planning to borrow from a bank at a future date at a rate equal to the LIBOR plus basis points could buy an interest rate call option from the bank with the call having an exercise rate equal to the current loan rate, expiration at the start of loan, and notional principal equal to the amount of the future loan. At expiration, the company would be entitled to a payoff if rates were higher than the exercise rate, offsetting the higher interest on the loan. On the other hand, if the rate is lower, there is no payoff, but the company does benefit from a lower interest rate on its loan. Thus, an interest rate call allows the company to cap its future interest rate. As a hedging tool, an interest rate call represents an alternative to purchasing a Eurodollar futures put option to cap a future loan rate.

Example: Hedging a Future Loan Rate with an OTC Interest Rate Call As an example, suppose the Bryce Manufacturing Company plans to finance one of its projects with a $20 million, 90-day loan from North Bank, with the loan rate to be set equal to the LIBOR + 100 bp when the project commences 60 days from now. Furthermore, suppose that the company expects rates to decrease in the future, but is concerned that they could increase. To obtain protection against higher rates, suppose the Bryce Company buys an interest rate call option from North Bank for $40,000 with the following terms:

- Exercise rate = 7%
- Reference rate = LIBOR
- Time period applied to the payoff = 90/360
- Notional principal = $20 million
- Payoff made at the maturity date on the loan (90 days after option's expiration)
- Interest rate call's expiration = T = 60 days (time of the loan)
- Interest rate call premium of $40,000 to be paid at the option's expiration with a 7% interest: Cost = $40,000(1 + (.07)(60/360)) = $40,467

TABLE 19.2 Hedging a Future $20 Million, 90-Day Loan with an OTC Interest Rate Call

Company's loan: $20 million at LIBOR+100 bp for 90 days (.25 per year)

Interest rate call option exercise rate = 7%, Reference rate = LIBOR, NP = $20m, Time period = .25, Option expiration = *T* = 60 days

Cost of option = $40,000, payable at *T*, plus 7% interest

(1) LIBOR	(2) Interest Rate Call Payoff: $20m[Max [LIBOR − .07,0] (.25)	(3) Cost of the Option at *T* $40,000 (1 + .07 (60/360))	(4) Interest Paid on Loan at Its Maturity (LIBOR + 100 bp) (.25)($20,000,000)	(5) Cost at Maturity Col (4) − Col (2)	(6) Annualized Hedged Rate 4[Col (5)/ ($20m − Col (3))]
0.0550	$0	$40,467	$325,000	$325,000	0.06513
0.0575	$0	$40,467	$337,500	$337,500	0.06764
0.0600	$0	$40,467	$350,000	$350,000	0.07014
0.0625	$0	$40,467	$362,500	$362,500	0.07265
0.0650	$0	$40,467	$375,000	$375,000	0.07515
0.0675	$0	$40,467	$387,500	$387,500	0.07766
0.0700	$0	$40,467	$400,000	$400,000	0.08016
0.0725	$12,500	$40,467	$412,500	$400,000	0.08016
0.0750	$25,000	$40,467	$425,000	$400,000	0.08016
0.0775	$37,500	$40,467	$437,500	$400,000	0.08016
0.0800	$50,000	$40,467	$450,000	$400,000	0.08016
0.0825	$62,500	$40,467	$462,500	$400,000	0.08016
0.0850	$75,000	$40,467	$475,000	$400,000	0.08016

Table 19.2 shows the company's cash flows from the call, interest paid on the loan, and effective interest costs that would result given different LIBORs at the starting date on the loan and the expiration date on the option. As shown in Column 6 of the table, the company is able to lock in a maximum interest cost of 8.016% if the LIBOR is 7% or greater at expiration, while still benefiting with lower rates if the LIBOR is less than 7%.

Interest Rate Put

An interest rate put, or ***floorlet***, gives the buyer a payoff on a specified payoff date if a designated interest rate is below the exercise rate, R_X. A financial or nonfinancial corporation that is planning to make an investment at some future date could hedge that investment against an interest rate decrease by purchasing a floorlet from a commercial bank, investment banking firm, or dealer. For example, suppose that instead of needing to borrow, the Bryce Company was expecting a net cash inflow in the future from its operations and was planning to invest the funds in a bank CD paying the LIBOR. To hedge against any interest rate decreases, the company could purchase an interest rate put (corresponding to the bank's CD it plans to buy) from the bank with the put having an exercise rate equal to the current LIBOR, expiration coinciding with the date it expects its cash inflow, and notional principal equal to the amount of funds it plans to invest. The interest rate put would provide a payoff for the company if the LIBOR were less than the exercise rate, giving the company a hedge against interest rate decreases. However, if the rate is higher, there is no payoff, but the company does benefit from a higher interest rate on its CD investment. Thus, an interest rate put allows the company to set a floor on its future interest earning. As a hedging tool, an interest rate put represents an alternative to purchasing a Eurodollar futures call option.

Example: Hedging a CD Rate with an OTC Interest Rate Put Suppose the Star Manufacturing Company was expecting a net cash inflow of $20 million in 60 days from its operations and was planning to invest the excess funds in a 90-day CD from Sun Bank paying the LIBOR. To hedge against any interest rate decreases occurring 60 days from now, suppose the company purchases an interest rate put from Sun Bank for $20,000, with the put having the following terms:

- Exercise rate = 7%
- Reference rate = LIBOR
- Time period applied to the payoff = 90/360
- Day count convention = 30/360
- Notional principal = $20 million
- Payoff made at the maturity date on the CD (90 days from option's expiration)
- Interest rate put's expiration = T = 60 days (time of CD purchase)
- Interest rate put premium of $20,000 to be paid at the option's expiration with 7% interest: Cost = $20,000[1 + (.07)(60/360)] = $20,233

As shown in Table 19.3, the purchase of the interest rate put makes it possible for the Star Company to earn a higher rate if the LIBOR is greater than 7% and to lock in a minimum rate of 6.993% if the LIBOR is 7% or less. For example, if 60 days later the LIBOR is at 6.5%, then the company would receive a payoff (90 days later at the maturity of its CD) on the interest rate put of $25,000 [= ($20,000,000)[.07−.065](90/360)]. The $25,000 payoff would offset the lower (than 7%) interest on the company's CD of $325,000 [= ($20,000,000) (.065) (90/360)]. At the maturity of the CD, the company would therefore receive CD interest and an interest rate put payoff equal to $350,000 (= $325,000 + $25,000). With the interest rate put's payoffs increasing the lower the LIBOR, the company would be able to hedge any lower CD interest and lock in a hedged dollar return of $350,000. Based on an investment of $20 million plus the $20,233 costs of the put, the hedged return equates to an effective annualized yield of 6.993% = [(4)($350,000)]/[$20,000,000 + $20,233]. On the other hand, if the LIBOR exceeds 7%, the company benefits from the higher CD rates, while its losses are limited to the $20,233 costs of the puts.

Cap

A plain-vanilla ***cap*** is a series of European interest rate call options—a portfolio of caplets. Caps are often written by financial institutions in conjunction with a floating-rate loan and are used by buyers as a hedge against interest rate risk. For example, a company with a floating-rate loan tied to the LIBOR could lock in a

TABLE 19.3 Hedging a CD Investment with an Interest Rate Put Option

Company's investment = $20 million at LIBOR for 90 days (.25 per year)

Interest rate put option: Exercise rate = 7%, Reference rate = LIBOR, NP = $20 million, Time period = .25, Option expiration = *T* = 60 days

Cost of option = $20,000, payable at *T*, plus 7% interest

1 LIBOR	2 Interest Rate Put Payoff: $20m[Max [.07 − LIBOR,0] (.25)	3 Cost of the Option at *T* $20,000 (1 + .07 (60/360))	4 Interest Received on CD at Its Maturity	5 Revenues at Maturity Col (2) + Col 4	6 Annualized Hedged Rate 4[Col (5)/ ($20m + Col (3))]
0.0550	$75,000	$20,233	$275,000	$350,000	0.06993
0.0575	$62,500	$20,233	$287,500	$350,000	0.06993
0.0600	$50,000	$20,233	$300,000	$350,000	0.06993
0.0625	$37,500	$20,233	$312,500	$350,000	0.06993
0.0650	$25,000	$20,233	$325,000	$350,000	0.06993
0.0675	$12,500	$20,233	$337,500	$350,000	0.06993
0.0700	$0	$20,233	$350,000	$350,000	0.06993
0.0725	$0	$20,233	$362,500	$362,500	0.07243
0.0750	$0	$20,233	$375,000	$375,000	0.07492
0.0775	$0	$20,233	$387,500	$387,500	0.07742
0.0800	$0	$20,233	$400,000	$400,000	0.07992
0.0825	$0	$20,233	$412,500	$412,500	0.08242
0.0850	$0	$20,233	$425,000	$425,000	0.08491

maximum rate on the loan by buying a cap corresponding to its loan. At each reset date, the company would receive a payoff from the caplet if the LIBOR exceeded the cap rate, offsetting the higher interest paid on the floating-rate loan; on the other hand, if rates decrease, the company would pay a lower rate on its loan while its losses on the caplet would be limited to the cost of the option. Thus, with a cap, the company would be able to lock in a maximum rate each quarter, while still benefiting from lower interest costs if rates decrease.

Floor

A plain-vanilla ***floor*** is a series of European interest rate put options—a portfolio of floorlets. Floors are often purchased by investors as a tool to hedge their floating-rate investments against interest rate declines. Thus, with a floor, an investor with a floating-rate security is able to lock in a minimum rate each period, while still benefiting with higher yields if rates increase.

19.3 HEDGING A SERIES OF CASH FLOWS: OTC CAPS AND FLOORS

In the last chapter, we examined how a strip of Eurodollar futures puts could be used to cap the rate paid on a floating-rate loan, and how a strip of Eurodollar futures calls could be used to set a floor on a floating-rate investment. Using such exchange-traded options to establish interest rate floors and ceilings on floating rate assets and liabilities, though, is subject to hedging risk. As a result, many financial and nonfinancial companies looking for such interest rate insurance prefer to buy OTC caps or floors that can be customized to meet their specific needs.

Financial institutions typically provide caps and floors with terms that range from one to five years, have monthly, quarterly, or semiannual reset dates or frequencies, and use the LIBOR as the reference rate. The notional principal and the reset dates usually match the specific investment or loan (the better the fit, though, the more expensive the cap or floor), and the settlement dates usually come after the reset dates. In cases where a floating-rate loan (or investment) and cap (or floor) come from the same financial institution, the loan and cap (or investment and floor) are usually treated as a single instrument so that when there is a payoff, it occurs at an interest payment (receipt) date, lowering (increasing) the payment (receipt). The exercise rate is often set so that the cap or floor is initially out of the money, and the payments for these interest rate products are usually made up front, although some are amortized.

Example: A Floating Rate Loan Hedged with an OTC Cap

As an example, suppose the Zuber Development Company borrows \$100 million from Commerce Bank to finance a two-year construction project. Suppose the loan is for two years, starting on March 1 at a known rate of 8%, then resets every three months—6/1, 9/1, 12/1, and 3/1—at the prevailing LIBOR plus 150 bp. In entering this loan agreement, suppose the Zuber Company is uncertain of future interest rates and therefore would like to lock in a maximum rate, while still benefiting from lower rates if the LIBOR decreases. To achieve this, suppose Zuber buys a cap corresponding to its loan from Commerce Bank for \$300,000, with the following terms:

- The cap consists of seven caplets with the first expiring on 6/1/Y1 and the others coinciding with the loan's reset dates
- Exercise rate on each caplet = 8%
- NP on each caplet = \$100 million
- Reference rate = LIBOR
- Time period to apply to payoff on each caplet = 90/360 (Typically the day count convention is defined by actual number of days between reset dates.)
- Payment date on each caplet is at the loan's interest payment date, 90 days after the reset date
- The cost of the cap = \$300,000; it is paid at beginning of the loan, 3/1/Y1.

On each reset date, the payoff on the corresponding caplet would be

$$\text{Payoff} = (\$100{,}000{,}000)(\text{Max}[\text{LIBOR} - .08, 0])(90/360)$$

With the 8% exercise rate (sometimes called the ***cap rate***), the Zuber Company would be able to lock in a maximum rate each quarter equal to the cap rate plus the basis points on the loan (9.5%), while still benefiting with lower interest costs if

TABLE 19.4 Hedging a Floating-Rate Loan with a Cap

Loan: Floating rate loan; Term = two years; Reset dates: 3/1, 6/1, 9/1, 12/1; Time frequency = .25; Rate = LIBOR + 150bp; Payment date = 90 days after reset date

Cap: Cost of cap = $300,000; Cap rate = 8%; Reference rate = LIBOR; Time frequency = .25; Caplets' expiration: On loan reset dates, starting at 6/1/Y1; Payoff made 90 days after reset date.

1 Reset Date	2 Assumed LIBOR	3 Loan Interest on Payment Date (LIBOR + 150bp) (.25)($100m)	4 Cap Payoff on Payment Date (Max[LIBOR − .08,0]) (.25)($100m)	5 Hedged Interest Payment Col. (3) − Col. (4)	6 Hedged Rate 4[Col (5)/ $100m]
3/1/Y1*	0.065				
6/1/Y1	0.070	$2,000,000	$0	$2,000,000	0.080
9/1/Y1	0.075	$2,125,000	$0	$2,125,000	0.085
12/1/Y1	0.080	$2,250,000	$0	$2,250,000	0.090
3/1/Y2	0.085	$2,375,000	$0	$2,375,000	0.095
6/1/Y2	0.090	$2,500,000	$125,000	$2,375,000	0.095
9/1/Y2	0.095	$2,625,000	$250,000	$2,375,000	0.095
12/1/Y2	0.100	$2,750,000	$375,000	$2,375,000	0.095
3/1/Y3		$2,875,000	$500,000	$2,375,000	0.095

*There is no cap on this date

rates decrease. This can be seen in Table 19.4, where the quarterly interests on the loan, the cap payoffs, and the hedged rate are shown for different assumed LIBORs at each reset date on the loan. For the four reset dates from 3/1/Y2 to the end of the loan, the LIBOR exceeds 8%. In each of these cases, the higher interest on the loan is offset by the payoff on the cap, yielding a hedged rate on the loan of 9.5% (the 9.5% rate excludes the $300,000 cost of the cap). For the first two reset dates on the loan, 6/1/Y1 and 9/1/Y1, the LIBOR is less than the cap rate. At these rates, there is no payoff on the cap, but the rates on the loan are lower with the lower LIBORs.

Example: A Floating Rate Asset Hedged with an OTC Floor

As noted, floors are purchased to create a minimum rate on a floating-rate asset. As an example, suppose Commerce Bank in the above example wanted to establish a minimum rate or floor on the rates it was to receive on the two-year floating-rate loan it made to the Zuber Development Company. To this end, suppose the bank purchased from another financial institution a floor for $200,000 with the following terms corresponding to its floating-rate asset:

- The floor consists of seven floorlets with the first expiring on 6/1/Y1 and the others coinciding with the reset dates on the bank's floating-rate loan to the Zuber Company
- Exercise rate on each floorlet = 8%

- NP on each floorlet = \$100 million
- Reference rate = LIBOR
- Time period to apply to payoff on each floorlet = 90/360. Payment date on each floorlet is at the loan's interest payment date, 90 days after the reset date
- The cost of the floor = \$200,000; it is paid at beginning of the loan, 3/1/Y1

On each reset date, the payoff on the corresponding floorlet would be

$$\text{Payoff} = (\$100{,}000{,}000)(\text{Max}[.08 - \text{LIBOR}, 0])(90/360)$$

With the 8% exercise rate, Commerce Bank would be able to lock in a minimum rate each quarter equal to the floor rate plus the basis points on the floating-rate asset (9.5%), while still benefiting with higher returns if rates increase. In Table 19.5, Commerce Bank's quarterly interests received on its loan to Zuber, its floor payoffs, and its hedged and unhedged yields on its loan asset are shown for different assumed LIBORs at each reset date. For the first two reset dates on the loan, 6/1/Y1 and 9/1/Y1, the LIBOR is less than the floor rate of 8%. At theses rates, there is a payoff on the floor that compensates the Commerce Bank for the lower interest it receives on the loan; this results in a hedged rate of return on the bank's loan asset of 9.5% (the cost of the floor excluded). For the five reset dates from 12/1/Y1 to the end of the loan, the LIBOR equals or exceeds the floor rate. At these rates, there is no payoff on the floor, but the rates the bank earns on its loan are greater, given the greater LIBORs.

19.4 FINANCING CAPS AND FLOORS: COLLARS AND CORRIDORS

The purchaser of a cap or a floor is, in effect, paying a premium for insurance against adverse interest rate movements. The cost of that insurance can be reduced by forming a collar, corridor, or reverse collar.

A ***collar*** is the combination of a long position in a cap and a short position in a floor with different exercise rates. The sale of the floor is used to defray the cost of the cap. For example, the Zuber Development Company in our previous case could reduce the cost of the cap it purchased to hedge its floating-rate rate loan by selling a floor. By forming a collar to hedge its floating-rate debt, the Zuber Company, for a lower net hedging cost, would still have protection against a rate movement against the cap rate, but it would have to give up potential interest savings from rate decreases below the floor rate. For example, suppose the Zuber Company decided to defray the \$300,000 cost of its 8% cap by selling a 7% floor for \$200,000, with the floor having similar terms to the cap (effective dates on floorlet = reset dates, reference rate = LIBOR, NP on floorlets = \$100 million, and time period for rates = .25). By using the collar instead of the cap, Zuber reduces its hedging cost from \$300,000 to \$100,000, and as shown in Table 19.6, the company can still lock in a maximum rate on its loan of 9.5%. However, when the LIBOR is less than 7%, the

TABLE 19.5 Hedging a Floating-Rate Asset with a Floor

Asset: Floating rate loan; Term = two years; Reset dates: 3/1, 6/1, 9/1, 12/1; Time frequency = .25; Rate = LIBOR + 150bp; Payment date = 90 days after reset date

Floor: Cost of floor = $200,000; Floor rate = 8%; Reference rate = LIBOR; Time frequency = .25; Floorlets' expiration: On loan reset dates, starting at 6/1/Y1; Payoff made 90 days after reset date

1 Reset Date	2 Assumed LIBOR	3 Interest Received on Payment Date (LIBOR + 150bp) (.25)($100m)	4 Floor Payoff on Payment Date (Max[.08 – LIBOR,0]) (.25)($100m)	5 Hedged Interest Income Col. (3) + Col. (4)	6 Hedged Rate 4 [Col (5)/$100m]	7 Unhedged Rate LIBOR + 150bp
3/1/Y1[n]	0.065					
6/1/Y1	0.070	$2,000,000	$0	$2,000,000	0.080	0.080
9/1/Y1	0.075	$2,125,000	$250,000	$2,375,000	0.095	0.085
12/1/Y1	0.080	$2,250,000	$125,000	$2,375,000	0.095	0.090
3/1/Y2	0.085	$2,375,000	$0	$2,375,000	0.095	0.095
6/1/Y2	0.090	$2,500,000	$0	$2,500,000	0.100	0.100
9/1/Y2	0.095	$2,625,000	$0	$2,625,000	0.105	0.105
12/1/Y2	0.100	$2,750,000	$0	$2,750,000	0.110	0.110
3/1/Y3		$2,875,000	$0	$2,875,000	0.115	0.115

[n]There is no floor on this date

TABLE 19.6 Hedging a Floating-Rate Loan with a Collar

Loan: Floating rate loan; Term = two years; Reset dates: 3/1, 6/1, 9/1, 12/1; Time frequency = .25; Rate = LIBOR + 150bp; Payment date = 90 days after reset date

Cap Purchase: Cost of cap = $300,000; Cap rate = 8%; Reference rate = LIBOR; Time frequency = .25;

Caplets' expiration: On loan reset dates, starting at 6/1/Y1; Payoff made 90 days after reset date.

Floor Sale: Sale of floor = $200,000; Floor rate = 7%; Reference rate = LIBOR; Time frequency = .25;

Floorlets' expiration: On loan reset dates, starting at 61/Y1; Payoff date = 90 days after reset date.

Loan interest, cap payoff, and floor payment made on payment date.

1 Reset Date	2 Assumed LIBOR	3 Loan Interest (LIBOR + 150 bp) (.25)($100m)	4 Cap Payoff Max[LIBOR – .08,0] (.25)($100m)	5 Floor Payment Max[.07 – LIBOR,0] (.25)($100m)	6 Hedged Interest Payment Col. (3) – Col. (4) + Col (5)	7 Hedged Rate 4[Col (6)/ $100m]	8 Unhedged Rate LIBOR + 150bp
3/1/Y1[n]	0.050						
6/1/Y1	0.060	$1,625,000	$0	$0	$1,625,000	0.065	0.065
9/1/Y1	0.065	$1,875,000	$0	$250,000	$2,125,000	0.085	0.075
12/1/Y1	0.070	$2,000,000	$0	$125,000	$2,125,000	0.085	0.080
3/1/Y2	0.075	$2,125,000	$0	$0	$2,125,000	0.085	0.085
6/1/Y2	0.080	$2,250,000	$0	$0	$2,250,000	0.090	0.090
9/1/Y2	0.085	$2,375,000	$0	$0	$2,375,000	0.095	0.095
12/1/Y2	0.090	$2,500,000	$125,000	$0	$2,375,000	0.095	0.100
3/1/Y3		$2,625,000	$250,000	$0	$2,375,000	0.095	0.105

[n]There is no cap or floor on this date.

company has to pay on the 7% floor, offsetting the lower interest costs it would pay on its loan. For example, when the LIBOR is at 6% on 6/1/Y1, Zuber has to pay $250,000 90 days later on its short floor position, and when the LIBOR is at 6.5% on 9/1/Y1, the company has to pay $125,000; these payments, in turn, offset the benefits of the respective lower interest of 7.5% and 8% (LIBOR + 150) it pays on its floating rate loan. Thus, for LIBORs less than 7%, the Zuber Company has a floor in which it pays an effective rate of 8.5% (losing the benefits of lower interest payments on its loan) and for rates above 8% it has a cap in which it pays an effective 9.5% on its loan.

In forming collars to finance capped floating rate loans, the borrower needs to determine the exercise rates on the caps and floors that best meet the cost of the hedge and his acceptable floor and cap rates. Specifically, if the exercise rate on the floor and cap are the same (e.g., 8% in our example), then the long cap and short floor will be equivalent to a forward contract. This low-cost (if not zero cost) collar makes the floating-rate loan combined with a collar a synthetic fixed-rate loan. For floors, the lower the exercise rate, the lower its premium. As a result, by selling a floor with a lower floor rate (e.g., 7% or 6%), the borrower's net costs of forming a collar will increase, but the floor rate at which the borrower gives up interest savings will be lower. On the other hand, for caps, the higher the cap rate, the lower the premium. Thus, by buying a cap with a higher cap rate (e.g., 9% or 10%), the borrower's net costs of forming a collar will decrease, but the effective maximum rate on his loan will be higher.

An alternative financial structure to a collar is a corridor. A ***corridor*** is a long position in a cap and a short position in a similar cap with a higher exercise rate. The sale of the higher exercise-rate cap is used to partially offset the cost of purchasing the cap with the lower strike rate. For example, the Zuber Company, instead of selling a 7% floor for $200,000 to partially finance the $300,000 cost of its 8% cap, could sell a 9% cap for, say, $200,000. If cap purchasers, however, believe there is a greater chance of rates increasing than decreasing, they will prefer the collar to the corridor as a tool for financing the cap. In practice, collars are more frequently used than corridors.

A ***reverse collar*** is the combination of a long position in a floor and a short position in a cap with different exercise rates. The sale of the cap is used to defray the cost of the floor. For example, the Commerce Bank in our above floor example could reduce the $200,000 cost of the 8% floor it purchased to hedge the floating-rate loan it made to the Zuber Company by selling a cap. By forming a reverse collar to hedge its floating-rate asset, the bank would still have protection against rates decreasing against the floor rate, but it would have to give up potential higher interest returns if rates increase above the cap rate. For example, suppose Commerce sold a 9% cap for $100,000, with the cap having similar terms to the floor. By using the reverse collar instead of the floor, the company would reduce its hedging cost from $200,000 to $100,000, and as shown in Table 19.7, would lock in an effective minimum rate on its asset of 9.5% and an effective maximum rate of 10.5%.

As with collars, in forming reverse collars to finance a floating rate asset with a floor, the investor needs to determine the exercise rates on the caps and floors that best meet the cost of the hedge and the investor's acceptable floor and cap rates. Also,

TABLE 19.7 Hedging a Floating-Rate Asset with a Reverse Collar

Asset: Floating rate loan made by bank; Term = two years; Reset dates: 3/1, 6/1, 9/1, 12/1; Time frequency = .25; Rate = LIBOR + 150bp; Payment date = 90 days after reset date

Floor Purchase: Cost of floor = $200,000; Floor rate = 8%; Reference rate = LIBOR; Time frequency = .25;

Floorlets' expiration: On loan reset dates, starting at 6/1/Y1; Payoff made 90 days after reset date.

Cap Sale: revenue from cap = $100,000; Cap rate = 9%; Reference rate = LIBOR; Time frequency = .25;

Caplets' expiration: On loan reset dates, starting at 6/1/Y1; Payoff made 90 days after reset date.

1 Reset Date	2 Assumed LIBOR	3 Interest Received (LIBOR + 150bp) (.25)($100m)	4 Floor Payoff Max[.08 − LIBOR,0] (.25)($100m)	5 Cap Payment Max[LIBOR − .09,0] (.25)($100m)	6 Hedged Interest Income Col. (3) + Col. (4) − Col (5)	7 Hedged Rate 4[Col (5)/ $100m]	8 Unhedged Rate LIBOR + 150 bp
3/1/Y1[n]	0.065						
6/1/Y1	0.070	$2,000,000	$0		$2,000,000	0.080	0.080
9/1/Y1	0.075	$2,125,000	$250,000	$0	$2,375,000	0.095	0.085
12/1/Y1	0.080	$2,250,000	$125,000	$0	$2,375,000	0.095	0.090
3/1/Y2	0.085	$2,375,000	$0	$0	$2,375,000	0.095	0.095
6/1/Y2	0.090	$2,500,000	$0	$0	$2,500,000	0.100	0.100
9/1/Y2	0.095	$2,625,000	$0	$0	$2,625,000	0.105	0.105
12/1/Y2	0.100	$2,750,000	$0	$125,000	$2,625,000	0.105	0.110
3/1/Y3		$2,875,000	$0	$250,000	$2,625,000	0.105	0.115

[n]There is no cap or floor on this date

instead of financing a floor with a cap, an investor could form a ***reverse corridor*** by selling another floor with a lower exercise rate.

19.5 OTHER INTEREST RATE PRODUCTS

Caps and floors are some of the more popular interest rate products offered by the OTC derivative market. In addition to these derivatives, a number of other interest rate products have been created over the last decade to meet the many different interest rate hedging needs. Many of these products are variations of the generic OTC caps and floors; two of these to note are barrier options and path-dependent options.

Barrier Options

Barrier options are options in which the payoff depends on whether an underlying security price or reference rate reaches a certain level. Barrier options can be classified as either knock-out or knock-in options: A ***knock-out option*** is one that ceases to exist once the specified barrier rate or price is reached; a ***knock-in option*** is one that comes into existence when the reference rate or price hits the barrier level. Both types of options can be formed with either a call or put and the barrier level can be either above or below the current reference rate or price when the contract is established (down-and-out or up-and-out knock-outs or up-and-in or down-and-in knock-in options). For example, a down-and-out, knock-out call is a call that ceases to exist once the reference price or rate reaches the barrier level, and the barrier level is below the reference rate or price when the option was purchased.

Barrier caps and floors with termination or creation features are offered in the OTC market at a premium above comparable caps and floors without such features. Down-and-out caps and floors are options that cease to exist once rates hit a certain level; for example, a two-year, 8% cap that ceases when the LIBOR hits 6.5%, or a two-year, 8% floor that ceases once the LIBOR hits 9%. By contrast, an up-and-in cap is one that becomes effective once rates hit a certain level: a two-year, 8% cap that that becomes effective when the LIBOR hits 9%, for example, or a two-year, 8% floor that become effective when rates hit 6.5%.

Path-Dependent Options

In the generic cap or floor, the underlying payoff on the caplet or floorlet depends only on the reference rate on the effective date. The payoff does not depend on previous rates; that is, it is independent of the path the LIBOR has taken. Some caps and floors, though, are structured so that their payoff is dependent on the path of the reference rate. An ***average cap***, for example, is one in which the payoff depends on the average reference rate for each caplet. If the average is above the exercise rate, then all the caplets will provide a payoff; if the average is equal or below, the whole cap expires out of the money. Consider a one-year average cap with an exercise rate of 7% with four caplets. If the LIBOR settings turned out to be 7.5%, 7.75%, 7%, and 7.5%, for an average of 7.4375%, then the average cap would be in the money: (.074375 − .07)(.25)(NP). If the rates, though, turned out to be 7%, 7.5%, 6.5%, and 6%, for an average of 6.75%, then the cap would be out of the money.

Another type of path-dependent interest rate option is a ***cumulative cap (Q-cap)***. In a Q-cap, the cap seller pays the holder when the periodic interest on the accompanying floating-rate loan hits or exceeds a specified level. As an example, suppose the Zuber Company in our earlier cap example decided to hedge its two-year floating rate loan (paying LIBOR + 150bp) by buying a Q-cap from Commerce Bank with the following terms:

- The cap consists of seven caplets with the first expiring on 6/1/Y1 and the others coinciding with the loan's reset dates
- Exercise rates on each caplet = 8%
- NP on each caplet = $100 million
- Reference rate = LIBOR
- Time period to apply to payoff on each caplet = 90/360
- For the period 3/1/Y1 to 12/1/Y1, the caplet will pay off when the cumulative interest starting from loan date 3/1/Y1 on the company's loan hits $6 million
- For the period 3/1/Y2 to 12/1/Y2, the caplet will pay off when the cumulative interest starting from date 3/1/Y2 on the company's loan hits $6 million
- Payment date on each caplet is at the loan's interest payment date, 90 days after the reset date.
- The cost of the cap = $250,000; it is paid at beginning of the loan, 3/1/Y1

Table 19.8 shows the quarterly interest, cumulative interest, Q-cap payment, and effective interest for assumed LIBORs. In the Q-cap's first protection period, 3/1/Y1 to 12/1/Y1, Commerce Bank will pay the Zuber Company on its 8% caplet when the cumulative interest hits $6 million. The cumulative interest hits the $6 million limit on reset date 9/1/Y1, but on that date the 9/1/Y1 caplet is not in the money. On the following reset date, though, the caplet is in the money at the LIBOR of 8.5%. Commerce would, in turn, have to pay Zuber $125,000 (90 days later) on the caplet, locking in a hedged rate on its loan of 9.5%. In the second protection period, 3/1/Y2 to 12/1/Y2, the assumed LIBOR rates are higher. The cumulative interest hits the $6 million limit on reset date 9/1/Y2. Both the caplet on that date and the next reset date (12/1/Y2) are in the money. As a result, with the caplet payoffs, Zuber is able to obtain a hedged rate of 9.5% for the last two payment periods on its loan.

When compared to a standard cap, the Q-cap provides protection for the one-year protection periods, whereas the standard cap provides protection for each period. As shown in the lower table of Table 19.8, a standard 8% cap provides more protection given the assumed increasing interest rate scenario than the Q-cap, capping the loan at 9.5% from date 12/1/Y1 to the end of the loan and providing payoffs on five of the seven caplets for a total of $1,375,000. In contrast, the Q-cap

TABLE 19.8 Hedging a Floating Rate Loan with a Q-Cap

Loan: Floating Rate Loan; Term = 2 years; Reset dates: 3/1, 6/1, 9/1, 12/1; Time frequency = .25; Rate = LIBOR + 150BP; Payment date = 90 days after reset date

Q-Cap: Cost of Q-cap = $250,000; Cap rate = 8%; Reference rate = LIBOR; Time frequency = .25; Caplets' expiration: On loan reset dates, starting at 6/1/Y1;

Payoff made 90 days after reset date; Cap become effective once cumulative interest reaches $6M; protection periods: Y1 and Y2.

1 Reset Date	2 Assumed LIBOR	3 Interest to Be Paid at Next Reset Date (LIBOR + 150BP) (.25)($100m)	4 Cumulative Interest	5 Q-Cap Payment to Be Paid at Next Reset Date	6 Hedged Interest Payment at Payment Date: Col (3) − Col (5)	7 Hedged Rate 4[Col (6)/ $100m]
3/1/Y1[n]	0.070	$2,125,000	$2,125,000	$0		
6/1/Y1	0.075	$2,250,000	$4,375,000	$0	$2,125,000	0.085
9/1/Y1	0.080	$2,375,000	$6,750,000	$0	$2,250,000	0.090
12/1/Y1	0.085	$2,500,000	$9,250,000	$125,000	$2,375,000	0.095
3/1/Y2	0.085	$2,500,000	$2,500,000	$0	$2,375,000	0.095
6/1/Y2	0.090	$2,625,000	$5,125,000	$0	$2,500,000	0.100
9/1/Y2	0.095	$2,750,000	$7,875,000	$375,000	$2,625,000	0.105
12/1/Y2	0.100	$2,875,000	$10,750,000	$500,000	$2,375,000	0.095
3/1/Y3					$2,375,000	0.095

Comparison of Cap and Q-Cap

Loan: Floating Rate Loan; Term = 2 years; Reset dates: 3/1, 6/1, 9/1, 12/1; Time frequency = .25; Rate = LIBOR + 150BP; Payment date = 90 days after reset date

Cap: Cost of cap = $300,000; Cap rate = 8%; Reference rate = LIBOR; Time frequency = .25; Caplets' expiration: On loan reset dates, starting at 6/1/Y1; Payoff made 90 days after reset date.

Q-Cap: Cost of Q-cap = $250,000; Cap rate = 8%; Reference rate = LIBOR; Time frequency = .25; Caplets' expiration: On loan reset dates, starting at 6/1/Y1;

Payoff made 90 days after reset date; Cap become effective once cumulative interest reaches $6m; protection periods: Y1 and Y2.

1 Reset Date	2 Assumed LIBOR	3 Loan Interest	4 Unhedged Loan Rate	5 Q-Cap Payment	6 Q-Cap Hedged Rate	7 Cap-Payments	8 Cap-Hedged Rate
3/1/Y1[n]	0.070	$2,125,000					
6/1/Y1	0.075	$2,250,000	0.085	$0	0.085	$0	0.085
9/1/Y1	0.080	$2,375,000	0.090	$0	0.090	$0	0.090
12/1/Y1	0.085	$2,500,000	0.095	$0	0.095	$125,000	0.095
3/1/Y2	0.085	$2,500,000	0.100	$125,000	0.095	$125,000	0.095
6/1/Y2	0.090	$2,625,000	0.100	$0	0.100	$250,000	0.095
9/1/Y2	0.095	$2,750,000	0.105	$0	0.105	$375,000	0.095
12/1/Y2	0.100	$2,875,000	0.110	$375,000	0.095	$500,000	0.095
3/1/Y3			0.115	$500,000	0.095		0.095

[n]There is no cap on this date

EXHIBIT 19.1 Exotic Options

Asian Option: An option in which the payoff depends on the average price of the underlying asset during some part of the option's life. Call: IV = Max[S_{av} – X,0]; put: IV = Max[X – S_{av},0].
Lookback Option: An option in which the payoff depends on the minimum or maximum price reached during the life of the option.
Binary Option: An option with a discontinuous payoff such as a payoff or nothing. For example, if the price is equal to or less than X, the option pays nothing; if the price exceeds X, the option pays a fixed amount.
Compound Option: An option on an option. Call on call, call on put, put on put, and put on call.
Chooser Option: Option that gives the holder the right to choose whether the option is a call or a put after a specified period of time.
Bermudan Option: An option in which early exercise is restricted to certain dates.
Forward Start Option: An option that will start at some time in the future.
Trigger Option: An option that depends on another index; that is, whether the option is in the money depends on value of another index.
Caption: An option on a cap.
Floortion: An option on a floor.
Yield Curve Option: An option between two points on a yield curve. For example, a yield curve with a exercise equal to 200 basis point on the difference between the yields on two-year and 10-year notes: Payoff = Max[(YTM_{10} – YTM_2) – .02, 0]NP.

pays on only three of the seven caplets for a total of only $1,000,000. Because of its lower protection limits, the Q-cap costs less than the standard cap.

Exotic Options

Q-caps, average caps, knock-in options, and knock-out options are sometimes referred to as exotic options. Exotic option products are nongeneric products that are created by financial engineers to meet specific hedging needs and return-risk profiles. Exhibit 19.1 defines some of the popular exotics options used in interest rate management.

19.6 HEDGING CURRENCY POSITIONS WITH FOREIGN CURRENCY OPTIONS

As we noted in Chapter 10, when investors purchase and hold foreign securities or when corporations and governments sell debt securities in external markets or incur foreign debt positions, they are subject to exchange-rate risk. In Chapter 10, we examined how forward contracts can be used by financial and nonfinancial corporations to hedge their international positions. These positions can also be hedged with currency options, futures, and futures options.

In 1982, the Philadelphia Stock Exchange (PHLX) became the first organized exchange to offer trading in foreign currency options (they are also the United States' first stock exchange). The contract sizes for many of PHLX's options are equal to half the size of the currency's futures listed on the International Monetary Market. For example, the foreign currency call option contract on the British pound requires (upon exercise) the purchase of 31,250 British pounds, whereas a long British pound futures contract requires the purchase of 62,500 BP. Foreign currency options also are traded on a number of derivative exchanges outside the United States. In addition to offering foreign currency futures, the Chicago Mercantile Exchange and other futures exchanges also offer options on foreign currency futures. Finally, there is a sophisticated dealer's market. This interbank currency options market is part of the interbank foreign exchange market. In this dealer's market, banks provide tailor-made foreign currency option contracts for their customers, primarily multinational corporations. Compared to exchange-traded options, options in the interbank market are larger in contract size, often European, and are available on more currencies. Because these options are tailor-made to fit customer needs, though, there is not a significant secondary market for these options.

Until the introduction of currency options, exchange-rate risk usually was hedged with foreign currency forward or futures contracts. Hedging with these instruments allows foreign exchange participants to lock in the local currency values of their international revenues or expenses. However, with exchange-traded currency options and dealer's options, hedgers, for the cost of the options, can obtain not only protection against adverse exchange rate movements, but (unlike forward and futures positions) benefits if the exchange rates move in favorable directions.

To illustrate the use of currency options as a hedging tool for international debt and investment positions, consider a U.S. fund with investments in Eurobonds that were to pay a principal in British pounds of £10 million next September. For the costs of BP put options, the U.S. fund could protect its dollar revenues from possible exchange rate decreases when it converts, while still benefiting if the exchange rate increases. For example, suppose a September BP put with an exercise price of $X =$ \$1.425/£ is available at $P =$ \$0.02/£. Given a contract size of 31,250 British pounds, the U.S. fund would need to buy 320 put contracts ($n_p =$ £10,000,000/£31,250 = 320) at a cost of \$200,000 [Cost = (320)(£31,250)(0.02/£)] to establish a floor for the dollar value of its £10,000,000 receipt in September. Table 19.9 shows the dollar cash flows the U.S. fund would receive in September from converting its receipts of £10,000,000 to dollars at the spot exchange rate (E_T) and closing its 320 put contracts at a price equal to the put's intrinsic value (assume the September payment date and option expiration date are the same). As shown in Table 19.9, if the exchange rate is less than $X =$ \$1.425/£, the company will receive less than \$14,250,000 when it converts its £10,000,000 to dollars; these lower revenues, however, would be offset by the profits from the put position. For example, at a spot exchange rate of \$1.325/£ the company would receive only \$13,250,000 from converting its £10,000,000, but would earn a profit of \$800,000 from the puts (\$800,000 = 320 Max[(\$1.425/£) − (\$1.325/£), 0](£31,250) − \$200,000); this would result in a combined receipt of \$14,050,000. As shown in the table, if the exchange rate is \$1.425/£ or less, the company would receive \$14,050,000. This amount equals the hedged amount (\$14,250,000) minus the \$200,000 costs

TABLE 19.9 Hedging £10,000,000 Cash Inflow with a British Pound Put Option

X = $1.425/£, P = .02, Contract Size = £32,250, Number of Puts = 320

1 Exchange Rate E_T = \$/BP	2 Dollar Revenue E_T(10,000,000BP)	3 Put Profit	4 Hedged Revenue Col(2) + Col(3)
$1.200	$12,000,000	$2,050,000	$14,050,000
$1.250	$12,500,000	$1,550,000	$14,050,000
$1.275	$12,750,000	$1,300,000	$14,050,000
$1.300	$13,000,000	$1,050,000	$14,050,000
$1.325	$13,250,000	$800,000	$14,050,000
$1.350	$13,500,000	$550,000	$14,050,000
$1.375	$13,750,000	$300,000	$14,050,000
$1.400	$14,000,000	$50,000	$14,050,000
$1.425	$14,250,000	−$200,000	$14,050,000
$1.450	$14,500,000	−$200,000	$14,300,000
$1.475	$14,750,000	−$200,000	$14,550,000
$1.500	$15,000,000	−$200,000	$14,800,000
$1.525	$15,250,000	−$200,000	$15,050,000
$1.500	$15,000,000	−$200,000	$14,800,000

Put Profit = (320)(31250BP)(Max[$1.425/BP − E_T,0] − $200,000

of the put options. On the other hand, if the exchange rate at expiration exceeds $1.425/£, the U.S. fund will realize a dollar gain when it converts the £10,000,000 at the higher spot exchange rate, while its losses on the put would be limited to the amount of the premium. Thus, by hedging with currency put options, the company is able to obtain exchange rate risk protection in the event the exchange rate decreases, while still retaining the potential for increased dollar revenues if the exchange rate rises.

Suppose that instead of receiving foreign currency, a U.S. company had a foreign liability requiring a foreign currency payment at some future date. To protect itself against possible increases in the exchange rate while still benefiting if the exchange rate decreases, the company could hedge the position by taking a long position in a currency call option. For example, suppose a U.S. company owed £10,000,000, with the payment to be made in September. To benefit from the lower exchange rates and still limit the dollar costs of purchasing £10,000,000 in the event the $/£ exchange rate rises, the company could buy September British pound call options. Table 19.10 shows the costs of purchasing £10,000,000 at different exchange rates and the profits and losses from purchasing 320 September British pound calls with X = $1.425/£ at $0.02/£ (contract size = £31,250) and closing them at expiration at a price equal to the call's intrinsic value. As shown in the exhibit, for cases in which the exchange rate is greater than $1.425/£, the company has dollar expenditures exceeding $14,250,000; the expenditures, though, are offset by the profits from the calls. On the other hand, when the exchange rate is less than $1.425/£, the dollar costs of purchasing £10,000,000 decrease as the exchange rate decreases, while the losses on the call options are limited to the option premium.

TABLE 19.10 Hedging £10,000,000 Cash Outflow with a British Pound Call Option

X = $1.425/£, C = .02, Contract Size = £32,250, Number of Calls = 320

1 Exchange Rate E_T = $/BP	2 Dollar Cost E_T(10,000,000BP)	3 Call Profit	4 Hedged Cost Col(2) − Col(3)
$1.200	$12,000,000	−$200,000	$12,200,000
$1.250	$12,500,000	−$200,000	$12,700,000
$1.275	$12,750,000	−$200,000	$12,950,000
$1.300	$13,000,000	−$200,000	$13,200,000
$1.325	$13,250,000	−$200,000	$13,450,000
$1.350	$13,500,000	−$200,000	$13,700,000
$1.375	$13,750,000	−$200,000	$13,950,000
$1.400	$14,000,000	−$200,000	$14,200,000
$1.425	$14,250,000	−$200,000	$14,450,000
$1.450	$14,500,000	$50,000	$14,450,000
$1.475	$14,750,000	$300,000	$14,450,000
$1.500	$15,000,000	$550,000	$14,450,000
$1.525	$15,250,000	$800,000	$14,450,000
$1.500	$15,000,000	$550,000	$14,450,000

Call Profit = (320)(31250BP)(Max[E_T − $1.425/BP,0] − $200,000

19.7 CONCLUSION

In this chapter, we have examined how OTC interest rate options are used to manage different types of fixed-income investment and debt management positions, as well as how foreign currency-denominated positions can be hedged using currency options. Like exchange-traded options, OTC options allow investors and borrowers to attain floor and cap limits on the rates they can earn on their investments or must pay on their loans, while still allowing them to benefit if rates move in a favorable direction. In the next part of the book, we examine another popular derivative used extensively in managing interest rate, currency, and credit positions—financial swaps.

KEY TERMS

average cap
barrier options
cap
cap rate
caplet
collar
corridor
cumulative cap (Q-cap)
floor
floorlet
forward rate agreement (FRA)
knock-in option
knock-out option
reverse collar
reverse corridor

PROBLEMS AND QUESTIONS

1. Given a forward rate agreement (FRA) with the following terms:
   - Notional principal = $20 million
   - Reference rate = LIBOR
   - Contract rate = $R_k = .05$ (annual)
   - Time period = 90 days
   - Day-count convention = Actual/365

   Show in a table the payments and receipts for long and short positions on the FRA given possible spot LIBORs at the FRA's expiration of 4%, 4.5%, 5%, 5.5%, and 6%.

2. Explain the similarities and differences between an FRA tied to the LIBOR and a Eurodollar futures contract.

3. The Glasgo Manufacturing Company forecasts a cash inflow of $20 million in two months from the sale of one of its assets. It is considering investing the cash in a First National Bank CD for 90 days. First National Bank's jumbo CD pays a rate equal to the LIBOR. Currently such rates are yielding 6%. Glasgo is concerned that short-term interest rates could decrease in the next two months and would like to lock in a rate now. As an alternative to hedging its investment with Eurodollar futures, First National suggests that Glasgo hedge with a forward rate agreement (FRA).
   a. Define the terms of the FRA that would effectively hedge Glasgo's future CD investment.
   b. Show in a table the payoffs that Glasgo and First National would pay or receive at the maturity of the FRA given the following LIBORs: 5.5%, 5.75%, 6%, 6.25%, and 6.5%.
   c. Show in a table Glasgo's cash flows from investing the $20 million cash inflow plus or minus the FRA receipts or payments at possible LIBORs of 5.5%, 5.75%, 6%, 6.25%, and 6.5%. What is the hedged rate of return Glasgo would earn from its $20 million investment?

4. The Fort Washington Money Market Fund expects interest rates to be higher in September when it plans to invest its $18 million cash flow in a 90-day CD offered by Sun Bank paying the LIBOR. Suppose the fund decides to hedge the September investment by buying an interest rate put from Provident bank. The floorlet has the following terms:
   - Exercise rate of 7%
   - Payoff at the maturity of the CD
   - Reference rate of LIBOR
   - Time period of 90 days (.25)
   - Notional principal of $18 million
   - Expiration at the time of the September cash flow investment
   - Cost of floorlet is $100,000, payable at the expiration

   Using the table below, determine the fund's hedged yield for possible spot LIBORs at the option's expiration date of 6%, 6.5%, 7%, 7.5%, and 8%.

1 LIBOR	2 Interest Rate Put	3 Cost of the Option at *T*	4 Interest Received on CD at Its Maturity	5 Total Revenue at Maturity	6 Annualized Hedged Rate
6.00					
6.50					
7.00					
7.50					
8.00					

5. In January, the O'Brien Development Company closed a deal with local officials to develop a new office building. The project is expected to begin in June and take 270 days to complete. The cost of the development is expected to be $32 million, with the Western Southern Insurance Company providing the permanent financing of the development once the construction is completed. O'Brien Development has obtained a 270-day construction loan from the Reinhart Financial Company. Reinhart Financing will disburse funds to O'Brien at the beginning of the project in June, with the interest rate on the loan being set equal to the LIBOR plus 150 bp. The principal and interest on the loan are to be paid at maturity. Reinhart Financial is also willing to sell O'Brien an interest rate call with the following terms:

- Exercise rate of 10%
- Payoff at the maturity of the loan
- LIBOR reference rate
- Time period of 270 days or .75 per year
- Notional principal of $32 million
- Expiration at the June start of the loan
- Cost of the caplet is $150,000

Using the table below, determine O'Brien's hedged loan rate for possible LIBORs at the June date of 8%, 9%, 10%, 11%, 11.5%, and 12%.

1 LIBOR	2 Interest Rate Call	3 Cost of the Option at *T*	4 Interest Paid on Loan at Its Maturity	5 Total Cost at Maturity	6 Annualized Hedged Rate
8%					
9%					
10%					
11%					
11.5%					
12%					

6. Suppose Eastern Bank offers Gulf Refinery a $150 million floating-rate loan to finance the purchase of its crude oil imports along with a cap. The floating-rate loan has a maturity of one year, starts on December 20, and is reset the next three quarters. The initial quarterly rate is equal to 10%/4, the other rates are set on 3/20, 6/20, and 9/20 equal to one fourth of the annual LIBOR on those dates plus 100 basis points: (LIBOR % + 1%)/4. The cap Eastern Bank is offering Gulf has the following terms:

- Three caplets with expiration dates of 3/20, 6/20, and 9/20
- The cap rate on each caplet is 9.5%
- The time period for each caplet is .25 per year
- The payoffs for each caplet are at the interest payment dates
- The reference rate is the LIBOR
- Notional principal is $150 million
- The cost of the cap is $500,000

Show in the table below the company's quarterly interest payments, caplet cash flows, hedged interest payments (interest minus caplet cash flow), and hedged rate as a proportion of a $150 million loan (do not include cap cost) for each period (12/20, 3/20, 6/20, and 9/20) given the following rates: LIBOR = 10% on 3/20, LIBOR = 9.5% on 6/20, and LIBOR = 9% on 9/20.

1 Date	2 LIBOR	3 Cap Payoff on Payment Date	4 Loan Interest on Payment Date	5 Hedged Debt	6 Hedged Rate	7 Unhedged Rate
12/20/Y1						
3/20/Y1						
6/20/Y1						
9/20/Y1						
12/20/Y2						

7. XU Trust is planning to invest $15 million in a Commerce Bank one-year floating-rate note paying LIBOR plus 150 basis points. The investment starts on 3/20 at 9% (when the LIBOR = 7.5%) and is then reset the next three quarters on 6/20, 9/20, and 12/20. XU Trust would like to establish a floor on the rates it obtains on the note. A money center bank is offering XU a floor for $100,000 with the following terms corresponding to the floating-rate note:

   - The floor consists of three floorlets coinciding with the reset dates on the note
   - Exercise rate on the floorlets = 7%
   - Notional principal = $15 million
   - Reference rate = LIBOR
   - Time period on the payoffs is .25
   - Payoff is paid on the payment date on the note
   - Cost of the floor is $100,000 and is paid on 3/20

   Calculate and show in the table below XU Trust's quarterly interest receipts, floorlet cash flow, hedged interest revenue (interest plus floorlet cash flow), and hedged rate as a proportion of the $15 million investment (do not include floor cost) given the LIBORs shown in the table.

1 Date	2 LIBOR	3 Interest on FRN on Payment Date	4 Floor Payoff on Payment Date	5 Hedged Interest Income	6 Hedged Rate	7 Unhedged Rate
3/20	.075					
6/20	.07					
9/20	.065					
12/20	.06					
3/20						

8. Suppose Commerce Bank sells XU Trust a two-year, $15 million FRN paying the LIBOR plus 150 basis points. The note starts on 3/20 at 9% and is then reset the next seven quarters on dates 6/20, 9/20, and 12/20. Suppose a money center bank offers Commerce Bank a cap for $200,000 with the following terms corresponding to its floating-rate liability:
   - The cap consists of seven caplets coinciding with the reset dates on the note
   - Exercise rate on the caplets = 7%
   - Notional principal = $15 million
   - Reference rate = LIBOR
   - Time period on the payoffs is .25
   - Payoff is paid on the payment date on the note
   - Cost of the cap is $200,000 and is paid on 3/20

   a. Show in a table Commerce Bank's quarterly interest payments, caplet cash flows, hedged interest cost (interest minus caplet cash flow), and hedged rate as a proportion of the $15 million FRN loan (do not include cap cost) for each period given the following rates: LIBOR = 7.5% on 3/20/Y1, 8% on 6/20/Y1, 9% on 9/20/Y1, 8% on 12/20/Y1, 7% on 3/20/Y2, 6.5% on 6/20/Y2, 6% on 9/20/Y2, and 5.5% on 12/20/Y2.

   b. To help defray part of the cost of the cap, suppose Commerce Bank decides to set up a collar by selling a floor to one of its customers with a floor rate of 6.5% for $150,000 with the following terms:
   - The floor consists of seven floorlets coinciding with the reset dates on the note
   - Exercise rate on the floorlets = 6.5%
   - Notional principal = $15 million
   - Reference rate = LIBOR
   - Time period on the payoffs is .25
   - Payoff is paid on the payment date on the note
   - Cost of the floor is $150,000 and is paid on 3/20

   c. Evaluate Commerce Bank's hedged interest costs from using the collar given the interest-rate scenario defined in 8a.

   d. Contrast Commerce Bank's cap hedge with its collar hedge.

   e. Define another interest rate option position Commerce Bank might use to defray the costs of its cap-hedged floating-rate liability.

9. Suppose Commerce Bank in Question 8 decides to hedge its two-year $15 million FRN it sold to XU Trust (FRN terms: Pays the LIBOR plus 150 basis points; starts on 3/20/Y1 at 9%; reset the next seven quarters on dates 6/20, 9/20, and 12/20) by buying a Q-cap (or cumulative cap) from a money center bank for $150,000 with the following terms corresponding to its floating-rate liability:
   - The cap consists of seven caplets coinciding with the reset dates on the note
   - Exercise rate on the caplets = 7%
   - Notional principal = $15 million
   - Reference rate = LIBOR
   - Time period on the payoffs is .25
   - For the period from 3/20/Y1 to 12/20Y1, the caplet will pay when the cumulative interest starting from the loan date 3/20/Y1 hits $700,000
   - For the period from 3/20/Y2 to 12/20/Y2, the caplet will pay when the cumulative interest starting from the loan date 3/20/Y2 hits $700,000
   - Payoff is paid on the payment date on the note
   - Cost of the cap is $150,000 and is paid on 3/20/Y1

   Show in the table below Commerce Bank's quarterly interest payments, caplet cash flows from the Q-cap, hedged interest cost (interest minus caplet cash flow), and hedged rate as a proportion of the $15million FRN loan (do not include Q-cap cost) for each period given the following interest rate scenario: LIBOR = 7.5% on 3/20/Y1, 8% on 6/20/Y1, 9% on 9/20/Y1, 8% on 12/20/Y1, 7% on 3/20/Y2, 8% on 6/20/Y2, 9% on 9/20/Y2, and 10% on 12/20/Y2.

1 Date	2 LIBOR	3 Interest Paid on FRN on Payment Date	4 Cumulative Interest	5 Q-cap Payoff on Payment Date	6 Hedged Interest Payment	7 Hedged Rate	8 Unhedged Rate
3/20/Y1	.075						
6/20/Y1	.08						
9/20/Y1	.09						
12/20/Y1	.08						
3/20/Y2	.07						
6/20/Y2	.08						
9/20/Y2	.09						
12/20/Y2	.10						
3/20/Y3							

10. Suppose UK Trust plans to invest $15 million in a two-year FRN paying LIBOR. The FRN starts on 3/20 at 7.5% and is then reset the next seven quarters on 6/20, 9/20, and 12/20. UK Trust would like to establish a floor on the rates it obtains on the note. A money center bank is offering UK a floor for $200,000 with the following terms corresponding to the floating-rate note:

- The floor consists of seven floorlets coinciding with the reset dates on the note
- Exercise rate on the floorlets = 7%
- Notional principal = \$15 million
- Reference rate = LIBOR
- Time period on the payoffs is .25
- Payoff is paid on the payment date on the note
- Cost of the floor is \$100,000 and is paid on 3/20

UK would like to finance the \$200,000 cost of the floor by forming a reverse collar by selling a cap. The money-center bank is willing to buy a cap with an exercise rate of 8% from UK for \$150,000 with similar terms to the floor.

a. Evaluate a reverse collar-hedged FRN investment UK could form with the cap and floor offered by the money center bank given the following interest-rate scenarios: LIBOR = 7.5% on 3/20/Y1, 7% on 6/20/Y1, 6.5% on 9/20/Y1, 6% on 12/20/Y1, 7% on 3/20/Y2, 8% on 6/20/Y2, 8.5% on 9/20/Y2, and 9% on 12/20/Y2. In your evaluation, include the quarterly interest receipts, cap and floor cash flows, hedged interest revenue, and hedged rate (do not include cost of the floor or revenue from selling the cap).

b. Define another interest rate option position UK Trust might use to defray the costs of its floor-hedged floating-rate investment.

NOTES

1. Some of the material in this chapter draws from Johnson, *Introduction to Derivatives*, 2009.
2. The FRA evolved from *forward-forward* contracts in which international banks would enter an agreement for a future loan at a specified rate.

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PART Five

Swaps

Bond Evaluation, Selection, and Management, Second Edition
by R. Stafford Johnson

CHAPTER 20

Interest Rate Swaps

20.1 INTRODUCTION

Today, there exists an interest rate swap market consisting of financial and nonfinancial corporations who annually conduct trillions of dollars (as measured by contract value) in swap contract trades. Financial institutions and corporations use the market to hedge their liabilities and assets more efficiently—transforming their floating-rate liabilities and assets into fixed-rate ones, or vice versa, and creating synthetic fixed or floating-rate liabilities and assets with better rates than the ones they can directly obtain. The strategy of swapping loans, though, is not new. In the 1970s, corporations began exchanging loans denominated in different currencies, creating a ***currency swap*** market. This market evolved from corporations who could obtain favorable borrowing terms in one currency but needed a loan in a different currency. To meet such needs, companies would go to swap dealers who would try to match their needs with other parties looking for the opposite positions.

Whether it is an exchange of currency-denominated loans or fixed and floating interest rate payments, a swap, by definition, is a legal arrangement between two parties to exchange specific payments. There are four types of financial swaps.

1. **Interest Rate Swaps**: The exchange of fixed-rate payments for floating-rate payments
2. **Currency Swaps**: The exchange of liabilities in different currencies
3. **Cross-Currency Swaps**: The combination of an interest rate and currency swap
4. **Credit Default Swaps**: Exchange of premium payments for default protection

In this chapter, we examine the features, markets, and uses of standard interest rate swaps and in the next chapter we will examine two interest rate swap derivatives—forward swaps and swaptions. In Chapter 22, we will explore the markets, uses, and pricing of currency and credit default swaps.

20.2 GENERIC INTEREST RATE SWAPS

Features

The simplest type of interest rate swap is called the ***plain vanilla swap*** or ***generic swap***. In this agreement, one party provides fixed-rate interest payments to another party, who provides floating-rate payments. The parties to the agreement are referred to as ***counterparties***: The party who pays fixed interest and receives floating is called the ***fixed-rate payer***; the other party (who pays floating and receives fixed) is the ***floating-rate payer***. The fixed-rate payer is also called the floating-rate receiver and is often referred to as having bought the swap or having a long position; the floating-rate payer is also called the fixed-rate receiver and is referred to as having sold the swap and being short.

On a generic swap, principal payments are not exchanged. As a result, the interest payments are based on a notional principal (NP). The interest rate paid by the fixed payer often is specified in terms of the yield to maturity (YTM) on a T-note plus basis points; the rate paid by the floating payer on a generic swap is the LIBOR. Swap payments on a generic swap are made semiannually and the maturities typically range from three to 10 years. In the swap contract, a trade date, effective date, settlement date, and maturity date are specified. The ***trade date*** is the day the parties agree to commit to the swap; the ***effective date*** is the date when interest begins to accrue; the ***settlement*** or ***payment date*** is when interest payments are made (interest is paid in arrears six months after the effective date); and the ***maturity date*** is the last payment date. On the payment date, only the interest differential between the counterparties is paid. That is, generic swap payments are based on a ***net settlement basis***: The counterparty owing the greater amount pays the difference between what is owed and what is received. Thus, if a fixed-rate payer owes $2 million and a floating-rate payer owes $1.5 million, then only a $0.5 million payment by the fixed payer to the floating payer is made. All of the terms of the swap are specified in a legal agreement called the ***confirmation,*** signed by both parties. The drafting of the confirmation often follows document forms suggested by the ***International Swap and Derivative Association (ISDA)*** in New York. This organization provides a number of master agreements delineating the terminology used in many swap agreements (e.g., what happens in the case of default, the business day convention, and the like).

WEB INFORMATION

For more information on the International Swap and Derivative Association and size of the markets, go to www.isda.org.

Interest Rate Swap: Example

Consider an interest rate swap with a maturity of three years, first effective date of 3/23/Y1 and a maturity date of 3/23/Y4. In this swap agreement, assume the fixed-rate payer agrees to pay the current YTM on a three-year T-note of 5% plus 50 basis points and the floating-rate payer agrees to pay the six-month LIBOR as determined on the effective dates with no basis points. Also assume the semiannual interest rates are determined by dividing the annual rates (LIBOR and 5.5%) by 2. Finally, assume the notional principal on the swap is $10 million. (The calculations will be slightly off because they fail to include the actual day count convention.)

Table 20.1 shows the interest payments on each settlement date based on assumed LIBORs on the effective dates. In examining the table, several points should

TABLE 20.1 Interest Rate Swap: 5.5%/LIBOR Swap with NP = $10 Million

1 Effective Dates	2 LIBOR	3 Floating-Rate Payer's Payment*	4 Fixed-Rate Payer's Payment**	5 Net Interest Received by Fixed-Rate Payer Column 3 − Column 4	6 Net Interest Received by Floating-Rate Payer Column 4 − Column 3
3/23/Y1	0.045				
9/23/Y1	0.050	$225,000	$275,000	−$50,000	$50,000
3/23/Y2	0.055	$250,000	$275,000	−$25,000	$25,000
9/23/Y2	0.060	$275,000	$275,000	$0	$0
3/23/Y3	0.065	$300,000	$275,000	$25,000	−$25,000
9/23/Y3	0.070	$325,000	$275,000	$50,000	−$50,000
3/23/Y4		$350,000	$275,000	$75,000	−$75,000

*(LIBOR/2)($10,000,000)
**(.055/2)*($10,000,000)

be noted. First, the payments are determined by the LIBOR prevailing six months prior to the payment date; thus, payers on swaps would know their obligations in advance of the payment date. Second, when the LIBOR is below the fixed 5.5% rate, the fixed-rate payer pays the interest differential to the floating-rate payer; when it is above 5.5%, the fixed-rate payer receives the interest differential from the floating-rate payer. The net interest received by the fixed-rate payer is shown in Column 5 of the table, and the net interest received by the floating-rate payer is shown in Column 6. As we will discuss later, the fixed-rate payer's position is very similar to a short position in a series of Eurodollar futures contracts, with the futures price determined by the fixed rate. The fixed payer's cash flows also can be replicated by the fixed payer buying a $10 million, three-year, floating-rate note (FRN) paying the LIBOR and shorting (issuing) a $10 million, 5.5% fixed-rate bond at par. The floating-rate payer's position, on the other hand, is similar to a long position in a Eurodollar strip, and it can be replicated by issuing (shorting) a three-year, $10 million FRN paying the LIBOR and purchasing a three-year, $10 million, 5.5% fixed-rate bond at par.

Synthetic Loans

One of the important uses of swaps is in creating a synthetic fixed- or floating-rate liability that yields a better rate than the conventional one. To illustrate, suppose a corporation with an AAA credit rating wants a three-year, $10 million fixed-rate loan starting on March 23, Y1. Suppose one possibility available to the company is to borrow $10 million from a bank at a fixed rate of 6% (assume semiannual payments) with a loan maturity of three years. Suppose, though, that the bank also is willing to provide the company with a three-year floating-rate loan, with the rate set equal to the LIBOR on March 23 and September 23 each year for three years. If a swap agreement identical to the one described above were available, then instead of a direct fixed-rate loan, the company alternatively could obtain a fixed-rate loan by borrowing $10 million on the floating-rate loan, then fix the interest rate by taking a fixed-rate payer's position on the swap:

Conventional floating-rate loan	Pay floating rate
Swap: Fixed-rate payer position	Pay fixed rate
Swap: Fixed-rate payer position	Receive floating rate
Synthetic fixed rate	Pay fixed rate

As shown in Table 20.2, if the floating-rate loan is hedged with a swap, any change in the LIBOR would be offset by an opposite change in the net receipts on the swap position. In this example, the company (as shown in the table) would end up paying a constant $275,000 every sixth month, which equates to an annualized borrowing rate of 5.5%: $R = 2(\$275{,}000)/\$10{,}000{,}000 = .055$. Thus the corporation would be better off combining the swap position as a fixed-rate payer with the floating-rate loan to create a synthetic fixed-rate loan than simply taking the straight fixed-rate loan.

In contrast, a synthetic floating-rate loan is formed by combining a floating-rate payer's position with a fixed-rate loan. This loan then can be used as an alternative to a floating-rate loan:

Conventional fixed-rate loan	Pay fixed rate
Swap: Floating-rate payer position	Pay floating rate
Swap: Floating-rate payer position	Receive fixed rate
Synthetic floating rate	Pay floating rate

An example of a synthetic floating-rate loan is shown in Table 20.3. The synthetic loan is formed with a 5% fixed-rate loan (semiannual payments) and the floating-rate payer's position on our illustrate swap. As shown in the table, the synthetic floating-rate loan yields a 0.5% lower interest rate each period (annualized rate) than a floating-rate loan tied to the LIBOR.

Note, in both of the above examples, the borrower is able to attain a better borrowing rate with a synthetic loan using swaps than with a direct loan. When differences between the rates on actual and synthetic loans do exist, then swaps provide an apparent arbitrage use in which borrowers and investors can obtain better rates with synthetic positions formed with swap positions than they can from conventional loans.

Similarities between Swaps and Bond Positions and Eurodollar Futures Strips

Bond Positions Swaps can be viewed as a combination of a fixed-rate bond and floating-rate note (FRN). A fixed-rate payer position is equivalent to buying an FRN paying the LIBOR and shorting (issuing) a fixed-rate bond at the swap's fixed rate.

TABLE 20.2 Synthetic Fixed-Rate Loan: Floating-Rate Loan Set at LIBOR and Fixed-Payer Position on 5.5%/LIBOR Swap

1 Effective Dates	2 LIBOR	3 Swap Floating-Rate Payer's Payment*	4 Swap Fixed-Rate Payer's Payment**	5 Swap Net Interest Received by Fixed-Rate Payer Column 3 − Column 4	6 Loan Interest Paid on Floating-Rate Loan*	7 Synthetic Loan Payment on Swap and Loan Column 6 − Column 5	8 Synthetic Loan Effective Annualized Rate***
3/23/Y1	0.0450						
9/23/Y1	0.0500	$225,000	$275,000	−$50,000	$225,000	$275,000	0.055
3/23/Y2	0.0550	$250,000	$275,000	−$25,000	$250,000	$275,000	0.055
9/23/Y2	0.0600	$275,000	$275,000	$0	$275,000	$275,000	0.055
3/23/Y2	0.0650	$300,000	$275,000	$25,000	$300,000	$275,000	0.055
9/23/Y3	0.0700	$325,000	$275,000	$50,000	$325,000	$275,000	0.055
3/23/Y4		$350,000	$275,000	$75,000	$350,000	$275,000	0.055

*(LIBOR/2)($10,000,000)
**(.055/2)*($10,000,000)
***2 (Payment on Swap and Loan)/$10,000,000

TABLE 20.3 Synthetic Floating-Rate Loan: 5% Fixed-Rate Loan and Floating-Payer Position on 5.5%/LIBOR Swap

1 Effective Dates	2 LIBOR	3 Swap Floating-Rate Payer's Payment*	4 Swap Fixed-Rate Payer's Payment**	5 Swap Net Interest Received by Floating-Rate Payer Column 4 − Column 3	6 Loan Interest Paid on 5% Fixed-Rate Loan	7 Synthetic Loan Payment on Swap and Loan Column 6 − Column 5	8 Synthetic Loan Effective Annualized Rate***
3/23/Y1	0.0450						
9/23/Y1	0.0500	$225,000	$275,000	$50,000	$250,000	$200,000	0.040
3/23/Y2	0.0550	$250,000	$275,000	$25,000	$250,000	$225,000	0.045
9/23/Y2	0.0600	$275,000	$275,000	$0	$250,000	$250,000	0.050
3/23/Y3	0.0650	$300,000	$275,000	−$25,000	$250,000	$275,000	0.055
9/23/Y3	0.0700	$325,000	$275,000	−$50,000	$250,000	$300,000	0.060
3/23/Y4		$350,000	$275,000	−$75,000	$250,000	$325,000	0.065

*(LIBOR/2)($10,000,000)
**(.055/2)*($10,000,000)
***2 (Payment on swap and loan)/$10,000,000

From the previous example, the purchase of $10 million worth of three-year FRNs with the rate reset every six months at the LIBOR and the sale of $10 million worth of three-year, 5.5% fixed-rate bonds at par would yield the same cash flow as the fixed-rate payer's swap. On the other hand, a floating-rate payer's position is equivalent to shorting (or issuing) an FRN at the LIBOR and buying a fixed-rate bond at the swap's fixed rate. Thus, the purchase of $10 million worth of three-year 5.5% fixed-rate bonds at par and the sale of $10 million worth of FRNs paying the LIBOR would yield the same cash flow as the floating-rate payer's swap in the above example.

Eurodollar Futures Strip A plain vanilla swap can also be viewed as a series of Eurodollar futures contracts. To see the similarities, consider a short position in a Eurodollar strip in which the short holder agrees to sell 10 Eurodollar deposits, each with a face value of $1 million and maturity of six months, at the IMM-index price of 94.5 (or discount yield of $R_D = 5.5\%$), with the expirations on the strip being March 23 and September 23 for a period of two and half years.

With the index at 94.5, the contract price on one Eurodollar futures contract is $972,500:

$$f_0 = \left[\frac{100 - (5.5)(180/360)}{100}\right](\$1{,}000{,}000)$$

$$= \$972{,}500$$

Table 20.4 shows the cash flows at the expiration dates from closing the 10 short Eurodollar contracts at the same assumed LIBOR used in the preceding swap example, with the Eurodollar settlement index being 100 – LIBOR. For example,

TABLE 20.4 Short Positions in Eurodollar Futures

1	2	3	4	5
Closing Dates	LIBOR	f_T	Cash Flow from Short Position $10[f_0 - f_T]$	Cash Flow from Long Position $10[f_T - f_0]$
3/23/Y1	0.050	$975,000	−$25,000	$25,000
9/23/Y1	0.055	$972,500	$0	$0
3/23/Y2	0.060	$970,000	$25,000	−$25,000
9/23/Y2	0.065	$967,500	$50,000	−$50,000
3/23/Y3	0.070	$965,000	$75,000	−$75,000
9/23/Y3	0.075	$962,500	$100,000	−$100,000

$f_0 = 972{,}500$

$f_T = \left[\frac{100-(\text{LIBOR})(180/360)}{100}\right]$ ($1,000,000) with the LIBOR at 5% on 9/23/Y1, a $25,000 loss occurs from settling the 10 futures contracts. That is:

$$f_T = \left[\frac{100 - (5)(180/360)}{100}\right](\$1,000,000) = \$975,000$$

$$\text{Futures cash flow} = 10[f_0 - f_T] = 10[\$972,500 - \$975,000] = -\$25,000$$

Comparing the fixed-rate payer's net receipts shown in Column 5 of Table 20.1 with the cash flows from the short positions on the Eurodollar strip shown in Table 20.4, one can see that the two positions yield the same numbers. There are, however, some differences between the Eurodollar strip and the swap. First, a six-month differential occurs between the swap payment and the futures payments. This time differential is a result of the interest payments on the swap being determined by the LIBOR at the beginning of the period, whereas the futures position's profit is based on the LIBOR at the end of its period. Second, we have assumed the futures contract is on a Eurodollar deposit with a maturity of six months instead of the standard three months. For the standard Eurodollar strip and swap to be more similar, we would need to compare the swap to a synthetic contract on a six-month Eurodollar deposit formed with two short positions on three-month Eurodollar deposits: one expiring at T and one at $T + 90$ days. In addition to these technical differences, other differences exist: Strips are guaranteed by a clearinghouse, whereas banks can act as guarantors for swaps; strip contracts are standardized, whereas swap agreements often are tailor made.

20.3 SWAP MARKET

Structure

Corporations, financial institutions, and others who use swaps are linked by a group of brokers and dealers who collectively are referred to as ***swap banks***. These swap banks consist primarily of commercial banks and investment bankers. As brokers, swap banks try to match parties with opposite needs (see Figure 20.1). Many of the first interest-rate swaps were customized brokered deals between counterparties, with the parties often negotiating and transacting directly between themselves. As a broker, the swap bank's role in the contract is to bring the parties together and provide information; swap banks often maintains lists of companies and financial institutions that are potential parties to a swap. Once the swap agreement is closed, the swap broker usually has only a minor continuing role. With some ***brokered swaps***, the swap bank guarantees one or both sides of the transaction. With many, though, the counterparties assume the credit risk and make their own assessment of the other party's default potential.

One of the problems with a brokered swap is that it requires each party to have knowledge of the other party's risk profile. Historically, this problem led to more swap banks taking positions as dealers instead of as brokers. With ***dealer swaps***, the swap dealer often makes a commitment to enter a swap as a counterparty before

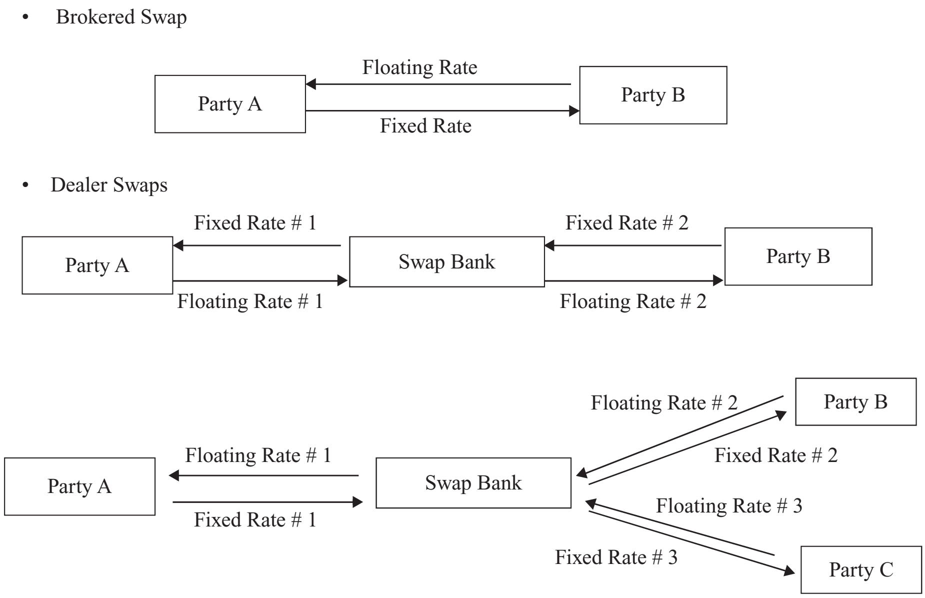

FIGURE 20.1 Swap Market Structure

the other end party has been located. Each of the counterparties (or in this context, the end parties) contracts separately with the swap bank, who acts as a counterparty to each. The end parties, in turn, assume the credit risk of the financial institution instead of that of the other end party, whereas the swap dealer assumes the credit risk of both of the end parties.

In acting as dealers, swap banks often match a swap agreement with multiple-end parties. For example, as illustrated in Figure 20.1, a $30 million fixed-for-floating swap between a swap dealer and Party A might be matched with two $15 million floating-for-fixed swaps. Ideally, a swap bank tries to maintain a perfect hedge. In practice, though, swap banks are prepared to enter a swap agreement without an opposite counterparty. This practice is sometimes referred to as ***warehousing***. In warehousing, swap banks will try to hedge their swap positions with opposite positions in T-notes and FRNs or using Eurodollar futures contracts. For example, a swap bank might hedge a $10 million, two-year floating-rate position by shorting $10 million worth of two-year T-notes, and then use the proceeds to buy FRNs tied to the LIBOR. In general, most of the commitments a swap bank assumes are hedged through a portfolio of alternative positions—opposite swap positions, spot positions in T-notes and FRNs, or futures positions. This type of portfolio management by swap banks is referred to as ***running a dynamic book***.

Swap Market Price Quotes

By convention, the floating rate on a swap is quoted flat without basis point adjustments. The fixed rate on a generic swap, in turn, is quoted in terms of the yield to

TABLE 20.5 Swap Bank Quote

Swap Maturity	Treasury Yield (%)	Bid Swap Spread (bp)	Ask Swap Spread (bp)	Fixed Swap Rate Spread	Swap Rate
2 year	4.95	54	64	5.49%–5.59%	5.540%
3 year	5.12	72	76	5.84%–5.88%	5.860%
4 year	5.32	69	74	6.01%–6.06%	6.035%
5 year	5.74	70	76	6.44%–6.50%	6.470%

maturity (YTM) on an on-the-run T-bond or T-note; that is, the most recent note or bond issued with a maturity matching the swap. In a dealer swap, the swap dealer's compensation comes from a markup or bid-ask spread extended to the end parties. The spread is reflected on the fixed-rate side. The swap dealer will provide a bid-ask quote to a potential party, and will in some cases post the bid-and-ask quotes. The quotes are stated in terms of the bid rate the dealer will pay as a fixed payer in return for the LIBOR, and the ask rate the dealer will receive as a floating-rate payer in return for paying the LIBOR. For example, a 70/75 swap spread implies the dealer will buy (take fixed-payer position) at 70 basis points over the T-note yield and sell (take floating-payer position) at 75 basis points over the T-note yield. Table 20.5 shows some illustrative quotes being offered by a swap bank on four generic swaps. The quote on the two-year swap indicates that the swap bank will take the fixed-rate payer's position at 5.49%/LIBOR (fixed rate = 4.95% + 54 bp), and the floating-rate payer's position at 5.59%/LIBOR (fixed rate = 4.95% + 64 bp). The average of the bid-and-ask rates is known as the ***swap rate***.

It should be noted that the fixed and floating rates quoted on a swap are not directly comparable. That is, the T-note assumes a 365-day basis, whereas the LIBOR assumes 360 days. In addition, these rates also need to be prorated to the actual number of days that have elapsed between settlement dates to determine the actual payments. These adjustments can be accounted for by using the following formulas:

$$\text{Fixed-rate settlement payment} = (\text{Fixed rate})\left[\frac{\text{No. of days}}{365}\right]\text{NP}$$

$$\text{Floating-rate settlement payment} = (\text{LIBOR})\left[\frac{\text{No. of days}}{360}\right]\text{NP}$$

To simplify the exposition, we will ignore the day count conventions in our other examples of generic swaps and simply divide annual rates by two (i.e., we will use a 180/360 day count convention).

Opening Swap Positions

Suppose a corporate treasurer wants to fix the rate on a five-year, $50 million floating-rate debt of the company by taking a fixed-rate payer's position on a five-year swap with an NP of $50 million. To obtain the swap position, suppose the treasurer calls a swap banker for a quote on a fixed-rate payer's position. After assessing the corporation's credit risk, suppose the swap banker gives the treasurer a swap quote of 100 basis points over the current five-year T-note yield, and the corporate treasurer, in turn, accepts. Thus, the treasurer would agree to take the fixed payer's position on the swap at 100 bp above the current five-year T-note. Except for the rate, both parties would mutually agree to the terms of the swap. After agreeing to the terms, the actual rate paid by the fixed payer would typically be set once the swap banker hedges her swap position by taking, for example, a position on an on-the-run T-note. After confirming a quote on a five-year T-note from her bond trader, the swap banker would then instruct the trader to sell (or short) $50 million of five-year T-notes with the proceeds invested in a five-year FRN paying LIBOR. The yield on the T-note purchased plus the 100 basis points would determine the actual rate on the swap. At a later date, the swap banker would most likely close the bond positions used to hedge the swap whenever she finds a floating-rate swap position to take on one or more swaps with similar terms.

Note, since swap banks can hedge an opening swap position with positions in an on-the-run T-note or an FRN, the rates they, in turn, set on a swap contract are determined by the current T-note yields (with bp added to reflect credit risk). Thus, opening swap contracts are tied to T-note yields. This relation is also reinforced by an arbitrage strategy consisting of positions in a swap, T-note, and FRN.

Closing Swap Positions

Prior to maturity, swap positions can be closed by selling the swap to a swap dealer or another party. If the swap is closed in this way, the new counterparty either pays or receives an upfront fee to or from the existing counterparty in exchange for receiving the original counterparty's position. Alternatively, the swap holder could hedge his position by taking an opposite position in a current swap or possibly by hedging the position for the remainder of the maturity period with a futures position. Thus, a fixed-rate payer who unexpectedly sees interest rates decreasing and, as a result, wants to change his position, could do so by selling the swap to a dealer, taking a floating-rate payer's position in a new swap contract, or by going long in an appropriate futures contract; this latter strategy might be advantageous if there is only a short period of time left on the swap.

If the fixed-payer swap holder decides to hedge his position by taking an opposite position on a new swap, the new swap position would require a payment of the LIBOR that would cancel out the receipt of the LIBOR on the first swap. The difference in the positions would therefore be equal to the difference in the higher fixed-interest rate that is paid on the first swap and the lower fixed-interest rate received on the offsetting swap. For example, suppose in our first illustrative swap example (Table 20.1), a decline in interest rates occurs one year after the initiation of the swap, causing the fixed-rate payer to want to close his position. To this end, suppose the fixed-rate payer offsets his position by entering a new two-year swap as a floating-rate payer in which he agrees to pay the LIBOR for a 5% fixed rate. The two positions would result in a fixed payment of $25,000 semiannually for two years [(.005/2) NP]. If interest rates decline over the next year, this offsetting position would turn out to be the correct strategy.

Offsetting Swap Positions

Original swap: Fixed payer's position	Pay 5.5%	−5.5%
Original swap: Fixed payer's position	Receive LIBOR	+LIBOR
Offsetting swap: Floating payer's position	Pay LIBOR	−LIBOR
Offsetting swap: Floating payer's position	Receive 5.0%	+5%
	Pay 0.5% (annual)	−0.5% (annual)

Instead of hedging the position, the fixed-rate payer is more likely to close his position by simply selling it to a swap dealer. In acquiring a fixed position at 5.5%, the swap dealer would have to take a floating payer's position to hedge the acquired fixed position. If the fixed rate on a new two-year swap were at 5%, the dealer would likewise lose $25,000 semiannually for two year on the two swap positions given an NP of $10 million. Thus, the price the swap bank would charge the fixed payer for buying his swap would be at least equal to the present value of $25,000 for the next four semiannual periods. Given a discount rate of 5%, the swap bank would charge the fixed payer a minimum of $94,049 for buying his swap:

$$SV_0^{Fix} = \sum_{t=1}^{4} \frac{-\$25{,}000}{(1 + (.05/2))^t} = -\$94{,}049$$

In contrast, if rates had increased, the fixed payer would be able to sell the swap to a dealer at a premium. For example, if the fixed rate on a new swap were 6%, a swap dealer would realize a semiannual return of $25,000 for the next two years by buying the 5.5%/LIBOR swap and hedging it with a floating position on a two-year, 6%/LIBOR swap. Given a 6% discount rate, the dealer would pay the fixed payer a maximum of $92,927 for his 5.5%/LIBOR swap:

$$SV_0^{Fix} = \sum_{t=1}^{4} \frac{\$25{,}000}{(1 + (.06/2))^t} = \$92{,}927$$

Note that the above case illustrates that the value of an existing swap depends on the rates on current swaps. Moreover, the fixed rate on current swaps depends on the yields on T-notes.

20.4 SWAP VALUATION

At origination, most plain vanilla swaps have an economic value of zero. This means that neither counterparty is required to pay the other in the agreement. An economic value of zero requires that the swap's underlying bond positions trade at par—a ***par value swap***. If this were not the case, then one of the counterparties would need to compensate the other. In this case, the economic value of the swap is not zero. Such a swap is referred to as an ***off-market swap***.

Although most plain vanilla swaps are originally par value swaps with economic values of zero, as the preceding example illustrates, the economic values of existing swaps change over time as rates change; that is, existing swaps become off-market swaps as rates change. In our example, the fixed payer's position on the 5.5%/LIBOR swap had a value of −$94,049 one year later when the fixed rate on new two-year par value swaps was 5%; that is, the holder of the fixed position would have to pay the swap bank at least $94,049 to assume the swap. On the other hand, the fixed payer's position on the 5.5%/LIBOR swap had a value of $92,927 when the fixed rate on new two-year par value swaps was 6%; that is, the holder of the fixed position would receive $92,927 from the swap bank.

Just the opposite values apply to the floating position. Continuing with our illustrative example, if the fixed rate on new two-year par value swaps were at 5%, then a swap bank who assumed a floating position on a 5.5%/LIBOR swap and then hedged it with a fixed position on a current two-year 5%/LIBOR swap would gain $25,000 semiannually over the next two years. As a result, the swap bank would be willing to pay $94,049 for the floating position. Thus, the floating position on the 5.5% swap would have a value of $94,049:

Offsetting Swap Positions

Original swap: Floating payer's position	Pay LIBOR	−LIBOR
Original swap: Floating payer's position	Receive 5.5%	+5.5%
Offsetting swap: Fixed payer's position	Pay 5%	−5%
Offsetting swap: Fixed payer's position	Receive LIBOR	+LIBOR
	Receive 0.5% (annual)	0.5% (annual)

$$SV_0^{FL} = \sum_{t=1}^{4} \frac{\$25{,}000}{(1 + (.05/2))^t} = \$94{,}049$$

If the fixed rate on new one-year par value swaps were at 6%, then a swap bank assuming the floating position on a 5.5%/LIBOR swap and hedging it with a fixed position on a current two-year 6%/LIBOR swap would lose $25,000 semiannually over the next year. As a result, the swap bank would charge $92,927 for assuming the floating position. Thus, the floating position on the 5.5% swap would have a negative value of $92,927:

$$SV_0^{FL} = \sum_{t=1}^{4} \frac{-\$25{,}000}{(1 + (.06/2))^t} = -\$92{,}927$$

Offsetting Swap Positions

Original swap: Floating payer's position	Pay LIBOR	−LIBOR
Original swap: Floating payer's position	Receive 5.5%	+5.5%
Offsetting swap: Fixed payer's position	Pay 6%	−6%
Offsetting swap: Fixed payer's position	Receive LIBOR	+LIBOR
	Pay 0.5% (annual)	−0.5% (annual)

In general, the value of an existing swap is equal to the value of replacing the swap (replacement swap), which depends on current T-note rates. Formally, the values of the fixed and floating swap positions are:

$$SV^{fix} = \left[\sum_{t=1}^{M} \frac{K^P - K^S}{(1+K^P)^t}\right] NP$$

$$SV^{FL} = \left[\sum_{t=1}^{M} \frac{K^S - K^P}{(1+K^P)^t}\right] NP$$

where K^S = Fixed rate on the existing swap
K^P = Fixed rate on current par-value swap
SV^{fix} = Swap value of the fixed position on the existing swap
SV^{FL} = Swap value of the floating position on the existing swap

Note that these values are obtained by discounting the net cash flows at the current YTM (K^P). As a result, this approach to valuing off-market swaps is often referred to as the ***YTM approach***. However, recall from our discussion of bonds that the equilibrium price of a bond is obtained not by discounting all of the bond's cash flows by a common discount rate, but rather by discounting each of the bond's cash flows by their appropriate spot rates—the rate on a zero-coupon bond. As we have discussed, valuing bonds by using spot rates instead of a common YTM ensures that there are no arbitrage opportunities from buying bonds and stripping them or buying zero-discount bonds and bundling them. The argument for pricing bonds in terms of spot rates also applies to the valuation of off-market swaps. Similar to bond valuation, the equilibrium value of a swap is obtained by discounting each of the swap's cash flows by their appropriate spot rates. The valuation of swaps using spot rates is referred to as the ***zero-coupon approach***. The approach, in turn, requires generating a spot yield curve for swaps.[1]

20.5 COMPARATIVE ADVANTAGE AND THE HIDDEN OPTION

Comparative Advantage

Swaps are often used by financial and nonfinancial corporations to take advantage of apparent arbitrage opportunities resulting from capital–market inefficiencies. To see this, consider the case of the Star Chemical Company who wants to raise \$300 million with a five-year loan to finance an expansion of one of its production plants. Based on its moderate credit ratings, suppose Star can borrow five-year funds at a 10.5% fixed rate or at a floating rate equal to LIBOR + 75 bp. Given the choice of financing, Star prefers the fixed-rate loan. Suppose the treasurer of the Star Company contacts his investment banker for suggestions on how to finance the acquisition. The investment banker knows that the Moon Development Company is also looking for five-year funding to finance its proposed \$300 million office park development. Given its high credit rating, suppose Moon can borrow the funds for 5 years at a fixed rate of 9.5% or at a floating rate equal to the LIBOR + 25 bp. Given the choice, Moon prefers a floating-rate loan. In summary, Star and Moon have the following fixed and floating rate loan opportunities:

Company	**Fixed Rate**	**Floating Rate**	**Preference**	**Comparative Advantage**
Star Company	10.5%	LIBOR + 75 bp	Fixed	Floating
Moon Company	9.5%	LIBOR + 25 bp	Floating	Fixed
Credit spread	100 bp	50 bp		

In this case, the Moon Company has an absolute advantage in both the fixed and floating markets because of its higher quality rating. However, after looking at the credit spreads of the borrowers in each market, the investment banker realizes that there is a ***comparative advantage*** for Moon in the fixed market and a comparative advantage for Star in the floating market. That is, Moon has a relative advantage in the fixed market where it gets 100 basis points less than Star; Star, in turn, has a relative advantage (or relatively less disadvantage) in the floating rate market where it only pays 50 basis points more than Moon. Thus, lenders in the fixed-rate market supposedly assess the difference between the two creditors to be worth 100 basis points, whereas lenders in the floating-rate market assess the difference to be only 50 basis points. Whenever a comparative advantage exists, arbitrage opportunities can be realized by each firm borrowing in the market where it has a comparative advantage and then swapping loans or having a swap bank set up a swap.

For the swap to work, the two companies cannot just pass on their respective costs: Star swaps a floating rate at LIBOR + 75 bp for a 10.5% fixed; Moon swaps a 9.5% fixed for a floating at LIBOR + 25 bp. Typically, the companies divide the differences in credit spreads, with the most creditworthy company taking the most savings. In this case, suppose the investment banker arranges a five-year, 9.5%/LIBOR generic swap with an NP of $300 million in which Star takes the fixed-rate payer position and Moon takes the floating-rate payer position.

The Star Company would then issue a $300 million FRN paying LIBOR + 75 bp. This loan, combined with the fixed-rate position on the 9.5%/LIBOR swap would give Star a synthetic fixed-rate loan paying 10.25%—25 basis points less than its direct fixed-rate loan:

Star Company's Synthetic Fixed-Rate Loan		
Issue FRN	Pay LIBOR + 75bp	−LIBOR − .75%
Swap: Fixed-rate payer's position	Pay 9.5%	−9.5%
Swap: Fixed-rate payer's position	Receive LIBOR	+LIBOR
Synthetic fixed rate	Pay 9.5% + .75%	−10.25%
Direct fixed rate	Pay 10.5%	−10.5%

The Moon Company, on the other hand, would issue a $300 million, 9.5% fixed-rate bond that, when combined with its floating-rate position on the 9.5%/LIBOR swap, would give Moon a synthetic floating-rate loan paying LIBOR, which is 25 basis points less than the rates paid on the direct floating-rate loan of LIBOR plus 25 bp:

Moon Company's Synthetic Floating-Rate Loan		
Issue 9.5% fixed-rate bond	Pay 9.5%	−9.5%
Swap: Floating-rate payer's position	Pay LIBOR	−LIBOR
Swap: Floating-rate payer's position	Receive 9.5%	+9.5%
Synthetic fixed rate	Pay LIBOR	−LIBOR
Direct fixed rate	Pay LIBOR + 25bp	−LIBOR − .25%

Thus, the swap makes it possible for both companies to create synthetic loans with better rates than direct ones.

As a rule, for a swap to provide arbitrage opportunities, at least one of the counterparties must have a comparative advantage in one market. The total arbitrage gain available to each party depends on whether one party has an absolute advantage in both markets or each has an absolute advantage in one market. If one party has an absolute advantage in both markets (as in this case), then the arbitrage gain is the difference in the comparative advantages in each market: 50 bp = 100 bp – 50 bp. In this case, Star and Moon split the difference in the 50 bp gain. In contrast, if each party has an absolute advantage in one market, then the arbitrage gain is equal to the sum of the comparative advantages.

Hidden Option

The comparative advantage argument has often been cited as the explanation for the dramatic growth in the swap market. This argument, though, is often questioned on the grounds that the mere use of swaps should over time reduce the credit interest rate differentials in the fixed and flexible markets, taking away the advantages from forming synthetic positions. With observed credit spreads and continuing use of swaps to create synthetic positions, some scholars (Smith, Smithson, and Wakeman, 1986) have argued that the comparative advantage that is apparently extant is actually a hidden option embedded in the floating-rate debt position that proponents of the comparative advantage argument fail to include. They argue that the credit spreads that exist are due to the nature of the contracts available to firms in fixed and floating markets. In the floating market, the lender usually has the opportunity to review the floating rate each period and increase the spread over the LIBOR if the borrower's creditworthiness has deteriorated. This option, though, does not usually exist in the fixed market.

In the preceding example, the lower quality Star Company is able to get a synthetic fixed rate at 10.25% (.25% less than the direct loan). However, using the hidden option argument, this 10.25% rate is only realized if Star can maintain its creditworthiness and continue to borrow at a floating rate that is 75 basis points above LIBOR. If its credit ratings were to subsequently decline and it had to pay

150 basis points above the LIBOR, then its synthetic fixed rate would increase. Moreover, studies have shown that the likelihood of default increases faster over time for lower quality companies than it does for higher quality. In our example, this would mean that the Star Company's credit spread is more likely to rise than the Moon Company's spread and that its expected borrowing rate is greater than the 10.25% synthetic rate. As for the higher quality Moon Company, its lower synthetic floating rate of LIBOR does not take into account the additional return necessary to compensate the company for bearing the risk of a default by the Star Company. If it borrowed floating funds directly, the Moon Company would not be bearing this risk.

20.6 SWAPS APPLICATIONS

Arbitrage Applications: Synthetic Positions

In the preceding case, the differences in credit spreads in the fixed-rate and floating-rate debt markets or the hidden options on the floating debt position made it possible for both corporations to obtain different rates with synthetic positions than they could with direct loans. The example represents what is commonly referred to as an arbitrage use of swaps. In general, the presence of comparative advantage or a hidden option makes it possible to create not only synthetic loans with lower rates than direct, but also synthetic investments with rates exceeding those from direct investments. To illustrate this, four cases showing how swaps can be used to create synthetic fixed-rate and floating-rate loans and investments are presented below.

Synthetic Fixed-Rate Loan

Suppose a company is planning to borrow $50 million for five years at a fixed rate. Given a swap market, suppose its alternatives are to issue a five-year, 10%, fixed-rate bond paying coupons on a semiannual basis or create a synthetic fixed-rate bond by issuing a five-year floating-rate medium-term note (MTN) paying LIBOR plus 100 bp and taking a fixed-rate payer's position on a swap with an NP of $50 million. The synthetic fixed-rate MTN will be equivalent to the direct fixed-rate loan if it is formed with a swap that has a fixed rate equal to 9%:

Synthetic Fixed-Rate Loan		
Issue FRN	Pay LIBOR + 1%	−LIBOR − 1%
Swap: Fixed-rate payer's position	Pay 9% fixed rate	−9%
Swap: Fixed-rate payer's position	Receive LIBOR	+LIBOR
Synthetic rate	Pay 9% + 1%	−10%
Direct loan rate	Pay 10%	−10%

If the company can obtain a fixed rate on a swap that is less than 9%, then the company would find it cheaper to finance with the synthetic fixed-rate MTN than the direct. For example, if the company could obtain an 8%/LIBOR swap, then the company would be able to create a synthetic 9% fixed-rate loan by issuing a floating-rate MTN at LIBOR plus 100 basis points and taking the fixed payer's position on the swap:

Synthetic Fixed-Rate Loan		
Issue floating-rate MTN (FRN)	Pay LIBOR + 1%	−LIBOR – 1%
Swap: Fixed-rate payer's position	Pay 8% fixed rate	−8%
Swap: Fixed-rate payer's position	Receive LIBOR	+LIBOR
Synthetic rate	Pay 8% + 1%	−9%
Direct loan rate	Pay 10%	−10%

Synthetic Floating-Rate Loan

Suppose a bank has just made a five-year, $30 million floating-rate loan that is reset every six months at the LIBOR plus 100 bp. The bank could finance this floating-rate asset by either selling CDs every six months at the LIBOR or by creating a synthetic floating-rate loan by selling a five-year fixed-rate note at 9% and taking a floating-rate payer's position on a five-year swap with an NP of $30 million. The synthetic floating-rate loan will be equivalent to the direct floating-rate loan paying LIBOR if the swap has a fixed rate that is equal to the 9% fixed rate on the note:

Synthetic Floating-Rate Loan		
Issue 9% fixed-rate note	Pay 9% fixed rate	−9%
Swap: Floating-rate payer's position	Pay LIBOR	−LIBOR
Swap: Floating-rate payer's position	Receive 9% fixed rate	+9%
Synthetic rate	Pay LIBOR	−LIBOR
Direct loan rate	Pay LIBOR	−LIBOR

Thus, if the bank can obtain a fixed rate on the swap that is greater than 9%, say, 9.5%, then it would find it cheaper to finance its floating-rate loan asset by issuing fixed-rate notes at 9% and taking the floating-rate payer's position on the swap. By doing this, the bank's effective interest payments are 50 basis points less than LIBOR with a synthetic floating-rate loan formed by selling the 9% fixed rate note and taking a floating-rate payer's position on a five-year, 9.5%/LIBOR swap with NP of $30 million:

Synthetic Floating-Rate Loan		
Issue 9% fixed-rate note	Pay 9% fixed rate	−9%
Swap: Floating-rate payer's position	Pay LIBOR	−LIBOR
Swap: Floating-rate payer's position	Receive 9.5% fixed rate	+9.5%
Synthetic rate	Pay LIBOR − .5%	−(LIBOR − .5%)
Direct loan rate	Pay LIBOR	−LIBOR

Synthetic Fixed-Rate Investment In the early days of the swap market, swaps were primarily used as a liability management tool. In the late 1980s, investors began to use swaps to try to increase the yield on their investments. A swap used with an asset is sometimes referred to as an ***asset-based interest rate swap*** or simply an asset swap. In terms of synthetic positions, asset-based swaps can be used to create either fixed-rate or floating-rate investment positions.

Consider the case of an investment fund that is setting up a $100 million collateralized debt obligation (CDO) consisting of five-year, AAA quality, option-free, fixed-rate bonds. If the YTM on such bonds is 6%, then the investment company could form the CDO by simply buying $100 million worth of 6% coupon bonds at par. Alternatively, it could try to earn a higher return by creating a synthetic fixed-rate bond by buying five-year, high-quality FRNs currently paying the LIBOR plus 100 bp and taking a floating-rate payer's position on a five-year swap with an NP of $100 million. If the fixed rate on the swap is equal to 5% (the 6% rate on the bonds minus the 100 bp on the FRN), then the synthetic fixed-rate investment will yield the same return as the 6% fixed-rate bonds:

Synthetic Fixed-Rate Investment

Purchase FRN	Receive LIBOR + 1%	+LIBOR + 1%
Swap: Floating-rate payer's position	Pay LIBOR	−LIBOR
Swap: Floating-rate payer's position	Receive 5% fixed rate	+5%
Synthetic rate	Receive 5% + 1%	+6%
Direct investment rate	Receive 6%	+6%

If the fixed rate on the swap is greater than 5%, then the synthetic fixed-rate loan will yield a higher return than the 6% bonds. For example, if the investment company in forming the CDO could take a floating payer's position on a 5.75%/LIBOR swap with maturity of five years, NP of $100 million, and effective dates coinciding with the FRNs' dates, then the investment company would earn a fixed rate of 6.75%:

Synthetic Fixed-Rate Investment

Purchase FRN	Receive LIBOR + 1%	+LIBOR + 1%
Swap: Floating-rate payer's position	Pay LIBOR	−LIBOR
Swap: Floating-rate payer's position	Receive 5.75% fixed rate	+5.75%
Synthetic rate	Receive 6.75%	+6.75%
Direct investment rate	Receive 6%	+6%

Synthetic Floating-Rate Investment This time consider an investment fund that is looking to invest $10 million for three years in a FRN. Suppose the fund can either invest directly in a high-quality, five-year FRN paying LIBOR plus 50 basis points, or it can create a synthetic floating-rate investment by investing in a five-year, 7% fixed-rate note selling at par and taking a fixed-rate payer's position. If the fixed rate on the swap is equal to 6.5% (the rate on the fixed-rate note minus the bp on the direct FRN investment), then the synthetic floating-rate investment will yield the same return as the FRN:

Synthetic Floating-Rate Investment		
Purchase fixed-rate note	Receive 7%	+7%
Swap: Fixed-rate payer's position	Pay 6.5% fixed rate	−6.5%
Swap: Fixed-rate payer's position	Receive LIBOR	+LIBOR
Synthetic rate	Receive LIBOR + .5%	+LIBOR + .5%
Floating investment rate	Receive LIBOR + .5%	+LIBOR + .5%

If the fixed rate on the swap is less than 6.5%, then the synthetic floating-rate investment will yield a higher return than the FRN. For example, the fund could obtain a yield of LIBOR plus 100 basis points from a synthetic floating-rate investment formed with an investment in the five-year, 7% fixed-rate note and fixed-rate payer's position on a 6%/LIBOR swap:

Synthetic Floating-Rate Investment		
Purchase fixed-rate note	Receive 7%	+7%
Swap: Fixed-rate payer's position	Pay 6% fixed rate	−6%
Swap: Fixed-rate payer's position	Receive LIBOR	+LIBOR
Synthetic rate	Receive LIBOR + 1%	+LIBOR + 1%
Floating investment rate	Receive LIBOR + .5%	+LIBOR + .5%

Hedging

Initially, interest rate swaps were used primarily in arbitrage strategies. Today, there is an increased use of swaps for hedging. Hedging with swaps is done primarily to minimize the market risk of positions currently exposed to interest rate changes. For example, suppose a company had previously financed its capital projects with intermediate-term FRNs tied to the LIBOR. Furthermore, suppose the company was expecting higher interest rates and wanted to fix the rate on its floating-rate debt. To this end, one alternative would be for the company to refund its floating-rate debt with fixed-rate obligations. This, though, would require the cost of issuing new debt (underwriting, registration, etc.), as well as the cost of calling the current FRNs or buying the notes in the market if they were not callable. Thus, refunding would be a relatively costly alternative. Another possibility would be for the company to hedge its floating-rate debt with a strip of short Eurodollar futures contracts. This alternative is relatively inexpensive, but there may be hedging risk. The third and perhaps obvious alternative would be to combine the company's FRNs with a fixed-rate payer's position on a swap, thereby creating a synthetic fixed-rate debt position. This alternative of hedging FRNs with swaps, in turn, is less expensive and more efficient than the first alternative of refinancing; plus, it can also effectively minimize hedging risk.

An opposite scenario to the above case would be a company that has intermediate to long-term fixed-rate debt that it wants to make floating either because of a change in its economic structure or because it expects rates will be decreasing. Given the costs of refunding fixed-rate debt with floating-rate debt and the hedging risk problems with futures, the most efficient way for the company to meet this objective would be to create synthetic floating rate debt by combining its fixed-rate debt with a floating-rate payer's position on a swap.

Speculation

Because swaps are similar to Eurodollar futures contracts, they can be used like them to speculate on short-term interest rate movements. Specifically, as an alternative to a Eurodollar futures strip, speculators who expect short-term rates to increase in the future can take a fixed-rate payer's position; in contrast, speculators who expect short-term rates to decrease can take a floating-rate payer's position. Note, though, that there are differences in maturity, size, and marketability between futures and swaps that need to be taken into account when considering which one to use.

For financial and nonfinancial corporations, speculative positions often take the form of the company changing the exposure of its balance sheet to interest rate changes. For example, suppose a fixed-income bond fund with a portfolio measured against a bond index wanted to increase the duration of its portfolio relative to the index's duration based on an expectation of lower interest rates across all maturities. The fund could do this by selling its short-term Treasuries and buying longer term ones or by taking long positions in Treasury futures. With swaps, the fund could also change its portfolio's duration by taking a floating-rate payer's position on a swap. If they did this and rates were to decrease as expected, then not only would the value of the company's bond portfolio increase but the company would also profit from the swap; on the other hand, if rates were to increase, then the company would see a decrease in the value of its bond portfolio, as well as losses from its swap positions. By adding swaps, though, the fund has effectively increased its interest rate exposure by increasing its duration.

Instead of increasing its portfolio's duration, the fund may want to reduce or minimize the bond portfolio's interest rate exposure based on an expectation of higher interest rates. In this case, the fund could effectively shorten the duration of its bond fund by taking a fixed-rate payer's position on a swap. If rates were to later increase, then the decline in the value of the company's bond portfolio would be offset by the cash inflow realized from the fixed-payer's position on the swap.

20.7 CREDIT RISK

When compared to their equivalent fixed and floating bond positions, swaps have less credit risk. To see this, suppose one party to a swap defaults. Typically, the swap contract allows the non-defaulting party the right to give up to a 20-day notice that a particular date will be the termination date.[2] This notice gives the parties time to determine a settlement amount. The settlement amount depends on the value of an existing swap or equivalently on the terms of a replacement swap. For example, suppose the fixed payer on a 9.5%/LIBOR swap with NP of $10 million runs into severe financial problems and defaults when there are three years and six payment dates remaining and the LIBOR is now relatively low. Suppose the current three-year swap calls for an exchange of 9% fixed for LIBOR. To replace the defaulted swap, the non-defaulting floating payer would have to take a new floating position on the 9%/LIBOR swap or sell it to a swap bank, who would hedge the assumed swap with the floating position. As a result, she or the swap bank would be receiving only \$450,000 each period instead of the \$475,000 on the defaulted swap. Thus, the default represents a semiannual loss of \$25,000 for three years. Using 9% as the discount rate, the present value of this loss would be \$128,947:

$$PV = \sum_{t=1}^{6} \frac{-\$25{,}000}{(1 + (.09/2))^t} = -\$128{,}947$$

Thus, given a replacement fixed swap rate of 9%, the actual credit risk exposure is \$128,947. The replacement value of \$128,947 is also the economic value of the original 9.5%/LIBOR swap. Note: If the replacement fixed swap rate had been 10% instead of 9%, then the floating payer would have had a positive economic value of \$126,892:

$$PV = \sum_{t=1}^{6} \frac{\$25{,}000}{(1 + (.10/2))^t} = \$126{,}892$$

Under a higher interest rate scenario, the fixed payer experiencing the financial distress would not have defaulted on the swap, although he may be defaulting on other obligations. The increase in rates in this case has made the swap an asset to the fixed payer instead of a liability.

The example illustrates that two events are necessary for default loss on a swap: an actual default on the agreement and an adverse change in rates. Credit risk on a swap is therefore a function of the joint probability of financial distress and adverse interest rate movements.

In practice, credit risk is often managed by adjusting the negotiated fixed rate on a swap to include a credit risk spread between the parties: A less risky firm will pay a lower fixed rate or receive a higher fixed rate the riskier the counterparty. The credit rate adjustment also takes into account the probability of rates increasing and decreasing and its impact on the future economic value of the couterparty's swap position. In addition to rate adjustments, swap dealers can also manage credit risk by requiring collateral and maintenance margins.

20.8 CONCLUSION

In this chapter, we have examined the market, uses, and valuations of generic interest rate swaps. Swaps provide investors and borrowers with a tool for more effectively managing their asset and liability positions. They have become a basic financial engineering tool to apply to a variety of financial problems. Over the years, the underlying structure of the generic swap has been modified in a number of ways to accommodate different uses. In the next chapter, we examine swap derivatives—forward swaps and swaptions—and show how they can be used to manage fixed-income positions.

KEY TERMS

asset-based interest rate swaps
brokered swaps
comparative advantage
confirmation
counterparties
currency swap
dealer swaps
effective date
fixed-rate payer
floating-rate payer
interest rate swap
International Swap and Derivative Association (ISDA)
maturity date
net settlement basis
off-market swap
par value swap
plain vanilla swap (or generic swap)
running a dynamic book
settlement (or payment date)
swap banks
swap rate
trade date
warehousing
YTM approach
zero-coupon approach

PROBLEMS AND QUESTIONS

1. Given the following interest-rate swap:
   - Fixed-rate payer pays half of the YTM on a T-note of 6.5%
   - Floating-rate payer pays the LIBOR
   - Notional principal is $10 million
   - Effective dates are 3/23 and 9/23 for the next three years

   a. Determine the net receipts of the fixed-rate payer and the floating-rate payer given the following LIBORs:
      - 3/23/Y1 .055
      - 9/23/Y1 .060
      - 3/23/Y2 .065
      - 9/23/Y2 .070
      - 3/23/Y3 .075
      - 9/23/Y3 .080

   b. Show in a table how a company with a three-year, $10 million floating-rate loan, with the rate set by the LIBOR on the dates coinciding with the swap, could make the loan a fixed-rate one by taking a position in the swap. What would be the fixed rate?

   c. Show in a table how a company with a two-year, $10 million fixed-rate loan at 6.0%, could make the loan a floating-rate one by taking a position in the swap.

2. Explain how the fixed-payer and floating-payer positions in Question 1 could be replicated with positions in fixed-rate and floating-rate bonds.

3. What positions in a Eurodollar futures strip would be similar to the fixed-payer and floating-payer positions in Question 1? Show in the following table the cash

flows for each strip position given the LIBOR scenario in Question 1. Note: The interest payments on the swap are determined by the LIBOR at the beginning of the period, whereas the futures position's cash flows are based on the LIBOR at the end of its period.

Eurodollar Closing Dates	LIBOR	f_T	Cash Flow from Short Position	Cash Flow from Long Position
3/23/Y1	.055			
9/23/Y1	.060			
3/23/Y2	.065			
9/23/Y2	.070			
3/23/Y3	.075			
9/23/Y3	.080			

4. Explain some of the differences between a plain vanilla swap position and a Eurodollar strip.
5. Using a table showing payments and receipts, prove that the following positions are equivalent:
   a. Floating-rate loan plus fixed-rate payer's position is equivalent to a fixed-rate loan.
   b. Fixed-rate loan plus floating-rate payer's position is equivalent to a floating-rate loan.
   c. Floating-rate note investment plus floating-rate payer's position is equivalent to a fixed-rate investment.
   d. Fixed-rate bond investment plus fixed-rate payer's position is equivalent to a floating-rate investment.
6. What are the dealer's bid and ask quotes on a five-year swap with a quoted 60/70 swap spread over a T-note with a yield of 6.5%?
7. The table below shows a swap bank's quotes on four generic swaps.

Swap	Swap Maturity	Treasury Yield (%)	Bid Swap Spread (bp)	Ask Swap Spread (bp)	Fixed Swap-Rate Spread	Swap Rate
1	2 years	4.95%	54	64		
2	3 years	5.12%	72	76		
3	4 years	5.32%	69	74		
4	5 years	5.74%	70	76		

Questions:

a. Complete the table by calculating the swap bank's fixed swap-rate spreads and swap rates for the four swaps.
b. Explain in more detail the positions the swap bank is willing to take on the first swap contract.

c. Describe the swap arrangement for a swap bank customer who took a floating-rate payer's position with a notional principal of $50 million on the third contract offered by the bank.

d. Suppose two of the swap bank's customers take fixed-rate payer positions on the third contract offered by the bank, each with notional principals of $25 million. Show in a flow diagram the contracts between the swap bank and these customers and its customer in Question 7.c.

8. The table below shows the effective dates on a 6%/LIBOR swap, the number of days between effective dates, and assumed LIBORs on effective dates. Complete the table by determining the swap's fixed payments, floating payments, and net receipts received by the fixed- and floating-rate payers.

Settlement Date	Number of Days	LIBOR	Fixed Payment	Floating Payment	Fixed Payer's Net Receipts	Floating Payer's Net Receipts
6/10/Y1		6.50%				
12/10/Y1	183	6.75%				
6/10/Y2	182	6.25%				
12/10/Y2	183	6.00%				
6/10/Y3	182	5.50%				
12/10/Y3	183	5.25%				
6/10/Y4	182					

9. Suppose a swap bank can go long and short in three-year FRNs paying LIBOR and three-year T-notes yielding 5%.

a. Given these securities, define the three-year generic par value swap the bank could offer its customers. Exclude the basis point that the swap bank might add to its bid and ask prices.

b. Explain how the swap bank determines basis points to add to the fixed rate on its fixed and floating positions.

c. Suppose the swap bank provided one of its customers with a 5%/LIBOR fixed-rate position. Explain how the bank would hedge its position with the above securities if it did not have a customer taking an opposite swap position.

d. Suppose the swap bank provided one of its customers with a three-year, 5%/LIBOR floating-rate position. Explain how the bank would hedge its position with the above securities if it did not have a customer taking an opposite swap position.

10. Given the 5% yield on three-year T-notes and LIBOR yields on FRNs in Question 9, how much would the swap bank pay or charge to buy an *existing* floating-rate payer's position on a three-year, 6%/LIBOR swap with a notional principal of $50 million, if it planned to hedge the purchase with positions in the T-notes and FRNs? How much would it pay or charge for an existing fixed-payer's position on a three-year, 6%/LIBOR with a notional principal of $50 million? Exclude the basis points that the swap bank might add to its bid and ask prices.

11. If the swap bank in Question 9 were offering three-year generic 5%/LIBOR par value swaps, how much would it pay or charge to buy a floating-rate payer's position on an *existing* three-year, 6%/LIBOR swap with a notional principal of $50 million, if it planned to hedge the purchase with a position on its par value swaps? How much would it pay or charge for an existing fixed-payer's position on a three-year, 6%/LIBOR with a notional principal of $50 million? Exclude the basis points that the swap bank might add to its bid and ask prices.
12. Explain the alternative ways a swap holder could close her swap position instead of selling it to a swap bank.
13. If the fixed rate on a new par value two-year swap were at 5%, how much would a swap dealer pay or charge to assume an existing fixed-payer's position on a 5.5%/LIBOR generic swap with two years left to maturity and notional principal of $20 million? How much would the dealer pay or charge if the fixed rate on a new par value two-year swap were at 6%?
14. If the fixed rate on a new par value two-year swap were at 5%, how much would a swap dealer pay or charge for assuming an existing 5.5%/LIBOR floating-rate position on a generic swap with two years left to maturity and notional principal of $20 million? How much would the dealer pay or charge if the fixed rate on a new par value two-year swap were at 6%?
15. The Star Chemical Company wants to finance an expansion of one of its production plants by borrowing $150 million for five years. Based on its moderate credit ratings, Star can borrow five-year funds at a 10.5% fixed rate or at a floating rate equal to the LIBOR + 75 bp. Given the choice of financing, Star prefers the fixed-rate loan. The Moon Development Company is also looking for five-year funding to finance its proposed $150 million office park development. Given its high credit rating, suppose Moon can borrow the funds for five years at a fixed rate of 9.5% or at a floating rate equal to the LIBOR + 25 bp. Given the choice, Moon prefers a floating-rate loan. In summary, Star and Moon have the following fixed- and floating-rate loan alternatives:

Company	Fixed Rate	Floating Rate
Star Company	10.5%	LIBOR + 75 bp
Moon Company	9.5%	LIBOR + 25 bp

Questions:

a. Describe Moon's absolute advantage and each company's comparative advantage.

b. What is the total possible interest rate reduction gain for both parties if both parties were to create synthetic positions with a swap?

c. Explain how a swap bank could arrange a five-year, 9.5%/LIBOR a swap that would benefit both the Star and Moon companies. What is the total interest rate reduction gain and how is it split?

16. Suppose the Star and Moon companies in Question 15 both have the same quality ratings with the following fixed- and floating-rate loan alternatives:

Company	Fixed Rate	Floating Rate
Star Company	9.50%	LIBOR + 50 bp
Moon Company	9.25%	LIBOR + 75 bp

Questions:

a. Describe each company's absolute and comparative advantages.
b. What is the total possible interest rate reduction gain for both parties if both parties were to create synthetic positions with a swap?
c. Explain how a swap bank could arrange a five-year swap in which Star and Moon split total interest-rate reduction gain.

17. Explain the idea of comparative advantage in terms of Questions 15 and 16.

18. Suppose a company wants to borrow $100 million for five years at a fixed rate. Suppose the company can issue both a five-year, 11%, fixed-rate bond paying coupons on a semiannual basis and a five-year FRN paying LIBOR plus 100 bp.

a. Explain how the company could create a synthetic five-year fixed-rate loan with a swap.
b. What would the fixed rate on the swap have to be for the synthetic position to be equivalent to the direct loan position? Show the synthetic position in a table.
c. Define the company's criterion for selecting the synthetic loan.

19. Suppose a financial institution wants to finance its three-year $100 million floating-rate loans by selling three-year floating-rate notes. Suppose the institution can issue a three-year, 7%, fixed-rate note paying coupons on a semiannual basis and also a three-year FRN paying LIBOR plus 100 bp.

a. Explain how the institution could create a synthetic three-year floating-rate note with a swap.
b. What would the fixed rate on the swap have to be for the synthetic position to be equivalent to the floating-rate note? Show the synthetic position in a table.
c. Define the institution's criterion for selecting the synthetic loan.

20. Suppose a financial institution wants to invest $100 million in a three-year fixed-rate note. Suppose the institution can invest in a three-year, 7%, fixed-rate note paying coupons on a semiannual basis and selling at par and also in a three-year FRN paying LIBOR plus 100 bp.

a. Explain how the institution could create a synthetic three-year fixed-rate note with a swap.
b. What would the fixed rate on the swap have to be for the synthetic position to be equivalent to the fixed-rate note? Show the synthetic position in a table.
c. Define the institution's criterion for selecting the synthetic investment.

21. Suppose a financial institution wants to invest $100 million in a three-year floating-rate note. Suppose the institution can invest in a three-year, 7%, fixed-rate note paying coupons on a semiannual basis and selling at par and also in a three-year FRN paying LIBOR plus 100 bp.
    a. Explain how the institution could create a synthetic three-year floating-rate note with a swap.
    b. What would the fixed rate on the swap have to be for the synthetic position to be equivalent to the floating-rate note? Show the synthetic position in a table.
    c. Define the institution's criterion for selecting the synthetic investment.

22. Given a generic five-year par value swap with a fixed rate of 6%, determine the values of the following off-market swap positions using the YTM approach:
    a. Fixed-rate position on a five-year, 5%/LIBOR generic swap with NP = $50 million.
    b. Floating-rate position on a five-year 5%/LIBOR generic swap with NP = $50 million.
    c. Fixed-rate position on a five-year 7%/LIBOR generic swap with NP = $50 million.
    d. Floating-rate position on a five-year 7%/LIBOR generic swap with NP = $50 million.

23. Short-Answer Questions
    a. Who generally assumes the credit risk in a brokered swap?
    b. Who assumes the credit risk in a dealer swap?
    c. What is one of the problems with brokered swaps that contributed to the growth in the dealer-swap market?
    d. What does the term *warehousing* mean?
    e. What does the term *running a dynamic book* mean?
    f. How do dealers typically quote the fixed rate and floating rate on swap agreements they offer?
    g. Describe the comparative advantage argument that is often advanced as the reason for the growth in the swap market.
    h. If one borrower has a comparative advantage in the fixed-rate market and another borrower has a comparative advantage in the floating-rate market, what is the total possible interest rate reduction gain for both borrowers from creating synthetic debt positions using swaps, given that one of the borrowers has an absolute advantage in both the fixed-rate and floating-rate credit markets?
    i. If one borrower has a comparative advantage in the fixed-rate market and another borrower has a comparative advantage in the floating-rate market, what is the total possible interest rate reduction gain for both borrowers from creating synthetic debt positions using swaps, given that each party has an absolute advantage in each market?
    j. What is the criticism that is often advanced against the comparative advantage argument?
    k. What is the hidden option and does it relate to a difference in credit spreads in the fixed and floating credit markets?

l. Explain how a company could take a swap position to replace its current floating-rate debt with a fixed-rate debt obligation.
m. Explain how a company could take a swap position to replace its current fixed-rate debt with a floating-rate debt obligation.
n. Explain how a company with investments in intermediate fixed-rate notes could take a swap position to create a floating-rate note. Why might a company want to do this?
o. Explain how an insurance company that expects lower rates in the future could change its currently immunized position to a more speculative one with a positive duration gap.
p. Define the YTM approach to valuing off-market swaps.
q. What is the problem with using the YTM approach to value off-market swaps?

WEB EXERCISES

1. Find out who the International Swap and Derivatives Association is by going to their Web site, www.isda.org, and clicking on "About ISDA." At the site, also click on "Educational" to find a bibliography of swaps and derivatives articles and links to other derivatives sites.
2. Examine the growth in the interest rate swap market by looking at the International Swap and Derivatives Association's Market Survey. Go to www.isda.org and click on "Survey and Market Statistics."

NOTES

1. For an analysis of the zero-coupon approach to valuing swaps, see Johnson, *Introduction to Derivatives*, Chapter 19.
2. Swaps fall under contract law and not security law. The mechanism for default is governed by the swap contract, with many patterned after International Swap and Derivatives Association (ISDA) documents.

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CHAPTER 21

Swap Derivatives: Forward Swaps and Swaptions

21.1 INTRODUCTION

Today, there are a number of nonstandard or nongeneric swaps used by financial and nonfinancial corporations to manage their varied cash flow and return-risk positions. Two of the most widely used nongeneric swaps are the ***forward swap*** and options on swaps or ***swaptions***. A forward swap is an agreement to enter into a swap that starts at a future date at an interest rate agreed upon today. A swaption, in turn, is a right, but not an obligation, to take a position on a swap at a specific swap rate. In this chapter, we examine these two interest rate swap derivatives.

21.2 FORWARD SWAPS

Like futures contracts on debt securities, forward swaps provide borrowers and investors with a tool for locking in a future interest rate. As such, they can be used to manage interest rate risk for fixed-income positions.

Hedging a Future Loan with a Forward Swap

Financial and nonfinancial institutions that have future borrowing obligations can lock in a future rate by obtaining forward contracts on fixed-payer swap positions. For example, a company wishing to lock in a rate on a five-year, fixed-rate $100 million loan to start two years from today could enter a two-year forward swap agreement to pay the fixed rate on a five-year 9%/LIBOR swap. At the expiration date on the forward swap, the company could issue floating-rate debt at LIBOR that, when combined with the fixed position on the swap, would provide the company with a synthetic fixed-rate loan paying 9%:

Instrument	Action	
Issue floating-rate note	Pay LIBOR	−LIBOR
Swap: Fixed-rate payer's position	Pay fixed rate	−9%
Swap: Fixed-rate payer's position	Receive LIBOR	+LIBOR
Synthetic fixed rate	Net payment	9%

Alternatively, at the forward swap's expiration date, the company is more likely to sell the five-year 9%/LIBOR swap underlying the forward swap contract and issue five-year fixed-rate debt. If the rate on five-year fixed-rate bonds were higher than 9%, for example at 10%, then the company would be able offset the higher interest by selling its fixed position on the 9%/LIBOR swap to a swap dealer for an amount equal to the present value of a five-year annuity equal to 1% (difference in rates: 10% − 9%) times the NP. For example, at 10% the value of the underlying 9%/LIBOR swap would be $3,860,867 using the YTM swap valuation approach:

$$SV^{fix} = \left[\sum_{t=1}^{10} \frac{(.10/2) - (.09/2)}{(1 + (.10/2))^t}\right] \$100{,}000{,}000 = \$3{,}860{,}867$$

With the proceeds of $3,860,867 from closing its swap, the company would only need to raise $96,139,133 (= $100,000,000 − $3,860,867). The company, though, would have to issue $96,139,133 worth of five-year fixed-rate bonds at the higher 10% rate. This would result in semiannual interest payments of $4,860,957 [= (.10/2)($96,139,133)], and the total return based on the $100 million funds needed would be approximately 9%.

In contrast, if the rate on five-year fixed-rate loans were lower than 9%, say 8%, then the company would benefit from the lower fixed-rate loan, but would lose an amount equal to the present value of a five-year annuity equal to 1% (difference in rates: 8% − 9%) times the NP when it closed the fixed position. Specifically, at 8%, the value of the underlying 9%/LIBOR swap is −$4,055,488 using the YTM approach:

$$SV^{fix} = \left[\sum_{t=1}^{10} \frac{(.08/2) - (.09/2)}{(1 + (.08/2))^t}\right] \$100{,}000{,}000 = -\$4{,}055{,}448$$

The company would therefore have to pay the swap bank $4,055,488 for assuming its fixed-payer position. With a payment of $4,055,488, the company would need to raise a total of $104,055,488 from its bond issue. The company, though, would be able to issue $104,055,488 worth of five-year fixed-rate bonds at the lower rate of 8%. Its semiannual interest payments would be $4,162,220 [= (.08/2)($104,055,488)], and its total return based on the $100 million funds needed would be approximately 9%.

Hedging a Future Investment

Instead of locking in the rate on a future liability, forward swaps can also be used on the asset side to fix the rate on a future investment. Consider the case of an institutional investor planning to invest an expected $10 million cash inflow one year from now in a three-year, high-quality fixed-rate bond. The investor could lock in the future rate by entering a one-year forward swap agreement to receive the fixed rate and pay the floating rate on a three-year, 9%/LIBOR swap with an NP of $10 million. At the expiration date on the forward swap, the investor could invest the $10 million cash inflow in a three-year FRN at LIBOR which, when combined with the floating position on the swap, would provide the investor with a synthetic fixed-rate investment paying 9%:

Instrument	**Action**	
Buy floating-rate note	Receive	LIBOR
Swap: Floating-rate payer's position	Pay LIBOR	−LIBOR
Swap: Floating-rate payer's position	Receive fixed rate	+9%
Synthetic fixed-rate investment	Net receipt	9%

Instead of a synthetic fixed investment position, the investor is more likely to sell the three-year 9%/LIBOR swap underlying the forward swap contract and invest in a five-year fixed rate note. If the rate on the three-year fixed-rate note were lower than the 9% swap rate, then the investor would be able sell his floating position at a value equal to the present value of an annuity equal to the $10 million NP times the difference between 9% and the fixed rate on three-year par value swap; this gain would offset the lower return on the fixed-rate bond. For example, if at the forward swap's expiration date, the rate on three-year, fixed-rate bonds were at 8%, and the fixed rate on a three-year par value swap were at 8%, then the investment firm would be able to sell its floating-payer's position on the three-year 9%/LIBOR swap underlying the forward swap contract to a swap bank for $262,107 (using the YTM approach with a discount rate of 8%):

$$SV^{fl} = \left[\sum_{t=1}^{6} \frac{(.09/2) - (.08/2)}{(1 + (.08/2))^t}\right] \$10{,}000{,}000 = \$262{,}107$$

The investment firm would therefore invest $10 million plus the $262,107 proceeds from closing its swap in three-year, fixed rate bonds yielding 8%. The total return on an investment of $10 million, though, would be approximately equal to 9%.

On the other hand, if the rate on three-year fixed-rate securities were higher than 9%, the investment company would benefit from the higher investment rate, but would lose when it closed its swap position. For example, if at the forward swap's expiration date, the rate on three-year, fixed rate bonds were at 10%, and the fixed rate on a three-year par value swap were at 10%, then the investment firm would have to pay the swap bank $253,785 for assuming its floating-payer's position on the three-year 9%/LIBOR swap underlying the forward swap contract:

$$SV^{fl} = \left[\sum_{t=1}^{6} \frac{(.09/2) - (.10/2)}{(1 + (.10/2))^t}\right] \$10{,}000{,}000 = -\$253{,}785$$

The investment firm would therefore invest $9,746,215 ($10 million minus the $253,785 costs incurred in closing its swap) in three-year, fixed-rate bonds yielding 10%. The total return on an investment of $10 million, though, would be approximately equal to 9%.

Other Uses of Forward Swaps

The examples illustrate that forward swaps are like futures on debt securities. As such, they are used in many of the same ways as futures: locking in future interest rates, speculating on future interest rate changes, and altering a balance sheet's exposure to interest rate changes. Different from futures, though, forward swaps can be customized to fit a particular investment or borrowing need and with the starting dates on forward swaps ranging anywhere from one month to several years, they can be applied to not only short-run but also long-run positions.

Consider for example a company with an existing 10%, fixed-rate debt having ten years remaining to maturity, but not callable for three more years. Suppose that as a result of the current low interest rates the company expects rates to increase in the future and would like to call its bonds now. Although the company cannot call its debt for three years, it can take advantage of the current low rates by using a forward swap. In this case, suppose the company enters into a three-year forward swap agreement to pay the fixed rate on a seven-year, 8%/LIBOR swap with an NP equal to the par value of its current 10-year, 10% fixed-rate debt (seven-year swap, three years forward). Three years later, if rates are lower than 10%, the company could issue floating-rate debt to finance the call of its 10% debt. This action, combined with its fixed payer's position on the 8%/LIBOR swap obtained from its forward swap, would give the company an effective fixed rate of 8%. On the other hand, if rates are higher than 10% on the call date, the company would not call its debt, but it would be able to offset its 10% debt by selling its 8%/LIBOR swap at a premium. Thus, by using a three-year forward agreement on a seven-year swap, the company is able to take advantage of a period of low interest rates to refinance its long-term debt; this opportunity would not have been possible using more standardized, shorter term, exchange-traded futures contracts.

Valuation of Forward Swaps

As with many nongeneric swaps, there can be an up-front fee for a forward swap. The value of a forward swap depends on whether the rate on the forward contract's underlying swap is different from its break-even forward swap rate. The break-even rate on a generic swap is that rate that equates the present values of the fixed and floating cash flows, with the floating cash flows estimated as implied forward rates generated from the zero coupon rates on swaps. The break-even rate on a forward swap is that rate that equates the present value of the fixed-rate flows to the present value of floating-rate flows corresponding to the period of the underlying swap.[1]

21.3 SWAPTIONS

As the name suggests, a swaption is an option on a swap. The purchaser of a swaption buys the right to start an interest rate swap with a specific fixed rate or exercise rate, and with a maturity at or during a specific time period in the future. If the holder exercises, she takes the swap position, with the swap seller obligated to take the opposite counterparty position. For swaptions, the underlying instrument is a forward swap and the option premium is the up-front fee. The swaption can be either a right to be a payer or the right to be receiver of the fixed rate. A ***receiver swaption*** gives the holder the right to enter a particular swap agreement as the fixed-rate receiver (and floating-rate payer), whereas a ***payer swaption*** gives the holder the right to enter a particular swap as the fixed-rate payer (and floating-rate receiver).

Swaptions are similar to interest rate options or options on debt securities. They are, however, more varied: They can range from options to begin a one-year swap in three months to a 10-year option on an eight-year swap (sometimes referred to as a 10 × 8 swaption); the exercise periods can vary for American swaptions; swaptions can be written on generic swaps or nongeneric. Like interest rate and debt options, swaptions can be used for speculating on interest rates, hedging debt and asset positions against market risk, and managing a balance sheet's exposure to interest rate changes. In addition, like swaps they also can be used in combination with other securities to create synthetic positions.

Speculation

Suppose a speculator expects the rates on high-quality, five-year fixed rate bonds to increase from their current 8% level. As an alternative to a short T-note futures or an interest rate call position, the speculator could buy a payer swaption. Suppose she elects to buy a one-year European payer swaption on a five-year, 8%/LIBOR swap with an NP of $10 million for 50 basis points times the NP; that is:

- 1 × 5 payer swaption
- Exercise date = one year
- Exercise rate = 8%
- Underlying swap = five-year, 8%/LIBOR with NP = $10 million
- Swap position = fixed payer
- Option premium = 50 bp times NP

On the exercise date, if the fixed rate on a five-year swap were greater than the exercise rate of 8%, then the speculator would exercise her swaption at 8%. To profit, she could take her 8% fixed-rate payer's swap position obtained from exercising and sell it to a swap bank. For example, if current five-year par value swaps were trading at 9% and swaps were valued by the YTM approach, then she would be able to sell her 8% swap for $395,636:

$$\text{Value of swap} = \left[\sum_{t=1}^{10} \frac{(.09/2) - (.08/2)}{(1 + (.09/2))^t}\right](\$10{,}000{,}000) = \$395{,}636$$

Alternatively, she could exercise and then enter into a reverse swap; for example, at the current swap rate of 8%, she could take the floating payer's position on a five-year, 8%/LIBOR swap. By doing this she would receive an annuity equal to 1% of the NP for five years (or .5% semiannually for 10 periods), which would be worth \$395,636:

From payer swaption:		
Swap: Fixed-rate payer's position	Pay	8% per year for five years
Swap: Fixed-rate payer's position	Receive	LIBOR
From replacement swap:		
Swap: Floating-rate payer's position	Receive	9% per year for five years
Swap: Floating-rate payer's position	Pay	LIBOR
Net position	Receive	1% per year for five years

If the swap rate at the expiration date were less than 8%, then the payer swaption would have no value and the speculator would simply let it expire, losing the premium she paid.

More formally, the intrinsic value or expiration value of the payer swaption is

$$\text{Value of payer swaption} = \left[\sum_{t=1}^{10} \frac{\text{Max}[(R/2) - (.08/2), 0]}{(1 + (R/2))^t}\right] (\$10{,}000{,}000)$$

For rates, R, on par value five-year swaps exceeding the exercise rate of 8%, the value of the payer swaption will be equal to the present value of the interest differential times the notional principal on the swap, and for rates less than 8%, the swap is worthless. Table 21.1 and Figure 21.1 show in a table and graphically the values and profits at expiration obtained from closing the payer swaption on the five-year 8%/LIBOR swap given different rates at expiration.

TABLE 21.1 Value and Profit at Expiration from 8%/LIBOR Payer Swaption

Rates on Five-year Par Value Swaps at Expiration *R*	Payer Swaption's Interest Differential Max((*R* – .08)/2,0)	Value of 8%/LIBOR Payer Swaption at Expiration PV(Max[(*R* – .08)/2, 0]($10m))	Payer Swaption Cost	Profit from Payer Swaption
0.060	0.0000	\$0	\$50,000	−\$50,000
0.065	0.0000	\$0	\$50,000	−\$50,000
0.070	0.0000	\$0	\$50,000	−\$50,000
0.075	0.0000	\$0	\$50,000	−\$50,000
0.080	0.0000	\$0	\$50,000	−\$50,000
0.085	0.0025	\$200,272	\$50,000	\$150,272
0.090	0.0050	\$395,636	\$50,000	\$345,636
0.095	0.0075	\$586,226	\$50,000	\$536,226
0.100	0.0100	\$772,173	\$50,000	\$722,173

$$\text{Value of swap} = \left[\sum_{t=1}^{10} \frac{\text{Max}[(R/2) - (.08/2), 0]}{(1 + (R/2))^t}\right] (\$10{,}000{,}000)$$

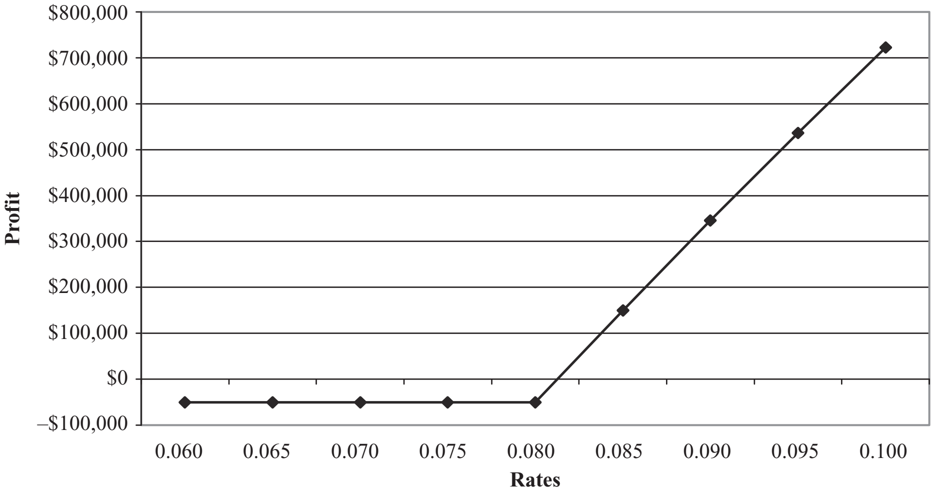

FIGURE 21.1 Value and Profit at Expiration from 8%/LIBOR Payer Swaption

Instead of higher rates, suppose the speculator expects rates on five-year high-quality bonds to be lower one year from now. In this case, her strategy would be to buy a receiver swaption. If she bought a receiver swaption similar in terms to the above payer swaption (a one-year option on a five-year, 8%/LIBOR swap for $50,000), and the swap rate on a five-year swap were less than 8% on the exercise date, then she would realize a gain from exercising and then either selling the floating-payer position or combining it with a fixed-payer's position on a replacement swap. For example, if the fixed rate on a five-year par value swap were 7%, the investor would exercise her receiver swaption by taking the 8% floating-rate payer's swap and then sell the position to a swap bank or another party. With the current swap rate at 7%, she would be able to sell the 8% floating-payer's position for $415,830:

$$\text{Value of swap} = \left[\sum_{t=1}^{10} \frac{(.08/2) - (.07/2)}{(1 + (.07/2))^t}\right](\$10{,}000{,}000) = \$415{,}830$$

Alternatively, the swaption investor could exercise and then enter into a reverse swap. At the swap rate of 8%, she could take the fixed payer's position on a five-year, 8%/LIBOR swap. By doing this, the investor would receive an annuity equal to 1% of the NP for five years. The value of the annuity would be $415,830:

From receiver swaption:		
Swap: Floating-rate payer's position	Pay	LIBOR
Swap: Floating-rate payer's position	Receive	8% per year for five years
From replacement swap:		
Swap: Fixed-rate payer's position	Receive	LIBOR
Swap: Fixed-rate payer's position	Pay	7% per year for five years
Net position	Receive	1% per year for five years

TABLE 21.2 Value and Profit at Expiration from 8%/LIBOR Receiver Swaption

Rates on Five-Year Par Value Swaps at Expiration *R*	Receiver Swaption's Interest Differential Max((.08 – *R*)/2,0)	Value of 8%/LIBOR Receiver Swaption at Expiration PV(Max[(.08 – *R*)/2, 0]($10m))	Receiver Swaption Cost	Profit from Receiver Swaption
0.060	0.0100	$853,020	$50,000	$803,020
0.065	0.0075	$631,680	$50,000	$581,680
0.070	0.0050	$415,830	$50,000	$365,830
0.075	0.0025	$205,320	$50,000	$155,320
0.080	0.0000	$0	$50,000	−$50,000
0.085	0.0000	$0	$50,000	−$50,000
0.090	0.0000	$0	$50,000	−$50,000
0.095	0.0000	$0	$50,000	−$50,000
0.100	0.0000	$0	$50,000	−$50,000

$$\text{Value of swap} = \left[\sum_{t=1}^{10} \frac{\text{Max}[(.08/2) - (R/2), 0]}{(1 + (R/2))^t}\right](\$10{,}000{,}000)$$

Thus, if rates were at 7%, then the investor would realize a profit of $365,830 (= $415,830 – $50,000) from the receiver swaption. If the swap rate were higher than 8% on the exercise date, then the investor would allow the receiver swaption to expire, losing, in turn, her premium of $50,000.

Formally, the value of the 8%/LIBOR receiver swaption at expiration is

$$\text{Value of receiver swaption} = \left[\sum_{t=1}^{10} \frac{\text{Max}[(.08/2) - (R/2), 0]}{(1 + (R/2))^t}\right](\$10{,}000{,}000)$$

For rates, *R*, on par value five-year swaps less than the exercise rate of 8%, the value of the receiver swaption will be equal to the present value of the interest differential times the notional principal on the swap, and for rates greater than 8%, the swap is worthless. Table 21.2 and Figure 21.2 show in a table and graphically the values and profits at expiration obtained from closing the receiver swaption on the five-year 8%/LIBOR swap given different rates at expiration.

Hedging

Caps and Floors on Future Debt and Investment Positions Like other option hedging tools, swaptions give investors and borrowers protection against adverse price or interest rate movements, while allowing them to benefit if prices or rates move in their favor. Since receiver swaptions increase in value as rates decrease below the exercise rate, they can be used to establish floors on the rates of return obtained from future fixed-income investments. In contrast, since payer swaptions increase in value as rates increase above the exercise rate, they can be used for capping the rates paid on debt positions.

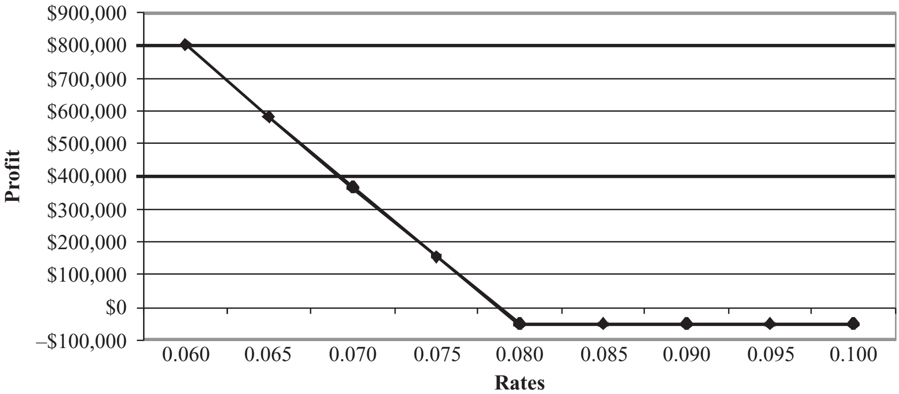

FIGURE 21.2 Value and Profit at Expiration from 8%/LIBOR Receiver Swaption

To illustrate how receiver swaptions are used for establishing a floor, consider the case of an investment fund that has an investment-grade fixed-rate portfolio worth $30 million in par value that is scheduled to mature in two years. Suppose the fund plans to reinvest the $30 million in principal for another three years in investment-grade bonds that are currently trading to yield 6%, but is worried that interest rates could be lower in two years. To establish a floor on its investment, suppose the fund purchases a two-year receiver swaption on a three-year 6%/LIBOR generic swap with a notional principal of $30 million from First Bank for $100,000. Two years later, the swaption's value will be greater if interest rates are lower (provided rates are less than 6%); this, in turn, would offset the lower yields the fund would obtain from investing in lower-yielding securities. On the other hand, for rates higher than 6%, the swaption is worthless, but the fund is able to invest in higher-yield securities. Thus, for the cost of $100,000, the receiver swaption provides the fund a floor.

In contrast to the use of swaptions to establish a floor on a future investment, suppose a firm had a future debt obligation whose rate it wants to cap. In this case, the firm could purchase a payer swaption. For example, suppose a company has a $60 million, 9% fixed-rate bond obligation maturing in three years that it plans to refinance at that time by issuing new five-year fixed-rate bonds. Suppose the company is worried that interest rates could increase in three years and as a result wants to establish a cap on the rate it would pay on its future five-year bond issue. To cap the rate, suppose the company purchases a three-year payer swaption on a five-year 9%/LIBOR generic swap with notional principal of $60 million from First Bank for $200,000. Two years later, the swaption's value will be greater if interest rates are higher (provided rates exceed 9%); this, in turn, would offset the higher borrowing rates the firm would have when it issues its new fixed-rate bonds. On the other hand, for rates less than 9%, the swaption is worthless, but the firm benefits with lower rate on new debt issues. Thus, for the cost of $200,000, the payer swaption provides the fund a cap on it future debt.

Investor Hedging the Risk of an Embedded Call Option The cap and floor hedging examples illustrate that swaptions are a particularly useful tool in hedging future investment and debt position against adverse interest rate changes. Swaptions can also be used to hedge against the impacts that unfavorable interest rate changes have on investment and debt positions with embedded options. Consider the case of a fixed-income manager holding $10 million worth of ten-year, high quality, 8% fixed-rate bonds that are callable in two years at a call price equal to par. Suppose the manager expects a decrease in rates over the next two years, increasing the likelihood that his bonds will be called and he will be forced to reinvest in a market with lower rates. To minimize his exposure to this call risk, suppose the manager buys a two-year receiver swaption on an eight-year, 8%/LIBOR swap with an NP of $10 million. If two years later, rates were to increase, then the bonds would not be called and the swaption would have no value. In this case, the fixed-income manager would lose the premium he paid for the receiver swaption. However, if two years later, rates on eight-year bonds were lower at, say, 6%, and the bonds were called at a call price equal to par, then the manager would be able to offset the loss from reinvesting the call proceeds at a lower interest rate by the profits from exercising the receiver swaption.

Borrower Hedging the Risk of an Embedded Put Option The contrasting case of a fixed-income manager hedging callable bonds would be the case of a borrower who issued putable bonds some time ago and was now concerned that rates might increase in the future. If rates did increase and bondholders exercised their option to sell the bonds back to the issuer at a specified price, the issuer may have to finance the purchase by issuing new bonds paying higher rates. To hedge against this scenario, the borrower could buy a payer swaption with a strike rate equal to the coupon rate on the putable bonds. Later, if the current swap rate exceeded the strike rate and the bonds were put back to the issuer/borrower, the borrower could exercise his payer swaption to take the fixed payer position at the strike rate and then sell the swap and use the proceeds to defray part of the higher financing cost of buying the bonds on the put. On the other hand, if rates were to decrease, then the put option on the bond would not be exercised and the payer swaption would have no value. In this case, the borrower would lose the swaption premium.

Arbitrage: Synthetic Positions

Like other swaps, swaptions can be used to create synthetic bond or debt positions with supposedly better rates than conventional ones. Consider for example a company that wants to finance a $50 million capital expenditure with seven-year, option-free, 9% fixed-rate debt. As we examined in Chapter 20, the company could issue seven-year, option-free, fixed-rate bonds or create a synthetic seven-year bond by issuing seven-year FRNs and taking a fixed-payer's position on a seven-year swap. With swaptions, as well as other nongeneric swaps, there are actually several other ways in which this synthetic fixed-rate bond could be created. For example, to obtain an option-free, fixed-rate bond, the company could issue a callable bond and then sell a receiver swaption with terms similar to the bond. This synthetic debt position will provide a lower rate than the rate on a direct loan if investors underprice the call option on callable debt.

21.4 CANCELABLE AND EXTENDABLE SWAPS

Generic swaps can have clauses giving the counterparty the right to extend the option or cancel it. These swaps are known as cancelable and extendable swaps. Analogous to bonds with embedded call and put options, these swaps are equivalent to swaps with embedded payer swaptions and receiver swaptions.

Cancelable Swap

A ***cancelable swap*** is a generic swap in which one of the counterparties has the option to terminate one or more payments. Cancelable swaps can be callable or putable. A ***callable cancelable swap*** is one in which the fixed payer has the right to early termination. Thus, if rates decrease, the fixed-rate payer on the swap with this embedded call option to early termination can exercise her right to cancel the swap. A ***putable cancelable swap***, on the other hand, is one in which the floating payer has the right to early cancellation. A floating-rate payer with this option may find it advantageous to exercise his early-termination right when rates increase.

If there is only one termination date, then a cancelable swap is equivalent to a standard swap plus a position in a European swaption. For example, a five-year putable cancelable swap to receive 6% and pay LIBOR that is cancelable after two years is equivalent to a floating position in a five-year 6%/LIBOR generic swap and a long position in a two-year payer swaption on a three-year 6%/LIBOR swap; that is, after two years, the payer swaption gives the holder the right to take a fixed-payer's position on a three-year swap at 6% that offsets the floating position on the 6% generic swap. On the other hand, a five-year callable swap to pay 6% and receive LIBOR that is cancelable after two years would be equivalent to a fixed position in a five-year 6%/LIBOR generic swap and a long position in a two-year receiver swaption on a three-year 6%/LIBOR swap.

Extendable Swaps

An ***extendable swap*** is just the opposite of a cancelable swap: It is a swap that has an option to lengthen the terms of the original swap. The swap allows the holder to take advantage of current rates and extend the maturity of the swap. Like cancelable swaps, extendable swaps can be replicated with a generic swap and a swaption. The floating payer with an extendable option has the equivalent of a floating position on a generic swap and a receiver swaption. That is, a three-year swap to receive 6% and pay LIBOR that is extendable at maturity to two more years would be equivalent to a floating position in a three-year 6%/LIBOR generic swap and a long position in a three-year receiver swaption on a two-year 6%/LIBOR. That is, in three years, the receiver swaption gives the holder the right to take a floating-payer's position on a two-year swap at 6%, which in effect extends the maturity of the expiring floating position on the 6% generic swap. In contrast, the fixed payer with an extendable option has the equivalent of a fixed position on a generic swap and a payer swaption.

Synthetic Positions

Since cancelable and extendable swaps are equivalent to generic swaps with a swaption, they can be used like swaptions to create synthetic positions. For example, the synthetic fixed-rate debt position formed by issuing FRNs and taking a fixed-payer's position on a generic swap or by issuing callable bonds and selling a receiver swaption could also be created by (1) issuing callable bonds, (2) taking a fixed payer's position on a generic swap, and (3) taking a floating payer's position on a callable cancelable swap. In general, with swaptions, forward swaps, and extendable and cancelable swaps there are a number of synthetic asset and liability permutations: callable and putable debt, callable and putable bonds, flexible rate securities, and flexible-rate debt. The interested reader may want to consider how some of those positions can be combined to create synthetic fixed-rate and floating-rate positions.

21.5 NONGENERIC SWAPS

Concomitant with this growth of swaps has been the number of innovations introduced in swaps contracts over the years. Today, there are a number of nonstandard or nongeneric swaps used by financial and nonfinancial corporations to manage their varied cash flow and return-risk positions. Nongeneric swaps usually differ in terms of their rates, principal, or effective dates. For example, instead of defining swaps in terms of the LIBOR, some swaps use the T-bill rate, prime lending rate, or the Federal Reserve's Commercial Paper Rate Index with different maturities. Similarly, the principals defining a swap can vary. An ***amortizing swap***, for example, is a swap in which the NP is reduced over time based on a schedule, whereas a ***set-up swap*** (sometimes called an ***accreting swap***) has its NP increasing over time. A variation of the amortizing swap is the ***index-amortizing swap*** (also called ***index-principal swap***). In this swap, the NP is dependent on interest rates; for example, the lower the interest rate the greater the reduction in principal. There are also ***zero-coupon swaps*** in which one or both parties do not exchange payments until the maturity on the swap. Finally, there are a number of ***non-U.S. dollar interest rate swaps***. These swaps often differ in terms of their floating rate: London rate, Frankfurt (FIBOR), Copenhagen (CIBOR), Madrid (MIBOR), or Vienna (VIBOR). Exhibit 21.1 summarizes some of the common nongeneric swaps.

21.6 CONCLUSION

In this chapter, we have examined the markets and uses of swap derivatives: forward swaps, swaptions, cancelable swaps, and extendable swaps. Like exchange-traded options and futures, these swap derivatives provide investors and borrowers with tools for more effectively managing their asset and liability positions. Like generic swaps, swap derivatives have also become a basic financial engineering tool to apply to a variety of financial problems. For financial and non-financial corporations who buy and sell swaps and their derivatives, one of their important decisions is how to price these contracts. Pricing forward swaps and swaptions is examined

EXHIBIT 21.1 Non-Generic Swaps

- **Non-LIBOR Swap**: Swaps with floating rates different from LIBOR. Example: T-bill rate, CP rate, or prime lending rate.
- **Delayed-Rate Set Swap** allows the fixed payer to wait before locking in a fixed swap rate—the opposite of a forward swap.
- **Zero-Coupon Swap**: Swap in which one or both parties do not exchange payments until maturity on the swap.
- **Prepaid Swap**: Swap in which the future payments due are discounted to the present and paid at the start.
- **Delayed-Reset Swap**: The effective date and payment date are the same. The cash flows at time t are determined by the floating rate at time t rather than the rate at time $t - 1$.
- **Amortizing Swaps**: Swaps in which the NP decreases over time based on a set schedule.
- **Set-Up Swap** or **Accreting Swap**: Swaps in which the NP increases over time based on a set schedule.
- **Index Amortizing Swap**: Swap in which the NP is dependent on interest rates.
- **Equity Swap**: Swap in which one party pays the return on a stock index and the other pays a fixed or floating rate.
- **Basis Swap**: Swaps in which both rates are floating; each party exchanges different floating payments. One party might exchange payments based on LIBOR and the other based on the Federal Reserve Commercial Paper Index.
- **Total Return Swap**: Returns from one asset are swapped for the returns on another asset.
- **Non-U.S. Dollar Interest Rate Swap**: Interest-rate swap in a currency different from U.S. dollars with a floating rate often different from the LIBOR: Frankfurt rate (FIBOR), Vienna (VIBOR), and the like.

in a number of derivative texts. In the next chapter, we conclude our analysis of swaps by looking at two other popular swaps: currency swaps and credit default swaps.

KEY TERMS

amortizing swap
callable cancelable swap
cancelable swap
extendable swap
forward swaps
index-amortizing swap (interest-principal swap)
non-U.S. dollar interest rate swaps
payer swaption
putable cancelable swap
receiver swaption
set-up swap (accreting swap)
swaptions
zero-coupon swaps

PROBLEMS AND QUESTIONS

1. Explain how a company planning to issue four-year, fixed-rate bonds in two years could use a forward swap to lock in the fixed rate it will pay on the bonds. Explain how the hedge works at the expiration of the forward contract.

2. The MEJ Development Company is constructing a $300 million office park development that it anticipates completing in two years. At the project's completion, the company plans to refinance its short-term construction and development loans by borrowing $300 million through the private placement of 10-year bonds. The MEJ Company has a BBB quality rating and its option-free, fixed-rate bonds trade 200 basis points above comparable Treasury bonds and its floating-rate bonds trade at 150 basis points above the LIBOR. Currently, 10-year T-bonds are trading to yield 6%. With current rates considered relatively low, MEJ is expecting interest rates to increase and would like to lock in a rate on the 10-year, fixed-rate bond two years from now. The company is considering locking in its rate by entering a forward swap with Star Bank. Star is willing to provide MEJ a two-year forward swap agreement on a 10-year, 7.25%/LIBOR swap.
   - **a.** Explain the forward swap position that MEJ would need to take in order to lock in the rate on its 10-year, fixed-rate bond to be issued two years from now.
   - **b.** Given MEJ hedges with a swap position, explain how it would obtain a fixed rate for 10 years at the forward swap's expiration date by issuing its floating rate notes at LIBOR plus 150 bp. What is the fixed rate MEJ would have to pay on its position?

3. Suppose the MEJ Development Company in Question 2 hedges its planned $300 million bond sale in two years by taking a position in the forward swap contract offered by Star Bank. Also suppose that at the forward swap's expiration date, 10-year T-bonds are trading at 7% and the fixed rate on 10-year par value swaps that Star Bank would offer MEJ is 150 bp above the T-bond yield.
   - **a.** What would be the value of the swap underlying MEJ's forward swap at the expiration date?
   - **b.** What would be the amount of funds MEJ would need in order to refinance its $300 million short-term loan obligation and close its swap position?
   - **c.** Given that MEJ's fixed-rate bonds trade at 200 basis points above T-bond rates, what would be MEJ's semiannual interest payments on the funds that it borrows?
   - **d.** What would be MEJ's annualized rate based on the $300 million refinancing funds it needs?

4. Suppose the MEJ Development Company in Question 2 hedges its planned $300 million bond sale in two years by taking a position in the forward swap contract offered by Star Bank. Also suppose that at the forward swap's expiration date, 10-year T-bonds are trading at 5% and the fixed rate on 10-year par value swaps that Star Bank would offer MEJ is 50 bp above the T-bond yield.
   - **a.** What would be the value of the swap underlying MEJ's forward swap at the expiration date?
   - **b.** Given that MEJ's fixed-rate bonds trade at 200 basis points above T-bond rates, what would be the amount of funds MEJ would need in order to refinance its $300 million short-term loan obligation and close its swap position?
   - **c.** What would be MEJ's annualized rate based on the $300 million funds it needs for refinancing?

5. XSIF Investment Trust has bonds worth $20 million in par value maturing in one year. To maintain the duration and quality ratings of its overall bond fund, the trust plans to reinvest the principal in three-year, option-free bonds with a quality rating of A. Such bonds are currently trading 200 basis points above comparable three-year Treasury notes. XSIF Trust also could invest in three-year, A-rated, floating-rate bonds. Such bonds are presently trading at 150 basis points above the LIBOR. Currently, three-year T-notes are trading to yield 6%. XSIF is worried, though, that the Fed will lower interest rate in the next year to stimulate a sluggish economy. As a result, the trust would like to lock in a rate on its $20 million investment. The Trust is considering locking in a rate by entering a forward swap agreement with Fort Washington Bank. To hedge its future loan, Fort Washington is willing to provide XSIF a one-year forward swap agreement on a three-year 6.5%/LIBOR swap.
   - **a.** Explain the forward swap position that XSIF would need to take in order to lock in the rate on a three-year, fixed-rate bond investment to be made in one year.
   - **b.** Given XSIF's swap position, explain how it would obtain a fixed rate for three years at the forward swap's expiration date by investing in A-rated floating-rate notes at LIBOR plus 150 bp. What is the fixed rate XSIF would earn from this hedged investment?
6. Suppose the XSIF Investment Trust in Question 5 hedges its planned $20 million bond investment in one year by taking a position in the one-year, 6.5%/LIBOR forward swap contract offered by Fort Washington Bank. Also suppose that at the forward swap's expiration date, three-year T-notes are trading at par and the fixed rate on three-year par value swaps that Fort Washington would offer XSIF is 50 bp above the T-note yield.
   - **a.** What would be the values of the swap underlying XSIF's forward swap at the expiration date if three-year T-notes are trading at 5% and 7%?
   - **b.** Determine XSIF's investments after it closes its swap position and its annualized rate based on the $20 million investment in T-notes yielding 5% and 7%.
7. Suppose a speculative hedge fund anticipating higher rates in several years purchased a two-year payer swaption on a three-year 6%/LIBOR generic swap with semiannual payments and a notional principal of $20 million for a price equal to 50 bp times the NP. Explain what the fund would do at the swaption's expiration if the fixed rate on a three-year par value swap were at 7% and at 5%. What would be the fund's profits or losses at those rates? Use the YTM-approach in valuing the swap's position.
8. Show graphically and in a table the values and profits/losses at expiration that the hedge fund in Question 7 would obtain from closing its payer swaption on a 6%/LIBOR swap with a notional principal of $20 million purchased at a price equal to 50 bp times the NP. Evaluate at fixed rates on three-year par value swap at expiration of 4%, 4.5%, 5%, 5.5%, 6%, 6.5%, 7%, 7.5%, and 8%. Use the YTM-approach in valuing the swap's position.
9. Suppose the speculative hedge fund in Question 7 was anticipating lower rates in several years and purchased a two-year receiver swaption on a three-year 6%/LIBOR generic swap with semiannual payments and notional principal of

$20 million for a price equal to 60 bp times the NP. Explain what the fund would do at the swaption's expiration if the fixed rate on three-year par value swaps were at 7% and at 5%. What would be the fund's profits or losses at those rates? Use the YTM-approach in valuing the swap's position.

10. Show graphically and in a table the values and profits/losses at expiration that the hedge fund in Question 9 would obtain from closing its receiver swaption on a 6%/LIBOR swap with a notional principal of $20 million purchased at a price equal to 60 bp times the NP. Evaluate at fixed rates on three-year par value swaps at expiration of 4%, 4.5%, 5%, 5.5%, 6%, 6.5%, 7%, 7.5%, and 8%. Use the YTM-approach in valuing the swap's position.

11. The Washington Investment Fund has a Treasury bond portfolio worth $100 million in par value that is scheduled to mature in one year. Washington plans to reinvest the $100 million of principal for another three years in similar fixed-income bonds. Currently, such bonds are trading to yield 6%. Washington is worried that interest rates could be lower in one year and would like to establish a floor on the rate it would obtain for its future three-year investment. The fund is considering purchasing a one-year receiver swaption on a three-year 6%/LIBOR generic swap with notional principal of $100 million from First American Bank for $500,000. Show in a table (1) the values and profits/losses at expiration that Washington would obtain from closing the receiver swaption and (2) the hedged rate based on $100 million investment that they would obtain from reinvesting for three years the $100 million plus the proceeds from selling the swaption (do not include $500,000 cost). Determine the values, profits, and rates given possible fixed rates on a three-year par value swap at expiration of 4%, 4.5%, 5%, 5.5%, 6%, 6.5%, 7%, 7.5%, and 8%. Assume the rate on the par value swaps and the three-year T-note rate are the same and that the yield curve is flat.

12. The Ebersole Software Development Company has a $25 million, 8% fixed-rate bond obligation maturing in one year. The company plans to finance the $25 million principal liability by issuing new five-year fixed-rate bonds. Currently, five-year T-notes are trading to yield 6% and Ebersole's bonds are trading at 200 basis points above the Treasury yields. Ebersole is worried that interest rates could increase in one year and would like to establish a cap on the rate it would pay on its future five-year bond issue. Ebersole is considering purchasing a one-year payer swaption on a five-year 8%/LIBOR generic swap with notional principal of $25 million from First South Bank for $250,000. Show in a table (1) the values and profits/losses at expiration that Ebersole would obtain from closing the swaption, and (2) the hedged rate (based on $25 million debt) they would pay from issuing five-year bonds to raise $25 million minus the proceeds from selling the swaption (do not include $250,000 cost). Determine the values, profits, and rates at fixed rates on five-year par value swap at expiration of 6%, 6.5%, 7%, 7.5%, 8%, 8.5%, 9%, 9.5%, and 10%. Assume the rate on the par value swaps and Ebersole's five-year bond rate are the same and that the yield curve is flat.

13. The Zuber Bottling Manufacturing Company is considering financing the construction of its new $50 million facility by selling seven-year, 10% fixed-rate, option-free bonds at par through a private placement. The company's investment banker has informed the company that it could also sell seven-year, 10.5% fixed-rate bonds at par with a call option giving the Zuber Company the right to buy back the bonds at par after two years. In addition, Zuber is also informed it can sell FRNs paying the LIBOR plus 25 bp. Zuber does not believe rates will decrease over the next two years and prefers the non-callable bonds. The Zuber Company, though, can take a long or short position with its investment banker on a two-year receiver swaption on a five-year, 10%/LIBOR swap selling at a price equal to 75 bp per year for seven years.

a. Explain how the Zuber Company could create a synthetic option-free bond with the callable bond and a position on the receiver swaption.

b. What would be the effective rate Zuber would pay on its synthetic option-free bond if rates two years later were greater than 10% and the swaption holder does not exercise and Zuber does not exercise the call option on its bonds?

c. What actions would Zuber have to take to fix its rate if rates two years later were less than 10% and the swaption holder exercises? What would be Zuber's effective rate for the remaining five years?

d. Based on your analysis in b. and c., what would be your financing recommendation to the Zuber Company: the synthetic option-free bond or the straight option-free bond?

14. Explain how a 10-year, 8%/LIBOR putable swap cancelable after five years can be created with positions in a 10-year 8%/LIBOR generic swap and a five-year payer swaption on a five-year 8%/LIBOR swap.

15. Explain how a 10-year, 8%/LIBOR callable swap cancelable after five years can be created with positions in a 10-year 8%/LIBOR generic swap and a five-year receiver swaption on a five-year 8%/LIBOR swap.

16. Given a borrower can issue floating-rate notes, callable bonds, and putable bonds and can take positions in comparable generic swaps, callable and putable swaps, and receiver and payer swaptions, define five ways the borrower could form synthetic fixed-rate positions.

17. Define the following swaps and give an example of their use: amortizing, accreting, and index amortizing swaps.

NOTES

1. For an analysis of break-even forward rates and the pricing of forward swaps and swaptions, see Johnson, *Introduction to Derivatives*, Chapter 19.

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Bond Evaluation, Selection, and Management, Second Edition
by R. Stafford Johnson

CHAPTER 22

Currency and Credit Default Swaps

22.1 INTRODUCTION

Since the mid-1970s there has been an active currency swap market amongst financial and nonfinancial corporations. Concomitant with this growth has been the growth of one of the more controversial swaps: credit default swap. This swap allows companies to trade credit risk and by so doing change the credit risk exposure of their debt and fixed-income assets. In this chapter, we conclude our analysis of swaps by examining currency swaps and credit default swaps.

22.2 CURRENCY SWAPS

In its simplest form, a ***currency swap*** involves an exchange of principal and interest in one currency for the principal and interest in another. For example, the Monsanto Company swaps a 9% loan in dollars to the British Petroleum Company for their 10% loan in sterling. The market for currency swaps comes primarily from corporations who can borrow in one currency at relatively favorable terms but need to borrow in another. For example, a U.S. multinational corporation that can obtain favorable borrowing terms for a dollar loan, but really needs a loan in sterling to finance its operations in London or to provide it with a sterling liability to hedge its income against exchange rate risk, might use a currency swap. To meet such a need, the company could go to a swap dealer who would try to match its needs with another party wanting the opposite position. For example, the dealer might match the U.S. corporation with a British multinational corporation with operations in the United States that it is financing with a sterling-denominated loan, but would prefer instead a dollar-denominated loan. If the loans are approximately equivalent, then the dealer could arrange a swap agreement in which the companies simply exchange their principal and interest payments. If the loans are not equivalent, the swap dealer may have to bring in other parties who are looking to swap, or the dealer could take the opposite position and warehouse the swap.[1]

Example

As a example, suppose the British Petroleum Company plans to issue five-year bonds worth £100 million at 7.5% interest, but actually needs an equivalent amount in dollars, $150 million (current $/£ rate is $1.50/£), to finance its new refining facility in the United States. Also, suppose that the Piper Shoe Company, a U.S. company, plans to issue \$150 million in bonds at 10% with a maturity of five years, but it really needs £100 million to set up its distribution center in London. To meet each other's needs, suppose that both companies go to a swap bank that sets up the following agreements.

Agreement 1:

1. The British Petroleum Company will issue five-year £100 million bonds paying 7.5% interest. It will then deliver the £100 million to the swap bank who will pass it on to the U.S. Piper Company to finance the construction of its British distribution center.
2. The Piper Company will issue five-year \$150 million bonds. The Piper Company will then pass the \$150 million to the swap bank who will pass it on to the British Petroleum Company who will use the funds to finance the construction of its U.S. refinery.

Agreement 2:

1. The British company, with its U.S. asset (refinery), will pay the 10% interest on \$150 million (\$15 million) to the swap bank who will pass it on to the U.S. company so it can pay its U.S. bondholders.
2. The U.S. company, with its British asset (distribution center), will pay the 7.5% interest on £100 million [(.075)(£100m) = £7.5 million], to the swap bank who will pass it on to the British company so it can pay its British bondholders.

Agreement 3:

1. At maturity, the British company will pay \$150 million to the swap bank who will pass it on to the U.S. company so it can pay its U.S. bondholders.
2. At maturity, the U.S. company will pay £100 million to the swap bank who will pass it on to the British company so it can pay its British bondholders.

Valuation

Equivalent Bond Position In the above swap agreement, the U.S. company will receive \$15 million each year for five years and a principal of \$150 million at maturity and will pay £7.5 million each year for five years and £100 million at maturity. To the U.S. company, this swap agreement is equivalent to a position in two bonds: a long position in a dollar-denominated, five-year, 10% annual coupon bond with a principal of \$150 million and trading at par and a short position in a sterling-denominated, five-year, 7.5% annual coupon bond with a principal of £100 million and trading at par. The dollar value of the U.S. company's swap position where dollars are received and sterling is paid is

$$\text{SV} = B_{\$} - E_0\ B_{£}$$

where $B_{\$}$ = Dollar-denominated bond value
$B_{£}$ = Sterling-denominated bond value
E_0 = Spot exchange rate = \$/BP

The dollar value of the swap to the U.S. company in terms of equivalent bond positions is zero:

$$SV = \$150 \text{ million} - (\$1.50/£)(£100 \text{ million}) = 0$$

The British company's swap position in which it will receive sterling and pay dollars is just the opposite of the U.S. company's position. It is equivalent to a long position in a sterling-denominated bond and short position in a dollar-denominated bond. In this example, it likewise has a value of zero.

If a dealer had been warehousing swaps and provided a swap to just the U.S. company, then it could have hedged its swap position of paying \$15 million and receiving £7.5 million by shorting the 7.5% sterling-denominated bond and buying the 10% U.S.-dollar-denominated bond. Given this hedge, a currency swap, like an interest rate swap, generally has an economic value of zero when it is created. The zero economic value of the swap positions, though, will change over time with changes in U.S. rates, British rates, and the spot exchange rate. In general, the value of a dollar received/foreign currency paid swap is inversely related to U.S. interest rates and the exchange rate and directly related to the foreign rate, whereas the value of a foreign currency received/dollar paid swap, valued in dollars, is directly related to U.S. rates and the exchange rate and inversely related to the foreign rate.

Equivalent Forward Exchange Position Instead of viewing its swap as a bond position, the British company could alternatively view its interest agreement to pay \$15 million for £7.5 million each year for five years and its principal agreement to pay \$150 million for £100 million at maturity as a series of long currency forward contracts in years 1, 2, 3, 4, and $5E_{ft}$. In contrast, the U.S. company could view its swap agreements to sell £7.5million each year for \$15 million and sell £100 million at maturity for \$150 million as a series of short currency forward contracts. In the absence of arbitrage, the value of the U.S. company's swap of dollars received/British pounds paid should be equal to (1) the sum of the present values of \$15 million received each year from the swap minus the dollar cost of buying £7.5 million at the forward exchange rate, and (2) the present value of the \$150 million received at year 5 minus the dollar cost of buying £100 million at the five-year forward exchange rate. The dollar value of the British position is just the opposite. Note that in the absence of arbitrage, the values of the swap positions as forward contracts are equal to their values as bond positions:

$$SV = \sum_{t=1}^{M} \frac{(\$\text{Received}) - E_{ft}(\text{FC Paid})}{(1 + R_{\text{US}})^t} = B_{\$} - E_0 B_{\text{FC}}$$

Comparative Advantage

The currency swap in the above example represents an exchange of equivalent loans. Most currency swaps, though, are the result of financial and nonfinancial corporations exploiting a comparative advantage resulting from different rates in different currencies for different borrowers. Recall, in the case of interest rate swaps, we pointed out that observed differences in credit spreads could be the result of either comparative advantage or hidden options on floating-rate loans. In the case of currency swaps, though, the existence of such differences is more likely to be the result of actual comparative advantages.

To see the implications of comparative advantage with currency swaps, suppose the U.S. and British companies in the preceding example both have access to each country's lending markets and that the U.S. company is more creditworthy, and as such, can obtain lower rates than the British company in both the U.S. and British markets. For example, suppose the U.S. company can obtain 10% U.S.-dollar-denominated loans in the U.S. market and 7.25% sterling-denominated loans in the British market, whereas the best the British company can obtain is 11% in the U.S. market and 7.5% in the British market:

Spot : $E_0 = \$/£ = \$1.50/£$

	Dollar Market (rate on $)	Pound Market (rate on £)
U.S. company	10%	7.25%
British company	11%	7.5%

With these rates, the U.S. company has a comparative advantage in the U.S. market: It pays 1% less than the British company in the U.S. market, compared to only 0.25% less in the British market. On the other hand, the British company has a comparative advantage in the British market: It pays 0.25% more than the U.S. company in Britain, compared to 1% more in the United States. When such a comparative advantage exists, a swap bank is in a position to arrange a swap to benefit one or both companies. For example, suppose in this case a swap bank sets up the following swap arrangement:

The U.S. company borrows $150 million at 10%, and then agrees to swap it for £100 million loan at 7%.

The British company borrows £100 million at 7.5%, and then agrees to swap it for $150 million loan at 10.6%.

Figure 22.1 shows the initial swap of principal, annual cash flows of interest, and final exchange of principal. In this swap arrangement, the U.S. company benefits by paying 0.25% less than it could obtain by borrowing British pounds directly in the British market, and the British company gains by paying 0.4% less than it could obtain directly from the U.S. market.

As shown in Figure 22.1, the swap bank in this case will receive $15.9 million each year from the British company, while only having to pay $15 million to the U.S. company, for a net dollar receipt of $ 0.9 million. On the other hand, the swap bank will receive only £7 million from the U.S. company, while having to pay £7.5 million to the British company, for a net sterling payment of £0.5 million:

Swap Bank's Dollar Position	Swap Bank's £ Position
Receives: (.106)($150,000,000) = $15,900,000	Receives: (.07)(£100,000,000) = £7,000,000
Pays: −(.10)($150,000,000) = −$15,000,000	Pays: (.075)(£100,000,000) = −£7,500,000
Net $ receipt: $900,000	Net £ payment: −£500,000

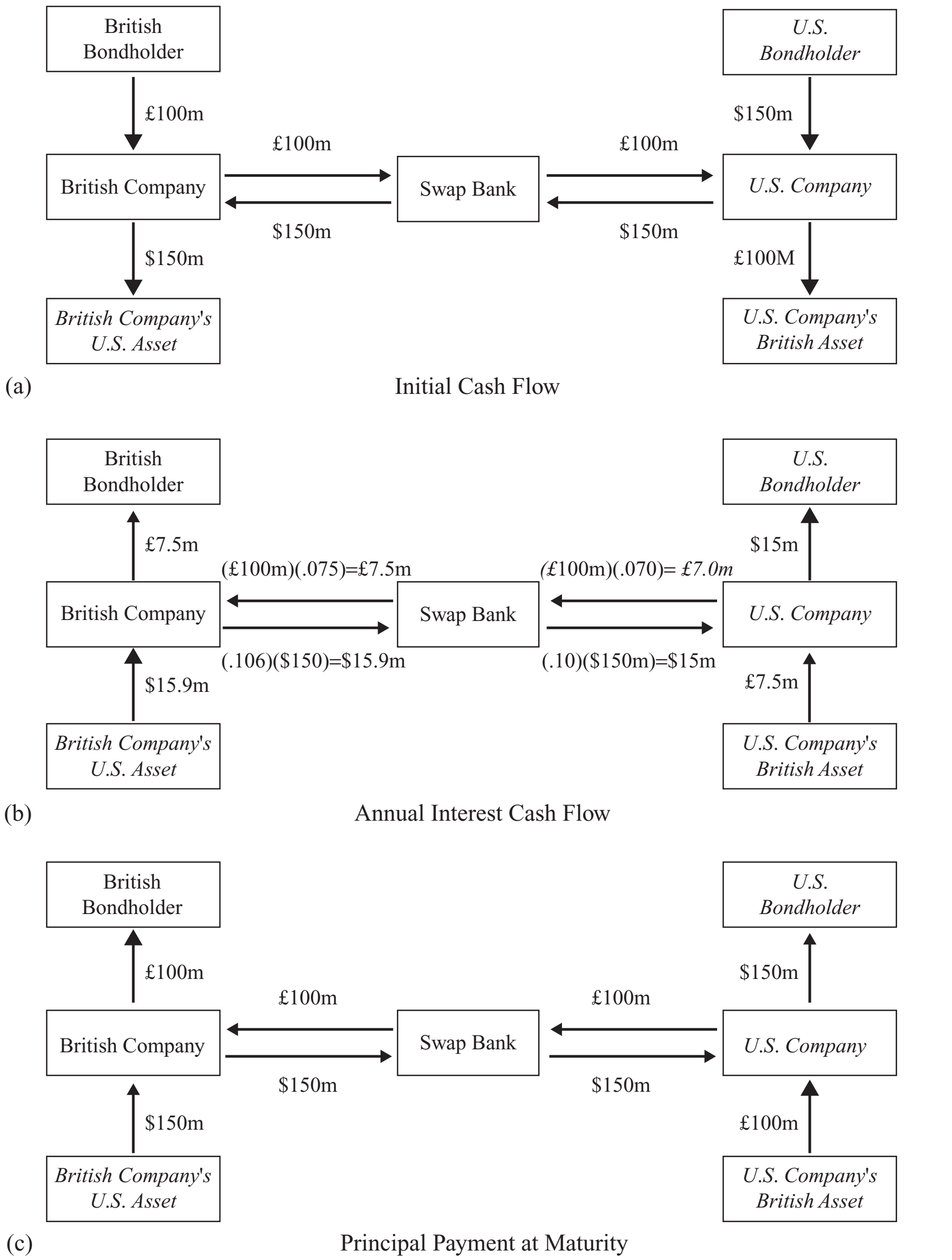

FIGURE 22.1 Currency Swap

Thus, the swap bank has a position equivalent to a series of long currency forward contracts in which it agrees to buy £500,000 for $900,000 each year. The swap bank's implied forward rate on each of these contracts is $1.80/£:

$$E_f = \frac{\$900{,}000}{\pounds 500{,}000} = \frac{\$1.80}{\pounds}$$

The swap bank can hedge its position with currency forward contracts. If the forward rate is less than \$1.80/£, then the bank could gain from hedging the swap agreement with forward contracts to buy £500,000 each year for the next five years. For example, suppose the yield curves applicable for the swap bank are flat at 9.5% in U.S. dollars and 7% in pounds (assume annual compounding). Using the interest rate parity relation that governs the equilibrium forward exchange rate relation, the one-, two-, three-, four-, and five-year forward exchange rates would be:

$$E_f = E_0\left(\frac{1+R_{\text{US}}}{1+R_{\text{GB}}}\right)^T$$

$$T=1\text{: } E_f = (\$1.50/\pounds)\left(\frac{1.095}{1.07}\right)^1 = \$1.535047/\pounds$$

$$T=2\text{: } E_f = (\$1.50/\pounds)\left(\frac{1.095}{1.07}\right)^2 = \$1.570912/\pounds$$

$$T=3\text{: } E_f = (\$1.50/\pounds)\left(\frac{1.095}{1.07}\right)^3 = \$1.607616/\pounds$$

$$T=4\text{: } E_f = (\$1.50/\pounds)\left(\frac{1.095}{1.07}\right)^4 = \$1.645177/\pounds$$

$$T=5\text{: } E_f = (\$1.50/\pounds)\left(\frac{1.095}{1.07}\right)^5 = \$1.683616/\pounds$$

The swap bank could enter into forward contracts to buy £500,000 each year for the next five years at these forward rates. With all of the forward rates less than the implied forward rate of \$1.80/£, the bank's dollar costs of buying £500,000 each year would be less than its \$900,000 annual inflow from the swap. By combining its swap position with forward contracts, the bank would be able to earn a total profit from the deal of \$478,815 (see Table 22.1).

Instead of forward contracts, the swap bank also could hedge its swap position by using a money market position. For example, on its first sterling liability of £500,000

TABLE 22.1 Swap Bank's Hedged Position

1 Year	2 \$ Cash Flow	3 £ Cash Flow	4 Forward Exchange: \$/£	5 \$ Cost of Sterling Column (4) × Column (3)	6 Net \$ Revenue Column (2) – Column (5)
1	\$900,000	£500,000	\$1.535047	–\$767,524	\$132,477
2	\$900,000	£500,000	\$1.570912	–\$785,456	\$114,544
3	\$900,000	£500,000	\$1.607616	–\$803,808	\$96,192
4	\$900,000	£500,000	\$1.645177	–\$822,589	\$77,412
5	\$900,000	£500,000	\$1.683616	–\$841,808	\$58,192
					\$478,816

due in one year, the bank would need to create a sterling asset worth £500,000 one year later (current value of £467,290M = £500,000/1.07) and a dollar liability worth $767,524 (based on the forward contract). The bank could do this by borrowing $700,935 [= ($1.50/£) (£467,290)] at 9.5%, converting it to £467,290, and investing the sterling at 7% interest for the next year. One year later, the bank would have £500,000 [= £467,290(1.07)] from the investment to cover its sterling swap liability and would have a dollar liability of $767,524 [= $700,935(1.095)], which is less than the $900,000 dollar inflow from the swap. The bank would thus earn a profit of $132,476 (= $900,000 – $767,524) from the hedged cash flow—the same profit it would earn from hedging with the forward exchange contracts if the interest rate parity relation holds. By forming the same types of money market positions for each sterling liability, the bank could obtain the same total profit of $478,815 that it would have received from the forward-hedged positions.

In summary, the presence of comparative advantage creates a currency swap market in which swap banks look at the borrowing rates offered in different currencies to different borrowers and at the forward exchange rates and money market rates that they can obtain for hedging. Based on these different rates, they will arrange swaps that provide each borrower with rates better than the ones they can directly obtain and a profit for them that will compensate them for facilitating the deal and assuming the credit risk of the counterparties.

Nongeneric Currency Swaps

The generic currency swap has been modified to accommodate different uses. Of particular note is the ***cross-currency swap*** that is a combination of the currency swap and interest rate swap. This swap calls for an exchange of floating-rate payments in one currency for fixed-rate payments in another. There are also currency swaps with amortizing principals, cancelable and extendable currency swaps, forward currency swaps, and options on currency swaps.

22.3 CREDIT DEFAULT SWAPS[2]

Traditionally, a bond portfolio manager or a financial institution with a portfolio of loans managed their portfolio's exposure to credit risk by the selection and allocation of credits (bonds or loans) in their portfolio. For example, a bond portfolio manager expecting recession and wanting to reduce her portfolio's exposure to credit risk would sell her lower-quality bonds and buy higher-quality ones. With the development of the credit default swap market, though, a bond manager or lender could change her credit risk by simply buying or selling swaps to change the credit risk profile on either an individual bond or loan or on a bond or loan portfolio.

Generic Credit Default Swap

Credit default swaps and other related credit swap derivatives are contracts in which the payoffs depend on the credit quality of a company. In a standard ***credit default swap (CDS)***, a counterparty, such as a bank, buys protection against default by a particular company from a counterparty (seller). The company is known as the reference entity and a default by that company is known as a credit event. The buyer of the CDS makes periodic payments or a premium to the seller until the end of the life of the CDS or until the credit event occurs. If the credit event occurs, the buyer, depending on the contract, has either the right to sell a particular bond (or loan) issued by the company for its par value (physical delivery) or receive a cash settlement based on the difference between the par value and the defaulted bond's market price.

To illustrate, suppose two parties enter into a five-year CDS with an NP of $100 million. The buyer agrees to pay 100 basis points annually for protection against default by the reference entity. If the reference entity does not default, the buyer does not receive a payoff and ends up paying $1,000,000 each year for five years. If a credit event does occur, the buyer will receive the default payment and pay a final accrual payment on the unpaid premium; for example, if the event occurs half way through the year, then the buyer pays the seller $500,000. If the swap contract calls for physical delivery, the buyer will sell $100 million par value of the defaulted bonds for $100 million. If there is a cash settlement, then an agent will poll dealers to determine a midmarket value. If that value were $30 per $100 face value, then the buyer would receive $70 million minus the $500,000 accrued interest payment.

Terms

In the standard CDS, payments are usually made in arrears either on a quarterly, semiannual, or annual basis. The par value of the bond or debt is the notional principal used for determining the payments of the buyer. In many CDS contracts, a number of bonds or credits can be delivered in the case of a default. A company like Kraft, for example, might have five bonds with similar maturities, coupons, and embedded option and protection features that a buyer of a CDS can select in the event of a default. In the event of a default, the payoff from the CDS is equal to the face value of the bond (or NP) minus the value of the bond just after the default. The value of the bond just after the default expressed as a proportion of the bond's face value is known as the ***recovery rate (RR)***. Thus, the payoff from the CDS is

$$\text{CDS payoff} = (1 - RR)\text{NP} - \text{Accrued payment}$$

If the recovery value on the $100 million CDS were $30 per $100 face value, then the recovery rate would be 30% and the payoff to the CDS buyer would be $70 million [= (1− .30)$100 million)]minus any accrued payment.

The payments on a CDS are quoted as an annual percentage of the NP. The payment is referred to as the ***CDS spread***. Swap bankers function as both brokers and dealers in the CDS market. As dealers, they will provide bid and ask quotes on a particular credit entry. For example, a swap bank might quote a five-year CDS on a GE credit at 270 basis points bid and 280 basis points offer; that is, the swap bank will buy protection on GE for 2.7% of the underlying credit's principal per year for five years; the swap bank will sell protection on GE for 2.8% of the principal.

Uses

CDSs are used primarily to manage the credit risk on debt and fixed-income positions. To see this, consider a bond fund manager who just purchased a five-year BBB corporate bond at a price yielding 8% and wanted to eliminate the credit risk on the bond. To do this, suppose the manager bought a five-year CDS on the bond. If the payments or spread on the CDS were equal to 3% of the bond's principal, then the purchase of the CDS would have the effect of making the 8% BBB bond a risk-free bond yielding approximately 5%. That is, if the bond does not default, then the bond fund manager will receive 5% from owning the bond and the CDS (8% yield on bond – 3% payment on CDS). If the bond defaults, then the bond manager will receive 5% from the bond and CDS up to the time of the default and then will receive the face value on the bond from the CDS seller, which the manager can reinvest for the remainder of the five-year period. Thus, the CDS allows the manager to reduce, or in this case, eliminate the credit risk on the bond.

In contrast, suppose a manager holding a portfolio of five-year U.S. Treasury notes yielding 5% expected the economy to improve and therefore was willing to assume more credit risk in return for a higher return by buying BBB corporate bonds yielding 8%. As an alternative to selling his Treasuries and buying the corporate bonds, the manager could sell a CDS. If he were to sell a five-year CDS on the above five-year BBB bond to a swap bank for the 3% spread, then the manager would be adding 3% to the 5% yield on his Treasuries to obtain an effective yield of 8%. Thus with the CDS, the manager would be able to obtain an expected yield equivalent to the BBB bond yield and would also be assuming the same credit risk associated with that bond.

As a second example, consider a commercial bank with a large loan to a corporation. Prior to the introduction of CDSs, the bank would typically have to hold on to the loan once it was created. During this period, its only strategy for minimizing its loan portfolio's exposure to credit risk was to create a diversified loan portfolio. With a CDS, such a bank can now buy credit protection for the loan. In general, CDSs allow banks and other financial institutions to more actively manage the credit risk on their loan portfolio, buying CDSs on some loans and selling CDSs on other. Today, commercial banks are the largest purchasers of CDSs and insurance companies are the largest sellers.

The Equilibrium CDS Spread

In equilibrium, the payment or spread on a CDS should be approximately equal to the credit spread on the CDS's underlying bond or credit. In terms of the above example, if the only risk on a five-year BBB corporate bond yielding 8% were credit risk (i.e., there is no option risk associated with embedded call options and the like, no liquidity risk, and no interest rate risk), and the risk-free rate on five-year investments were 5%, then the BBB bond would be trading in the market with a 3% credit spread. If the spread on a five-year CDS on a BBB-quality bond or credit were 3%, then an investor could obtain a five-year risk-free investment yielding 5% by either buying a five-year Treasury or buying the five-year BBB corporate yielding 8% and purchasing the CDS on the underlying credit at a 3% spread.

If the spread on a CDS is not equal to the credit spread on the underlying bond, then an arbitrage opportunity would exist by taking positions in the bond, risk-free security, and the CDS. For example, suppose a swap bank were offering the preceding CDS for 2% instead of 3%. In this case, an investor looking for a five-year risk-free investment would find it advantageous to create the synthetic risk-free investment with the BBB bond and the CDS. That is, the investor could earn 1% more than the yield on the Treasury by buying the five-year BBB corporate yielding 8% and purchasing the CDS on the underlying credit at 2%. In addition to the investor gaining, an arbitrageur could also realize a free lunch equivalent to a five-year cash flow of 1% of the par value of the bond by shorting the Treasury at 5% and then using the proceeds to buy the BBB corporate and the CDS. These actions by investors and arbitrageurs, in turn, would have the impact of pushing the spread on the CDS towards 3%—the underlying bond's credit risk spread.

On the other hand, if the swap bank were offering the CDS at a 4% spread, then an investor looking for a five-year risk-free investment would obviously prefer a Treasury yielding 5% to a synthetic risk-free investment formed with the five-year BBB corporate yielding 8% and a CDS on the credit requiring a payment of 4%. A more aggressive investor looking to invest in the higher-yielding five-year BBB bonds, though, could earn 1% more than the 8% on the BBB bond by creating a synthetic five-year BBB bond by purchasing the five-year Treasury at 5% and selling the CDS at 4%. Similarly, a bond portfolio manager holding five-year BBB bonds yielding 8% could pick up an additional 1% yield with the same credit risk exposure by selling the bonds along with the CDS at 4% and then using the proceeds from the bond sale to buy the five-year Treasuries yielding 5%. Finally, an arbitrageur could realize a free lunch equivalent to a five-year cash flow of 1% of the par value on the bond by shorting the BBB bond, selling the CDS, and then using proceeds to purchase five-year Treasuries. With these positions, the arbitrageur for each of the next five years would receive 5% from her Treasury investment and 4% from her CDS, while paying only 8% on her short BBB bond position. Furthermore, her holdings of Treasury securities would enable her to cover her obligation on the CDS if there was a default. That is, in the event of a default she would be able to pay the CDS holder from the net proceeds from selling her Treasuries and closing her short BBB bond by buying back the corporate bonds at their defaulted recovery price. As noted in Chapter 12, this strategy was the basis of many synthetic collateral debt obligation (CDO) structures. That is, the manager or sponsor of the CDO would issue a CDO (similar to shorting the BBB bond), sell CDSs, and purchase investment grade bonds. Collectively, the actions of the investors, bond portfolio managers, CDO sponsors, and arbitrageur would have the effect of pushing the spread on the CDS from 4% to 3%.

In equilibrium, arbitrageurs and investors should ensure that the spreads on CDSs are approximately equal to spreads on the underlying bond or credit. This spread can be defined as the equilibrium spread or the ***arbitrage-free spread***. The arbitrage-free spread, Z, on a bond or CDS can also be thought of as the bond investor's or CDS buyer's expected loss from the principal from default. To see this, consider a portfolio of five-year BBB bonds trading at a 3% credit spread. The 3% premium that investors receive from the bond portfolio represents their compensation for an implied expected loss of 3% per year of the principal from the defaulted bonds. If the spread were 3% and bond investors believed that the expected loss from default on such bonds would be only 2% per year of the principal, then the bond investors would want more BBB bonds, driving the price up and the yield down until the premium reflected a 2% spread. Similarly, if the spread were 3% and bond investors believed the default loss on a portfolio of BBB bonds would be 4% per year, then the demand and price for such bonds would decrease, increasing the yield to reflect a credit spread of 4%. Thus, in an efficient market, the credit spread on bonds and the equilibrium spreads on CDSs represent the market's implied expectation of the expected loss per year from the principal from default. In the case of a CDS, the equilibrium spread can therefore be defined as the implied probability of default loss of the principal on the contract.

CDS Valuation

The total value of a CDS's payments is equal to the sum of the present values of the periodic CDS spread (Z) times the NP over the life of the CDS, discounted at the risk-free rate (R):

$$\text{PV(CDS payments)} = \sum_{t=1}^{M} \frac{Z\,\text{NP}}{(1+R)^t}$$

The present value of the payment on a five-year CDS with a spread equal to an equilibrium spread of 2% and with an NP of \$100 would be \$8.425 (assuming annual compounding and 6% risk-free rate):

$$\text{PV(CDS payments)} = \sum_{t=1}^{5} \frac{(.02)(\$100)}{(1.06)^t} = \$8.425$$

The buyer (seller) of this five-year CDS would therefore be willing to make (receive) payments over five years that have a present value of \$8.425 per \$100 of NP. Since the spread can also be viewed as an expected loss of principal, the present value of the payments is also equal to the expected default protection the buyer (seller) receives (pays). The value of the CDS protection, in turn, is equal to the present value of the expected payout in the case of default:

$$\text{PV(Expected payout)} = \sum_{t=1}^{M} \frac{p_t\,\text{NP}(1-RR)}{(1+R)^t}$$

where p_t = probability of default in period t conditional on no earlier default
RR = recovery rate (as a proportion of the face value) on the bond at the time of default
NP = notional principal equal to the par value of the bond

Note that the probability of default, p_t, is the conditional probability of no prior defaults that was defined in Chapter 5. Thus the conditional probability of default in year 3 is based on the probability that the bond will survive until year 3. In contrast, an unconditional probability is the likelihood that the bond will default at a given time as seen from the present. Conditional default probabilities are referred to as ***default intensities***. Over a period of time, these probabilities will change, increasing or decreasing depending on the quality of the credit.

Instead of defining a CDS's expected payout in terms of periodic probability density, p_t, the CDS's expected payout can alternatively be defined by the average conditional default loss probability, $\overline{p}$:

$$\text{PV(Expected payout)} = \sum_{t=1}^{M} \frac{\overline{p}\ \text{NP}(1-RR)}{(1+R)^t} = \overline{p}\text{NP}(1-RR)\sum_{t=1}^{M} \frac{1}{(1+R)^t}$$

Given an equilibrium spread of .02 and a recovery rate of 30%, the implied probability density for our illustrative CDS would be .02857. This implied probability is obtained by solving for $\overline{p}$ that makes the present value of the expected payout equal to the present value of the payments of \$8.425:

$$\text{PV(Expected payout)} = \text{PV(Payments)}$$

$$\sum_{t=1}^{M} \frac{\overline{p}\ \text{NP}(1-RR)}{(1+R)^t} = \sum_{t=1}^{M} \frac{Z\ \text{NP}}{(1+R)^t}$$

$$\overline{p} = \frac{Z}{(1-RR)}$$

$$\overline{p} = \frac{.02}{(1-.30)} = .02857$$

$$\text{PV(Expected payout)} = \sum_{t=1}^{M} \frac{\overline{p}\ \text{NP}(1-RR)}{(1+R)^t}$$

$$\text{PV(Expected payout)} = \sum_{t=1}^{5} \frac{(.02857)\ (\$100)(1-.30)}{(1.06)^t} = \$8.425$$

Note that if there were no recovery ($RR = 0$), then the implied probability would be equal to the spread Z, which as we noted earlier can be thought of as the probability of default of principal. The probability density implied by the market is referred to as the risk-neutral probability since it is based on an equilibrium spread that is arbitrage free.[3]

Alternative CDS Valuation Approach

Suppose in our illustrative example, the estimated default intensity, sometimes referred to as the ***real world probability***, on the five-year BBB bond were .02 and not the implied probability of .02857. In this case, the present value of the CDS expected payout would be \$5.897 instead of \$8.425:

$$\text{PV(Expected payout)} = \sum_{t=1}^{5} \frac{(.02)\ (\$100)(1 - .30)}{(1.06)^t} = \$58.97$$

Given the spread on the CDS is at 2% and the present value of the payments are \$8.425, buyers of the CDS would have to pay more on the CDS than the value they receive on the expected payoff (\$5.897). If the real-world probability density of .02 is accurate, then buyers of the CDS would eventually push the spread down until it is equal to the value of the protection. For the payment on the CDS to match the expected protection, the spread would have to equal .014. This implied spread is found by solving for the Z that equates the present value of the payments to the present value of the expected payout given the real-world probability of $\overline{p} = .02$ and the estimated recovery rate of $RR = .30$. That is:

$$\text{PV(Payments)} = \text{PV(Expected payout)}$$

$$\sum_{t=1}^{M} \frac{Z\ \text{NP}}{(1+R)^t} = \sum_{t=1}^{M} \frac{\overline{p}\ \text{NP}(1 - RR)}{(1+R)^t}$$

$$Z = \overline{p}\ (1 - RR)$$

$$Z = (.02)(1 - .30) = .014$$

$$\text{PV(Payments)} = \sum_{t=1}^{M} \frac{Z\ \text{NP}}{(1+R)^t} = \sum_{t=1}^{5} \frac{(.014)\ (\$1.00)}{(1.06)^t} = \$5.897$$

We now have two alternative methods for pricing a CDS. On the one hand, we can value the CDS swap given the credit spread in the market and then determine the present value of the payments; thus in terms of our example, we would use the market spread of 2% and value the swap at \$8.425 with the implied probability density (or risk-neutral probability) being .02857. On the other hand, we can value the swap given estimated probabilities of default and then determine the present value of the expected payout; in terms of our example, we would use the estimated real-world probability of .02 and value the CDS at \$5.897 with the implied credit spread being .014. The question becomes, what valuation method should be used?

While cogent arguments can be made for either case, many scholars argue for the use of valuing swaps with risk-neutral probabilities (or equivalently pricing swaps in terms of credit spreads on the underlying bonds) because it reflects an arbitrage-free price. Thus, they would price the five-year CDS equal to the 2% spread with a total value of \$8.425. On the other hand, though, if one can estimate real-world probabilities on bonds and CDSs that are more accurate than the probabilities implied by the current credit spread, then eventually the bond and CDS market will price such claims so that the credit spread reflects the real-world probability. If this is the case, then in our example one should price the five-year CDS at \$5.897 given the estimated probability density of .02. The argument for pricing CDSs using real-world probabilities ultimately depends on the ability of practitioners to estimate default probabilities.

Estimating Default Rates and Valuing CDSs Based on Estimated Default Intensities

As just noted, the alternative to pricing a CDS in terms of its credit spread is to determine the spread that will equate the present value of the payments to the present value of the expected payout based on estimates of the conditional probability. That is, given estimated periodic default intensities and the recovery rate (RR), the objective is to find the Z where

$$\sum_{t=1}^{M} \frac{Z\,\mathrm{NP}}{(1+R)^t} = \sum_{t=1}^{M} \frac{p_t \mathrm{NP}(1-RR)}{(1+R)^t}$$

There are several approaches for estimating conditional probabilities. The simplest and most direct one is to estimate the probabilities based on historical default rates.

Estimating Probability Intensities from Historical Default Rates Table 22.2 shows the cumulative default rates, unconditional probability rates, and conditional probability rates (probability intensities) for corporate bonds with quality ratings of Aaa, Baa, B, and Caa. The probabilities shown in the table are the average historical cumulative default rates from 1970–2006 as complied by Moody's. As explained in Chapter 5, the unconditional probabilities are the probabilities of default in a given year as viewed from time zero. The unconditional probability of a bond defaulting during year t is equal to the difference in the cumulative probability in year t minus the cumulative probability of default in year $t-1$. As shown in the table, the probability of a Caa bond default during year 4 is equal to 7.18% (= 46.9%% − 39.72%). Finally, the conditional probability is the probability of default in a given year conditional on no prior defaults. This probability is equal to unconditional probability of default in time t as a proportion of the bond's probability of survival at the beginning of the period. The probability of survival is equal to 100 minus the cumulative probability. For example, the probability that a Caa bond will survive until the end of year 3 is 60.28% (100 minus its cumulative probability, 39.72%), and the probability that the Caa bond will default during year 4 conditional on no prior defaults is 11.91% (= 7.18%/60.28%). As noted earlier, conditional probabilities of default are known as default intensities. These probabilities, in turn, can be used to determine the expected payoff on a swap.

Using the conditional probabilities generated from the historical cumulative default rates, the values and spreads for four CDSs with quality ratings of Aaa, Baa, B, and Caa are shown in Table 22.2. Each swap is assumed to have a maturity of five years, annual payments, NP of \$100, and recovery rate of 30%. The values are obtained by determining the present values of the expected payoff, with the discount rate assumed to be 6% and with the possible defaults assumed to occur at the end of each year (implying there are no accrued payments). The spreads on the CDSs are the spreads that equate the present value of the payments to the present value of the

TABLE 22.2 Cumulative Default Rates, Probability Intensities, and CDS Values and Spreads

Average Cumulative Default Rates 1970–2006 (Moody's) in %

Year	1	2	3	4	5	PV(Expected Payoff) NP = $100 and RR = .3	CDS Spread Z
Aaa							
Cumulative Probability (%)	0.00000	0.00000	0.00000	0.03000	0.10000		
Unconditional Probability (%)	0.00000	0.00000	0.00000	0.03000	0.07000		
Conditional Probability p (%)	0.00000	0.00000	0.00000	0.03000	0.07002		
Present Value of p at 6%	0	0	0	0.0237628	0.052324	0.053260605	0.00013
Baa							
Cumulative Probability (%)	0.18000	0.51000	0.93000	1.43000	1.94000		
Unconditional Probability (%)	0.18000	0.33000	0.42000	0.50000	0.51000		
Conditional Probability p (%)	0.18000	0.33060	0.42215	0.50469	0.51740		
Present Value of p at 6%	0.169811321	0.294228	0.354448	0.3997646	0.38663	1.123417867	0.002666953
B							
Cumulative Probability (%)	5.24000	11.30000	17.04000	22.05000	26.79000		
Unconditional Probability (%)	5.24000	6.06000	5.74000	5.01000	4.74000		
Conditional Probability p (%)	5.24000	6.39510	6.47125	6.03905	6.08082		
Present Value of p at 6%	4.943396226	5.691619	5.433387	4.7834972	4.543943	17.77709035	0.04220217
Caa							
Cumulative Probability (%)	19.48000	30.49000	39.72000	46.90000	52.62000		
Unconditional Probability (%)	19.48000	11.01000	9.23000	7.18000	5.72000		
Conditional Probability p (%)	19.48000	13.67362	13.27866	11.91108	10.77213		
Present Value of p at 6%	18.37735849	12.16947	11.14902	9.4346923	8.049561	41.42607634	0.098344009

$$\text{PV(Expected payoff)} = \sum_{t=1}^{M} \frac{p_t \, \text{NP}(1 - RR)}{(1+R)^t} \qquad Z = \frac{\sum_{t=1}^{M} \frac{p_t(1 - RR)}{(1+R)^t}}{\sum_{t=1}^{M} \frac{1}{(1+R)^t}}$$

expected payoff. For example, the present value of the expected payoff for the CDS with a B quality rating is 17.78:

$$\text{PV(Expected payoff)} = \sum_{t=1}^{M} \frac{p_t \, \text{NP}(1 - RR)}{(1 + R)^t}$$

$$\text{PV(Expected payoff)} = (\$100)(1 - .3)$$

$$\times \left[\frac{.0524}{(1.06)} + \frac{.06395}{(1.06)^2} + \frac{.0647125}{(1.06)^3} + \frac{.0603905}{(1.06)^4} + \frac{.0608082}{(1.06)^5}\right]$$

$$\text{PV(Expected payoff)} = 17.78$$

The spread on the B-quality CDS that equates the present value of its payments to the expected payoff of \$17.78 is .0422:

$$\sum_{t=1}^{M} \frac{Z \, \text{NP}}{(1 + R)^t} = \sum_{t=1}^{M} \frac{p_t \, \text{NP}(1 - RR)}{(1 + R)^t}$$

$$Z \sum_{t=1}^{5} \frac{\$100}{(1.06)^t} = \$17.78$$

$$Z = \frac{\$17.78}{\sum_{t=1}^{5} \frac{\$100}{(1.06)^t}} = \frac{\$17.78}{\$421.2364} = .0422$$

As shown in the table, the present value of the expected payoffs on the Caa-quality CDS is \$41.43 and its spread is 0.0983. As expected, the CDS values and spreads are greater, the greater the default risk.

Estimating Expected Default Rates: Implied Default Rates The above default rates are based on historical frequencies. Past frequencies are often not the best predictors of future probabilities. If the market is efficient such that prices of bonds reflect the market's expectation of future economic conditions, then the implied probabilities based on a CDS's risk spread would represent an expected future default probability. We previously calculated the average implied probability by solving for $\overline{p}$ that equated the present value of the expected payoff to the present value of the payments based on the current bond's credit spread. Using this methodology, one could estimate the implied conditional probabilities for each year for a given quality rating using the CDS spread on one-year to m-year swaps. This would result in a set of implied default probabilities that could be used to determine the spread on any m-year swap. As an example, the table below shows the spreads (Z) and the implied probability densities given an assumed recovery rate of 30% on five B-rated CDSs with maturities ranging from one to five years:

Maturity t	Spread	Implied Probabilty
1	0.0400	0.0571
2	0.0425	0.0607
3	0.0450	0.0643
4	0.0475	0.0679
5	0.0500	0.0714

The implied probability densities are equal to $Z/(1 - RR)$. Given these probabilities, the value of a five-year CDS based on its expected payoff would be \$18.83 and its spread would be .0447:

$$\text{PV(Expected payoff)} = \sum_{t=1}^{M} \frac{p_t \text{NP}(1 - RR)}{(1 + R)^t}$$

$$\text{PV(Expected payoff)} = (\$100)(1 - .3)$$

$$\times \left[\frac{.0571}{(1.06)} + \frac{.0607}{(1.06)^2} + \frac{.0643}{(1.06)^3} + \frac{.0679}{(1.06)^4} + \frac{.0714}{(1.06)^5} \right]$$

$$\text{PV(Expected payoff)} = \$18.83$$

$$\times \sum_{t=1}^{M} \frac{Z \text{ NP}}{(1 + R)^t} = \sum_{t=1}^{M} \frac{p_t \text{ NP}(1 - RR)}{(1 + R)^t}$$

$$Z \sum_{t=1}^{5} \frac{\$100}{(1.06)^t} = \$18.83$$

$$Z = \frac{\$18.83}{\sum_{t=1}^{5} \frac{\$100}{(1.06)^t}} = \frac{\$18.83}{\$421.2364} = .0447$$

Summary of the Two Valuation Approaches

We previously noted that many scholars argue for valuing CDSs in terms of credit spreads on the underlying bonds because it results in an arbitrage-free price. Pricing CDSs in terms of bond credit spreads also implies that the default probability for determining the expected payout by the seller is a probability implied by the credit spread of bonds traded in the market and not a real-world estimated probability. The alternative to pricing CDSs in terms of bond credit spreads is to determine the spread that will equate the present value of the payments to the present value of the expected payout based on estimates of the conditional probability. As we just discussed, default probabilities can be estimated using historical cumulative default rates and implied default rates. There are also a number of other more advanced estimating techniques that practitioners can use to determine default probabilities. Several of these estimating approaches are referenced at the end of this chapter. Of particular note is the Gaussian copula model.

The Value of an Off-Market CDS Swap

Similar to a generic par value interest rate swap, a swap rate on a new CDS is generally set so that there is not an initial exchange of money. Over time and as economic conditions change the credit spreads on new CDSs, the value of an existing CDS will change. For example, suppose one year after a bond fund manager bought our illustrative five-year CDS on BBB bonds at the 2% spread, the economy became weaker and credit spreads on four-year BBB bonds and new CDSs on such bonds were 50 bp greater at 2.5% (assume for this discussion that CDS spreads are determined by bond credit spreads in the market). Suppose the bond fund manager sold her 2% CDS to a swap bank who hedged the CDS by selling a new 2.5% CDS on the four-year BBB bond. With a buyer's position on the assumed 2% CDS and a seller's position on the 2.5% CDS, the swap bank, in turn, would gain .5% of the NP for the next four years. Given a discount rate of 6%, the present value of this gain would be \$1.73 per \$100 NP. The swap banks would therefore pay the bond manager a maximum of \$1.73 for assuming the swap.

Offsetting Swap Positions		
Buyer of 2% CDS Swap	Pay 2% of NP	Receive default protection
Seller of 2.5% CDS Swap	Receive 2.5%	Pay default protection
	Receive .5% per year	

$$SV = \sum_{t=1}^{4} \frac{(\text{Current Spread} - \text{Existing Spread})(\text{NP})}{(1+R)^t} = \sum_{t=1}^{4} \frac{0.005\ (\$100)}{(1.06)^t} = \$1.73$$

With four years left on the current swap, the increase in the credit spread in the market has increased the value of the buyer's position on the CDS swap by \$1.73 from \$6.93 to \$8.66:

$$\text{Existing CDS}:\ \text{PV(CDS payments)} = \sum_{t=1}^{4} \frac{(.02)(\$100)}{(1.06)^t} = \$6.93$$

$$\text{Current PV(CDS payments)} = \sum_{t=1}^{4} \frac{(.025)(\$100)}{(1.06)^t} = \$8.66$$

$$\text{Change in value} = \$1.73$$

The increase in value on the buyer's position of the existing swap reflects the fact that with poorer economic conditions the 2% swap payments now provide greater default protection (i.e., the present value of the expect payout is greater).

For the initial seller, the increase in credit spreads causes a decrease in the value on the seller's positions. For example, suppose that an insurance company was the one who sold the five-year CDS on the BBB bond at the 2% spread to the bond portfolio manager (via a swap bank) and that one year later the credit spread on new four-year CDSs on BBB bonds was again at 2.5%. If the insurance company were to sell its seller's position to a swap bank, the swap bank could hedge the assumed position by taking a buyer's position on a new four-year, 2.5% CDS on the BBB bond. With the offsetting positions, the swap bank would lose .5% of the NP for the next four years. Given a discount rate of 6%, the present value of this loss would be \$1.73 per \$100 NP. The swap banks would therefore charge the insurance company at least \$1.73 for assuming the seller's position on the swap:

Offsetting Swap Positions		
Seller of 2% CDS swap	Receive 2% of NP	Pay default protection
Buyer of 2.5% CDS swap	Pay 2.5%	Receive default protection
	Pay .5% per year	

$$SV = \sum_{t=1}^{4} \frac{(\text{Existing spread} - \text{Current spread})(\text{NP})}{(1+R)^t} = \sum_{t=1}^{4} \frac{-0.005\ (\$100)}{(1.06)^t} = -\$1.73$$

For the insurance company, the increase in the credit spread has decreased the value of their seller's position on the CDS swap by \$1.73. That is, for the increase in credit risk, the seller should be receiving \$8.66 instead of \$6.93. Alternatively stated, the poorer economic conditions reflected in the greater credit spreads have increased the probability of default on the BBB bonds and as a result have increased the present value of the seller's expected payoff. Specifically, with the credit spread increasing from 2% to 2.5%, the implied conditional default rate on the bond has increased from .02857 to .035714, increasing the present value of the seller's expected payout from \$6.93 to \$8.66:

$$\text{Existing } \overline{p} = \frac{Z}{(1-RR)} = \frac{.02}{(1-.30)} = .02857$$

$$\text{Current: } \overline{p} = \frac{Z}{(1-RR)} = \frac{.025}{(1-.30)} = .035714$$

$$\text{PV(Expected payout)} = \sum_{t=1}^{M} \frac{\overline{p}\ \text{NP}(1-RR)}{(1+R)^t}$$

$$\text{Existing: PV(Expected payout)} = \sum_{t=1}^{4} \frac{(.02857)\ (\$100)(1-.30)}{(1.05)^t} = \$6.93$$

$$\text{Current: PV(Expected payout)} = \sum_{t=1}^{4} \frac{(.035714)\ (\$100)(1-.30)}{(1.05)^t} = \$8.66$$

To summarize, an increase in the credit spread will increase the value of the buyer's position on an existing CDS and decrease the seller's position. Just the opposite occurs if economic conditions improve and credit spreads decrease.

Other Credit Derivatives

The market for CDSs has grown dramatically over the last decade, although it has slowed in the aftermath of 2008 financial crisis. With that growth there has been an increase in the creation of other credit derivatives. The most noteworthy of these other credit derivatives are the binary swap, the credit swap basket, CDS forward contracts, CDS option contracts, contingent swaps, and total return swaps.

Binary CDS A ***binary CDS*** is identical to the generic CDS except that the payoff in the case of a default is a specified dollar amount. Often the fixed payoff is the principal on the underlying credit. When this is the case, then the only difference between the generic and binary swap is that the generic CDS adjusts the payoff by subtracting the recovery value, whereas the binary CDS does not. Without the recovery value, the value of a binary CDS is more sensitive to changes in credit spreads or default probabilities.

Basket CDS In a ***basket credit default swap***, there is a group of reference entities or credits instead of one, and there is usually a specified payoff whenever one of the reference entities defaults. Basket CDSs can vary by the type of agreement governing the payout. For example, an ***add-up basket CDS*** provides a payout when any reference credit in the basket defaults; a ***first-to-default CDS*** provides a payout only when the first entry defaults; a ***second-to-default CDS*** provides a payout when the second default occurs; an ***nth-to-default CDS*** provides a payout when the nth credit entry defaults. Typically, after the relevant entry defaults, the swap is terminated.

CDS Forward Contracts There is a dealer's market for CDS forward and option contracts. Like any futures or forward contract, a ***CDS forward contract*** is a contract to take a buyer's position or a seller's position on a particular CDS at a specified spread at some future date. CDS forward contracts provide a tool for locking in the credit spread on a future credit position.

CDS Option Contracts A ***CDS option*** is an option to buy or sell a particular CDS at a specified swap rate at a specified future time; for example, a one-year option to buy a five-year CDS on GE for 300 basis points. At expiration, the holder of this option would exercise her right to take the buyer's position at 300 basis points if current five-year CDSs on GE were greater than 300 basis points; in contrast, she would allow the option to expire and take the current CDS on GE if it is offered at 300 basis points or less.

Contingent CDS A ***contingent CDS*** provides a payout that is contingent on two or more events occurring. For example, the payoff might require both a credit default of the reference entity and an additional event such as a credit event with another entity or a change in a market variable.

Total Return Swaps

In a ***total return swap***, there is an agreement to exchange the return on an asset (such a bond, bond portfolio, stock, or stock portfolio) for some benchmark rate such as LIBOR plus basis points. In the case of an exchange of the return on a bond or bond portfolio for LIBOR and basis points, the return on the bond includes coupons and gains and losses on the bond. Such a swap allows parties to trade different risks, including credit risk.

A variation of the total return swap is the ***equity swap.*** In an equity swap, one party agrees to pay the return on an equity index, such as the S&P 500, and the other party agrees to pay a floating rate (LIBOR) or fixed rate. For example, on an S&P 500/LIBOR swap, the equity payer would agree to pay the six-month rate of change on the S&P 500 (e.g., proportional change in the index between effective dates) times an NP in return for LIBOR times NP, and the debt payer would agree to pay the LIBOR in return for the S&P 500 return. Equity swaps are useful to fund managers who want to increase or decrease the equity or bond exposure of their portfolios as part of their overall asset allocation strategy.

CAT Bond

Somewhat related to credit risk is the catastrophic (CAT) risk that insurance companies face in providing protection against hurricanes, earthquakes, and other natural disasters. Insurance companies often hedge CAT risk through reinsurance. However, one CAT hedging product that insurance companies are increasingly using is the issuance of CAT bonds. A ***CAT bond*** pays the buyer a higher-than-normal interest rate. In return for the additional interest, the CAT bondholder agrees to provide protection for losses from a specified event up to a specified amount or when the losses exceed a specified amount. For example, an insurance company could issue a CAT bond with a principal of $200 million against a hurricane cost exceeding $300 million. The CAT bondholders would then lose some or possibly all principal if the event occurs and the cost exceeds $300 million.

22.4 CONCLUSION

In this final chapter, we completed our analysis of swaps by examining the markets, uses, and valuation of currency and credit swaps. Like interest rate options and futures, swaps provide investors and borrowers with a tool for hedging asset and liability positions against interest rate and exchange rate fluctuations, speculating on interest rate and exchange rate movements, and improving the returns received on fixed income investments or paid on debt positions. We, of course, have not exhausted all derivative securities, just as we have not covered all the strategies, uses, markets, and pricing of debt securities. What we hope we have done here and in this last part of the book, though, is develop a foundation for the understanding of derivative products and their important applications in debt management. To this extent, we also hope our odyssey into the world of bonds and their derivatives has established a foundation and methodology for understanding the markets and uses of fixed-income securities.

KEY TERMS

add-up basket CDS
arbitrage-free spread
basket credit swap
binary CDS
CAT bond
CDS forward contract

CDS option
CDS spread
contingent CDS
credit default swap (CDS)
cross-currency swap
currency swap
default intensities
equity swap
first-to-default CDS
nth-to-default CDS
real-world probability
recovery rate (RR)
second-to-default CDS
total return swap

PROBLEMS AND QUESTIONS

1. The table shows the annual loan rates U.S. and Canadian multinational companies can each obtain on a five-year, $100 million loan in U.S. dollars and an equivalent five-year, C$142.857 million loan in Canadian dollars.

   Loan Rates for U.S. and Canadian Companies in U.S. Dollars and Canadian Dollars

Spot: E_0 = $/C$ = $0.70/C$		
	U.S Dollar Market (rate on $)	**Canadian Dollar Market (rate on C$)**
U.S. company	10%	7.25%
Canadian company	11%	7.5%

   a. Suppose the U.S. multinational wants to borrow C$142.857 million for five years to finance its Canadian operations, while the Canadian company wants to borrow $100 million for five years to finance its U.S. operations. Explain how a swap bank could arrange a currency swap that would benefit the U.S. company by lowering the rate on its Canadian dollar loan by .25% and would benefit the Canadian company by lowering its dollar loan by .4%. Describe the financial market conditions that allow the swap bank to provide such rates.
   b. Show the swap arrangements in terms of U.S. dollar and Canadian dollar interest payments and receipts in a diagram.
   c. Describe the swap bank's U.S. and Canadian dollar positions.
   d. Explain how the swap bank's position is equivalent to a series of long currency forward contracts. What is the swap bank's implied forward exchange rate on the contracts?
   e. Assume that forward rates are governed by the interest rate parity theorem, that the swap bank can borrow and lend dollars at 9.5% and Canadian dollars at 7%, and that the yield curves for rates in both currencies are flat. Explain how the bank could hedge its swap position using currency forward contracts. What would be the swap bank's profit from its swap and forward positions?

2. The table shows the annual loan rates U.S. and Mexican companies can each obtain on a five-year, $20 million loan in the United States and/or equivalently on a five-year 114.2857 million peso loan in the Mexican market.

Loan Rates for U.S. and Mexican Companies in United States and Mexico

Spot: E_0 = \$/peso = \$0.175/peso		
	U.S. Market	**Mexican Market**
Risk-free rate	8%	6%
U.S. company	11%	8.5%
Mexican company	12%	9.0%

a. Explain the comparative advantages that exist for the U.S. and Mexican companies.

b. Suppose the U.S. company wants to borrow 114.2857 million pesos for five years to finance its Mexican operations, while the Mexican company wants to borrow $20 million for five years to finance its U.S. operations. Explain how a swap bank could arrange a currency swap that would benefit the U.S. company by lowering its peso loan by .25% and would benefit the Mexican company by lowering its dollar loan by .1%. Show the initial cash flow, interest rate, and principal swap arrangements in a diagram.

c. Describe how the swap bank's position is similar to a series of peso forward contracts.

d. What would the bank's dollar position be if it hedged the swap position using the forward market at forward rates determined by the IRPT and at the risk-free rates shown in the table? Assume a flat yield curve. Determine the swap bank's profit from its swap position and forward exchange rate position.

3. Short-Answer Questions:

a. What is the bond equivalent of a currency swap position in which the counterparty agrees to swap a three-year, 10% loan of $14.6 million for a three-year, 7% loan of £10 million?

b. What is the bond equivalent of a currency swap position in which the counterparty agrees to swap a three-year, 7% loan of £10 million for a three-year, 10% loan of $14.6 million?

c. What is the forward exchange rate equivalent of a currency swap position in which the counterparty agrees to swap a three-year, 10% loan of $14.6 million for a three-year, 7% loan of £10 million?

d. What is the forward exchange rate equivalent of a currency swap position in which the counterparty agrees to swap a three-year, 7% loan of £10 million for a three-year, 10% loan of $14.6 million?

e. What is the value of an existing currency swap position in which the counterparty agrees to swap a two-year, 10% loan of $14.6 million for a two-year, 7% loan of £10 million if the current dollar rate is 9%, sterling rate is 7.5%, and spot $/£ exchange rate is $1.45/£?

f. Describe comparative advantage in terms of U.S. and British multinational companies who can each obtain loans in dollars and pounds at the following rates:

	Dollar Market (rate on $)	Pound Market (rate on £)
U.S. company	11%	8.25%
British company	12%	8.5%

4. A U.S. company agrees to exchange a five-year, $10 million, 9% fixed-rate loan to a swap bank for a 6%, five-year, 25 million euro loan, and a German company agrees to exchange a five-year, € 25 million, 6.5% fixed-rate euro loan to a swap bank for a five-year, $10 million, 9.5% loan.
   Questions:
   a. What would the U.S. company exchange each year?
   b. What would the German company exchange each year?
   c. What would the swap bank's position be each year?
   d. How could the swap bank hedge its position?
5. Define a credit default swap and its terms. Explain how the swap works with an example.
6. Suppose a bond fund manager has a portfolio consisting of A- to AAA- quality bonds with an average maturity of seven years and currently yielding 6%. Furthermore, suppose the manager expects the economy to be strong over the next year and would like to improve the yield on her portfolio by swapping 10% of her portfolio for lower-quality B-rated bonds currently trading at a 3% credit spread. Explain how the manager could alternatively use a CDS on a B-rated credit with a spread equal to that on B-rated bonds to achieve similar results.
7. Suppose the vice president of Sun Bank is in the process of structuring a five-year, $100 million loan to the Jetgreen Company, a major airline carrier. Suppose the vice president assesses Jetgreen's credit rating as B quality and believes that a credit spread of 5% is required on the loan. Explain the type of CDS the vice president would need to buy in order to eliminate the credit risk on his pending loan to Jetgreen.
8. Given the following:
   - The yield on a five-year risk-free Treasury-note = 5%
   - The yield on a five-year BB-quality bond = 8%, with the 3% spread reflecting only credit risk
   - The credit spread on a five-year CDS on the five-year BB-quality bond is 2%

   a. Explain how a bond investor looking for a five-year risk-free investment could gain a 1% yield over the risk-free investment by using a CDS.
   b. Explain what an arbitrageur would do.
   c. Comment on the impact the actions by investors and arbitrageurs would have on determining the equilibrium spread on a CDS.
9. Given the following:
   - The yield on a five-year risk-free Treasury-note = 5%
   - The yield on a five-year BB-quality bond = 8%, with the 3% spread reflecting only credit risk

- The credit spread on a five-year CDS on the five-year BB-quality bond is 4%
  - **a.** Explain how a bond investor looking to invest in the five-year BB-rated bond could gain a 1% yield over that investment by using a CDS.
  - **b.** Explain what an arbitrageur would do.
  - **c.** Comment on the impact the actions by investors and arbitrageurs would have on determining the equilibrium spread on a CDS.

10. Explain how a 3% credit spread on five-year, BB-quality bonds can be viewed as the expected loss from the principal resulting from default.

11. Given a discount rate of 5%, determine the present value of the payments on a five-year CDS with a spread of 3% and an NP of $1. If the recovery rate on the underlying credit is 30%, what is the probability intensity implied by the spread?

12. Given an estimated five-year average probability intensity of .0375 on a five-year, BB-rated CDS, a recovery rate of 30%, and discount rate of 5%, determine the value and spread on the CDS. Assume NP = $1.

13. Comment on the two alternative approaches to valuing a CDS.

14. The table below shows the historical cumulative probabilities for corporate bonds with quality ratings of AA and B:

Cumulative Probabilities (%)

Year	1	2	3	4	5
AA	0.10	0.40	1.10	1.50	1.75
B	6.00	13.00	20.00	28.00	36.00

a. Determine the unconditional and conditional default probabilities from the cumulative probabilities shown in the table.

b. Given your probability calculations, determine the values and spreads on a five-year CDS with a B-quality rating. Assume each swap has an NP of $1 and a recovery rate of 30% and that the appropriate discount rate is 6%.

15. The table below shows the current spreads on one- to five-year CDSs, each with a quality rating of B.

Maturity t	Spread
1	0.0490
2	0.0500
3	0.0510
4	0.0520
5	0.0530

a. Determine the implied probability intensities for years 1–5.

b. Given your probability calculations, determine the value and spread on a five-year CDS with a B-quality rating. Assume each swap has an NP of $1 and a recovery rate of 30% and that the appropriate discount rate is 6%.

16. How much would a swap bank pay or require as compensation for assuming the buyer's position on an existing four-year BBB-rated CDS with a spread of 2.5%, if current four-year BBB-rated CDSs are trading at a spread of 2%? Assume the appropriate discount rate is 6%, NP is $1, and the recovery rate is 30%. Explain why the value of the swap position changes.

17. How much would a swap bank pay or require as compensation for assuming the seller's position on an existing four-year BBB-rated CDS with a spread of 2.5%, if current four-year BBB-rated CDSs were trading at a spread of 2%? Assume the appropriate discount rate is 6%, NP is $1, and the recovery rate is 30%. Explain why the value of the swap position changes.

18. Define the following swaps: binary swap, basket CDS, total return swap, and equity swap.

WEB EXERCISES

1. Examine the growth in currency swaps and credit default swaps by looking at the International Swaps and Derivatives Association's market survey. Go to www.isda.org and click on "Survey and Market Statistics."
2. Determine the recent spreads on credit derivative indexes by going to www.wsj.com/free. Click "Bonds, Rates, and Credit Markets" and "Credit Derivatives."

NOTES

1. Currency swaps evolved from back-to-back loans and parallel loans. In a back-to-back loan, companies exchange loans denominated in different currencies; in a parallel loan, one corporation loans to the subsidiary of a foreign multinational and vice versa. For example, a British parent company provides a sterling loan to a British subsidiary of a U.S.-based multinational, while a U.S. parent company provides a dollar loan to a U.S. subsidiary of a British company.
2. The material in this section draws from Johnson, *Introduction to Derivatives*, Chapter 20.
3. The estimated recovery rate, *RR*, is generally treated as a given. For generic CDSs, the value of the CDS is not as sensitive to *RR* as it is to *Z*. Default studies by Moody's found that from 1982 to 2003, recovery rates on corporate bonds as a percentage of face value have ranged from 51.6% for senior secured debt to 24.5% for junior subordinated debt.

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APPENDIX A

Uses of Exponents and Logarithms

A.1 EXPONENTIAL FUNCTIONS

An exponential function is one whose independent variable is an exponent. For example:

$$y = b^t$$

where y = dependent variable
t = independent variable
b = base ($b > 1$)

In calculus, many exponential functions use as their base the irrational number 2.71828, denoted by the symbol e:

$$\mathrm{e} = 2.71828$$

An exponential function that uses e as its base is defined as a natural exponential function. For example:

$$y = e^2$$
$$y = Ae^{Rt}$$

These functions also can be expressed as

$$y = \exp(t)$$
$$y = A\ \exp\ (Rt)$$

In calculus, natural exponential functions have the useful property of being their own derivative. In addition to this mathematical property, e also has a finance meaning. Specifically, e is equal to the future value (FV) of \$1 compounded continuously for one period at a nominal interest rate (R) of 100%.

To see e as a future value, consider the future value of an investment of A dollars invested at an annual nominal rate of R for t years, and compounded m times per year. That is:

$$\text{FV} = A\left(1 + \frac{R}{m}\right)^{mt} \qquad \text{(A.1)}$$

If we let $A = \$1$, $t =$ one year, and $R = 100\%$, then the FV would be

$$\text{FV} = \$1\left(1 + \frac{1}{m}\right)^m \tag{A.2}$$

If the investment is compounded one time ($m = 1$), then the value of the \$1 at end of the year will be \$2; if it is compounded twice ($m = 2$), the end-of-year value will be \$2.25; if it is compounded 100 times ($m = 100$), then the value will be 2.7048138.

As m becomes large, the FV approaches the value of \$2.71828. Thus, in the limit:

$$\text{FV} = \lim_{m\to\infty}\left(1 + \frac{1}{m}\right)^m = 2.71828 \tag{A.3}$$

If A dollars are invested instead of \$1, and the investment is made for t years instead of one year, then given a 100% interest rate, the future value after t years would be

$$\text{FV} = Ae^t \tag{A.4}$$

Finally, if the nominal interest rate is different than 100%, then the FV is

$$\text{FV} = Ae^{Rt} \tag{A.5}$$

To prove Equation (A.5), rewrite Equation (A.1) as follows:

$$\begin{aligned} \text{FV} &= A\left(1 + \tfrac{R}{m}\right)^{mt} \\ \text{FV} &= A\left[\left(1 + \tfrac{R}{m}\right)^{m/R}\right]^{Rt} \end{aligned} \tag{A.6}$$

If we invert *R*/*m* in the inner term, we get

$$\text{FV} = A\left[\left(1 + \frac{1}{m/R}\right)^{m/R}\right]^{Rt} \tag{A.7}$$

The inner term takes the same form as Equation (A.2). As shown earlier, this term, in turn, approaches e as m approaches infinity. Thus, for continuous compounding the FV is

$$\text{FV} = Ae^{Rt}$$

Thus, a two-year investment of \$100 at a 10% annual nominal rate with continuous compounding would be worth \$122.14 at the end of year 2:

$$\text{FV} = \$100^{(.10)(2)} = \$122.14$$

A.2 LOGARITHMS

A logarithm (or log) is the power to which a base must be raised to equal a particular number. For example, given:

$$5^2 = 25$$

the power (or log) to which the base 5 must be raised to equal 25 is 2. Thus, the log of 25 to the base 5 is 2:

$$\log_5 25 = 2$$

In general:

$$y = b^t \Leftrightarrow \log_b y = t$$

Two numbers that are frequently used as the base are 10 and the number e. If 10 is used as the base, the logarithm is known as the common log. Some of the familiar common logs are:

$$\begin{array}{ll}
\log_{10} 1000 = 3 & (10^3 = 1000) \\
\log_{10} 100 = 2 & (10^2 = 100) \\
\log_{10} 10 = 1 & (10^1 = 10) \\
\log_{10} 1 = 0 & (10^0 = 1) \\
\log_{10} 0.1 = -1 & (10^{-1} = \dfrac{1}{10^1} = .10) \\
\log_{10} 0.01 = -2 & (10^{-2} = \dfrac{1}{10^2} = \dfrac{1}{100} = .01)
\end{array}$$

When e is the base, the log is defined as the natural logarithm (denoted $\log_e$ or ln).

For the natural log we have:

$$y = e^t \Leftrightarrow \log_e y = \ln y = t$$
$$\ln e^t = t$$

Thus, given an expression such as $y = e^t$, the exponent t is automatically the natural log.

A.3 RULES OF LOGARITHMS

Like exponents, logarithms have a number of useful algebraic properties. The properties are stated below in terms of natural logs; the properties, though, do apply to any log regardless of its base.

Equality:	If $X = Y$, then $\ln X = \ln Y$
Product rule:	$\ln(XY) = \ln X + \ln Y$
Quotient rule:	$\ln(X/Y) = \ln X - \ln Y$
Power rule:	$\ln(X^a) = a \ln X$

A.4 USES OF LOGARITHMS

The above properties of logarithms make logarithms useful in solving a number of algebraic problems.

Solving for R

In finance, logs can be used to solve for R when there is continuous compounding. That is, from Equation (A.5):

$$\text{FV} = A\text{e}^{Rt}$$

Using the above log properties, R can be found as follows:

$$A\text{e}^{Rt} = \text{FV}$$

$$\text{e}^{Rt} = \frac{\text{FV}}{A}$$

$$\ln\left(\text{e}^{Rt}\right) = \ln\left(\frac{\text{FV}}{A}\right)$$

$$Rt = \ln\left(\frac{\text{FV}}{A}\right)$$

$$R = \frac{\ln(\text{FV}/A)}{t}$$

Thus, a \$100 investment that pays \$120 at the end of two years would yield a nominal annual rate of 9.12% given continuous compounding: $R = \ln(\$120/\$100)/2 = .0912$. Similarly, a pure discount bond selling for \$980 and paying \$1,000 at the end of 91 days would yield a nominal annual rate of 8.10% given continuous compounding:

$$R = \frac{\ln(\$1{,}000/\$980)}{91/365} = 0.810$$

Logarithmic Return

The expression for the rate of return on a security currently priced at S_0 and expected to be S_T at the end of one period ($t = 1$) can be found using Equation (A.5). That is:

$$S_T = S_0 e^{Rt}$$
$$R = \ln\left(\frac{S_T}{S_0}\right)$$

When the rate of return on a security is expressed as the natural log of S_T/S_0, it is referred to as the security's logarithmic return. Thus, a security currently priced at \$100 and expected to be \$110 at the end of the period would have an expected logarithmic return of 9.53%: $R = \ln(\$110/100) = .0953$.

Time

Using logarithms, one can solve for t in either the discrete or continuous compounding cases. That is:

$$\text{FV} = A(1+R)^t$$
$$A(1+R)^t = \text{FV}$$
$$\ln\left[(1+R)^t\right] = \ln\left(\frac{\text{FV}}{A}\right)$$
$$t\ln[1+R] = \ln\left(\frac{\text{FV}}{A}\right)$$
$$t = \frac{\ln(\text{FV}/A)}{\ln(1+R)}$$

$$Ae^{Rt} = \text{FV}$$
$$e^{Rt} = \frac{\text{FV}}{A}$$
$$\ln\left(e^{Rt}\right) = \ln\left(\frac{\text{FV}}{A}\right)$$
$$Rt = \ln\left(\frac{\text{FV}}{A}\right)$$
$$t = \frac{\ln(\text{FV}/A)}{R}$$

The equations can be used in problems in which one knows the interest or growth rate and wants to know how long it will take for an investment to grow to equal a certain terminal value. For example, given an annual interest rate of 10%

(no annual compounding) an investment of \$800 would take 2.34 years to grow to \$1,000:

$$t = \frac{\ln(\$1{,}000/\$800)}{\ln(1.10)} = 2.34 \text{ years}$$

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APPENDIX B

Mathematical Statistical Concepts

In this appendix we define some of the important statistical concepts used in analyzing investment strategies.

Random Variable: A random variable is a variable whose value is uncertain. Signified with a ∼ (tilde) over the symbol of the variable, a random variable is sometimes referred to as a stochastic variable. The opposite of a random variable is a deterministic or controlled variable, referred to as a non-stochastic variable.

Probability Distribution: A probability distribution is a function that assigns probabilities to the possible values of a random variable. The function can be objective (such as using past frequencies or assuming the distribution takes a certain form) or subjective. Also, the distribution can be either continuous, where it takes on all possible values over the range of the distribution with the probabilities being defined for a particular range, or discrete, where the distribution takes on only a few possible values with probabilities assigned to each possible value. In Table B.1 a probability distribution is shown for next period's interest rates (random variable r). This discrete distribution is defined by five possible interest rate values (column 1) and their respective probabilities (column 2) and is shown graphically in Figure B.1.

The most common way to describe the probability distribution is in terms of its parameters: expected value or mean, variance, and skewness.

Expected Value: The expected value of a random variable is the weighted average of the possible values of the random variable with the weights being the probabilities assigned to each possible value (P_i). The expected value or mean, along with the median and the mode, is a measure of the central tendency of the distribution. The expected value for random variable $\tilde{r}$ is

$$\mathrm{E}(\tilde{r}) = \sum_{i=1}^{T} P_i r_i = P_1 r_1 + P_2 r_2 + \cdots + P_T r_T$$

$$\mathrm{E}(\tilde{r}) = (.1)(4\%) + (.2)(5\%) + (.4)(6\%) + (.2)(7\%) + (.1)(8\%) = 6\%$$

A random variable may be described in terms of an algebraic equation, for example:

$$\tilde{r} = \mathrm{a} + \mathrm{b}\tilde{Y}$$

TABLE B.1 Probability Distribution

(1) r_i	(2) P_i	(3) $P_i\,r_i$	(4) $[r_i - E(r)]$	(5) $[r_i - E(r)]^2$	(6) $P_i[r_i - E(r)]^2$	(7) $[r_i - E(r)]^3$	(8) $P_i[r_i - E(r)]^3$
4%	0.1	0.4	−2	4	0.4	−8	−0.8
5%	0.2	1.0	−1	1	0.2	−1	−0.2
6%	0.4	2.4	0	0	0.0	0	0.0
7%	0.2	1.4	1	1	0.2	1	0.2
8%	0.1	0.8	2	4	0.4	8	0.8
	1	$E(r) = 6\%$			$V(r) = 1.2$		$S_k(r) = 0$

where a and b are coefficients and $\tilde{Y}$ is the independent variable. To describe the expected value of $\tilde{r}$ as $E(a + b\tilde{Y})$, one can make use of the following expected value operator rules:

1. Expected value of a constant (a) is equal to the constant:

$$\text{EV Rule1} : E(a) = a.$$

2. Expected value of a constant times a random variable is equal to the constant times the expected value of the random variable:

$$\text{EV Rule 2} : E(b\tilde{X}) = bE(\tilde{X})$$

3. Expected value of a sum is equal to the sum of the expected values:

$$\text{EV Rule 3} : E(\tilde{X} + \tilde{Y}) = E(\tilde{X}) + E(\tilde{Y})$$

Applying the three rules to the equation $\tilde{r} = a + b\tilde{Y}$, $E(\tilde{r})$ can be expressed as

$$E(\tilde{r}) = a + bE(\tilde{Y})$$

Variance: The variance of a random variable ($V(\tilde{r})$) is the expected value of the squared deviation from the mean:

$$V(\tilde{r}) = E[\tilde{r} - E(\tilde{r})]^2$$

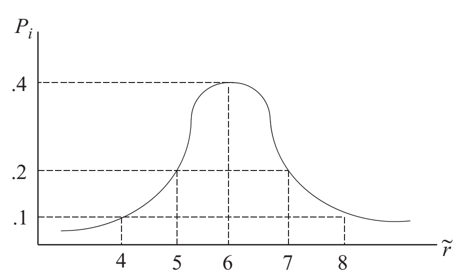

FIGURE B.1 Probability Distribution

The variance is defined as the second moment of the distribution. It is a measure of the distribution's dispersion, measuring the squared deviation most likely to occur. As an expected value, the variance is obtained by calculating the weighted average of each squared deviation, with the weights being the relative probabilities:

$$V(\tilde{r}) = \sum_{i=1}^{T} p_i[\tilde{r}_i - E(\tilde{r})]^2$$

$$V(\tilde{r}) = p_1[\tilde{r}_1 - E(\tilde{r})]^2 + p_2[\tilde{r}^2 - E(\tilde{r})]^2 + \cdots + p_T[\tilde{r}_T - E(\tilde{r})]^2$$

The random variable described in Table B.1 has a variance of 1.2.

$$V(\tilde{r}) = (.1)[4\% - 6\%]^2 + (.2)[5\% - 6\%]^2 + (.4)[6\% - 6\%]^2 + (.2)[7\% - 6\%]^2 + (.1)[8\% - 6\%]^2 = 1.2$$

Standard Deviation: The standard deviation, $\sigma(\tilde{r})$, is the square root of the variance:

$$\sigma(\tilde{r}) = \sqrt{V(\tilde{r})}$$

The standard deviation provides a measure of dispersion that is on the same scale as the distribution's deviations. The standard deviation of the random variable in Table B.1 is 1.0954451; this indicates the distribution has an average deviation of plus or minus 1.0954451.

Note, the risk of a security is defined as the possibility that the actual return earned from investing in a security will deviate from the expected. By definition, the variance and standard deviation of a security's rate of return define the security's relative risk. That is, the greater a security's variance relative to another security, the greater that security's actual return can deviate from its expected return and thus the greater the security's risk relative to the other security.

Skewness: Skewness measures the degree of symmetry of the distribution. A distribution that is symmetric about its mean is one in which the probability of $\tilde{r} = E(\tilde{r}) + x$ is equal to the probability of $\tilde{r} = E(\tilde{r}) - x$, for all values of x. Skewness, $S_k(\tilde{r})$, is defined as the third moment of the distribution and can be measured by calculating the expected value of the cubic deviation:

$$S_k(\tilde{r}) = \sum_{i=1}^{T} p_i[\tilde{r}_i - E(\tilde{r})]^3$$

$$S_k(\tilde{r}) = p_1[\tilde{r}_1 - E(\tilde{r})]^3 + p_2[\tilde{r}_i - E(\tilde{r})]^3 + \cdots + p_T[\tilde{r}_T - E(\tilde{r})]^3$$

The skewness of the distribution in Table B.1 is zero.

Covariance: The covariance is a measure of the extent to which one random variable is above or below its mean at the same time or state that another

TABLE B.2 Correlation between Random Variables

State	P_i	$\tilde{r}_{1i}$	$\tilde{r}_{2i}$	$P_i\tilde{r}_{1i}$	$P_i\tilde{r}_{2i}$	$P_i[\tilde{r}_{1i} - E(\tilde{r}_1)]^2$	$P_i[\tilde{r}_{2i} - E(\tilde{r}_2)]^2$	$P_i\,[\tilde{r}_{1i} - E(\tilde{r}_1)]\,[\tilde{r}_{2i} - E(\tilde{r}_2)]$
A	1/8	6%	24%	0.75	3.0	(1/8)(144	(1/8)(64)	$(1/8)(-12)(8) = -12$
B	6/8	18%	16%	13.5	12.0	(6/8)(0)	(6/8)(0)	$(6/8)(0)(0) = 0$
C	1/8	30%	8%	3.75	1.0	(1/8)(144)	(1/8)(64)	$(1/8)(12)(-8) = -12$
				$E(\tilde{r}_1) = 18$	$E(\tilde{r}_2) = 16$	$V(\tilde{r}_1) = 36$ $\sigma(\tilde{r}_1) = 6$	$V(\tilde{r}_2) = 16$ $\sigma(\tilde{r}_2) = 4$	$Cov(\tilde{r}_1\tilde{r}_2) = -24$ $\rho_{12} = -1$

random variable is above or below its mean. The covariance measures how two random variables move with each other. If two random variables, on average, are above their means at the same time and, on average, are below at the same time, then the random variables would be positively correlated with each other and would have a positive covariance. In contrast, if one random variable, on average, is above its mean when another is below and vice versa, then the random variables would move inversely or negatively to each other and would have a negative covariance.

The covariance between two random variables, $\tilde{r}_1$ and $\tilde{r}_2$, is equal to the expected value of the product of the variables' deviations:

$$Cov(\tilde{r}_1\tilde{r}_2) = E[\tilde{r}_1 - E(\tilde{r}_1)][\tilde{r}_2 - E(\tilde{r}_2)]$$

$$Cov(\tilde{r}_1\tilde{r}_2) = \sum_{i=1}^{T} p_i[\tilde{r}_{1i} - E(\tilde{r}_1)][\tilde{r}_{2i} - E(\tilde{r}_2)]$$

In Table B.2, the possible rates of return for securities 1 and 2 are shown for three possible states (A, B, and C) along with the probabilities of occurrence of each state. As shown in the table, $E(\tilde{r}_1) = 18\%$, $V(\tilde{r}_1) = 36$, $E(\tilde{r}_2) = 16\%$, and $V(\tilde{r}_2) = 16$. In addition, the table also shows that in State A security 1 yields a return below its mean while security 2 yields a return above its mean; in State B both yield rates of return equal their mean; in State C security 1 yields a return above its mean while security 2 yields a return below. Securities 1 and 2 therefore are negatively correlated and, as shown in Table B.2, have a negative covariance of –24.

Correlation Coefficient: The correlation coefficient between two random variables $\tilde{r}_1$ and $\tilde{r}_1$ (ρ_{12}) is equal to the covariance between the variables divided by the product of each random variable's standard deviation:

$$\rho_{12} = \frac{Cov(\tilde{r}_1\tilde{r}_2)}{\sigma(\tilde{r}_1)\sigma(\tilde{r}_2)}$$

The correlation coefficient has the mathematical property that its value must be within the range of minus and plus one:

$$-1 \leq \rho_{12} \leq 1$$

If two random variables have a correlation coefficient equal to one, they are said to be perfectly positively correlated; if their coefficient is equal to minus one, they are said to be perfectly negatively correlated; if their correlation coefficient is equal to zero, they are said to be zero correlated and statistically independent. That is:

$$\text{If } \rho_{12} = -1 \Rightarrow \text{Perfect negative correlation}$$
$$\text{If } \rho_{12} = 0 \Rightarrow \text{Uncorrelated}$$
$$\text{If } \rho_{12} = 1 \Rightarrow \text{Perfect positive correlation}$$

Parameter Estimates Using Historical Averages: In most cases we do not know the probabilities associated with the possible values of the random variable and must therefore estimate the parameter characteristics. The simplest way to estimate is to calculate the parameter's historical average value from a sample. For the rate of return on a security, this can be done by calculating the average rate of return per period or the holding period yield, HPY_t (stock $\text{HPY} = [(P_t - P_{t-1}) + \text{dividend}]/P_{t-1}$) over n historical periods:

$$\bar{r} = \frac{1}{n}\sum_{t=1}^{n} \text{HPY}_t$$

Similarly, the variance of a security can be estimated by averaging the security's squared deviations, and the covariance between two securities can be estimated by averaging the product of the securities' deviations. Note, in estimating variances and covariances, averages usually are found by dividing by $n - 1$ instead of n in order to obtain better unbiased estimates:

$$\hat{V}(r) = \frac{1}{n-1}\sum_{t=1}^{n} (\text{HPY}_t - \bar{r})^2$$

$$\hat{\text{Cov}}(r_1 r_2) = \frac{1}{n-1}\sum_{t=1}^{n} (\text{HPY}_{1t} - \bar{r}_1)(\text{HPY}_{2t} - \bar{r}_2)$$

An example of estimating parameters is shown in Table B.3 in which the average HPY, variances, and covariance are computed for a stock and a stock index (S_m).

Linear Regression: Regression involves estimating the coefficients of an assumed algebraic equation. A linear regression model has only one explanatory variable; a multiple regression model has more than one independent variable. As an example, consider a linear regression model relating the rate of return on a security (dependent variable) to the market rate of return (R_m) (independent variable), where R_m is measured by the proportional change in a stock index. That is:

$$\tilde{r}_j = \alpha + \beta \tilde{R}_{mj} + \varepsilon_j$$

TABLE B.3 Historical Averages and Regression Estimates

Time	S	Dividend	HPY	(HPY − Av.)	$(\text{HPY} - \text{Av.})^2$
1	100	0			
2	105	0	0.050000	0.032324	0.0010448
3	110	1	0.057143	0.039467	0.0015576
4	115	0	0.045455	0.027779	0.0007716
5	110	1	−0.034783	−0.052459	0.0027519
6	105	0	−0.045455	−0.063131	0.0039855
7	100	1	−0.038095	−0.055771	0.0031104
8	105	0	0.050000	0.032324	0.0010448
9	110	1	0.057143	0.039467	0.0015576
			0.141408		0.0158244
			Av.= .017676		Var = .0022606 Stan. Dev = .0475457

Time	S_m	$R_m = HPY$	$(R_m - \text{Av.})$	$(R_m - \text{Av.})^2$	$(R_m - \text{Av.})(\text{HPY} - \text{Av.})$
1	300				
2	315	0.050000	0.035583	0.001266	0.0011502
3	333	0.057143	0.042726	0.001825	0.0016863
4	346	0.039039	0.024622	0.000606	0.0006840
5	334	−0.034682	−0.049099	0.002411	0.0025757
6	319	−0.044910	−0.059327	0.003520	0.0037454
7	306	−0.040752	−0.055169	0.003044	0.0030769
8	320	0.045752	0.031335	0.000982	0.0010129
9	334	0.043750	0.029333	0.000860	0.0011577
		0.115339		0.014514	0.0150888
		Av. = 0.014417		Var =.0020735	Cov = .002156

$$\hat{\alpha} = \bar{r} - \hat{\beta}\bar{R}_m$$
$$= .017676 - 1.04(.014417) = .00268$$
$$\hat{\beta} = \frac{\hat{Cov}(r\ R_m)}{\hat{V}(R_m)} = \frac{.002156}{.002073} = 1.04$$
$$V(\varepsilon) = V(R) - \hat{\beta}^2\, V(R_m)$$
$$= .0022606 - (1.04)^2 .002073 = .0000184$$

$$E(r) = \alpha + \beta E(R_m)$$
$$= .00268 + 1.04\ E(R_m)$$
$$V(r) = \beta^2 V(R_m) + V(\varepsilon)$$
$$= (1.04)^2 V(R_m) + .0000184$$

where α = intercept
β = slope = $\Delta r/\Delta R_m$
j = observation
ε = error

In the above equation, ε_j is referred to as the error term or stochastic disturbance term. Thus, the model assumes that for each observation j, errors in the relationship between r and R_m can exist, causing r to deviate from the algebraic relation defined by α and β. Since, a priori, the errors are not known, the regression model needs to provide assumptions concerning ε. The standard assumptions are

$$E(\varepsilon_j) = 0$$

$$V(\varepsilon_j) \text{ does not change}$$

$$Cov(\varepsilon, R_m) = 0$$

Using the above assumptions and the expected value operator rules, the expected value and variance can be defined in terms of the regression model as follows:

$$E(r) = E[\alpha + \beta\, R_m + \varepsilon]$$

$$E(r) = \alpha + \beta\, E(R_m) + E(\varepsilon)$$

$$E(r) = \alpha + \beta\, E(R_m)$$

$$V(r) = E[r - E(r)]^2$$

$$V(r) = \beta^2 V(R_m) + V(\varepsilon)$$

The first term on the right for the equation V(r) defines systematic risk: the amount of variation in r that can be attributed to the market (factors that affect all securities); the second term defines unsystematic risk: the amount of variation in r that can be attributed to factors unique to that security (industry and firm factors).

If two securities (1 and 2) both are related to R_m such that

$$r_1 = \alpha_1 + \beta_1\, R_m + \varepsilon_1$$

$$r_2 = \alpha_2 + \beta_2\, R_m + \varepsilon_2$$

(the j subscript is deleted) and ε_1 and ε_2 are independent ($Cov(\varepsilon_{1,}\ \varepsilon_2) = 0$), then the $Cov(r_1\ r_2)$ simplifies to

$$Cov(r_1 r_2) = \beta_1 \beta_2 V(R_m)$$

The intercept and slope of the regression model can be estimated by the ordinary least squares estimation procedure. This technique uses sample data for the dependent and independent variables (time series data or cross-sectional data) to find the estimates of α and β that minimizes the sum of the squared errors. The estimates for α and β in which the errors are minimized are

$$\hat{\alpha} = \bar{r} - \hat{\beta}\,\bar{R}_m$$

$$\hat{\beta} = \frac{\hat{\mathrm{Cov}}(r\ R_m)}{\hat{\mathrm{V}}(R_m)}$$

where $\hat{\mathrm{Cov}}(r, R_m)$, $\hat{\mathrm{V}}(R_m)$, $\bar{r}$, *and* $\bar{R}_m$ are estimates (averages)

An estimate of unsystematic risk, $\mathrm{V}(\varepsilon)$, can be found using the equation for $\mathrm{V}(r)$. That is:

$$\hat{\mathrm{V}}(\varepsilon) = \hat{\mathrm{V}}(r) - \hat{\beta}^2 \mathrm{V}(\hat{R}_m)$$

where $\mathrm{V}(r)$ and $\mathrm{V}(R_m)$ can be estimated using the sample averages and β can be estimated using the ordinary least squares estimating equation. In Table B.3, a regression model relating the rate of return on the security to the market rate as measured by the rate of change in the index is shown.

It should be noted that the coefficients between any variables can be estimated using a regression model. Whether the relationship is good or not depends on the quality of the regression model. All regression models, therefore, need to be accompanied by information about the quality of the regression results. Regression qualifiers include the coefficient of determination (R^2), *t*-tests, and *F*-tests.

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by R. Stafford Johnson

APPENDIX C

Primer on Return, Present Value, and Future Value

C.1 RATE OF RETURN

Holding Period Yield

The rate of return an investor earns from holding a security is equal to the total dollar return received from the security per period of time (e.g., year) expressed as a proportion of the price paid for the security. The total dollar return includes income payments (coupon interest or dividends), interest earned from reinvesting the income during the period, and capital gains or losses realized when the security is sold or matures. For example, an investor who purchased XYZ stock for S_0 = \$100, then received \$10 in dividends (D) two years later when he sold the stock for S_T = \$110, would realize a rate of return for this two-year period of 20%:

$$\text{Rate of return} = \frac{D + S_T - S_0}{S_0} = \frac{\$10 + \$110 - \$100}{\$100} = 0.20$$

Similarly, a bond investor who bought a Treasury bond on April 20 for P^B_0 = \$95,000 and then sold it for P^B_T = \$96,000 on October 20 just after receiving a coupon of C = \$4,000, would earn a rate of return for this six-month period of 5.263%:

$$\text{Rate of return} = \frac{C + P^B_T - P^B_0}{P^B_0} = \frac{\$4,000 + \$96,000 - \$95,000}{\$95,000} = 0.05263$$

Both the bond and stock rates of return are measured as ***holding period yields (HPY)***. The HPY is the rate earned from holding the security for one period (e.g., one year or six months). The HPY can alternatively be expressed in terms of the security's ***holding period return*** (***HPR***) minus one, where the HPR is the ratio of the ending-period value (e.g, $D + S_T$) to the beginning period value (S_0). That is:

$$\text{HPY} = \frac{\text{Ending value}}{\text{Beginning value}} - 1$$

$$\text{HPY} = \text{HPR} - 1$$

$$\text{HPY} = \frac{D + S_T}{S_0} - 1; \ \text{HPY} = \frac{C + P^B_T}{P^B_0} - 1$$

Annualized HPY

In order to evaluate alternative investments with different holding periods, investment analysts often annualize the rate of return. The simplest way to annualize a return is to multiply the periodic rate of return by the number of periods of that length in a year. Thus, to annualize the HPY on the bond investment, we would multiply the six-month HPY of .05263 by 2 to obtain .10526; to annualize the stock's rate of return, we would multiply the two-year HPY of 0.20 by 1/2 to get .10. This method for annualizing rates of return, though, does not take into account the interest that could be earned from reinvesting the cash flows. That is, a \$1 investment in the bond would yield \$1.0526 after six months, which could be reinvested. If it is reinvested for six months at the same six-month rate of 5.26%, then the dollar investment would be worth \$1.108 after one year. Thus, the ***effective annual rate*** (i.e., the rate that takes into account the reinvestment of interest or the compounding of interest) is 10.8%. The ***effective annualized HPY***, HPY^A, can be calculated using the following formula:

$$\text{HPY}^A = \text{HPR}^{1/M} - 1$$

where M = Number of years the investment is held.

Thus, the effective annualized HPY for the bond investment would be 10.8%, and the effective HPY^A for the stock investment would be 9.54%:

$$\text{HPY}^A = \text{HPR}^{1/M} - 1$$

$$\text{HPY}^A = \left[\frac{\$4{,}000 + \$96{,}000}{\$95{,}000}\right]^{1/0.5} - 1 = .108$$

$$\text{HPY}^A = \left[\frac{\$10 + \$110}{\$100}\right]^{1/2} - 1 = .0954$$

The two-year HPY on the stock of 20% reflects annual compounding. That is, \$1 after one year would be worth \$1.0954, which reinvested for the next year would equal \$1.20:

$$\$1.00(1.0954)(1.0954) = (1.0954)^2 = \$1.20$$

Required Rates of Return and Value for a Single-Period Cash Flow

The price of a security and its rate of return are related. When an investor knows the price of a security and its cash flow, she can determine the rate of return. Alternatively, when the investor knows her ***required rate of return*** and the security's cash flow, she can determine the value of the security or the price she is willing to pay. For example, an investor who requires an annual 10% rate of return in order to invest in a one-year, AAA bond paying a single cash flow of coupon interest of

$10 and a principal of $100 at maturity would value the bond at $100. This price can be found by expressing the equation for the HPY in terms of its price. That is:

$$\text{Rate of return} = R = \frac{C + P_T^B}{P_0^B} - 1$$

$$P_0^B(1 + R) = C + P_T^B$$

$$P_0^B = \frac{C + P_0^B}{(1 + R)}$$

$$P_0^B = \frac{\$10 + \$100}{1.10} = \$100$$

If the bond were priced in the market below $100, the investor would consider it underpriced, yielding a rate of return that exceeds her required rate of 10%; if the bond were priced above $100, she would consider the bond to be overpriced, yielding a rate of return less than 10%.

C.2 FUTURE AND PRESENT VALUES

Future Value

The above value and return relations can be described in terms of the present values and future values of investments and future receipts. More formally, the future value of any amount invested today is

$$P_N = P_0(1 + R)^N$$

where

- N = Number of periods of the investment
- P_N = Future value of investment N periods from present (future value, FV)
- P_0 = Initial investment value (present value, PV)
- R = Rate per period (periodic rate)
- $(1 + R)^N$ = Future value of $1 invested today for N periods at a compound rate of R

In terms of the preceding bond example, the future value (FV) of the bond investment of $100 at 10% is $110 (coupon and principal):

$$P_1 = P_0(1 + R)^1$$

$$P_1 = \$100(1.10) = \$110$$

An investment fund that invested $1,000,000 in a security that paid 10% per year for three years would, in turn, have $1,331,000 at the end of three years:

$$P_N = P_0(1 + R)^N$$

$$P_3 = \$1{,}000{,}000(1.10)^3 = \$1{,}331{,}000$$

If the interest is paid more than once a year, then the rate of return and the number of periods must be adjusted. Specifically, let

- n = Number of times interest is paid per year
- M = Number of years of the investment
- Period rate $= R = \dfrac{\text{Annual rate}}{n}$
- N = Number of periods of the investment $= (n)\ (M)$

If an investment fund invested \$1,000,000 in a three-year security that paid annual interest at 10% for three years with the interest paid semiannually, then the investment would be worth \$1,340,095.64 after three years:

- $n = 2$
- $M = 3$ years
- Period rate $= R = \dfrac{\text{Annual rate}}{n} = \dfrac{.10}{2} = .05$
- N = Number of periods of the investment $= (n)\ (M) = (2)(3) = 6$

$$P_N = P_0(1 + R)^N$$
$$P_6 = \$1{,}000{,}000(1.05)^6 = \$1{,}340{,}095.64$$

Note that with semiannual interest payments, there are more opportunities for reinvesting the interest received. As a result, the future value of the investment is greater with interest paid semiannually than annually.

Future Value of an Annuity

An annuity is a periodic investment or receipt. For example, an investment of \$1,000,000 each year for three years would be an example of an investment annuity, and a security paying \$50 every six months for 10 years would be an example of a receipt annuity. The future value of an annuity (A) is equal to the sum of the future values of each investment at the investment horizon:

$$P_N = A(1 + R)^{N-1} + A(1 + R)^{N-2} + A(1 + R)^{N-3} + \cdots + A(1 + R)^{N-N}$$
$$P_N = \sum_{t=1}^{N} A(1 + R)^{N-t}$$

As an example, suppose an investment fund owns \$50,000,000 of bonds maturing in three years that promise to pay 10% per year and \$50,000,000 at the end of three years. If the fund reinvested the annual interest of \$5,000,000 at a rate of 10%, then at the end of three years the sum of the annual interest payments would be worth \$16,550,000:

Year	0	1	2	3	3	Values
A		\$5,000,000.00			\$5,000,000$(1.10)^2$	\$6,050,000.00
A			\$5,000,000.00		\$5,000,000$(1.10)^1$	\$5,500,000.00
A				\$5,000,000.00	\$5,000,000$(1.10)^0$	\$5,000,000.00
					Horizon value = P_N	\$16,550,000.00

$$P_N = \sum_{t=1}^{N} A(1+R)^{N-t}$$

$$P_3 = \sum_{t=1}^{3} \$5{,}000{,}000(1+R)^{3-t}$$

$$P_N = \$5{,}000{,}000(1.10)^{3-1} + \$5{,}000{,}000(1.10)^{3-2} + \$5{,}000{,}000(1.10)^{3-3}$$

$$P_N = \$16{,}550{,}000$$

At the end of three years, the fund would have \$500,000,000 in principal, \$15,000,000 in interest, and \$1,550,000 (= \$16,550,000 – \$15,000,000) in interest earned from reinvesting the interest.

The equation for the future value of an annuity is equal to the annuity times the future value of \$1 invested each period for N periods:

$$P_N = \sum_{t=1}^{N} A(1+R)^{N-t}$$

$$P_N = A\sum_{t=1}^{N} (1+R)^{N-t}$$

The future value of \$1 invested each period for N periods is defined as the future value interest factor of an annuity, FVIF_a. The formula for determining FVIF_a is

$$\text{FVIF}_a = \sum_{t=1}^{N} (1+R)^{N-t} = \left[\frac{(1+R)^N - 1}{R}\right]$$

Substituting the formula for the FVIF_a into the equation for P_N, the future value of annuity can alternatively be expressed as:

$$P_N = A\sum_{t=1}^{N} (1+R)^{N-t}$$

$$P_N = A\left[\frac{(1+R)^N - 1}{R}\right]$$

In terms of our example:

$$P_3 = \$5{,}000{,}000\sum_{t=1}^{3} (1.10)^{3-t}$$

$$P_3 = \$5{,}000{,}000\left[\frac{(1.10)^3 - 1}{.10}\right]$$

$$P_3 = \$16{,}550{,}000$$

Note: If the bond investment fund received interest semiannually, then the fund would receive $2,500,000 every six months. If the semiannual reinvestment rate were 5%, then the sum of the future values of the interest payments would be $17,004,782:

Year	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3	Values
N		1	2	3	4	5	6	6	
A		$2,500,000						$2,500,000$(1.05)^5$	$3,190,704
A			$2,500,000					$2,500,000$(1.05)^4$	$3,038,766
A				$2,500,000				$2,500,000$(1.05)^3$	$2,894,063
A					$2,500,000			$2,500,000$(1.05)^2$	$2,756,250
A						$2,500,000		$2,500,000$(1.05)^1$	$2,625,000
A							$2,500,000	$2,500,000$(1.05)^0$	$2,500,000
								Horizon Value = P_N	$17,004,782

$$P_N = A \sum_{t=1}^{N} (1 + R)^{N-t}$$

$$P_6 = \$2{,}500{,}000 \sum_{t=1}^{6} (1.05)^{6-t}$$

$$P_6 = \$2{,}500{,}000 \left[\frac{(1.05)^6 - 1}{.05} \right]$$

$$P_6 = \$17{,}004{,}782$$

Present Value

The present value is the amount that must be invested today to realize a specific future value. The present value of one future receipt is:

$$P_0 = \frac{P_N}{(1 + R)^N}$$

Thus, $1,331,000 received three years from now would be worth $1,000,000 given a rate of return of 10% and annual compounding:

$$P_0 = \frac{P_N}{(1 + R)^N}$$

$$P_0 = \frac{\$1{,}331{,}000}{(1.10)^3} = \$1{,}000{,}000$$

$1,340,095.64 received three years from now ($M = 3$) would be worth $1,000,000 given a 5% semiannual rate and two compoundings per year ($n = 2$ and $N = nM = (2)(3) = 6$):

$$P_0 = \frac{P_N}{(1 + R)^N}$$

$$P_0 = \frac{\$1{,}340{,}095.64}{(1.05)^6} = \$1{,}000{,}000$$

The method of computing the present value is referred to as ***discounting***, and the interest rate used to discount is referred to as the ***discount rate***.

Present Value of an Annuity When a fixed-dollar annuity is received each period, the series is also called an annuity. If the first payment is received one period from the present, the annuity is referred to as an ***ordinary annuity***; if the first payment is immediate, then the annuity is called an ***annuity due***. The present value of an ordinary annuity is the sum of the present values of each annuity received:

$$P_0 = \frac{A}{(1+R)^1} + \frac{A}{(1+R)^2} + \frac{A}{(1+R)^3} + \cdots + \frac{A}{(1+R)^N}$$

$$P_0 = \sum_{t=1}^{N} \frac{A}{(1+R)^t}$$

$$P_0 = A\sum_{t=1}^{N} \frac{1}{(1+R)^t}$$

$\sum_{t=1}^{N} \frac{1}{(1+R)^t}$ is the present value of $1 received each period for N periods. It is referred to as the present value interest factor of an annuity, $PVIF_a$. $PVIF_a$ is equal to

$$PVIF_a = \sum_{t=1}^{N} \frac{1}{(1+R)^t} = \left[\frac{1-(1/(1+R)^N}{R}\right]$$

Thus, an investor who received $100 at the end of each year for three years would have an investment currently worth $248.69 given a discount rate of 10%:

Year	0	0	1	2	3
A	$90.91	$100/(1.10)^1	$100.00		
A	$82.64	$100/(1.10)^2		$100.00	
A	$75.13	$100/(1.10)^3			$100.00
	$248.69	Present value = P_0			

$$P_0 = \sum_{t=1}^{3} \frac{\$100}{(1.10)^t} = \frac{\$100}{(1.10)^1} + \frac{\$100}{(1.10)^2} + \frac{\$100}{(1.10)^3}$$

$$P_0 = \$100\sum_{t=1}^{3} \frac{1}{(1.10)^t}$$

$$P_0 = \$100\left[\frac{1-(1/(1.10)^3}{.10}\right]$$

$$P_0 = \$248.69$$

If the investment paid \$50 every six months and the appropriate six-month rate were 5%, then the present value of the \$50 annuity would be \$253.78:

Year	0	0	0.5	1.0	1.5	2.0	2.5	3.0
Number of Semiannual Periods from Present	0	0	1	2	3	4	5	6
A	\$47.62	$\$50/(1.05)^1$	\$50					
A	\$45.35	$\$50/(1.05)^2$		\$50				
A	\$43.19	$\$50/(1.05)^3$			\$50			
A	\$41.14	$\$50/(1.05)^4$				\$50		
A	\$39.18	$\$50/(1.05)^5$					\$50	
A	\$37.31	$\$50/(1.05)^6$						\$50
	\$253.78	Present value = P_0						

$$P_0 = \$50 \sum_{t=1}^{6} \frac{1}{(1.05)^t}$$

$$P_0 = \$50 \left[\frac{1 - (1/(1.05)^6}{.10} \right]$$

$$P_0 = \$253.78$$

Valuing a Bond

The price of any security is equal to the present value of its expected cash flows. For many bonds, the cash flow consists of fixed coupon payments paid over a specified number of periods and a principal payment at maturity. The fixed coupon payments are an annuity and their value can be found by computing the present value of annuity. The value of the principal payment, in turn, can be found by simply computing the present value of the principal.

For example, suppose an investor planned to buy a three-year bond, paying a coupon (C) of \$100 at the end of each year plus a \$1,000 principal (F) at end of year 3. If she required an annual rate of return of 10%, then using the present value approach she would value the bond at \$1,000:

$$P_0^B = \sum_{t=1}^{N} \frac{C}{(1+R)^t} + \frac{F}{(1+R)^N}$$

$$P_0^B = \sum_{t=1}^{3} \frac{\$100}{(1.10)^t} + \frac{\$1{,}000}{(1.10)^3}$$

$$P_0^B = \frac{\$100}{(1.10)} + \frac{\$100}{(1.10)^2} + \frac{\$100}{(1.10)^3} + \frac{\$1{,}000}{(1.10)^3} = \$1{,}000$$

or

$$P_0^B = C \sum_{t=1}^{N} \frac{1}{(1+R)^t} + \frac{F}{(1+R)^N}$$

$$P_0^B = C\left[\frac{1 - 1/(1+R)^N}{R}\right] + \frac{F}{(1+R)^N}$$

$$P_0^B = \$100\left[\frac{1 - 1/(1.10)^3}{.10}\right] + \frac{\$1{,}000}{(1.10)^3}$$

$$P_0^B = \$248.69 + 751.31 = \$1{,}000$$

If the investor required a higher rate of return on the bond of 12%, then she would value the bond at \$951.96, less than \$1,000:

$$P_0^B = \sum_{t=1}^{N} \frac{C}{(1+R)^t} + \frac{F}{(1+R)^N}$$

$$P_0^B = \sum_{t=1}^{3} \frac{\$100}{(1.12)^t} + \frac{\$1{,}000}{(1.12)^3}$$

$$P_0^B = \$100 \sum_{t=1}^{3} \frac{1}{(1.12)^t} + \frac{\$1{,}000}{(1.12)^3}$$

$$P_0^B = \$100\left[\frac{1 - 1/(1.12)^3}{.12}\right] + \frac{\$1{,}000}{(1.12)^3}$$

$$P_0^B = \$240.18 + \$711.78 = \$951.96$$

If we reverse the case and assume that our investor actually paid \$951.96 for the three-year bond, then her annualized rate of return from the investment would be the discount rate of 12%—the rate that equates the present value of the security's cash flows to the current price of the security:

$$\$951.96 = \frac{\$100}{(1+R)^1} + \frac{\$100}{(1+R)^2} + \frac{\$100 + \$1{,}000}{(1+R)^3} \Rightarrow R = .12$$

As we discuss in Chapter 2, for multiple-period investments, the discount rate is the most acceptable way of determining the rate of return or yield on an investment. It is analogous to the internal rate of return used in capital budgeting. As a rate of return measure, it includes the return from the income (coupons or dividends), the interest earned from reinvesting the income, and any capital gains or losses. Unfortunately, unless the cash flows are equal, solving for R requires an iterative (trial and error) procedure: substituting different R values until an R is found that equates the present value of the cash flows to the security's price.

Finally, note that if the three-year bond paying \$100 annual coupons paid the coupons semiannually and the investor required a semiannual rate of return of 5%, then the value of the bond would be \$1,000:

$$P_0^B = \sum_{t=1}^{N} \frac{C}{(1+R)^t} + \frac{F}{(1+R)^N}$$

$$P_0^B = \sum_{t=1}^{6} \frac{\$50}{(1.05)^t} + \frac{\$1{,}000}{(1.05)^6}$$

$$P_0^B = \$50\left[\frac{1 - 1/(1.05)^6}{.05}\right] + \frac{\$1{,}000}{(1.05)^6}$$

$$P_0^B = \$253.78 + \$746.22 = \$1{,}000$$

If we reverse this case and assume that our investor actually paid $1,000 for the three-year bond, then her semiannual rate of return from the investment would be the discount rate of 5%, her simple annualized rate would be 10% [= (2)(5%)], and her effective annualized rate (the rate that accounts for reinvesting the 5% earned) would be 10.25%:

$$\$1{,}000 = \sum_{t=1}^{6} \frac{\$50}{(1+R)^t} + \frac{\$1{,}000}{(1+R)^6} \Rightarrow R = 5\%$$

$$\text{Simple annual rate} = (n)(R) = (2)(.05) = .10$$

$$\text{Effective annual rate} = (1+R)^n - 1 = (1.05)^2 - 1 = .1025$$

APPENDIX D

Web Site Information and Examples

D.1 YAHOO! BOND SEARCH AND VALUE AND YIELD CALCULATION ILLUSTRATION

Yahoo.com

Yahoo.com provides information on Treasury, corporate, and municipal bonds and notes, as well as a bond screener.

http://finance.yahoo.com/bonds

Finance Calculator—FICALC

The FICALC calculator computes a bond's price given its yield or its yield given its price, as well as other information, such as cash flows and total returns.

www.ficalc.com/calc.tips

1. The Yahoo! "Advanced Bond Search" on January 16, 2009 for T-notes with maturities of three and five years and coupons between 3% and 5% yielded:

Issue	Price	Coupon (%)	Maturity [down]	YTM (%)	Current Yield(%)
T-NOTE 4.250 15-Nov-2013	114.27	4.25	15-Nov-13	1.193	3.719
T-NOTE 3.125 30-Sep-2013	107.94	3.125	30-Sep-13	1.37	2.894
T-NOTE 3.125 31-Aug-2013	107.87	3.125	31-Aug-13	1.357	2.897
T-NOTE 4.250 15-Aug-2013	113.77	4.25	15-Aug-13	1.149	3.735
T-NOTE 3.375 31-Jul-2013	109.23	3.375	31-Jul-13	1.272	3.089
T-NOTE 3.375 30-Jun-2013	109.17	3.375	30-Jun-13	1.247	3.091
T-NOTE 3.500 31-May-2013	109.75	3.5	31-May-13	1.197	3.189
T-NOTE 3.625 15-May-2013	110.23	3.625	15-May-13	1.187	3.288
T-NOTE 3.125 30-Apr-2013	108.16	3.125	30-Apr-13	1.164	2.889
T-NOTE 3.875 15-Feb-2013	111.09	3.875	15-Feb-13	1.081	3.488
T-NOTE 3.625 31-Dec-2012	109.71	3.625	31-Dec-12	1.102	3.304
T-NOTE 3.375 30-Nov-2012	108.53	3.375	30-Nov-12	1.11	3.109
T-NOTE 4.000 15-Nov-2012	110.79	4	15-Nov-12	1.105	3.61
T-NOTE 3.875 31-Oct-2012	110.03	3.875	31-Oct-12	1.152	3.521
T-NOTE 4.250 30-Sep-2012	111.12	4.25	30-Sep-12	1.164	3.824

2. Using the online FICALC finance calculator (www.ficalc.com/calc.tips), the price and cash flow information on the T-note with coupon of 4.25% and maturity of November 2013 were:

Finance Calculator: http://www.ficalc.com/calc.tips

Input

Coupon: 4.25

Maturity: 11/15/2013

of: 1.193

Given: Yield

Odd Coupon Periods (optional)

Issue/Dated: 01/16/2009

First Coupon: 05/15/2009

Settings...

Semi-Annual, Act/Act, Adjust

Settlement: 01/16/2009

Selected Output

Reinvestment Rate: 1.193000

Total Return: 1.193000

Price: 114.304312

Yield: 1.193

AI: 0.000000

Cash Flows

05/15/2009	$ 1.40	11/15/2011	$ 2.13
11/15/2009	$ 2.13	05/15/2012	$ 2.13
05/15/2010	$ 2.13	11/15/2012	$ 2.13
11/15/2010	$ 2.13	05/15/2013	$ 2.13
05/15/2011	$ 2.13	11/15/2013	$102.13

3. Value calculation based on the YTM using the bond valuation formula:

$$V_0^b(9 - \text{semiannual periods}) = \sum_{t=1}^{9} \frac{2.125}{(1 + (.01193/2))^t} + \frac{100}{(1 + (.01193/2))^9} = 113.350279$$

$$V_0^b = 2.125\left[\frac{1 - [1/(1.005965)]^9}{.005965}\right] + \frac{100}{(1.005965)^9} = 113.350279$$

$$V_0^b = \frac{113.350279 + 2.125}{(1.01193)^{178/365}} = \frac{115.4800279}{(1.01193)^{178/365}} = 114.81$$

4. Total return calculation to maturity:

$$\text{Days to next coupon date} = 178 \text{ days or } 178/365 = .4876 \text{ years}$$

$$\text{Time to maturity} = 4.4876 \text{ years}$$

$$\text{Coupon value} = \frac{\sum_{t=0}^{9-1} 2.125\ (1.005965)^t}{1.005965)^{178/365}}$$

$$= \frac{2.125\left[\dfrac{(1.005965)^9 - 1}{.005965}\right]}{(1.005965)^{178/365}} = 19.5281$$

$$\text{HD value} = \$100 + 19.5281 = 119.5281$$

$$\text{Price} = \sum_{t=1}^{9} \frac{2.125}{(1 + (.01193/2))^t} + \frac{100}{(1 + (.01193/2))^9} = 113.350279$$

$$\text{TR} = \left[\frac{119.5281}{113.350279}\right]^{1/4.4876} - 1 = .0119$$

5. Price-yield table generated using the Finance Calculator www.ficalc.com/ calc.tips

YTM	Price
.50%	117.869
.75%	116.568
1.00%	115.284
1.25%	114.0167
1.50%	112.765
1.75%	111.5296
2.00%	110.3098
2.25%	109.1056
2.50%	107.9166
2.75%	106.7428
3.00%	105.5833

D.2 FINRA BOND SEARCH

Financial Industry Regulatory Authority Site

The Financial Industry Regulatory Authority (FINRA) is the largest nongovernmental regulator for all securities firms doing business in the United States. FINRA oversees nearly 5,000 brokerage firms, about 172,000 branch offices, and approximately 665,000 registered securities representatives. Created in July 2007 through the consolidation of NASD and the member regulation, enforcement, and arbitration functions of the New York Stock Exchange, FINRA is dedicated to investor protection and market integrity through effective and efficient regulation and complementary compliance and technology-based services. Their site provides information, market data, and information on bonds, stocks, mutual funds and other securities:

www.finra.org/

For bond information:

www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds"

Bond Search Illustration Using FINRA Site

1. To do a bond search, go to www.finra.org/index.htm, "Sitemap," "Market Data," and "Bonds." Use "Quick Bond Search" to find bonds with certain features.
   - Example: Corporate; Kraft (kft); Coupon: 5.01%-10%; Yield: Select; Maturity: 5–10 yrs
   - On January 8, 2009, this search provided four Kraft bonds:

KRAFT FOODS INCORPORATED							
			Ratings			Last Sale	
Coupon	Maturity	Callable	Moody's	S&P	Fitch	Price	Yield
6.75	2/19/2014	No	Baa2	BBB+	BBB	106.25	5.335
6.5	8/11/2017	No	Baa2	BBB+	BBB	102.824	6.071
6.13	2/1/2018	No	Baa2	BBB+	BBB	102.399	5.78
6.13	8/23/2018	No	Baa2	BBB+	BBB	100.797	6.012

2. One can click one of the Kraft bonds to get more information.
3. At the bottom of the information page, one can do a search of the bond's trading activity (click "Show Results"). This will access TRACE, which provides information on price and quantity traded at all execution dates.
   - Example: Kraft, 6.13%, 2018 bond from 01/07/09 to 01/08/09:

Execution Date	Time	Quantity	Price	Yield
1/8/2009	10:28:37	10000	102.399	5.78
1/8/2009	10:28:29	10000	102.399	5.78
1/8/2009	10:27:32	42000	102.45	5.773
1/8/2009	10:27:19	42000	102.45	5.773
1/8/2009	9:53:30	25000	102.399	5.78
1/8/2009	9:53:18	25000	102.399	5.78
1/8/2009	9:18:44	4000	102.471	5.77
1/8/2009	9:18:09	4000	102.471	5.77
1/8/2009	8:57:18	5000	103.5	5.625
1/8/2009	8:57:09	5000	103.5	5.625
1/8/2009	8:54:00	40000	102.783	5.726
1/8/2009	8:43:30	40000	102.621	5.748
1/7/2009	17:07:44	1000000	101.648	5.886
1/7/2009	16:50:37	3800000	99.954	6.131
1/7/2009	16:01:51	70000	99.182	6.244
1/7/2009	15:56:34	70000	99.486	6.199
1/7/2009	15:45:56	3000	104.721	5.456
1/7/2009	15:45:46	3000	104.721	5.456
1/7/2009	15:33:07	750000	102.04	5.831
1/7/2009	15:25:56	10000	104.625	5.469
1/7/2009	15:25:00	10000	103.083	5.684
1/7/2009	15:08:02	50000	104.122	5.539
1/7/2009	15:06:09	50000	102.685	5.739
1/7/2009	15:03:00	11000	103.747	5.591
1/7/2009	14:49:02	1000000	100.574	6.041

D.3 INVESTINGINBONDS.COM BOND SEARCH

The investinginbonds.com site was created by the Securities Industry and Financial Markets Association to help educate investors.

www.sifma.org

www.investinginbonds.com

Corporate Bond Search

1. Go to http://investinginbonds.com/.
2. Click "Corporate Price Date."
3. Click "Corporate Market Data-At-A-Glance."
4. On Search box: Enter corporation, and then select bond.
   - Example: On 01/09/09: Caterpillar, 4.7% coupon, 2012 maturity (CUSIP: 14912L2RI)

Graph Trade Data

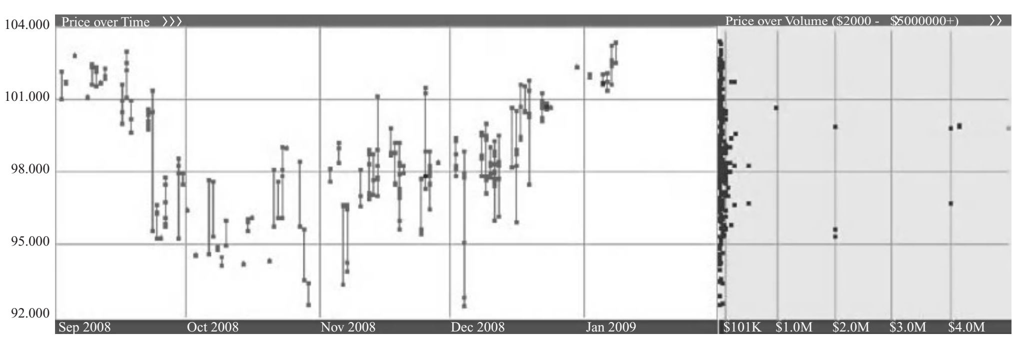
1 2 3 4 5 6 7 8 9 10 ...

TRADE DATE TRADE TIME	PRICE	YIELD	SIZE	NOTES	MORE INFO SPECIAL PRICE INDICATOR
01/09/2009 15:53:39	99.802	4.765	25K	@	Run calculations
01/09/2009 15:53:39	99.619	4.828	25K	@	Run calculations

5. On Trade Date Information table, click "Graph Trade Data" (above box) to obtain the last five months of trading activity:

   Last Five Months of Activity for CUSIP: 14912L2R1

   The graph on the left below shows trades at prices according to the price scale on the left for each day. Darker color dots indicate more trades at that price. Grey vertical bars serve to align each day's trades.



6. On Trade Date Information table, click "Run Calculations" to obtain more information on the bond and a price/yield calculator set at the price for the trade time you selected. The information on the issue and its cash flows can be exported to Excel.
   - Example: For the 15:53:30 time when 25,000 issues of Caterpillar were trading at 99.802:

Data	Amount	Date	Amount
Current Yield	4.709	Accrued Interest	$15.54
Annual Yield	4.823	3/15/2009	$23.50
Semiannual Yield	4.766	9/15/2009	$23.50
Yield-to-Maturity	4.766	3/15/2010	$23.50
Duration	2.937	9/15/2010	$23.50
Modified Duration	2.869	3/15/2011	$23.50
Convexity	10.065	9/15/2011	$23.50
		3/15/2012	$1,023.50

Calculation of value:

$$V_0^b(5 - \textit{Semiannual periods}) = \sum_{t=1}^{5} \frac{\$47/2}{(1 + (.04766/2))^t} + \frac{\$1000}{(1 + (.04744/2))^{10}} = \$977.9102956$$

$$V_0^b = \$23.50 \left[\frac{1 - [1/(1.02383)]^5}{.02383} \right] + \frac{\$1000}{(1.0283)^5} = \$977.9102956$$

$$V_0^b = \frac{\$977.9102956 + \$23.50}{(1.04766)^{65/365}} = \frac{\$1{,}001.410296}{(1.04766)^{65/365}} = \$993.1415819$$

D.4 EQUILIBRIUM BOND PRICE AND SPOT RATES

Yahoo.com provides information on Treasury strips:

http://finance.yahoo.com/bonds, "Advanced Bond Search," select "Treasury Zero Coupon."

Information on Treasury strips can also be found on the *Wall Street Journal* site:

Go to http://online.wsj.com/public/us, "Market Data," "Bonds, Rates & Credit Markets," and "Treasury Strips."

The FICALC calculator computes a bond's price given its yield or its yield given its price, as well as other information, such as cash flows, total returns, and price-yield curves:

www.ficalc.com/calc.tips

Strip Securities and Equilibrium Bond Price

1. On 1/16/2009, a T-note maturing August 15, 2012 was selected using Yahoo.com:

Price:	111.82
Coupon (%):	4.375
Maturity Date:	15-Aug-12
Yield to Maturity (%):	0.994
Current Yield (%):	3.912
Coupon Payment Frequency:	Semiannual
First Coupon Date:	15-Feb-03
Type:	Treasury

2. Using the FICALC calculator, the cash flows on the Treasury note were:

2/15/2009	$0.3600	2/15/2011	$2.1875
8/15/2009	$2.1875	8/15/2011	$2.1875
2/15/2010	$2.1875	2/15/2012	$2.1875
8/15/2010	$2.1875	8/15/2012	$102.1875

- Settlement date = 1/16/2009
- Next coupon = 2/15/2009

3. Finding spot rates from Treasury strips
   - On date 1/16/2009, selected Treasury strips using yahoo.com
   - http://finance.yahoo.com/bonds, "Advanced Bond Search," and "Treasury Zero Coupon"
   - Selected the following strips:

Price:	99.88	Price:	99.78
Maturity Date:	15-Aug-09	Maturity Date:	15-Feb-10
Yield to Maturity (%):	0.211	Yield to Maturity (%):	0.21
Type:	Treasury Zero	Type:	Treasury Zero
Price:	99.43	Price:	98.74
Maturity Date:	15-Aug-10	Maturity Date:	15-Feb-11
Yield to Maturity (%):	0.365	Yield to Maturity (%):	0.615
Type:	Treasury Zero	Type:	Treasury Zero
Price:	98.32	Price:	97.4
Maturity Date:	15-Aug-11	Maturity Date:	15-Feb-12
Yield to Maturity (%):	0.66	Yield to Maturity (%):	0.86
Type:	Treasury Zero	Type:	Treasury Zero
Price:	95.17		
Maturity Date:	15-Aug-12		
Yield to Maturity (%):	1.391		
Type:	Treasury Zero		

4. Equilibrium price of 4.375% Treasury, maturing on 2/15/2012, at date 2/15/2009 (using 30/360 day count convention):

$$P_0^* = \frac{2.1875}{(1.00211)^{.5}} + \frac{2.1875}{(1.0021)^{1}} + \frac{2.1875}{(1.00365)^{1.5}} + \frac{2.1875}{(1.00615)^{2}}$$

$$+\frac{2.1875}{(1.0066)^{2.5}} + \frac{2.1875}{(1.0086)^{3}} + \frac{102.1875}{(1.01391)^{3.5}}$$

$$P_0^* = 110.35269$$

Equilibrium price at 1/16/2009 (using 30/365 day count convention):

$$P_0^* = \frac{0.36 + 110.35269}{(1.0069234)^{30/365}} = 110.6499$$

5. Note: The market price of the 4.375% Treasury maturing on 2/15/2009 was 111.92.

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APPENDIX E

Global Investments and Exchange Rates

Figure 3.9 shows the dollar/British Pound spot exchange rate and the dollar/euro rate from 2000 to 2008. For a number of subperiods between 2000 and 2008, the dollar prices of the BP and euro were increasing (dollar depreciation). For example, from June 2001 to December 2004, the dollar/British pound exchange rate increased 41.8% from \$1.3781/£ to \$1.9549/£, and from December 2005 to November 2007, the dollar/British pound exchange rate increased 18.2% from \$1.7807/£ to \$2.1048/£. For dollar investors, the dollar depreciations over these periods made investments in foreign securities very attractive. For example, from January 3, 2006 to January 3, 2007, the dollar/British pound exchange rate increased 12.03% from \$1.7404/£ to \$1.9498/£. In January 2006, the one-year U.S. Treasury rate was yielding 4.46% and the one-year British Treasury rate was at 4.30%. With perfect foresight, a dollar investor would have earned 17.027% from investing in the British Treasury security. To attain 17.027%, the investor would have had to convert each of her investment dollars to $1/\ E_0 = 1/\$1.7404/£, = 0.57458£/\$$ and invested the 0.57458£ at $R_F = 4.46\%$. One year later, the investor would have 0.6002£ [= 0.57458£ (1.0446)], which she would have been able to convert at the spot exchange rate of \$1.9498/£ to earn to \$1.17027 [= (0.6002£) (\$1.9498/£)]. Thus, the dollar investment in the foreign security would have yielded a dollar rate of 17.027%, compared to the U.S. Treasury yield of 4.30%.

$$\text{Rate} = \frac{(\$1.9498/\text{BP})\ [0.57458\text{BP}(1.0446)]}{\$1} - 1 = \frac{\$1.17027}{\$1} - 1 = .17027$$

Note that the same strategy would have yielded a negative rate if implemented on September 9, 2007, when the dollar/British pound exchange rate was at \$2.0278/£, and then liquidated one year later on September 9, 2008 when the exchange rate was at \$1.7543/£ (–13.49%).

In contrast, British pound investors would find investing in dollar-denominated assets less attractive when the dollar is depreciating (or the pound is appreciating). For example, from January 3, 2006 to January 3, 2007, a British pound investor would find the British pound/dollar exchange rate (£/\$) decreasing from 0.57458£/\$ (1/\$1.7404/£) to 0.512873£/\$ (1/1.9498/£; 10.74% decrease). Such an investor would have seen a loss of 6.90% if he were to have converted £1 to \$1.7404, invested the dollar at 4.30% for the year to earn \$1.8152, and then converted back to pounds at 0.512873£/\$ to receive only £0.93098615:

$$\text{Rate} = \frac{[(\$1.7404\text{BP})(1.043)]\ [0.512873\text{BP}/\$]}{1\text{BP}} - 1$$

$$= \frac{0.93098615\text{BP}}{1\text{BP}} - 1 = -0.069$$

Whether investors find it advantageous to invest in foreign securities or domestically depends on the rates earned on investments in different countries and their expectations of future exchange rates relative to the exchange rates that they can lock in using a futures or forward contract. An important and useful relationship to guide investors in such decisions is the interest rate parity relation.

WEB EXERCISE

1. In this appendix, we examine the impact of exchange rates on dollar investment returns. Calculate the dollar return you would have earned by investing in either a 5-year euro-denominated bond or a 5-year yen-denominated bond.
   - Go to www.research.stlouisfed.org/fred2.
   - Click "Exchange Rates" tab, "By Country" tab, and Euro or Japan.
   - Download data to Excel.
   - Example: For euro series, graph Click EXUSEU and then click "Download Data" to send to Excel.
   - Identify an investment period (e.g. 5-year) when the dollar was depreciating (for example, the \$/euro was increasing from 1/2002–1/2007).
   - Find yields on a euro-denominated bond or a yen-denominated bond for the period. Note that historical 5-year euro-denominated and yen-denominated bond yields can be found by going to www.fxstreet.com, "Rates and Charts," and "Bond Yields."
   - Determine the annualized dollar return you would have earned over the period.
   - Compare your dollar return from the euro investment or yen investment to a comparable yield from a U.S. dollar-denominated bond (5-year dollar-denominated bond yield also can be found on www.fxstreet.com).
   - Comment on the yield differences you observe.

APPENDIX F

Arbitrage Features of the Calibration Model

In addition to satisfying an arbitrage-free condition on option-free bonds, the calibration model also values a bond's embedded options as arbitrage-free prices. To see this, consider a three-year, 9% callable bond priced in terms of the calibrated binomial tree shown in Exhibit 15.7.

The values of the bond and its call option are the same as the ones shown in the lower tree in the figure. As shown, the value of the call option in period 1 is .3095 when the rate is at 11% and 1.2166 when the rate is at 9.5%, and the value of the option in the current period is .6937. Each of these values was determined by calculating the present value of each option's expected value. These values are also equal to the values of their replicating portfolios. For example, the current call price of .6937 is equal to the value of a portfolio consisting of a one-year, 9% option-free bond and a two-year, 9% option-free bond constructed so that next year the portfolio is worth .3095 if the spot rate is 11% and 1.2166 if the rate is at 9.5%. Specifically, given the possible cash flows on the two-year bond of $B_u + C = (109/1.11) + 9 = 107.198198$ and $B_d + C = (109/1.095) + 9 = 108.543379$, and the cash flow on the one-year bond of 109, the replicating portfolio is formed by solving for the number of one-year bonds, n_1, and the number of two-year bonds, n_2, where:

$$n_1(109) + n_2(107.198198) = .3095$$
$$n_1(109) + n_2(108.543379) = 1.2166$$

Solving for n_1 and n_2, we obtain $n_1 = -.66035$ and $n_2 = .674333$.

Thus, a portfolio formed by buying .674333 issues of a two-year bond and shorting .66035 issues of a one-year bond will yield possible cash flows next year of .3095 if the spot rate is at 11% and 1.2166 if the rate is at 9.5%. Moreover, given the current one-year and two-year bond prices of 99.09091 and 98.06435, the value of this replicating portfolio is .6937:

$$B_1 = \frac{109}{1.10} = 99.09091$$

$$B_2 = \frac{9}{1.10} + \frac{9}{(1.1012238)^2} = 98.06435$$

$$V_0^{RP} = (-.66035)(99.09091) + (.674333)(98.06435) = .6937$$

Since the replicating portfolio and the call option have the same cash flows, by the law of one price, they must be equally priced. Thus, in the absence of arbitrage, the price of the call is equal to .6937, which is the same price we obtained by discounting the option's expected value using the calibrated binomial interest rate tree.

The two call values in period 1 of .3095 and 1.2166 are likewise equal to the values of their replicating portfolios. That is, at the 11% rate a replicating portfolio of .4730827 of the two-year, 9% bond (original three-year) and −.46108027 issues of a one-year, 9% bond (original two-year bond) will yield cash flows in period 2 equal to 0 if the spot rate is 12.1% and .6872 if the rate is 10.45%. At that node, the price of the one-year bond is 109/1.11 = 98.198198 and the price of the two-year bond is 96.3612. At these prices, the value of the replicating portfolio is .30956[= (−.46108027)(98.198198) + (.4730827)(96.3612)], which matches the value of the call. At the lower node, the replicating portfolio consists of −.98165 one-year, 9% bonds priced at 99.54338 (= 109/1.095) and one two-year, 9% bond priced at 98.9335. This portfolio's possible cash flows in period 2 match the possible call values of .6872 and 1.9771 and its period 1 value is equal to the call value of 1.2166.

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APPENDIX G

T-Bond Delivery Procedure and Equilibrium Pricing

G.1 CHEAPEST-TO-DELIVER BOND

The T-bond futures contract gives the party with the short position the right to deliver, at any time during the delivery month, any bond with a maturity of at least 15 years. When a particular bond is delivered, the price received by the seller is equal to the quoted futures price on the futures contract times a conversion factor, CFA, applicable to the delivered bond. The invoice price, in turn, is equal to that price plus any accrued interest on the delivered bond:

$$\text{Invoice price} = (f_0)\,(\text{CFA}) + \text{Accrued interest}$$

The CBT uses a conversion factor based on discounting the deliverable bond by a 6% YTM. The CBT's rules for calculating the CFA on the deliverable bond are as follows:

- The bond's maturity and time to the next coupon date are rounded down to the closest three months.
- After rounding, if the bond has an exact number of six-month periods, then the first coupon is assumed to be paid in six months.
- After rounding, if the bond does not have an exact number of six-month periods, then the first coupon is assumed to be paid in three months and the accrued interest is subtracted.

Using these rules, a 5.5% T-bond maturing in 18 years and 1 month would be (1) rounded down to 18 years; (2) the first coupon would be assumed to be paid in six months; and (3) the CFA would be determined using a discount rate of 6% and face value of $100. The CFA for the bond would be .945419:

$$V = \sum_{t=1}^{36} \frac{2.75}{(1.03)^t} + \frac{100}{(1.03)^{36}} = 94.5419$$

$$\text{CFA} = \frac{94.5419}{100} = .945419$$

If the bond matured in 18 years and four months, the bond would be assumed to have a maturity of 18 years and three months. Its CFA would be found by determining the value of the bond three months from the present, discounting that value to the current period, and subtracting the accrued interest [(3/6)(2.75) = 1.375]:

$$V = \frac{94.5419}{(1.03)^{.5}} = 93.1549$$

$$\text{CFA} = \frac{93.1549 - 1.375}{100} = .917799$$

During the delivery month, there are a number of possible bonds that can be delivered. The party with the short position will select the bond that is cheapest to deliver. The CBT maintains tables with possible deliverable bonds. The tables show the bond's current quoted price and CFA. For example, suppose three possible bonds are

Bond	Quoted Price	CFA
1	110.75	1.15
2	97.5	1.05
3	125.75	1.35

If the current quoted futures price were 90 -16 or 90.5, the costs of buying and delivering each bond would be:

Bond	Cost of Bond Minus Revenue from Selling Bond on Futures
1	110.5 – (90.5)(1.15) = 6.425
2	97.5 – (90.5)(1.05) = 2.475
3	125.75 – (90.5)(1.35) = 3.575

Thus, the cheapest bond to deliver would be number 2. Over time and as rates change, the cheapest-to-deliver bond can change. In general, if rates exceed 6%, the CBT's conversion system favors bonds with higher maturities and lower coupons; if rates are less than 6%, the system tends to favor higher-coupon bonds with shorter maturities.

G.2 WILD-CARD PLAY

Under the CBT's procedures, a T-bond futures trader with a short position who wants to deliver on the contract has the right to determine during the expiration month not only the eligible bond to deliver, but also the day of the delivery. The delivery process encompasses the following three business days:

- **Business Day 1**, *Position Day*: The short position holder notifies the clearinghouse that she will deliver.

- **Business Day 2**, *Notice of Intention Day*: The clearinghouse assigns a long position holder for the contract (typically the holder with the longest outstanding contract).
- **Business Day 3**, *Delivery Day*: The short holder delivers an eligible T-bond to the assigned long position holder who pays the short holder an invoice price determined by the futures price and a conversion factor.

Since a short holder can notify the clearinghouse of her intention to deliver a bond by 8 p.m. (Chicago time) at the end of the position day (not necessarily at the end of the futures trading day), an arbitrage opportunity has arisen because of the futures exchange's closing time being 2:00 (Chicago time) and the closing time on spot T-bond trading being 4:00. Thus, a short holder knowing the settlement price at 2:00 p.m. could find the price of an eligible T-bond decreasing in the next two hours on the spot market. If this occurred, she could buy the bond at the end of the day at the lower price, and then notify the clearinghouse of her intention to deliver that bond on the futures contract; if the bond price does not decline, the short holder can keep her position and wait another day. This feature of the T-bond futures contract is known as the ***wild-card option***. This option tends to lower the futures price.

G.3 EQUILIBRIUM T-BOND FUTURES PRICE

The pricing of a T-bond futures contract is more complex than the pricing of T-bill or Eurodollar futures because of the uncertainty over the bond to be delivered and the time of the delivery. Like T-bill futures, the price on a T-bond futures contract depends on the spot price on the underlying T-bond (S_0) and the risk-free rate. If we assume that we know the cheapest-to-deliver bond and the time of delivery, the equilibrium futures price is

$$f_0 = [S_0 - \text{PV}(C)](1 + R_f)^T$$

where S_0 = current spot price of the cheapest-to-deliver T-bond (clean price plus accrued interest)
PV(C) = present value of coupons paid on the bond during the life of the futures contract

Example

As an example, suppose the following:

- The cheapest-to-deliver T-bond underlying a futures contract pays a 10% coupon, has a CFA of 1.2, and is currently trading at 110 (clean price).
- The cheapest-to-deliver T-bond's last coupon date was 50 days ago, its next coupon is 132 days from now, and the coupon after that comes 182 days later.
- The yield curve is flat at 6%.
- The T-bond futures estimated expiration is $T = 270$ days.

The current T-bond spot price is 111.37 and the present value of the \$5 coupon received in 132 days is 4.8957:

$$S_0 = 110 + \frac{50}{50 + 132}(5) = 111.37$$

$$\text{PV}(C) = \frac{5}{(1.06)^{132/365}} = 4.8957$$

The equilibrium futures price based on a 10% deliverable bond is therefore 111.16 per \$100 face value:

$$f_0^* = [S_0 - \text{PV}(C)](1 + R_f)^T$$

$$f_0^* = [111.37 - 4.8957](1.06)^{270/365} = 111.16$$

The quoted price on a futures contract written on the 10% delivered bond would be stated net of accrued interest at the delivery date. The delivery date occurs 138 days after the last coupon payment (270 – 132). Thus, at delivery, there would be 138 days of accrued interest. Given the 182-day period between coupon payments, accrued interest would therefore be 3.791 [= (138/182)(5)]. The quoted futures price on the delivered bond would be 107.369 (= 111.16 – 3.791), and with a CFA of 1.2, the equilibrium quoted futures price would be 89.47:

Quoted futures price on bond = 111.16 − (138/182)5 = 107.369

Quoted futures price = 107.369/1.2 = 89.47

G.4 ARBITRAGE

Like T-bill futures, cash-and-carry arbitrage opportunities will exist if the T-bond futures are not equal to 111.16 (or their quoted price of 89.47). For example, if futures were priced at $f^M = 113$, an arbitrageur could go short in the futures at 113 and then buy the underlying cheapest-to-deliver bond for 111.37, financed by borrowing 106.4743 [= S_0 – PV(C) = 111.37 – 4.8957] at 6% for 270 days and 4.8957 at 6% for 132 days. 132 days later, the arbitrageur would receive a \$5 coupon that he would use to pay off the 132-day loan of 5 [= $4.8957(1.06)^{132/365}$]. At expiration, the arbitrageur would sell the bond on the futures contract at 113 and pay off his financing cost on the 270-day loan of 111.16 [= $106.4743(1.06)^{270/365}$]. This, in turn, would equate to an arbitrage profit of $f^M - f_0{}^* = 113 - 111.16 =$ \$1.84 per \$100 face value. This risk-free return would result in arbitrageurs pursuing this strategy of going short in the futures and long in the T-bond, causing the futures price to decrease to 111.16 where the arbitrage disappears. If the futures price were below 111.16, arbitrageurs would reverse the strategy, shorting the bond, investing the proceeds, and going long in the T-bond futures contract.

Bond Evaluation, Selection, and Management, Second Edition
by R. Stafford Johnson

APPENDIX H

Pricing Interest Rate Options with a Binomial Interest Rate Tree

In Chapter 14, we examined how the binomial interest rate model can be used to price bonds with embedded call and put options, sinking fund arrangements, and convertible clauses, and in Chapter 15, we looked at two approaches to estimating the tree. In this appendix, we show how the binomial interest rate tree can be used to price interest rate options.

H.1 VALUING T-BILL OPTIONS WITH A BINOMIAL TREE

Figure H.1 shows a two-period binomial tree for an annualized risk-free spot rate (S) and the corresponding prices on a T-bill (B) with a maturity of .25 years and face value of \$100 and also a futures contract (f) on the T-bill, with the futures expiring at the end of period 2. The length of each period is six months (six-month steps); the upward parameter on the spot rate (u) is 1.1 and the downward parameter (d) is 1/1.1 = 0.9091; the probability of the spot rate increasing in each period is .5; and the yield curve is assumed flat. As shown in the figure, given an initial spot rate of 5% (annual), the two possible spot rates after one period (six months) are 5.5% and 4.54545%, and the three possible rates after two periods (one year) are 6.05%, 5%, and 4.13223%. At the current spot rate of 5%, the price of the T-bill is $B_0 = 98.79$ [$= 100/(1.05)^{.25}$]; in period 1, the price is 98.67 when the spot rate is 5.5% [$= 100/(1.055)^{.25}$] and 98.895 when the rate is 4.54545% [$= 100/(1.0454545)^{.25}$]. In period 2, the T-bill prices are 98.54, 98.79, and 99 for spot rates of 6.05%, 5%, and 4.13223%, respectively.

The futures prices shown in Figure H.1 are obtained by assuming a risk-neutral market. If the market is risk neutral, then the futures price is an unbiased estimator of the expected spot price: $f_t = \mathrm{E}(S_T)$.

The futures prices at each node in the exhibit are therefore equal to their expected price next period. Given the spot T-bill prices in period 2, the futures prices in period 1 are 98.665 [$= \mathrm{E}(B) = .5(98.54) + .5(98.79)$] and 98.895 [$= \mathrm{E}(B) = .5(98.79) + .5(99)$]. Given these prices, the current futures price is $f_0 = 98.78$ [$= \mathrm{E}(f_1) = .5(98.665) + .5(98.895)$].

Given the binomial tree of spot rates, prices on the spot T-bill, and prices on the T-bill futures, we can determine the values of call and put options on spot and futures T-bills. For European options, the methodology for determining the price is to start at

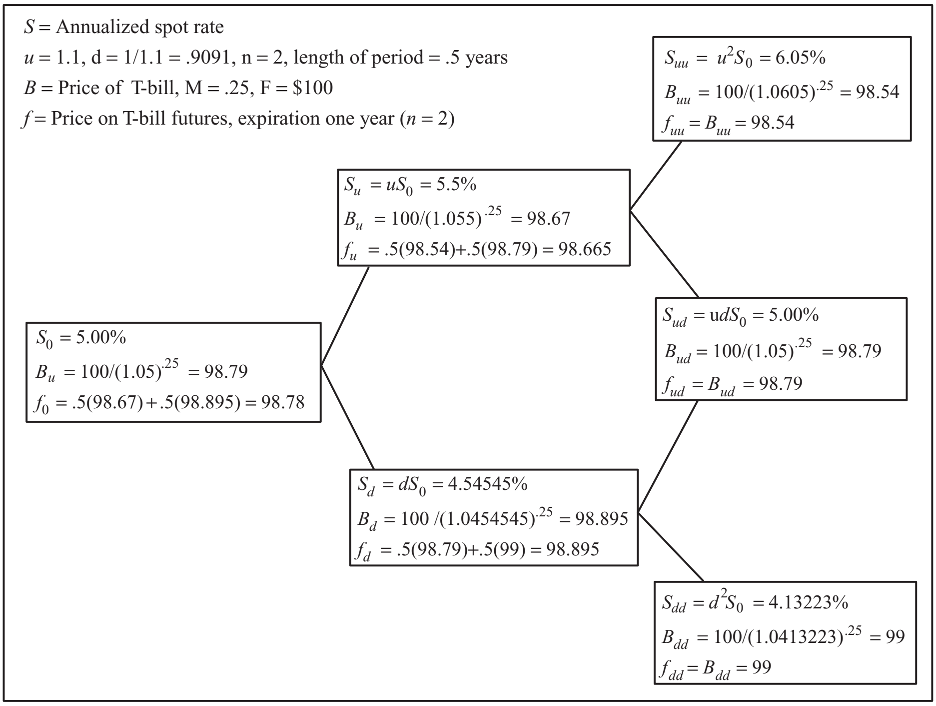

FIGURE H.1 Binomial Tree of Spot Rates, T-Bill Prices, and T-Bill Futures Prices

expiration where we know the possible option values are equal to their intrinsic values, IVs. Given the option's IVs at expiration, we then move to the preceding period and price the option to equal the present value of its expected cash flows for next period. Given these values, we then roll the tree to the next preceding period and again price the option to equal the present value of its expected cash flows. We continue this recursive process to the current period. If the option is American, then its early exercise advantage needs to be taken into account by determining at each node whether or not it is more valuable to hold the option or exercise. This is done by starting one period prior to the expiration of the option and constraining the price of the American option to be the maximum of its binomial value (present value of next period's expected cash flows) or the intrinsic value (i.e., the value from exercising). Those values are then rolled to the next preceding period, and the American option values for that period are obtained by again constraining the option prices to be the maximum of the binomial value or the IV; this process continues to the current period.

Spot T-Bill Call

Suppose we want to value a European call on a spot T-bill with an exercise price of 98.75 per \$100 face value and expiration of one year. To value the call option on the T-bill, we start at the option's expiration, where we know the possible call values are equal to their intrinsic values, IVs. In this case, at spot rates of 5% and 4.13223%, the call is in the money with IVs of .04 and .25, respectively, and at the spot rate of 6.05% the call is out of the money and thus has an IV of zero (see Figure H.2).

Given the three possible option values at expiration, we next move to period 1 and price the option at the two possible spot rates of 5.5% and 4.54545% to equal the present values of their expected cash flows next period. Assuming there is an equal probability of the spot rate increasing or decreasing in one period ($q = .5$), the two possible call values in period 1 are .01947 and .1418:

$$C_u = \frac{.5(0) + .5(.04)}{(1.055)^{.5}} = .01947$$

$$C_d = \frac{.5(.04) + .5(.25)}{(1.0454545)^{.5}} = .1418$$

Rolling these call values to the current period and again determining the option's price as the present value of the expected cash flow, we obtain a price on the European T-bill call of .0787:

$$C_0 = \frac{.5(.01947) + .5(.1418)}{(1.05)^{.5}} = .0787$$

If the call option were American, its two possible prices in period 1 are constrained to be the maximum of the binomial value (present value of next period's expected cash flows) or the intrinsic value (i.e., the value from exercising):

$$C_t^A = \text{Max}[C_t, \text{IV}]$$

In period 1, the IV slightly exceeds the binomial value when the spot rate is 4.54545%. As a result, the American call price is equal to its IV of .145 (see Figure H.2.

Rolling this price and the upper rate's price of .01947 to the current period yields a price for the American T-bill call of .08. This price slightly exceeds the European value of .0787, reflecting the early exercise advantage of the American option.

Futures T-Bill Call

If the call option were on a European T-bill futures contract instead of a spot T-bill, with the futures and option having the same expiration, then the value of the futures option would be the same as the spot option. That is, at the expiration spot rates of 6.05%, 5%, and 4.13223%, the futures prices on the expiring contract would be equal to the spot prices (98.54, 98.79, and 99), and the corresponding IVs of the European futures call with an exercise price of 98.75 would be 0, .04, and .25—the same as the spot call's IV. Thus, when we roll these call values back to the present period, we end up with the price on the European futures call of .0787—the same as the European spot.

If the futures call option were American, then the option prices at each node need to be constrained to be the maximum of the binomial value or the futures option's IV. Since the IV of the futures call in period 1 is zero when the spot rate is 5.5% (IV = Max[98.665 – 98.75, 0] = 0) and .145 when the rate is 4.54545% (IV = Max[98.895 – 97.75, 0] = .145), the corresponding prices of the American futures option would therefore be the same as the spot option: .01947 and .145. Rolling

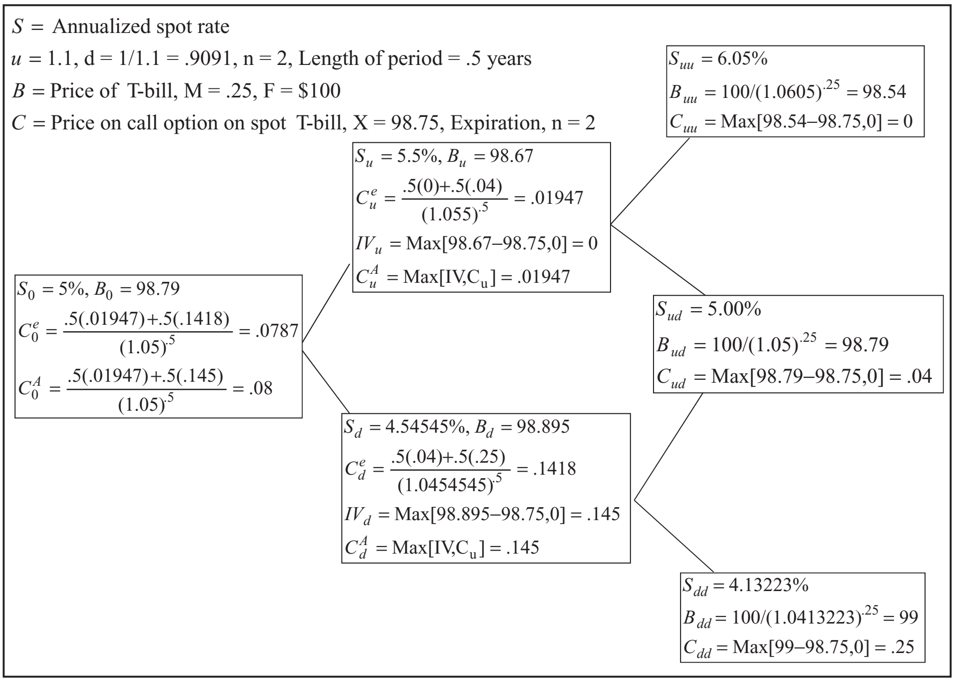

FIGURE H.2 Binomial Tree of Spot Rates and T-Bill Call Prices

theses prices to the current period yields a price on the American T-bill futures call of .08—the same price as the American spot option.[1]

T-Bill Put

In the case of a spot or futures T-bill put, their prices can be determined given a binomial tree of spot rates and their corresponding spot and futures prices. Figure H.3 shows the binomial valuation of a European T-bill futures put contract with an exercise price of 98.75 and expiration of one year (two periods).

At the expiration spot rate of 6.05%, the put is in the money with an IV of .21, and at the spot rates of 5% and 4.13223% the put is out of the money. In period 1, the two possible values for the European put are .1022 and 0. Since these values exceed or equal their IV, they would also be the prices of the put if it were American. Rolling these values to the current period, we obtain the price for the futures put of .05.

H.2 VALUING A CAPLET AND FLOORLET WITH A BINOMIAL TREE

The price of a caplet or floorlet can also be valued using a binomial tree of the option's reference rate. For example, consider an interest rate call on the spot rate defined by our binomial tree, with an exercise rate of 5%, time period applied to the payoff of $\phi = .25$, and notional principal of NP = 100. As shown in Figure H.4,

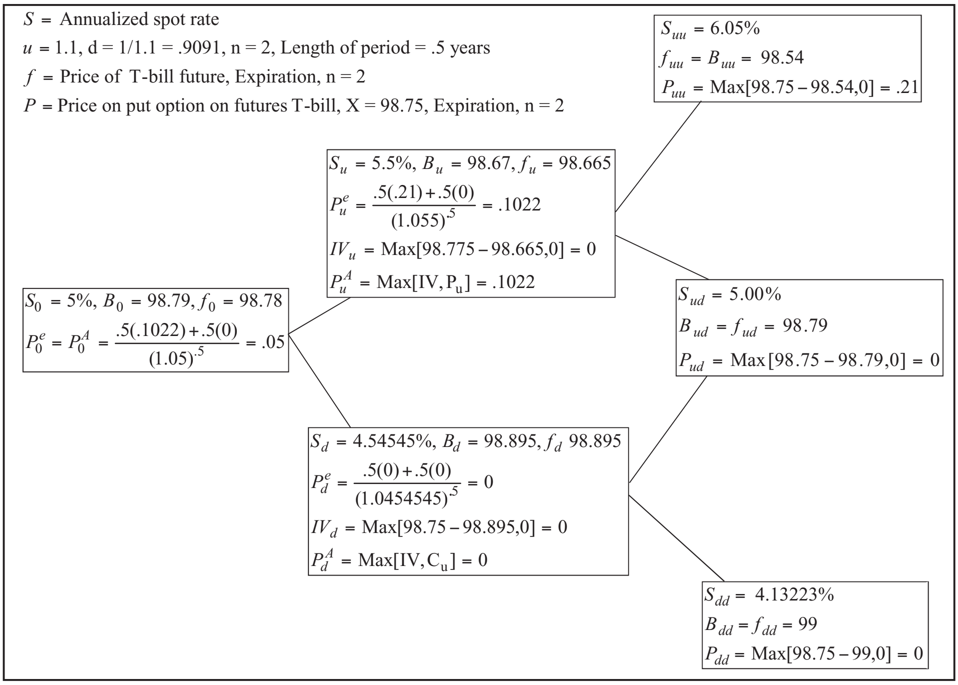

FIGURE H.3 Binomial Tree of Spot Rates and T-Bill Futures Put Prices

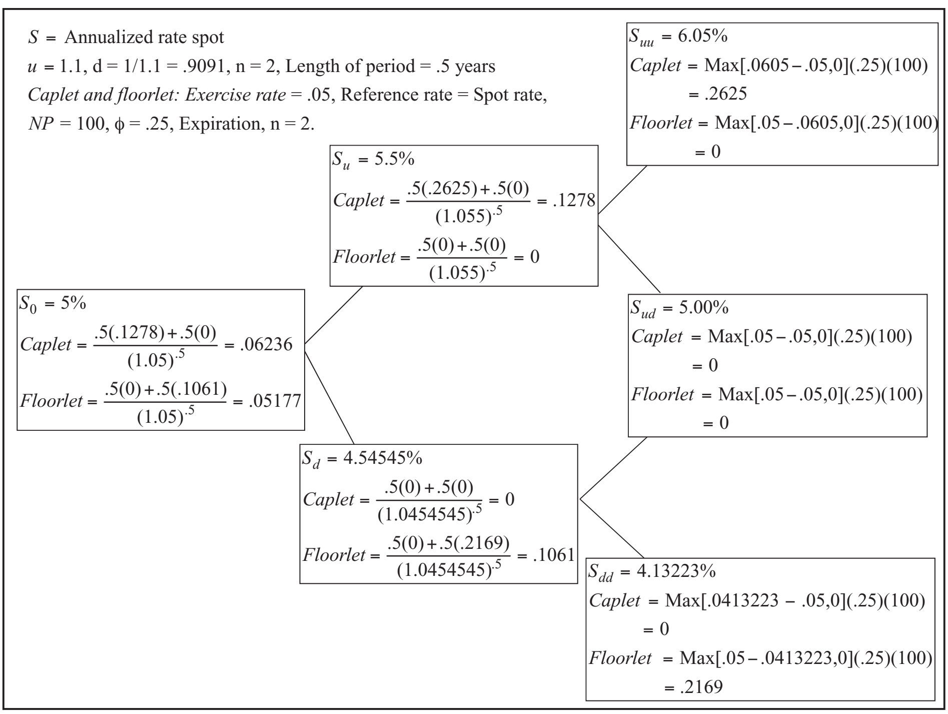

FIGURE H.4 Binomial Tree: Caplets and Floorlets the interest rate call is in the money at expiration only at the spot rate of 6.05%. At this rate, the caplet's payoff is .2625 [= (.0605 – .05)(.25)(100)]. In period 1, the value of the caplet is .1278 (= [.5(.2625) + .5(0)]/(1.055)$^{.5}$) at spot rate 5.5% and 0 at spot rate 4.54545%. Rolling theses values to the current period, in turn, yields a price on the interest rate call of .06236 (= [.5(.1278) + .5(0)]/(1.05)$^{.5}$). In contrast, an interest rate put with similar features would be in the money at expiration at the spot rate of 4.13223%, with a payoff of .2169 [= (.05 – .0413223)(.25)(100) and out of the money at spot rates 5% and 6.05%. In period 1, the floorlet's values would be .1061 (= [.5(0) + .5(.2169)]/(1.0454545)$^{.5}$) at spot rate 4.454545% and 0 at spot rate 6.05%. Rolling these values to the present period, we obtain a price on the floorlet of .05177 (= [.5(0) + .5(.1061)]/(1.05)$^{.5}$).

Since a cap is a series of caplets, its price is simply equal to the sum of the values of the individual caplets making up the cap. To price a cap, we can use a binomial tree to price each caplet and then aggregate the caplet values to obtain the value of the cap. Similarly, the value of a floor can be found by summing the values of the floorlets comprising the floor.

H.3 VALUING T-BOND OPTIONS WITH A BINOMIAL TREE

The T-bill underlying the spot or futures T-bill option is a fixed-deliverable bill; that is, the features of the bill (maturity of 91 days and principal of $1 million) do not change during the life of the option. In contrast, the T-bond or T-note underlying a T-bond or T-note option or futures option is a specified T-bond or note or the bond from an eligible group that is most likely to be delivered. Because of the specified bond clause on a T-bond or note option or futures option, the first step in valuing the option is to determine the values of the specified T-bond (or bond most likely to be delivered) at the various nodes on the binomial tree, using the same methodology we used in Chapter 14 to value a coupon bond.

As an example, consider an OTC spot option on a T-bond with a 6% annual coupon, face value of $100, and with three years left to maturity. In valuing the bond, suppose we have a two-period binomial tree of risk-free spot rates, with the length of each period being one year, the estimated upward and downward parameters being $u = 1.2$ and $d = .8333$, and the current spot rate being 6% (see Figure H.5).

To value the T-bond, we start at the bond's maturity (end of period 3) where the bond's value is equal to the principal plus the coupon, 106. We next determine the three possible values in period 2 given the three possible spot rates. As shown in Figure H.5, the three possible values of the T-bond in period 2 are 97.57 (= 106/1.084), 100 (= 106/1.06), and 101.760 (= 106/1.0416667). Given these values, we next roll the tree to the first period and determine the two possible values. The values in that period are equal to the present values of the T-bond's expected cash flows in period 2; that is:

$$B_u = \frac{.5[97.57 + 6] + .5[100 + 6]}{1.072} = 97.747$$

$$B_d = \frac{.5[100 + 6] + .5[101.760 + 6]}{1.05} = 101.79$$

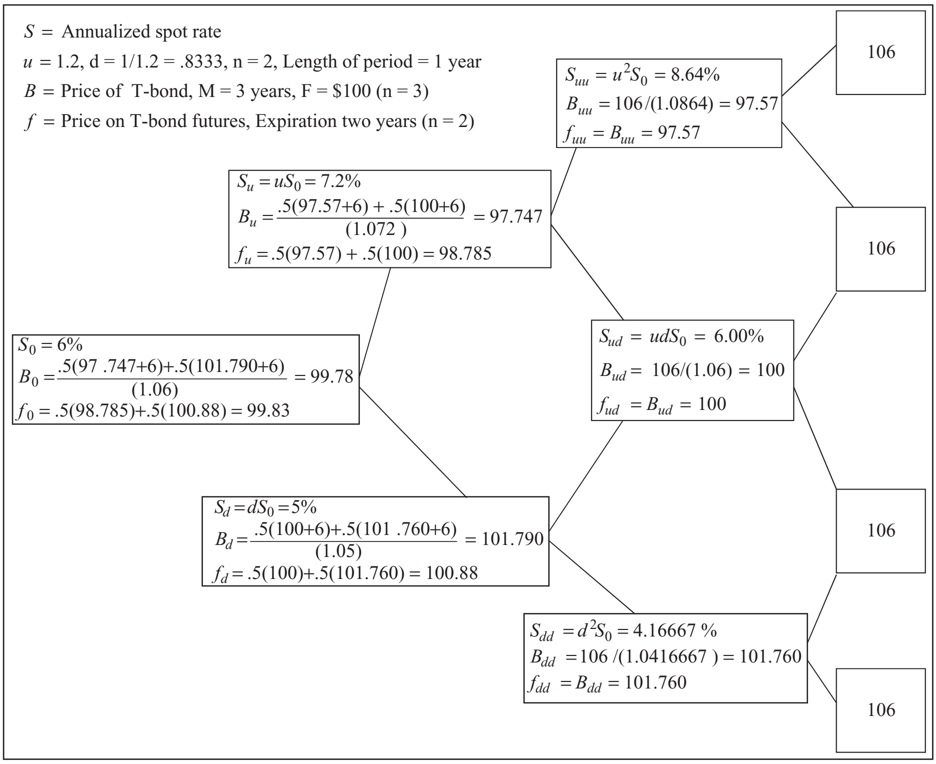

FIGURE H.5 Binomial Tree: Spot Rates, T-Bond Prices, and T-Bond Futures Prices

Finally, using the bond values in period 1, we roll the tree to the current period where we determine the value of the T-bond to be 99.78:

$$B_0 = \frac{.5[97.747 + 6] + .5[101.79 + 6]}{1.06} = 99.78$$

Figure H.5 also shows the prices on a two-year futures contract on the three-year, 6% T-bond. The prices are generated by assuming a risk-neutral market. As shown, at expiration (period 2) the three possible futures prices are equal to their spot prices: 97.57, 100, 101,76; in period 1, the two futures prices are equal to their expected spot prices: $f_u = E(B_T) = .5(97.57) + .5(100) = 98.875$ and $f_d = E(B_T) = .5(100) + .5(101.76) = 100.88$; in the current period, the futures price is $f_0 = E(f_1) = .5(98.785) + .5(100.88) = 99.83$.

Spot T-Bond Call

Suppose we want to value a European call on the T-bond, with the call having an exercise price of 98 and expiration of two years. At the option's expiration, the underlying T-bond has three possible values: 97.57, 100, and 101.76. The 98 T-bond call's respective IVs are therefore 0, 2, and 3.76 (see Figure H.6).

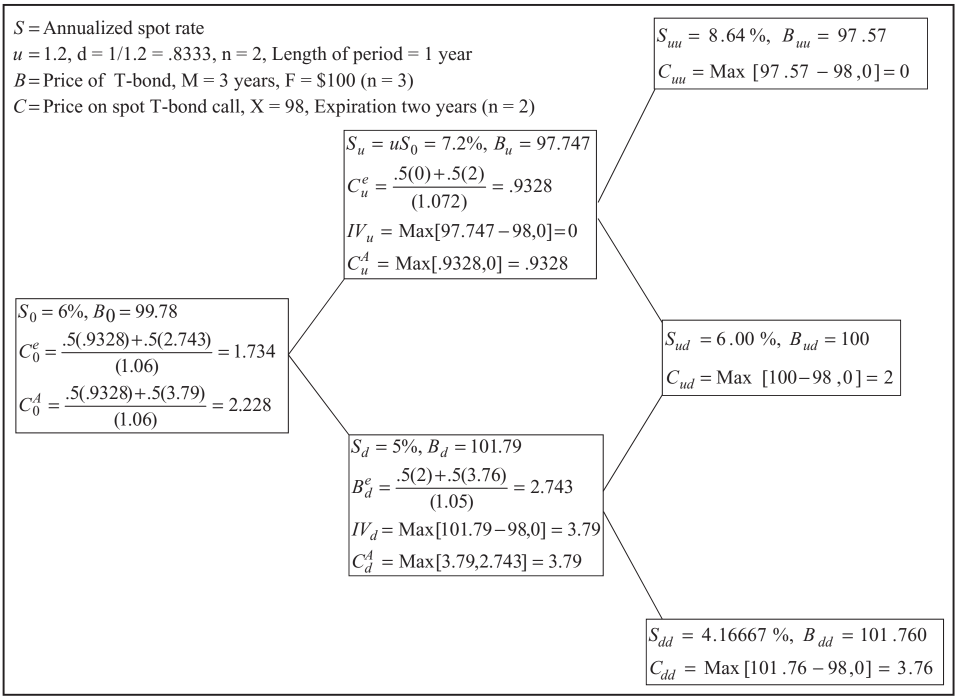

FIGURE H.6 Binomial Tree: T-Bond Call Prices

Given these values, the call's possible values in period 1 are .9328 (= [.5(0) + .5(2)]/1.072) and 2.743 (= [.5(2) + .5(3.76)]/1.05). Rolling these values to the current period, we obtain the price on the European T-bond call of 1.734 (= [.5(.9328) + .5(2.743)]/1.06). If the call option were American, then its value at each node is the greater of the value of holding the call or the value from exercising. As we did in valuing American T-bill options, this valuation requires constraining the American price to be the maximum of the binomial value or the IV. In this example, if the T-bond option were American, then in period 1 the option's price would be equal to its IV of 3.79 at the lower rate. Rolling this price and the upper rate's price of .9328 to the current period yields a price of 2.228.

Futures T-Bond Call

If the European call were an option on a futures contract on the three-year, 6% T-bond (or if that bond were the most-likely-to-be-delivered bond on the futures contract), with the futures contract expiring at the same time as the option (end of period 2), then the value of the futures option will be the same as the spot. That is, at expiration the futures prices on the expiring contract would be equal to the spot prices, and the corresponding IVs of the European futures call would be the same as the spot call's IV. Thus, when we roll these call values back to the present period, we end up with the price on the European futures call being the same as the European spot: 1.734. If the futures call were American, then at the spot rate of 5% in period

1, its IV would be 2.88 (= Max[100.88 – 98,0]), exceeding the binomial value of 2.743. Rolling the 2.88 value to the current period yields a price on the American futures option of 1.798 [= (.5(.9328) + .5(2.88)/1.06]—this price differs from the American spot option price of 2.228.

T-Bond Put

Suppose we want to value a European put on a spot or futures T-bond with the put having similar terms as the call. Given the bond's possible prices at expiration of 97.57, 100, and 101.76, the corresponding IVs of the put are .43, 0, and 0. In period 1, the put's two possible values are .2006 (= [.5(.43) + .5(0)]/1.072) and 0. Rolling these values to the current period yields a price on the European put of .0946 (= [.5(.2006) + .5(0)]/1.06). Note that if the spot put were American, then its possible prices in period 1 would be .253 and 0, and its current price would be .119 (= [.5(.253) + .5(0)]/1.06); if the futures put were American, there would be no exercise advantage in period 1 and thus the price would be equal to its European value of .0946.

Realism In the above examples, the binomial trees' six-month and one-year steps were too simplistic. Ideally, binomial trees need to be subdivided into a number of periods. In pricing a 5-year T-bond, for example, we might use one-month steps and a five-year horizon. This would, in turn, translate into a 60-period tree with the length of each period being one month. By doing this, we would have a distribution of 60 possible T-note values one period from maturity.

NOTE

1. It should be noted that if the futures option expired in one period while the T-bill futures expire in two, then the value of the futures option would be .071:

$$C_0 = \frac{.5(IV_u) + .5(IV_d)}{(1.05)^{.5}} = \frac{.5\,\text{Max}(98.665 - 98.75, 0) + .5\,\text{Max}(98.895 - 98.75, 0)}{(1.05)^{.5}} = .071$$

APPENDIX I

Pricing Interest Rate Options with the Black Futures Option Model

I.1 BLACK FUTURES MODEL

An extension of the B-S OPM that is sometimes used to price interest rate options is the Black futures option model. The model is defined as follows:

$$C_0^* = [\,f_0 N(d_1) - XN(d_2)]\, e^{-R_f T}$$

$$P_0^* = [X(1 - N(d_2)) - f_0(1 - N(d_1))]\, e^{-R_f T}$$

$$d_1 = \frac{\ln(f_0/X) + (\sigma_f^2/2)T}{\sigma_f\sqrt{T}}$$

$$d_2 = d_1 - \sigma_f\sqrt{T}$$

where σ_f^2 = variance of the logarithmic return of futures prices = $V(\ln(f_n/f_0)$
T = time to expiration expressed as a proportion of a year
R_f = continuously compounded annual risk-free rate [if simple annual rate is R, the continuously compounded rate is $\ln(1+R)$]
$N(d)$ = cumulative normal probability; this probability can be looked up in a standard normal probability table or by using the following formula:

$$N(d) = 1 - n(d), \text{ for } d < 0$$
$$N(d) = n(d), \text{ for } d > 0,$$

where

$$n(d) = 1 - .5[1 + .196854(|d|) + .115194(|d|)^2 + .0003444(|d|)^3 + .019527(|d|)^4]^{-4}$$

$$|d| = \text{absolute value of } d$$

Example: T-Bill Futures

Consider the European futures T-bill call options we priced in Appendix H in which the futures option had an exercise price of 98.75 and expiration of one year and the current futures price was $f_0 = 98.7876$. If the simple risk-free rate is 5%, implying a continuously compounded rate of 4.879% [= ln(1.05)], and the annualized standard deviation of the futures logarithmic return, $\sigma(\ln(f_n/f_0))$, is .00158, then using the Black futures model the price of the T-bill futures call would be .07912.

$$C_0^* = [98.7876(.595462) - 98.75(.594847)]\, e^{-(.04879)(1)} = .07912$$

where

$$d_1 = \frac{\ln(98.7876/98.75) + (.00158)^2/2)(1)}{.00158\sqrt{1}} = .24175$$

$$d_2 = .24175 - .00158\sqrt{1} = .24017$$

$$N(.24175) = .595462$$

$$N(.24017) = .594847$$

Example: T-Bond Futures

As a second example, consider one-year put and call options on a CBOT T-bond futures contract, with each option having an exercise price of $100,000. Suppose the current futures price is $96,115, the futures volatility is $\sigma(\ln(f_n/f_0)) = .10$, and the continuously compounded risk-free rate is .065. Using the Black futures option model, the price of the call option would be $2,137 and the price of the put would be $5,777:

$$C_0^* = [\$96{,}115\,(.36447) - \$100{,}000\,(.327485)]\, e^{-(.065)(1)} = \$2{,}137$$

$$P_0^* = [\$100{,}000(1 - .327485) - \$96{,}115(1 - .36447)]\, e^{-(.065)(1)} = \$5{,}777$$

where

$$d_1 = \frac{\ln(96115/100{,}000) + (.01/2)(1)}{.10\sqrt{1}} = . - .34625$$

$$d_2 = -.34625 - .10\sqrt{1} = -.44625$$

$$N(-.34625) = .36447$$

$$N(-.44625) = .327485$$

It should be noted that the call and futures prices are also consistent with put-call futures parity:

$$P_0^* - C_0^* = \text{PV}(X - f_0)$$

$$P_0^* = (X - f_0)\, e^{-R_f T} + C_0^*$$

$$P_0^* = (\$100{,}000 - \$96{,}115)\, e^{-(.065)(1)} + \$2{,}137 = \$5{,}777$$

Also, note that the Black model can be used to price a spot option. In this case, the current futures price, f_0, is set equal to its equilibrium price as determined by the carrying cost model: $f_0 = S_0(1+R_f)^T$ – (Accrued interest at T). If the carrying cost model holds, the price obtained using the Black model will be equal to the price obtained using the B-S OPM.

Pricing Caplets and Floorlets with the Black Futures Option Model

The Black futures option model also can be extended to pricing caplets and floorlets by (1) substituting T^* for T in the equation for C^* (for a caplet) or P^* (for a floorlet), where T^* is the time to expiration on the option plus the time period applied to the interest rate payoff time period, ϕ: $T^* = T + \phi$; (2) using an annual continuously compounded risk-free rate for period T^* instead of T; (3) multiplying the Black adjusted-futures option model by the notional principal times the time period: (NP) ϕ.

$$C_0^* = \phi(\text{NP})\,[\text{RN}(d_1) - R_X\,N(d_2)]\,e^{-R_f T^*}$$

$$P_0^* = \phi(\text{NP})\,[R_X(1 - N(d_2)) - R(1 - N(d_1))]\,e^{-R_f T^*}$$

$$d_1 = \frac{\ln(R/R_X) + (\sigma^2/2)T}{\sigma - \sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Example: Pricing a Caplet Consider a caplet with an exercise rate of $X = 7\%$, NP = \$100,000, $\phi = .25$, expiration = $T = .25$ year, and reference rate = LIBOR. If the current LIBOR were $R = 6\%$, the estimated annualized standard deviation of the LIBOR's logarithmic return were .2, and the continuously compounded risk-free rate were 5.8629%, then using the Black model, the price of the caplet would be 4.34.

$$C_0^* = .25(\$100{,}000)\,[.06(.067845) - .07\,(.055596)]\,e^{-(.058629)(.5)} = 4.34$$

where

$$d_1 = \frac{\ln(.06/.07) + (.04/2)(.25)}{.2\sqrt{.25}} = -1.49151$$

$$d_2 = d_1 - .2\sqrt{.25} = -1.59151$$

$$N(-1.49151) = .067845$$

$$N(-1.59151) = .055596$$

Example: Pricing a Cap Suppose the caplet represented part of a contract that caps a two-year floating-rate loan of \$100,000 at 7% for a three-month period. The cap consists of seven caplets, with expirations of $T = .25$ years, .5, .75, 1, 1.25, 1.5, and 1.75.

The value of the cap is equal to the sum of the values of the caplets comprising the cap. If we assume a flat yield curve such that the continuous rate of 5.8629% applies, and we use the same volatility of .2 for each caplet, then the value of the cap would be $254.38:

Expiration	Price of Caplet
0.25	4.34
0.50	15.29
0.75	26.74
1.00	37.63
1.25	47.73
1.50	57.04
1.75	65.61
	254.38

In practice, different volatilities for each caplet are used in valuing a cap or floor. The different volatilities are referred to as spot volatilities. They are often estimated by calculating the implied volatility on a comparable Eurodollar futures option.

Note

Even though the B-S OPM and the Black model can be used to estimate the equilibrium price of interest rate options and futures options, there are at least two problems. First, the OPM is based on the assumption that the variance of the underlying asset is constant. In the case of a bond, though, its variability tends to decrease as its maturity becomes shorter. Second, the OPM assumes the interest rate is constant. This assumption does not hold for options on interest-sensitive securities. In spite of these problems, the B-S OPM and the Black futures model are still used to value interest rate options.

PROBLEMS AND QUESTIONS

Note: The appendix problems can be done using the B-S OPM Excel program available on the Web site.

1. Suppose a T-bill futures is priced at $f_0 = 99$ and has an annualized standard deviation of .00175, and that the continuously compounded annual risk-free rate is 4%.
   - **a.** Using the Black futures option model, calculate the equilibrium price for a three-month T-bill futures call option with an exercise price of 98.95.
   - **b.** Using the Black futures option model, calculate the equilibrium price for a three-month T-bill futures put option with an exercise price of 98.95.

2. Suppose a T-bond futures expiring in six months is priced at $f_0 = 95{,}000$ and has an annualized standard deviation of .10, and that the continuously compounded annual risk-free rate is 5%.
   a. Using the Black futures option model, calculate the equilibrium price for a six-month T-bond futures call option with an exercise price of 100,000.
   b. Using the Black futures option model, calculate the equilibrium price for a six-month T-bond futures put option with an exercise price of 100,000.

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Additional Resources The PDF provides guides to information and uses for such sites and platforms as Bloomberg, FINRA, Moody's, Investinginbonds, the Federal Reserve, the Treasury, and government agencies.

PowerPoint Slides There are 22 chapters of slides for download.

Instructional Manual This is available for download only for professors.

Answers and Solutions to Select End-of-Chapter Problems

CHAPTER 1

1. In the private sector, real assets consist of both the tangible and intangible capital goods, as well as human capital, which are combined with labor to form the business. The business, in turn, transforms ideas into the production and sale of goods or services that will generate a future stream of earnings. The financial assets, on the other hand, consist of the financial claims on the earnings. These claims are sold to raise the funds necessary to acquire and develop the real assets. In the public sector, the federal government's capital expenditures and state and local governments' capital expenditures represent the development of real assets that these units of government often finance through the sale of financial claims on either the revenue generated from a particular public sector project or from future tax revenues.

2. The financial market can be described as a market for loanable funds; that is, a market where there is a supply and demand for loanable funds. The supply of loanable funds comes from the savings of households, the retained earnings of businesses, and the surpluses of units of government. The demand for loanable funds comes from businesses who need to raise funds to finance their capital purchases of equipment, plants, and inventories; households who need to purchase houses, cars, and other consumer durables; and the Treasury, federal agencies, and municipal governments who need to finance the construction of public facilities, projects, and operations. The exchange of loanable funds from savers to borrowers is done either directly through the selling of financial claims (stock, bonds, commercial paper, etc.) or indirectly through financial institutions.

11. Securitized assets are claims on a portfolio of loans. In creating a securitized asset, an intermediary will put together a package of loans of a certain type (mortgages, auto loans, credit cards, etc.). The institution then sells claims on the package to investors, with the claim being secured by the package of assets—securitized assets. The package of loans, in turn, generates interest and principal that is passed on to the investors who purchased the securitized asset.

16. An efficient market can be defined as one in which all information on which investors base their investment decisions are reflected in the security's price. In an efficient market, speculators, on average, would not earn abnormal returns. An inefficient market is one in which the information the market receives is asymmetrical: some investors have information that others don't, or some investors receive information earlier than others. In this market, the market price

is not equal to its equilibrium value at all times, and there are opportunities for speculators to earn abnormal returns.

18. For an instrument to be liquid it must be highly marketable and have little, if any, short-run risk. Thus, the difference between marketability and liquidity is the latter's feature of low or zero risk that makes the security cashlike.

CHAPTER 2

1.

$$V_0 = \sum_{t=1}^{5} \frac{80}{(1.08)^t} + \frac{1{,}000}{(1.08)^5} = 80\left[\frac{1-(1/1.08)^5}{.08}\right] + \frac{1{,}000}{(1.08)^5} = \$1{,}000$$

$$V_0 = \sum_{t=1}^{5} \frac{80}{(1.06)^t} + \frac{1{,}000}{(1.06)^5} = 80\left[\frac{1-(1/1.06)^5}{.06}\right] + \frac{1{,}000}{(1.06)^5} = \$1{,}084.25$$

$$V_0 = \sum_{t=1}^{5} \frac{80}{(1.10)^t} + \frac{1{,}000}{(1.10)^5} = 80\left[\frac{1-(1/1.10)^5}{.10}\right] + \frac{1{,}000}{(1.10)^5} = \$924.18$$

The problem shows the inverse relationship that exists between the price of a bond and its rate of return. From 8% to 6%, the percentage change in value is 8.425%; from 8% to 10%, the percentage change in value is –7.582%.

2. $1,000, $1,147.20, $877.11.
From 8% to 6%, the percentage change in value is 14.72%; from 8% to 10%, the percentage change in value is –12.289%.
Comment: The percentage changes in value for given changes in yields are greater for the 10-year bond than the 5-year bond. This illustrates the bond price relation that the greater the maturity on a bond the greater its price sensitivity to interest rate changes.

3. $680.58, $747.26, $620.92.
From 8% to 6%, the percentage change in value is 9.7975%; from 8% to 10%, the percentage change in value is –8.766%. Comment: The percentage changes in value for given changes in yields are greater for the 5-year, zero-coupon bond than the 5-year, 8% coupon bond than. This illustrates the bond price relation that the lower the coupon rate on a bond, the greater its price sensitivity to interest rate changes.

4. Values, Effective rates: Semiannual: $960.44, 9.2025%; Monthly: $959.86, 9.3807%; Weekly: $959.76, 9.4089%.

5. Monthly: 81.941; Weekly: 81.8889; Daily: 81.875; Continuous: 81.873.

6.

Yields	0.04	0.0425	0.045	0.0475	0.05	0.0525	....	0.06	....	0.07
Values	97.2449	97.0787	96.9132	96.7484	96.5842	96.4208	....	95.9343	....	95.2948

7.

a.

Yields	0.050	0.055	0.060	...	0.100
Values	123.384	119.034	114.877	...	87.538

b. The price change when the yield increases from 8% to 8.5% is –3.32.
c. The price change when the yield decreases from 8% to 7.5% is +3.47.
d. The capital gain and capital loss in b and c are not equal in absolute value. This suggests that gains and losses are not symmetrical.

8. Full price = $1,147.45; Accrued interest = $50; Clean price = $1,097.45.

9. Purchase price = $968.30, Selling price (full) = $1,048.85

10.

a. The dealer would offer to buy the bonds from investors at $951.25: ((95 + (4/32))/100) ($1,000) = (95.125/100)($1,000) = $951.25.
b. The dealer would be willing to sell the bond for $1,100.625: (110.0625/100)($1,000) = $1,100.625.
c. The dealer would be willing to buy the bond for $975: (97.5/100)($1,000) = $975.
d. The dealer would sell the bond for $947.87: $1,000/(1.055) = $947.87.
e. The dealer is willing to sell the bond for $9,943: $10,000[1−.04(52/360)] = $9,943.

11. 10%, $924.18; 10.5%, 906.43.

12. V_0 = 898.94; Effective yield = 12.36%

14. ARTM = 7.8776%, V_0 = $913.04.

15. ARTC = 9.42634%

16. a: Discount yield = 4.8%; b: YTM = 5.039%; c: Logarithmic return = 4.916%.

17. b: Yield = 8%:

18. c: Bond equivalent yield = 9.013%

19. a: $938.55; c: 1,046.23; d: HD value = $1,451.78, TR = 11.52%

20. a: TR = 10%; b: TR = 10.67%; c: TR = 9.43%

21. a: $98.75; b: $96.83

22. a: S_1 = .06; S_2 = .08; b: 100.14

23.

$$YTM_4 = \sqrt[4]{(1+y_1)(1+f_{11})(1+f_{12})(1+f_{13})} - 1$$

$$YTM_4 = \sqrt[4]{(1+y_2)^2(1+f_{12})(1+f_{13})} - 1$$

$$YTM_4 = \sqrt[4]{(1+y_1)(1+f_{21})^2(1+f_{13})} - 1$$

$$YTM_4 = \sqrt[4]{(1+y_1)(1+f_{31})^3} - 1$$

24. a: f_{11} = 8.62%; b: (1) Sell X short at \$945; (2) buy 945/870 = 1.0862 issues of Y; (3) after one year, cover short bond: cost = \$1,000; (4) at the end of year 2 (one year after covering the short position) receive (1.0862)1,000 = 1086.20 from original 2-year bond. Rate on one-year investment made one year from the present period: R_{11} = (1,086.20 – 1,000)/1,000 = 8.62%.

25. Use f_{mt} rule: Short in t-bond and long in $m + t$ bond. For example, for f_{11}: Short in 1-year bond (t) and long in 2-year ($m + t$) bond. After one year, you would cover your short position using your funds and then one year later you would collect on the original 2-year bond investment. Thus, the strategy results in a one-year investment to be made one year from now.

26. $f_{91,91} = 4.25\%$
Strategy: Prices on the T-bills per \$100 face value are $P(91) = 100/(1.0375)^{91/365} = 99.08637$ and $P(182) = 100/(1.04)^{182/365} = 98.0633$. To lock in an implied forward rate on a 91-day investment to be made 91 days from now, one could: (1) Short the 91-day bill: borrow the bill and sell it for 99.08637 (or borrow 99.08637 at 3.75% interest). (2) Use 99.08637 proceeds from the short sale to buy 99.08637/98.0633 = 1.0104327 issues of the 182-day bill. (3) At the end of 91 days, cover the short position: cost = 100 (equivalent of an 100 investment made 91 days in the future. (4) At the end of 182 days (91 days after covering the short position) collect on the 182-day investment: 1.0104327(100) = 101.04327. Annualized rate on 91-day investment made 91 days from now would be 4.25%.

CHAPTER 3

2.

a. Expansionary open market operation: Central bank buys bonds, decreasing the bond supply and shifting the bond supply curve to the left. The impact would be an increase in bond prices and a decrease in interest rates. Intuitively, as the central bank buys bonds, they will push the price of bond up and the interest rate down.
b. Economic recession: In an economic recession, there is less capital formation and therefore fewer bonds are sold. This leads to a decrease in bond supply and a leftward shift in the bond supply curve. The decrease in supply initially leads to an excess demand for bonds, given fewer bonds; this excess demand increases bond prices and lowers interest rates. The recession may also lead to a decrease in bond demand, shifting the bond demand curve to the left. This would, in turn, cause bond prices to fall and rates to increase. If the supply effect dominates the demand effect, then rates will fall; if the demand effect dominates, then rates will increase.
c. Treasury financing of a deficit: With a government deficit, the Treasury will have to sell more bonds to finance the shortfall. Their sale of bonds will increase the supply of bonds, shifting the bond supply curve to the right, and initially creating an excess supply of bonds. This excess supply will force bond prices down and interest rates up.

d. Economic expansion: In a period of economic expansion, there is an increase in capital formation and therefore more bonds are being sold to finance the capital expansion. This leads to an increase in bond supply and a rightward shift in the bond supply curve. The increase in supply initially leads to an excess supply for bonds, decreasing bond prices and increasing interest rates. The expansion may also lead to an increase in bond demand, shifting the bond demand curve to the right. This would, in turn, cause bond prices to rise and rates to decrease. If the supply effect dominates the demand effect, then rates will increase; if the demand effect dominates, then rates will decrease.

3. The increased riskiness on the one bond would cause its demand to decrease, shifting its bond demand curve to the left. That bond's riskiness would also make the other bond more attractive, increasing its demand and shifting its demand curve to the right. At the new equilibriums, the riskier bond's price is lower and its rate greater than the other. The different risk associated with bonds leads to a market adjustment in which at the new equilibrium there is a positive risk premium.

4. The markets are defined in terms of their risk premium, RP: RP = YTM on risky bond − YTM on risk-free bond. By definition, a risk-neutral market is one in which the RP is zero, a risk-averse market is one with a positive RP, and a risk-loving market is one in which the RP is negative.

5.

a. The expected dollar return on the risky bond is $775 [.75($1,000) + .25($100)]. In a risk-neutral market, the risk premium is equal to zero. As a result, the price of the risky bond is found by discounting its $775 by the risk-free rate of 5%. In this problem, the price of the risky bond is $738.0952 (= 775/1.05).

b. In a risk-averse market, investors would not want the risky bond when it is priced to equal the risk-free rate. This would cause the demand for the risky bond to decrease, decreasing its price and increasing its yields, and also cause the demand for the risk-free bond to increase, causing its price to increase and its yield to fall. In this problem, the price of the risky bond would fall below $738.0952, resulting in an expected yield that exceeds the initial risk-free yield of 5%, and the price of the risk-free bond would rise above $952.35, resulting in a risk-free yield lower than 5%. Combined, these market adjustments would result in a positive risk premium: RP = *E*(YTM on risky bond) – YTM on risk-free bond > 0.

c. In a risk-loving market, investors would want the risky bond when it is priced to equal the risk-free rate. This would cause the demand for the risky bond to increase, increasing its price and lowering its yields, and also cause the demand for the risk-free bond to decrease, causing its price to decrease and its yield to rise. In this problem, the price of the risky bond would increase above $738.0952, resulting in an expected yield that is less than the initial risk-free yield of 5%, and the price of the risk-free bond would fall below $952.35, resulting in a risk-free yield greater than 5%. Combined, these market adjustments would result in a negative risk premium: RP = E(YTM on risky bond) – YTM on risk-free bond < 0.

6. The decrease in the liquidity on the one bond would cause its demand to decrease, shifting its bond demand curve to the left. The decrease in that bond's liquidity would also make the other bond relatively more liquid, increasing its demand and shifting its demand curve to the right. Once the markets adjust to the liquidity difference between the bonds, then the less liquid bond's price would be lower and its rate greater than the more liquid bond. Thus, the difference in liquidity between the bonds leads to a market adjustment in which there is a difference between rates due to their different liquidity features.

7. If investors know with certainty the probabilities and payoffs on the risky bond, then they would know that by buying a number of such bonds they would earn \$775 per bond. That is, if they bought 100 risky bonds, 75 of the bonds would pay \$1,000 and 25 would pay \$100, yielding an average return of \$775 ([(75)(\$1,000) + (25)(\$100)]/100 = \$775). Or, if they were to buy risky bonds for the next 100 periods, then in 75 of the periods they would get a payoff of \$1,000 and in 25 of the periods, they would get \$100, yielding an average payoff of \$750. To be assured that they would earn an average of \$750 per bond, investors would have to buy a sufficient number of bonds (e.g., 100) or have sufficient number of time periods (e.g., 100). Buying a sufficient number of bonds now or over time can therefore eliminate the risk of a loss. For there to be a sufficient number of such bonds, in turn, requires a big enough market, which is a liquidity or marketability requirement. Such a requirement is not the case for the risk-free bond. Thus, for liquidity or marketability reasons, investors may price the so-called risky bond at a price to yield a rate above the risk-free bond of 5%.

8. ATY = 5.2%; P_0 = 1,051.92.

CHAPTER 4

1.
   a. Outline: Decrease in capital formation (S-T and L-T) ⇒ Fewer bonds sold (S-T and L-T) ⇒ Excess demand for bonds (S-T and L-T) ⇒ Bond prices increase and rates decrease. ∴ Downward shift in YCs
   b. Outline: Increase in capital formation (S-T and L-T) ⇒ More bonds sold (S-T and L-T) ⇒ Excess supply of bonds (S-T and L-T) ⇒ Bond prices decrease and rates increase. ∴ Upward shift in YCs
   c. Outline: Central bank buys S-T Treasuries (T-bills) ⇒ T-bill prices increase and rates decrease ⇒ Substitution effect in which the demand for S-T corporate securities increase, causing their prices to increase and their yields to decrease. ∴ Tendency for YCs to become positively sloped.
   d. Outline: Treasury sells L-T Treasuries (T-bonds) ⇒ T-bond prices decrease and yields increase ⇒ Substitution effect in which the demand for L-T corporate securities decreases, causing their prices to decrease and their rates to increase. ∴ Tendency for YCs to become positively sloped.

e. Outline: Treasury buys L-T Treasuries (T-bonds) ⇒ T-bond prices increase and yields decrease ⇒ Substitution effect in which the demand for L-T corporate securities increases, causing their prices to increase and their rates to decrease. ∴ Tendency for YCs to become negatively sloped.

2.

a. The combination of investors preferring short-term bonds investments, while corporations prefer to sell long-term bonds, would lead to an excess demand for short-term bonds and an excess supply for long-term claims. An equilibrium adjustment would have to occur in both markets. Specifically, the excess supply in the long-term market would force issuers to lower their bond prices, thus increasing bond yields and inducing some investors to change their short-term investment demands. In the short-term market, the excess demand would cause bond prices to increase and rates to fall, inducing some corporations to finance their long-term assets by selling short-term claims. Ultimately, equilibriums in both markets would be reached with long-term rates higher than short-term rates, a premium necessary to compensate investors and borrowers/issuers for the risk they've assumed.
b. In this case, the combination of investors preferring long-term bonds, while corporations prefer to sell short-term bonds, would lead to an excess supply for short-term bonds and an excess demand for long-term claims. The excess supply in the short-term market would force issuers to lower their bond prices, thus increasing bond yields and inducing some investors to change their long-term investment demands. In the long-term market, the excess demand would cause bond prices to increase and rates to fall, inducing some corporations to finance their long-term assets by selling long-term claims. Equilibriums in both markets would be reached with long-term rates lower than short-term rates.

3. Investors with horizon dates of two years can buy the two-year bond with an annual rate of 6%, or they can buy the one-year bond yielding 6%, then reinvest the principal and interest one year later in another one-year bond expected to yield 8%. In a risk-neutral market, such investors would prefer the latter investment since it yields a higher expected average annual rate for the two years of 7% ($[(1.06)(1.08)]^{1/2} - 1$). Similarly, investors with one-year horizon dates would also find it more advantageous to buy a one-year bond yielding 6% than a two-year bond (priced at \$890 = \$1,000/1.06^2) that they would sell one year later to earn an expected rate of only 4.037% ($E(P_{11}) = 1{,}000/1.08 = \$925.93$; $E(R) = (925.93/890) - 1 = .0437$). Thus, in a risk-neutral market with an expectation of higher rates next year, both investors with one-year horizon dates and investors with two-year horizon dates would purchase one-year instead of two-year bonds. If enough investors do this, an increase in the demand for one-year bonds and a decrease in the demand for two-year bonds would occur until the average annual rate on the two-year bond is equal to the equivalent annual rate from the series of one-year investments (or the one-year bond's rate is equal to the rate expected on the two-year bond held one year).

5.

a. Outline: (1) Given HD = 2 years, investors would prefer a series of one-year bonds at 10% ($= [(1.08)(1.12)]^{1/2} - 1$) to a two-year bond at 8% (also holds for investors with HD = 1 year). (2) Market Response: Demand for 2-year bonds would decrease, causing their price to decrease and their yield to increase, and the demand for 1-year bonds would increase, causing their price to increase and their yield to decrease. Impact: Tendency for YC to become positively sloped.

Outline: (1) Given the expectation of higher rates in the future, borrowers wishing to finance 2-year assets would prefer to issue two-year bonds at 8% rather than sell a series of one-year bonds at 10% = $[(1.08)(1.12)]^{1/2} - 1$. (2) Market Response: The supply of 2-year bonds would increase, causing their price to decrease and their yield to increase, and the supply of 1-year bonds would decrease, causing their price to increase and their yield to decrease. Impact: Tendency for YC to become positively sloped, complementing the impact of investors' response to the expectation.

b. Outline: (1) Given HD = 2 years, investors would prefer a 2-year bond at 10% to a series of one-year bonds at 9% (= $[(1.10)(1.08)]^{1/2} - 1$) (also holds for investors with HD = 1 year). (2) Market Response: Demand for 2-year bonds would increase, causing their price to increase and their yield to decrease, and the demand for 1-year bonds would decrease, causing their price to decrease and their yield to increase. Impact: Tendency for YC to become negatively sloped.

Outline: (1) Given the expectation of lower rates in the future, borrowers wishing to finance 2-year assets would prefer to issue a series of one-year bonds at 9% (= $[(1.08)(1.10)]^{1/2} - 1$) to two-year bonds at 10%. (2) Market response: The supply of 2-year bonds would decrease, causing their price to increase and their yield to decrease, and the supply of 1-year bonds would increase, causing their price to decrease and their yield to increase. Impact: Tendency for YC to become negatively sloped, complementing the impact of investors' response to the expectation.

c. Outline: (1) Given HD = 2 years, investors would be indifferent to a 2-year bond at 7% and a series of one-year bonds at 7% (= $[(1.06)\ (1.08)]^{1/2} - 1$) (also holds for investors with HD = 1 year). (2) Market response: No change in the yield curve.

Outline: (1) Given HD = 2 years, borrowers would be indifferent to issuing a 2-year bond at 7% and a series of one-year bonds at 7% (= $[(1.06)(1.08)]^{1/2} - 1$). (2) Market response: No change in the yield curve.

6.

a. $f_{11} = 9\%, f_{21} = 8.5\%, f_{31} = 7\%, f_{41} = 5.751\%$.

b. $f_{12} = 8\%, f_{22} = 6.01\%, f_{32} = 4.69\%$.

c. The expected rate of return from buying a three-year bond and selling it one year later is equal to the yield on the one-year bond of 7% if the implied forward rate, f_{21}, is used as the expected rate on a two-year bond one year later: $P_3 = 100/(1.08)^3 = 79.383$; $E(P_{21}) = 100/(1.085)^2 = 84.9455$; $E(R) = (84.9455/79.383) - 1 = .07$.

d. The expected rate of return from buying a bond of any maturity and selling it one year later is equal to the yield on the one-year bond of 7% if the implied forward rate is used as the expected rate on the bond one year later.

e. The expected rate of return from buying a four-year bond and selling it two years later is equal to the yield on the two-year bond of 8% if the implied forward rate, f_{22}, is used as the expected rate on a two-year bond two years later: $P_4 = 100/(1.07)^4 = 76.2895$; $E(P_{22}) = 100/(1.0601)^2 = 88.98285$; $E(R) = (88.9825/76.2895)^{1/2} - 1 = .08$.

f. The expected rate of return from buying a bond of any maturity and selling it two years later is equal to the yield on the two-year bond of 8% if the implied forward rate is used as the expected rate on the bond two years later.

7.

a. $P_0 = 98.61113$

b. $f_{11} = 7.002\%, f_{21} = 7.5035\%, f_{31} = 8.0047\%$

c. $E(P_{31}) = 97.5278$

d. If bonds are equal to their equilibrium prices and the implied forward rates are used as estimate of future rates, then the one-year expected rates will be equal to the current one-year rate. The one-year expected rate on the four-year, 7% bond held one year is 6%—the same as the one-year rate:

$$E(R) = \frac{7 + 97.5278}{98.61113} - 1 = .06$$

e. $f_{12} = 8.0071\%; f_{22} = 8.5094\%$

f. The expected price on the four-year, 7% coupon bond two years later is 97.357 $(= [7/(1.080071) + (107/(1.085094)^2] = 97.357)$. Assuming that the \$7 coupon is reinvested to year 2 at f_{11} of 7%, the expected rate of return for holding the four-year, 7% bond two years is 6.5%—the same as the two-year spot rate:

$$E(R_{22}) = \left[\frac{7(1.07) + 7 + 97.357}{98.61113}\right]^{1/2} - 1 = .065$$

8. (1) A contractionary open market operation in which the central bank sells short-term government securities. (2) A Treasury purchase of long-term Treasury securities. (3) A poorly hedged economy in which investors prefer long-term investments and borrowers prefer short-term. (4) A market expectation of lower interest rates.

9. The liquidity premium theory (LPT) posits that there is a liquidity premium for long-term bonds over short-term bonds because the prices of long-term securities tend to be more volatile and therefore more risky than short-term securities. According to LPT, if investors were risk averse, then they would require some additional return (liquidity premium) in order to hold long-term bonds instead of short-term ones. The yield curves in Questions 6 and 7 had no risk premium factored in to compensate investors for the additional volatility they assumed from buying long-term bonds. Thus a liquidity premium would need to be added to longer-term yields to reflect the additional risk associated with the longer-term bonds.

10. (1) The market expects higher interest rates in the future. (2) The market expects lower interest rates in the future. (3) Expected rates for holding bonds for a specified period (e.g., one year, two years) will be equal to the applicable current rate if expected rates are equal to implied forward. If investors expect future rate to be less than the applicable implied forward rate, then they would expect their holding period yields to exceed the current rates. Thus, the implied forward rates can be used as a cutoff rate in evaluating and selecting bonds. (4) The preferred habitat theory posits that investors and borrowers will move away from their preferred maturity segment if rates are attractive enough to compensate them for foregoing their preferences.

CHAPTER 5

1. During recessions investors are more concerned with safety than during expansionary times. As a result, a relatively low demand for lower grade bonds and a high demand for higher grades occur, leading to lower prices for the lower grade bonds and thus a higher interest premium. On the other hand, during periods of economic expansion there is usually less concern about default. This tends to increase the demand for lower grade bonds relative to higher grade, causing a smaller premium.

2. The negatively sloped yield curve for low-grade bonds suggests that investors have more concern over the repayment of principal (or the issuer's ability to refinance at favorable rates) than they do about the issuer meeting interest payments. This concern would explain the low demand and higher yields for short-term bonds, in which principal payment is due relatively soon compared to long-term bonds.

8. $TR_4 = 12.78448\%$; $TR_{10} = 9.889\%$

10. a. 9.806%; b. 8.232%

11. a. 10%; b. 10%. c. Since the TR stays at 10% at the different rates, there is no market risk.

12. The eight-year, 8.5% coupon bond given a flat yield curve at 10% has Macaulay duration of 6.03. The bond's modified duration is 5.482. Comment: This problem illustrates how an investor with an HD = 6 years can eliminate, or at least minimize, market risk by buying a bond that has a duration equal to the HD of 6.

13. a. For zero-coupon bonds, Macaulay's duration is equal to the bond's maturity. b. $w_1 = 1/3$ and $w_2 = 2/3$.

14.

Bond	Period Coupon, C	P	F	Period Yield, y	Periods to Maturity, M	Payments per Year, n	Modified Duration	Macaulay Duration	Convexity
a	90	1000.00	1000	0.0900	4	1	3.2397	3.5313	14.2221
b	0	708.42	1000	0.0900	4	1	3.6698	4.0000	16.8337
c	90	1000.00	1000	0.0900	5	1	3.8897	4.2397	20.1848
d	35	1000.00	1000	0.0350	20	2	7.1062	7.3549	64.2998
e	35	1000.00	1000	0.0350	6	2	2.6643	2.7575	8.7515
f	0	816.30	1000	0.0700	3	1	2.8037	3.0000	10.4812

CHAPTER 6

4. Even though it is true that a sinking fund provision benefits bondholders by allaying their concern over the ability of the issuer to pay the principal, many sinking funds have a provision that allows the issuer to buy up the requisite amount of bonds either at a stipulated call price or in the secondary market at its market price. This sinking fund call option provision benefits the issuer and is a disadvantage to the bondholder.

5. The average life is 8.5 years.

7. YTM = 14.64%

8. Call protection means that the bond cannot be called. Refunding protection means that the bond cannot be called from the proceeds of certain types of refunding debt. Bonds that have refunding protection may still be called.

9. The initial year's call price is equal to the offering price of $950 plus the $100 coupon: $1,050. The call price will then decrease by $2.50 each year [= ($1,050 − $1,000)/20] to equal $1,000 at the end of year 20. The call price will be equal to $1,000 for years 21 through 25.

11. A release and substitution provision allows the issuer of a mortgage bond to sell the collateral in order to retire the bond; that is, the proceeds from the asset sale are used to retire the bond.

13. (1) A provision requiring the issuer to deliver to the trustee the pledged securities. (2) A provision allowing the company to retain its voting right if the collateral is the stock of one of its subsidiaries. (3) A provision requiring that the company maintains the value of its securities, positing addition collateral if the collateral decreases in value. (4) A provision allowing the company to withdraw the collateral provided there is an acceptable substitute.

15. The creditworthiness of debentures can be improved with protective covenants, subordination, and credit enhancements.

16. The guarantee does not eliminate the risk but shifts it from the corporation to the insurer.

19. The board of directors hires the managers and officers of a corporation. Since the board represents the stockholders, this arrangement can create a moral hazard in which the managers may engage in activities that could be detrimental to the bondholders. Since bondholders cannot necessarily seek redress from managers after they've made decisions that could harm them, they need to include in the bond indenture protective covenants that place restrictions on the company.

20. Some of the standard covenants specify the financial criterion that must be met before borrowers can incur additional debt (debt limitation) or pay dividends (dividend limitations). Other possible covenants include limitations on liens, borrowing from subsidiaries, asset sales, mergers and acquisitions, and leasing.

22. (a) A company is considered insolvent if the value of its liabilities exceeds the value of its assets; it is considered in default if it cannot meet its obligations.

Technically, default and insolvency are dependent: A company with liabilities exceeding assets will inevitably be in default when the future income from its assets is insufficient to cover future obligations on its liabilities. (b) A company can be illiquid and not insolvent. A company with assets that are not expected to generate a return for some time in the futures may very well be a solvent company.

25. The investment banker underwriting a bond issue bears the risk that the price of the issue could decrease during the time the bonds are being sold. A classic example discussed in the chapter was the $1 billion bond issue of IBM in 1979.

30. (1) Commercial paper is usually sold as a zero-coupon bond; (2) many have maturities less than 270 days to avoid registration; (3) CP is sold either directly (direct paper) or through dealers (dealer paper); (4) many CP issues include credit enhancements such as lines of credit.

32. Reverse inquiry is when institutional investors indicate to agents the type of maturity they want. The agent will inform the corporation of the investor's request; the corporation could then agree to sell the notes with that maturity from its MTN program, even if they are not posted.

33. As a source of financing, MTNs provide corporations with *flexibility* in their financing choices. Corporations selling an MTN through a shelf registration are able to enter the market constantly or intermittently, with the flexibility to finance a number of different short-, intermediate-, and long-term projects over a two-year period.

CHAPTER 7

2.

Year	Inflation	Inflation-Adjusted Principal	TIP Cash Flow
1	2%	$1020.00	$30.60
2	2%	$1040.40	$31.20
3	2%	$1061.21	$31.84
4	2%	$1082.43	$32.47 + $1082.43

4.

a. $S_{.5} = 5\%$, $S_1 = 5.25\%$, $S_{1.5} = 6.03\%$, $S_2 = 6.55\%$, $S_{2.5} = 7.08\%$, $S_3 = 7.62\%$.

b. $P_{.5} = 3.658537$, $P_1 = 3.560614$, $P_{1.5} = 3.431781$, $P_2 = 3.299674$, $P_{2.5} = 3.157399$, $P_3 = 83.18777$; total value of the strips = 100.2958.

c. An arbitrage exists by buying the three-year note for 100, stripping it into six zero coupons, and selling the strips for 100.2958.

d. If the actual yield curve matches the theoretical spot yield curve, then the proceeds from the sale of the strips would be equal to the cost of the three-year note of 100. Thus, no arbitrage exists: $P_{.5} = 3.658537$, $P_1 = 3.560614$,

$P_{1.5} = 3.430510$, $P_2 = 3.296742$, $P_{2.5} = 3.151571$, $P_3 = 82.902026$; total value of the strips = 100.

e. If the yield curve had yields equal to the YTMs on the Treasury securities, arbitrageurs could buy a T-note, strip it, and sell the strips for a risk-free profit. In contrast, if the yields on the yield curve were equal to the spot rates estimated from bootstrapping, then the arbitrage profit would be zero. Thus, the process of stripping will change the supply and demand for bonds, causing their yields to move to their theoretical spot yield curve levels.

6. a. 2.84% or 99.28.21; b. 11.9 billion; c. 68.75%; d. 2.798%; e. .042%.

8. Income comes from two sources: carry income and position profit. Carry income is the difference between the interest dealers earn from holding the securities and the interest they pay on the funds they borrow to purchase the securities. The position profit of dealers comes from long positions, as well as short positions. In a long position, a dealer purchases the securities and then holds them until a customer comes along. The dealer will realize a position profit if rates decrease and prices increase during the time she holds the securities. In contrast, in a short position, the dealer borrows securities and sells them hoping that rates will subsequently increase and prices will fall by the time he purchases the securities.

9. *On-the-run issues* are recently issued Treasury securities; they are the most liquid securities with a very narrow bid-ask spread. *Off-the-run issues* are Treasury securities issued earlier; they are not quite as liquid and can have slightly wider spreads.

10. The interdealer market is a market in which dealers trade amongst themselves. The market functions through government security brokers.

11. Under a repurchase agreement (RP), the holder of a security, such as a dealer, sells securities to another party, often a financial institution, with an agreement to buy the securities back at a later date and price. To the seller/repurchaser, the RP represents a collateralized loan, with the underlying securities serving as the collateral. To the buyer/reseller, the RP represents a secured investment. Their position is referred to as a reverse repo.

12.

a. The dealer would buy the notes and then per the repurchase agreement sell the acquired notes on the repurchase agreement with an agreement to buy them back the next day.

b. Interest = ($100,000,000)(.03)(1/360) = $8,333; Selling price = $99,991,667; Repurchase price = $100,000,000.

c. The dealer could use an open repo.

CHAPTER 8

2. In evaluating the creditworthiness of a GO bond, investors need to review the legal opinion to determine the state or local government's unlimited taxing authority. The legal opinion should identify if there are any statutory or constitutional limitations on the jurisdiction's taxing power, as well as any priority of claims on general funds. The legal opinion should also specify what the bondholders' redress is in the case of a default and whether there are any statutory or constitutional questions involved. Municipal defaults are usually handled through a restructuring, which makes the security and priorities as defined in the indenture and explained in the legal opinion important in establishing the type of new debt the holder will receive.

3. Municipal anticipation notes are municipal securities sold to obtain funds in lieu of anticipated revenues. They include tax-anticipation notes (TANs), revenue-anticipation notes (RANs), grant-anticipation notes (GANs), bond-anticipation notes (BANs), and municipal tax-exempt commercial paper. TANs and RANs are used to cover regular recurring government expenses. BANs are used as temporary financing or construction financing for long-term projects, with the principal paid from the proceeds from the sale of a long-term municipal bond.

4. If the tax rate increases, then more investors will find tax-exempt bonds relatively attractive; by contrast, if tax rates decrease, tax-exempt bonds become less attractive.

5. Until the Tax Reform Act of 1986, commercial banks were one of the larger purchasers of municipals. The lowering of the top corporate tax rate from 46% to 35%, though, led to a sizable reduction in their investments in municipal bonds.

6. Revenue bonds are used to finance capital projects such as roads, bridges, tunnels, airports, hospitals, power-generating facilities, water treatment plants, municipal buildings, university buildings, educational programs, inner-city housing development, and student loan programs.

7. Industrial revenue bonds are used by state and local governments to support private-public sector projects. They are often sold to finance the expansion of an area's industrial base or to attract new industries. Typically, the government or authority floats a bond issue and then uses the proceeds to build a plant or an industrial facility; it then leases the facility to a company or provides a low interest loan for the company to acquire the asset.
   Comment: Because of the tax-exempt status of municipal bonds, this type of financial arrangement benefits all parties: investors receive a higher after-tax yield, corporations receive lower interest rates on loans or lower rental rates, and the area benefits from a new or expanding industry.

8. For revenue bonds, some of the important provisions specified in the official statement and legal opinion that investors should consider are:
   (1) Whether the issuer can increase the tax or user's fee underlying the revenue source.
   (2) Whether the issuer can incur additional debt secured by the revenue of the project (referred to as minimum revenue clauses) or under what conditions new debt can be incurred.
   (3) How the revenues of the project are to be directed—if they are to be paid to bondholders after operating expenses but before other expenses (this is called

a net revenue structured revenue bond) or to bondholders first (this is called a gross revenue structured bond).

(4) Whether there are any additional collateral or guarantees.

9. (a) *Serial issue*: Many municipals are sold as a serial issue, with the bond issue broken into different maturities. (b) *Insured bonds*: Insured municipal bonds are ones secured by an insurance company. (c) *Letter-of-credit-backed municipal bonds* are municipals secured by a letter of credit from a commercial bank. (d) *Mello-Roos bonds* are municipal securities issued by local governments in California that are not backed by the full faith and credit of the government. (e) *Refunded bonds* are municipal bonds secured by an escrow fund consisting of high-quality securities such as Treasuries and federal agencies. There are also refunded municipals backed by an escrow fund consisting of a mix of Treasuries and non-Treasuries such as municipals. (f) *Moral obligation bonds* are bonds issued without the legislature approving appropriation. The bonds are therefore considered backed by the permissive authority of the legislature to raise funds, but not the mandatory authority.

CHAPTER 9

2. Today, dealers and brokers form the core of the primary and secondary markets for CDs, selling new CDs and trading and maintaining inventories in existing ones. Money-market funds, banks, bank trust departments, state and local governments, foreign governments and central banks, and corporations are the major investors in CDs.

3. In 1961, First Bank of New York issued a negotiable CD that was accompanied by an announcement by First Boston Corporation and Salomon Brothers that they would stand ready to buy and sell the CDs, thus creating a secondary market for CDs. The secondary market provided a way for banks to circumvent Regulation Q and offer investors rates competitive with other money market securities. Specifically, with Federal Reserve Regulation Q setting the maximum rates on longer term CDs (e.g., six months), and with those rates set relatively higher than shorter term CDs (e.g., three months), the existence of a secondary market meant that an investor could earn a rate higher than either a short- or longer-term CD by buying a longer term CD and selling it later in the secondary market at a higher price associated with the short-term maturity.

4. Bank notes are similar to medium-term notes. They are sold as a program consisting of a number of notes with different maturities typically ranging from one to five years and offered either continuously or intermittently. They differ from corporate MTNs in that they are not registered with the SEC, unless it is the bank's holding company, and not the individual bank, issuing the MTN.

5. Banker's acceptances (BAs) are time drafts (postdated checks) guaranteed by a bank—guaranteed postdated checks. The guarantee of the bank improves the credit quality of the draft, making it marketable. BAs are used to finance the purchase of goods that have to be transferred from a seller to the buyer.

They are often created in international business transactions where finished goods or commodities have to be shipped. The use of BAs to finance transactions is known as *acceptance financing* and banks that create BAs are referred to as *accepting banks*.

6. The U.S. company would obtain a letter of credit (LOC) from its bank. The LOC would say that the bank would pay the German company $20 million if the U.S. company failed to do so. The LOC would then be sent by the U.S. bank to the German company's bank. Upon receipt of the LOC, the German bank would notify the German manufacturing company who would then ship the drilling equipment. The German company would then present the shipping documents to the German bank and receive the present value of $20 million in local currency from the bank. The German bank would then present a time draft to the U.S. bank who would stamp 'accepted' on it, thus creating the BA. The U.S. company would sign the note and receive the shipping documents. At this point, the German bank is the holder of the BA. The bank can hold the BA as an investment or sell it to the American bank at a price equal to the present value of $20 million. If the German bank opts for the latter, then the U.S. bank holds the BA and can either retain it or sell it to an investor such as a money market fund or a BA dealer. If all goes well, at maturity the U.S. company will present the shipping documents to the shipping company to obtain the drilling equipment, as well as deposit the $20 million funds in its bank; whoever is holding the BA on the due date will present it to the U.S. bank to be paid.

10. A unit investment trust has a specified number of fixed-income securities that are rarely changed, and the fund usually has a fixed life. A unit investment trust is formed by a sponsor who buys a specified number of securities, deposits them with a trustee, and then sells claims on the security, known as redeemable trust certificates, at their net asset value plus a commission fee. The financial institution would purchase $100 million worth of 10-year Treasury bonds, place them in a trust, and then issue 100,000 redeemable trust certificates at net asset value of $1,000 plus commission: NAV = ($100,000,000/100,000) = $1,000. The financial institution's sale of trust certificates provides the proceeds to purchase the $100 million of T-bonds.

11. Hedge funds are structured so that they can be largely unregulated. To achieve this, they are often set up as limited partnerships. By federal law, as limited partnerships, hedge funds are limited to no more than 99 limited partners each with annual incomes of at least $200,000 or a net worth of at least $1 million (excluding home), or to no more than 499 limited partners each with a net worth of at least $5 million. Many funds or partners are also domiciled offshore to circumvent regulations. Hedge funds acquire funds from many different individual and institutional sources; the minimum investments range from $100,000 to $20 million, with the average investment being $1 million.

18. $1,425,761

22. (a) *Bank investment contracts* (BIC) are deposit obligations with a guaranteed rate and fixed maturity; they are similar to a GIC. (b) *Stable value investment* is the term used to describe investments in bank investment contracts and

guaranteed investment contracts. (c) *Bullet contract* is the term used to describe a generic GIC. (d) *Window GIC* refers to a GIC that allows for premium deposits to be made over a specified period, such as a year; they are designed to attract the annual cash flow from a pension or 401(k) plan. (e) *Floating-rate GIC contract* is a GIC in which the rate is tied to a benchmark rate.

CHAPTER 10

3. **Interest Equalization Tax**: In 1963, the U.S. government imposed the Interest Equalization Tax (IET) on the price of foreign securities purchased by U.S. investors. The tax was aimed at reducing the interest-rate difference between higher yielding foreign bonds and lower yielding U.S. bonds. Predictably, it led to a decline in the Yankee bond market. It also contributed to the development of the Eurobond market as more foreign borrowers began selling dollar-denominated bonds outside the U.S. The IET was repealed in 1974.
   Foreign Withholding Tax: In the 1970s there was a U.S. foreign withholding tax that imposed a 30% tax on interest payments made by domestic U.S. firms to foreign investors. There was a tax treaty, though, that exempted the withholding tax on interest payments from any Netherlands Antilles subsidiary of a U.S. incorporated company to non-U.S. investors. This tax-treaty led to many U.S. firms issuing dollar-denominated bonds in the Eurobond market through financing subsidiaries in the Netherlands Antilles. During this time, Germany also imposed a withholding tax on German DM-denominated bonds held by nonresidents. Even though the U.S and other countries with withholding taxes granted tax credits to their residents when they paid foreign taxes on the incomes from foreign security holdings, the tax treatments were not always equivalent. In addition, many tax-free investors, such as pension funds, could not take advantage of the credit (or could, but only after complying with costly filing regulations). As a result, during the 1970s and early 1980s, Eurobonds were often more attractive to foreign investors and borrowers than foreign bonds. In 1984, the U.S. and Germany rescinded their withholding tax laws on foreign investments and a number of other countries followed their lead by eliminating or relaxing their tax codes. Even with this trend, though, the Eurobond market had already been established.
4. A corporation or government wanting to issue a Eurobond will usually contact a multinational bank that will form a syndicate of other banks, dealers, and brokers from different countries. The members of the syndicate usually agree to underwrite a portion of the issue, which they usually sell to other banks, brokers, and dealers. The multinational makeup of the syndicate allows the issue to be sold in many countries.
7. In 1984, U.S. corporations were allowed to issue bearer bonds directly to non-U.S. investors. To accommodate U.S. investors, the SEC allows U.S. investors to purchase nonregistered Eurobonds after they are ‘seasoned.’ Thus, U.S. investors are locked out of initial offerings of Eurobonds, but can acquire them in the secondary market.

8. In a number of countries, commercial banks, instead of investment bankers, underwrite new bonds. In the secondary market, some countries trade bonds exclusively on exchanges, whereas others trade bonds on both the exchanges and through market makers on an OTC market. Bonds sold in different countries also differ in terms of whether they are sold as either registered bonds or bearer bonds. A foreign investor buying a domestic bond may also be subject to special restrictions. These can include special registrations, exchange controls, and foreign withholding taxes.

10. The Eurocurrency market is a market in which funds are intermediated (deposited or loaned) outside the country of the currency in which the funds are denominated. For example, a certificate of deposit denominated in dollars offered by a subsidiary of a U.S. bank incorporated in the Bahamas is a Eurodollar CD. Similarly, a loan made in yen from a bank located in the United States would be an American-yen loan.

    The Eurocurrency market is one of the largest financial markets. The underlying reason for this is that Eurocurrency loan and deposit rates are often better than the rates on similar domestic loans and deposits because of the differences that exist in banking and security laws among countries. Foreign lending or borrowing, regardless of what currency it is denominated in and what country the lender or borrower is from, is subject to the rules, laws, and customs of the foreign country where the deposits or loans are made. Accordingly, if a country's banking laws are less restrictive, then it is possible for a foreign bank or a foreign subsidiary of a bank to offer more favorable rates on its loans and deposits than it could in its own country by simply intermediating the deposits and loans in that country.

11.
    (a) In the 1950s, the Soviet Union maintained large dollar deposits in U.S. banks in order to participate in world trade. However, poor political relations, as well as U.S. claims on the Soviet Union originating from the Lend-Lease Policy, led to fears by the Soviet Union that the U.S. government could expropriate their deposits. As a result, the USSR transferred their dollar deposits to banks in Paris and London, thus creating the first modern-day Eurodollar deposit.
    (b) Under the old Bretton Woods exchange rate system, the central banks of many countries maintain holding of currencies in order to intervene in the foreign currency market to support their exchange rate. The dollar was the major currency held by central banks. This contributed to dollar deposits outside the U.S.
    (c) In the mid-1960s, U.S. banks began to go after Eurodollar deposits and loans by establishing foreign subsidiaries. What attracted many U.S. banks to the Eurodollar market were the opportunities around Federal Reserve rules.
    (d) In the late 1970s, many oil-exporting countries used the Eurodollar market, depositing large dollar deposits. Some of these petrodollars were used to make loans to oil-importing counties, leading the dollar deposits from oil revenues to be recycled.

(e) By the 1980s, the Eurodollar market had become the second largest market in the world, extending beyond Europe and intermediating in currencies other than the dollar. Accordingly, the market gave rise to the offshore banking centers in such areas as Nassau, Singapore, Luxembourg, and Kuwait. These areas had less-restrictive banking laws and thus became a place for intermediation between both foreign lenders and foreign borrowers.

12. The interbank Eurocurrency market is a market where Eurocurrency deposits are bought and sold. For example, a bank holding a $10 million Eurocurrency deposit might sell the deposit at a discount to a London Eurobank who might be arranging a $100 million loan by buying Eurocurrency deposits.

CHAPTER 11

1.

Item	Month 1	Month 2
Balance	100,000,000	99,907,884
Interest	666,667	666,053
p	733,765	733,581
Scheduled principal	67,098	67,528
CPR	.003	.006
SMM	.0002503	.0005014
Prepaid principal	25,018	50,058
Total principal	92,116	117,586
Cash flow	758,782	783,639

2. a.

Item	Month 1	Month 2
Balance	100,000,000	99,907,884
Interest	583,333	582,796
p	733,765	733,581
Scheduled principal	67,098	67,528
CPR	.003	.006
SMM	.0002503	.0005014
Prepaid principal	25,018	50,058
Total principal	92,116	117,586
Cash flow	675,449	700,382

b. The monthly fees on the MBS issue are equal to .08333% = (8% − 7%)/12 of the monthly balance. In the first month this is $83,333 and in the second month it is $83,257.

3. c.

	Month 1
Balance	50,000,000
Interest	354,167
p	633,379
Scheduled principal	258,379
CPR	.004
SMM	.000333946
Prepaid principal	16,611
Total principal	274,990
Cash flow	629,157

13.

Period Month	Collateral Balance	Collateral Interest	Collateral Scheduled Principal	Collateral Prepaid Principal	Collateral Total Principal	Stripped PO Cash Flow	Stripped IO Cash Flow
1	100,000,000	625,000	67,098	25,018	92,116	92,116	625,000
2	99,907,884	624,424	67,528	50,058	117,586	117,586	624,424

14. a.

Month	Collateral Balance	Collateral Interest	Collateral Principal	A Balance	A Interest	A Principal	B Balance	B Principal	B Interest
1	100,000,000	625,000	92,116	50,000,000	312,500	92,116	50,000,000	0	312,500
92	50,324,347	314,527	460,885	324,347	2,027	324,347	50,000,000	136,538	312,500
93	49,863,462	311,647	457,196	0	0	0	49,863,462	457,196	311,647

b.
- The first month's interest for Tranche A = (.07/12)($50,000,000) = $291,667
- The first month's interest for Tranche B = (.065/12)($50,000,000) = $270,833
- The first month's cash flow on the notional principal is $62,500: [(.075 − .07)/12]($50,000,000) + [(.075 − .065)/12]($50,000,000) = $62,500
- The quoted principal on the notional principal tranche is $10,000,000: [(.075 − .07)($50,000,000) + (.075 − .065)($50,000,000)]/.075 = $10,000,000

16.

Month	Collateral Balance	Collateral Interest	Collateral Principal	PAC Low PSA Principal	PAC High PSA Principal	PAC Minimum Principal	Support Principal
1	100,000,000	625,000	92,116	83,769	117,202	83,769	8,347

22.

Discount Rate/PSA	50	150
	Value	Value
5%	$57,565,779	$55,167,657
6%	$52,884,127	$52,005,608
7%	$48,833,080	49,174,840
8%	$45,306,641	46,629,042
Average Life	14.28	8.74

27. b.

PSA	Collateral	PAC	Support
	Average Life	Average Life	Average Life
50	14.28	7.48	20.86
100	10.98	6.60	18.89
150	8.74	6.60	12.59
200	7.16	6.60	8.17
250	6.01	6.60	4.96
300	5.16	6.60	2.58
350	4.51	5.97	2.39

CHAPTER 12

2.

SDA	Senior Principal	Subordinate
100	$480,965,975	$19,034,124
200	$462,613,559	$37,386,441
300	$444,920,709	$55,079,291

3. a. Senior interest = 83.33%. b. Subordinate interest = 16.67%. c. Tranches 5 and 6 would absorb $10 million (100% of their principal) and Tranche 3 would absorb $7 million (70% of its principal).

4.

Years after Issuance	Shifting Interest Percentage	% Prepayment to Senior Tranche
1–7	100%	100%
8	80%	96.67%
9	60%	93.33%
10	40%	90.00%
After 10	0%	83.33%

5. In a senior-subordinated structured deal, the step-down provision allows for reductions in the credit support over time provided there is no deterioration in the credit quality of the collateral. For a shifting-interest schedule in Question 4, the shift-down of credit starts in year 8 when the percentage of interest on the prepayment to senior goes down from 100% in year 7 to 80% in year 8; in year 9 from 80% to 60%; in year 10 from 60% to 40%; and after year 10 from 40% to 0. A step-down provision would, in turn, prohibit the step-downs if the credit quality of the collateral, as measured by certain tests (e.g., principal-loss test or delinquency test), is not met.

6. Excess spread (or excess interest) is the interest from the collateral that is not being used to meet liabilities (pass-through rate) and fees (mortgage servicing and administrative services). The excess spread is used to offset any default losses from the collateral. If the excess spread is retained in the structure rather than paid out, it can be accumulated in a reserve account and used to offset current and future losses. Excess interest could be set up as an excess interest-only (IO) class, with the proceeds going to a reserve account and paid out to IO holders at some future date if there is an excess.

7. Overcollateralization is common with subprime MBSs. The problem created with the 2008 financial crisis was the drop in housing values that left many MBSs undercollateralized.

8. Commercial loans are non-recourse. This means that the lender can only look to the income-producing property backing the loan for interest and principal. As a result, the lender monitors the credit performance in terms of key ratios such as the debt-to-service ratio and the loan-to-value ratio.

   Commercial mortgages often have call or prepayment protection, such as prepayment penalties, and defeasance (a contractual agreement that requires that the borrowers to invest in Treasury securities that would replicate the cash flows from of a projected prepayment schedule).

   Commercial mortgage loans are typically balloon loans. At the balloon date, the borrower must pay off the remaining balance. This is typically done by refinancing.

9.

   1. Commercial MBSs have greater prepayment protection than residential MBSs. There is more prepayment protection in the loans (lockout period, prepayment points, defeasance, etc.) and there can be prepayment protection via the tranches that redistribute prepayment (sequential-pay tranches or PACs).
   2. The underlying collateral on a commercial MBS are non-recourse, whereas the collateral for a residential MBS are recourse.
   3. Unlike residential MBSs, commercial MBSs benefit from a loan servicer. With many commercial property loans, a special servicer takes over if the loan is in imminent default. The servicer can modify the loan terms. The servicer can reduce the chance of default.
   4. Commercial MBS credit and prepayment tranches (sequential-pay, PACs, NIO, floaters, and senior-subordinated structures) are similar to nonagency residential MBSs.

5. Potential buyers of a commercial MBS may request the removal of certain loans, which is not the case for residential MBSs.

10. Two types of commercial MBS deals are single borrower/multiproperty deals and conduit deals. A single borrower/multiproperty deal involves one borrower, such as a developer, who has multiple development properties that are collectively financed by the commercial MBS issue. In a conduit deal, there is one conduit and a number of originators with different types of properties and from different geographical areas.

11. Credit analysis should be performed on a loan-by-loan basis and on an ongoing basis.

12. XYZ could set up a special purpose vehicle (SPV)—XYZ Trust. XYZ Inc. would then sell XYZ Trust $500 million of installment loans for $500 million cash. XYZ Trust would then sell $500 million in securities backed by the $500 million in loans.
XYZ Inc. alternatively could have raised $500 million by issuing corporate bonds, either as a debenture or collateralized by the installment loans. If XYZ were to default, all of its creditors would be able to go after all of its assets. If XYZ Inc. sells the installment loans to its SPV, XYZ Trust, then XYZ Trust owns the loans/assets and not XYZ Inc. If XYZ Inc. were forced into bankruptcy, the creditors of XYZ Inc. would not be able to recover the installment loans of XYZ Trust. When XYZ Trust issues ABSs, the investors will look only at the credit risk associated with the installment loans; i.e., credit risk is based only on that collateral. As a result, by financing with securitization via an SPV, the bonds or ABS may have a better credit rating and lower rate than would XYZ Inc. bonds.

13. The collateral backing an ABS are car loans or home equity loans in which the principal is amortized. As a result, the principal is paid off to the ABS holders. In contrast, the collateral backing an ABS is not amortized. Amortized ABSs are, in turn, subject to prepayment risk, whereas nonamortized ABSs are not.

14. Because the payment of principal on nonamortized ABSs does not follow a specified schedule, principal payments are managed by setting up periods when principal received from the collateral is invested (lockout period) and periods when the principal is paid to the ABS holder (amortizing period).

15. CARs differ from MBSs in that they have much shorter lives, their prepayment rates are less influenced by interest rates than mortgage prepayment rates, and they can include more credit enhancements.

16. 3.846%

17. a.

Tranche A2	Pay LIBOR + 100 Basis Points	− (LIBOR + 1%)
Swap	Pay 5%	− 5%
Swap	Receive LIBOR	+ LIBOR
Net	Pay 5% + 1%	− 6%

b.

Projected First-Year Cash Flow	
1. Interest from collateral: (.07)($300m)	$21m
2. Payment to A1 tranche: ($150m)(.05 + .015)	− $9.75m
3. Payment to A2 tranche: (LIBOR + .01)($100m)	− (LIBOR + .01)($100m)
4. Interest paid to swap counterparty: (.05)($100m) = $5m	−$5.0m
5. Interest received from swap counterparty: (LIBOR)($100m)	+ (LIBOR)($100m)
6. Payment to B tranche: (.05 + .02)($20m)	− $1.4m
Net	$3.85m
7. Payment to subordinate/equity tranche	− $3.85m

18. A common structure for a synthetic CDO is to issue CDOs to finance the purchase of high-quality bonds with the CDO manager then entering into credit default swap contracts as the seller to enhance the return.

CHAPTER 13

1. a.

	Current Bond: Five Yr, 10% Coupon Bond	**Substitute Bond: 10 Yr, 10% Coupon Bond**
Current value	100	100
Current Macaulay duration	4.17	6.76
Coupons	10	10
Bond price one year later	103.24	106.00
Dollar return one year later	13.24	16.00
One-year TR	13.24%	16%

b.

	Current Bond: Five Yr, 10% Coupon Bond	**Substitute Bond: 10 Yr, 10% Coupon Bond**
Current value	100	100
Current Macaulay duration	4.17	6.76
Coupons	10	10
Bond price one year later	96.90	94.96
Dollar return one year later	6.90	4.46
One-year TR	6.90%	4.46%

c. If interest rates are expected to decrease across all maturities, then a rate anticipation swap in which the investor sells her lower duration bonds and buys higher duration ones would provide a greater upside gain in value if rates decrease but also a greater loss in value if rates increase.

2. a.

	Current Bond: 15 Yr, 7% Coupon Bond	**Substitute Bond: 3 Yr, 10% Coupon Bond**
Current value per 100 face value	109.80	110
Coupons	7	10
Interest on interest +	0.105	0.15
Bond price one year later	100	105.51
Dollar return one year later	–2.695	5.66
One-year TR	–2.454%	5.145%

+3.5(1.03) + 3.5 – 7 = .105; 5(1.03) + 5 – 10 = .15

4.

a. The price on the AAA two-year zero-coupon bond is 89, and the price on the BBB two-year zero-coupon bond is 86.533. The strategy is go short in AAA bond and long in BBB bond: Short AAA Bond at 89; use 89 proceeds to buy n = 89/86.533 = 1.0285 issues of the two-year BBB bond.
b. Cash flow if the yield on the AAA bond were 7% and the spread were 100bp a year later would be 1.774.
c. Cash flow if the yield on the AAA bond were 5% and the spread were 100bp a year later would be 1.791.
d. If the spread widens, profit on the position decreases with losses occurring when the spread is above 300 points.

5. Bond A is trading at par, 100, and Bond B is trading at 98.48. To take advantage of the mispricing, a yield pickup swap could be formed by going long in Bond B at 98.48 (the underpriced bond) and shorting Bond A at 100 (the overpriced bond) to realize an initial cash flow of 1.52. Since the bonds are identical, their prices will eventually converge. When this occurs, the arbitrageur can sell bond A, and then use the proceeds to buy Bond B and return the borrowed bond to the bond lender to close her short position. The risk in a yield pickup swap is that the bonds are not identical.

6. The fundamental objective of many credit analysis strategies is to determine expected changes in default risk. If changes in quality ratings of a bond can be projected prior to an upgrade or downgrade announcement, bond investors can realize significant gains by buying bonds they project will be upgraded, and they can avoid significant losses by selling or not buying bonds they project will be downgraded.

12. Suppose a bond investor realized a capital gain and also a capital gains tax liability. One way for the investor to negate the tax liability would be to offset the capital gain with a capital loss. If the investor were holding bonds with current capital losses, he could sell those to incur a capital loss to offset his gain. Except for the offset feature, though, the investor may not otherwise want to sell the bond. If this were the case, then the investor could execute a bond swap in which he sells the bond needed for creating a capital loss and then uses the proceeds to purchase a similar, though not identical, bond.

14. Using three durations ranges ($D < 4$; $4 \leq D \leq 7$; $D > 7$), two quality ratings (investment grade, IG, and speculative grade, SG), and the two sectors (municipals and corporate), the following 12 cells can be created:

C1 = D < 4, IG, Corp	C7 = D < 4, SG, Corp
C2 = 4 ≤ D ≤ 7, IG, Corp	C8 = 4 ≤ D ≤ 7, SG, Corp
C3 = D > 7, IG, Corp	C9 = D > 7, IG, SG, Corp
C4 = D < 4, IG, Mun	C10 = D < 4, SG, Mun
C5 = 4 ≤ D ≤ 7, IG, Mun	C11 = 4 ≤ D ≤ 7, SG, Mun
C6 = D > 7, IG, Mun	C12 = D > 7, IG, SG, Mun

To form a bond index portfolio with these cells, one would purchase bonds matching each of the cells with the funds allocated to each bond based on the cell's proportion to the index.

17.

(a) Match strategy (1) The \$10 million liability at the end of year 4 is matched by buying \$9,433,962 worth of four-year bonds: \$9,433,962 = \$10,000,000/1.06. (2) The \$7 million liability at the end of year 3 is matched by buying \$6,069,775 of three-year bonds: \$6,069,776 = (\$7,000,000 – (.06)(\$9,433,962))/1.06. (3) The \$12 million liability at the end of year 2 is matched by buying \$10,443,185 of two-year bonds: \$10,443,185 = (\$12,000,000 – (.06)(\$9,433,962) – (.06)(\$6,069,776))/1.06. (4) The \$2 million liability at the end of year 1 is matched by buying \$418,099 of one-year bonds: \$418,099 = (\$2,000,000 –(.06)(\$9,433,962) (.06)(\$6,069,776) – (.06)(\$10,443,185))/1.06.

(b) The total investment: \$9,433,962 + \$6,069,776 + \$10,443,185 + \$418,099 = \$26,365,021

1 Year	2 Total Bond Value Outstanding	3 Coupon Income	4 Maturing Principal	5 Liability	6 Ending Balance (3) + (4) – (5)
1	\$26,365,021	\$1,581,901	\$418,099	\$2,000,000	0
2	\$25,946,923	\$1,556,815	\$10,443,185	\$12,000,000	0
3	\$15,503,738	\$930,224	\$6,069,776	\$7,000,000	0
4	\$9,433,962	\$566,038	\$9,433,962	\$10,000,000	0

19.

Cash Flow at Year 8 4%	Cash Flow at Year 8 8%
Price = 92.64	Price = 92.64
$5(1.04)^7 = 6.58$	$5(1.08)^7 = 8.57$
$5(1.04)^6 = 6.33$	$5(1.08)^6 = 7.93$
$5(1.04)^5 = 6.08$	$5(1.08)^5 = 7.35$
$5(1.04)^4 = 5.85$	$5(1.08)^4 = 6.80$
$5(1.04)^3 = 5.62$	$5(1.08)^3 = 6.30$
$5(1.04)^2 = 5.41$	$5(1.08)^2 = 5.83$
$5(1.04)^1 = 5.20$	$5(1.08)^1 = 5.40$
$5 = 5.00$	$5 = 5.00$
$P_8 = \frac{5}{1.04} + \frac{105}{(1.04)^2} = 101.89$	$P_8 = \frac{5}{1.08} + \frac{105}{(1.08)^2} = 94.65$
Target Value = 147.96 TR = .0603	Target Value = 147.83 TR = .0602

20.

a. *Duration-matching strategy*: At 4%: Target value = 134.12, TR = .0602; at 6%: Target value = 133.94, TR = .06; at 8%: Target value = 133.90, TR = .06. *Maturity-matching strategy*: At 4%: Target value = 139.80, TR = .0574; at 6%: Target value = 141.85, TR = .06; at 8%: Target value = 144.02, TR = .0627.

b. The duration-matching strategy yields the same target value and TR, whereas a maturity-matching strategy does not. A duration-matching strategy works by having offsetting price and reinvestment effects. In contrast, a maturity-matching strategy has no price effect and therefore no way to offset the reinvestment effect.

c.

Cash Flow at Year 6 after Yield Curve Shifts to 4% at End of Year 2	Cash Flow at Year 6 after Yield Curve Shifts to 8% at End of Year 2
Price = 94.42	Price = 94.42
$5(1.06)2(1.04)^3 = 6.32$	$5(1.06)2(1.08)^3 = 7.08$
$5(1.06)2(1.04)^2 = 6.08$	$5(1.06)2(1.08)^2 = 6.55$
$5(1.04)^3 = 5.62$	$5(1.08)^3 = 6.30$
$5(1.04)^2 = 5.41$	$5(1.08)^2 = 5.83$
$5(1.04) = 5.20$	$5(1.08). = 5.40$
$5 = 5.00$	$5 = 5.00$
$105/(1.04)^1 = 100.96$	$105/(1.08)^1 = 97.22$
Target Value = 134.59 TR = .06086	Target Value = 133.38 TR = .0593

After two years, the bond would have a maturity of five years and would be priced at 104.45 if rates were at 4% and 88.02 if rates were at 8%. At the 4% yield, the bond's duration would be 4.56 and at 8%, the duration would be 4.52; neither duration matches the remaining horizon period of five years. Comment: The total returns now differ given different rates, indicating market risk. In addition, the duration no longer matches the horizon period. The implication is that for a position to be immunized, the duration of the asset and the liability need to be matched at all times.

26.

a. MTV = $68,024,448; Safety margin = $3,538,387.
b. Value of the fund = $63,245,000; Safety margin = $9,245,000; TR = 12.35.
c. Value of the fund = $48,461,200; Safety margin = $3,813; TR = 8%.

CHAPTER 14

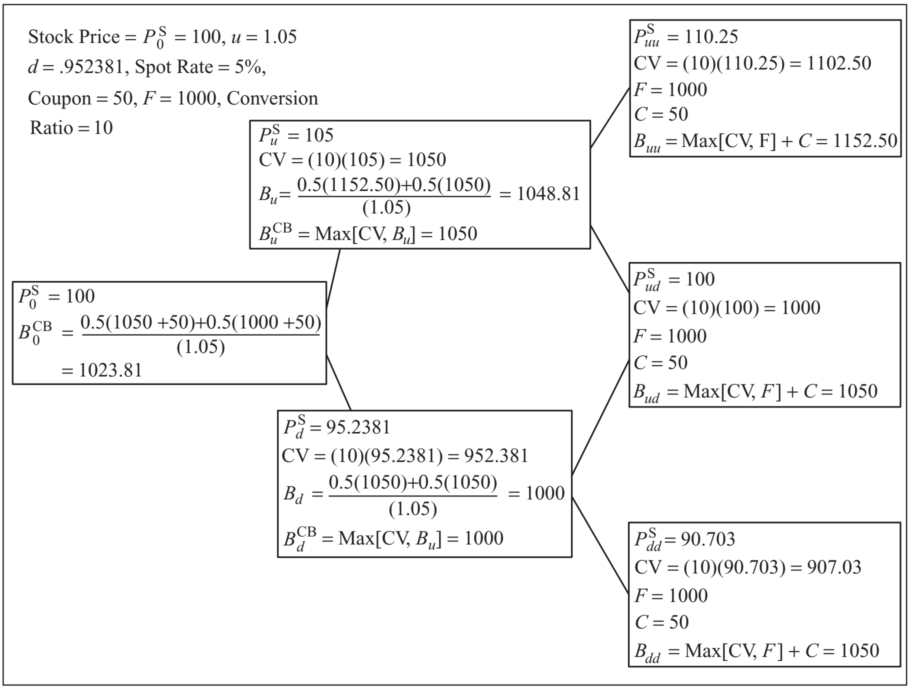
1.

a. S_{uu} = 12.10%, S_{ud} = 10%, S_{dd} = 8.2646%, S_u = 11%, S_d = 9.091%, S_0 = 10%.
b. 98.234
c. 97.81736
d. Call option values: $V_u{}^C = 0$, $V_d{}^C = .9166$, and $V_0{}^C = .416636$; callable bond values: $B_u{}^C = 98.1982$, $B_d{}^C = 99$, and $B_0{}^C = 97.81736$.
e. At spot rate 9.091% in period 1, the issuer could buy the bond back at 99 on the call option, financing the refunding by issuing a one-year bond at 9.091% interest. One period later the issuer would owe 99(1.09091) = 108; this represents a savings of 109 – 108 = 1. The value of that savings in period 1 is 1/1.09091 = .9166, which is equal to the value of the call option: 99.9166 – 99 = 0.9166.
f. 98.59845
g. Put option values: $V_u{}^P = .8018$, $V_d{}^P = 0$, and $V_0{}^P = .364454$; putable bond values: $B_u{}^P = 99$, $B_d{}^P = 99.9166$, and $B_0{}^P = 98.59845$.

2.

Period	Path 1	Geometic Mean	CF	PV
1	0.10000	0.100000	9	8.181818
2	0.09091	0.095446	109	90.833258
				99.015076
Period	**Path 2**	**Geometic Mean**	**CF**	**PV**
1	0.10000	0.10000	9	8.181818
2	0.11000	0.10499	109	89.271089
				97.452907
Weighted average value = (99.015076 + 97.452907)/2 = 98.234				

3. The value of the sinking-fund call option per \$100 face value is .416636. Since each option represents 1/3 of the issue, the value of the bond's sinking fund option is 0.13888, and the value of the sinking fund bond per \$100 face value is 98.09512. Thus, the total value of the \$9 million face value issue is \$8,828,561.
4. The value of the convertible bond is 1,023.81:



5.

   a. Binomial tree of spot rates: $S_{uu} = 6.05\%$, $S_{ud} = 5\%$, $S_{dd} = 4.13223\%$, $S_u = 5.5\%$, $S_d = 4.5454\%$, and $S_0 = 5\%$.
   b. Binomial tree of bond values: $B_{uu} = 99.009901$, $B_{ud} = 100$, $B_{dd} = 100.83333$, $B_u = 99.056825$, $B_d = 100.833339$, and $B_0 = 99.94772$.
   c. The value of three-period, option-free bond paying a 5% coupon per period and callable with a call price = CP = 100 is 99.550869. Binomial tree values: $B_{uu} = 99.009901$, $B_{ud} = 100$, $B_{dd} = 100$, $B_u = 99.056825$, $B_d = 100$, and $B_0 = 99.550869$.
   d. The value of three-period, option-free bond paying a 5% coupon per period and putable with put price = PP = 100 is 100.39684. Binomial tree values: $B_{uu} = 100$, $B_{ud} = 100$, $B_{dd} = 100.8333$, $B_u = 100$, $B_d = 100.833339$, and $B_0 = 100.39684$.

6.

Period	Path 1	Geometic Mean	CF	PV
1	0.050000	0.050000	5	4.761905
2	0.045454	0.047725	5	4.554868
3	0.041322	0.045586	105	91.856502
				101.173274

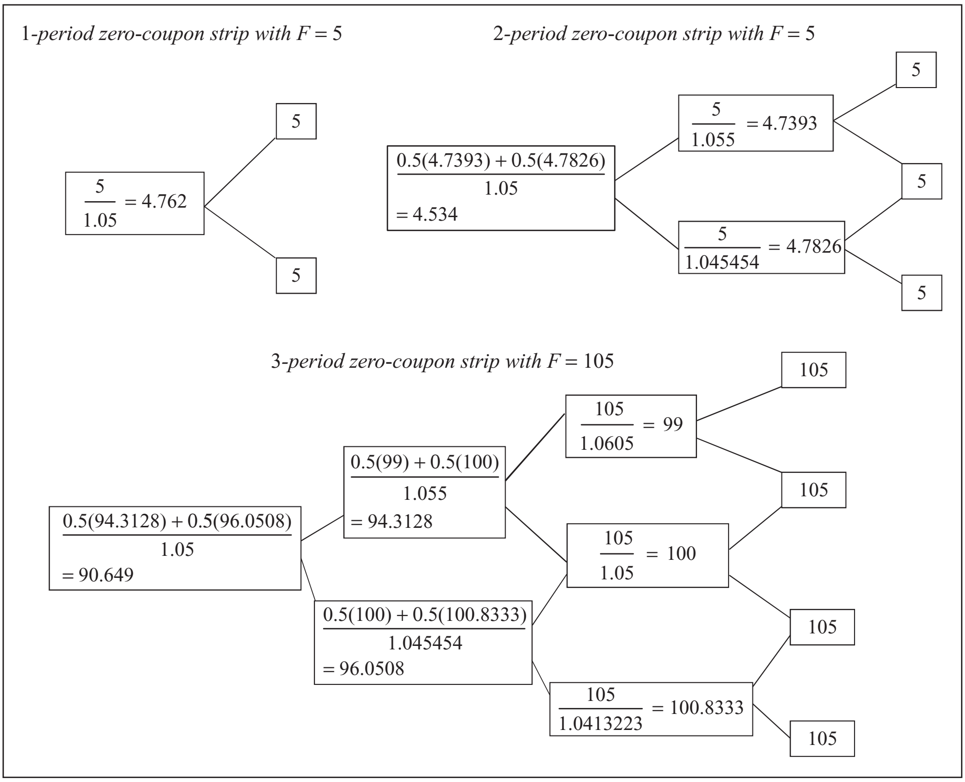
Period	Path 2	Geometic Mean	CF	PV
1	0.050000	0.050000	5	4.761905
2	0.045454	0.047725	5	4.554868
3	0.050000	0.048482	105	91.097356
				100.414129

Period	Path 3	Geometic Mean	CF	PV
1	0.050000	0.050000	5	4.761905
2	0.055000	0.052497	5	4.513654
3	0.050000	0.051664	105	90.273076
				99.548635

Period	Path 4	Geometic Mean	CF	PV
1	0.050000	0.050000	5	4.761905
2	0.055000	0.052497	5	4.513654
3	0.060500	0.055158	105	89.379283
				98.654842

Weighted average value = (101.173274 + 100.414129 + 99.548635 + 98.654842)/4 = 99.94772

7. The value of the one-period zero-coupon bond paying $F = 5$ is 4.762, the value of the two-period zero-coupon bond paying $F = 5$ is 4.534, and the value of the three-period zero-coupon bond paying $F = 105$ is 90.649. The sum of the values of these three stripped securities is 99.94—the same value found for the three-period 5% coupon bond.



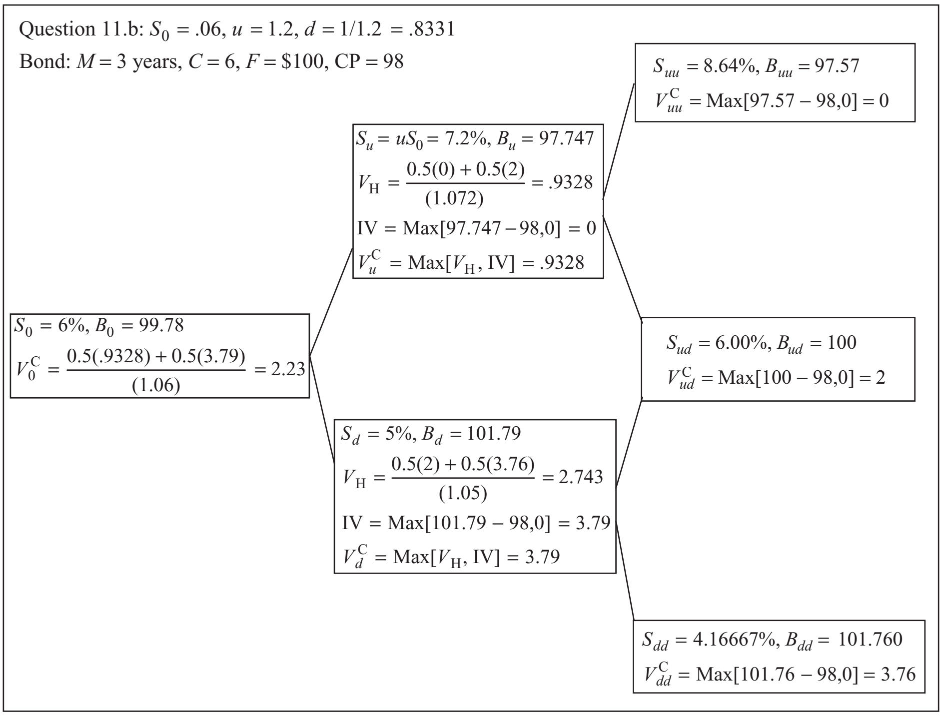
8. The value of the first sinking-fund call option per \$100 face value is .3968524 and the value of the second is .189785. Since each option represents 1/3 of the issue, the value of the bond's sinking fund option value is .19554, and the value of the sinking fund bond per \$100 face value is 99.752174. Thus, the total value of the \$9 million face value issue is \$8,977,696.

9. a. Conversion price = 100; b. Conversion value = 900; c. Straight debt value = 887; d. Minimum price of the convertible = 900; e. Arbitrage strategy: Buy convertible for \$880, convert to 10 shares of XYZ stock, then sell the stock at S_0 = \$90 per share for a profit of \$20: Profit = (10)(\$90) – \$880 = \$20.

10. a. 1,073.20; b. 1,071.47

11. a. $B_{uu} = 97.57$, $B_{ud} = 100$, $B_{dd} = 101.760$, $B_u = 97.747$, $B_d = 101.790$, and $B_0 = 99.78$.

b.



c. $B_{uu} = 97.57$, $B_{ud} = 98$, $B_{dd} = 98$, $B_u = 96.8142$, $B_d = 98$, and $B_0 = 97.55$.

12. Excel problems:

a. 95.50
b. 95.36
c. 100.31
d. 87.84
e. 87.76
f. 100.35

13.

Spot Rates	Bond Values Option Free	Bond Values Callable
0.040	115.33	100.00
0.045	110.93	100.00
0.050	106.75	99.63
0.055	102.76	98.08
0.060	98.97	95.85
0.065	95.36	95.83
0.070	91.92	90.48
0.075	88.65	87.60
0.080	85.52	84.84

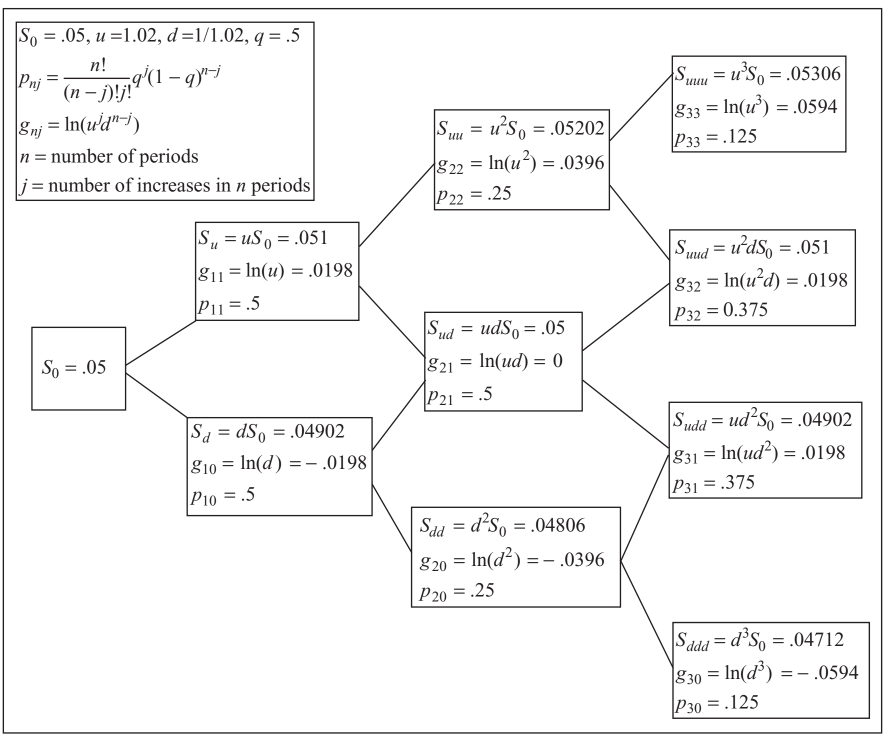
14. Paths:

Period	Path 1	Path 2	Path 3	Path 4
0	6.0000%	6.0000%	6.0000%	6.0000%
1	5.4545%	5.4545%	6.6000%	6.6000%
2	4.9587% balloon	6.0000% balloon	6.6000% balloon	7.2600% balloon

Note: With the balloon payment in period 2, Path 1 and Path 2 have the same cash flow, and Path 3 and Path 4 have the same cash flow.
Cash flows: Paths 1 and 2: CF_1 = \$335,223, CF_2 = \$804,359; Paths 3 and 4: CF_1 = \$195,578, CF_2 = \$955,176.
Discount rates: Paths 1 and 2: $Z_{10} = .08$, $Z_{20} = .0773$; Paths 3 and 4: $Z_{10} = .08$, $Z_{20} = .083$.
Path values and theoretical value: Path 1 = 1,003,461; Path 2 = 1,003,461; Path 3 = 995,470; Path 4 = 995,470; Theoretical value = 999,466.

CHAPTER 15

2. a.



b. $n = 1$: $E(r_1) = 0$, $V(r_1) = .000392$; $n = 2$: $E(r_2) = 0$, $V(r_2) = .000784$; $n = 3$: $E(r_3) = 0$, $V(r_3) = .001176$.
d. Suppose the estimated mean and variance for a period equal in length to $n = 3$ periods were $\mu_e = 0$ and $V_e = .001176$. Substituting these values into the formulas for u and d, we obtain $u = 1.02$ and $d = 1/1.02$.

5.

a. The logarithmic return is $g_n = \ln(S_T/S_0)$. Given the distribution of spot rates, the parameter values are $\mu_e = E(g_n) = .008952$ and $V_e = .002017$.
b. Since the distribution is for future spot rates after four months, the parameter values in (a) reflect average logarithmic returns for a four-month period. To annualize the parameters, one needs to multiply each by 3. Doing this yields: ${\mu_e}^A = 3(.008952) = .026856$ and ${V^A}_e = 3(.002016) = .006051$.
c.

Length	h	u	d
One Month	1/12	1.025001	.97999
One Week	1/52	1.011368	.98978
One Day	1/360	1.004183	.99598

d.

Length	h	u	d
One Month	1/12	1.02271	.97780
One Week	1/52	1.01084	.98927
One Day	1/360	1.00411	.99591

There is very little difference between the u and d values with the mean of .026856 and with a mean of zero when $n = 360$. This is consistent with the equations for u and d that show that as n gets large, the impact of the mean on u and d is small. Thus for large n the mean is not important in estimating u and d.

6.

a. The average logarithmic return is zero and the variance is .0057027.
b. The annualized mean and variance are ${\mu_e}^A = 4\ {\mu_e}^q = 4(0) = 0$ and ${V^A}_e = 4\ {V_e}^q = 4(.0057027) = .0228108$.
c.

Length of Period	u	d
Quarter: $h = 1/4$	1.078441	.927265
Month: $h = 1/12$	1.044564	.957337
Week: $h = 1/52$	1.021165	.979273
Day: $h = 1/360$	1.007992	.992071

7. Option-free bond: $B = 99.50$; callable bond: $B^C = 98.21$

8. a. 98.15. b. 97.75. c. 100.47.

10.

a. Given $\sigma = .10$ and $h = 1$, the variability condition governing the upper and lower rates is

$$S_u = S_d e^{2\sqrt{hV_e}}$$
$$S_u = S_d e^{2\sigma} = S_d e^{2(.10)}$$

Given the variability condition, the tree is generated by solving for the S_d value that makes the value of a two-period zero-discount bond obtained from the tree equal to its current equilibrium price. Given the spot rate on a two-period bond of $y_2 = .0804$, this requires solving for the S_d value where:

$$\frac{1}{(1.0804)^2} = \frac{.5[1/(1+S_d e^{2(.10)})] + .5[1/(1+S_d)]}{1.07}$$

Solving the above equation for S_d yields a rate of 8.148%. At $S_d = 8.148\%$, S_u is equal to 9.952%. A one-period binomial tree with upper and lower spot rates in period 1 of $S_d = 8.148\%$ and $S_u = 9.9952\%$ yields a price for a two-period zero-coupon bond with a face value of \$1 of .857—the same value as the equilibrium price.

b. Both S_u and S_d are greater than the current rate of 7%. This implies that we have calibrated the tree to a positively sloped yield curve.

c. Using the tree, the price of a two-period, 10.5% coupon bond is 104.5.

d. The equilibrium price on a two-period, 10.5% coupon bond is 104.5—the same as the price obtained using the calibrated binomial tree:

$$B_2^M = \frac{10.50}{1.07} + \frac{110.50}{(1.0804)^2} = 104.5$$

This illustrates that one of the features of a calibrated binomial tree is that it yields a value on an option-free bond that is equal to the bond's equilibrium price.

e. Using the tree, the price of a two-period, 10.5% coupon bond callable at CP =101 is 103.97.

12.

a. Given $\sigma = .10$ and $h = 1$, the variability condition governing the upper and lower rates is

$$S_u = S_d e^{2\sqrt{hV_e}}$$
$$S_u = S_d e^{2\sigma} = S_d e^{2(.10)}$$

Given the variability condition, the tree is generated by first solving for the S_d value that makes the value of a two-year zero-discount bond obtained from the tree equal to its current equilibrium price of .857 (= $1/(1.0804)^2$):

$$\frac{1}{(1.0804)^2} = \frac{.5[1/(1+S_d e^{2(.10)})] + .5[1/(1+S_d)]}{1.07}.$$

Solving the above equation for S_d yields a rate of 8.148%. Thus, at $S_d =$ 8.148%, S_u is equal to 9.952% and the binomial tree yields a bond price of .857 for two-period zero-coupon bond with $F = 1$.
Given the variability condition $S_d = 8.148\%$ and $S_u = 9.952\%$, the spot rates for period 2 are next found by solving for the S_{dd} value that makes the value of a three-year zero-discount bond obtained from the tree equal to its current equilibrium price of .77113 ($= 1/(1.0904952)^3$). The S_{dd} value that satisfies the above equation can be found iteratively (or take the hint); in this case, S_{dd} is 9.06%. Given $S_{dd} = 9.06\%$, by the variability condition S_{ud} is 11.06% and S_{uu} is 13.516%. This tree yields a price for a three-period zero-coupon bond with a face value of \$1 of .77113—the same value as the equilibrium price.

b. Using the tree, the price of a three-period, 10.5% coupon bond is 104.02.
c. The equilibrium price on a three-period, 10.5% coupon bond is 104.02—the same as the price obtained using the calibrated binomial tree.
d. Using the tree, the price of a three-period, 10.5% coupon bond callable at CP = 101 is 103.30.
e. Option-adjusted spread is .2772%.

14. If the market price is equal to 103.30 (the binomial value using the calibration model), then the option spread, k, is equal to zero. This reflects the fact that we have calibrated the tree to the yield curve and have considered all of the possibilities. If the market price is 102.80, the option-adjusted spread is 1%.

16. a. 1,000. b. The value of the bond's call feature is \$208.19. c. The callable bond is worth \$791.81.

17. a. 1,000. b. The value of bond's put feature is \$28.56. c. The value of the putable bond is \$1,028.56.

CHAPTER 16

6.

(1) Contract	(2) IMM Index	(3) f_0	(4) Implied YTM
March	93.764	\$984,410	.065
June	93.3092	\$983,273	.07
September	91.8607	\$979,652	.086

7.

LIBOR at Expiration %	Settlement Price $f_T = [(100 - \text{LIBOR}(.25))/100](\$1m)$	Short Futures Profit [$986,250 − f_T]	Long Futures Profit [f_T − $986,250]
4.75	988,125	−1,875	1,875
5	987,500	−1,250	1,250
5.25	986,875	−625	625
5.5	986,250	0	0
5.75	985,625	625	−625
6	985,000	1,250	−1,250
6.25	984,375	1,875	−1,875

8. Loss of $840.
9. 112.50 per $100 face value
10. 1.875 per $100 face value
12.

LIBOR	Long FRA Agreement	Short FRA Agreement
0.040	−48,833	48,833
0.045	−24,387	24,387
0.050	0	0
0.055	24,328	−24,328
0.060	48,596	−48,596

15. a. $49,375
b. and c.

Date	R_D %	Futures Price	Initial Margin	Value of Equity Account M_0 + (Futures Price − 987,500)	Deposit Required on Maintenance Margin	Value of Equity Account with Deposit
1-Mar	5.0	$987,500	$49,375	$49,375	$0	$49,375
2-Mar	5.1	$987,250	$49,375	$49,125	$250	$49,375
3-Mar	5.2	$987,000	$49,375	$48,875	$500	$49,375
4-Mar	5.0	$987,500	$49,375	$49,375	$0	$49,375
5-Mar	4.8	$988,000	$49,375	$49,875	$0	$49,875
8-Mar	4.7	$988,250	$49,375	$50,125	$0	$50,125
9-Mar	5.0	$987,500	$49,375	$49,375	$0	$49,375

16.

a. Ms. Hunter needs to go short in 10 September T-bill futures contracts at $f_0 = \$985{,}000$.
b. At delivery, Ms. Hunter closes the futures contracts at $f_T = S_T$ and sells the 10 T-bills on the spot market at S_T. Her hedged revenue is \$9.85 million regardless of rates.

17.

a. Ms. Hunter needs to go long in 10 September T-bill futures contracts at $f_0 = \$987{,}500$.
b. At delivery, Ms. Hunter closes the futures contracts at $f_T = S_T$ and buys 10 T-bills on the spot market at S_T. Her hedged cost is \$9.875 million.

18.

a. FRA Terms: (1) FRA would mature in two months (T) and would be written on a 90-day (three-month) LIBOR $T \times (T + M) = 2 \times 5$ agreement; (2) NP = \$10 million; (3) Contract rate = $R_k = 5.5\%$; (4) Day count convention = 90/365; (5) Cagle would take the short position on the FRA, receiving the payoff from Sun National if the LIBOR were less than $R_k = 5.5\%$; (6) Sun National would take the long position on the FRA, receiving the payoff from Cagle if the LIBOR were greater than $R_k = 5.5\%$.
b. Cagle's and Sun National's payoffs:

LIBOR	Sun National Payoff	Cagle Payoff
0.0500	−12,179	12,179
0.0525	−6,086	6,086
0.0550	0	0
0.0575	6,078	−6,078
0.0600	12,149	−12,149

c. Cash flows = \$10,135,616; Hedged rate = 5.5%

19.

a. Cagle's position would be equivalent to a long position in 10 Eurodollar futures at a futures price of 100 − (5.5%(.25)) = 98.625 per \$100 face value. If the LIBOR were 5% at expiration, then the settlement price would be $S_T = 98.75$ and Cagle's cash flow would be \$12,500:

$$10\left[\frac{f_0 - S_T}{100}\right](\$1{,}000{,}000) = 10\left[\frac{98.75 - 98.625}{100}\right](\$1{,}000{,}000) = \$12{,}500$$

At 5%, the present value of \$12,500 is \$12,500/(1+(.05(90/365)) = \$12,348, which is approximately equal to the value of the short FRA position. In contrast, if the LIBOR were 6%, then the settlement price would be $S_T = 98.5$ and Cagle's cash flow would be −\$12,500. At 6%, the present value of −\$12,500 is −$12,500/(1 + (.06(90/365)) = −$12,318, which is approximately equal to the value of the short FRA position.

b. Sun National's position would be equivalent to a short position in 10 Eurodollar futures at a futures price of 100 − (5.5%(.25)) = 98.625 per $100 face value. If the LIBOR were 5% at expiration, then the settlement price would be S_T = 98.75 and Sun National's cash flow would be −$12,500. At 5%, the present value of −$12,500 is −$12,500/(1 + (.05(90/365)) = −$12,348, which is approximately equal to the value of the long FRA position. In contrast, if LIBOR were 6%, then the settlement price would be S_T = 98.5 and Sun's cash flow would be $12,500. At 6%, the present value of $12,500 is $12,500/(1 + (.06(90/365)) = $12,318, which is approximately equal to the value of the long FRA position.

c. To hedge, Sun National would need to go long in 10 Eurodollar futures. With the hedge, Sun National's net position (Eurodollar + FRA) would be no more than $351 at the possible rates at expiration of 5%, 5.25%, 5.5%, 5.75%, and 6%.

LIBOR	Sun National Payoff FRA	Eurodollar Settlement Price: S_T = 100 − LIBOR(.25)	Cash Flow from Eurodollar Futures (10)([S_T − 98.625]/100) ($1m)	Net Position Eurodollar + FRA
0.0500	−$12,179	98.75	$12,500	$321
0.0525	−$6,086	98.69	$6,250	$164
0.0550	$0	98.63	$0	$0
0.0575	$6,078	98.56	−$6,250	−$172
0.0600	$12,149	98.50	−$12,500	−$351

21. Short-Answer Questions: (a) The introduction of financial futures; (b) Cross hedge; (c) Quality and quantity risk; (d) $4,750; (e) Arbitrageurs; (f) Open interest; (g) Through locals: scalpers, day-traders, and position-traders; (h) Dual trading; (i) $980,000; (j) 7.34%; (k) 98.75; (l) The Eurodollar contracts have cash settlement at delivery while T-bill contracts call for actual delivery; (m) $2,500; (n) 6.52%.

22.

a. f_0 = 98.7246 per 100 face value; YTM_f = 5.283%.

b. Create a synthetic 30-day investment by buying a 121-day bill and then going short at 98.8 in the T-bill futures contract expiring in 30 days. Her return would be 6.13%, compared to 5.15% from the 30-day spot T-bill.

c. A money manager currently holding 121-day T-bills could obtain an arbitrage by selling the bills for 98.318 and investing the proceeds at 5.15% for 30 days, then going long in the T-bill futures contract expiring in 30 days. Thirty days later the manager would receive 98.7246 from the investment and would pay 98 on the futures to reacquire the bills for a cash flow of .7246 per $100 par.

23. a.

(1) Contract	(2) Days to Expiration	(3) IMM Index	(4) f_0	(5) Implied YTM
March	91	93.7640	$984,410	.065
June	182	93.3092	983,273	.07
September	273	91.8607	979,652	.086

b. The implied 91-day repo rate is 6%.

c. Given a 91-day repo rate of 6%, the price on a 91-day spot T-bill with a face value of $1million would be $985,578 = $1,000,000/(1.06)$^{91/365}$.

d. Given the price of $970,223 on the spot 182 T-bill, the equilibrium price on the March contract would be $981,513 given a repo rate of 4.75%. If the IMM index price were 93.764, then the March futures would be trading at $984,410. In this case, the T-bill futures would be overpriced. To exploit this, an arbitrageur would short the March futures contract and borrow $970,223 at 4.75% for 91 days to finance the purchase of a 182-day T-bill. At expiration (91 days later), the arbitrageur would sell the T-bill on the futures contract for $f^M{}_0$ = $984,410, and repay the debt of $981,513 = $970,223 (1.0475)$^{91/365}$, for a positive cash flow of $2,897: Cash flow = $984,410 − $981,513 = $2,897.

CHAPTER 17

1. a. 8,000, 3,000, −2,000, 3,000, 8,000
b. −8,000, −3,000, 2,000, −3,000, −8,000
c. −10,000, −5,000, 0, 5,000, 10,000
d. 10,000, 5,000, 0, −5,000, −10,000
e. 17,000, 7,000, −3,000, 2,000, 7,000
f. −7,000, −2,000, 3,000, −7,000, −17,000

2. a. −500, −500, −500, 500, 500, 500
b. 500, 500, 500, −500, −500, −500

3. a. −250, −250, −250, 0, 250, 500, 750, 500, 250, 0, −250, −250, −250
b. 250, 250, 250, 0, −250, −500, −750, −500, −250, 0, 250, 250, 250

4. a. 1250, 0, −1250, −2500, −1250, 0, 1250
b. −1250, 0, 1250, 2500, 1250, 0, −1250
c. −3750, −2500, −1250, 0, 1250, 2500, 37500
d. 3750, 2500, 1250, 0, −1250, −2500, −3750

5. a. −25000, −25000, −25000, −25000, −25000, 0, 25000, 50000, 75000
b. 75000, 50000, 25000, 0, −25000, −25000, −25000, −25000, −25000
c. 25000, 25000, 25000, 25000, 25000, 0, −25000, −50000, −75000
d. −75000, −50000, −25000, 0, 25000, 25000, 25000, 25000, 25000

6. In this case, the American call option is selling below its intrinsic value ($C_t <$ Max$[f_T - X, 0]$ = Max[\$988,500 − \$987,500, 0] = \$1,000). An arbitrage opportunity exists by buying the call for \$900, exercising the futures call to obtain a long futures position at \$988,500 and a margin account of \$1,000, and then closing the futures position at \$988,500. By implementing this strategy, an arbitrageur would receive a positive cash flow of \$100. However, as arbitrageurs try to exploit this situation they will increase the price of the call as they try to go long in the option. The call price will increase until it is at least equal to the intrinsic value of \$1,000; at that price the arbitrage opportunity disappears. Note that this strategy requires an immediate exercising of the call. Thus, the strategy and condition applies only to an American call option and not European in which exercise only can occur at expiration.
7. In this case, the American put option is selling below its intrinsic value ($P_t <$ Max$[X - f_T, 0]$ = Max[\$987,500 − \$986,500, 0] = \$1,000). An arbitrage opportunity exists by buying the put for \$900, exercising the futures put to obtain a short futures position at \$986,500 and a margin account of \$1,000, and then closing the futures position at \$986,500. By implementing this strategy, an arbitrageur would receive a positive cash flow of \$100. However, as arbitrageurs try to exploit this situation they will increase the price of the put as they try to go long in the option. The put price will increase until it is at least equal to the intrinsic value of \$1,000; at that price the arbitrage opportunity disappears.
8. Options are derivative securities. They derive their values from the underlying asset. Thus, regardless of demand, the price of a call would increase if the price of the call's underlying security increased, and the price of a put would decrease if the price of the put's underlying security decreased.

12. Profit: −1187.50, −1187.50, −1187.50, −187.50, 812.50, 1812.50, and 2812.50. Break-even price = 98.1875.
13. Profit: 2218.75, 1218.75, 218.75, −781.25, −781.25, −781.25, and −781.25. Break-even price = 96.21875.
14. Given a risk-free rate of 6.0154%, expiration of .25 per year, spot T-bond priced at 96, and a T-bond call priced at 1.1875, the price on the 97 T-bond put should be .78125 using put call parity:

$$P_0 = \text{PV}(X) + C_0 - S_0$$

$$P_0 = \frac{97}{(1.060154)^{.25}} + 1.1875 - 96 = .78125$$

17. If a holder sells the option, she will receive a price that is equal to the intrinsic value plus the time value premium; if she exercises, though, her exercise value is only equal to the intrinsic value. Thus, by exercising instead of closing, she loses the time value premium. Thus, an option holder in most cases should close instead of exercise. An exception to the rule of closing instead of exercising would be a case in which the underlying security pays a coupon (or in the case of a stock, a dividend) that exceeds the time value premium.
18. Short-Answer Questions: (a) Limited profit and unlimited loss. (b) Open interest. (c) AMEX. (d) Upon exercise, the right to go long in a September T-bill futures

contract at the current futures price, with the assigned writer paying the holder the difference between the current futures price and the exercise price: $f_t - X = f_t - \$980{,}000$. (e) Upon exercise, the right to go short in a September T-bill futures contract at the current futures price, with the assigned writer paying the holder the difference between the exercise price and the current futures price: $X - f_t = \$980{,}000 - f_t$. (f) Long position in a futures, long position in a put, and short position in a call. (g) The options are equivalent if the futures contract expires at the same time as the option contracts, if the carrying-cost model holds, and the options are European. (h) True. (i) A cap is a series of interest rate call options. They can be purchased by borrowers in order to place a cap on a floating-rate loan. (j) A floor is a series of interest rate put options. They can be purchased by an investor in order to place a floor on a floating-rate investment.

CHAPTER 18

2. b.

1 Spot Discount Rates R_D %	2 $f_T = S_T$	3 Call Profit/Loss	4 Hedged Investment Funds \$18m + Col 3	5 Number of Bills Col (4)/Col(2)	6 YTM
6.00	\$985,000	\$22,500.00	\$18,022,500.00	18.30	0.0678
6.25	\$984,375	\$11,250.00	\$18,011,250.00	18.30	0.0679
6.50	\$983,750	\$0.00	\$18,000,000.00	18.30	0.0679
7.00	\$982,500	−\$22,500.00	\$17,977,500.00	18.30	0.0680
7.50	\$981,250	−\$22,500.00	\$17,977,500.00	18.32	0.0735
8.00	\$980,000	−\$22,500.00	\$17,977,500.00	18.34	0.0790

Call Profit $= 18[\text{Max}(f_T - \$982{,}500, 0] - \$22{,}500$
YTM $= [(\text{Number of Bills})(\$1\text{M})/\$18\text{M}]^{365/91} - 1$

3. b.

LIBOR (R)	.075	.09
(1) Spot 90-day CD price and closing Eurodollar futures price: $S_T = f_T = \$1{,}000{,}000/(1 + R)^{90/365}$	\$982,326	\$978,975
(2) Futures Profit: $\pi_f = n_f[f_0 - f_T] = 10.4055[\$980{,}000 - f_T]$	−\$24,203	\$10,666
(3) Debt on 1st CD: $\$10{,}000{,}000(1.0825)^{90/365}$	\$10,197,391	\$10,197,391
(4) Total funds to finance for next 90 days: Row 4 − Row 3	\$10,221,594	\$10,186,725
(5) Debt at end of the next 90 days: $[\text{Row 4}](1 + R)^{90/365}$	\$10,405,505	\$10,405,502
(6) Rate paid for 180-day period: Rate $= [\text{Row (5)}/\$10{,}000{,}000]^{365/180}$ (allow for slight rounding differences)	8.394%	8.394%

5.

a. To hedge his June bond portfolio purchase, Mr. Devine would go short in 87 T-bond contracts:

$$n_f = \frac{\text{Dur}_S}{\text{Dur}_f} \frac{S_0}{f_0} \frac{(1+\text{YTM}_f)^T}{(1+\text{YTM}_S)^T}$$

$$n_f = \frac{7}{9} \frac{\$10{,}000{,}000}{\$80{,}500} \frac{(1.095)^{15}}{(1.1025)^{15}} = 87 \text{ short contracts}$$

b. With the bond portfolio selling at 96 of par, Mr. Devine would sell the portfolio for \$9.6 million: (.96)(\$10,000,000) = \$9,600,000. Mr. Devine, though, would realize a profit of \$391,500 from the T-bond futures position, leaving him with revenue of \$9,991,500:

$$\text{Portfolio value} = \$10{,}000{,}000(.96) = \$9{,}600{,}000$$

$$\text{Futures profit} = 87[\$80{,}500 - \$76{,}000] = \$391{,}500$$

$$\text{Revenue} = \$9{,}600{,}000 + \$391{,}500 = \$9{,}991{,}500$$

6.

a. To obtain a synthetic fixed-rate loan, Zuber would obtain the floating-rate loan and go short in Eurodollar strip: short in 10 3/20 Eurodollar futures contracts and short in 10 6/20 contracts.

$$\text{Locked-in rate} = [100 - \text{index}] + \text{BP}/100$$

$$3/20{:}\, R_{3/20} = [100 - 91] + 1\% = 10\%$$

$$6/20{:}\, R_{6/20} = [100 - 92] + 1\% = 9\%$$

$$\text{Current rate} = 9.5\%$$

b.

1 Date	2 LIBOR	3 Futures Settlement Price: $f_T = 100 -$ LIBOR(.25)	4 Futures Profit*** Profit = $[(f_0 - f_T)/100]$ (\$1m)(10)	5 Quarterly Interest .25(LIBOR + .01) (\$10m)	6 Hedged Debt Col 5 − Col 4	7 Hedged Rate [(4)(Col 6)]/ \$10m
12/20	0.085			237,500	237,500	0.095
3/20	0.100	97.50	25,000	275,000	250,000	0.100
6/20	0.090	97.75	25,000	250,000	225,000	0.090

***$f_0(3/20) = 100 - 9(.25) = 97.75$
***$f_0(6/20) = 100 - 8(.25) = 98$

7. b.

1 Date	2 LIBOR	3 Futures and Spot Price $S_T = f_T$	4 Put Cash Flow at Option's Expiration 10(Max [977500 − f_T,0]	5 Value of Put Cash Flow at Payment Date (Put CF at T) $(1 + \text{LIBOR})^{.25}$	6 Quarterly Interest at Payment Date .25[(LIBOR + .01)]($10m)	7 Hedged Debt Col 6 − Col 5	8 Hedged Rate [(4)(Col 7)]/ $10m	9 Unhedged Rate LIBOR + 100bp
12/20	0.085							
3/20	0.100	$979,811.57	$0.00		$237,500.00	$237,500.00	0.0950	0.0950
6/20	0.090	$976,454.09	$10,459.10	$0.00	$275,000.00	$275,000.00	0.1100	0.1100
9/20		$978,686.00		$10,686.88	$250,000.00	$239,313.12	0.0957	0.1000

8.

a. To obtain a synthetic fixed-rate investment, XSIF could invest in the floating-rate note and go long in 10 Eurodollar strips: long 3/20, 6/20, and 9/20 Eurodollar futures contract.

$$\text{Locked-in rate} = [100 - \text{index}] + \text{bp}/100$$

$$3/20{:}\, R_{3/20} = [100 - 91] + 1\% = 10\%$$

$$6/20{:}\, R_{6/20} = [100 - 92] + 1\% = 9\%$$

$$9/20{:}\, R_{3/20} = [100 - 92.5] + 1\% = 8.5\%$$

$$\text{Current rate} = 9\%$$

$$\text{Synthetic fixed rate} = \left[(1.09)^{.25}(1.10)^{.25}(1.09)^{.25}(1.085)^{.25}\right]^{1} - 1 = .0912364$$

9. b.

1 Reset Date	2 Assumed LIBOR	3 Futures and Spot Prices $S_T = f_T$	4 Call Cash Flow at T 10(Max[f_T − 982500,0]	5 Value of Call CF at Payment Date (Call CF at T) $(1 + \text{LIBOR})^{.25}$	6 Interest Received on Payment Date (LIBOR + .01) (.25)($10m)	7 Hedged Interest Income Col (5) + Col (6)	8 Hedged Rate 4[Col (6)/ $10m]	9 Unhedged Rate LIBOR + 100bp
3/20	0.075							
6/20	0.070	$983,227.59	$7,275.88		$212,500.00	$212,500.00	0.0850	0.0850
9/20	0.065	$984,379.58	$18,795.84	$7,399.99	$200,000.00	$207,399.99	0.0830	0.0800
12/20	0.060	$985,538.36	$30,383.62	$19,094.10	$187,500.00	$206,594.10	0.0826	0.0750
3/20				$30,829.46	$175,000.00	$205,829.46	0.0823	0.0700

CHAPTER 19

1.

$$\text{Payoff} = (\$20{,}000{,}000)\frac{[\text{LIBOR} - .05](90/365)}{1 + \text{LIBOR}(90/365)}$$

LIBOR	Long FRA Agreement	Short FRA Agreement
0.040	−48,833	48,833
0.045	−24,387	24,387
0.050	0	0
0.055	24,328	−24,328
0.060	48,596	−48,596

3.

a. FRA terms:

- FRA would mature in two months (T) and would be written on a 90-day (three-month) LIBOR ($T \times (T + M) = 2 \times 5$ agreement
- NP = \$20M
- Contract rate = $R_k = 6\%$
- Day count convention = 90/365
- Glasgo would take the short position on the FRA, receiving the payoff from First National if the LIBOR were less than $R_k = 6\%$;
- First National would take the long position on the FRA, receiving the payoff from Glasgo if the LIBOR were greater than $R_k = 6\%$.

b and c. Cash flow = \$20,295,890; Hedged rate = 6%

LIBOR	First National Bank Payoff	Clasgo Payoff	Clasgo CD Investment \$20m + FRA Payoff	CF at CD Maturity CD Investment $(1 + \text{LIBOR})^{(90/365))}$	Hedged Rate $[(\text{CF at Maturity}/\$10\text{m}) - 1]^{(365/90)}$
0.0550	−\$24,328	\$24,328	\$20,024,328	\$20,295,890	0.0600
0.0575	−\$12,156	\$12,156	\$20,012,156	\$20,295,890	0.0600
0.0600	\$0	\$0	\$20,000,000	\$20,295,890	0.0600
0.0625	\$12,142	−\$12,142	\$19,987,858	\$20,295,890	0.0600
0.0650	\$24,269	−\$24,269	\$19,975,731	\$20,295,890	0.0600

4. Interest Rate Put Option: Exercise Rate = 7%, Reference Rate = LIBOR, NP = $18m, time period = .25,
Cost of Option = $100,000, payable at T

1 LIBOR	2 Interest Rate Put Payoff $18m[Max[.07 − LIBOR,0](.25)	3 Cost of the Option at T $100,000	4 Interest Received on CD at Its Maturity (LIBOR)(.25) ($18,000,000)	5 Revenues at Maturity Col (2) + Col 4	6 Annualized Hedged Rate 4[Col (5)/ ($18m + Col (3))]
0.060	$45,000	$100,000	$270,000	$315,000	0.0696
0.065	$22,500	$100,000	$292,500	$315,000	0.0696
0.070	$0	$100,000	$315,000	$315,000	0.0696
0.075	$0	$100,000	$337,500	$337,500	0.0746
0.080	$0	$100,000	$360,000	$360,000	0.0796

5. Company's Loan: $32m at LIBOR + 150BP for 270 days (.75 per year)
Interest Rate Call Option: Exercise Rate = 10%, Reference Rate = LIBOR, NP = $32m, Time Period = .75
Cost of Option = $150,000, payable at T

1 LIBOR	2 Interest Rate Call Payoff: $32m[Max[LIBOR − .10,0](.75)	3 Cost of the Option at T $150,000	4 Interest Paid on Loan at its Maturity (LIBOR + 150BP) (.75)($32m)	5 Cost at Maturity Col (4) − Col (2)	6 Annualized Hedged Rate (1/.75)[Col (5)/ ($32m−Col (3))]
0.080	$0	$150,000	$2,280,000	$2,280,000	0.0954
0.090	$0	$150,000	$2,520,000	$2,520,000	0.1055
0.100	$0	$150,000	$2,760,000	$2,760,000	0.1155
0.110	$240,000	$150,000	$3,000,000	$2,760,000	0.1155
0.115	$360,000	$150,000	$3,120,000	$2,760,000	0.1155
0.120	$480,000	$150,000	$3,240,000	$2,760,000	0.1155

6.

1	2	3	4	5	6	7
Date	LIBOR	Cap Payoff on Payment Date Max[LIBOR − .095,0]($150m)(.25)	Quarterly Interest .25(LIBOR + .01)($150m)	Hedged Debt Col 4 − Col 3	Hedged Rate [(4)(Col 5)]/ $150m	Unhedged Rate [(4)(Col 4)]/ $150m
12/20/Y1	0.090					
3/20/Y1	0.100	$0	$3,750,000	$3,750,000	0.100	0.100
6/20/Y1	0.095	$187,500	$4,125,000	$3,937,500	0.105	0.110
9/20/Y1	0.090	$0	$3,937,500	$3,937,500	0.105	0.105
12/20/Y2		$0	$3,750,000	$3,750,000	0.100	0.100

9.

1 Reset Date	2 Assumed LIBOR	3 Interest Paid on FRN on Payment Date (LIBOR + 150BP) (.25)($15M)	4 Cumulative Interest	5 Q-Cap Payoff on Payment Date If Cum. Int. > $700,000 Max[LIBOR − .07,0](.25)($15M)	6 Q-Cap-Hedged Interest Payment Col. (3) − Col. (5)	7 Hedged Rate 4[Col (6)/ $15M]	8 Unhedged Rate LIBOR + 150BP
3/20/Y1	0.075		$337,500				
6/20/Y1	0.080	$337,500	$693,750		$337,500	0.090	0.090
9/20/Y1	0.090	$356,250	$1,087,500	$0	$356,250	0.095	0.095
12/20/Y1	0.080	$393,750	$1,443,750	$75,000	$318,750	0.085	0.105
3/20/Y2	0.070	$356,250	$318,750	$37,500	$318,750	0.085	0.095
6/20/Y2	0.080	$318,750	$675,000	$0	$318,750	0.085	0.085
9/20/Y2	0.090	$356,250	$1,068,750	$0	$356,250	0.095	0.095
12/20/Y2	0.100	$393,750	$1,500,000	$75,000	$318,750	0.085	0.105
3/20/Y3		$431,250		$112,500	$318,750	0.085	0.115

CHAPTER 20

1. a.

1 Effective Dates	2 LIBOR	3 Floating-Rate Payer's Payment*	4 Fixed-Rate Payer's Payment**	5 Net Interest Received by Fixed-Rate Payer Column 3 − Column 4	6 Net Interest Paid by Floating-Rate Payer Column 4 − Column 3
3/23/y1	0.0550				
9/23/y1	0.0600	$275,000	$325,000	−$50,000	$50,000
3/23/y2	0.0650	$300,000	$325,000	−$25,000	$25,000
9/23/y2	0.0700	$325,000	$325,000	$0	$0
3/23/y3	0.0750	$350,000	$325,000	$25,000	−$25,000
9/23/y3	0.0800	$375,000	$325,000	$50,000	−$50,000
		$400,000	$325,000	$75,000	−$75,000

*(LIBOR/2)($10,000,000)
**(.065/2)*($10,000,000)

b.

1 Effective Dates	2 LIBOR	3 Floating-Rate Interest Payment*	4 Net Interest Received by Fixed-Rate Payer	5 Net Interest Paid Column 3 − Column 4	6 Rate 2(Column 5)/ 10M
3/23/y1	0.0550				
9/23/y1	0.0600	$275,000	−$50,000	$325,000	0.065
3/23/y2	0.0650	$300,000	−$25,000	$325,000	0.065
9/23/y2	0.0700	$325,000	$0	$325,000	0.065
3/23/y3	0.0750	$350,000	$25,000	$325,000	0.065
9/23/y3	0.0800	$375,000	$50,000	$325,000	0.065
		$400,000	$75,000	$325,000	0.065

*(LIBOR/2)($10,000,000)

c.

1 Effective Dates	2 LIBOR	3 Net Interest Received by Floating-Rate Payer	4 Interest Paid on Fixed-Rate Loan*	5 Net Interest Paid	6 Rate (2(Column(5))/ $10M
3/23/y1	0.0550				
9/23/y1	0.0600	$50,000	$300,000	$250,000	0.050
3/23/y2	0.0650	$25,000	$300,000	$275,000	0.055
9/23/y2	0.0700	$0	$300,000	$300,000	0.060
3/23/y3	0.0750	−$25,000	$300,000	$325,000	0.065
9/23/y3	0.0800	−$50,000	$300,000	$350,000	0.070
		−$75,000	$300,000	$375,000	0.075

*(.06/2)($10,000,000)

5. a. Synthetic Fixed-Rate Loan

Conventional Floating-Rate Loan	Pay Floating Rate
Swap: Fixed-Rate Payer Position	Pay Fixed Rate
Swap: Fixed-Rate Payer Position	Receive Floating Rate
Synthetic Fixed Rate	Pay Fixed Rate

c. Synthetic Fixed-Rate Investment

Floating-Rate Note Investment	Receive Floating Rate
Swap: Floating-Rate Payer Position	Pay Floating Rate
Swap: Floating-Rate Payer Position	Receive Fixed Rate
Synthetic Fixed Rate	Receive Fixed Rate

6. The swap bank will take the fixed-rate payer's position at 7.10%/LIBOR and the floating-rate payer's position at 7.2%/LIBOR.

7. a. Swap rate spreads: 5.49%–5.59%, 5.84%–5.88%, 6.01%–6.06%, and 6.44%–6.50%. Swap rates: 5.54%, 5.86%, 6.035%, and 6.47%.

8. Floating payer's net receipts: −59,189.50, −84,143.84, −33,772.83, −8,310.50, 42,477.17, and 67,522.83.

9.

a. With 3-year T-notes trading at 5%, the swap bank could offer a 5%/LIBOR par value swap.

b. The bank would add basis points to the fixed rate, with the basis points on its fixed payer's position being less than the points on its floating payer's position.

c. The swap bank would have a floating-payer's position on a 5%/LIBOR. To hedge its swap's 5% fixed-rate receipt and payment of LIBOR, the bank would short the three-year T-note and use funds to buy a three-year FRN paying LIBOR.

d. The swap bank would have a fixed-payer's position on a 5%/LIBOR. To hedge its swap's 5% fixed-rate payment and LIBOR receipt, the bank would short or issue a three-year FRN and use the funds to buy a three-year T-note yielding 5%.

10. To hedge the acquired 6%/LIBOR floating-payer position, the swap dealer would have to go short in \$50 million worth of three-year T-notes yielding 5% and long in \$50 million worth of three-year FRNs paying LIBOR. The combined swap and bonds positions would provide the dealer with \$250,000 cash inflow per semiannual period for three years.

Purchased Floating-Rate Payer Position on 6%/LIBOR Swap	
Swap: Floating-Rate Payer Position	Receive 6% Fixed Rate
Swap: Floating-Rate Payer Position	Pay LIBOR
Hedge	
Short: 5% T-note	Pay 5%
Long: FRN paying LIBOR	Receive LIBOR
	Receive 1% Semiannual receipts: \$50,000,000(.01)/2 = \$250,000

Thus, the maximum price the swap bank *would pay* the fixed payer for buying her swap would be equal to the present value of \$250,000 cash inflows received for the next six semiannual periods. Given a discount rate of 5%, the swap bank would be willing to pay the fixed payer up to \$1,377,031 for her swap.

$$SV_0 = \sum_{t=1}^{6} \frac{\$250,000}{(1+(.05/2))^t} = \$1,377,031$$

To hedge the acquired 6%/LIBOR fixed-payer position, the swap dealer would have to go short in \$50 million worth of three-year FRNs paying LIBOR and long in \$50 million worth of three-year T-notes yielding 5%. The combined swap and bonds positions would cost the dealer a \$250,000 per semiannual period for three years.

Purchased Fixed-Rate Payer Position on 6%/LIBOR Swap	
Swap: Fixed-Rate Payer Position	Pay 6% Fixed Rate
Swap: Fixed-Rate Payer Position	Receive LIBOR
Hedge	
Long: 5% T-note	Receive 5%
Short: FRN paying LIBOR	Pay LIBOR
	Pay 1% Semiannual payments: \$50,000,000(.01)/2 = \$250,000

Thus, the minimum price the swap bank *would charge* the fixed payer for buying her swap would be equal to the present value of the $250,000 loss each semiannual period for the next six periods. Given a discount rate of 5%, the swap bank would charge the fixed payer at least $1,377,031 for assuming her swap.

$$SV_0 = \sum_{t=1}^{6} \frac{-\$250,000}{(1+(.05/2))^t} = -\$1,377,031$$

11. In the first case, the maximum price the swap bank would pay the fixed payer for buying her swap is $1,377,031. In the second case, the swap bank would charge the fixed payer at least $1,377,031.

13. To hedge the assumed 5.5%/LIBOR fixed-payer's position, the swap dealer would have to take a floating-payer's position on new par value swap with a notional principal of $20 million. If the fixed rate on a new two-year swap were at 5%, the dealer would lose $50,000 per period for two years on the two swap positions given an NP of $20 million.

Purchased Fixed-Rate Payer Position on 5.5%/LIBOR Swap	
Swap: Fixed-Rate Payer Position	Pay 5.5% Fixed Rate
Swap: Fixed-Rate Payer Position	Receive LIBOR
Hedge	
Floating Payer Position on New 5%/LIBOR Par Value Swap	
Swap: Floating-Rate Payer Position	Pay LIBOR
Swap: Floating-Rate Payer Position	Receive 5%
	Pay 5% Semiannual payments: $20,000,000(.005)/2 = $50,000

Thus, the price the swap bank *would charge* the fixed payer for assuming (buying) his swap would be at least equal to the present value of $50,000 paid semiannually for the next four semiannual periods. Given a discount rate of 5%, the swap bank would charge the fixed payer at least $188,099 for assuming his swap.

$$SV_0 = \sum_{t=1}^{4} \frac{\$50,000}{(1+(.05/2))^t} = \$188,099$$

If the fixed rate on a new two-year swap were at 6%, the dealer would receive $50,000 per semiannual period for two years on the two swap positions given a NP of $20 million. Thus, the maximum price the swap bank *would pay* the fixed payer for buying his swap would be equal to the present value of $50,000 for the next four semiannual periods. Given a discount rate of 6%, the swap bank would be willing to pay the fixed-payer up to $185,855 for his swap.

14. The swap dealer would have to take a fixed-payer's position (NP = \$20 million) to hedge the acquired floating position. If the fixed rate on a new two-year swap were at 5%, the dealer would gain \$50,000 per period for two years on the two swap positions given an NP of \$20 million. Thus, the maximum price the swap bank would pay the fixed payer for buying his swap would be \$188,099. If the fixed rate on a new two-year swap were at 6%, the dealer would lose \$50,000 per period for two years on the two swap positions given an NP of \$20 million. Thus, the amount the swap bank would charge the fixed payer for assuming his swap would be equal to the present value of \$50,000 for the next four semiannual periods. Given a discount rate of 6%, the swap bank would charge the fixed payer \$185,855 for assuming his swap.

15.

a. The Moon Company has an absolute advantage in both the fixed and floating markets because of its higher quality rating. Moon, though, has a relative advantage in the fixed market where it gets 100bp less than Star, whereas Star has a relative advantage (or relatively less disadvantage) in the floating rate market where it only pays 50bp more than Moon. Thus, lenders in the fixed-rate market supposedly assess the difference between the two creditors to be worth 100bp, whereas lenders in the floating-rate market assess the difference to be only 50bp.

b. 50 basis points.

c. With this swap, the Star Company could then issue a \$150 million FRN paying LIBOR + 75bp. This loan, combined with the fixed-payer swap, would give Star a synthetic fixed rate loan paying 10.25%—25bp less than its direct fixed-rate loan. The Moon Company, on the other hand, could issue a \$150 million, 9.5% fixed-rate bond which, when combined with the floating-payer swap, would give Moon a synthetic floating-rate loan paying LIBOR—25bp less than the direct floating-rate of LIBOR plus 25bp.

17. In Question 15, lenders in the fixed-rate market supposedly assess the differences between the two creditors to be worth 100 basis points, whereas lenders in the floating-rate market assess the differences to be only 50 basis points. In Question 16, lenders in the fixed-rate market supposedly assess the Moon Company to be more creditworthy, offering the company a loan 25bp less than Star, whereas lenders in the floating-rate market assess Star to be more creditworthy, offering them a floating loan 25bp less than they offer Moon. According to the comparative advantage argument, whenever comparative advantage exists, arbitrage opportunities can be realized by each firm borrowing in the market where it has a comparative advantage and then swapping loans or having an swap bank set up a swap. The total arbitrage gain available to each party depends on whether one party has an absolute advantage in both markets or each has an absolute advantage in one market.

18.

a. The company needs to issue a five-year FRN paying LIBOR plus 100bp and take a fixed-rate payer's position on a swap with an NP of \$100 million.

b. 10%

c. For the synthetic loan to be preferred over the direct, the fixed rate on the swap must be less than 10%.

19.

a. To form a synthetic floating-rate note, the financial institution needs to issue $100 million worth of three-year fixed-rate notes at 7% and take a floating-rate payer's position on a three-year swap with a NP of $100 million.
b. 6%
c. For the synthetic floating-rate loan to be preferred over the direct, the fixed rate on the swap must be less than 6%.

20.

b. 6%
c. For the synthetic fixed-rate investment to exceed 7%, the swap rate must be greater than 6%.

21.

b. 6%
c. For the synthetic FRN to yield a rate greater that the LIBOR + 1%, the rate on the swap has to be less than 6%.

22. a. 2,132,551, b. –2,132,551, c. –2,132,551, and d. 2,132,551

CHAPTER 21

2.

a. To lock in the rate on its 10-year fixed-rate bonds two years from now, MEJ would need to enter a two-year forward contract to take a fixed-payers position on a 10-year, 7.25%/LIBOR swap with NP = $300 million.
b. At the forward swap's expiration, MEJ would issue a 10-year floating-rate note at 150 basis points above the LIBOR and assume its fixed-payer's position on the swap underlying the forward contract. Combined, the floating-rate note and fixed-payer's position on the swap represent the equivalent of a 10-year fixed-rate loan at 8.75%.

At the expiration date on the forward swap:

Instrument	Action	
Issue Flexible Rate Note	Pay	−LIBOR−150BP
Swap: Fixed-Rate Payer's Position	Pay Fixed Rate	−7.25%
Swap: Fixed-Rate Payer's Position	Receive LIBOR	+LIBOR
Synthetic Fixed Rate	Net Payment	8.75%

3.

a. With 10-year T-bonds trading at 7% and the fixed rate on 10-year par value swaps that Star Bank offers MEJ set at 150 bp above that yield, the current par value swap available to MEJ would have a fixed rate of 8.5%. At the forward swaps' expiration date, MEJ would therefore be able to sell its fixed-payer position on its 10-year 7.25%/LIBOR swap underlying the forward swap contract to Star for $24,926,936 given the swap is valued using the YTM approach with a discount rate of 8.5%:

$$SV^{fix} = \left[\sum_{t=1}^{20} \frac{(.085/2) - (.0725/2)}{(1 + (.085/2))^t}\right] \$300{,}000{,}000 = 24{,}926{,}936$$

b. With the $24,926,936 proceeds from closing its swap, MEJ would only need to raise $275,073,064 (= $300,000,000 − $24,926,936). Given MEJ bonds sell at 200 bp above T-bonds, MEJ would therefore issue $275,073,064-worth of 10-year fixed-rate bonds at 9% given T-bond rates are at 7%.

c. Its semiannual interest payments would be $12,378,288 (= (.09/2) ($275,073,064).

d. MEJ's interest payments equate to an annualized rate of 8.25% based on a $300 million debt.

Funds Needed	$300,000,000
−Proceeds from Swap	−$24,926,936
= Amount Borrowed	= $275,073,064
Semiannual Interest	(.09/2)($275,073,064) = $12,378,288
Annualized Rate Based on Funds Needed	(2)($12,378,288)/$300,000,000 = .0825

The forward swap position has therefore lowered the effective debt rate from 9% to 8.25%.

5.

a. To lock in the rate on a three-year $20 million fixed-rate bond investment one year from now, XSIF would need to enter a one-year forward contract to take a floating-payer's position on a three-year, 6.5%/LIBOR swap.

b. At the forward swap's expiration, XSIF would buy a three-year floating-rate note at 150 basis points above the LIBOR and assume its floating-payer position on the swap underlying the forward contract. Combined, the floating-rate note and floating-payer position on the swap represent the equivalent of a three-year fixed-rate investment at 8.0%:

At the expiration date on the forward swap:

Instrument		
Buy Flexible Rate Note	Receive: LIBOR + 150bp	LIBOR + 150bp
Swap: Floating-Rate Payer's Position	Pay LIBOR	−LIBOR
Swap: Floating-Rate Payer's Position	Receive 6.5%	+6.5%
Synthetic Fixed Rate	Net Receipt	8.0%

6.

a. With three-year T-notes trading at 5% and the fixed rate on a three-year par value swap that Fort Washington Bank offers XSIF set at 50bp above that yield, the current par value swap available to XSIF would have a fixed rate of 5.5%. At the forward swap's expiration date, XSIF would therefore be able to sell its floating-payer position on its three-year 6.5%/LIBOR swap underlying the forward swap contract to Fort Washington at \$546,237 given the swap is valued using the YTM approach with a discount rate of 5.5%:

$$SV^{fl} = \left[\sum_{t=1}^{6} \frac{(.065/2) - (.055/2)}{(1 + (.055/2))^t}\right] \$20{,}000{,}000 = \$546{,}237$$

If three-year T-notes were trading at 7%, then the current par value swap available to XSIF would have a fixed rate of 7.5%. At the forward swap's expiration date, XSIF would have to pay the swap bank \$528,507 for assuming its floating-payer position on its three-year, 6.5%/LIBOR swap:

$$SV^{fl} = \left[\sum_{t=1}^{6} \frac{(.065/2) - (.075/2)}{(1 + (.075/2))^t}\right] \$20{,}000{,}000 = -\$528{,}507$$

8.

Rates on Three-Year Par Value Swaps at Expiration R	Payer Swaption's Interest Differential Max((*R* − .06)/2,0)	Value of 6%/LIBOR Payer Swaption at Expiration PV(Max[(*R* − .06)/2, 0]($10M))	Payer Swaption Cost	Profit from Payer Swaption
0.040	0.000	\$0	\$100,000	−\$100,000
0.045	0.000	\$0	\$100,000	−\$100,000
0.050	0.000	\$0	\$100,000	−\$100,000
0.055	0.000	\$0	\$100,000	−\$100,000
0.060	0.000	\$0	\$100,000	−\$100,000
0.065	0.003	\$268,629	\$100,000	\$168,629
0.070	0.005	\$532,855	\$100,000	\$432,855
0.075	0.008	\$792,761	\$100,000	\$692,761
0.080	0.010	\$1,048,427	\$100,000	\$948,427

$$\text{Value of swap} = \left[\sum_{t=1}^{6} \frac{\text{Max}[(\text{R}/2) - (.06/2), 0]}{(1 + (\text{R}/2))^t}\right](\$20{,}000{,}000)$$

10.

Rates on Three-Year Par Value Swaps at Expiration *R*	Receiver Swaption's Interest Differential Max((.06 − *R*)/2,0)	Value of 6%/LIBOR Receiver Swaption at *T* PV(Max[(.06 − *R*)/2, 0]($10m))	Receiver Swaption Cost	Profit from Receiver Swaption
0.040	0.010	$1,120,286	$120,000	$1,000,286
0.045	0.008	$833,172	$120,000	$713,172
0.050	0.005	$550,813	$120,000	$430,813
0.055	0.003	$273,118	$120,000	$153,118
0.060	0.000	$0	$120,000	−$120,000
0.065	0.000	$0	$120,000	−$120,000
0.070	0.000	$0	$120,000	−$120,000
0.075	0.000	$0	$120,000	−$120,000
0.080	0.000	$0	$120,000	−$120,000

$$\text{Value of swap} = \left[\sum_{t=1}^{6} \frac{\text{Max}[(.06/2) - (\text{R}/2), 0]}{(1 + (\text{R}/2))^t}\right](\$20{,}000{,}000)$$

CHAPTER 22

1.

a. Swap arrangement:

(1) The U.S. company borrows $100 million at 10%, then agrees to swap it for C$142.857 million loan at 7%.

(2) The Canadian company borrows C$142.857 million at 7.5%, then agrees to swap it for $100 million loan at 10.6%.

The swap arrangement is made possible by the existence of a comparative advantage. The U.S. company has a comparative advantage in the U.S. market: it pays 1% less than the Canadian Company in the U.S. dollar market, compared to only .25% less in the Canadian dollar market. On the other hand, the Canadian Company has a comparative advantage in the Canadian market: it pays .25% more than the U.S. company on Canadian dollar loans, compared to 1% more on U.S dollars loans. When such a comparative advantage exists, a swap bank is in a position to arrange a swap to benefit one or both companies.

In this swap, the U.S. company benefits by paying .25% less than it could obtain by borrowing Canadian dollars directly in the Canadian market, and the Canadian company gains by paying .4% less than it could obtain directly from the U.S. market.

c. The swap bank in this case will receive $10.6 million each year from the Canadian company, while only having to pay $10 million to the U.S. company, for a net dollar receipt of $0.6 million. On the other hand, the swap bank will receive only C$10 million from the U.S. company, while having to pay C$10.714 million to the Canadian company, for a net Canadian dollar payment of C$0.714 million:

Swap Bank's Dollar Position	Swap Bank's C$ Position
Receives: (.106)($100m) = $10.6m	Receives: (.07)(C$142.857m) = C$10m
Pays: (.10)($100m) = $10m	Pays: (.075)(C$142.857m) = C$10.714m
Net $ Receipt: $10.6m − $10m = $0.6m	Net C$ Payment: C$10.714m–C$10m = C$0.714m

d. The swap bank has a position equivalent to a series of long currency forward contracts in which it agrees to buy C$0.714 million for $0.6 million each year. The swap bank's implied forward rate on each of these contracts is $0.84/C$:

$$E_f = \frac{\$0.6\text{m}}{\text{C}\$0.714\text{m}} = \frac{\$0.84}{\text{C}\$}$$

e. The swap bank could enter into forward contracts to buy C$0.714 million each year for the next five years at the forward rates shown in column 4. The forward rates as determined by IRPT are all less than $0.84/C$. As a result, the bank's dollar costs of buying C$0.714 million each year would be less than its $0.6 million annual inflow from the swap. By combining its swap position with forward contracts, the bank would be able to earn a total profit from the deal of $320,285:

1	2	3	4	5	6
Year	$ CF (millions)	C$ CF (millions)	Forward Exchange: $/C$	$ Cost of C$ (millions) Column (4) × Column (3)	Net $ Revenue (millions) Column (2) − Column (5)
1	$0.60	0.714	0.716355	$0.511477	$0.088523
2	$0.60	0.714	0.73309	$0.523426	$0.076574
3	$0.60	0.714	0.75022	$0.535657	$0.064343
4	$0.60	0.714	0.76775	$0.548174	$0.051827
5	$0.60	0.714	0.785687	$0.560981	$0.039019
					$0.320285

6. As an alternative to selling 10% of her higher quality bonds and buying seven-year B-rated ones, the manager could sell an equivalent amount of CDSs with seven-year terms and a credit spread of 3%. With the credit spread, the manager would be adding 3% to the 6% yield on her current bond holdings to obtain an effective yield of 9%. Thus with the CDS, the manager would be able to obtain an expected yield equivalent to the B-quality bond yield and would also be assuming the same credit risk associated with that quality of bond.

7. The vice president would need to buy a five-year CDS on the Jetgreen loan. The NP on the CDS would be $100 million and the spread would be approximately 5%.

8.
   a. An investor looking for a five-year risk-free investment would find it advantageous to create the synthetic risk-free investment with the BB bond and the CDS. That is, the investor could earn 1% more than the yield on the Treasury by buying the five-year BB corporate yielding 8% and purchasing the CDS on the underlying credit at a 2% spread.
   b. An arbitrageur could realize a free-lunch equivalent to a five-year cash flow of 1% of the par value of bond by shorting the Treasury at 5% and then using the proceeds to buy the BB corporate and then buying the CDS.
   c. Collectively, the actions of the investors and arbitrageurs would have the effect of pushing the spread on CDS from 2% to 3%—the same spread as the BB-bond.

9.
   a. The investor looking to invest in five-year BB bonds could earn 1% more than the 8% on the BB bonds by creating a synthetic five-year BB bond by purchasing the five-year Treasury at 5% and selling the CDS at 4%.
   b. An arbitrageur could realize a free-lunch equivalent to a five-year cash flow of 1% of the par value on the bond by shorting the BB-bond, selling the CDS, and then using proceeds to purchase five-year Treasuries. That is, for each of the next five years, the arbitrageur would receive 5% from her Treasury investment and 4% from her CDS, while paying only 8% on her short BB bond position. Furthermore, the arbitrageur's holdings of Treasury securities would enable her to cover her obligation on the CDS if there was a default. That is, in the event of a default she would be able to pay the CDS holder from the net proceeds from selling her Treasuries and closing her short BB bond by buying back the corporate bonds at their defaulted recovery price.
   c. Collectively, the actions of the investors and arbitrageurs would have the effect of pushing the spread on CDS from 4% to 3%—the same spread as the BB-bond.

11.

$$\text{PV(CDS payments)} = \sum_{t=1}^{5} \frac{(.03)(\$1)}{(1.05)^t} = \$0.129884$$

This implied probability is obtained by solving for the $\bar{p}$ that makes the present value of the expected payout equal to present value of the payments of $0.129884. In this problem, the implied probability is .042857:

$$\text{PV(Expected Payout)} = \text{PV(Payments)}$$

$$\sum_{t=1}^{M} \frac{\bar{p}\text{NP}(1-\text{RR})}{(1+R)^t} = \sum_{t=1}^{M} \frac{Z\text{NP}}{(1+R)^t}$$

$$\bar{p} = \frac{Z}{(1-\text{RR})}$$

$$\bar{p} = \frac{.03}{(1-.30)} = .042857$$

$$\text{PV(Expected payout)} = \sum_{t=1}^{M} \frac{\bar{p}\text{NP}(1-\text{RR})}{(1+R)^t} = \sum_{t=1}^{5} \frac{(.042857)(\$1)(1-.30)}{(1.05)^t}$$
$$= \$0.129884$$

12. The value of the CDS is equal to the present value of the expected payoff, which is \$0.113649:

$$\text{PV(Expected payout)} = \sum_{t=1}^{M} \frac{\bar{p}\text{NP}(1-\text{RR})}{(1+R)^t} = \sum_{t=1}^{5} \frac{(.0375)(\$1)(1-.30)}{(1.05)^t}$$
$$= \$0.113649$$

This implied spread is found by solving for the Z that equates the present value of the payments to the present value of the expected payout given the real-world probability of $\bar{p} = .0375$. In this problem, the implied spread is .026250.

14.

a. **Probabilities (%)**

Year	1	2	3	4	5
AA					
Cumulative Probability (%)	0.10	0.40	1.10	1.50	1.75
Unconditional Probability (%)	0.10	0.30	0.70	0.40	0.25
Conditional Probability p (%)	0.100000	0.300300	0.702811	0.404449	0.253807
Present Value of p at 6%	0.094339623	0.267266198	0.59009387	0.320361	0.189659

Probabilities (%)

Year	1	2	3	4	5
B					
Cumulative Probability (%)	6.00	13.00	20.00	28.00	36.00
Unconditional Probability (%)	6.00	7.00	7.00	8.00	8.00
Conditional Probability p (%)	6.0000	7.4468	8.0460	10.0000	11.1111
Present Value of p at 6%	5.660377358	6.627633064	6.7555574	7.92094	8.302869

b. B-Quality CDS

$$\text{PV(Expected payoff)} = \sum_{t=1}^{M} \frac{p_t \text{NP}(1 - \text{RR})}{(1 + R)^t}$$

$$\text{PV(Expected payoff)} = (\$1)(1 - .3)\left[\frac{.06}{(1.06)} + \frac{.074468}{(1.06)^2} + \frac{.080460}{(1.06)^3} + \frac{.10}{(1.06)^4} + \frac{.11111}{(1.06)^5}\right]$$

$$\text{PV(Expected payoff)} = .2468716$$

$$\sum_{t=1}^{M} \frac{\text{ZNP}}{(1 + R)^t} = \sum_{t=1}^{M} \frac{p_t \text{NP}(1 - \text{RR})}{(1 + R)^t}$$

$$Z\sum_{t=1}^{5} \frac{\$1}{(1.06)^t} = \$0.2468716$$

$$Z = \frac{\$0.2468716}{\sum_{t=1}^{5} \frac{\$1}{(1.06)^t}} = \frac{\$0.2468716}{\$4.212364} = .058606$$

15.

a.

Maturity	Spread	Implied Probabilty
t		
1	0.0490	0.0700
2	0.0500	0.0714
3	0.0510	0.0729
4	0.0520	0.0743
5	0.0530	0.0757

b.

$$\text{PV(Expected payoff)} = \sum_{t=1}^{M} \frac{p_t \text{NP}(1 - \text{RR})}{(1+R)^t}$$

$$\text{PV(Expected payoff)} = (\$1)(1 - .3)\left[\frac{.07}{(1.06)} + \frac{.0714}{(1.06)^2} + \frac{.0729}{(1.06)^3} + \frac{.0743}{(1.06)^4} + \frac{.0757}{(1.06)^5}\right]$$

$$\text{PV(Expected payoff)} = \$0.21434$$

$$\sum_{t=1}^{M} \frac{Z\text{NP}}{(1+R)^t} = \sum_{t=1}^{M} \frac{p_t \text{NP}(1 - \text{RR})}{(1+R)^t}$$

$$Z \sum_{t=1}^{5} \frac{\$1}{(1.06)^t} = \$0.21434$$

$$Z = \frac{\$0.21434}{\sum_{t=1}^{5} \frac{\$1}{(1.06)^t}} = \frac{\$0.21434}{\$4.212364} = .050884$$

16. With the credit spreads on new four-year BBB bonds at 2%, the swap bank would lose .5% of the NP for the next four years by assuming the buyer's position on the existing four-year, 2.5% CDS and hedging it with a seller's position on the new CDS:

Offsetting Swap Positions		
Buyer of 2.5% CDS Swap	Pay 2.5% of NP	Receive Default Protection
Seller of 2% CDS Swap	Receive 2.0%	Pay Default Protection
	Pay .5% per year	

$$\text{SV} = \sum_{t=1}^{4} \frac{(\text{Current spread} - \text{Exisitng spread})(\text{NP})}{(1+R)^t} = \sum_{t=1}^{4} \frac{-0.005(\$1)}{(1.06)^t} = -\$0.017326$$

With four years left on the current swap, the decrease in credit spread in the market has increased the value of the seller's position on the CDS swap by \$0.017326 from \$0.069302 to \$0.086628. The swap banks would therefore charge the swap holder at least \$0.017326 for assuming the buyer's swap.

APPENDIX I

1: a. Call value = .065; b. Put value = .0154.
2: a. Call value = 911; b. Put value = 5,788

GLOSSARY OF TERMS

accreting swap Swap in which the notional principal increases over time based on a set schedule.

accrual bond class A sequential-pay tranche whose interest is not paid but accrues until its principal payments are made.

accrued interest The interest on a bond or fixed-income security that has accumulated since the last coupon date.

active strategies Bond strategies that involve taking speculative positions.

after-acquired property clause Provision in a mortgage bond that dictates that all property or assets acquired after the issue be added to the property already pledged.

agency pass-throughs Mortgage-backed securities created by agencies: Federal National Mortgage Association, Government National Mortgage Association, and Federal Home Loan Mortgage Corporation.

American option An option which can be exercised at any time on or before the exercise date.

amortizing swap Swap in which the notional principal is reduced over time based on a schedule.

annual realized return (ARR) The annual rate earned on a bond for the period from when the bond is bought to when it is converted to cash (which could be either maturity or a date prior to maturity if the bond is sold), with the assumption that all coupons paid on the bond are reinvested to that date.

annualized discount yield Annualized return specified as a proportion of the bill's principal.

annualized mean The mean obtained by multiplying a mean defined for certain length of period (e.g., one week) by the number of periods of that length in a year (e.g., 52).

annualized variance The variance obtained by multiplying a variance defined for certain length of period (e.g., one week) by the number of periods of that length in a year (e.g., 52).

annuity An insurance product that pays the holder a periodic fixed income for as long as the policyholder lives in return for an initial lump-sum investment.

anticipation notes Municipal securities sold to obtain funds in lieu of anticipated revenues. They include tax-anticipation notes, revenue-anticipation notes, grant-anticipation notes, bond-anticipation notes, and municipal tax-exempt commercial paper.

arbitrage Transaction that provides a positive cash flow with no liabilities—a free lunch. An arbitrage opportunity exists when positions generating identical cash flows are not equally priced. In such cases the arbitrage is formed by buying the lower-priced position and selling the higher-priced one.

arbitrageur An individual who engages in arbitrage.

asked price The price at which a dealer offers to sell a security.

asset-based interest rate swap A swap used with an asset. In terms of synthetic positions, asset-based swaps can be used to create either fixed-rate or floating-rate investment positions.

assignment Procedure in which a brokerage firm or clearing firm selects one of its customers who is short in an option to fulfill the terms of the option after a holder has exercised.

assumed bond Bond whose obligations are taken over or assumed by another company or economic entity. In many cases such bonds are the result of a merger.

average cap Cap in which the payoff depends on the average reference rate for each caplet. For example, if the average is above the exercise rate, then all the caplets will provide a payoff; if the average is equal or below, the whole cap expires out of the money.

average life The average amount of time the debt will be outstanding. It is equal to the weighted average of the time periods, with the weights being relative principal payments.

average rate to maturity (ARTM) The average return per year as a proportion of the average price of the bond per year. It is used as an estimate of the YTM.

bank investment contract (BIC) Bank deposit obligation with a guaranteed rate and fixed maturity.

banker's acceptances (BAs) Time drafts (postdated checks) guaranteed by a bank.

banker's discount yield *See* annualized discount yield.

barbell strategy Bond strategy in which investments are in both short-term and long-term bonds.

barrier options Options in which the payoff depends on whether an underlying security price or reference rate reaches a certain level.

basis The difference between futures and spot prices.

basis points (bp) 1/100 of a percentage point. Fractions on bond yields are often quoted in terms of basis points.

basis risk *See* timing risk.

bearer bond Bond that pays coupons and principal to whoever has possession of the bond.

best effort Term used to refer to investment bankers who sell a security for the issuer for a commission.

beta A measure of the responsiveness of a change in a security's rate of return to a change in the rate of return of the market.

bid-asked spread The difference between the bid and ask prices.

bid price The price at which a dealer offers to buy a security.

Black model A model for pricing a futures option contract.

Black-Scholes option pricing model (B-S OPM) A model for valuing European options.

bond-equivalent yield The yield obtained by multiplying the semiannual periodic rate by two. Bonds with different payment frequencies often have their rates expressed in terms of their bond-equivalent yield so that their rates can be compared to each other on a common basis.

bond immunization Bond strategy aimed at minimizing market risk.

bond portfolio yield The yield for a portfolio of bonds that will make the present value of the portfolio's cash flow equal to the market value of the portfolio.

bond swap Active bond strategy that involves liquidating one bond group and simultaneously purchasing another.

bootstrapping A sequential process of estimating spot rates. The approach requires having at least one pure discount bond; it solves for the implied spot rates on coupon bonds.

Brady bond Bond issued by a number of emerging countries in exchange for rescheduled bank loans. The bonds were part of a U.S. government program started in 1989 to address the Latin American debt crisis of the 1980s.

break-even swap rate (also called the ***market rate***) The swap rate that equates the present value of the swap's fixed-rate payments to the present value of the swap's floating payments estimated using implied forward swap rates.

bull call money spread A vertical spread formed by purchasing a call at a certain exercise price and selling another call on the same security at a higher exercise price.

bull put money spread A vertical spread formed by purchasing a put at a certain exercise price and selling another put on the same security at a higher exercise price.

bulldog bonds Foreign bonds sold in the United Kingdom.

bullet strategy Strategy of constructing a bond portfolio that concentrates on one maturity area.

burnout factor Term used to refer to the tendency for mortgages to hit some maximum prepayment rate and then level off.

calendar spread *See* horizontal spread.

calibration model Binomial model that generates a binomial interest rate tree by solving for the spot rate that satisfies a variability condition and a price condition that ensures that the binomial tree is consistent with the term structure of current spot rates.

call An option that gives the holder the right to buy an asset or security at a specified price on or possibly before a specific date.

call market Market set up so that those wishing to trade in a particular security can do so only at that time when the exchange calls the security for trading.

call provision Provision in an indenture that gives the issuer the right to redeem some or all of the issue for a specific amount before maturity.

call risk The risk that the issuer/borrower will buy back a bond, forcing the investor to reinvest in a market with lower interest rates.

call spread A strategy in which one simultaneously buys a call and sells another call on the same stock but with different terms.

callable bond A bond that gives the issuer the right to buy back the bond from the bondholders at a specified price before maturity.

cancelable swap Swap in which one of the counterparties has the option to terminate one or more payments. Cancelable swaps can be callable or putable: A callable cancelable swap is one in which the fixed payer has the right to early termination; a putable cancelable swap is one in which the floating payer has the right to early cancelation.

cap A series of European interest rate calls that expire at or near the interest payment dates on a loan.

capital market Market where long-term securities (original maturities over one year) are traded.

caplet or ***interest rate call*** *See* interest rate option.

carry income Difference between the interest dealers earn from holding securities and the interest they pay on the funds they borrow to purchase the securities.

carrying-cost model (or ***cost-of-carry model***) A model for determining the equilibrium price on a futures contract. In this model the forward price equals the net costs of carrying the underlying asset to expiration.

cash-and-carry arbitrage A riskless strategy formed by taking opposite positions in spot and forward contracts on a security. This strategy underlies the carrying-cost model.

cash-flow matching strategy Strategy of constructing a bond portfolio with cash flows that match the outlays of the liabilities.

cash flow waterfalls The rules for the distribution of the cash flows that include the distribution of losses.

cash market *See* spot market.

cash-settlement A feature on some futures and options contracts in which the contract is settled in cash at delivery instead of an exchange of cash for the underlying asset.

certificates of deposit Short-term bank notes usually sold at their face value, with the principal and interest paid at maturity if the CD is less than one year.

cell matching A methodology for constructing an bond index fund based on decomposing the index into cells with each cell defining a different mix of features of the index (duration, credit rating, sector, etc.).

Chapter 11 fund A fund consisting of the bonds of bankrupt or distressed companies.

cheapest-to-deliver bond The least expensive bond (or note) among the Chicago Board of Trade's eligible bonds (or notes) a short holder of a Treasury bond (or note) futures contract can deliver.

classical immunization Bond immunization strategy of equating the duration of the bond to the duration of the liability.

clearinghouse A corporation associated with a futures or options exchange that guarantees the performance of each contract and acts as intermediary by breaking up each contract after the trade has taken place.

closed-end bond Bond that prohibits the company from incurring any additional debt secured by a first lien on the assets already being used as security.

closed-end fund Fund that has a fixed number of nonredeemable shares sold at its initial offering. Unlike an open-end fund, the closed-end fund does not stand ready to buy existing shares or sell new shares.

closing transactions Term used to describe closing an option or futures position. It requires taking an opposite position: selling an option or futures contract to close an initial long position; buying an option or futures contract to close an initial short position.

collar Combination of a long position in a cap and a short position in a floor with different exercise rates. The sale of the floor is used to defray the cost of the cap.

collateral trust bond Bond secured by a lien on equity shares of a company's subsidiary, holdings of other companies' stocks and bonds, government securities, and other financial claims.

collateralized debt obligations (CDOs) Securities backed by a diversified pool of one or more fixed-income assets or derivatives.

collateralized mortgage obligations (CMOs) Derivative securities formed by dividing the cash flow of an underlying pool of mortgages or a mortgage-backed security issue into several classes, with each class having a different claim on the mortgage collateral and with each sold separately to different types of investors.

combination matching A bond strategy that combines cash flow matching and immunization strategies.

commercial paper Short-term debt obligation usually issued by large, well-known corporations.

Commodity Futures Trading Commission (CFTC) The federal agency that oversees and regulates futures trading.

commodity-linked bond Bond that has its coupons and possibly principal tied to the price of a particular commodity.

conditional prepayment rate (CPR) Term used to define the annualized prepayment speed on a mortgage portfolio or mortgage-backed security.

confirmation The legal agreement governing a swap.

consul *See* perpetuity.

contingent immunization An enhanced immunization strategy that combines active management to achieve higher returns and immunization strategies to ensure a floor.

continuous market A market that provides constant trading in a security. Such markets operate through specialists or market makers who are required by the exchange to take temporary positions in a security whenever there is a demand.

continuously compounded return The rate of return in which the value of the asset grows continuously. The rate is equal to the natural logarithm of one plus the simple (noncompounded) rate.

contractual institutions Institutions such as life insurance companies, property and casualty insurance companies, and pension funds that obtain their funds from savings plans and from legal contracts to protect businesses and households from risk (premature death, accidents, etc.).

convenience yield A term used to describe when the benefits from holding an asset exceed the costs of holding the asset.

conventional pass-throughs Mortgage-backed securities created by private entities.

conversion A risk-free portfolio formed by going long in an underlying security, short in a European call, and long in a European put. The portfolio yields a certain cash flow equal to the exercise price at expiration regardless of the price of the underlying security.

conversion price A convertible bond's par value divided by its conversion ratio.

conversion ratio (CR) The number of shares of stock that can be acquired when a convertible bond is tendered for conversion.

conversion value (CV) A convertible bond's value as a stock. It is equal to the convertible bond's conversion ratio times the market price of the stock.

convertible bond A bond in which the holder can covert the bond to a specified number of shares of stock.

convexity A measure of the change in the slope of the price-yield curve for a small change in yield; it is the second-order derivative.

corpus Term used to describe a principal-only security.

corridor Long position in a cap and a short position in a similar cap with a higher exercise rate.

counterparties The parties to a swap agreement.

coupon rate The contractual rate the issuer agrees to pay each period. It is expressed as a proportion of the annual coupon payment to the bond's face value.

covered interest arbitrage An arbitrage strategy consisting of long and short positions in currency spot and futures contracts, and positions in domestic and foreign risk-free securities. The strategy is used by arbitrageurs when the interest-rate parity condition does not hold.

credit analysis strategy A strategy involving credit analysis of corporate, municipal, or foreign bonds in order to identify potential changes in default risk. This information is then used to identify bonds to include or exclude in a bond portfolio or bond investment strategy.

credit default swap(CDS) Swap in which one counterparty buys protection against default by a particular company from another counterparty. The company is known as the reference entity and a default by that company is known as a credit event. The buyer of the swap makes periodic payments to the seller until the end of the life of the swap or until the credit event occurs.

credit risk *See* default risk.

credit-sensitive bond Bond with coupons that are tied to the issuer's credit ratings.

cross-border risk Foreign investment risk resulting from concerns over changes in political, social, and economic conditions.

cross-collateralization Property used to secure one loan is also used to secure the other loans in the pool.

cross-currency swap Combination of a currency swap and interest rate swap.

cross hedge A futures hedge in which the futures' underlying asset is not the same as the asset being hedged.

cumulative cap (Q-cap) A cap in which the seller pays the holder when the periodic interest on an accompanying floating-rate loan hits a certain level.

currency swap A contract in which one party agrees to exchange a liability denominated in one currency to another party who agrees to exchange a liability denominated in a different currency.

cushion bond A callable bond with a coupon that is significantly above the current market rate.

day count convention The time measurement used in valuing bonds; it defines the convention used to define the time to maturity (actual days or 30-day months) and days in the year (360 days or 365 days).

day trader A trader who holds a position for a day.

dealer paper Commercial paper sold through dealers.

dealers Traders who provide a market for investors to buy and sell a security by taking a temporary position in the security.

debenture Corporate bond backed by a general creditor's claim but not by a specified asset.

dedicated portfolio strategy *See* cash-flow matching strategy.

deep-discount bonds Bonds that pay low coupon interest.

default risk The risk that the issuer/borrower will fail to meet the bond's contractual obligations to pay interest and principal, as well as other obligations specified in the indenture.

deferred call Provision in a bond that prohibits the issuer from calling the bond before a certain period of time has expired.

deferred coupon bond Bond with a deferred coupon structure that allows the issuer to defer coupon interest for a specified period.

deficit economic unit An economic entity whose current expenditures exceed its income.

demand loans Short-term loans to dealers, secured by the dealers' securities.

depository institutions Financial institutions such as commercial banks, credit unions, savings and loans, and savings banks that obtain large amounts of their funds from deposits, which they use primarily to fund commercial and residential loans and to purchase Treasury, federal agency, and municipal securities.

derivative security A security whose value depends on another security or asset.

direct financial market Market where surplus units purchase claims issued by the ultimate deficit unit. This market includes the trading of stocks, corporate bonds, Treasury securities, federal agency securities, and municipal bonds.

direct hedge A futures hedge in which the futures' underlying asset is the same as the asset being hedged.

direct paper Commercial paper sold by the issuing company directly to investors, instead of through dealers. The issuing companies include the subsidiaries of large companies (captive finance companies), bank holding companies, independent finance companies, and nonfinancial corporations.

divisibility The smallest denomination in which an asset is traded.

dollar repo Repurchase agreement that permits the borrower to repurchase with securities similar, but not identical, to the securities initially sold.

domestic bond Bond of a foreign government or foreign corporation that is issued in the foreign country or traded on that country's exchange.

double-barreled Term used to describe revenue bonds that are issued with some general obligation backing and thus have characteristics of both general obligations and revenue bonds.

dual-currency Eurobonds Eurobonds that pay coupon interest in one currency and principal in another.

dual trading A security trading practice in which an exchange member trades for both her client and herself.

duration The average date that cash is received on a bond. It can be measured by calculating the weighted average of the bond's time periods, with the weights being the present value of each year's cash flows expressed as a proportion of the bond's price. It is also a measure of a bond's price sensitivity to interest rate changes.

duration gap The difference in the duration of assets and the duration of liabilities.

early exercise The exercise of an American option before its expiration.

economic surplus The difference between the market value of assets and the present value of liabilities.

effective date The date when interest begins to accrue.

efficient market A market in which the actual price of a security is equal to its intrinsic (true economic) value.

embedded option An option characteristic that is part of the features of a debt security. Features include call and put features on debt securities, the call provisions in a sinking fund, and the conversion clauses on convertible bonds.

enhanced bond indexing A bond-indexing approach that allows for minor deviations of certain features and some active management in order to attain a return better than the index.
equilibrium price of a bond Price obtained by discounting the bond's cash flows by their appropriate spot rates.
equipment trust bond Bond secured by a lien on specific equipment, such as airplanes, trucks, or computers.
equity swap Swap in which one party agrees to pay the return on an equity index, such as the S&P 500, and the other party agrees to pay a floating rate or fixed rate.
Eurobonds Bonds issued in a number of countries through an international syndicate.
Eurocurrency market Market in which funds are intermediated (deposited or loaned) outside the currency's country.
event risk Bond risk resulting from specific actions such as a merger or capital structure change.
excess spread The interest from the collateral that is not being used to pay MBS investors and fees (mortgage servicing and administrative services). The excess spread can be used to offset any losses.
exercise price The price specified in the option contract at which the underlying asset or security can be purchased (call) or sold (put).
extendable bond Bond that has an option to extend the maturity of the bond.
extendable swap Swap that has an option to lengthen the terms of the original swap.
external bond market Term used to refer to a market where Eurobonds and Eurodeposits are bought and sold.
federal funds Deposits of banks and deposit institutions with the Federal Reserve that are used to maintain the bank's reserve position required to support their deposits.
federal funds market Market in which depository institutions with excess reserves lend to institutions that are deficient.
federally sponsored agencies Privately owned companies with a federal charter.
financial engineering A term used to describe strategies of buying and selling derivatives and their underlying securities in order to create portfolios with certain desired features.
financial swap An agreement between two parties to exchange the cash flows from each party's liabilities.
fixed-rate payer The party in a financial swap that agrees to pay fixed interest and receive variable interest.
floater *See* floating-rate notes.
floating-rate notes Notes that pay a coupon rate that can vary in relation to another bond, benchmark rate, or formula. Typically the rate is based on a short-term index and is reset more than once a year.
floating-rate payer The payer in a financial swap that agrees to pay variable interest in return for fixed interest.
floating-rate tranches A sequential-pay tranche that pays a floating rate.
floor A series of interest rate puts that expire at or near the effective dates on a loan. They are often used as a hedging tool by financial institutions.

floorlet **or** ***interest rate put*** *See* interest rate option.
foreign bonds Bonds of a foreign government or corporation being issued or traded in a local country.
foreign currency futures A futures contract on a foreign currency.
foreign currency option An option on a foreign currency.
forward contract An agreement between two parties to trade a specific asset or security at a future date with the terms and price agreed upon today.
forward rate agreement (FRA) A contract that requires a cash payment or provides a cash receipt based on the difference between a realized spot rate such as the LIBOR and a prespecified rate.
forward swaps An agreement to enter into a swap that starts at a future date at an interest rate agreed upon today.
full faith and credit obligations General obligation bonds issued by states and large municipal governments that have a number of tax revenue sources.
fungible Term used to refer to the feature of an asset in which it can be converted into cash or other assets.
fusion conduit deal CMS deal that has one large borrower or property combined with a number of smaller borrowers.
futures contract A marketable forward contract.
futures fund A mutual fund that pools investors' monies and uses them to set up futures positions.
futures hedge ratio The optimal number of futures contracts needed to hedge a position.
futures options **(or** ***options on futures*** **or** ***commodity options)*** An option contract that gives the holder the right to take a position in a futures contract on or before a specific date. A call option on a futures contract gives the holder the right to take a long position in the underlying futures contract when she exercises, and requires the writer to take the short position in the futures if he is assigned. A put option on a futures option entitles the holder to take a short futures position and the assigned writer the long position.
futures spread Futures position formed by simultaneously taking long and short positions in different futures contracts.
general obligation bonds (GOs) Intermediate- and long-term municipal debt obligations that are secured by the issuing government's general taxing power and can pay interest and principal from any revenue source.
generic swap An interest rate swap in which fixed interest payments are exchanged for floating interest payments.
geometric mean The yield to maturity expressed as the geometric average of the current spot rate and implied forward rates.
gilt Bond that does not mature although it can be redeemed after a specified date.
global bond Bond that is issued and traded as a foreign bond and also sold through a syndicate as a Eurobond.
global funds Funds with stocks and bonds from different countries.
GLOBEX A computer trading system in which bids and asks are entered into a computer which matches them.
government-sponsored enterprises (GSE) Term used to refer to federally sponsored agencies and federal agencies. Collectively, the claims sold by federal agencies and federally sponsored companies are referred to as GSE securities.

graduated payment mortgages (GPM) Mortgages that start with low monthly payments in earlier years and then gradually increase.

guaranteed bonds Bond issued by one company and guaranteed by another economic entity.

guaranteed investment contract (GIC) An obligation of an insurance company to pay a guaranteed principal and rate on an invested premium. For a lump-sum payment, the insurance company guarantees a specified dollar amount will be paid to the policyholder at a specified future date.

hedge A strategy in which an investor protects the future value of a position by taking a position in a futures contract, option, or other security.

hedge funds Special types of investment funds often structured so that they are largely unregulated. Minimum investment in such funds ranges from $100,000 to $20m, with the average investment being $1m. Many of the funds invest or set up investment strategies reflecting pricing aberrations.

humpedness A non-parallel yield curve shift in which short-term and long-term rates change by greater magnitudes than intermediate rates. An increase in both short- and long-term rates relative to intermediate rates is referred to as a ***positive butterfly***, and a decrease is known as a ***negative butterfly***.

IMM index The quoted index price for futures on Treasury bill contracts and Eurodollar contracts traded on the International Monetary Market. The index is equal to 100 minus the annual percentage discount yield.

implied forward rate The rate in the future that is implied by current rates. The implied forward rate can be attained by a locking-in strategy consisting of a position in a short-term bond and an opposite position in a long-term one.

implied futures rate The rate implied on an interest rate futures contract.

implied repo rate The rate where the arbitrage profit from implementing a cash-and-carry arbitrage strategy with futures contracts is zero. This rate is also the one earned from an investment in a synthetic Treasury bill.

implied variance Variance that equates the OPM's price to the market price. Conceptually, it can be thought of as the market's consensus of the stock's volatility.

implied volatility *See* implied variance.

income bond Corporate bond that pays interest only if the earnings of the firm are sufficient to meet the interest obligations.

indenture Contract between the borrower and the lender (all the bondholders).

index funds Investment funds constructed so that returns are highly correlated with the market.

indexing The construction of bond portfolios whose returns over time replicate those of some specified bond index.

initial (**or** ***performance) margin*** The amount of cash or cash equivalents that must be deposited by the investor on the day a futures or options position is established.

interbank market A spot and forward currency exchange market consisting primarily of major banks who act as currency dealers.

intercommodity spread A spread formed with futures contracts with the same expiration dates but on different underlying assets.

interdealer market Market in which dealers trade amongst themselves.

Interest Equalization Tax (IET) Tax on income from foreign securities purchased by local investors.

interest on interest The interest earned from reinvesting coupons.

interest-only (IO) security Zero-discount bond that pays a principal received from the coupon interest from another security.

interest rate option An option which gives the holder the right to a payoff if a specific interest rate is greater (call) or less (put) than the option's exercise rate.

interest rate parity (IRPT) The carrying cost model that governs the relationship between spot and forward exchange rates.

interest rate swap An agreement between parties to exchange interest payments on loans.

intermediary financial market Market where financial institutions, such as commercial banks, savings and loans, credit unions, insurance companies, pension funds, trust funds, and mutual funds sell securities and then use the proceeds to provide loans or buy securities.

intermediary securities Securities such as certificates of deposit and mutual fund shares that are created in the intermediary financial markets.

internal bond market Term used to refer to markets where there are domestic bonds and foreign bonds.

intracommodity spread A spread formed with futures contracts on the same underlying asset but with different expiration dates.

intrinsic value of a call The maximum of zero or the difference between the call's underlying security price and its exercise price.

intrinsic value of a put The maximum of zero or the difference between a put's exercise price and its underlying security's price.

inverted market Market where the futures price is less than the spot price.

investment banker Middleman who for a fee or share in the trading profit finds surplus units who want to buy securities being offered by a deficit unit.

investment-grade bonds Bonds with relatively low chance of default; they have quality rating of Baa (or BBB) or higher.

junk bonds *See* speculative-grade bonds.

knock-in option Option that comes into existence when the reference rate or price hits the barrier level.

knock-out option Option that ceases to exist once the specified barrier rate or price is reached.

Kolb-Chiang Price-Sensitivity Model A price sensitivity model for hedging interest rate positions. The model determines the number of futures contracts that will make the value of a portfolio consisting of fixed-income securities and an interest rate futures contract invariant to small changes in interest rates.

ladder strategy Strategy of constructing a bond portfolio with equal allocations in each maturity group.

law of one price An economic principle that two assets with the same future payouts will be priced the same.

legal opinion Document accompanying a municipal issue that interprets legal issues related to the bond's collateral, priority of claims, and the like.

limited-tax general obligation bonds General obligation bonds issued by smaller municipalities or authorities whose revenues are limited to only one or two sources.

liquidity The cashlike property of a security.

liquidity premium The difference in the yield of a less-liquid bond and the yield of a more-liquid one.

liquidity premium theory (LPT) Term structure theory that posits that there is a liquidity premium for long-term bonds over short-term bonds.

locals Members of an exchange who trade from their own accounts, acting as speculators or arbitrageurs.

lockout period Period specified in ABS when the principal payments made on the receivables are retained and either reinvested in other receivables or invested in other securities.

logarithmic return The continuously compounded return. It is equal to the natural logarithm of the security price relatives.

London interbank bid rate (LIBID) The rate paid on funds purchased by large London Eurobanks in the interbank market.

London interbank offer rate (LIBOR) The rate on funds offered for sale by London Eurobanks. The average LIBOR among London Eurobanks is a rate commonly used to set the rate on bank loans, deposits, and floating-rate notes and loans. There are also similar rates for other currencies (e.g., Sterling LIBOR) and areas (e.g., Paris interbank offer rate, PIBOR, or the Singapore interbank offer rate, SIBOR).

Long futures hedge A long position in a futures contract taken in order to protect against an increase in the price of the underlying asset or commodity.

Long futures position A position in which one agrees to buy the futures' underlying asset at a specified price, with the payment and delivery to occur on the expiration date.

LYON Zero-coupon bond that has the features of being convertible into the issuer's stock, callable, with the call price increasing over time, and putable with the put price increasing over time.

Macaulay's duration Duration as measured by the weighted average of the time periods.

maintenance (**or** ***variation) margin*** The value of the commodity equity account that must be maintained.

mark (**or** ***marking) to market*** Process of adjusting the equity in a commodity or margin account to reflect the daily changes in the market value of the account.

market risk The risk that interest rates will change, changing the price of the bond and the return earned from reinvesting coupons.

market segmentation theory (MST) Term structure theory that posits financial markets are segmented into a number of smaller markets by maturity, with supply and demand forces unique to each segment determining the equilibrium yields for each segment.

marketability The speed in which an asset can be bought and sold.

market maker A dealer on an exchange who specializes in the trading of a specific security.

marketability An asset characteristic that defines the ease or speed with which the asset can be traded.

matador bonds Foreign bonds sold in Spain.

maturity The length of time from the present until the last contractual payment is made.

medium-term note (MTN) Debt instrument sold on a continuing basis to investors who are allowed to choose from a group of bonds from the same corporation, but with different maturities.

Mello-Roos bonds Municipal securities issued by local governments in California that are not backed by the full faith and credit of the government.

modified convexity Convexity estimated by determining the price of the bond when the yield increases by a small number of basis points (e.g., 2–10 basis points) and when the yield decreases by the same number of basis points.

modified duration Duration measured as the percentage change in a bond's price given a small change in yield.

money market Market where short-term instruments (by convention defined as securities with original maturities of one year or less) are traded.

money spread *See* vertical spread.

moral-obligation bonds Bonds issued without the legislature approving appropriation. The bonds are considered backed by the permissive authority of the legislature to raise funds, but not the mandatory authority.

mortgage-backed securities (MBSs) Asset-backed securities formed with mortgages.

Mortgage-Backed Security Dealers Association Association of mortgage-backed securities dealers who operate in the over-the-counter market. These dealers form the core of the secondary market for the trading of existing pass-throughs.

mortgage bond Bond secured by a lien on real property or buildings.

mortgage pass-throughs *See* mortgage-backed securities.

multiple disciminant analysis A statistical technique that can be used to forecast default or changes in credit ratings.

multiple listings The listing of a security on more than one exchange.

municipal bond index futures contract A futures contract on the municipal bond index; an index based on the average value of 40 municipal bonds.

Municipal Securities Rule Board (MSRB) Self-regulatory board responsible for establishing rules for brokers, dealers, and banks operating in the municipal bond market.

mutual fund *See* open-end fund.

naive hedging ratio A hedge ratio in which one unit of a futures position hedges one unit of a spot position. The ratio is found by dividing the value of the spot position to be hedged by the price of the futures contract.

naked call write An option position in which an option trader sells a call but does not own the underlying stock.

naked position A term used to describe a long or short speculative futures position.

naked put write An option position in which an option trader sells a put but does not cover the put obligation by selling the underlying stock short.

national market *See* internal bond market.

negotiated market Market in which securities are privately placed.

net settlement basis Feature of a swap in which the counterparty owing the greater amount pays the difference between what is owed and what is received.

net worth maintenance clause Provision in the indenture requiring that the issuer redeem all or part of the debt or give bondholders the right to sell (offer-to-redeem clause) their bonds back to the issuer if the company's net worth falls below a stipulated level.

NOB spread A futures spread formed with Treasury note and Treasury bond futures contracts.

nonprime certificates of deposit Certificates of deposit of smaller banks.

normal market Market where the futures price exceeds the spot price.

notional interest-only (IO) class A tranche that receives the excess rate from other tranches' principals, with the excess rate being equal to the difference in the collateral rate minus the tranches' coupon rates.

notional principal The principal used to determine the amount of interest paid on a swap agreement. This principal is not exchanged.

off-balance sheet restructuring A method of changing a balance sheet's return-risk exposure by using derivatives such that the original composition of assets and liabilities is not changed.

off-market swap Swap that has a value. Many existing swaps are off-market swaps.

official statement Document similar to the prospectus for a stock or corporate bond, which details the return, risk, and other characteristics of a municipal issue and provides information on the issuer.

offsetting order *See* closing order.

on-the-run issues Recently issued Treasury securities trading on the secondary market.

open-end bond Bond that allows for more debt to be secured by the same collateral.

open-end fund (mutual fund) Fund that stands ready to buy back shares of the fund any time the fund's shareholders want to sell, and sell new shares any time an investor wants to buy into the fund. Technically, a mutual fund is an open-end fund.

open interest The number of option or futures contracts that are outstanding at a given point in time.

open market Term used to refer to securities that are issued to the public at large.

open outcry Term used to describe the process of shouting bids and offers in an exchange-trading area.

open repo An overnight repo that is automatically rolled over into another overnight repo until one party closes.

opening transaction The transaction in which an investor initially buys or sells an option or futures contract.

option A security that gives the holder the right to buy (call) or sell (put) an asset at a specified price on or possibly before a specific date.

option-adjusted spread (OAS) analysis An analysis that solves for the option spread that makes the average of the present values of the bond's cash flows from the possible interest rate paths equal to the bond's market price.

option clearing corporation (OCC) A firm whose primary function is to facilitate the marketability of option contracts. It does this by intermediating each option transaction that takes place on the exchange and by guaranteeing that all option writers fulfill the terms of their option contracts.

option-currency Eurobonds Eurobonds that offer investors a choice of currency.

option holder The buyer of an option. The holder buys the right to exercise or evoke the terms of the option claim. An option buyer is said to have a long position in the option.

option premium The price of the option (call premium and put premium).

option writer The seller of an option. The writer is responsible for fulfilling the obligations of the option if the holder exercises. The option writer is said to have short position in the option.

order book official An employee of an exchange who keeps the limit order book.

original-issue discount (OID) The difference between the bond's face value and the offering price when the bond is issued.

over-the-counter market An informal exchange for the trading of stocks, corporate and municipal bonds, investment fund shares, mortgage-backed securities, shares in limited partnerships, and Treasury and federal agency securities.

over-the-counter options Options provided by dealers in the over-the-counter market. The option contracts are negotiable, with buyers and sellers entering directly into an agreement.

par value swap Swap that has a value of zero. An economic value of zero requires that the swap's underlying bond positions trade at par. Many plain vanilla swaps are originally par value swaps.

parallel shift Yield curve shift in which the yields on all maturities change by the same magnitude.

participating bond Corporate bond that pays minimum rate plus an additional interest up to a certain point if the company achieves a certain earnings level.

participation certificates (PC) *See* pass-through securities.

pass-through rate* or *coupon rate Rate paid to mortgage-backed security holders.

pass-through securities Securities formed by pooling a group of mortgages and other financial assets and then selling a security representing interest in the pool and entitling the holder to the income generated from the pool of assets.

passive strategy Bond strategy in which there is no change in the investment strategy once it is set up.

payer swaption Option that gives the holder the right to enter a particular swap as the fixed-rate payer (and floating-rate receiver).

payment-in-kind (PIK) bond Bond that gives the issuer the option on the interest-payment date to pay the coupon interest either in cash or in kind, usually by issuing the bondholder a new bond.

perfect market A market in which the price of the security is equal to its equilibrium value at all times.

perpetuity (consul) A coupon bond with no maturity.

plain vanilla swap *See* generic swap.

planned amortization class (PAC) A tranche formed by generating two monthly principal payment schedules from the collateral; one schedule is based on assuming a relatively low PSA speed, while the other is obtained by assuming a relatively high PSA speed. The PAC bond is then set up so that it will receive a monthly principal payment schedule based on the minimum principal from the two principal payments. The PAC bond is designed to have no prepayment risk provided the actual prepayment falls within the minimum and maximum assumed PSA speeds.

poison put Clause in the indenture giving the bondholders the right to sell the bonds back to the issuer at a specified price under certain conditions arising from a specific event such as a takeover, change in control, or an investment ratings downgrade.

position limit The maximum number of option or futures contracts an investor can buy and sell on one side of the market. A side of the market is either a bullish or bearish position.

position profit The profit dealers realize from taking long and short positions in securities in which they deal.

position trader A futures dealer who holds a position for a period longer than a day.

preferred habitat theory (PHT) Term structure theory that posits that investors and borrowers may stray away from desired maturity segments if there are relatively better rates to compensate them.

prepayment risk The risk that a loan will be paid off early and the lender or bond holder will have to invest or create new loans in a market with a lower rate.

prepayment speed* or *speed Term used to define the estimated prepayment rate on a portfolio of mortgages or a mortgage-backed security.

price compression Term that refers to limitations on a bond's price. For example, the percentage increases in the prices of a callable bond may be limited when interest rates decrease, given that the market expects the bonds to be redeemed at the call price.

price limits The maximum and minimum prices that a futures contract can trade.

primary market Market where financial claims are created.

primary securities Claims traded in the direct financial market.

prime certificates of deposit Certificates of deposit of larger banks.

principal-only (PO) security Zero-discount bond that pays a principal received from another security.

principal pay-down window Term used to describe the period between the beginning and ending principal payment.

private labels *See* conventional pass-throughs.

private placement Term used to refer to securities that are issued by economic entities under a private contract.

probability intensity Conditional probability of default.

prospectus Document that summarize the main provisions included in the indenture.

PSA models (Public Securities Association) The prepayment models of the Public Securities Association prepayment model. In the standard PSA model for a 30-year mortgage, known as 100 PSA, the CPR starts at .2% for the first month and then increases at a constant rate of .2% per month to equal 6% at the 30th month; then after the 30th month the CPR stays at a constant 6%.

pure discount bond (PDB) A bond which pays no coupon interest. The bond sells at a price below its face value. It is also called a zero-coupon bond.

pure expectations theory (PET) Term structure theory in which the term structure of interest rates is based on the impact of investors' and borrowers' expectations about future interest rates.

put An option that gives the holder the right to sell an asset or security at a specified price on or possibly before a specific date.

put-call-futures parity The equilibrium relationship between the prices on put, call, and futures contracts on the same asset. If the equilibrium condition for put-call-futures parity does not hold, then an arbitrage opportunity will exist by taking a position in the put and futures contract and an opposite position in the call and a riskless bond with a face value equal to the difference in the exercise price and futures price.

put-call parity The equilibrium relationship governing the prices on put and call contracts. If the equilibrium condition for put-call parity does not hold, then an arbitrage opportunity will exist by taking a position in the put and the underlying security and an opposite position in the call and a riskless bond with face value equal to the exercise price.

putable bond A bond that gives the bondholder the right to sell the bond back to the issuer at a specified price.

quality risk A hedging risk that precludes one from obtaining zero risk because the commodity or asset being hedged is not identical to the one underlying the futures contract.

quality swap A strategy of moving from one quality group to another in anticipation of a change in economic conditions.

quantity risk A hedging risk that precludes one from obtaining zero risk because the size of the standard futures contract differs from the number of units of the underlying asset to be hedged.

rate-anticipation strategies Active strategies of selecting bonds or a bond portfolio with specific durations based on interest rate expectations.

rate-anticipation swap Active strategies involving simultaneously selling and buying bonds with different durations based on interest rate expectations.

rate of return The total dollar return received from the asset per period of time expressed as a proportion of the price paid for the asset.

real estate investment trust (REIT) Fund that specializes in investing in real estate and real estate mortgages.

rebalancing Term used to refer to resetting the bond position when a bond's duration is no longer equal to the duration of the liability.

rebundling Term used to refer to the buying of zero coupon bonds and stripped securities and then forming coupon bond to sell. This process is also known as ***reconstruction***.

receiver swaption Swaption that gives the holder the right to pay a specific fixed rate and receive the floating rate; that is, the right to take a fixed payer's position.

red herring A preliminary prospectus that details all the pertinent information the official prospectus will have, except the price.

refunded bonds Municipal bonds secured by an escrow fund consisting of high-quality securities such as Treasuries and federal agencies.

registered bond Bond in which the bondholders are registered with the issuer or the trustee; the issuer pays coupons and principal to those registered.

regression hedging model A hedging model where the estimated slope coefficient from a regression equation is used to determine the hedge ratio. The coefficient, in turn, is found by regressing the spot price on the security to be hedged against its futures price.

release and substitution provision Provision in a mortgage bond that allows the mortgaged asset to be sold provided it is replaced with a suitable substitute or allows for the asset to be sold with the proceeds used to retire the bonds.

repo rate The rate on a repurchase agreement.

repos (RPs) or ***repurchase agreements*** A transaction in which one party sells a security to another party with the obligation of repurchasing it at a later date.

To the seller, the repurchase agreement represents a secured loan in which he receives funds from the sale of the security, with the responsibility of purchasing the security later at a higher price that reflects the shorter time remaining to maturity.

reset mortgages Mortgages that allow the borrower to renegotiate the terms of the mortgage at specified future dates.

revenue bonds Municipal securities paid by the revenues generated from specific public or quasi-public projects, by the proceeds from a specific tax, or by a special assessment on an existing tax.

reverse collar Combination of a long position in a floor and a short position in a cap with different exercise rates. The sale of the cap is used to defray the cost of the floor.

reverse corridor Long position in a floor and a short position in a similar floor with a lower exercise rate.

reverse inquiry Term used to refer to cases in which institutional investors indicate to the agents of a medium-term note the type a maturity they want.

risk The possibility that the rate of return an investor will obtain from holding an asset will be less than expected.

risk-averse market Market in which investors require compensation in the form of a positive risk premium over a risk-free investment to pay them for the risk they are assuming.

risk-loving market Market in which investors enjoy the excitement of the gamble and are willing to pay for it by accepting an expected return from a risky investment that is less than the risk-free rate. The market has a negative risk premium.

risk-neutral market A market in which investors accept the same expected rate of return from a risky investment as a risk-free one.

risk premium The difference in the yield on a risky bond and the yield on a less-risky or risk-free bond. The risk premium indicates how much additional return investors must earn in order to induce them to buy the riskier bond

risk premium theory (RPT) *See* liquidity premium theory.

risk spread *See* risk premium.

rolling the hedge forward A hedging strategy that involves taking a futures position, then at expiration closing the position and taking a new one.

running a dynamic book Term used to describe how swap banks hedge their positions through a portfolio of alternative positions—opposite swap positions, spot positions in T-notes and FRNs, or futures positions.

samurai bonds Foreign bonds sold in Japan.

scalper A floor trader who buys and sells securities on her own account, holding them for a short period.

seasoning Term used to define the age of a mortgage.

seat Term used to describe membership on an exchange.

SEC Rule 144A SEC rule that allows the issuer to sell unregistered securities to one or more investment bankers who could resell the securities to qualified investment buyers.

SEC Rule 415 SEC rule that allows a firm to register an inventory of securities of a particular type for up to two years.

secondary market Market for the buying and selling of existing assets and financial claims.

secondary securities *See* intermediary securities.

sector rotation Term used to describe the reallocation of funds to a specific quality sector in anticipation of a price change.

secured bond Bond that has a lien giving the bondholder the right to sell the pledged asset in order to pay the bondholders if the company defaults.

securitization Process in which the assets of a corporation or financial institution are pooled into a package of securities backed by the assets. The most common types of asset-backed securities are those secured by mortgages, automobile loans, credit card receivables, and home equity loans.

securitized assets Claim on cash flows from a portfolio of loans or assets.

selling group agreement The agreement between the investment banker and the selling group on a security issue. The agreement can define the period of time the members of the group have to sell their portion of the issue, commissions that can be charged, and restrictions such as prohibiting members from selling below a certain price.

senior interest The proportion of the mortgage balance of the senior bond class to the total mortgage deal.

sequential-pay collateralized mortgage obligation A collateralized mortgage obligation that is divided into classes with different priority claims on the collateral's principal. The tranche with the first priority claim has its principal paid entirely before the next priority class, which has its principal paid before the third class, and so on.

serial bond issue Bond issue consisting of a series of bonds with different maturities.

shelf registration rule *See* SEC Rule 415.

shifting interest schedule Schedule that is used to determine the allocation of prepayment that goes to the senior and subordinate tranches.

short futures hedge A short position in a futures contract that is taken in order to protect against a decrease in the price of the underlying asset.

short futures position A position in which an investor agrees to sell the underlying asset on a futures contract.

short sale The sale of a security now, then purchasing it later. To implement this strategy the investor must borrow the security, sell it in the market, then repay his debt obligation by later buying the security and returning it to the share lender.

single monthly mortality rate (SMM) Term used to define the monthly prepayment rate.

sinking fund Provision in the indenture requiring that the issuer make scheduled payments into a fund. Many sinking fund agreements require an orderly retirement of the issue, commonly handled by the issuer being required to buy up a certain portion of bonds each year either at a stipulated call price or in the secondary market at its market price.

sovereign risk Foreign investment risk on sovereign securities resulting from concern that the government is unable, or in some cases unwilling, to service its debt.

specialist A dealer on the exchange who specializes in the trading of a specific security and who is responsible for maintaining the order book.

speculative-grade bonds Bonds with relatively high chance of default; they have quality rating below Baa.
spot market A market in which there is an immediate sale and delivery of the asset or commodity.
spot option (also called options on actuals) Term used to refer to option contracts on stocks, debt securities, foreign currency, and indexes. The term is used to distinguish them from options on futures contracts.
spot price The price an asset or commodity trades for in the spot market.
spot rates The rate on a zero-coupon bond.
spread An option or futures position consisting of a long position in one contract and a short position in a similar, but not identical, contract. Also, the price paid on CDs.
stable value investment Term used to refer to guaranteed investments contracts and bank investment contracts.
standby underwriting agreement An agreement in which the investment banker sells an issue on a commission, but agrees to buy all unsold securities at a specified price.
step-down provision A provision in a ABS that allows for reductions in the credit support over time.
stop price* or *stop-out price The lowest price at a Treasury securities auction at which at least some securities are awarded to bidders. Those bidding above the stop price are awarded the quantity they requested, while those with bids below the stop price do not receive any bills.
straddle purchase A strategy formed by buying a put and a call with the same underlying security, exercise price, and expiration date.
straddle write A strategy formed by selling a put and a call with the same underlying security, exercise price, and expiration date.
straight debt value (SDV) A convertible bond's value as a nonconvertible bond. It is found by discounting the convertible bond's cash flows by the yield to maturity on an identical, but nonconvertible, bond.
strap purchase A strategy formed by purchasing more calls than puts, with the calls and puts having the same terms.
strap write A strategy formed by selling more calls than puts, with the calls and puts having the same terms.
strike price *See* exercise price.
strip A series of futures contracts with different maturities. Also, a term used to describe a combination of long or short call and put positions in which the number of puts exceeds the number of calls.
strip bills A package of T-bills with different maturities in which the buyer agrees to buy bills at their bid price for several weeks.
strip purchase A strategy formed by purchasing more puts than calls, with the calls and puts having the same terms.
strip write A strategy formed by selling more puts than calls, with the calls and puts having the same terms.
stripped mortgage-backed securities Stripped mortgage-backed securities consisting of principal-only class and an interest-only class.
STRIPS, Separate Trading of Registered Interest and Principal of Securities Treasury strip securities that are the direct obligations of the government, and for

clearing and payment purposes, the names of the buyers of these securities are included in the book entries of the Treasury.

subordinate interest The proportion of the mortgage balance of the subordinated bond classes to the total mortgage deal.

surplus economic unit An economic entity whose income from its current production exceeds its current expenditures; it is a saver or net lender.

surplus management The management of the surplus value of assets over liabilities.

swap *See* financial swap.

swap banks Group of brokers and dealers who intermediate swap agreements between swap users. As brokers, swap banks try to match parties with opposite needs; as dealers, swap banks take positions as counterparties.

swap rate The average of the bid and ask rates offered by a swap bank on an interest rate swap.

swaptions Options on a swap. The purchaser of a swaption buys the right to start an interest rate swap with a specific fixed rate or exercise rate, and with a maturity at or during a specific time period in the future. If the holder exercises, she takes the swap position, with the swap seller obligated to take the opposite counterparty position.

support class (also called the support bond or the companion bond) Tranche formed with a PAC bond that receives principal equal to the collateral's principal minus the PAC's principal.

systematic risk The risk of a security that is attributed to market factors (i.e., factors which affect all securities).

targeted amortization class (TAC) bonds A PAC-structured collateralized mortgage obligation with PACs with just one PSA rate.

tax-exempt corporate bond Corporate bond issued to finance projects such as the construction of solid and hazardous waste disposal facilities that qualify for tax-exemption; the holders of the bond do not have to pay federal income tax on the interest they receive.

tax swap Strategy in which an investor sells one bond and purchases another in order to take advantage of the tax laws.

taxability The claims that the federal, state, and local governments have on the cash flows of an asset.

TED spread A spread formed with Treasury bill and Eurodollar contracts.

term certificates of deposit Certificates of deposit issued with maturities greater than one year.

term repo A repurchase agreement that is not overnight.

term structure of interest rates The relationship between the yields on financial assets and their maturities.

time value premium (TVP) The difference between the price of an option and its intrinsic value.

timing risk A hedging risk that precludes one from obtaining zero risk because the delivery date on the futures contract does not coincide with the date the hedged assets or liabilities need to be purchased or sold.

TIPS (***Treasury Inflation Protection Securities***) Inflation-adjusted securities issued by the U.S. Treasury.

total return (TR) *See* annual realized return.

total return analysis A method for identifying an active bond position based on possible yield-curve shifts. In this approach, potential returns from several yield-curve strategies are evaluated for a number of possible interest-rate changes over different horizon periods to identify the best strategy.

total return swap Swap in which the return from one asset or portfolio of assets is swapped for the return on another asset or portfolio.

tracking errors The difference between the returns on the index and the index fund.

trade date Day that counterparties agree to commit to the swap.

trademark Term used to refer to various types of strip securities offered by investment firms.

tranches Term used to define the different classes making up a collateralized mortgage obligation. There are two general types of tranches—sequential-pay tranches and planned amortization class tranches.

Treasury-bill futures A futures contract that calls for the delivery or purchase of a Treasury bill.

Treasury-bill option An option that gives the holder the right to buy (call) or sell (put) a Treasury bill.

Treasury bills Short-term Treasury instruments sold on a pure discount basis.

Treasury-bond futures contract A futures contract which calls for the delivery or purchase of a Treasury bond.

Treasury-bond option An option on a Treasury bond.

Treasury bonds and notes The Treasury's coupon issues. Both are identical except for maturity: T-notes have original maturities up to 10 years (currently, original notes are offered with maturities of two, five, and 10 years), while T-bonds have maturities ranging between 10 and 30 years.

Treasury-note futures contract A futures contract which calls for the delivery or purchase of a Treasury note.

Treasury strips Zero-discount bond that pays a principal received from the interest or principal from a T-bond or T-note.

trustee Third party to a bond contract. The party represents the bondholders and is responsible for ensuring that the bond issue has been drawn up in accordance with all legal requirements and that the issuer meets all of the prescribed functions specified in the indenture. The trustee can take legal action against the corporation if it fails to meet its interest and principal payments or satisfy other terms specified in the indenture.

twist A non-parallel yield curve shift in which there is either a flattening or steepening of the yield curve.

unbiased expectations theory *See* pure expectations theory.

underwrite Action of investment bankers of buying securities from the issuer and then selling them.

underwriting syndicate Group of investment bankers who underwrite an issue.

unit investment trust An investment fund that has a specified number of fixed-income securities and a fixed life.

unsystematic risk The risk of a security that is attributed to factors other than market factors.

voting bond Bond that gives voting privileges to the holders. The vote is usually limited to specific corporate decisions under certain conditions.

vulture funds Funds consisting of debt securities of companies that are in financial trouble or in bankruptcy.

warehousing Practice of swap banks in which they enter a swap agreement without another counterparty. Swap banks often will hedge their swap positions with opposite positions in T-notes and FRNs or using Eurodollar futures contracts.

warrant A security or a provision in a security that gives the holder the right to buy a specified number of shares of stock or another designated security at a specified price.

wash sale The sale of a security at a loss and its subsequent repurchase. Tax laws disallow claiming such losses for tax purposes.

wild-card option The right on a Chicago Board of Trade's Treasury bond futures contract to deliver the bond after the close of trading on the exchange.

Yankee bonds Foreign bonds sold in the U.S.

yield approximation formula *See* average rate to maturity.

yield curve A graph showing the relationship between the yields to maturity on comparable bonds and their maturities.

yield pickup swap Bond strategy in which investors or arbitrageurs try to find bonds that are identical, but are mispriced and trading at different yields.

yield to call (YTC) The yield obtained by assuming the bond is called on the first call date.

yield to maturity (YTM) The discount rate on a bond. It is the rate that equates the price of the bond to the present value of its coupons and principal.

yield to put (YTP) The yield obtained by assuming the bond is put on the first put date.

yield to worst The lowest of the yield to maturity, yield to call and yield to put.

YTM approach The valuation of off-market swaps by discounting the net cash flows at the current YTM.

zero-coupon approach The valuation of off-market swaps by discounting the net cash flows using spot rates.

zero-coupon bond *See* pure discount bond.

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